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Corporate self-regulation of imperfect competition*

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Abstract

We consider Cournot competition in general equilibrium. Decisions in firms are taken by majority voting. Naturally, interests of voters–shareholders or stakeholders–depend on their endowments and portfolios. Indeed, voters in every firm are concerned about the return on their portfolios rather than their shares in the firm. We introduce two notions of local Cournot-Walras equilibria to overcome difficulties arising from non-concavity of profit functions and multiplicity of equilibrium prices. We show existence of local Cournot-Walras equilibria, and characterize distributions of voting weights for which equilibrium allocations are Pareto optimal. We discuss the efficiency of various governance modes and highlight the importance of financial markets in regulating large firms.

Keywords Cournot-Walras equilibrium · Majority voting · Pareto optimality · Shareholder governance · Stakeholder democracy · Walrasian equilibria

JEL Classification D41 · D43 · D51 · D61 · D71

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1 Introduction

Overview of the paper: The systemic dominance of few gigantic corporations, for example the GAFA, has recently resurrected the debate on regulation. These corporations brought to the market revolutionary products which deeply penetrate personal lives, putting at threat the privacy of their users. Hence the debate on regulation highlights the protection of privacy. It remains though that these vast conglomerates are traditional oligopolies; so the good old questions of pricing of their product and efficiency of their output are back to light. These are the question addressed in this paper.

What is unprecedented however about these oligopolies is how wildly they transcend national borders. Even the most powerful federal or continent-sized regulators (for example the US Federal Trade Commission or the EU Directorate General for Competition) seem out of their depth to regulate them. The GAFA are not above the law, but the new challenges that they raise call for new laws, and novel coordination between regulators at a transnational level. Given how fast their products evolve and how slow are the responses of governments and regulatory bodies are, the question of the possibility of self-regulation comes naturally to mind.

By self-regulation we mean the ability of the decision-making process in corporations to autonomously lead to actions that would typically be imposed by the regulator. In the present paper we consider self-regulation under a straightforward form: we search for a governance mode that leads decision-makers to collectively choose efficient actions.

In search for maximal profit, a corporation with market power will overprice and underproduce compared to perfect competition and the outcome will be inefficient. However, individuals do not want maximal profit, they want maximal profit *as well as* prices that maximize the value of their other assets and favour their trades. Market clearing implies there is a trade-off between the two (Gabszewics & Vial, 1972).

Hence typically shareholders do not want plain profit maximization; moreover they disagree on what they want. A wide variety of outcomes can result from collective decision-making. We focus here on a fundamental question: can efficient production spontaneously arise from majority voting among shareholders or stakeholders?

We address the question in an Arrow-Debreu model with many commodities, consumers and firms. For fixed production plans proposed by firms, consumers maximize their utilities and markets clear in a quite standard way. Production plans in firms are chosen through majority voting: when contemplating a change of production in some firm, every consumer computes the impact of the considered change on profits and prices, and subsequently on her net demand and utility level. This leads her either to support the change or not. Her voting right depends on the governance. At a Cournot-Walras equilibrium, no alternative production plan receives sufficient support against the status quo, consumers maximize utility and markets clear. Hence a notion that rests both on economic equilibrium and political stability.

The fundamental question of the paper, namely whether efficient production can spontaneously arise from majority voting among stakeholders, translates into the following: are Cournot-Walras equilibrium allocations Pareto optimal?

Overview of the results: The contribution of the paper is threefold. First, we solve long-standing technical issues facing the concept of Cournot-Walras equilibrium. Second, we identify conditions under which equilibrium allocations are Pareto optimal. Third, we show that splitting up firms in smaller firms does not change welfare at Cournot-Walras equilibria. Indeed, splitting up firms in smaller firms has no effect unless ownership is split up and shareholders are limited to own shares in few small firms.

We face two technical issues: profits are not necessarily convex in production plans; and, there can be multiple equilibrium prices. We deal with the first difficulty by using the notion of *local* Cournot-Walras equilibrium according to which consumers use linear approximations of inverse demand when computing the impact of a change in production. This strategy has already been introduced in Gary-Bobo (1989); we extend it to modes of governance based on voting. The way we deal with the second difficulty is more novel; it goes by proposing an extension of the notion of local Cournot-Walras equilibrium to random inverse demand functions. Our construction keeps the simplicity of the standard approach without assuming away multiplicity of equilibrium prices.

Finally, we overcome a conceptual difficulty: the multi-dimensionality of the collective choices can prevent simple majority voting from giving rise to stable production plans. This is tackled by resorting to super majority voting. Once these technical and conceptual difficulties are overcome, we turn toward the fundamental question of the paper, that of the governance for which some version of the first welfare theorem can be restored.

Intuitively, a consumer with lots of shares in a firm typically has an excess supply of the goods produced by the firm, and therefore prefers prices of these goods to be above competitive prices. Hence a governance that gives lots of voting weight to consumers with lots of shares, e.g. the shareholder governance (one share-one vote), could foster excessive prices. The ideal modification of the shareholder governance would be to let non-shareholders have a say in firms. External board members such as government officials, employee representatives or consumers associations can be seen as proxies for non-shareholders.

Alternatively, a governance that gives voting weight to non-shareholders, e.g. the stakeholder democracy (one stakeholder-one vote), could foster excessive regulation. Lobbying can be interpreted as an activity that enables shareholders to increase their voting weights, and potentially mitigate excessive regulation. Though neither the stakeholder democracy nor the shareholder governance is likely to result in perfectly competitive behaviour, there is one big difference between the two governance modes. For the stakeholder democracy, *the mean voter supports perfectly competitive behaviour*. On the contrary, for the shareholder governance, the mean voter typically does not support perfect competition.

The difference gives an edge to the stakeholder democracy over the shareholder governance as to supporting efficient production, as it has been argued that the mean voter is a natural proxy for a median voter in a multi-dimensional setting as in the present paper. In particular, Caplin & Nalebuff (1988, 1991) provide conditions on the distribution of voters under which every decisive coalition includes the mean voter for a super majority rate of 0.64.

Finally we show that splitting up firms in smaller firms has no effect on the equilibrium unless ownership is split up too and consumers are limited to own shares in few small firms. The intuition is that shareholders with shares in many small firms support coordination between these firms and thereby can thwart the effect of Sherman Act-like regulation.

Related literature: In Gabszewics & Vial (1972) the notion of Cournot-Walras equilibrium was introduced. Consumers consider prices and production plans to be fixed and maximize utilities. Firms take into account that their production plans influence prices. At Cournot-Walras equilibria, consumers maximize utility, firms maximize profits and markets clear. However it is well-known that for some production plans there can be multiple market clearing prices and that the profit function need not be concave. In order to get existence of Cournot-Walras equilibria it is assumed that there is an inverse demand function and that profit functions are concave.

In Gary-Bobo (1989), the notion of k-consistent Cournot-Walras equilibrium was introduced. At these equilibria firms perceive k'th-order Taylor expansions of their inverse demand functions instead of their actual inverse demand functions. In our notion of local Cournot-Walras equilibria, firms perceive first-order Taylor expansions of their inverse demand functions. In comparison with Gabszewics & Vial (1972) and Gary-Bobo (1989), production plans are determined by majority voting in our setup instead of by profit maximization. As in Gary-Bobo (1989), to get existence of local Cournot-Walras equilibrium it has to be assumed that inverse demand is a continuously differentiable function.

However, to get existence of local random Cournot-Walras equilibrium, a weaker assumption is needed, namely that for all production plans there is a price vector at which inverse demand is continuously differentiable. Random inverse demand functions are briefly discussed in Mas-Colell & Nachbar (1991) where it is shown that if for all production plans there are countable many price vectors in the inverse demand, then there exist continuous random inverse demand functions. The two assumptions, namely at least one continuously differentiable price vector and at most countable many price vectors, are independent.

Profit maximization as the objective for firms has at least two drawbacks as explained in Grodal (1996). It is typically not in the interest of shareholders, and it depends on price normalization. In Grodal (1996) it is shown that for all efficient production plans there are price normalizations such that these production plans are Cournot-Walras equilibria where firms maximize profits. In Bejan (2008), Dierker & Dierker (2006) and Dierker & Grodal (1999) alternative objectives are suggested and studied. The common idea behind these suggestions is to compare aggregate demands of shareholders at different production plans and associated prices.

For the comparisons to be relevant there need to be transfers between shareholders. One possible path is to go deeper into how these transfers between shareholders are organized and how they influence behaviour. We take a different path studying majority voting in firms and focusing on existence of equilibria and welfare properties of equilibrium allocations without transfers between shareholders. On a related theme Zierhut (2021) studies the indeterminacy of Cournot-Walras equilibria with transferable utility, sole proprietorship and incomplete financial markets.

The case of imperfect competition due to market power is not the only instance in which shareholders or stakeholders fail to agree on what the firm should do. Two other classic instances are: the case of incomplete financial markets, and the case of production externalities. There is a long-standing literature on modeling the firm's decision making process based on majority voting for the case of incomplete financial markets, e.g. Gevers (1974), Drèze (1985), DeMarzo (1993), Kelsey & Milne (1996) and Tvede & Crès (2005). Crès & Tvede (2013) have extended it to the case of production externalities. See also Crès & Tvede (2021) for a survey on this approach.

Related to our study of splitting up firms, there has been some recent interest in common ownership in IO. Schmalz (2019) contains an overview of the literature. In Backus, Conlon & Sinkinson (2019) it is shown that common ownership has increased a lot in the period 1980-2017. Indeed, it is suggested that the internalization of profits of other firms has increased from around 0.2 in the early 1980's to 0.7 in the late 2010's for S&P500. In Azar, Schmalz and Tecu (2018) it is shown that price changes and changes in common ownership correlate in the US airline industry. On a different but related theme Ma et al. (2021) study the incentives for engaging into cross-holdings and their welfare effects.

Plan of the paper: In Section 2 we set up the model and consider the problems of the consumers and the firms. In Section 3 we introduce the notions of local Cournot-Walras equilibrium, show existence of equilibrium. In Section 4, we characterize distributions of portfolios for which Walras equilibria are local Cournot-Walras equilibria, compare the performance toward efficiency of the stakeholder democracy vs the shareholder governance and consider the effect on equilibrium of splitting up firms. We conclude with some final remarks.

2 The model

We introduce our setup and consider the problems of consumers and firms.

Setup

Consider an economy with ℓ goods, m consumers and n firms. Let $p = (p_1, \ldots, p_\ell)$ be a price vector. Prices are normalized to sum to one. The price simplex is

$$S^{\ell-1} = \{ p \in \mathbb{R}^\ell_+ \mid \sum_h p_h = 1 \}.$$

Consumers are described by their utility functions $u_i : \mathbb{R}^\ell \to \mathbb{R}$, initial endowments $\omega_i \in \mathbb{R}^\ell$ and shares in firms $(\theta_{i1}, \ldots, \theta_{in})$ where $(\theta_{ij})_i \in S^{m-1}$ for every j. Since firms are not necessarily maximizing profit, profits need not be positive so consumer income need not be positive. Therefore consumption is unbounded. Utility functions u_i are assumed to satisfy the following assumptions.

(C.1) $u_i \in C^2(\mathbb{R}^\ell, \mathbb{R})$ where $Du_i(x_i) \in \mathbb{R}^\ell_{++}$ and $v^T D^2 u_i(x_i) v < 0$ for all $v \neq 0$ and all x_i.

(C.2) The set $\{ x_i \in \mathbb{R}^\ell \mid u_i(x_i) = a \}$ is bounded from below for all $a \in \mathbb{R}$.

The first assumption implies utility functions are strongly monotone and strictly concave. The two assumptions imply the sets of affordable consumption bundles preferred to initial allocations are compact for price vectors with strictly positive coordinates.

Firms are described by their production sets $Y_j \subset \mathbb{R}^\ell$. Production sets are assumed to satisfy the following assumptions.

(F.1) Y_j is non-empty, compact and convex.

(F.2) There is an r-dimensional subspace $L_j \subset \mathbb{R}^\ell$ such that $Y_j \subset L_j$.

Assumptions (F.1) and (F.2) are standard.

Let $y = (y_1, \ldots, y_n)$ with $y_j \in Y_j$ for every j be a list of production plans and $Y = \times_j Y_j$ the product of the production sets.

Consumer behaviour

Consumers consider the list of production plans y and the price vector p to be fixed and choose consumption bundles to maximize their utilities subject to their budget constraints. The problem of consumer i is

$$\begin{array}{rl} \max\limits_{x_i} & u_i(x_i) \\ \text{s.t.} & p\cdot x_i = p\cdot(\omega_i + \sum_j \theta_{ij} y_j). \end{array}$$

There are no strategic considerations in the choice of consumption bundles. Assumptions (C.1) and (C.2) ensure there is a unique solution to the problem of consumer i for all price price vectors and incomes and the solution is a differentiable function of price vectors and income. Let $f_i : \mathbb{R}^\ell_{++} \times \mathbb{R} \to \mathbb{R}^\ell$ be the demand function as a function of price vectors and income.

Walrasian price vectors and equilibria

At Walrasian price vectors for fixed productions plans, consumers maximize utility and markets clear, but firms do not necessarily maximize profits.

Definition 1 *A* ***Walrasian price vector*** *for y is a price vector $\bar{p}$ such that markets clear:*

$$\sum_i f_i(\bar{p}, \bar{p}\cdot(\omega_i + \sum_j \theta_{ij} y_j)) \;=\; \sum_i \omega_i + \sum_j y_j.$$

The first welfare theorem does not apply to lists of production plans and their Walrasian price vectors.

At Walrasian equilibria consumers maximize utilities, firms maximize profits and markets clear.

Definition 2 *A* ***Walrasian equilibrium*** *is a list of production plans and a price vector $(\bar{y}, \bar{p})$ such that*

- *Markets clear: $\bar{p}$ is a Walrasian price vector for $\bar{y}$.*
- *Firms maximize profits for the price vector $\bar{p}$: for every j and all y_j,*

$$\bar{p}\cdot y_j \;\le\; \bar{p}\cdot \bar{y}_j.$$

The first welfare theorem does apply to Walrasian equilibria. In general firms do not aim at maximizing profits for fixed price vectors because they have market power and decisions in firms are made by collectives of consumers.

Firm behaviour

Firm compete à la Cournot so they take into account that their production plans influence prices. In the context of imperfect competition, profit maximization is typically not in the interest of the shareholders, and shareholders typically disagree on which production plan is best. Hence, a mechanism for collective decision making is necessary. In the present paper, we study the effect of majority voting among consumers on the choice of production plan.

Voting weights for consumers are $v \in (S^{m-1})^n$. For the *shareholder governance* (one share-one vote) voting weights are equal to shares, $v_{ij} = \theta_{ij}$ for every i and every j, and, for the *stakeholder democracy* (one consumer-one vote) voting weights are equal, $v_{ij} = 1/m$ for every i and every j. The majority rate is $\rho \in]0,1[$ where: $\rho \to 0$ corresponds to every voter having the power to decide for a change (or equivalently, only unanimity having veto power against a change); and, $\rho \to 1$ corresponds to only unanimity having the power to decide for a change (or equivalently, every voter having veto power against a change).

Naturally, consumer i as a voter in firm j is interested in the combination of production plans and prices that maximizes her indirect utility $u_i \circ f_i(p, p \cdot (\omega_i + \sum_j \theta_{ij} y_j))$. At (y, p), for a change of production in firm j and prices to (z_j, q), the first-order change in the indirect utility of consumer i is:

$$Du_i^T D_{w_i} f_i D_{y_j} w_i (z_j - y_j) + Du_i^T (D_p f_i + D_{w_i} f_i D_p w_i)(q - p)$$

where $w_i = p \cdot (\omega_i + \sum_k \theta_{ik} y_k)$, $f_i = f_i(p, w_i)$ and $u_i = u_i(f_i)$. From the first-order condition of the consumer problem, it follows that $Du_i = \lambda_i p$. From $p \cdot x_i = w_i$, it follows that $f_i^T + p^T D_p f_i = 0$ and $p^T D_{w_i} f_i = 1$. Hence the first-order change in the indirect utility has the same sign as

$$\theta_{ij} p \cdot (z_j - y_j) + (q - p) \cdot (\omega_i + \textstyle\sum_k \theta_{ik} y_k - f_i). \tag{1}$$

The first term is the income effect of a change in production, and, the second term is the price effect of a change in prices. The price effect can be decomposed into three price effects. Indeed, the first-order change in utility has the same sign as

$$\theta_{ij} p \cdot (z_j - y_j) + \theta_{ij} (q - p) \cdot y_j + \sum_{k \neq j} \theta_{ik} (q - p) \cdot y_k + (q - p) \cdot (\omega_i - f_i).$$

The first term is the income effect for profit in firm j of a change in production. Taking the first effect into account, consumer i would like the firm to behave like a perfectly competitive firm for $\theta_{ij} > 0$. The second term is the price effect for profit in firm j. Taking the two first effects into account, consumer i would like the firm to behave like a monopoly for $\theta_{ij} > 0$. The third term is the price effect for profits in other firms $k \neq j$. Taking the three first effects into account, consumer i would like the firm to behave like a monopoly of an artificial firm with production set $\sum_k \theta_{ik} Y_k$. Therefore, price effects on goods transferred inside the artificial firm are not important for consumer i. The fourth term is the price effect on the difference between initial endowments and demand. It represents the preferences of consumer i.

The income effect has the same sign for all consumers with shares in firm j, but it can be more or less strong depending on how many shares they have. The price effect depends on the sign of the (individual) excess supply: the consumer wants high prices for goods she has in excess supply, and low prices for goods she has in excess demand. Hence consumers' preferences over production plans typically differ with respect to the size of the income effect and the size and sign of excess supplies.

For example, a consumer with shares only in firm j and no shares in other firms has a strong income effect and is likely to have excess supply of goods produced by firm j and excess demand of goods used by firm j. On the contrary, a consumer owning no shares at all has no income effect and is likely to have excess demand of goods produced by the firms and excess supply of goods used by the firms. How the governance of firms combines the interests of various stakeholders will impact the production decision.

3 The local Cournot-Walras approach

We consider two notions of local Cournot-Walras equilibrium. In the first notion, firms face a well behaved inverse demand function so voters know first-order approximations of the relation between production plans and Walrasian price vectors. In the second notion, firms face a well behaved random inverse demand function, so voters know the first-order approximations of the relation between production plans and probability distributions on Walrasian price vectors. We present results on existence of equilibrium for both notions of which the latter demands milder assumptions than the former.

Local Cournot-Walras equilibrium

A few steps are needed to define the notion of local Cournot-Walras equilibrium. The first step computes the effect of a change of production and prices on aggregate excess demand. The second step deals with production and identifies feasible changes for firms. The third step analyses decision-making in the firm and subsequently defines local Cournot-Walras equilibria.

First, assume p is a Walrasian price vector for y. Consider changes, from p to q in prices and from y_j to z_j in firm j. Then the first-order change of aggregate excess demand is

$$\sum_i (D_p f_i + D_{w_i} f_i (\omega_i + \sum_i \theta_{ij} y_j)^T)(q-p) + \sum_i \theta_{ij} D_{w_i} f_i p^T (z_j - y_j) - (z_j - y_j).$$

Second, let us identify the feasible changes. Let the correspondence $\Gamma_j(y,p) \subset Y_j \times S^{\ell-1}$ be the set of production plans in firm j and prices such that the first-order change of aggregate excess demand is zero:

$$\Gamma_j(y,p) = \Big\{ (z_j, q) \in Y_j \times S^{\ell-1} \mid \sum_i (D_p f_i + D_{w_i} f_i (\omega_i + \sum_k \theta_{ik} y_k)^T)(q-p) + \sum_i \theta_{ij} D_{w_i} f_i p^T (z_j - y_j) - (z_j - y_j) = 0 \Big\}.$$

Because of Walras' law, there are $\ell - 1$ independent equations and $r + \ell - 1$ variables. At *regular Walrasian price vectors*, where $\sum_i (D_p f_i + D_{w_i} f_i (\omega_i + \sum_j \theta_{ij} y_j)^T)$ has rank $\ell - 1$, $\Gamma_j(y,p)$ has dimension r and for all z_j there is either a unique q or no q such that $(z_j, q) \in \Gamma_j(y,p)$.

Third, let $D_j(y,p,z_j,q) \subset \{1,\ldots,m\}$ be the set of consumers who have a positive change in indirect utility, and therefore vote in favor of the change:

$$D_j(y,p,z_j,q) = \{ i \in \{1,\ldots,m\} \mid \theta_{ij} p^T (z_j - y_j) + (\omega_i + \sum_k \theta_{ik} y_k - f_i)^T (q-p) > 0 \}.$$

At local Cournot-Walras equilibria consumers maximize utility, markets clear and no firm has a feasible change that is preferred by a majority of voters.

Definition 3 *A* ***local Cournot-Walras equilibrium*** *is a list of production plans and a price vector* $(\bar{y}, \bar{p})$ *such that*

- *Markets clear:* $\bar{p}$ *is a Walrasian price vector for* $\bar{y}$*.*
- *There is no feasible change supported by a majority in any firm: for every* j *and all* $(y_j, p) \in \Gamma_j(\bar{y}, \bar{p})$*,*
$$\sum_{i \in D_j(\bar{y}, \bar{p}, y_j, p)} v_{ij} \leq \rho.$$

At Cournot-Walras equilibria, excess demand instead of first-order approximation of excess demand is used to evaluate how changes in production change prices. However, if there are multiple Walrasian price vectors for some y, then excess demand does not determine how changes in production change prices. On the contrary, if there is a unique Walrasian price vector for all y and it is a differentiable function, then Cournot-Walras equilibria are local Cournot-Walras equilibria too.

Example: There are two goods, two consumers and two firms. The consumers have identical log-linear utility functions:

$$u(x^1,x^2) \;=\; \ln x^1 + \ln x^2.$$

Consumer 1 (respectively 2) has an initial endowment of $n > 0$ units of good 1 (respectively 2), $\omega_1 = (n,0)$ and $\omega_2 = (0,n)$, and owns firms 1 (respectively 2), $\theta_{11} = 1$ and $\theta_{22} = 1$. The two firms have production sets:

$$Y_1 \;=\; \{y \in \mathbb{R}^2_+ \mid y^1 + 2y^2 \leq 2\} \;\text{ and }\; Y_2 \;=\; \{y \in \mathbb{R}^2_+ \mid 2y^1 + y^2 \leq 2\}.$$

Note firm 1 is twice as efficient at producing good 1 as at producing good 2 and conversely for firm 2.

Consider the list of production plans $\bar{y} = ((0,1),(1,0))$ where both firms specialize in producing the good they are inefficient at producing. Easy computations show that the associated Walrasian price vector is $\bar{p} = (1/2, 1/2)$. Next it is checked $(\bar{y},\bar{p})$ is a local Cournot-Walras equilibrium, at which there is an obvious productive inefficiency.

Suppose firm 1 considers another production plan: $y_1 = (2t, 1-t)$ with $t \in [0,1]$, while firm 2 produces $\bar{y}_2$. The demand of consumer 1 as a function of the price of good 1 is:

$$x_1 \;=\; \begin{pmatrix} \dfrac{(n-1+3t)p_1+1-t}{2p_1} \\ \dfrac{(n-1+3t)p_1+1-t}{2(1-p_1)} \end{pmatrix}$$

Market clearing results in the following expression for the corresponding Walrasian equilibrium price as a function of t:

$$p_1 \;=\; \frac{n+1-t}{2(n+1)+t} \;\text{ and }\; p_2 \;=\; 1-p_1.$$

Hence the consumption at equilibrium of consumer 1, as a function of (t,n), is

$$x_1(t,n) \;=\; \begin{pmatrix} \dfrac{\phi(t)}{2(1-t-n)} \\ \dfrac{\phi(t)}{2(2n+2+t)} \end{pmatrix} \;\text{ with }\; \phi(t) \;=\; (n+1)^2 + 3t - 4t^2.$$

The derivative with respect to t at $t = 0$ of the indirect utility of consumer 1 $v_1(t,n) = u(x_1(t,n))$ is

$$\frac{\partial v_1(t,n)}{\partial t} \;=\; \frac{5-n}{(n+1)^2}.$$

Obviously, the derivative is strictly negative for $n > 5$, in which case consumer 1 has no interest in a marginal change of production. Indeed, producing a bit of good 1 would decrease the price p to an extent that is overall detrimental for consumer 1 given her large endowment in this good.

Actually, plotting the graph of $v_1(t,n)$ on a computer shows that when $n > 5$, $v_1(t,n)$ is a decreasing function of t over the whole interval $[0,1]$, hence $(\bar{y},\bar{p})$ is a global Cournot-Walras equilibrium.

Existence of local Cournot-Walras equilibrium

Trivially, existence of local Cournot-Walras equilibria for $\rho = 1$ boils down to existence of Walrasian price vectors, as for $\rho = 1$ the second condition of Definition 3 is always satisfied. Two questions naturally come to mind: are there local Cournot-Walras equilibria for some $\rho < 1$? What is the *minimum* value of ρ for which there always exists a local Cournot-Walras equilibrium?

To answer the first question, remember that the first-order change in the indirect utility of consumer i for a change to (z_j,q) has the same sign as Equation (1). The income effect $\theta_{ij}p\cdot(z_j - y_j)$ has the same sign for all consumers with shares in firm j. But at a Walrasian price vector, it cannot be the case that the price effect has the same sign for all consumers, since the sum of price effects over all consumers is zero. Hence some consumers have a non-positive price effect, and therefore do not support the change, unless the income effect is positive and exceeds the price effect.

Moreover, at a Walrasian equilibrium $(\bar{y},\bar{p})$, for any change the income effect is non-positive. Hence consumers with non-positive price effects do not support the change. If some of these consumers have positive voting weights, as in the stakeholder democracy, then $(\bar{y},\bar{p})$ is a local Cournot-Walras equilibrium for some $\rho < 1$.

Let us turn now to the second question: that of the *minimum* value of ρ for which there always exists a local Cournot-Walras equilibria. Theorem 1 below gives a partial answer, by providing an *upper bound* to this minimum value. This upper bound is: $1 - 1/(r+1)$.

The intuition about this upper bound can easily be grasped in the case of $m = r+1$ consumers and the stakeholder democracy. Consider then, again, a Walrasian equilibrium $(\bar{y},\bar{p})$. As it was just argued, for any change in firm j, there must be at least one consumer with non-positive price effect who does not support the change; her voting weight is $1/(r+1)$; hence the result.

The following assumptions ensure the existence of local Cournot-Walras equilibrium for non-trivial rates of majority.

(E.1) For all y, there is a regular Walrasian price vector p.

(E.2) For all y, there is a unique Walrasian price vector p.

Assumption (E.1) states that for all y there is at least one Walrasian price vector that locally is a continuously differentiable function of y. It implies that locally at (p,y) there is a unique continuously differentiable inverse demand function. Assumption (E.2) states that for all y there is at most one Walrasian price vector. It implies that globally there is a unique inverse demand function. Obviously the two assumptions imply there is a unique continuously differentiable inverse demand function.

If the equilibrium manifold is S-shaped as illustrated in Figure 1.1 in Balasko (1988), then (E.1) is satisfied, but (E.2) is not. If equilibrium manifold is similar to $\{\,(y,p) \mid y = (p-1)^3\,\}$, then (E.2) is satisfied, but (E.1) is not. Consider the affine function from lists of production plans to lists of endowments $\Gamma : Y \to \mathbb{R}^{\ell m}$ defined by $\Gamma_i(y_1,\ldots,y_n) = \omega_i + \sum_j \theta_{ij} y_j$ for every $i \in \{1,\ldots,m\}$. Then the image of Y is contained in an affine set of dimension ℓn. Hence, if there are more consumers than firms, $m > n$, then the image of Γ has measure zero in the set of possible endowments $\mathbb{R}^{\ell m}$. Suppose the set of initial endowments, $(\omega_i)_i \in \mathbb{R}^{\ell m}$, for which there is a unique and singular equilibrium has dimension less than or equal to $\ell(m-n)-1$. Then we guess that Assumption (E.1) is satisfied for a residual set of utility functions and initial endowments $(u_i,\omega_i)_i$ with the set of utility functions being endowed with the Whitney topology.

Theorem 1 *Suppose Assumptions (E.1) and (E.2) are satisfied. If*

$$\rho \;\geq\; \frac{r}{r+1},$$

then there is a local Cournot-Walras equilibrium.

Proof: Assumptions (E.1) and (E.2) ensure there is a continuously differentiable inverse demand function $\phi : Y \to S^{\ell-1}$ defined by

$$\phi(y) \;= \{\, p \in S^{\ell-1} \mid \textstyle\sum_i \omega_i + \sum_j y_j - f_i(p, p\cdot(\omega_i + \sum_j \theta_{ij} y_j)) = 0\,\}.$$

For every j let the correspondence $P_j : Y \to Y_j$ be the defined by

$$P_j(y) \;=\; \left\{\, z_j \in Y_j \mid \exists q \in S^{\ell-1} : (z_j,q) \in \Gamma(y,\phi(y)) \text{ and } \sum_{i \in D_j(y,\phi(y),z_j,q)} v_{ij} \;>\; \rho \,\right\}.$$

Then P_j has open graph and y_j is not in convex hull of $P_j(y)$ provided $\rho \geq r/(r+1)$ for all y_j according to the proof of Theorem 2 in Greenberg (1979). Therefore there is y such that $P_j(y) = \emptyset$ for every j according to the theorem in Gale and Mas-Colell (1975). Clearly (y,p) with $p = \phi(y)$ is a local Cournot-Walras equilibrium. □

Majority voting is a potential source of indeterminacy. The intuition is clear in case of one-dimensional production sets and even numbers of voters with equal voting weights in firms. The median voters in firms are typically a continuum of production plans. In case of multi-dimensional production set and any number of voters, the median voters are not necessarily well defined. However the intuition carries over by using the notion of d-majority equilibrium in Greenberg (1979) as generalized median voters.

Local random Cournot-Walras equilibrium

Suppose there are multiple Walrasian price vectors for some production plans as in the case of an S-shaped equilibrium manifold. Then Assumption (E.2) is violated and there need not be a continuously differentiable inverse demand function in form of a selection of Walrasian price vectors. However there can be well behaved selections of probability measures with support on the set of Walrasian price vectors. Intuitively, selections of probability measures are random inverse demand functions. The realized prices are determined by extrinsic uncertainty, but they clear markets. With firms facing random inverse demand functions and consumers facing prices, consumers do not know prices when they vote over production plans, but they do know the probability distribution on prices, and, they do know prices when they trade.

Let $\mathbb{P}$ be the set of probability measures on $S^{\ell-1}$ and $\operatorname{supp}\psi$ the support of $\psi \in \mathbb{P}$. There are well behaved random inverse demand functions $\Phi : Y \to \mathbb{P}$ provided Assumption (E.1) is satisfied.

Lemma 1 *Suppose Assumption (E.1) is satisfied. Then there is a random inverse demand function $\Phi : Y \to \mathbb{P}$ such that for all y, $\operatorname{supp}\Phi(y)$ is finite and for every $p \in \operatorname{supp}\Phi(y)$:*

- *p is a Walrasian price vector.*
- *There is a neighborhood $U \subset Y$ of y and two functions $\phi \in C^1(U, S^{\ell-1})$ with $p = \phi(y)$ and $\pi \in C^1(U,]0,1[)$ such that $\phi(z) \in \operatorname{supp}\Phi(z)$ and $\pi(z) = \Phi(z)(\phi(z))$ for all $z \in U$.*

Proof: Suppose p is a regular Walrasian price vector for y. Then there is an open neighborhood U_y of y and a continuously differentiable function $\phi_y \in C^1(U_y, S^{\ell-1})$ with $\phi_y(y) = p$ such that $\phi_y(z)$ is a Walrasian price for $z \in U_y$ and $\sum_i (D_p f_i + D_{w_i} f_i (\omega_i + \sum_j \theta_{ij} y_j)^T)$ has rank $\ell - 1$ at $(z, \phi_y(z))$ for all $z \in U_y$. Obviously Assumption (E.1) implies $(U_y)_y$ is an open cover of Y. Since Y is compact, there is a finite subcover $(U_a)_a$ of Y. According to Corollary 4.2 on p. 538 in Lang (1993) there is a smooth partition of unity $(\pi_a)_a$ subordinated to $(U_a)_a$: $\pi_a \in C^\infty(Y, [0,1])$ with $\pi_a(y) = 0$ for all $y \notin F_a$, where $F_a \subset U_a$ is some closed set, and $\sum_a \pi_a(y) = 1$ for all y. Let the map $\Phi : Y \to \mathbb{P}$ be defined by $y \in U_a$ implies the price is $\phi_a(y)$ with probability $\pi_a(y)$. □

According to Lemma 1 for all $y \in Y$ there is a neighborhood U_y and a finite number of price functions $\phi_a \in C^1(U_y, S^{\ell-1})$ and probability functions $\pi_a \in C^1(U_y, \mathbb{R}_{++})$ such that for all production plans $z \in U_y$ prices will be $\phi_a(z)$ with probability $\pi_a(z)$. For simplicity, consumers are assumed to maximize their expected utilities $\sum_a \pi_a(y) u_i(x_{ia})$. At y, for a change in firm j to z_j, the first-order change in the indirect utility of consumer i is:

$$\sum_a D_{y_j} \pi_a^T (z_j - y_j) u_{ia} + \sum_a \pi_a D u_{ia}^T (D_p f_{ia} + D_{w_i} f_{ia} D_p w_{ia}) D_{y_j} \phi_a (z_j - y_j)$$

where $\phi_a = \phi_a(y)$, $\pi_a = \pi_a(y)$, $w_{ia} = \phi_a \cdot (\omega_i + \sum_j \theta_{ij} y_j)$, $f_{ia} = f_i(\phi_a, w_{ia})$ and $u_{ia} = u_i(f_{ia})$. The first term is new compared to the first-order change in indirect utility without randomness. It is the change in expected utility caused by the changes in probabilities of the different Walrasian price vectors. The second term is identical to the term without randomness because $Du_{ia} = \lambda_{ia} p_a$ for some $\lambda_{ia} > 0$ and every a.

Let $E_j(y, z_j) \subset \{1, \ldots, m\}$ be the set of consumers who have a positive change in expected indirect utility for a change of production plan in firm j to z_j,

$$E_j(y, z_j) = \Bigg\{ i \in \{1, \ldots, m\} \mid \sum_a D_{y_j} \pi_a^T (z_j - y_j) u_{ia} + \sum_a \pi_a D u_{ia}^T (D_p f_{ia} + D_{w_i} f_{ia} D_p w_{ia}) D_{y_j} \phi_a (z_j - y_j) > 0 \Bigg\}$$

where $\phi_a = \phi_a(y)$, $\pi_a = \pi_a(y)$, $w_{ia} = \phi_a \cdot (\omega_i + \sum_j \theta_{ij} y_j)$, $f_{ia} = f_i(\phi_a, w_{ia})$ and $u_{ia} = u_i(f_{ia})$.

At a local random Cournot-Walras equilibrium consumers maximize expected utility, markets clear and no firm has a feasible change that is preferred by a majority of voters for a fixed random inverse demand function.

Definition 4 *A* ***local random Cournot-Walras equilibrium*** *for a random inverse demand function $\Phi : Y \to \mathbb{P}$ is a list of production plans $\bar{y}$ such that no feasible change in any firm is supported by a majority: for every j and all y_j,*

$$\sum_{i \in E_j(\bar{y}, y_j)} v_{ij} \leq \rho.$$

Assumption (E.1) does not ensure there is a unique random inverse demand function. Hence, we need to define local random Cournot-Walras equilibria for a fixed random inverse demand function. However, Assumption (E.1) ensures there is a local random Cournot-Walras equilibrium.

Theorem 2 *Suppose Assumption (E.1) is satisfied. If*

$$\rho \geq \frac{r}{r+1},$$

then there is a local random Cournot-Walras equilibrium.

Proof: Assumption (E.1) ensures there is a random inverse demand function $\Phi : Y \to \mathbb{P}$ having the properties listed in Lemma 1. For every j let the correspondence $P_j : Y \to Y_j$ be the defined by

$$P_j(y) \;=\; \left\{ z_j \in Y_j \mid \sum_{i \in E_j(y,z_j)} v_{ij} \;>\; \rho \right\}.$$

Then P_j has open graph and y_j is not in convex hull of $P_j(y)$ provided $\rho \geq r/(r+1)$ for all y_j according to the proof of Theorem 2 in Greenberg (1979). Therefore there is y such that $P_j(y) = \emptyset$ for every j according to the theorem in Gale and Mas-Colell (1975). Clearly y is a local random Cournot-Walras equilibrium. □

Random inverse demand functions are a source of inefficiency and indeterminacy. If there is randomness at a local random Cournot-Walras equilibrium $\bar{y}$, then the equilibrium allocation is not Pareto optimal. Indeed, since utility functions are strictly concave, consumers prefer the average consumption bundle to the random consumption bundle. If Assumption (E.2) is not satisfied, then there is a continuum of different random inverse demand functions having the properties listed in Lemma 1.

Our model can be seen as a game with endogenous sharing rules, where strategy sets are production sets and payoffs are expected indirect utilities of consumers. In Carmona and Podczeck (2018) sufficient conditions for invariance of equilibrium sets for games with endogenous sharing rules are provided. One of the conditions is the payoff correspondence is virtually continuous. Intuitively, virtually continuity implies the set of lists of production plans for which there are multiple Walrasian price vectors is small in the set of lists of production plans. There are open sets of economies for which there are multiple Walrasian price vectors as shown in Ghiglino and Tvede (1997) so it is possible that there are multiple Walrasian price vectors for all lists of production plans.

An example can illustrate the indeterminacy. Suppose some economy has three regular price vectors for all lists of production plans. Then there are three deterministic and differentiable inverse demand functions $p_1, p_2, p_3 : Y \to S^{\ell-1}$. For each of these three inverse demand functions there is an equilibrium. Moreover, there is a continuum of differentiable random inverse demand functions: for all differentiable partitions of unity $\pi_1, \pi_2, \pi_3 \in C^1(Y, \mathbb{R}_+)$ there is a random and differentiable inverse demand function with $\pi_k(y)$ being the probability the price is $p_k(y)$ for $y \in Y$ and $k \in \{1,2,3\}$. Suppose the sets of local Cournot-Walras equilibria are pairwise disjoint for the three deterministic and differentiable inverse demand functions. Consider three pairwise disjoint and open sets $U_1, U_2, U_3 \subset Y$ and open set V such that (U_1, U_2, U_3, V) is an open cover of Y. Then there is a random and differentiable inverse demand function such that if y is an equilibrium with deterministic prices, then $y \in U_1 \cap U_2 \cap U_3$ and if the price vector is deterministic, then it is

$p_k(y)$ for $y \in U_k$ with $k \in \{1,2,3\}$, and if $y \in V \setminus (U_1 \cap U_2 \cap U_3)$ is an equilibrium, then the price is random. Consequently, the set of equilibria as well as whether the equilibrium price vector is deterministic or random depend on the inverse demand function.

Special cases where simple majority is enough: $\rho = 1/2$

Of course, simple majority is enough for $r = 1$. Then the space of political issues (change in production plan) is unidimensional and Theorem 1 is a version of the median voter theorem.

Beyond this simple case, it is possible to get existence of local Cournot-Walras equilibria for simple majority, but the assumptions needed are strong: identical homothetic utility functions; collinear initial endowments; and collinear portfolios.

Corollary 1 *Suppose:*

- *There is a homothetic utility function* $u : \mathbb{R}^\ell_{++} \to \mathbb{R}$ *such that* $u_i = u$ *for every* i.
- *There is* $\omega \in \mathbb{R}^\ell_{++}$ *such that* $\omega_i = \tau_i \omega$ *for every* i *and some* $\tau_i > 0$.
- *For every* i *there is* $\upsilon_i \geq 0$ *such that* $\theta_{ij} = \upsilon_i$ *for every* j.
- $Y_j \subset L_j \cap \mathbb{R}^\ell_+$ *for every* j.

If $\rho \geq 1/2$, *then there is a local Cournot-Walras equilibrium.*

Proof: Let $h : \mathbb{R}^\ell_{++} \to \mathbb{R}^\ell_{++}$ be the demand function depending on prices with the income normalized to one. Then the demand of consumer i is $h(p)w_i$, where $w_i = p \cdot (\tau_i \omega + \upsilon_i \sum_j y_j)$, because the utility function is homothetic. At (y, p), for a change of production in firm j to z_j and a change of prices to q, the first-order change in utility of consumer i has the same sign as

$$\frac{\upsilon_i}{\tau_i}\left(p^T(z_j - y_j) + (\textstyle\sum_k y_k - h(p)p \cdot \sum_k y_k)^T\right)(q-p) + (\omega - h(p)p \cdot \omega)^T(q-p)$$

Since variations in the first-order changes in utility of consumers depend on υ_i/τ_i and nothing else, voters can be ordered by υ_i/τ_i. Suppose $\upsilon_1/\tau_1 \geq \ldots \geq \upsilon_m/\tau_m$. For the shareholder governance, the median voter g is defined by $\sum_{i<g}\upsilon_i, \sum_{i>g}\upsilon_i \leq 1/2$. For the stakeholder democracy, the median voter d is defined by $d-1, m-d \leq m/2$. The first-order change in utility of the median voter in firm j can be used as objective for firm j. □

The assumptions in Corollary 1 ensure the relevant heterogeneity among consumers is one-dimensional, namely the ratio between income coming from shares in firms and income coming from initial endowments. The first assumption ensures that income levels are without importance. The second and third assumptions state that initial endowments and portfolios are collinear. The fourth assumption ensures that profits are non-negative for all prices so consumption can be restricted to be positive.

4 Voting and Pareto optimality

The core question of the paper was phrased as follows in the introduction: can efficient production spontaneously result from majority voting? A technical formulation of the question is akin to the second welfare theorem: under what conditions are Pareto optimal allocations supported at local Cournot-Walras equilibria?

When voting supports perfect competition

A useful link to Pareto optimality goes through Walrasian equilibrium. According to the first welfare theorem, if $(\bar{y},\bar{p})$ is a Walrasian equilibrium, then the Walrasian equilibrium allocation $(\bar{f},\bar{y})$, where $\bar{f}_i = f_i(\bar{p},\bar{p}\cdot(\omega_i+\sum_i\theta_{ij}\bar{y}_j))$ for every i and $\bar{f}=(\bar{f}_i)_i$, is Pareto optimal. Hence the previous question becomes: when are Walrasian equilibria also local Cournot-Walras equilibria?

Suppose that at a Walrasian equilibrium, for every firm and every majority of consumers, the convex hull of excess supplies $\omega_i+\sum_j\theta_{ij}y_j-f_i$ contains the zero excess supply. Then the Walrasian equilibrium is also a local Cournot-Walras equilibrium.

Theorem 3 *Assume $(\bar{y},\bar{p})$ is a Walrasian equilibrium. Suppose that for every j and every $C\subset\{1,\dots,m\}$ with $\sum_{i\in C}v_{ij}>\rho$ there is $(\alpha_i)_{i\in C}\in S^{|C|-1}$ such that*

$$\sum_{i\in C}\alpha_i(\omega_i+\textstyle\sum_k\theta_{ik}\bar{y}_k-\bar{f}_i) \;=\; 0.$$

Then $(\bar{y},\bar{p})$ is a local Cournot-Walras equilibrium.

Proof: Suppose

$$\sum_{i\in C}\alpha_i(\omega_i+\textstyle\sum_k\theta_{ik}\bar{y}_k-\bar{f}_i) \;=\; 0.$$

Then

$$\sum_{i\in C}\alpha_i(\theta_{ij}\bar{p}^T(y_j-\bar{y}_j)+(\omega_i+\textstyle\sum_k\theta_{ik}\bar{y}_k-\bar{f}_i)^T(p-\bar{p})) \;=\; \displaystyle\sum_{i\in C}\alpha_i\theta_{ij}\bar{p}^T(y_j-\bar{y}_j).$$

Since $\bar{p}\cdot y_j\le\bar{p}\cdot\bar{y}_j$ for all y_j, $\sum_{i\in C}\alpha_i\theta_{ij}\bar{p}^T(y_j-\bar{y}_j)\le 0$ so there is $i\in C$ such that

$$\theta_{ij}\bar{p}^T(y_j-\bar{y}_j)+(\omega_i+\textstyle\sum_k\theta_{ik}\bar{y}_k-\bar{f}_i)^T(p-\bar{p}) \;\le\; 0$$

so $i \notin D_j(\bar{y}, \bar{p}, y_j - \bar{y}_j, p - \bar{p})$. Repeated use of the argument implies that

$$\sum_{i \in D_j(\bar{y}, \bar{p}, y_j - \bar{y}_j, p - \bar{p})} v_{ij} \leq \rho$$

for all $(y_j - \bar{y}_j, p - \bar{p}) \in \Gamma_j(\bar{y}, \bar{p})$. □

The characterization in Theorem 3 is generalized to all $(\bar{y}, \bar{p})$, where $\bar{p}$ is a Walrasian price vector for $\bar{y}$, in the appendix.

Suppose shares are not assumed to be non-negative: $\theta_{ij} \in \mathbb{R}$ for every i and every j with $\sum_i \theta_{ij} = 1$ for every j. Then voting weights for the shareholder governance should be $v_{ij} = \max\{\theta_{ij}, 0\} / \sum_k \max\{\theta_{kj}, 0\}$ for every i and j. Theorems 1 and 2 on existence of local and local random Cournot-Walras equilibria extend without modifications. However, Theorem 3 has to be modified.

Corollary 2 *Assume $(\bar{y}, \bar{p})$ is a Walrasian equilibrium. Suppose that for every j and every $C \subset \{1, \ldots, m\}$ with $\sum_{i \in C} v_{ij} > \rho$ there is $(\alpha_i)_{i \in C} \in S^{|C|-1}$ such that*

$$\begin{cases} \sum_{i \in C} \alpha_i (\omega_i + \sum_k \theta_{ik} \bar{y}_k - \bar{f}_i) = 0 \\ \sum_{i \in C} \alpha_i \theta_{ij} \geq 0. \end{cases}$$

Then $(\bar{y}, \bar{p})$ is a local Cournot-Walras equilibrium.

Proof: Suppose

$$\begin{cases} \sum_{i \in C} \alpha_i (\omega_i + \sum_k \theta_{ik} \bar{y}_k - \bar{f}_i) = 0 \\ \sum_{i \in C} \alpha_i \theta_{ij} \geq 0. \end{cases}$$

First, $\sum_{i \in C} \alpha_i (\omega_i + \sum_k \theta_{ik} \bar{y}_k - \bar{f}_i) = 0$ implies

$$\sum_{i \in C} \alpha_i (\theta_{ij} \bar{p}^T (y_j - \bar{y}_j) + (\omega_i + \sum_k \theta_{ik} \bar{y}_k - \bar{f}_i)^T (p - \bar{p})) = \sum_{i \in C} \alpha_i \theta_{ij} \bar{p}^T (y_j - \bar{y}_j).$$

Second, since $\bar{p} \cdot y_j \leq \bar{p} \cdot \bar{y}_j$ for all y_j, $\sum_{i \in C} \alpha_i \theta_{ij} \geq 0$ implies $\sum_{i \in C} \alpha_i \theta_{ij} \bar{p}^T (y_j - \bar{y}_j) \leq 0$. Therefore, there is $i \in C$ such that

$$\theta_{ij} \bar{p}^T (y_j - \bar{y}_j) + (\omega_i + \sum_k \theta_{ik} \bar{y}_k - \bar{f}_i)^T (p - \bar{p}) \leq 0$$

so $i \notin D_j(\bar{y}, \bar{p}, y_j - \bar{y}_j, p - \bar{p})$. Repeated use of the argument implies that

$$\sum_{i \in D_j(\bar{y}, \bar{p}, y_j - \bar{y}_j, p - \bar{p})} v_{ij} \leq \rho$$

for all $(y_j - \bar{y}_j, p - \bar{p}) \in \Gamma_j(\bar{y}, \bar{p})$. □

Clearly, if shares are non-negative, then the second condition in Corollary 2, namely $\sum_{i \in C} \alpha_i \theta_{ij} \geq 0$, is superfluous.

Interpretation of Theorem 3

Remember the income effect is non-positive for any change to (z_j, q) in firm j at a Walrasian equilibrium $(\bar{y}, \bar{p})$. Hence consumers with non-positive price effects do not support the change.

No-trade consumers with $\omega_i + \sum_j \theta_{ij} \bar{y}_j - \bar{f}_i = 0$ have zero price effects. Consequently, they support profit maximization for fixed prices and will not support any deviation from that. As a consequence, if every decisive coalition in every firm *has to include a no-trade consumer*, then no decisive coalition will *unanimously* support a change, and therefore no change will ever be adopted. Hence the Walrasian equilibrium is a local Cournot-Walras equilibrium too. The assumption of Theorem 3 that for every j and every decisive coalition C in firm j,

$$\sum_{i \in C} \alpha_i (\omega_i + \textstyle\sum_k \theta_{ik} \bar{y}_k - \bar{f}_i) \ = \ 0$$

can be rephrased: every decisive coalition in every firm has to include a (potentially virtual) no-trade consumer.

Shareholder governance or stakeholder democracy?

Intuitively, a consumer with lots of (resp. without) shares in a firm would probably have negative (resp. positive) excess demand for the goods produced by the firm. Therefore they would prefer prices of outputs to be above (resp. below) competitive prices and prices of inputs to be below (resp. above) competitive prices.

A governance that gives large voting weight to consumers with lots of shares, e.g. the shareholder governance, could foster excessive prices and consequently not support Pareto optimality. The ideal modification would be to ensure non-shareholders have a say in firms such that majorities have to include non-shareholders. External board members such as government officials or workers representatives or consumer associations can be seen as proxies for non-shareholders.

Alternatively, a governance that gives large voting weight to non-shareholders, e.g. the stakeholder democracy, could foster excessive regulation and consequently not support Pareto optimality either. Lobbying can be interpreted as an activity that enables shareholders to increase their voting weights. The ideal modification would be to ensure that majorities have to include shareholders.

Though neither the stakeholder democracy nor the shareholder governance are likely to result in perfectly competitive behaviour, there is one big difference between the two governances. For the stakeholder democracy, *the mean voter supports perfectly competitive* *behaviour because she is a no-trade consumer*. Indeed,

$$\sum_i (\theta_{ij}\bar{p}^T(y_j - \bar{y}_j) + (\omega_i + \textstyle\sum_k \theta_{ik}\bar{y}_k - \bar{f}_i)^T(p - \bar{p})) \;=\; \bar{p}^T(y_j - \bar{y}_j).$$

On the contrary, the mean voter for the shareholder governance is typically not a no-trade consumer, because consumers with many shares have many votes. Hence, the mean voter for the shareholder governance typically does not support perfect competition.

The difference gives an edge to the stakeholder democracy over the shareholder governance as to support efficient production. To understand why, let us make a detour through the median voter theorem: if the space of political issues is one-dimensional (all excess supplies are distributed over a line), then for any coalition in favor of a change to rally a simple majority of 50% of the voters, it has to include the median voter. It has been argued that the average voter is a natural proxy for a median voter in a multi-dimensional setting as in the present paper. In particular, Caplin & Nalebuff (1988, 1991) provides conditions on the distribution of voters under which every decisive coalition includes the mean voter for a super majority rate of 0.64.

Casual evidence in support of the stakeholder democracy

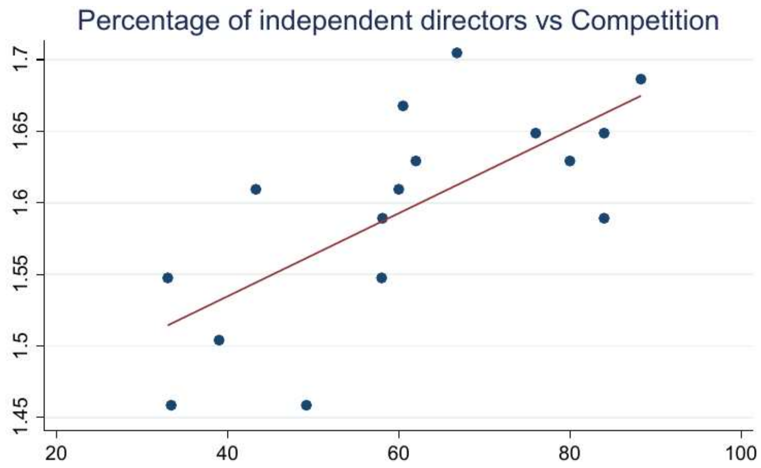
Employees of the corporation are primordial stakeholders. Making sure that they are part of the decision making process is a step toward the stakeholder democracy. This can be realized either by having employee representatives in the board, or opening the capital of the corporation to employees.

Employee shareholding could thus be taken as a proxy for stakeholders inclusion in the decision process. The European Federation of Employee Share Ownership publishes an Annual Survey of Employee Share Ownership in European Countries. It measures the average stake in percentages held by all employee shareholders in large European companies. These percentages can be plotted against the measure of perceived competition as measured by the World Economic Forum in its Global Competitive Index (GCI)−using an index ranging from 1-7, whereby a higher score represents greater competition.

The first figure plots, on the ordinate axis, the log of GCI in 2017 against, on the abscissa axis the percentage of shares held by employees for the same year, by country for the EU countries. It appears that countries competitive prospects correlates with greater employee shareholding as illustrated by the red line found by a simple OLS regression.

Another proxy for the inclusion of non-shareholders in the governance could be the percentage of independent members in the boards of directors. In the second figure we keep the log of GCI on the ordinate axis, and plot it against the average percentage of independent board members across EU countries as measured by Spencer Stuart (both in

2015). It appears that having more independent board members within a company correlates to greater competitive success on a country wide level. The same exercise can be done using



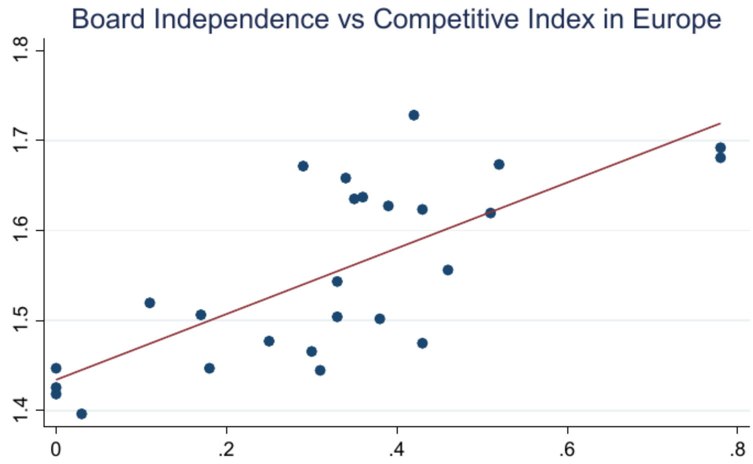
different data for 2010 from a different source (Ferreira & Kirchmaier, 2013) as shown in the third figure. It yields the same positive correlation.

Local Cournot-Walras equilibria and the number of firms

To consider how the number of firms influences equilibria, we split every firm j into a finite number of smaller identical firms $j1,\dots,jA$ with shareholders having the same number of shares in each of the small firms as they have in the big firms. By doing so, the aggregate production set is unchanged, but the number of firms is increased.

Definition 5 *The* A*-**split economy*** $\mathscr{E}^A = ((u_i, \omega_i, (\phi_{ija})_{j,a})_i, (Z_{ja})_{ja})$ *with* $a \in \{1,\dots,A\}$ *of an economy* $\mathscr{E} = ((u_i, \omega_i, (\theta_{ij})_j)_i, (Y_j)_j)$ *is defined by*

- $Z_{ja} = (1/A)Y_j$ *for every* j *and* a*.*



- $\phi_{ija} = \theta_{ij}$ *for every i, j and a.*

- $v_{ija} = v_{ij}$ *for every i, j and a.*

The sets of equilibria in an economy and the A-split economy are identical in terms of aggregate output and prices provided production is identical in identical firms. Consider a local Cournot-Walras equilibrium for an economy. Then there is a local Cournot-Walras equilibrium of the A-split economy where production in every small firm is equal to production in the big firms scaled down and prices are the same. Conversely, consider a local Cournot-Walras equilibrium for an A-split economy, where production in every small firm is equal. Then there is a local Cournot-Walras equilibrium for the economy, where production in the big firms is equal to the sum of productions in the small firms and prices are the same.

Theorem 4 *Consider $\mathscr{E}$ and $\mathscr{E}^A$.*

- *Suppose $(\bar{y}, \bar{p})$ is a local Cournot-Walras equilibrium for $\mathscr{E}$. Then $(\bar{z}, \bar{p})$ with $\bar{z}_{ja} = (1/A)\bar{y}_j$ for every j and a is a local Cournot-Walras equilibrium for $\mathscr{E}^A$.*

- *Suppose $(\bar{z}, \bar{p})$ is a local Cournot-Walras equilibrium for $\mathscr{E}^A$ with $\bar{z}_{ja} = \bar{z}_{jb}$ for every j, a and b. Then $(\bar{y}, \bar{p})$ with $\bar{y}_j = \sum_a \bar{z}_{ja}$ for every j is a local Cournot-Walras equilibrium for $\mathscr{E}$.*

Proof: Suppose $(\bar{y}, \bar{p})$ is a local Cournot-Walras equilibrium for $\mathscr{E}$. Then $\bar{p}$ is a Walrasian price vector for $\bar{z}$ and $\mathscr{E}^A$. Moreover, $(z_{ja}, p) \in \Gamma_{ja}(\bar{z}, \bar{p})$ for some a implies $(\sum_a z_{ja}, p) \in \Gamma_j(\bar{y}, \bar{p})$. Hence, $(\bar{z}, \bar{p})$ with $\bar{z}_{ja} = (1/A)\bar{y}_j$ for every j and every a is a local Cournot-Walras equilibrium for $\mathscr{E}^A$.

Suppose $(\bar{z}, \bar{p})$ is a local Cournot-Walras equilibrium for $\mathscr{E}^A$ with $\bar{z}_{ja} = \bar{z}_{jb}$ for every j, a and b. Then $\bar{p}$ is a Walrasian price vector for $\bar{y}$ and $\mathscr{E}$. Moreover, $(y_j, p) \in \Gamma_j(\bar{y}, \bar{p})$ implies

$((1/A)y_j, p) \in \Gamma_{ja}(\bar{z}, \bar{p})$ for every a. Hence, $(\bar{y}, \bar{p})$ with $\bar{y}_j = \sum_a \bar{z}_{ja}$ for every j is a local Cournot-Walras equilibrium for $\mathscr{E}$. □

Theorem 4 shows that splitting up firms has no effect on the equilibrium. Even though every small firm is indeed small compared to aggregate demand and supply, coordination of behaviour in firms is supported by voters as in Crès & Tvede (2013) in case of perfect competition and production externalities. Firms are not cooperating, but shareholders are supporting coordination. Consequently to make firms change behaviour both firms and ownership have to be split.

Suppose shareholders in firm j are split into shareholders with shares in one and only one of the A small firms ja. Then for every firm j there is a unique small firm a such that $\phi_{ija} = A\theta_{ij}$ and $\phi_{ijb} = 0$ for every $b \neq a$. Therefore shareholders in the small firms ja compared to shareholders in j put more weight on the income effect relative to the price effect. However, with asset markets consumers could diversify their portfolios by selling some of their shares in the small firms ja and buying shares in the other small firms jb. And with idiosyncratic risk in firms, consumers would want to diversify their portfolios. Consequently, it could be necessary to limit consumers to own shares in few small firms to ensure that splitting up firms changes firm behaviour.

5 Final Remarks

To sum up, we would like to highlight four contributions of the present paper. First we provide a Cournot-Walras model where decision making in firms is based on shareholder or stakeholder voting instead of profit maximization or some form of wealth maximization. Second we overcome the thorny issue of multiplicity of equilibrium prices in the Cournot-Walras model by introducing randomness in the notion of equilibrium. Third we characterize conditions under which self-regulation supports perfect competition. Fourth we highlight the importance of financial markets in the regulation of large corporations.

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Appendix: A generalization of Theorem 3

It is straightforward to generalize the characterization in Theorem 3 to all $(\bar{y}, \bar{p})$, where $\bar{p}$ is a Walrasian price vector for $\bar{y}$, but the characterization is more complicated.

First, assume there is a function $g \in C^1(L_j, \mathbb{R})$, where $Dg_j(y_j) \neq 0$ for all $y_j \in L_j$ with $g_j(y_j) = 0$, such that $Y_j = \{y_j \in L_j \mid g_j(y_j) \leq 0\}$. Second, for every j let $(a_{jh})_h$ be $\ell - r$ linearly independent vectors orthogonal to L_j.

Theorem 5 *Assume $\bar{p}$ is a Walrasian price vector for $\bar{y}$. Suppose that for every j and every $C \subset \{1, \ldots, m\}$ with $\sum_{i \in C} v_{ij} > \rho$, there are $(\alpha_i)_{i \in C} \in \mathbb{R}_+^{|C|-1}$, $\beta \geq 0$, $(\gamma_h)_h \in \mathbb{R}^{\ell - q}$ and $(\delta_b)_b \in$*

$\mathbb{R}^\ell$ *such that*

$$\left\{\begin{array}{rcl} \sum_i \alpha_i \theta_{ij} p - \mathbf{1}_{g_j=0} \beta D_j g_j + \sum_h \gamma_h a_{jh} + \sum_b \delta_b (\sum_i \theta_{ij} D_{w_i} f_i^b p - e_b) & = & 0 \\ \sum_i \alpha_i (\omega_i + \sum_k \theta_{ik} y_k - f_i) + \sum_b \delta_b \sum_i (D_p f_i^b + D_{w_i} f_i^b (\omega_i + \sum_k \theta_{ik} y_k))^T & = & 0. \end{array}\right.$$

Then $(\bar{y}, \bar{p})$ *is a local Cournot-Walras equilibrium.*

Proof: According to Theorem 22.2 in Rockafellar (1970) either there are $(\alpha_i)_{i \in C} \in \mathbb{R}_+^{|C|-1}$, $\beta \geq 0$, $(\gamma_h)_h \in \mathbb{R}^{\ell - q}$ and $(\delta_b)_b \in \mathbb{R}^\ell$ such that

$$\left\{\begin{array}{rcl} \sum_i \alpha_i \theta_{ij} p - \mathbf{1}_{g_j=0} \beta D_j g_j + \sum_h \gamma_h a_{jh} + \sum_b \delta_b (\sum_i \theta_{ij} D_{w_i} f_i^b p - e_b) & = & 0 \\ \sum_i \alpha_i (\omega_i + \sum_k \theta_{ik} y_k - f_i) + \sum_b \delta_b \sum_i (D_p f_i^b + D_{w_i} f_i^b (\omega_i + \sum_k \theta_{ik} y_k))^T & = & 0. \end{array}\right.$$

or there are $(z_j, q) \in Y_j \times S^{\ell-1}$ such that

$$\left\{\begin{array}{rcl} \theta_{ij} p^T (z_j - y_j) + (\omega_i + \sum_k \theta_{ik} y_k - f_i)^T (q - p) & > & 0 \text{ for every } i \\ \mathbf{1}_{g_j=0} D_j g_j^T (z_j - y_j) & \leq & 0 \\ a_{jh}^T (z_j - y_j) & = & 0 \text{ for every } h \\ \sum_i (D_p f_i^b + D_{w_i} f_i^b (\omega_i + \sum_k \theta_{ik} y_k)^T)(q - p) & & \\ + (\sum_i \theta_{ij} D_{w_i} f_i p^T - e_b)(z_j - y_j) & = & 0 \text{ for every } b. \end{array}\right.$$

The first block of equations states that first-order change in utility is positive for every consumer. The second equation states that the first-order change in production is negative. The third block of equations state that the change in production is in L_j. The fourth block of equations states that the first-order change in excess demand is zero for every good. □