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# Static assessment of notched additively manufactured polymers based on the Theory of Critical Distances

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**Abstract.** The aim of this paper is to review the work we have done in recent years to investigate the accuracy of the Theory of Critical Distances (TCD) in estimating static strength of notched additively manufactured acrylonitrile butadiene styrene (ABS) and polylactide (PLA). The TCD takes as its starting point the assumption that the extent of damage under static loading can be assessed successfully by using two different material parameters, i.e. (i) a critical distance whose length is closely related to the material micro/meso/macro-structural features and an inherent (i.e., a defect free) material strength. Plain and notched specimens of 3D-printed PLA and FDM were manufactured (with an in-fill level of 100%) by making the deposition angle vary in the range 0°-90°. Using the TCD, failures were predicted by directly post-processing the linear-elastic stress fields determined by solving standard linear-elastic Finite Element (FE) models. Independently of notch sharpness and printing direction, the estimates being obtained were found to be highly accurate, falling within an error interval of about 20%. This result fully supports the idea that the TCD can successfully be used in situations of practical interest to design against static loading notched components of additively manufactured polymers by directly post-processing the results from simple linear-elastic FE models.

**Keywords:** Additive Manufacturing, PLA, FDM, Notch, Critical Distance.

## 1 Introduction

According to ASTM F42, additive manufacturing (AM) is “*the process of joining materials to make objects from 3D-model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies*”. Thanks to its specific features, AM allows complex shapes to easily be incorporated in structural components.

If attention is focussed on thermoplastic polymers, certainly acrylonitrile butadiene styrene (ABS) and polylactide (PLA) are the most common materials that are used to additively manufacture objects at a relatively low cost. Polymers can be additively manufactured (AM) from powders, wires and flat sheets that are melted using a variety of different technological processes. PLA is a biodegradable, absorbable and biocompatible polymer that is widely employed to manufacture biomedical components. Owing to its specific mechanical response and resistance to corrosion, ABS is commonly used to

manufacture rigid and lightweight components. PLA and ABS can easily be additively manufactured by using low-cost commercial 3D-printers. Other than these two materials, polyphenyl sulfone and polycarbonate as well can be 3D-printed effectively, although they can be used provided that more advanced AM techniques are used.

One of the most relevant features of AM is that objects can be fabricated in a very accurate way, with this holding true in terms of both shape and dimensions. As far as structural integrity is concerned, the fact that 3D-printed components can contain geometrical features of all kinds results in localised stress concentration phenomena. Since stress raisers can reduce markedly the overall mechanical strength of structural components, engineers need reliable design methodologies suitable for performing the static assessment of notched 3D-printed objects accurately.

As far as static assessment of notched components is concerned, examination of the state of the art strongly supports the idea that the Theory of Critical Distances (TCD) [1] is the most powerful candidate to be used to design AM components against static loading since:

- the TCD assesses the detrimental effect of notches independently from their profile and sharpness;
- the TCD models the material morphology explicitly via suitable length scale parameters;
- the TCD can be used by directly post-processing the results from simple linear-elastic Finite Element (FE) models (where the same numerical solid models can be used also to inform the manufacturing process).

In this scenario, the aim of the present paper is to review the research work we have supervised in recent years [2-7] to investigate whether the linear-elastic TCD is successful in performing the static assessment of notched 3D-printed components of PLA and ABS subjected to in service static loading.

## 2 Fundamentals of the Theory of Critical Distances (TCD)

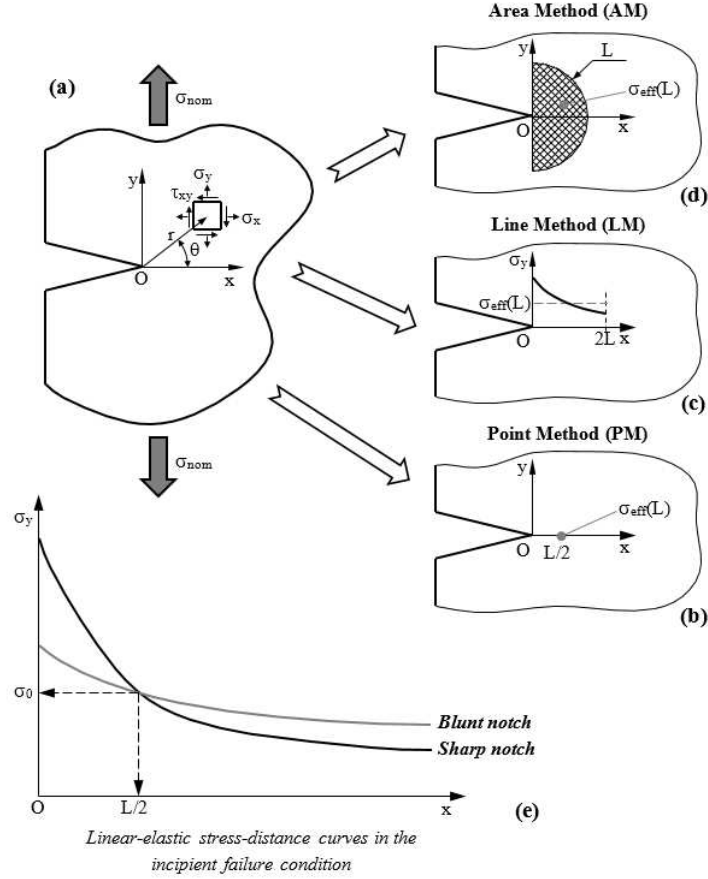
The TCD postulates that static breakage of notched/cracked components subjected to Mode I loading takes place as soon as a critical distance-dependent effective stress,  $\sigma_{\text{eff}}(L)$ , becomes equal to the material inherent strength,  $\sigma_0$  [1, 8-11], i.e.:

$$\sigma_{\text{eff}}(L) = \sigma_0 \Leftrightarrow \text{failure} \quad (1)$$

According to Eq. (1), the TCD is bi-parametrical design method where the critical distance,  $L$ , and the inherent material strength,  $\sigma_0$ , are the two material parameters being used to assess the extent of damage. In this setting,  $L$  is a length that is closely related to the micro/meso/macro-structural features of the material being designed.

An important feature of the TCD is that it performs static assessment by directly post-processing the linear-elastic stress fields acting on the material in the vicinity of the notch being designed, with this holding true independently of the ductility level of the material under investigation [9, 10].

Under static loading, the TCD critical distance can directly be estimated via the following well-known relationship [1]:



**Fig. 1.** Notch component and local system of coordinates (a); effective stress calculated according to the Point Method (b), to the Line Method (c) and to the Area Method (d); inherent strength  $\sigma_0$  and critical distance  $L$  determined from experimental results generated by testing notches of different sharpness (e).

$$L = \frac{1}{\pi} \left( \frac{K_{Ic}}{\sigma_0} \right)^2 \quad (2)$$

where  $K_{Ic}$  is the plane strain fracture toughness. Given the specific critical distance for the material under investigation, the design effective stress,  $\sigma_{eff}(L)$ , can be calculated directly according to either the Point Method, the Line Method or the Area Method as follows, respectively [1]:

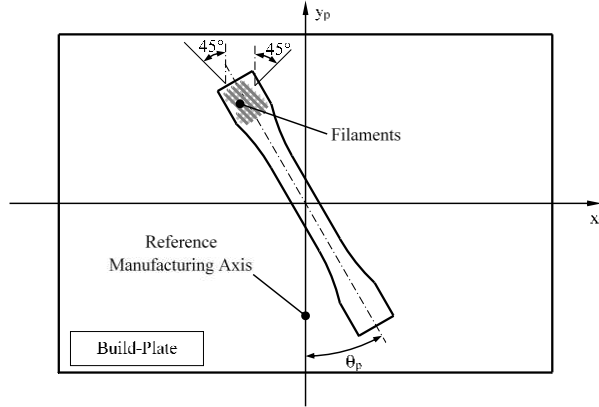
$$\sigma_{eff}(L) = \sigma_y \left( \theta = 0, r = \frac{L}{2} \right) \text{ - Point Method (Fig. 1b)} \quad (3)$$

$$\sigma_{eff}(L) = \frac{1}{2L} \int_0^{2L} \sigma_y(\theta = 0, r) \cdot dr \text{ - Line Method (Fig. 1c)} \quad (4)$$

$$\sigma_{\text{eff}}(L) = \frac{4}{\pi L^2} \int_0^{\pi/2} \int_0^L \sigma_1(\theta, r) \cdot r \cdot dr \cdot d\theta - \text{Area Method (Fig. 1d)} \quad (5)$$

The meaning of the used symbols as well as the visual explanation of the effective stress determined according to definitions (3) to (5) are reported in Figs 1a to 1d.

Eqs. (1) and (2) make it evident that inherent material strength  $\sigma_0$  plays a role of primary importance when the TCD is employed to perform static assessment of notched components. In particular, while for brittle materials [1, 8]  $\sigma_0$  is seen to approach the ultimate tensile strength,  $\sigma_{\text{UTS}}$ , for ductile materials (or non-linear materials)  $\sigma_0$  is instead quantified to be larger than  $\sigma_{\text{UTS}}$  [1, 9]. Further,  $\sigma_0$  is seen to take on a value that is higher than  $\sigma_{\text{UTS}}$  also when the plain material fails by different mechanisms to those leading to the final breakage in the presence of notches [1]. These considerations make it evident that the most accurate way to determine  $\sigma_0$  is by running specific experiments that involve notches whose presence results in different local stress distributions (Fig. 1e) [1, 9, 10].



**Fig. 2.** Orientation of the deposition filaments.

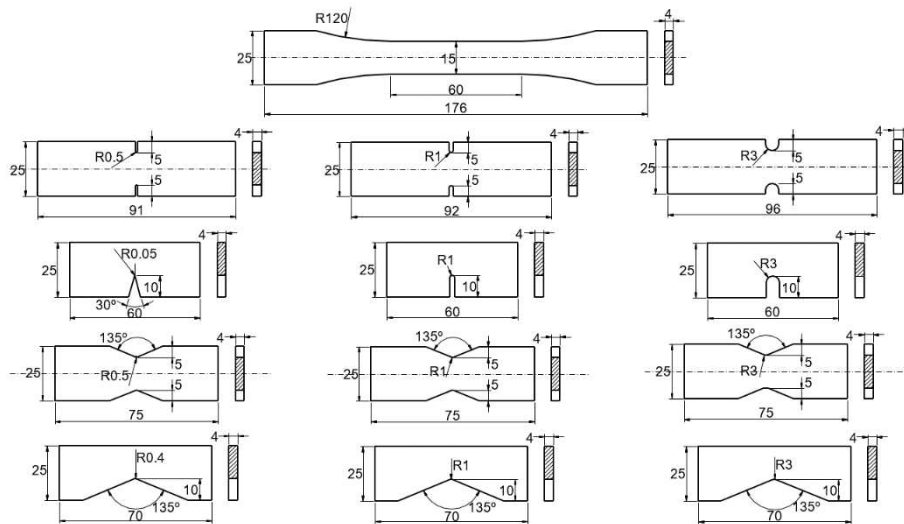
### 3 Fabrication and testing of laboratory specimens

The specimens employed to assess the accuracy of the TCD in performing static assessment of notched AM polymers were additively manufactured by using 3D-printer Ultimaker 2 Extended+. White New Verbatim filaments and grey 750g PRIMA filaments with initial diameter of 2.85mm were employed to fabricate the samples of PLA [2] and ABS [5], respectively. For both materials, the specimens were manufactured flat on the build-plate using a 0.4 mm-diameter extrusion nozzle. For the PLA specimens, the nozzle temperature was set equal to 240°C and the build-plate temperature to 60°C [2]. In contrast, the ABS specimens were fabricated by setting the temperature of the nozzle equal to 255°C and the temperature of the build plate to 90°C [5]. All the specimens were manufactured at a printing rate of 30 mm/s. The level of density was set equal to 100%, the height of the layers equal to 0.1 mm, and the thickness of the

shell equal to 0.4 mm [2, 5]. As per Fig. 2, the plain and notched specimens were fabricated horizontally on the build-plate by making printing angle  $\theta_p$  vary in the range  $0^\circ$ - $90^\circ$ . Since the used 3D-printer extruded the filaments forming the filling volume always at  $\pm 45^\circ$  to the principal manufacturing direction, setting angle  $\theta_p$  equal either to  $0^\circ$  or to  $90^\circ$  returned specimens having a  $\pm 45^\circ$  lay-up. Using a similar stratagem, specimens with a  $-15^\circ/75^\circ$  lay-up were then fabricated by taking  $\theta_p$  equal to both  $30^\circ$  and  $60^\circ$ , whereas samples with a  $0^\circ/90^\circ$  lay-up were manufactured by taking  $\theta_p$  equal to  $45^\circ$ . The technical drawings of the samples being fabricated and tested are seen in Fig. 3.

The un-notched samples and the specimens containing two opposite notches were tested under axial loading, whereas the samples with a single notch were tested under three point bending. For the bending tests the span between the two lower supports was set equal to 50 mm for the specimens with U-notches and to 60 mm for the samples containing open notches.

All the specimens were tested under a displacement rate equal to 2 mm/min up to complete breakage by using a Shimadzu axial machine. Local axial strains in the un-notched samples were gathered at a frequency of 10 Hz via an extensometer having gauge length equal to 50 mm. Three different specimens were tested for any geometry/manufacturing configuration that was investigated.

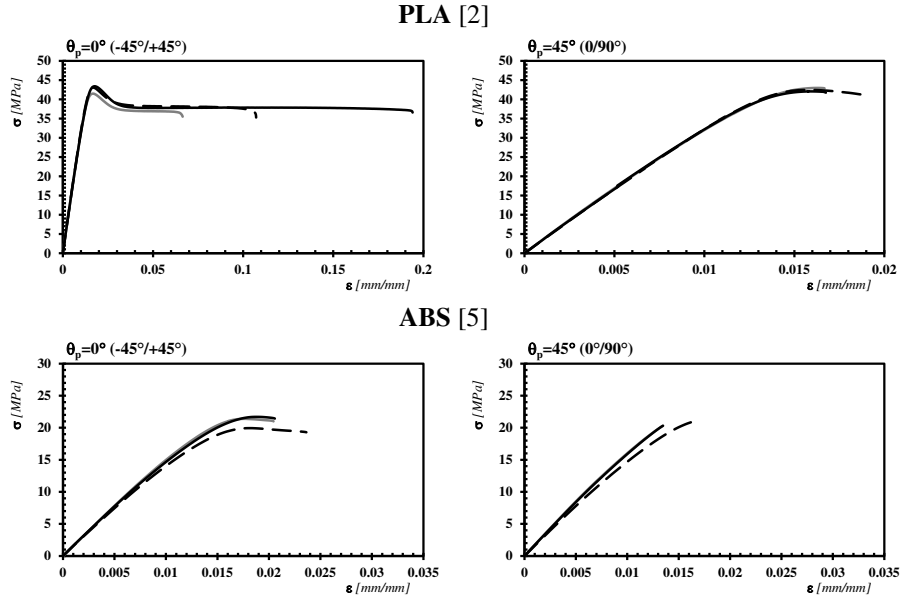


**Fig. 3.** Geometries and dimensions (in millimetres) of the tested specimens [2, 5].

## 4 Material mechanical properties

The stress,  $\sigma$ , vs. strain,  $\epsilon$ , diagrams reported in Fig. 4 show some examples of the mechanical behaviour displayed by the tested AM materials under tensile loading. These examples make it evident that the  $\sigma$  vs.  $\epsilon$  relationship was seen to be predominantly linear up to the maximum stress recorded during testing, with this holding true

independently of the value of manufacturing angle  $\theta_p$  [2, 5]. Taking as a starting point this important outcome, the force-displacement curves determined from the plain specimens under tensile loading were then re-analysed to derive the elastic modulus,  $E$ , the 0.2% proof stress,  $\sigma_{0.2\%}$ , and the tensile strength,  $\sigma_{UTS}$ .



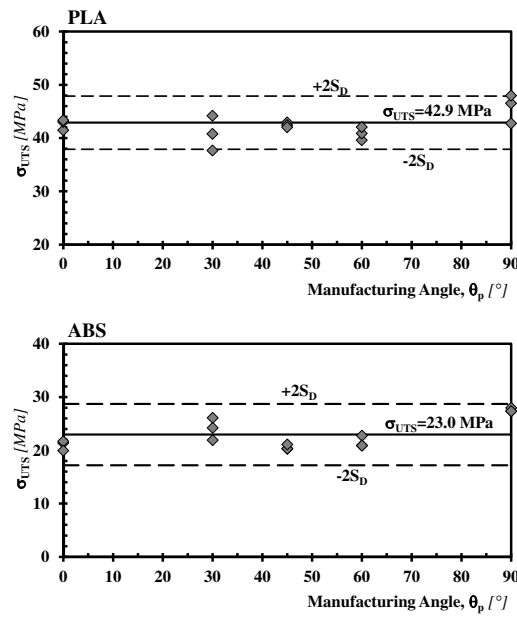
**Fig. 4.** Examples of the stress vs. strain curves displayed by the tested materials for two different values of manufacturing angle  $\theta_p$  [2, 5].

The obtained results for the ultimate tensile strength are summarised in the diagrams of Fig. 5 where the experimental values of  $\sigma_{UTS}$  are plotted against infill angle  $\theta_p$ . These two diagrams make it evident that the  $\sigma_{UTS}$  values experimentally determined by making  $\theta_p$  vary in the range  $0^\circ$ - $90^\circ$  were all within two standard deviations of the mean ( $\pm 2S_D$ ). Since both the 0.2% proof stress and Young's modulus were characterised by a similar trend, it was possible to conclude that the effect of  $\theta_p$  on the mechanical behaviour of the two 3D-printed materials under investigation could be neglected with this resulting just in a little loss of accuracy [2, 5].

## 5 Overall accuracy of the TCD

To use the TCD effective stress to assess the static strength of the notched samples shown in Fig. 2, the local linear-elastic stress fields in the notch regions were estimated by using commercial FE software ANSYS®. This was done by solving simple bi-dimensional linear-elastic FE modes, with the mesh density in the highly stressed regions being increased progressively until convergence occurred.

As far as ABS is concerned, a TCD critical distance,  $L$ , equal to 4.1 mm was estimated according to definition (2) by taking  $\sigma_0 = \sigma_{UTS} = 23$  MPa and  $K_{Ic} = 2.6$  MPa·m<sup>1/2</sup> [5]. Turning to PLA, since it was not possible to determine a consistent value for the plain strain fracture toughness [2],  $L$  was directly estimated according to a procedure similar to the one shown in Fig. 1e. In particular,  $L$  was determined by averaging the distance resulting from the points of intersection between the straight horizontal lines modelling the plain material ultimate tensile strength ( $\sigma_0 = \sigma_{UTS}$  in Fig. 1e) and the stress-distance curves describing the local stress fields in the notched specimens (tested under tensile loading) having root radius,  $r_n$ , equal to 0.5 mm. This simple procedure resulted in an average value of the critical distance equal to 4.6 mm.



**Fig. 5.** Influence of manufacturing angle  $\theta_p$  on the ultimate tensile strength of the two AM materials being investigated.

These two values for  $L$  allowed us to post-process the notch results being generated according to the Point Method, Eq. (3), and the Area Method, Eq. (5). The Line Method, Eq. (4), instead could not be used because the integration length (i.e.,  $2L$ ) was larger than half net-width of the tested notched specimens [1].

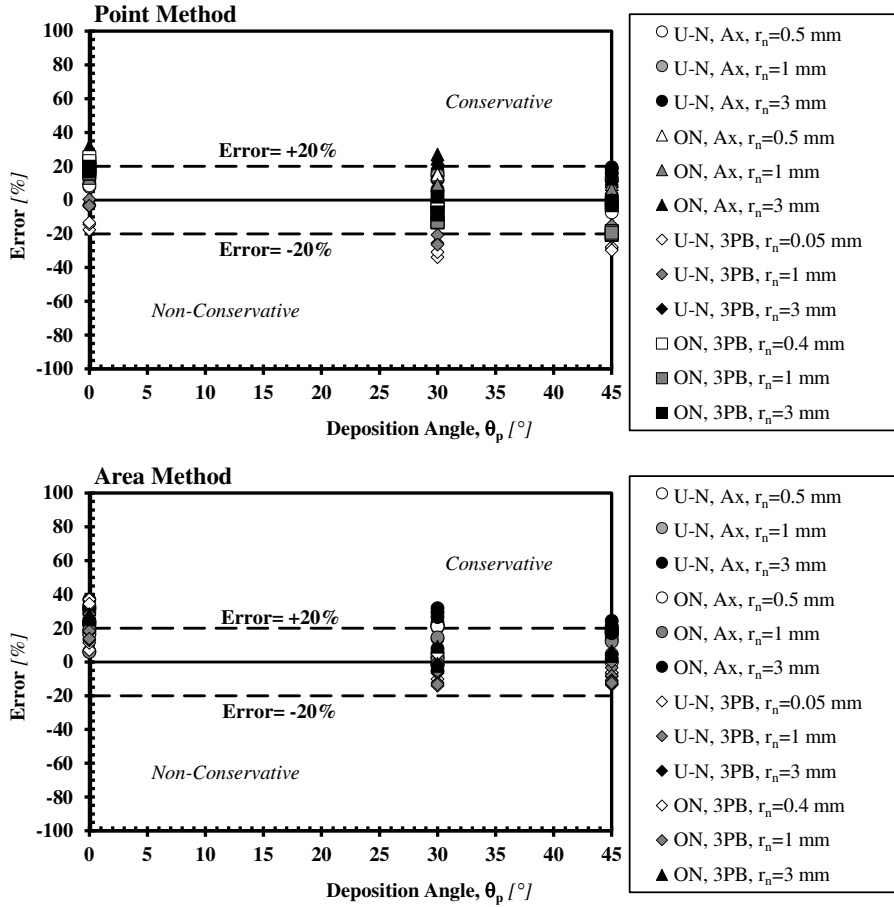
The diagrams reported in Figs 6 and 7 summarise the level of accuracy that was reached by using the TCD in terms of the Point and Area Method. In these charts the error was calculated as:

$$\text{Error} = \frac{\sigma_{\text{eff}} - \sigma_{UTS}}{\sigma_{UTS}} [\%] \quad (6)$$



According to this definition, a positive value for the error denotes conservative estimates, whereas, obviously, non-conservative predictions return negative errors.

The error diagrams reported Figs 6 and 7 confirm that, as for other conventional engineering materials [1], the systematic usage of the Point and Line Method returned estimates mainly falling within an error interval of  $\pm 20\%$ .

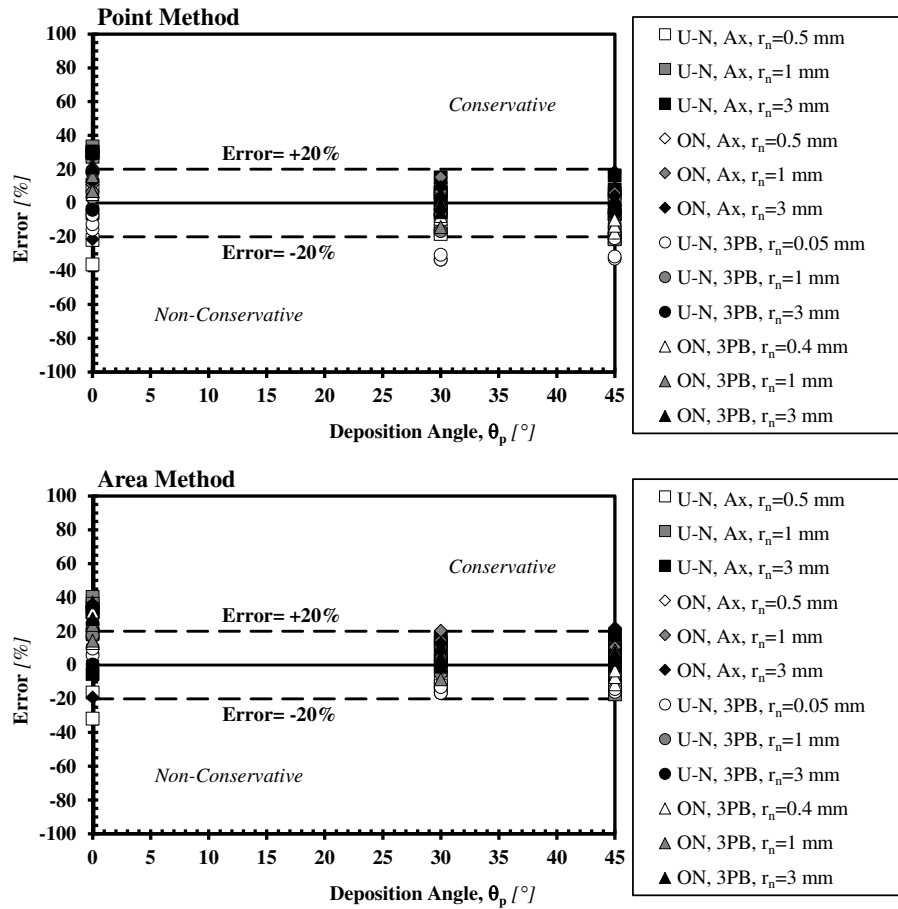


**Fig. 6.** Accuracy of the TCD used in the form of the Point and Area Method in estimating static strength of notched AM PLA (U-N=U-notched; ON=open notch; Ax=axial Loading; 3PB=three-point bending;  $r_n$ =notch root radius) [2].

## 6 Conclusions

The present paper reviews the accuracy of the TCD in designing notched AM polymers against static loading. To consistently extend the use of this powerful theory to the static assessment of 3D-printed polymers weakened by stress raisers of different sharpness,

the mechanical/cracking behaviour of two specific AM materials (i.e., PLA and ABS) were investigated by considering the effect of the manufacturing angle. The accuracy and reliability of the TCD was checked against a large number of experimental results generated by testing, under both tensile and bending loading, specimens of AM PLA as well as of AM ABS containing different geometrical features. Based on the work reviewed in the present paper, the most relevant conclusions are summarised in what follows.



**Fig. 7.** Accuracy of the TCD used in the form of the Point and Area Method in estimating static strength of notched AM ABS (U-N=U-notched; ON=open notch; Ax=axial Loading; 3PB=three-point bending;  $r_n$ =notch root radius) [5].

- The linear-elastic TCD is successful in estimating static strength of notched AM PLA as well as of notched AM ABS.
- The TCD allows notched components of AM polymers to be designed against static loading by directly post-processing the relevant stress fields determined

through conventional linear-elastic FE models. This implies that static assessment can be performed without the need for explicitly modelling the non-linear mechanical behaviour displayed by AM polymers.

- With AM polymers as well, the TCD can be used to design notched components against static loading by treating the required critical distance as a material property whose value is not affected by the sharpness of the notch being assessed.

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