Conference or Workshop Item:

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Combining Multiple Reservoirs on Multiple Timescales

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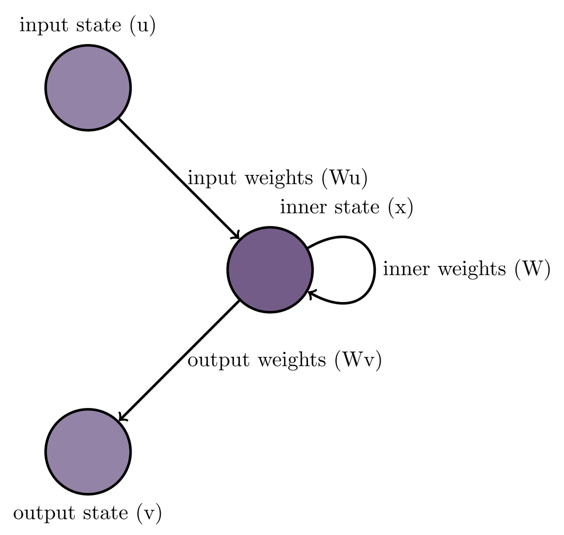
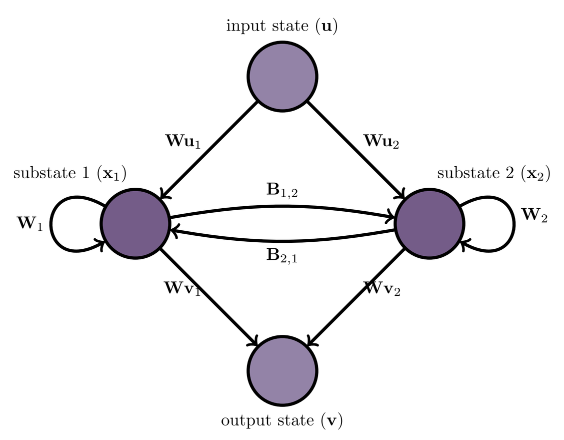
1. Defining a Restricted ESN Model





When exploring combining ESNs with each other, many works in the literature[1, 2, 4] propose what we call here a restricted ESN model, where the inner state of the reservoir is split into several clustered regions, or substates. One advantage of this model is that these ESNs can be described by the standard ESN equations:

$$\mathbf{x}(t+1) = f(\mathbf{W}_u\mathbf{u}(t) + \mathbf{W}\mathbf{x}(t)) \qquad \mathbf{v}(t+1) = \mathbf{W}_v\mathbf{x}(t) \tag{1}$$

The difference in the model comes from the topology of the inner state, described by the state vector $\mathbf{x}$ and the weight matrix $\mathbf{W}$.

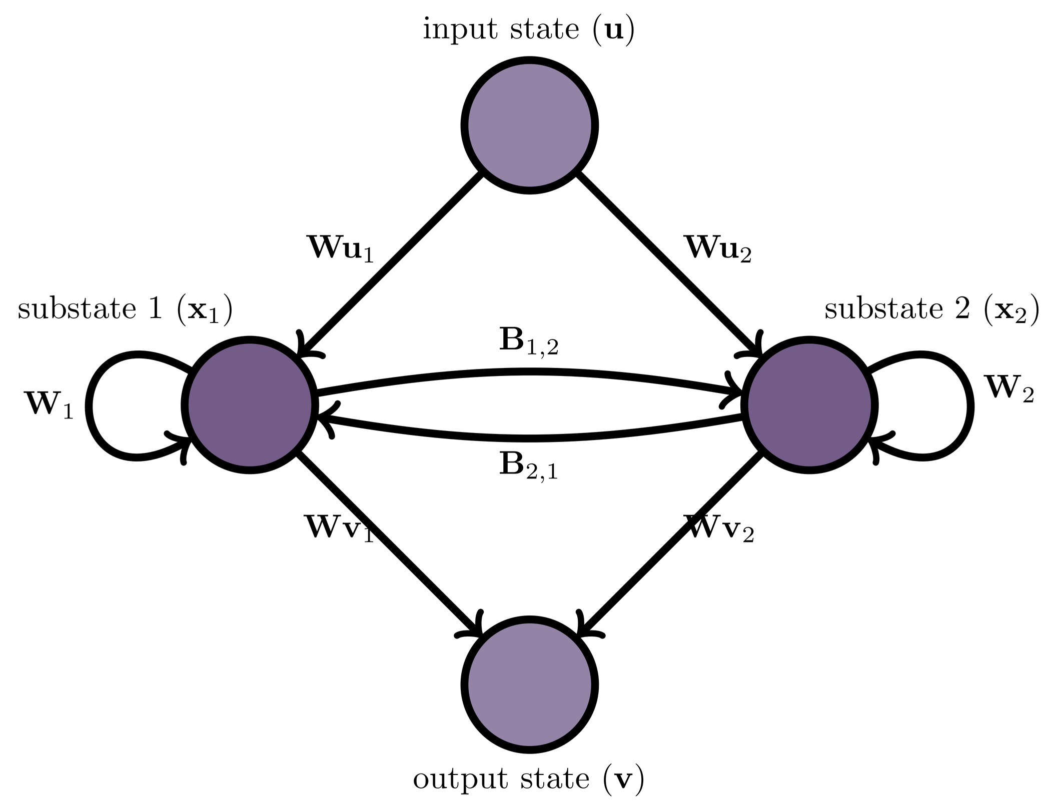
$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \dots \\ \mathbf{x}_n \end{pmatrix} \quad \mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{B}_{1,2} & \dots & \mathbf{B}_{1,n} \\ \mathbf{B}_{2,1} & \mathbf{W}_2 & \dots & \mathbf{B}_{2,n} \\ \dots & & & \\ \mathbf{B}_{n,1} & \mathbf{B}_{n,2} & \dots & \mathbf{W}_n \end{bmatrix} \tag{2}$$

$\mathbf{x}_1 \dots \mathbf{x}_2$ are the state vectors of the substates, the matrix $\mathbf{W}_n$ represents the edges within the substate n, and the matrix $\mathbf{B}_{n,m}$ represents the edges from substate n to substate m.

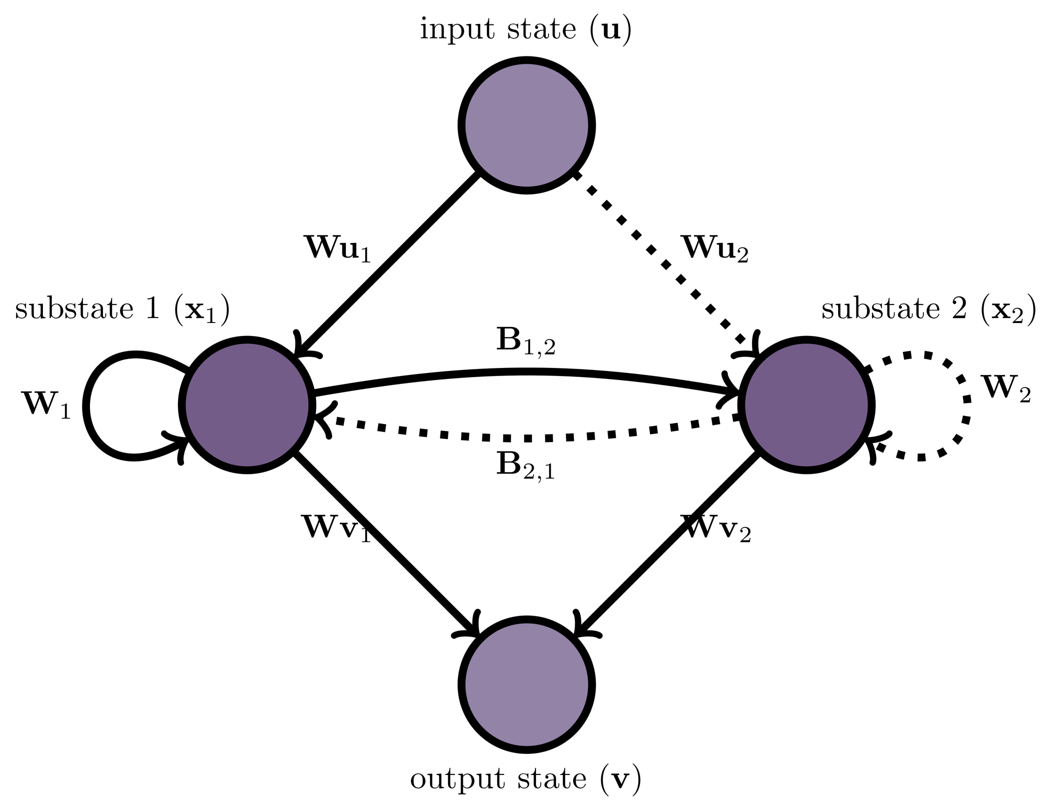
2. Using Leakage Rates for Multiple Timescales

Multiple timescales in a restricted ESN can be simulated using leakage rates[3]. Where an ESN is expected to have a constant leakage rate α, one can instead introduce a vector of leakage rates α, each of which corresponds to the leakage of one substate. If we assume that the ESN has a constant leakage over physical time, we can then simulate different amounts of physical time per iteration of the simulation.

3. Multi-Timescale Transfer Equations



(a) Elements of a restricted ESN on two timescales when substate $\mathbf{x}_2$ is awake.



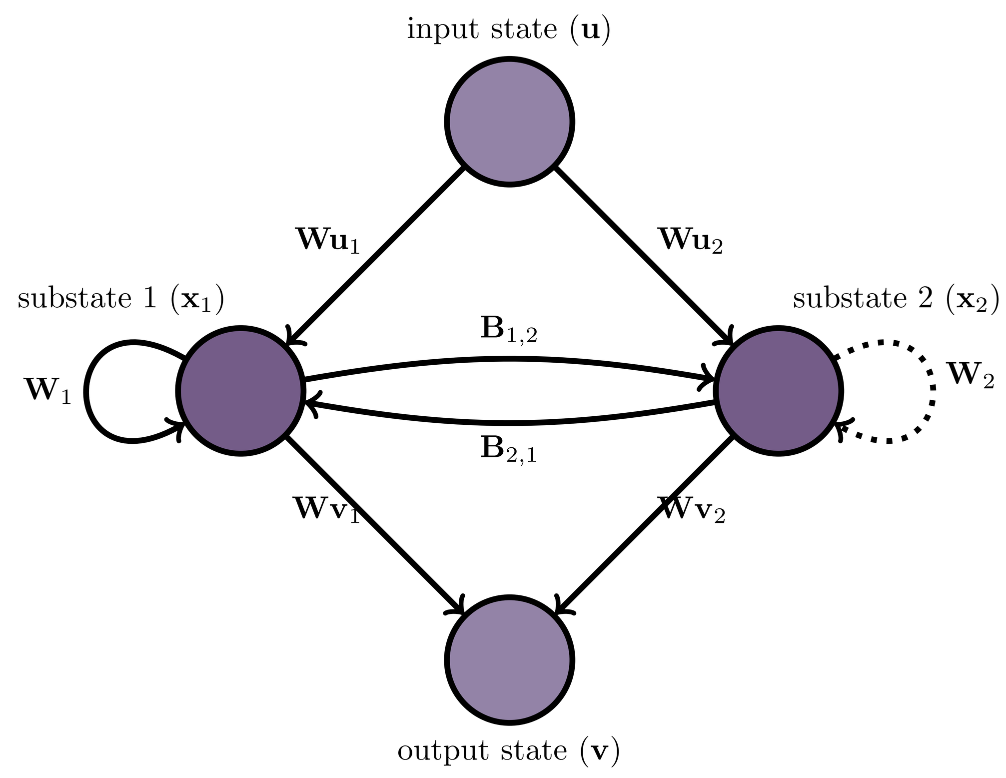
(b) Elements of a restricted ESN on two timescales when the substate $\mathbf{x}_2$ is asleep.

We propose a way of modelling different timescales, which allows for a more granular approach than that of modifying leakage rates. This approach involves modifying the ESN's transfer equation (eq.1). In this model, we introduce a "sleep" mode for one or more of the substates of the reservoir. If a substate is asleep on a timestep t, it does not update during that timestep. We introduce a new transfer function, one that ensures substate $\mathbf{x}_n$ does not update when $t = t_{sleep}$.

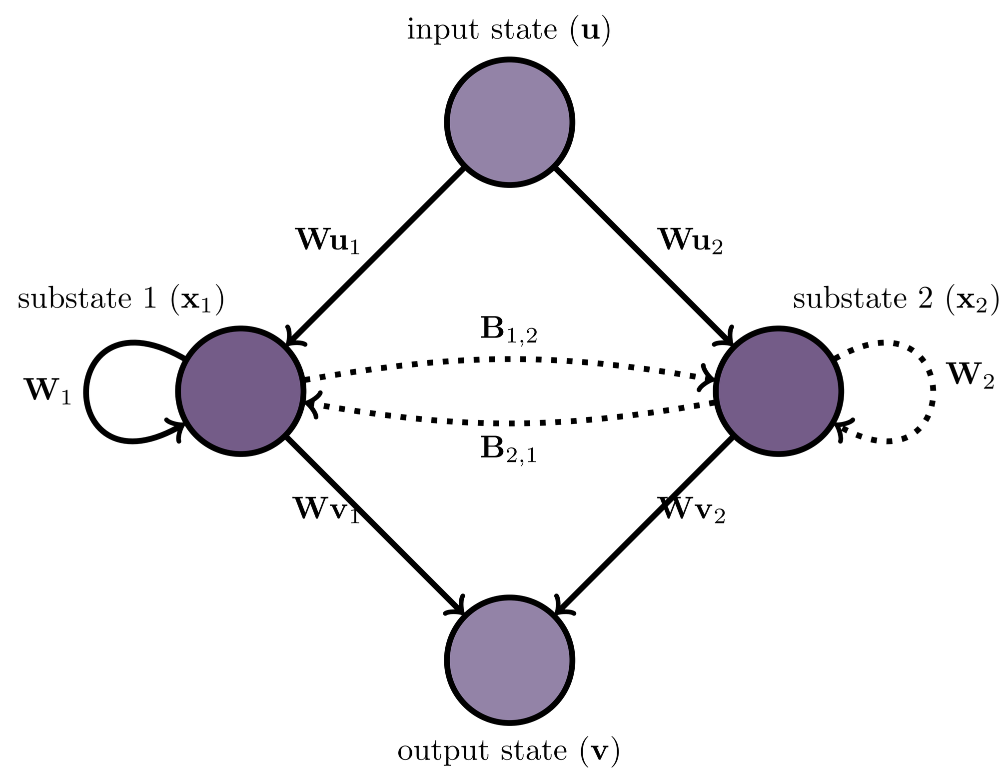
$$\begin{aligned} &\mathbf{x}(t+1): \\ t \mod 2 = 1 &\mapsto \mathbf{W}_u^{t1}\mathbf{u}(t) + \mathbf{W}^{t1}\mathbf{x}(t) \\ t \mod 2 = 0 &\mapsto \mathbf{W}_u^{t2}\mathbf{u}(t) + \mathbf{W}^{t2}\mathbf{x}(t) \\ &\textit{where} \\ \mathbf{W}_u^{t1} \mapsto \mathbf{W}_u \ &\& \ \mathbf{W}^{t1} \mapsto \mathbf{W} \\ \mathbf{W}_u^{t2} \mapsto \mathbf{0} \ &\& \ \mathbf{W}^{t2} \mapsto I \end{aligned} \tag{3}$$

t_{sleep} need not be a single value, but can also be a condition, such as $t \mod 2 = 0$. We can also introduce as many extensions as there are separate timescales.
We additionally introduce different types of "sleep" modes based on different physical models.

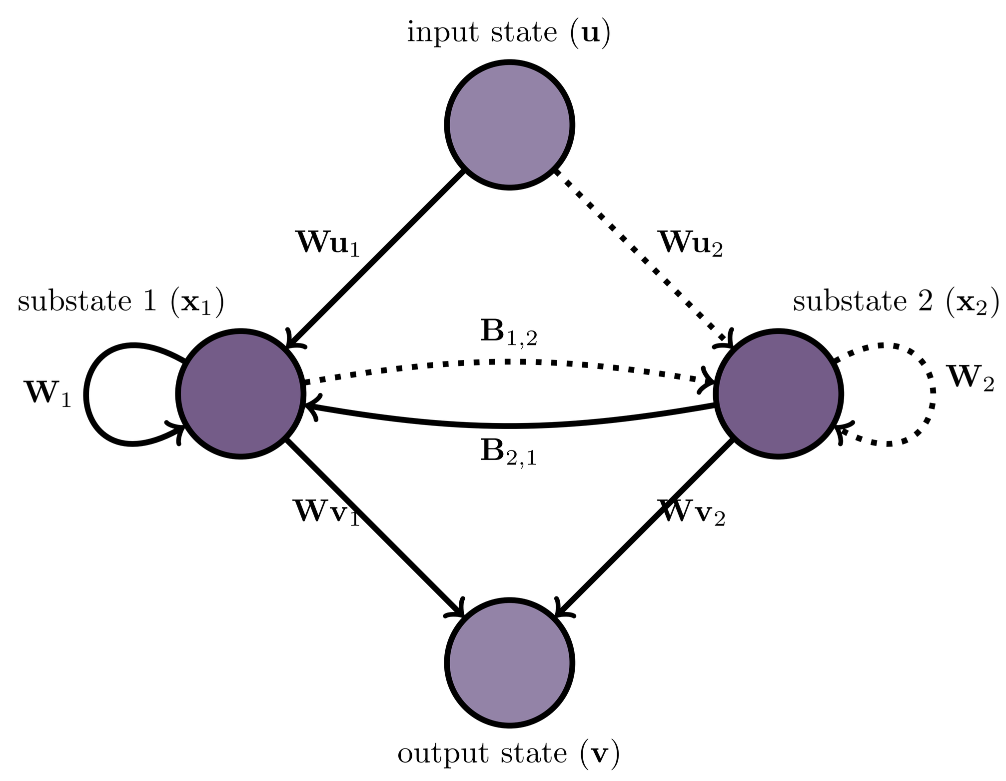
4. Circuit Model



5. Swarm Model



6. "Beacons of Gondor" Model



7. References

[1] Z. Deng and Y. Zhang. Collective behavior of a small-world recurrent neural system with scale-free distribution. *IEEE Trans. Neural Netw.*, 18(5):1364–1375, Sept. 2007.

[2] S. Jarvis, S. Rotter, and U. Egert. Extending stability through hierarchical clusters in echo state networks. *Front. Neuroinform.*, 4, July 2010.

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[4] Y. Xue, L. Yang, and S. Haykin. Decoupled echo state networks with lateral inhibition. *Neural Netw.*, 20(3):365–376, Apr. 2007.