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## RESEARCH ARTICLE

# A Utility Maximization Model of Pedestrian and Driver Interactions

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**ABSTRACT** Many models account for the traffic flow of road users, but few take the details of local interactions into consideration and how they could deteriorate into safety-critical situations. Building on an existing model of human sensorimotor control, we develop an agent-based modeling framework applying the principles of utility maximization, motor primitives, and intermittent action decisions to account for the details of interactive behaviors among road users. The framework connects the three principles to the decision theory and is tested to determine whether such an approach can reproduce the following four phenomena. Firstly, when two pedestrians travel on crossing paths, their interaction is sensitive to initial kinematic asymmetries, and secondly, based on the asymmetries, the two pedestrians rapidly resolve collision conflict by adapting their behaviors. Thirdly, when a pedestrian crosses a road while facing an approaching car, the pedestrian adapts his or her crossing behavior according to the time-to-arrival of the car, and fourthly, either the pedestrian or the driver of the car may yield to the other to resolve their conflict. We show that these phenomena emerge naturally from our modeling framework. We believe the proposed behavior model and phenomenon-centered approach of analysis offer promising tools to examine road user interactions. We conclude with a discussion on how the model can be generalized to safety-critical situations and to include other variables affecting road-user interactions.

**INDEX TERMS** Utility maximization, adaptive control, agent-based modeling, behavior model, decision theory.

## I. INTRODUCTION

Human locomotion, alone or in relation with others, is a long-standing topic of empirical and modeling research, not least in the context of road traffic [1]. Recently, the push towards increasingly fully automated vehicles has highlighted further the importance of this line of research. Crucially, one of the remaining challenges hindering automated vehicles from operating in urban environments is the lack of a sufficiently detailed understanding of how humans interact with others in such conditions and how the understanding could be quantified in computational models [2]. These models will be informative both as components in online algorithms, to make

real-time predictions about road user trajectories [3], [4] and as virtual agents in offline simulations, for safety testing of the algorithms driving automated vehicles before deployment [5], [6]. Automated vehicles, like human drivers, must trade between being sufficiently cautious to keep the risk of crashes minimal and being assertive to make meaningful progress in traffic. This balance is achieved in human drivers by being sensitive to subtle cues in the behavior of others, enabling rapid resolution of conflicts when needed [7], [8]. The balancing task is often successful in human drivers, but occasionally their interactions deteriorate into safety-critical situations. For automated vehicles to be deployed safely in an urban environment, they must resolve not only routine but also safety-critical interactions. In sum, a further understanding in details of how human drivers resolve these situations

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and how they interact with pedestrians can help the design of the algorithms driving automated vehicles to realistically capture both the subtleties of conflicts between road users and the local problems in the unresolved conflicts leading to crashes.

The current lack of widespread deployment of automated vehicles on urban roads, could, to some extent, be attributable to these gaps in existing models. This suggests many road-user models in examining interactive behaviors have yet meet the two above-mentioned criteria. Many models describe the interactions of a multitude of road users, allowing for predictions of traffic flow at the infrastructure level [9], [10], and some of them generalize to simulate the crashes caused by human error [11], [12]. These models have not focused on the details of local interactions. For instance, they do not explain how individuals collect information over time to reach decisions and how these decisions trigger different actions. Furthermore, models which do examine local interactions among road users, mostly assume that agents move to keep in lane and maneuver around obstacles [13], [14], [15] and assume that surrounding agents are static obstacles that do not interact. Few models started to examine subtle cues in road user interactions in some specific scenarios, such as routine local interactions between two or more road users in the highway merging [16], pedestrian road crossing [17], or locomotion in shared space [18]. Our paper aims to build on these models and to develop a modeling framework that could examine interactive scenarios, including safety-critical interactions, so as to enable a good understanding of the rationale in road-user interactions.

A different genre of models, namely machine-learning (ML) models, most prominently aim to bring automated vehicles to commercial uses and, in some cases, have also been used to study more complex interactions [19], [20]. The success of these models depends considerably on training them with sufficient data of good quality and their success does not necessarily inform understanding of road user interactions. Furthermore, the data of safety-critical situations are scarce and usually of poor quality so that ML models frequently rely on extrapolations to predict these high-stakes situations. Moreover, validation of ML models often uses the metrics that are also used in optimizing them. These metrics are statistically meaningful, but do not always result in an optimized model that is also meaningful for human road users, even in routine, non-critical interaction [21], [22].

Here, we leverage a cognitive theory for modeling sensorimotor control [23], which describes movement as consisting of discrete action primitives, triggered after one collects sufficient sensory information favoring a chosen action. This framework has been utilized in modeling driver behavior, investigating both near-optimal, closed-loop movement control (as is typical of routine driving) and sub-optimal open-loop maneuvers triggered after long, uncontrolled delays (as is typical of near-crash driving). This framework has been shown capable of accounting for driver behavior across a

range of situations including both routine and near-crash driving [23], [24], [25], [26].

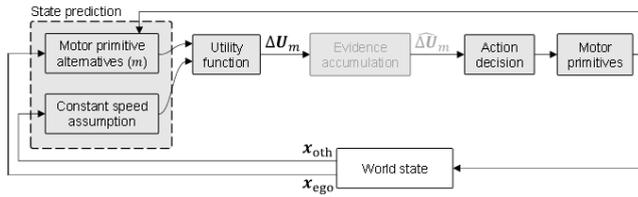
Recently, we and others have begun expanding this modeling approach to interactive traffic situations. Boda et al. [27] showed the framework could be extended to account for how drivers control longitudinal maneuvers when cyclists crossed the drivers' path. Others researchers have shown the onset latency of the road-crossing decisions in drivers and pedestrians could be modeled by the principle of analyzing sensory cues [28], [29], [30]. However, these models had only studied one agent in their interactive scenarios. Our paper presents a first step in the direction of a scenario-agnostic framework to model multiple road-user agents in interactive, time-evolving scenarios.

One modeling decision that we make is to formulate the framework, using the utility maximization (UM) principle. Our previous modeling work has instead emphasized the association between perceptual quantities (e.g., optical invariants [14]) and motor primitives for deciding on actions (i.e., action decisions), but perceptual quantities tend to be highly situation-specific, whereas the UM principle allows a more general model formulation.

When aiming to account for complex behaviors, the models themselves necessarily become complex, with a rapidly branching tree of modeling decisions, and therefore a primary concern becomes: Which model assumptions are needed to capture what behavioral phenomena? Instead of optimizing models around various data sets, we take a phenomenon-centric approach and take advantage of the computation power of modern computers by surveying many candidate models and examining them against a selection of typical road-user behavior phenomena reported in literature. This allows us to study the entire parameter space of our model to determine whether our model is at all capable of exhibiting the road-user behavior phenomena in question and if so in which parameter regions. This approach resembles the parameter space partitioning method used by cognitive modelers [31], [32], [33], which to our knowledge has not been previously used in the road user modeling domain.

This paper provides a first step in this direction, where we start with simple routine interactions, where the interacting agents are on straight crossing paths. We adopt a very simple utility function, and focus on deterministic simulations, but also show how the evidence accumulation mechanisms can be incorporated to generate stochastic model behavior. Using this approach, we show that our framework, despite its relative simplicity, can naturally account for several empirical phenomena that have been previously reported in interactions between pedestrians and between pedestrians and drivers, and in each case, we also provide a more detailed demonstration of how the model reproduces the road user behaviors in question, by comparing to empirical data in the literature.

Section II introduces the modeling framework, two target interactive scenarios and how we quantify behavioral data to test the model in these scenarios. Section III presents the results of the model analyses, and Sections IV and V discuss



**FIGURE 1. Outline of the assumptions in the modeling framework proposed in this paper. Evidence accumulation is present only in the stochastic version of the model, being represented in grey color.  $x_{ego}$  refers to the kinematic state, such as speed, acceleration, and positions, of an “ego” agent. The term, “ego” refers to a first-person point of view.  $x_{oth}$  refer to an “other” agent, relative to the “ego” agent.  $\Delta U_m$  is the accumulated evidence for a motion primitive,  $m$ .  $\Delta U_m$  signifies that the accumulated evidence is an estimated value subject to noise influence.**

the results, focusing on how the principle of utility maximization and our model can be extended to account for further variables in the road user interactions both in routine and safety-critical situations.

## II. METHODOLOGY

### A. MODELING FRAMEWORK

#### 1) MOTOR PRIMITIVES

The sensorimotor control framework on which we base our model [23] assumes agents move by initiating an intermittent, discrete, and ballistic motor primitive [34]. The motor primitive acts as a limited-duration motoric adjustment to an ongoing motion. The motoric adjustment cannot be changed once initiated but can act atop any ongoing adjustments, for example allowing midway changes of action before another primitive completes (see Figure 1 for an illustration of the modeling framework).

We assume the agent possesses a limited discrete set of motor primitives, with vehicle agents adjusting acceleration and yaw rate, and pedestrian agents speed and yaw angle. In the simulations below, we consider only longitudinal movement; therefore, any changes in yaw rates or angles become inconsequential. This simplifying assumption, as shown in the Result section, is applicable to a suite of data from both laboratory and field studies. Furthermore, we assume vehicle agents possessing the repertoire of motor primitives,  $-1, -0.5, 0, 0.5, \text{ and } 1 \text{ m/s}^2$  and pedestrian agents,  $-1, -0.5, 0, 0.5, \text{ and } 1 \text{ m/s}$ . These primitives adjust the ongoing action. All primitives are implemented as a constant pulse of signal of a duration  $\Delta T$ , here set to 0.3 s. For example, if pedestrian agents at standstill decide to apply a motor primitive of  $1 \text{ m/s}$ , their speed will increase linearly from zero to  $1 \text{ m/s}$  over  $\Delta T$  s, and then remain at  $1 \text{ m/s}$ . While that adjustment is taking place, the agent could decide to apply another motor primitive acting on top of the ongoing action.

#### 2) AGENT’S UTILITY FUNCTION

To estimate the utility of a motor primitive, the agent makes a prediction for  $T_p$  s into the future and assesses what states the motor primitive will lead to. Meanwhile, it assumes other agents remain in their current kinematic states. The agent then

calculates the utility corresponding to each of the primitives it can muster. The agent then favors the primitive resulting in the maximum utility at that (time) step. Here, we assume the action duration is equivalent to that of the prediction ( $T_p = \Delta T$ ). The agent associates a future state with a utility prediction, assuming the following utility function:

$$U = -k_g \dot{d}_g - k_{dv} v^2 - k_{da} a^2 - \sum_{i=1}^n C_i \quad (1)$$

The velocity and acceleration are denoted by,  $v$  and  $a$ ;  $\dot{d}_g$  denotes the rate of change of the distance  $d_g$  to a goal, reflecting an agent’s desire to reach its destination. The agent, in this paper, engages only in longitudinal control, so  $\dot{d}_g$  is further simplified to  $-v$ . The agent assumes the velocity and the acceleration components as energy consumption in the vehicle agent and as discomfort in experiencing high speeds and rapid accelerations in the pedestrian agent. Such assumption is common in the models of minimizing cost in sensorimotor control [35], [36]. The agent assumes a collision discomfort term,  $C_i$ , when it is on a collision course with other agents or obstacles  $i$ . In this paper we only consider a single other agent, therefore,  $n = 1$ . If a motor primitive does not predict a collision course with others,  $C_i$  reduces to zero, whereas if it does predict a collision course, then for the speed-controlling pedestrian agent,  $C_i$  becomes  $k_c \frac{1}{\tau_i}$ , where  $\tau_i$  is the time left to collision from a predicted state. A collision course is defined as that the two agents’ projected future paths come within a tolerance distance  $d_c$ . This is to assume agents maintain their state. These assumptions are based on the literature examining the role of the inverse time to collision in locomotion [37], [38]. The acceleration-control vehicle agent assumes this term as  $C_i = k_{sc} a_{sc,i}^2$ , where  $a_{sc,i} = \frac{v^2}{2d_i} = \frac{v}{2\tau_i}$ , the acceleration needed to stop before reaching the collision point a distance  $d_i$  away [39]. Theoretically, by partial differentiating (1) regarding to  $v$  and assuming  $C_i = 0$ , the maximum utility is achieved when agents are traveling at their theoretical free speed  $v_{free} = \frac{k_g}{2k_{dv}}$ . In summary, our utility function expresses the agents’ desire to progress toward their goals with minimal discomfort from excessive kinematics and collision threats, as expressed by the utility function parameters,  $k_g, k_{dv}, k_{da}$ , and  $k_c$  or  $k_{sc}$ . The parameters can be optimized to fit individuals with different characteristics in a wide range of situations. Therefore, a change in any of the parameters tunes an agent’s specific characteristics for speed preference, acceleration tendency, and collision avoidance; for example an increase in  $k_g$  increase the agent’s relative preference for making quick progress.

#### 3) DETERMINISTIC ACTION DECISIONS

An action decision to apply a motor primitive is only taken when its utility gains a positive margin over the others. In every time step, a favored action is derived by sweeping through all motor primitives,  $m = 0, 1, \dots, n$ , and estimating its utility,  $U_m$ , as described above. This sweep includes  $U_0$ , the utility of not applying any motor primitive. If one or more motor primitives  $m \neq 0$  is identified with

$\Delta U_m \equiv U_m - U_0 > 0$ , the motor primitive with the highest  $\Delta U_m$  is selected, and the corresponding action is then initiated.

#### 4) STOCHASTIC ACTION DECISIONS

An important feature of the sensorimotor control framework on which we base our model [23] is that it assumes that sensorimotor decisions are made not deterministically as described above, but by accumulation of noisy sensory evidence to a decision threshold, in line with extensive research in decision theory [40]. This assumption has proven crucial for describing full probability distributions of driver behavior in both routine and near-crash situations [23], [24], [25], [26], and has also been recently adopted in models of pedestrian and driver road-crossing decisions [27], [28], [29], [30]. We incorporate evidence accumulation into our framework by generalizing the deterministic decision variable above to the stochastically accumulated decision variable. This generalization thus enables the probabilistic human errors that might happen in near-crash situations [30].

$$\hat{\Delta U}_m(k) = \hat{\Delta U}_m(k-1) + \frac{d\hat{\Delta U}_m(k)}{dt} \Delta t + \sigma \epsilon(k) \sqrt{\Delta t} \quad (2)$$

where  $\hat{\Delta U}(k)$  is now the accumulated evidence in favor of motor primitive  $m$  at time step  $k$  of a simulation with time step length  $\Delta t$ , and where  $\sigma \epsilon(k)$  is Gaussian noise with standard deviation  $\sigma$ . The agent gathers evidence up to a threshold,  $E_a$ . When the accumulated evidence surpasses the threshold, namely  $\hat{\Delta U}_m > E_a$ , the motor primitive  $m$  is triggered, and all  $\hat{\Delta U}_m$  are reset to zero. The rate of change of the evidence could be implemented several different ways; here we have opted for a formulation which makes  $\Delta U$  interpretable as a low-pass filtered version of a noisy  $\Delta U_m$ :

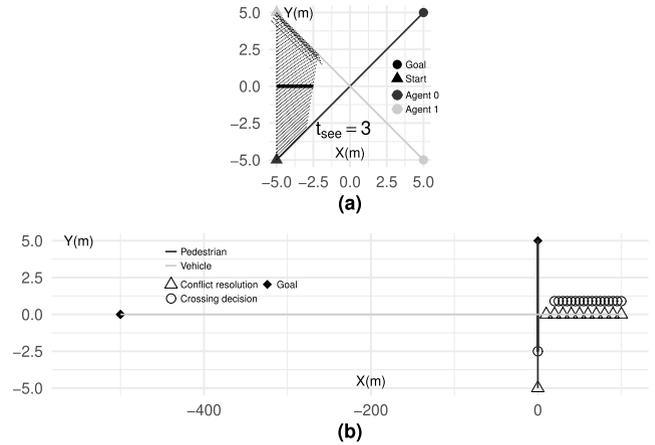
$$\frac{d\hat{\Delta U}_m(k)}{dt} = \frac{1}{T} [\Delta U_m(k) - \hat{\Delta U}_m(k-1)] \quad (3)$$

where  $T$  is a scaling parameter on the rate of evidence growth. Thus, we transform the idea of evidence accumulation, proposed for instance in [25], to not only enable the agent to travel but also motivate it to maximize the utility.

In the deterministic version of the model, the exact utility values  $U_m$  of individual motor primitives are not very important; only their relative magnitude compared to each other. However, in the stochastic formulation of the model, the exact values of  $U_m$  clearly affect the model dynamics, and in particular the  $C_i$  term can cause arbitrarily large negative  $U_m$ . For this reason, the utility values are squashed to the  $[-1, 1]$  range before being used in (2) and (3), by applying an arctan function.

#### B. INTERACTIVE CONFLICTS AND MANIFESTED BEHAVIORS

In this section, we first divide the interactions into two categories, pedestrian-pedestrian and pedestrian-vehicle interactions. In each category, we describe the assumptions that facilitate the investigation of each interactive conflict, and



**FIGURE 2.** Two interactive scenarios, depicted from a top-view perspective. In panel (a), the dotted lines show from the line-of-sight of Agent 0 to Agent 1.  $t_{see}$  represents the time when the agents first see each other behind the dividing curtain (the horizontal thick black line). Panel (b) shows the paths of a pedestrian and a vehicle. The triangle and circle represent different starting positions in the two scenarios, conflict resolution and crossing decision. The open symbols show different starting positions. The solid diamonds show the agent's destinations. We avoid clutter by drawing the starting positions in Crossing Decision scenario off the axes.

the modeling set-ups that enable the examination of the parameter regions generating the naturally-observed behaviors. Second, we operationalize the behavior generated by the model simulations, based on the empirical behaviors reported in literature. Third, we describe the experimental designs, assumptions and metrics used in the previous studies [28], [37], [41], [42], and the model set-ups used to reproduce the qualitative patterns observed in these studies.

##### 1) PEDESTRIAN-PEDESTRIAN CONFLICT

When two pedestrians are crossing each other's paths, they adapt to avoid collision. This was studied in a pair of publications by Olivier et al. [41], [43], where each of the two human participants walked along crossing diagonal lines in a square-shaped room in which a curtain initially occluded them from seeing each other. We illustrated their experimental setting in the upper panel in Figure 2. A main finding was that the pedestrian who had the initial lead, i.e., who was closer in time to the prospective collision zone at the time  $t_{see}$ , tended to speed up and pass the midpoint first, and the other participant contributed to the collision resolution accordingly by decelerating to give way.  $t_{see}$  refers to the time when two pedestrians were first able to see each other. These behaviors imply that the participants negotiated the priority by observing the other's kinematic state, without verbal communication.

We assumed two homogeneous pedestrians by assigning them identical model parameters and symmetrical initial positions to simulate the pedestrian-conflict paradigm. To create asymmetric kinematics at  $t_{see}$ , we assigned the agents with different initial speeds. In this first scenario, we investigated the parameter regions of the discomfort of speeding up,  $k_{dv}$  and of colliding,  $k_c$ , with an aim to reproduce

the empirical observations [41], [43]. We fixed the parameter  $k_g$  reflecting the desire of reaching the destination at 1, without loss of generality since the utility function is in arbitrary units (all parameters can be scaled by an arbitrary constant without changing the relative utilities of different actions), and for simplicity, when not noted, we assumed agents are free of acceleration discomfort,  $k_{da} = 0$ . We then tested different combinations of  $k_{dv}$ , and  $k_c$ , with the former restricted to the range of 0.28 to 0.71 (chosen to produce pedestrian free walking speeds  $v_{free}$  between 0.7 and 2 m/s [44]) and the latter to the range of 0 to 10. We varied the agents' initial speeds from 0 to 0.9 m/s with a step size of 0.1 m/s. In total, for each model parameterization ( $k_{dv}$ ,  $k_c$ ) we tested 100 different pairs of initial speeds.

We operationalize the key observations in [43] and [41] using the following Boolean metrics: (1) Lead agent passed first (LAPF) is true, if the lead agent passes the crossing point earlier than the other agent. The lead agent is the one with the shortest time to the crossing point at  $t_{see}$ , assuming constant speeds; (2) First-passer accelerated (FPA) is true, if the lead agent increases its speed just after  $t_{see}$ , operationalized as a speed  $v(t_{see} + T_a) > v(t_{see})$ , with  $T_a = 0.5$  s; (3) Second-passer decelerated (SPD) is true, if the lag agent decreases its speed just after  $t_{see}$ .

We consider a model parameterization effectively reproducing the natural behaviors if the LAPF is true in more than 80%<sup>1</sup> among a set of 100 simulations of the different initial speeds and if the FPA and SPD are both true in more than 20%<sup>2</sup>. The exact values of these thresholds are not important; a slight change in the percentages, for instance changing LAPF to 75% or to 85% or changing FPA to 15%, does not alter the qualitative pattern of the result. The main purpose of using these criteria is to investigate regions in the parameter space that produce the qualitative patterns in the empirical data [31], [32], [33].

Moreover, we define three excluding metrics, to discard model parameterizations yielding behaviors that were in direct conflict with the natural behaviors: (1) First-passer decelerated (FPD) (2) Second-passer accelerated (SPA), and (3) Collision occurred (CO). The first two are defined analogously to FPA and SPD above and the latter one is true if the agents collided (distance below  $d_c = 1$  m). We consider the model parameterizations reproducing unnatural behaviors when any of the three metrics exceeds 5%.

In addition to the excluding metrics, we compare the resulting trajectories from our simulations quantitatively with the empirical observations in [41], by calculating the minimal predicted distance (MPD),  $d_{min}$ . The metric was defined in [41] as the smallest future distance at any point in time  $t$  between the agents, assuming that they will travel in constant speeds from  $t$  onward.

<sup>1</sup>100% is not to be expected, because the times to crossing at  $t_{see}$  will sometimes be nearly or completely identical.

<sup>2</sup>The FPA and SPD are expected to be considerably lower than 100%, because for many combinations of initial speed, most cases the lead agent was in a higher speed than that of the other agent.

## 2) PEDESTRIAN-VEHICLE CONFLICT

Comparing to the interaction in pedestrians, the interaction between pedestrians and vehicles often manifests a different set of behaviors, for instance, the lead agent might not pass first [42]. Importantly, the time gap influences these behaviors. Specifically, the time gap refers to the moment-to-moment difference in the time to the crossing point between the two agents. The accelerating agents, like vehicles, often asserted the lead role to pass earlier even when they were not in a lead position [42], [45] (see also Figure 2).

In the second part of the investigation in the road-user interactive conflicts, we again harness the modeling framework to examine two questions: (1) how the time gap affects crossing decisions (CD) and (2) how pedestrians and drivers reach conflict resolution (CR). We define the natural behavior of time-gap dependent CD as: When the time gaps are small, the pedestrian yields to the vehicle, whereas, when the time gaps are large, the pedestrian may cross in front of the vehicle. In between, the crossing decisions depend on pedestrians' evaluation of the vehicle kinematics. The parameter,  $k_c$  reflects such evaluations. CR, on the other hand, was observed as either road user noticeably altering their behavior either to slow down or to speed up [7], [17]. These CR actions are often read as gauging the uncertainty in decelerations, or as asserting priority in accelerations. Therefore, we define the natural behavior of CR in the pedestrian as yielding in the "encounter" (see the following for its definition) time gaps.

To examine these two sets of natural behaviors, we set up two scenarios to investigate how the time gaps associate with the model parameters and how they associate with the CD and CR behaviors. The first, crossing decision scenario places a vehicle at 17 different locations on the x axis, starting from 20 m to 100 m by a 5-m step. These starting positions result in 17 time gaps between 1.4 s and 7.2 s, when the vehicle starts at its free-flow speed at 13.9 m/s (about 50 km/h). Meanwhile, a pedestrian stands still on the negative side of the y axis at 2.5 m. The second, conflict resolution scenario places a free-speed vehicle at ten different starting positions, from 10 m to 100 m by a 10-m step on the x axis. The pedestrian in this scenario walks at a speed of 1.1 m/s, starting from the negative side of the y axis at 5 m away from the crossing point. One major difference between the two scenarios is the pedestrian starts at zero speed in the former and, in the latter starts at a walking speed.

These two scenarios focus on the behavior of collision aversion and thus examine how the model parameters,  $k_c$  in the pedestrian and  $k_{sc}$  in the vehicle jointly affect a pedestrian to make crossing decisions and to resolve collision conflicts. The two parameters were selected because how different road users – pedestrians, human drivers, and automated drivers – perceive and weight the risk of collision has significantly societal impact both on current and near-future traffic. Similarly to the approach in the pedestrian interaction, we systematically examine 100 evenly spaced  $k_c$  and  $k_{sc}$  values from 0 to 5, across the time gaps in the CD scenario. We reduce the upper limit to 2.5 in the CR scenario and

keep surveying 100 evenly spaced parameters, because in the deterministic case, the values above 2.5 did not change the result substantially.

After testing deterministic model in the CD scenario to compare it to pedestrian gap acceptance data [37], we carried on investigating whether the stochastic version of the model can reproduce the empirically observed distributions of pedestrian crossing initiation times [28], showing an association between the response time distribution and the time gaps. We reused data from the study [28], which invited 20 human participants to perform a road-crossing task in a virtual reality environment where a constant-speed vehicle drove towards the crossing zone. The study divided the data set into three groups based on the time gaps, 2.29, 4.58 and 6.87 s.

Similarly, the parameter survey in the CR scenario was followed by an investigation to see whether the utility-maximization model is capable of reproduce the CR data in a field study [42]. The drivers in this study were found to accelerate to assert their priority, even when they were in a lag position. Following [42], we define an “encounter” as a situation, which a pedestrian and a vehicle would reach the crossing point within 1 s of each other’s prediction time, assuming they maintain their speeds. The encounter time then includes both the cases when either the vehicle or the pedestrian is closer to the crossing point (i.e., lead position). We create 859 encounter time gaps in this scenario by placing a pedestrian agent on 60 evenly spaced positions between  $-10$  m to  $-5$  m on the y axis and a vehicle agent also on 60 evenly spaced positions between 10 m to 79 m. The two agents assumed their speeds at  $1.4$  m/s and  $13.9$  m/s similar to those reported in [42] and reacted to each other immediately after the simulation started.

### III. RESULT

#### A. PEDESTRIAN-PEDESTRIAN CONFLICT

Figure 3 summarizes the parameters producing the natural behavior that the pedestrian agents resolve interaction conflicts. These are the parameter meeting all six, exclusion and inclusion, criteria. These “accepted” parameters cover a non-trivial region, including two strips and some scattered areas. Specifically, when the values of  $k_c$  and  $k_{dv}$  are close to zero and when  $k_{dv}$  are large, the parameters do not produce natural behaviors. Figure 4 shows the result of individual criterion. Almost all surveyed parameters could reproduce LAFP irrespective of the initial speeds. The two associated behaviors, the FPA and the SPD, filtered out most parameters, when  $k_c$  values were near zero and when  $k_{dv}$  values were very large. The former reflects the pedestrian agent does not overlook collisions completely, and the latter reflects the pedestrian does not increase speed drastically. The contrasting behaviors, the FPD and the SPA, excluded the parameters, associated with atypical pedestrian behaviors. When  $k_{dv}$  values were small, the parameters resulted in collisions.

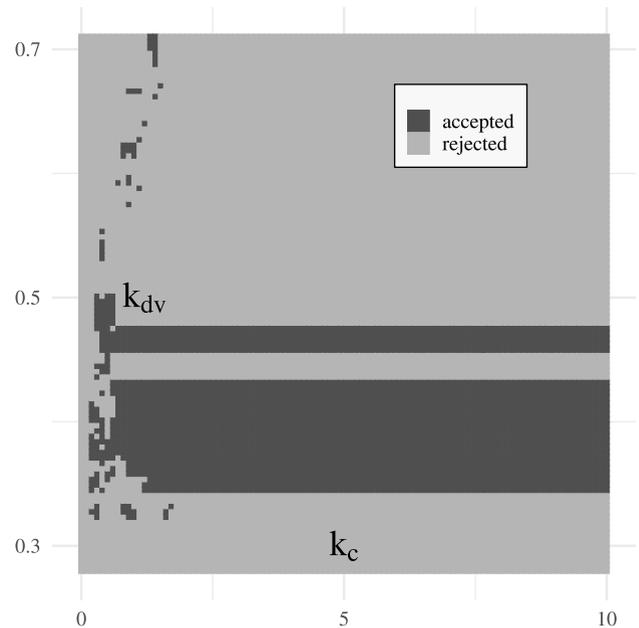


FIGURE 3. Parameters reproducing the behavior of leading agent passing first.

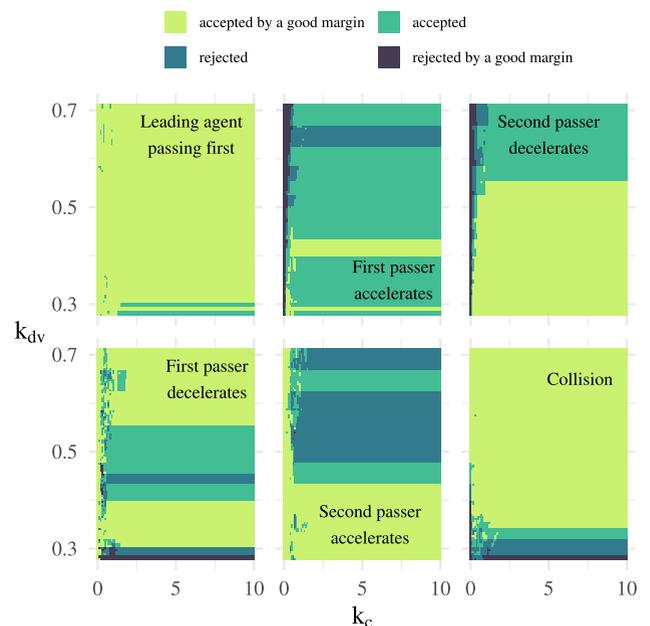
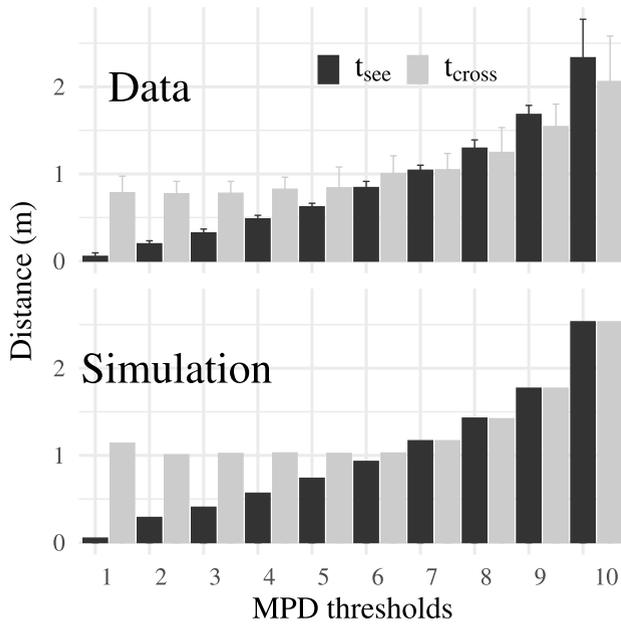


FIGURE 4. Individual criterion on the parameters reproducing the typical behavior of pedestrian interaction.

Figure 5 is the result of the model simulations compared to the empirical data, collected in a controlled experiment, using human participants [41], [43]. We compared the data with the trajectory simulations based on the accepted model parameters. First, we identified the 3072 accepted parameters shown in Figure 3 and used them to calculate the MPD between the agents at every time point, before they reached the intersection. Thereafter, following the same approach as in [41], these trajectories were then divided into ten groups, according to an ascending order of MPD at the time of  $t_{see}$ . The model simulations show that the simulated MPDs



**FIGURE 5.** Comparison of minimal predicted distances between the empirical data in [41] and the simulations, at both  $t_{see}$  and  $t_{cross}$ .

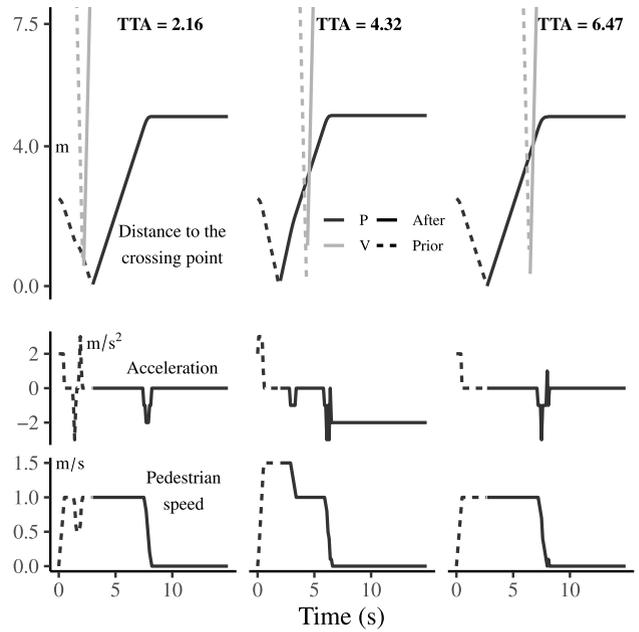
at the  $t_{cross}$  are in line with the empirically observed data, suggesting that the model resolves the collision conflicts in a similar way as the natural behaviors observed in humans participating in a controlled experiment.

**B. PEDESTRIAN-VEHICLE CONFLICT**

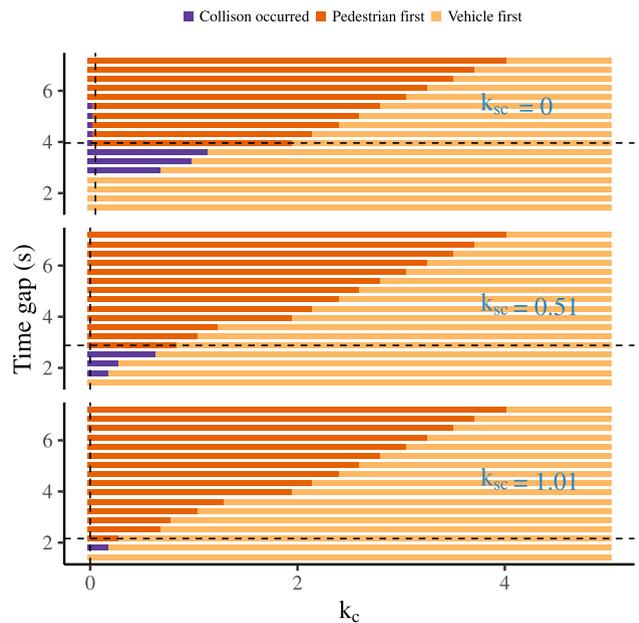
**1) CROSSING DECISION**

Figure 6 shows the evolution of speed, acceleration, and distance to the crossing zone in three simulation examples, where a pedestrian reacted to a constant-speed vehicle. When the time gap was 2.16 s, the pedestrian decelerated to yield. When the time gap was 4.32 s, the pedestrian passed the crossing earlier than the vehicle by stepping up the speed to 1.5 m/s. These simulations are in line with empirical observations of pedestrians accelerating when crossing in front of approaching vehicles [46]. When the time gap was 6.47 s, the pedestrian passed without accelerating drastically. In other words, the model naturally exhibits a number of typical features in the behaviors of pedestrian gap acceptance.

Figure 7 summarizes the results of the CD scenario. The top panel shows the results for simulations with  $k_{sc} = 0$ , i.e., with a constant-speed vehicle. As long as the pedestrian model had a minimal degree of collision aversion ( $k_c > 1$ ), it was able to cross without collision, with accepted time gaps starting from 3.96 s. This is in line with Experiment 2 in [37], where the participants in the age groups of 20-30 and 60-70 made crossing decision between 3 to 4 s. It can be noted that increasing  $k_c$  creates increasingly cautious pedestrians, requiring larger time gaps to cross. The middle and lower panel of Figure 7 show results with a vehicle who is responsive to the pedestrian,  $k_{sc} > 0$ . The shortest time gap safely accepted by the pedestrian decreased to 2.88 s when the vehicle was moderately cautious ( $k_{sc} = 0.51$ ) and



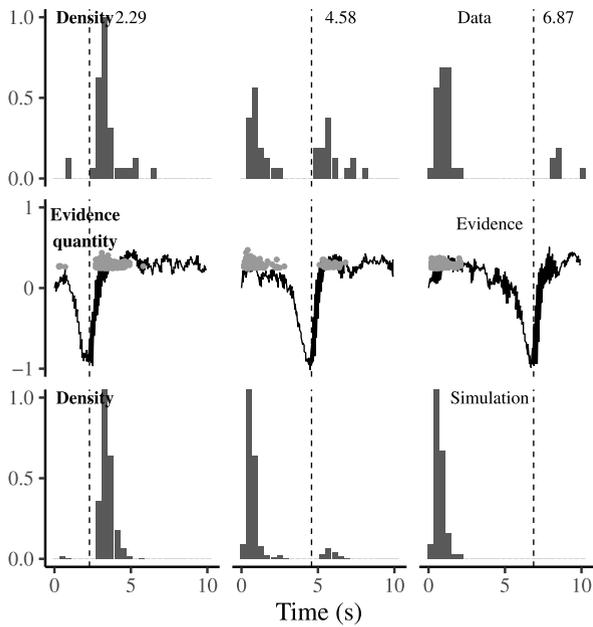
**FIGURE 6.** Behavior of crossing decision in three time gaps. The parameters were extracted from the top panel in fig 6 where no collision occurred. The dashed and solid lines show, respectively, the observations prior the crossing and those after.



**FIGURE 7.** Association between the time gaps and the collision aversion in the pedestrian and the vehicle agent. The crosshairs highlight the time gaps described in text.

decreased further to 2.16 s when the vehicle was very cautious ( $k_{sc} = 1.01$ ).

Next, we tested whether applying the utility maximization in the decision process results in a model that is capable of mimicking empirical data. That is, we tested the ability of the stochastic version of the model to reproduce full probability distributions of crossing response times of human pedestrians for different time gaps. We chose the parameters,  $k_g = 1$ ,  $k_c = 1$ ,  $k_{dv} = 0.38$ , based on the parameter survey in

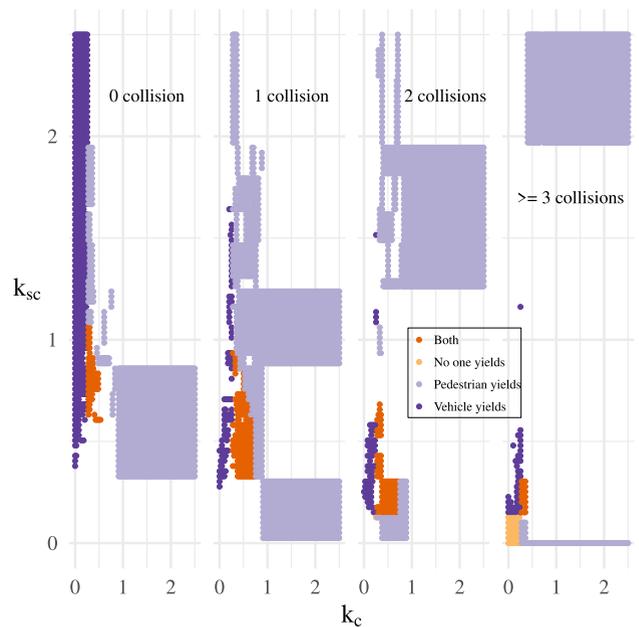


**FIGURE 8.** Model simulations of stochastic road-crossing decisions. The time gaps are highlighted by the dashed lines. The black traces in the middle row are the evolution of evidence values, and the grey dots are the resultant responses. The upper and lower rows show the density histogram of the response times.

deterministic simulations and applied (2) and (3) to generate stochastic simulations of pedestrian road-crossing, for the same road crossing scenarios as in the empirical study in [28], and applied some limited manual tuning to the stochastic model assumptions setting  $T = 2$ ,  $E_a = 0.25$ . As can be seen in Figure 8, when the time gap was 2.29 s, the empirical data showed the participants mostly crossed after the vehicle had passed, with some rare responses before the vehicle, and the same was true for the model. Secondly, when the time gap was long, namely 6.87 s, the data showed most human crossing responses occurred before the vehicle passed, and the model also reproduced this pattern. Third, when the time gap was in between (4.58 s), the data showed a balance of pre- and post-vehicle pass responses. The model in this case predicted a small proportion of post-vehicle responses. For all three time gaps, the model was able to capture the shape and scale of the distribution of crossing responses. In summary, the model has shown a good capacity to account for both the field- and laboratory-observed data in crossing decisions.

## 2) CONFLICT RESOLUTION

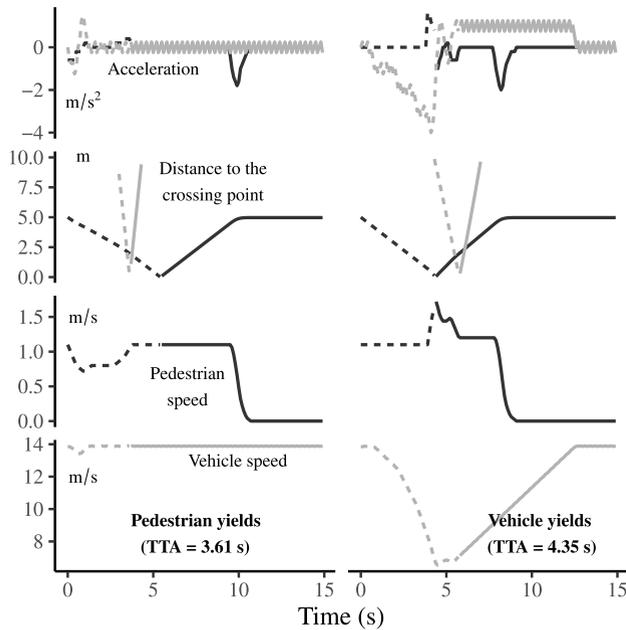
Figure 9 summarizes the results for the CR scenario, surveying the region of the two collision-discomfort parameters,  $k_{sc}$  and  $k_c$ . We investigated ten time gaps and divided the surveyed region by the frequency of collisions. The low-left corners in the fourth column shows neither agent yielded, leading to frequent collisions. These are the parameters enabling the agents to be more willing to take risks (i.e.,  $k_c < 0.28$  and  $k_{sc} < 0.13$ ). Of course, a risk insensitive road user does not always run into other, for example cautious, road users. For example, in the first column, when the pedestrian was



**FIGURE 9.** Parameter survey of conflict resolution. The number of collisions was tallied in the ten tested time gaps.

prone to risk-taking ( $k_c < 0.28$ ), but the vehicle was rendered to be risk-averse, ( $k_{sc}$  larger than 0.5), the vehicle yielded. Overall, the pedestrian and vehicle agents yielded when their collision-discomfort parameters,  $k_c$  and  $k_{sc}$ , are greater than 0.28 and 0.38. Interestingly, there were model parameterizations for which both pedestrian and vehicle yielding was observed (shown in orange in Figure 9), depending on the time gap. Figure 10 shows two time-series simulations for one such chosen pair of  $k_c$  and  $k_{sc}$  parameters, which result in collision-free interaction for all investigated time gaps (the orange area of the leftmost panel in Figure 9). The model is able to exhibit yielding both by the pedestrian (for the shorter TTA) and by the vehicle (for the longer TTA).

These results show that the model is generally capable of reproducing conflict resolution that is human-like at least in a qualitative sense. One empirically observed behavioral phenomenon that the model did not replicate in these tests, however, was priority assertion by vehicle drivers, who have been observed to increase speed to pass in front of pedestrians, even in situations where the pedestrian has the kinematic lead [42]. As mentioned in the Methods, we also ran a number of encounter scenarios specifically targeting this phenomenon. We found that in order to reproduce this phenomenon, we needed to reduce the  $k_{dv}$  parameter of the driver agent from the initial value of 0.45 to 0.018, thus effectively raising its free speed from 13.9 m/s to 27.8 m/s. All other parameters were unchanged. This implies that the model assumes that an assertive driver will drive as fast as 27.8 m/s when no other vehicles and obstacles are present, effectively defining the characteristic of aggressive as someone who tends to drive faster when no others are present. With this parameterization, we observed vehicle acceleration in the encounter scenarios to assert its priority.



**FIGURE 10.** Two examples of deceleration to yield. Dashed line showed the period before the intersection. TTA stands for the time to arrival. The parameters in the examples were for the pedestrian,  $k_c = 0.61$ , for the vehicle,  $k_{sc} = 0.64$ .

The CR simulation thus reproduced drivers' assertive behavior. Amongst the 859 possible time gaps of encounter, the vehicle agent accelerated in 616 (i.e., about 72%) occasions. The vehicle started from a lag position in 331 of the 859 time gaps, within which 286 occasions, the vehicle agent accelerated. This result is in line with what the field study observed, where 73% of the drivers maintained the same speed or accelerated [42].

To sum up, by tuning different combinations of parameters, the model reproduced natural behaviors both in pedestrian-pedestrian conflict and in pedestrian-vehicle conflict, revealing several interesting questions regarding the subtle interactions in road users. The model predictions derived from the parameter-space investigation enable one to explain and to examine further questions, for instance, in what situations a pedestrian may yield to the other pedestrian and in what situations, a driver with the characteristic of driving faster than a regular free speed, may assert his / her position even not in a lead position.

#### IV. DISCUSSION

We developed a modeling framework by applying the principle of maximizing one's psychological utility in order to understand interactions among road users in traffic. By applying the three principles – motor primitives, utility maximization, and intermittent action decisions – our model enables an expandable framework that helps to examine several questions in road user interactions. For example, a plausible hypothesis is that individuals assess collision risk differently and data from different people will result in different optimized  $k_c$  parameters. In its current form, the model has

successfully reproduced different scenarios of traffic interactions, where road users resolve interactive conflicts and make critical decisions. Computational models that capture these subtleties are needed to help achieve safe and acceptable vehicle automation [47], [48]. Previous work focused on accounting for the perceptual and attentional aspects in processing of sensory information with the traffic tasks in mind [48], [49]. These models resulted in quantitative descriptions in the traffic task performance but depend on identifying many perceptual quantities in different scenarios. Complementary to describing perceptual quantities in traffic tasks, in this paper, we applied the principle of utility maximization to not only account for deterministic and stochastic interactions in road-crossing decisions, but also organize the descriptions of perceptual quantities in a general framework by associating them with a linear utility function. The approach mitigates the problem of non-linearly growing model complexity and makes a conceptual shift from just quantifying perceptual quantities to maximizing utility.

Specifically, the proposed model is a useful tool to help to examine and understand how subtle details in road user interactions affect individual traffic decisions. We started from the existing framework for modeling sustained sensorimotor control [23], previously applied to modeling of non-interactive control behavior of single drivers. Then we generalized it to model road user interactions, including both driver and pedestrian agents. The rationale for building on this specific framework is that it has been shown capable of accounting for not only routine driving control, but also near-crash behavior, including probability distributions of open-loop defensive reactions, which are enabled by the evidence accumulation and motor primitive assumptions of the framework [23], [24], [25], [26] – notably the same assumptions used here allowed us to model entire probability distributions of pedestrian crossing onset (Figure 8). An important avenue for future work will be to leverage these capabilities of the framework to study and to model near-crash behavior in multi-agent interactions, involving road users with different, such as cyclists, kinematic profiles.

One requirement for doing so is to expand further the framework to model agent beliefs about the intentions of other agents, since this is an important cause of safety-critical interaction failures [50], [51]. Such an extension may also improve the model's account of non-critical interactions, because both empirical observation [2], [8], [30], and everyday experience suggests that belief about others' intentions plays an important role in the nuances in road user interactions. For example, a utility function in a vehicle agent can factor in a driver's belief in a pedestrian's crossing intention, and another utility function in the pedestrian agent can also factor in the pedestrian intention and how it is affected by the driver's actions. This may help to explain why some drivers tend to engage in priority-asserting acceleration. The hypothesis would be that drivers believe that an increase in vehicle speed decreases the probability of pedestrians to cross in front of the car. One example to model this hypothesis has been

framed in game-theoretic models, with beliefs not only about others' intentions but also about how those intentions are affected by one's own actions [52]. For some first indications of how beliefs can be incorporated in the present framework, see [26], [30].

Here, we started by testing model candidates that generate simulations close to empirical behaviors and studied parameter space to understand the ability of the model to reproduce the behavior in question between two pedestrians and between a pedestrian and a vehicle. An immediate next step is to study the concept of utility maximization in the interactions between drivers, where we found, using the neural network of convolutional social pooling, the drivers in the lead position stepped up speed to assert their way when driving on a merging lane [21]. Although it remains an open question as to whether the UM principle can be applied to naturalistic data, we believe the detailed behavior of driver interactions could also be accounted for by applying the principle. An important avenue for future work will be to perform more complete tests of the model against datasets from both controlled and naturalistic studies of road user interaction.

Although, in this paper, we focused only on the influence of time gap on crossing decisions, there are also other variables that might play critical roles in road users' crossing decisions. These include the distance gap, the perception of yielding deceleration by the vehicle, the size of the approaching agent and explicit signals of communication [30]. The capacity of the model can be further extended by adding these situation variables and assuming their corresponding model parameters. Next, we sketch the plan to investigate these variables and how the UM model may help to understand their roles in traffic interactions.

Despite the time gap, distance, and the speeds are bounded by classical mechanics, they are not always processed in strict physics principle by human road users. For example, to make a left turn maneuver, a driver seemed to rely more on the perceived distance to an opposing vehicle than on the perceived speed and the time gap [53]. In a similar example, drivers of over 56 years old accepted the same distance gap, irrespective of whether an approaching vehicle was at different speeds [54]. These findings imply that the perceived distance and decisions thereof depend not only on the physical but also on the psychological factors. Second, whether an observation of deceleration is read as a yielding sign also affects the crossing decisions. A study, analyzing a series of traffic videos, showed the clear sign of a vehicle to decelerate associated with an increase in the probabilities of pedestrians' crossing decision [17]. Third, smaller vehicles, for instance cars versus motorcycles, are perceived as approaching later. This cognitive illusion, dubbed size-arrival effect, resulted in an increase in the risk of dangerous crossing decision in pedestrians when a mixture of different vehicles are in traffic [55]. A similar observation was also reported, comparing trucks with motorcycles, even when both arrived at the same time [56]. These variables can be also tested in the UM model, which will help to discern their associations

with crossing decisions. A further step could be to examine how these many traffic-related variables contribute to human road-crossing decisions when, for instance, machine-driven vehicles are on the road. We expect the knowledge will help to design a safer environment when human and autonomous vehicles coexist in future traffic.

Another key problem in bridging psychological decision theory to traffic decision is that the time resolution typically applied in traffic simulation is vastly different from the time resolution typically used in the decision theory, such as the diffusion decision model [40], which was built upon here for the stochastic model formulation. To predict meaningful traffic flow, 0.1 s is the usual lower bound on simulation time step; however, in the decision theory, the decision process in humans is often examined in the time scale of 1 ms. This difference in time resolution makes bridging the traveling path of an agent to its decision point challenging. At this point, our model errs on the side of coarse time resolution to enable the agent to travel with natural progress as in real traffic. However, in some real critical incidents, a fatal decision happens in the range of tens to hundreds of milliseconds, as observed in many human decision-making tasks. We hope the next development of the UM model could enable such dynamic changes in time resolutions as in what might happen in real life traffic scenarios.

## V. CONCLUSION

We extend an existing framework for modeling sensorimotor control, incorporating the principle of utility maximization. This effort results in a general modeling framework that not only enables examination of detailed road user interactions in different scenarios, but which is also based on model assumptions that have been previously shown to explain road user behavior across both routine and safety-critical situations. One advantage of the utility maximization formulation, compared to previous mechanistic models emphasizing perceptual cues, is that the model is more easily expandable to new scenarios, without the need to identify the exact perceptual cues being used by road users. At this stage, we keep the model in a simple form to investigate the two-agent interaction. We have shown that the model can reproduce several empirically observed phenomena in pedestrian-pedestrian and driver-pedestrian interactions. Important next steps, toward generalizing the model and making it useful in automated vehicle applications, include testing the model in safety-critical scenarios (where our mechanistic approach has some potential advantages over machine-learned models), and investigating what types of model extensions (e.g., beliefs about other road users' intentions, more advanced utility functions, and so on) are needed to cover other interaction phenomena.

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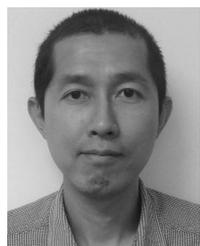
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