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A Generalized Closed-From Solution for Wide-Area Fault Location by Characterizing the Distributions of Superimposed Errors

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***Abstract*—Wide-area fault location (WAFL) refers to the estimation of fault distance on the faulted line using PMU data. A system of linear equations is formulated for WAFL, taking advantage of both voltage and current synchrophasors. This results in a generalized closed-form solution for the fault distance using the weighted least-squares method. The main contribution of the letter is the rigorous derivation of the equation weights based on the statistical distributions of the superimposed errors, i.e. the differences between the errors of the corresponding pre- and post-fault synchrophasors. The method's effectiveness, robustness against different factors, and superiority over existing methods are demonstrated by extensive simulations and comparison studies conducted on the IEEE 39-bus test system.**

***Index Terms*— Wide-area fault location (WAFL), measurement error, superimposed circuit, synchrophasors.**

I. Introduction

ACCURATE fault location (FL) reduces the outage time and enhances power system reliability. In this context, the proliferation of PMUs has paved the way for wide-area fault location (WAFL). Most existing WAFL methods suffer from technical difficulties associated with nonlinear formulations and iterative solving processes, such as divergence and multiplicity of solutions. Some of these methods even place constraints on the PMU locations. In practice, however, budget limits and the availability of communication infrastructure are the key factors determining PMU locations [1].

The WAFL methods proposed in [1] and [2] are linear but can only utilize voltage synchrophasors. Nevertheless, voltage transformers located far from the FL are likely to experience small voltage variations following a fault. Hence, the resulting superimposed synchrophasors could be of the order of noise and phasor estimation error. Thus, the relative errors of voltage-related equations might be exceedingly high, making them counterproductive. In contrast, the amounts of currents flowing through transmission lines greatly increase in fault conditions [3]. It follows that the relative errors of superimposed current synchrophasors are, in general, smaller than those of voltage synchrophasors. On the other hand, one can derive only a single equation for the voltage synchrophasor at a substation, while several equations can typically be formed based on current synchrophasors (for the multiplicity of the lines connected to each substation). The inclusion of current measurements has great potential to significantly improve the FL accuracy by adding a greater number of equations in the equation set [4].

In this letter, both voltage and current synchrophasors are incorporated into the WAFL formulation while maintaining its linearity. The weighted least-squares (WLS) method is used to solve the system of linear equations to provide a closed-form solution for the fault distance. As the main contribution of the letter, the derivations of the mean and variance of superimposed errors enable a rigorous establishment of the weight matrix required for the WLS estimation. To achieve this, the statistical distributions of the magnitude and phase-angle errors of pre- and post-fault synchrophasors are taken into account.

In principle, CTs located farther from the fault location experience smaller current variations upon a fault. Hence, the accuracy of current measurements taken farther away from the fault location would be hardly affected by saturation [5]. The linearity of the formulation, along with the derivations of superimposed errors, allows for the application of well-established bad data detection and identification methods to deal with erroneous measurements, e.g. current measurements of saturated CTs during close-in faults. Extensive simulations confirm that the FL accuracy is considerably improved by incorporating current synchrophasors into the formulation and taking account of the distributions of superimposed errors.

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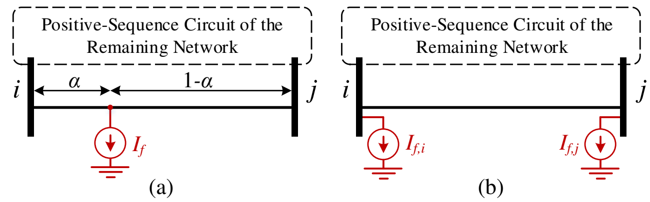

Fig. 1. (a) Superimposed positive-sequence circuit of the faulted power system with one current source, and (b) its equivalent circuit with two current sources.

II. Generalized Expressions for Fault Location

Fig. 1(a) shows the superimposed positive-sequence circuit of a network with a fault on line *i-j* at the distance α from bus *i*. A nodal current injection is used to represent the fault current in this superimposed circuit. As shown in Fig. 2(b), the current source I_f can be resolved into two nodal current injections placed at the faulted line terminals, i.e. $I_{f,i}$ and $I_{f,j}$, where [2]

$$I_{f,i} = \frac{\sinh(l_{ij}\gamma_{ij}(1-\alpha))}{\sinh(l_{ij}\gamma_{ij}\alpha)} I_{f,j} \tag{1}$$

in which l_{ij} and γ_{ij} are the length and propagation constant of the faulted line, respectively. As per the circuit of Fig. 1(b), the superimposed voltage at an arbitrary bus u is obtained from

$$\Delta V_u = Z_{ui} I_{f,i} + Z_{uj} I_{f,j} \tag{2}$$

where Z_{ui} is the (u,i)th entry of the bus impedance matrix.

To take advantage of the information provided by current measurements, they are also incorporated into the formulation. Let ΔJ_{uw} denote the sending-end superimposed current of line *u-w*, which satisfies the following equation:

$$\Delta J_{uw} = C_{uw,i} I_{f,i} + C_{uw,j} I_{f,j} \tag{3}$$

where $C_{uw,k}$ for a nodal current source at bus k is calculated as

$$C_{uw,k} = \frac{z_{u,k}}{Z_{uw}^{c}\tanh(l_{uw}\gamma_{uw})} - \frac{z_{w,k}}{Z_{uw}^{c}\sinh(l_{uw}\gamma_{uw})} \quad (4)$$

where Z_{uw}^{c} denotes the characteristic impedance of line u-w.

Writing (2) and (3) for the synchrophasors provided by PMUs, one can form the following system of equations:

$$\boldsymbol{m} = [\Delta J_1 \cdots \Delta J_L \ \Delta V_1 \cdots \Delta V_N]^T = \boldsymbol{H}\left[I_{f,i} \ \ I_{f,j}\right]^T + \boldsymbol{\varepsilon} \quad (5)$$

where indices 1 to L and 1 to N refer to the PMU-measured superimposed currents and voltages, respectively. Moreover, $\boldsymbol{m}$, $\boldsymbol{H}$, and $\boldsymbol{\varepsilon}$ are the measurement vector, coefficient matrix, and vector of measurement errors, respectively. System of equations (5) has two unknowns and thus will be uniquely solvable if it involves two independent equations [1]. Hence, it is almost impossible to be underdetermined in practice because each PMU typically provides several independent equations. The unknowns can be estimated using the WLS method as

$$\left[\hat{I}_{f,i}\,,\hat{I}_{f,j}\right]^T = (\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{H})^{-1}\boldsymbol{H}^*\boldsymbol{W}\boldsymbol{m} \quad (6)$$

where the asterisk refers to the conjugate transpose of the matrix and $\boldsymbol{W}$ is the weight matrix. Let β denote the ratio between $\hat{I}_{f,i}$ and $\hat{I}_{f,j}$ obtained from (6). With proper mathematical manipulations, a generalized closed-form solution for the fault distance can be derived from (1) as below [2]

$$\alpha = \frac{1}{2l_{ij}\gamma_{ij}}\ln\left[\frac{e^{l_{ij}\gamma_{ij}}+\beta}{e^{-l_{ij}\gamma_{ij}}+\beta}\right] \quad (7)$$

Fault location is carried out offline, and the faulted line may or may not be known to the process [2]. If not, one can easily use the residual-based technique proposed in [1] and [3] to identify the faulted line.

III. Weight Matrix and Superimposed Errors

Due to measurement errors, there may be more confidence in some synchrophasors than others. WLS is a generalization of ordinary least-squares (OLS) in which measurement errors are incorporated into the estimation to achieve the best linear unbiased prediction of unknowns [4]. If the measurements are independent, the weight of each measurement will be set equal to the reciprocal of its variance [4]. In the context of the proposed formulation, the term superimposed error refers to the error of a superimposed synchrophasor, which is the difference between the random errors of the corresponding post- and pre-fault synchrophasors. Thus, the superimposed error is a function of the error of the corresponding pre- and post-fault synchrophasors. A salient contribution of this letter is formulating the mean and variance of the superimposed error w.r.t the statistical distributions of the errors of pre- and post-fault synchrophasors.

As verified in [6], synchrophasors have independent distributions of errors in magnitude and phase-angle. Pre- and post-fault synchrophasors, in general, can have different distributions of errors. Let $y = re^{j\theta}$ and $y' = r'e^{j\theta'}$ denote random variables (RVs) representing the pre- and post-fault fundamental-frequency synchrophasors of a variable. Similar to many other studies, errors are assumed to have normal distributions [2]. The errors in the magnitudes and phase-angles of y and y' can be defined by four real-valued independent

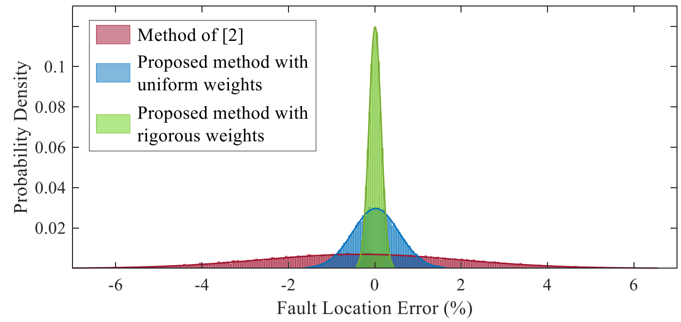

Fig. 2. Distribution of the fault location error using different methods.

Gaussian random variables as $\varepsilon_{r'} \sim \mathcal{N}(0,\sigma_{r'}^2)$, $\varepsilon_{\theta'} \sim \mathcal{N}(0,\sigma_{\theta'}^2)$, $\varepsilon_{r} \sim \mathcal{N}(0,\sigma_{r}^2)$, and $\varepsilon_{\theta} \sim \mathcal{N}(0,\sigma_{\theta}^2)$, where σ denotes the standard deviation of errors. Let $y_m = r_m e^{j\theta_m}$ and $y'_m = r'_m e^{j\theta'_m}$ denote the measured samples of these RVs. Let us denote the expected value of the error of the superimposed synchrophasor $\Delta y_m = y'_m - y_m$, i.e. the mean of superimposed error, by μ_{ϵ_m}. As will be justified in Appendix, this can be obtained from

$$\mu_{\epsilon_m} = y'_m\left(e^{-\sigma_{\theta'}^2} - e^{-\sigma_{\theta'}^2/2}\right) - y_m\left(e^{-\sigma_{\theta}^2} - e^{-\sigma_{\theta}^2/2}\right) \quad (8)$$

It follows from (8) that although the mean of $\varepsilon_{r'}$, $\varepsilon_{\theta'}$, ε_r, and ε_θ are zero, the mean of the superimposed error can be non-zero. This means WLS estimation using superimposed synchrophasors will be biased [4], [7]. To counteract this, the means should be subtracted from the corresponding superimposed synchrophasors in (5). As will be demonstrated in Appendix, the variance of superimposed errors will be

$$\sigma_{\Delta y_m}^2 = \sigma_{\epsilon_m}^2 = {r'_m}^2\left(1 - e^{-\sigma_{\theta'}^2}\right) + \sigma_{r'}^2\left(2 - e^{-\sigma_{\theta'}^2}\right) + {r_m}^2\left(1 - e^{-\sigma_{\theta}^2}\right) + \sigma_{r}^2\left(2 - e^{-\sigma_{\theta}^2}\right) \quad (9)$$

The weight associated with each equation in (6) is equal to the reciprocal of the variance of its superimposed error. The knowledge of the variance of superimposed errors, given in (9), is also a prerequisite for effective bad data detection [4].

IV. Performance Evaluation

Extensive simulations are conducted on the IEEE 39-bus test system to evaluate the performance of the proposed method. The phasors of the time-domain waveforms generated in PowerFactory are estimated using a real PMU model [8]. PMUs are typically placed in power systems to provide network observability [9]. Thus, using the method of [9], 12 PMUs are placed at buses 3, 5, 8, 11, 14, 16, 19, 23, 25, 27, 29, and 39 to make the network fully observable. However, the method does not require full network observability. The performance with partial network observability and different numbers of PMUs will also be studied.

The errors of magnitudes and phase-angles of synchrophasors are assumed to have a variation range of ±1% with normal distribution. The variation ranges of errors are reported based on the three-sigma criterion [7]. In other words, the variation range of a normally distributed error with standard deviation σ is [-3σ,+3σ]. In principle, one phasor reported before and one after the fault onset would be sufficient to obtain the superimposed phasors [2]. In the simulations conducted, these are calculated at 60 ms following the fault onset.

TABLE I
FAULT LOCATION ERROR (%) BY DIFFERENT METHODS

Fault Location Method	Fault Distance			
	97.5%	95%	90%	80%
Conventional [10]	9.88	9.53	8.75	7.92
Proposed	0.43	0.41	0.39	0.38

TABLE II
WAFL RESULTS WITH DIFFERENT NUMBERS OF PMUS

Number of PMUs		12	9	6	12	9	6
Fault Case	FL Error (%)	Proposed Method			Method of [2]		
2-ph-g at 25% Line 26-28	Mean	0.04	0.06	0.09	0.07	0.08	0.12
	Std. Dev.	0.22	0.21	0.25	1.38	1.77	2.35
3-ph-g at 95% Line 10-13	Mean	0.38	0.44	0.64	1.04	1.56	2.45
	Std. Dev.	0.36	0.75	0.97	2.56	3.96	5.45

First, the method's performance is studied for an arbitrary 1-ph-g fault at 20% of line 3-18. To obtain solid results, the fault case is repeated 50,000 times. Fig. 2 shows the distribution of the FL error by different methods, i.e. that of [2] (which ignores current measurements), the proposed method with uniform weights (OLS), and the proposed method with rigorous weights (WLS). It can be seen that the standard deviation of FL errors by OLS is smaller than that by the method of [2]. This demonstrates that the inclusion of current synchrophasors, even without the knowledge of variances of superimposed errors, enhances the accuracy of FL. As expected, the FL accuracy is significantly improved when the WLS is minimized using the rigorous weights obtained in the previous section.

To demonstrate the method's capability in dealing with CT saturations and close-in faults, all 12 PMUs are connected to magnetic-core CTs/VTs with an accuracy class of 0.5. A solid 1-ph-g fault at different locations of line 7-8 is considered. For faults near bus 8, the CT feeding the PMU at this bus becomes saturated, resulting in an erroneous current measurement. Accordingly, the largest normalized residual test is used for bad data detection and elimination [4]. Table I compares the FL error by the proposed method and the well-known two-terminal method of [10]. As seen, the presence of extra data (redundant equations) in the proposed method enables it to reduce the impact of erroneous measurements to a great extent.

Now, the impact of the number of PMUs on the FL error is investigated. To this end, 20 random PMU placements leading to a solvable system of equations are considered for each certain number of PMUs. The simulation is repeated 10,000 times for each PMU placement. Table II reports the mean and standard deviation of the FL errors for faults on two arbitrary lines using various methods. As can be seen, the proposed method outperforms the method of [2], especially with fewer PMUs.

Here, a total of 2,000 fault cases of different types are simulated at different locations in the system. For each fault case, the fault resistance is varied between 0 Ω to 50 Ω in 10-Ω steps. Measurement errors are set to have normal distributions, and each fault case is repeated 10,000 times. The voltage and current phasors from the 12 PMUs [9] are contaminated with up to 8% errors before calculating the fault distance from (7). Fig. 3 shows the results obtained by the proposed method using uniform weights and the rigorous weights derived. As seen, the mean and standard deviation of the FL error are noticeably smaller using the rigorously established weight matrix. This

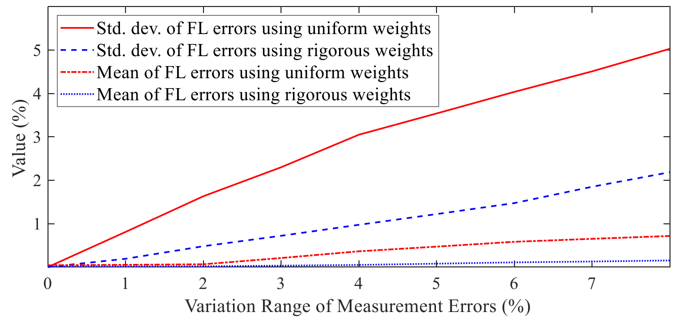

Fig. 3. Mean and standard deviation of the FL error by different methods.

TABLE III
SENSITIVITY TO THE PRESENCE OF PMU AT THE FAULTED LINE TERMINALS

Faulted Line Terminals with PMUs	Neither	One	Both
Mean of FL Errors (%)	0.10	0.09	0.06
Std. Dev. of FL Erros (%)	0.56	0.53	0.45

confirms that accounting for the distributions of superimposed errors can considerably improve WAFL accuracy.

Two independent equations are sufficient to solve (5) by (6). The method does not require any specific set of PMU data, such as PMUs at either or both terminals of the faulted line. To demonstrate the mentioned point, simulations are repeated for the lines with no PMUs at either terminal by adding PMUs at their terminals. In this study, the variation range of the errors is considered to be ±1%. The results are tabulated in Table III, showing that the proposed WAFL method can provide accurate results with and without PMUs at the faulted line terminals.

V. CONCLUSION

This letter proposes a generalized closed-form solution for wide-area fault location (WAFL) using sparse PMU measurements. To enable the application of the weighted least-squares method, the mean and variance of superimposed errors are rigorously calculated based on the statistical distributions of the errors of pre- and post-fault synchrophasors. Incorporating the established weight matrix into the formulation, as the main contribution of the letter, and taking advantage of both voltage and current synchrophasors make the results considerably more accurate than that of similar WAFL methods. The linearity of the formulation and the rigorous derivations of superimposed errors facilitate the application of well-established bad data detection methods. Extensive simulations conducted confirm the effectiveness of the proposed method and its robustness against different factors such as fault type and resistance.

APPENDIX: MATHEMATICAL PROOF FOR (8) AND (9)

Let μ_r, μ_θ, $\mu_{r'}$, and $\mu_{\theta'}$ denote the true values of the magnitudes and phase-angles of the pre- and post-fault synchrophasors. The RV for the corresponding superimposed synchrophasor can be expressed as

$$\Delta y = (\mu_{r'} + \varepsilon_{r'})e^{j(\mu_{\theta'}+\varepsilon_{\theta'})} - (\mu_r + \varepsilon_r)e^{j(\mu_\theta+\varepsilon_\theta)} \quad \text{(A1)}$$

Equation (A1) can be rewritten as

$$\Delta y = \mu_{r'} e^{j\mu_{\theta'}} - \mu_r e^{j\mu_\theta} + \epsilon \quad \text{(A2)}$$

where $\mu_{r'} e^{j\mu_{\theta'}} - \mu_r e^{j\mu_\theta}$ is the true value of Δy, and

$$\epsilon = \mu_{r'} e^{j\mu_{\theta'}} (e^{j\varepsilon_{\theta'}} - 1) + \varepsilon_{r'} e^{j(\mu_{\theta'}+\varepsilon_{\theta'})}$$

$$-\mu_r e^{j\mu_\theta}(e^{j\varepsilon_\theta}-1)-\varepsilon_r e^{j(\mu_\theta+\varepsilon_\theta)} \quad \text{(A3)}$$

It is well known that for two independent RVs, any function of one RV is independent of any function of the other RV. Since ε_r and ε_θ are independent,

$$\mathbb{E}\big(f(\varepsilon_r)\cdot g(\varepsilon_\theta)\big)=\mathbb{E}(f(\varepsilon_r))\,\mathbb{E}\big(g(\varepsilon_\theta)\big) \quad \text{(A4)}$$

As $\mathbb{E}(\varepsilon_r)=0$, the expected value of the multiplication of ε_r by any function of ε_θ is zero. Accordingly, the expected values of the second and the fourth terms in (A3) are zero. Moreover, for a Gaussian random variable, e.g. ε_θ, we have [7]

$$\mathbb{E}(e^{j\varepsilon_\theta})=\mathbb{E}(e^{-j\varepsilon_\theta})=e^{-\sigma_\theta^2/2} \quad \text{(A5)}$$

Using (A4) and (A5), the expected value of ϵ is obtained as

$$\mu_\epsilon=\mu_{r'}e^{j\mu_{\theta'}}(e^{-\sigma_{\theta'}^2/2}-1)-\mu_r e^{j\mu_\theta}(e^{-\sigma_\theta^2/2}-1) \quad \text{(A6)}$$

The variance of ϵ can be obtained by (A3) and (A6) as [7]

$$\sigma_\epsilon^2=\mathbb{E}(|\epsilon-\mu_\epsilon|^2)=\mathbb{E}([\epsilon-\mu_\epsilon][\epsilon-\mu_\epsilon]^*) \quad \text{(A7)}$$

Using (A4) and (A5) and some mathematical manipulations, one can obtain the variance of ϵ as below

$$\sigma_\epsilon^2=\mu_{r'}^2(1-e^{-\sigma_{\theta'}^2})+\sigma_{r'}^2+\mu_r^2(1-e^{-\sigma_\theta^2})+\sigma_r^2 \quad \text{(A8)}$$

Since the values of $\mu_{r'}$, $\mu_{\theta'}$, μ_r, and μ_θ are not available in practice, (A6) and (A8) cannot be directly utilized to calculate the mean and variance of the error of a superimposed phasor measurement. Therefore, the expected values of μ_ϵ and σ_ϵ^2 should be obtained conditioned on the measured values, i.e. $y_m=r_m e^{j\theta_m}$ and $y'_m=r'_m e^{j\theta'_m}$. We have $\mu_{r'}=r'_m-\varepsilon_{r'}$, $\mu_{\theta'}=\theta'_m-\varepsilon_{\theta'}$, $\mu_r=r_m-\varepsilon_A$, and $\mu_\theta=\theta_m-\varepsilon_\theta$. These equations are replaced in (A6) and (A8) to calculate the expected values of the resulting expressions. Using (A4) and (A5) and after some mathematical manipulations, the mean and the variance of the superimposed error associated with the superimposed synchrophasor $\Delta y_m=y'_m-y_m$, conditioned on the measured values, are obtained as (8) and (9), respectively.

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