Proceedings Paper:

Ni, P, Fragkoulis, VC, Kong, F et al. (2 more authors) (Accepted: 2022) Response determination of a nonlinear energy harvesting device under combined stochastic and deterministic loads. In: UNSPECIFIED 13th International Conference on Structural Safety & Reliability (ICOSSAR 2021-2022), 13-17 Sep 2022, Tongji University, Shanghai, China. (In Press)

ICOSSAR 2021
The 13th International Conference on Structural Safety and Reliability (ICOSSAR 2021-2022), June 20-24, 2022, Shanghai, P.R. China
J. Li, Pol D. Spanos, J.B. Chen & Y.B. Peng (Eds)

Response determination of a nonlinear energy harvesting device under combined stochastic and deterministic loads

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ABSTRACT: An approximate analytical technique for determining the response statistics of a nonlinear piezoelectric energy harvesting device is proposed. This is attained by resorting to a recently developed method for determining the response of multi-degree-of-freedom dynamical systems with singular matrices subject to combined deterministic and stochastic loads. Such systems are often met in engineering applications, for instance, as a result of modeling the governing equations of motion of complex multi-body systems by utilizing dependent coordinates. In this regard, the governing equations of the harvesting system dynamics are treated separately. Specifically, the harmonic balance method is used for treating the deterministic component of the response, while the corresponding stochastic response component is treated by combining the stochastic averaging and the statistical linearization methodologies. A numerical example is used to demonstrate the validity of the proposed technique. The obtained results are verified by using pertinent MCS data.

1 INTRODUCTION

In general, formulating the system governing equations of motion of engineering systems relies on the use of the minimum number of (generalized) coordinates (Roberts & Spanos 2003). This, in turn, results system parameter matrices with some appealing properties, such as positive definiteness and symmetry. However, for several classes of complex engineering systems and/or systems subject to constraint equations, it is often more efficient to derive the governing equations based on a dependent coordinates modeling, i.e., by considering additional degrees-of-freedom (DOF) (e.g., Udwadia & Kalaba 2001, Udwadia & Phohomsiri 2006, Schutte & Udwadia 2011). As a result the aforementioned appealing properties of the system parameter matrices do not apply anymore, since the latter are singular. Subsequently, this aspect necessitates the development of pertinent methodologies for conducting response analyses of such systems.

In this regard, considering the problem of multi-DOF linear and nonlinear systems with singular matrices, as well as with constraint equations, has led to the development of pertinent solution frameworks for determining the stochastic response of such system in time and frequency domains, as well as for conducting a joint time-frequency response analysis; see indicatively, Fragkoulis et al. 2016a; b, Kougioumtzoglou et al. 2017, Antoniou et al. 2017, Fragkoulis et al. 2015, Pantelous & Pirrotta 2017, Pirrotta et al. 2019, Pasparakis et al. 2021, Pirrotta et al. 2021, Karageorgos et al. 2021. This has been attained by resorting to the theory of generalized matrix inverses (Ben-Israel & Greville 2003), and particularly, by considering the concept of the Moore-Penrose inverse of a matrix.

In this paper, a recently proposed generalized matrix inverses-based framework for deriving the response of MDOF nonlinear systems with singular matrices subject to combined periodic and stochastic excitations (Ni et al. 2021) is used to compute in a direct way the stochastic response of a nonlinear piezoelectric energy harvesting device (Petromichelakis et al. 2018; 2021, Karageorgos et al. 2021). This is attained by considering the harmonic balance method for treating the periodic component of the response (e.g., Mickens 2010; Spanos et al. 2019; Kong et al. 2022) in conjunction with the statistical linearization methodology for systems with singular matrices for treating the corresponding stochastic response component (Fragkoulis et al. 2016b, Kougioumtzoglou et al. 2017). The obtained results are compared with pertinent Monte Carlo simulation data.

2 MATHEMATICAL FORMULATION

2.1 *Governing equations of motion*

The governing equations of motion of an l-DOF nonlinear system subjected to combined stochastic $\mathbf{Q_x}(t)$ and deterministic $\mathbf{f}_{d,\mathbf{x}}(t)$ excitations have the form (Fragkoulis et al. 2016b; Spanos et al. 2019)

$$\mathbf{M_x}\ddot{\mathbf{x}} + \mathbf{C_x}\dot{\mathbf{x}} + \mathbf{K_x}\mathbf{x} + \boldsymbol{\Phi}_\mathbf{x}(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}}) = \mathbf{f}_{d,\mathbf{x}}(t) + \mathbf{Q_x}(t). \quad (1)$$

In Eq. (1), $\mathbf{x}$ is an l dependent coordinates vector, $\mathbf{M_x}, \mathbf{C_x}$ and $\mathbf{K_x}$ denote the $l \times l$ system parameter matrices, whereas $\boldsymbol{\Phi}_\mathbf{x}(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}})$ corresponds to the l vector of the system nonlinearities. Next, the system of Eq. (1) is subject to additional constraint equations, which are written for simplicity in the form (Schutte & Udwadia 2011)

$$\mathbf{A}\ddot{\mathbf{x}} + \mathbf{E}\dot{\mathbf{x}} + \mathbf{L}\mathbf{x} = \mathbf{F}, \quad (2)$$

where $\mathbf{A}, \mathbf{E}, \mathbf{L}$ are $m \times l$ matrices and $\mathbf{F}$ is an l vector. In this regard, Eq. (1) is equivalently written as (Kougioumtzoglou et al. 2017)

$$\bar{\mathbf{M}}_\mathbf{x}\ddot{\mathbf{x}} + \bar{\mathbf{C}}_\mathbf{x}\dot{\mathbf{x}} + \bar{\mathbf{K}}_\mathbf{x}\mathbf{x} + \bar{\boldsymbol{\Phi}}_\mathbf{x}(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}}) = \bar{\mathbf{f}}_{d,\mathbf{x}}(t) + \bar{\mathbf{Q}}_\mathbf{x}(t), \quad (3)$$

where

$$\bar{\mathbf{M}}_\mathbf{x} = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\mathbf{M_x} \\ \mathbf{A} \end{bmatrix}, \quad (4)$$

$$\bar{\mathbf{C}}_\mathbf{x} = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\mathbf{C_x} \\ \mathbf{E} \end{bmatrix} \quad (5)$$

and

$$\bar{\mathbf{K}}_\mathbf{x} = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\mathbf{K_x} \\ \mathbf{L} \end{bmatrix}, \quad (6)$$

are the $(l+m) \times l$ parameter matrices of the system, whereas

$$\bar{\boldsymbol{\Phi}}_\mathbf{x} = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\boldsymbol{\Phi}_\mathbf{x} \\ \mathbf{0} \end{bmatrix} \quad (7)$$

and

$$\bar{\mathbf{Q}}_\mathbf{x}(t) = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\mathbf{Q_x}(t) \\ \mathbf{F} \end{bmatrix}, \quad (8)$$

$$\bar{\mathbf{f}}_{d,\mathbf{x}}(t) = \begin{bmatrix} (\mathbf{I}_l - \mathbf{A}^+\mathbf{A})\mathbf{f}_{d,\mathbf{x}}(t) \\ \mathbf{0} \end{bmatrix}, \quad (9)$$

are, respectively, the $(l+m)$ vectors of the system nonlinearities, as well as the stochastic and deterministic excitations. Also, $\mathbf{I}_l$ denotes the $l \times l$ identity matrix and "$+$" is used for the Moore-Penrose (M-P) matrix inverse operation. A detailed derivation of Eqs. (3-9) is found in Kougioumtzoglou et al. (2017).

2.2 *Determination of the system response*

Considering that $\bar{\mathbf{Q}}_\mathbf{x}(t)$ and $\bar{\mathbf{f}}_{d,\mathbf{x}}(t)$ in Eq. (3) correspond to the stochastic and deterministic excitations of the system, where the former is modeled as a zero-mean Gaussian process and the latter is modeled as a monochromatic function of period $T = \frac{2\pi}{\omega_d}$; i.e.,

$$\bar{\mathbf{f}}_{d,\mathbf{x}}(t) = \bar{\mathbf{f}}_{d_1,\mathbf{x}} \cos(\omega_d t) + \bar{\mathbf{f}}_{d_2,\mathbf{x}} \sin(\omega_d t), \quad (10)$$

where $\bar{\mathbf{f}}_{d_1,\mathbf{x}}$ and $\bar{\mathbf{f}}_{d_2,\mathbf{x}}$ are constants. It is assumed that the system response has also a stochastic and a periodic component. These are denoted by $\mathbf{x}_s(t)$ and $\mathbf{x}_d(t)$, respectively. Therefore, ensemble averaging Eq. (3), an expression consisting of a periodic and a stochastic component arises. This is given by

$$\bar{\mathbf{M}}_\mathbf{x}\ddot{\mathbf{x}}_d + \bar{\mathbf{C}}_\mathbf{x}\dot{\mathbf{x}}_d + \bar{\mathbf{K}}_\mathbf{x}\mathbf{x}_d + \mathbb{E}[\bar{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s + \mathbf{x}_d, \dot{\mathbf{x}}_s + \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_s + \ddot{\mathbf{x}}_d)] = \bar{\mathbf{f}}_{d,\mathbf{x}}(t), \quad (11)$$

which is used next for deriving the system response. To this end, a framework is proposed which is based on the combination of the harmonic balance method (for treating the deterministic component), and the statistical linearization methodology for systems with singular matrices (for treating the stochastic component).

2.2.1 *Application of the harmonic balance and statistical linearization treatments*

First, considering the system in Eq. (3), the harmonic balance method is applied for determining the periodic component of the response. It is assumed for simplicity that the nonlinear vector $\bar{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s + \mathbf{x}_d, \dot{\mathbf{x}}_s + \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_s + \ddot{\mathbf{x}}_d)$ in Eq. (3) contains polynomial nonlinear functions. This assumption facilitates the derivation of closed form solutions for the system response (Spanos et al. 2019), as well as simplifies the application of the harmonic balance method (Mickens 2010).

In this regard, the deterministic response becomes

$$\mathbf{x}_d(t) = \mathbf{x}_{d_1} \cos(\omega_d t) + \mathbf{x}_{d_2} \sin(\omega_d t), \quad (12)$$

where $\mathbf{x}_{d_1}, \mathbf{x}_{d_2}$ are constant l vectors. Next, applying the harmonic balance method yields

$$\mathbf{P}\mathbf{u} = \mathbf{v}. \quad (13)$$

In Eq. (13), $\mathbf{P}$ is a $2(l+m) \times 2l$ matrix whose elements are functions of ω_d and the augmented parameter matrices defined in Eqs. (4)-(6). Further, $\mathbf{v}$ is a $2(l+m)$ vector containing the deterministic excitation, as well as the ensemble average of the stochastic excitation, whereas the $2l$ vector

$$\mathbf{u} = \begin{bmatrix} \mathbf{x}_{d_1} \\ \mathbf{x}_{d_2} \end{bmatrix} \quad (14)$$

contains the deterministic response of the system.

Then, employing the M-P inverse of the matrix $\mathbf{P}$ (Ben-Israel & Greville 2003), the solution to the overdetermined system of equations defined in Eq. (14) is given by

$$\mathbf{u} = \mathbf{P}^+\mathbf{v} + (\mathbf{I} - \mathbf{P}^+\mathbf{P})\mathbf{y}. \quad (15)$$

In Eq. (15), $\mathbf{y}$ is an arbitrary $2l$ vector, and thus, this expression corresponds to a family of possible solutions for the deterministic response component of the system. However, a unique solution is attained when $\mathbf{P}$ has full column rank. Specifically, it such case the M-P inverse matrix of $\mathbf{P}$ is given by $\mathbf{P}^+ = (\mathbf{P}^*\mathbf{P})^{-1}\mathbf{P}^*$, and substituting the latter into Eq. (15), a simplified expression is derived.

Next, the stochastic response component is treated by resorting to the statistical linearization methodology for systems with singular matrices (Fragkoulis et al. 2016; Kougioumtzoglou et al. 2017); see also Mitseas et al. (2016, 2018); Fragkoulis et al. (2019); Mitseas & Beer (2019); Pasparakis et al. (2021); Mitseas & Beer (2021); Ni et al. (2022) for indicative application frameworks of the method.

In this regard, considering Eqs. (3) and (11) leads to

$$\bar{\mathbf{M}}_\mathbf{x}\ddot{\mathbf{x}}_s + \bar{\mathbf{C}}_\mathbf{x}\dot{\mathbf{x}}_s + \bar{\mathbf{K}}_\mathbf{x}\mathbf{x}_s + \tilde{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s, \mathbf{x}_d) = \bar{\mathbf{Q}}_\mathbf{x}(t), \quad (16)$$

where

$$\tilde{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s, \mathbf{x}_d) = \bar{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s + \mathbf{x}_d, \dot{\mathbf{x}}_s + \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_s + \ddot{\mathbf{x}}_d) - \mathbb{E}[\bar{\mathbf{\Phi}}_\mathbf{x}(\mathbf{x}_s + \mathbf{x}_d, \dot{\mathbf{x}}_s + \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_s + \ddot{\mathbf{x}}_d)] \quad (17)$$

is the zero-mean nonlinear vector of the system, to be replaced by equivalent linear elements. Specifically, applying the statistical linearization yields the equivalent linear system

$$(\bar{\mathbf{M}}_{\mathbf{x}}+\bar{\mathbf{M}}_e)\ddot{\mathbf{x}}_s+(\bar{\mathbf{C}}_{\mathbf{x}}+\bar{\mathbf{C}}_e)\dot{\mathbf{x}}_s+(\bar{\mathbf{K}}_{\mathbf{x}}+\bar{\mathbf{K}}_e)\mathbf{x}_s = \bar{\mathbf{Q}}_{\mathbf{x}}(t), \tag{18}$$

where $\bar{\mathbf{M}}_e$, $\bar{\mathbf{C}}_e$ and $\bar{\mathbf{K}}_e$ denote the unknown equivalent linear $(l+m)\times l$ matrices of the system, which are used to account for neglecting from Eq. (16) the nonlinear vector. It is noted that closed form expressions for the equivalent linear matrices are found in Fragkoulis et al. (2016b) and Kougioumtzoglou et al. (2017). Further, it is noted that since the nonlinear vector in Eq. (17) is written in terms of both the stochastic and deterministic response components, this will also hold for the equivalent linear elements. However, considering that the elements of the equivalent matrices are slowly varying over a period T of oscillation, they are approximated by their average over T (Spanos et al. 2019). Therefore, Eq. (18) becomes

$$(\bar{\mathbf{M}}_{\mathbf{x}}+\bar{\mathbf{M}}_e^a)\ddot{\mathbf{x}}_s+(\bar{\mathbf{C}}_{\mathbf{x}}+\bar{\mathbf{C}}_e^a)\dot{\mathbf{x}}_s+(\bar{\mathbf{K}}_{\mathbf{x}}+\bar{\mathbf{K}}_e^a)\mathbf{x}_s = \bar{\mathbf{Q}}_{\mathbf{x}}(t). \tag{19}$$

Eq. (19) corresponds to the equivalent linear system, whose solution is derived by following either a time-domain treatment, where the system response is derived by solving a Lyapunov equation (Fragkoulis et al. 2016a). Alternatively, applying a frequency-domain treatment, the system response is determined by (Roberts & Spanos 2003)

$$\mathbb{E}[\mathbf{x}\mathbf{x}^{\mathrm{T}}]=\int_{-\infty}^{\infty}\mathbf{S}_{\mathbf{x}}(\omega)d\omega, \tag{20}$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator and $\mathbf{S}_{\mathbf{x}}(\omega)$ is the response power spectrum. The latter is determined by resorting to the input-output expression

$$\mathbf{S}_{\mathbf{x}}(\omega)=\boldsymbol{\alpha}_{\mathbf{x}}(\omega)\mathbf{S}_{\bar{\mathbf{Q}}_{\mathbf{x}}}(\omega)\boldsymbol{\alpha}_{\mathbf{x}}^{\mathrm{T}*}(\omega), \tag{21}$$

where $\boldsymbol{\alpha}_{\mathbf{x}}(\omega)$ is the frequency response matrix and $\mathbf{S}_{\bar{\mathbf{Q}}_{\mathbf{x}}}(\omega)$ the power spectrum of the excitation; see Kougioumtzoglou et al. (2017) for a detailed presentation.

3 NUMERICAL EXAMPLE

In this section, the M-P generalized inverse matrix-based framework is used to compute the response of a piezoelectric energy harvesting device. An indicative piezoelectric energy harvester, consists of a mechanical system, such as a cantilever beam moving as a result of applied excitation, and a corresponding piezoelectric system, which is used for transforming the mechanical energy into electric current. Such devices are used in several applications, mostly for powering adjoining low power level devices. Specifically, they often operate in tandem with large scale infrastructure, such as bridges and high-rise buildings (e.g., Roccia et al. 2020), which are potentially subjected to combined deterministic and stochastic excitations.

The equations governing the dynamics of the system are given by (Daqaq et al. 2014; Petromichelakis et al. 2018; Karageorgos et al. 2021)

$$\ddot{q}+2\zeta\dot{q}+\frac{dU(q)}{dq}+\kappa^2 y=w(t)+f_d(t), \tag{22}$$

$$\dot{y}+\alpha y-\dot{q}=0. \tag{23}$$

In the coupled system of Eqs. (22) and (23), q denotes the response displacement of the mechanical part and y is either the induced voltage or the induced current. Further, ζ is the damping coefficient of the mechanical system, κ denotes a coupling coefficient, α is a constant and $U(q)$ denotes the potential function (He & Daqaq 2016). The system is subjected to the stochastic excitation $w(t)$, which is modeled as a Gaussian white noise stochastic process with constant spectral density S_0, and also to the deterministic component, which is given by $f_d = f_{d_1}\cos\omega_d t + f_{d_2}\sin\omega_d t$. It is assumed that the nonlinear function of the system has the form (Petromichelakis et al. 2018)

$$\frac{dU(q)}{dq}=q+\lambda q^2+\delta q^3, \tag{24}$$

where λ and δ denote parameters which control the intensity of the nonlinearity. The following set of parameter values are used: $\alpha =$

$0.8,\ S_0 = 0.05,\ \delta = 0.1,\ \kappa = 3.25,\ \omega_d = \pi,$ $f_{d_1} = 0$ and $f_{d_2} = 0.1$.

Setting

$$\mathbf{x}(t) = \begin{bmatrix} q(t) \\ y(t) \end{bmatrix} \tag{25}$$

and also considering Eq. (24), the system of Eqs. (22) and (23) is written in the form of Eq. (1), where the parameter matrices are given by

$$\mathbf{M_x} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \mathbf{C_x} = \begin{bmatrix} 2\zeta & 0 \\ -1 & 1 \end{bmatrix} \tag{26}$$

and

$$\mathbf{K_x} = \begin{bmatrix} 1 & k^2 \\ 0 & \alpha \end{bmatrix}, \tag{27}$$

whereas the deterministic and stochastic excitation vectors become, respectively,

$$\mathbf{f}_{d,\mathbf{x}} = \begin{bmatrix} f_d(t) \\ 0 \end{bmatrix} \tag{28}$$

and

$$\mathbf{Q_x} = \begin{bmatrix} w(t) \\ 0 \end{bmatrix}. \tag{29}$$

Clearly, the matrix $\mathbf{M_x}$ in Eq. (26) is singular, which hinders the direct treatment of the system. However, considering that Eq. (23) denotes the constraint equation of the harvester (see also Petromichelakis et al. 2018) facilitates the ensuing analysis. Specifically, differentiating Eq. (23) once with respect to time, Eq. (2) is formulated, where

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \end{bmatrix},\ \mathbf{E} = \begin{bmatrix} 0 & \alpha \end{bmatrix},\ \mathbf{L} = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{30}$$

and

$$F = 0. \tag{31}$$

In this regard, the system of Eqs. (22) and (23) is equivalently written in the form of Eq. (3), where

$$\bar{\mathbf{M}}_\mathbf{x} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ -1 & 0 \end{bmatrix},\ \bar{\mathbf{C}}_\mathbf{x} = \begin{bmatrix} -0.5\alpha & 0.5 \\ -0.5\alpha & 0.5 \\ 0 & \alpha \end{bmatrix} \tag{32}$$

and

$$\bar{\mathbf{K}}_\mathbf{x} = \begin{bmatrix} 0.5 & 0.5k^2+\alpha \\ 0.5 & 0.5k^2+\alpha \\ 0 & 0 \end{bmatrix}. \tag{33}$$

Further, Eq. (7) becomes

$$\bar{\boldsymbol{\Phi}}_\mathbf{x}(\mathbf{x}) = (\lambda q^2 + \delta q^3) \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}, \tag{34}$$

and Eqs. (8) and (9) yield, respectively,

$$\bar{\mathbf{Q}}_\mathbf{x} = w(t) \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix} \tag{35}$$

and

$$\bar{\mathbf{f}}_{d,\mathbf{x}} = f_{d_2} \sin(\omega_d t) \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}. \tag{36}$$

Next, considering that the voltage process $y(t)$ has zero mean (e.g., Grigoriu 2021), the herein generalized harmonic balance method for systems with singular matrices is employed. Considering further that the system response in Eq. (25) has a stochastic and a deterministic component, i.e.,

$$\mathbf{x}_s(t) = \begin{bmatrix} q_s(t) \\ y_s(t) \end{bmatrix},\ \mathbf{x}_d(t) = \begin{bmatrix} q_d(t) \\ y_d(t) \end{bmatrix} \tag{37}$$

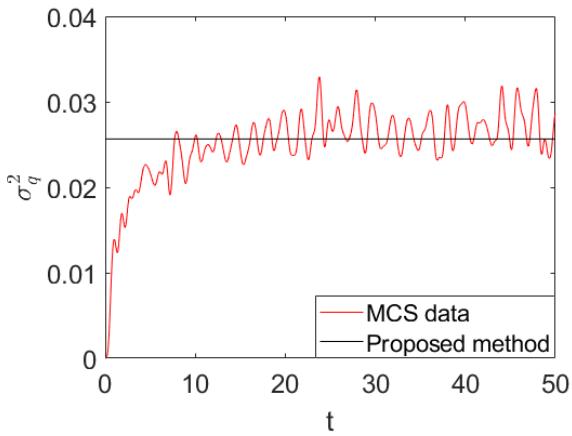
and ensemble averaging Eq. (34), leads to

$$\mathbb{E}[\bar{\boldsymbol{\Phi}}_\mathbf{x}] = \left(\lambda \sigma_{q_s}^2 + \lambda q_d^2 + 3\delta \sigma_{q_s}^2 q_d + \delta q_d^3\right) \times \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}. \tag{38}$$

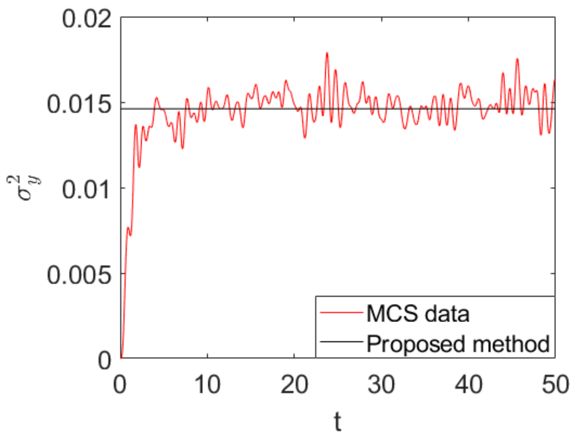
Then, the 6×4 matrix $\mathbf{P}$ in Eq. (13) is formed and since it has full rank, a unique solution for the periodic response vector is found by solving Eq. (15). Further, applying the generalized statistical linearization method, the equivalent matrices $\bar{\mathbf{M}}_e^a, \bar{\mathbf{C}}_e^a$ and $\bar{\mathbf{K}}_e^a$ are derived and the equivalent linear system in Eq. (19) is formed. Indicatively, the matrix $\bar{\mathbf{K}}_e^a$ is given by

$$\bar{\mathbf{K}}_e^a = 1.5\delta\sigma_{q_s}^2 \begin{bmatrix} H(1,1) & H(2,1) \\ H(1,1) & H(2,1) \\ 0 & 0 \end{bmatrix} + 1.5\delta \begin{bmatrix} \frac{q_{d_1}^2+q_{d_2}^2}{2} & 0 \\ \frac{q_{d_1}^2+q_{d_2}^2}{2} & 0 \\ 0 & 0 \end{bmatrix}. \quad (39)$$

In Eq. (39), $H(i,j)$, $i,j=1,2$, denote the (i,j) element of the matrix $\mathbb{E}[\hat{\mathbf{x}}\hat{\mathbf{x}}^{\mathrm{T}}]^{+}\mathbb{E}[\hat{\mathbf{x}}\hat{\mathbf{x}}^{\mathrm{T}}]$, where $\hat{\mathbf{x}}^{\mathrm{T}} = \begin{bmatrix}\mathbf{x} & \dot{\mathbf{x}}\end{bmatrix}$ and $\mathbf{x}$ is defined in Eq. (25); see Fragkoulis et al. (2016b) for a detailed discussion.



(a)



(b)

Figure 1. Response variance of the energy harvesting system of Eqs. Eqs. (22) and (23) subjected to combined stochastic and deterministic excitations ($S_0 = 0.05$, $f_{d_2} = 0.4$, $\omega_d = \pi$). Analytical solution vis-à-vis MCS estimate (500 realizations): (a) response displacement variance; (b) response voltage variance.

Finally, the variance of the stochastic response is computed by solving the coupled set of Eqs. (15), (20) and (21). In addition, considering Eqs. (12), (25) and (37), and successively ensemble and temporal averaging to treat, respectively, the stochastic and deterministic components of the response, yields

$$\langle \mathbb{E}[x_i^2] \rangle = \sigma_{x_{s,1}}^2 + \frac{\omega_d (x_{d_1,i}^2 + x_{d_2,i}^2)}{2}, \quad (40)$$

$i = 1,2$, where $\langle\cdot\rangle$ denotes the temporal averaging operation.

The response displacement variance and the variance of the response voltage of the nonlinear harvester of Eqs. (22) and (23) subjected to combined stochastic and deterministic excitations are shown, respectively, in Figs. 1(a) and 1(b). The validity of the results obtained by the proposed method is verified by also considering pertinent MCS data. Specifically, 500 realizations are generated by the spectral representation method (Shinozuka & Deodatis 1991) for duration $T_0 = 50$ s and cut-off frequency equal to 2π. Then, the system response variance is derived by utilizing a standard 4^{th} order Runge-Kutta numerical integration scheme to solve the governing equations of the system.

4 CONCLUSIONS

In this paper, the problem of determining the response statistics of a nonlinear piezoelectric energy harvesting device subjected to combined stochastic and deterministic excitation has been considered. The system response has been computed in a direct way by utilizing a recently developed method for determining the response of multi-degree-of-freedom nonlinear systems with singular parameter matrices (Ni et al. 2021). The method relies on the combination of the generalized statistical linearization treatment for systems with singular matrices and the harmonic balance method. Specifically, since the system excitation consists of a periodic and a stochastic component, the system response has been decomposed into two corresponding components. Then, the statistical linearization and harmonic balance methods have been utilized to treat, respectively, the former and latter. The validity of the obtained results has been verified by considering pertinent MCS data.

5 ACKNOWLEDGMENTS

The authors gratefully acknowledge the support from the German Research Foundation under Grant Nos. BE 2570/7-1, MI 2459/1-1 and FR 4442/2-1. Further, the support from the Hellenic Foundation for Research and Innovation under Grant No. 1261 is gratefully acknowledged.

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