Review of Political Economy

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/crpe20

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To cite this article: Bill Gerrard (2024) The Road Less Travelled: Keynes and Knight on Probability and Uncertainty, Review of Political Economy, 36:3, 1253-1278, DOI: 10.1080/09538259.2022.2114291

To link to this article: https://doi.org/10.1080/09538259.2022.2114291

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Published online: 16 Sep 2022.

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REVIEW OF POLITICAL ECONOMY
2024, VOL. 36, NO. 3, 1253–1278
https://doi.org/10.1080/09538259.2022.2114291

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The Road Less Travelled: Keynes and Knight on Probability and Uncertainty

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ABSTRACT

Knight's risk/uncertainty distinction is reviewed in its original context as a contribution to the theory of profit. Knight's approach to probability is paralleled by Ludwig von Mises, as emphasised by recent developments in strategic entrepreneurship theory. Von Mises distinguishes between class probability (i.e., risk) and case probability (i.e., uncertainty) in contrast to the frequentist approach of his brother, Richard von Mises. Keynes's contribution to probability and uncertainty is reviewed, focusing on his logical theory of probability in *A Treatise on Probability* which he more fully contextualised subsequently in the *General Theory*. Keynes's fragmentary later philosophical writings are reviewed to provide some insight into the contextual issues encountered. The key contributions of Knight and Keynes are summarised as signposts for 'The Road Less Travelled'. The possibilities of a Keynesian-Knightian synthesis as a way forward are considered by comparing these signposts. However, it is concluded that, although there is some common ground between Knight and Keynes, there are fundamental differences particularly associated with the definition of confidence that preclude any meaningful synthesis.

ARTICLE HISTORY
Received 22 April 2022
Accepted 13 August 2022

KEYWORDS
Risk; uncertainty; probability; relative frequency; degree of belief; weight of argument; human logic

JEL CODES
B5; D8; E12 Keynes

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I -
I took the one less travelled by,
And that has made all the difference. (Robert Frost, The Road Not Taken, 1920)

1. Introduction

The centenary of the publication of both Knight's *Risk, Uncertainty and Profit* (1921) and Keynes's *A Treatise on Probability* (1921) provided a timely opportunity for their re-appraisal both individually and comparatively. As Dimand (2021, pp. 578–579) argues in his contribution to this journal's symposium on the subject, 'the two literatures on

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fundamental uncertainty, Knightian and Keynesian, can benefit from communication, which they have engaged in only intermittently'. Crucially, as Dimand recognises, any appreciation of the respective contributions of Knight and Keynes to our understanding of probability and uncertainty must start with the recognition that they came at these problems from very different perspectives. Whereas Keynes in *A Treatise on Probability* is principally addressing the philosophical problem of extending logic into the non-demonstrative realm of partial knowledge, Knight's *Risk, Uncertainty and Profit* is motivated by the economic problem of the nature of profit, specifically the proposition that profit represents the entrepreneur's reward for successfully bearing uncertainty. The basic argument of this paper is that, despite their very different intellectual journeys leading to some significant divergences in their respective approaches, there are some commonalities that provide clear direction for an alternative approach to the understanding of human action under conditions of uncertainty. However, there are fundamental differences particularly associated with the definition of confidence that preclude any meaningful synthesis. This alternative approach, although becoming more widely recognised as an important source of insight for the understanding of economic behaviour, as exemplified by the discussions of animal spirits by Akerlof and Shiller (2009) and radical uncertainty by Kay and King (2020), has still been largely ignored in mainstream economic theory and remains 'The Road Less Travelled'.

The paper is structured as follows. The next section sets out Knight's approach to probability and uncertainty, and the parallel development by Ludwig von Mises which has been highlighted by recent developments in strategic entrepreneurship theory. These developments are contrasted with the frequentist approach of Ludwig's brother, Richard von Mises. Section Three reviews Keynes's contributions to probability and uncertainty in *A Treatise on Probability* and, subsequently, in *The General Theory of Employment, Interest and Money* and other related economic writings as well as his fragmentary later philosophical writings. In both these sections, the key contributions of Knight and Keynes are summarised as signposts for 'The Road Less Travelled'. Section Four considers the possibilities for Keynesian-Knightian synthesis as a way forward by comparing the signposts identified in the earlier sections. It is argued that, although there is some common ground between Knight and Keynes, there are fundamental differences particularly associated with the definition of confidence that preclude any meaningful synthesis. Section Five provides a short summary of the key arguments.

2. Knight on Probability and Uncertainty

2.1. *Knight's* Risk, Uncertainty and Profit

Knight Signpost 1: The Context of Probability and Uncertainty

Probability and uncertainty are contextualised in the economic theory of profit as the return to entrepreneurs for bearing uncertainty successfully

The starting point for Knight's *Risk, Uncertainty and Profit* is the economics context. Knight's principal purpose in proposing the risk/uncertainty distinction is to develop the economic theory of profit as the reward to successful entrepreneurship under

conditions of uncertainty. Knight argues that profit has been wrongly attributed as a reward to bearing risk but, unlike uncertainty, risk is insurable and should, therefore, be treated as a cost to business measured by the price of insurance (irrespective of whether the risk is actually insured).

> It will appear that a measurable uncertainty, or 'risk' proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term 'uncertainty' to cases of the non-quantitative type. It is this 'true' uncertainty, and not risk, as has been argued, which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition.
> (Knight 1921, p. 20)

In developing the risk/uncertainty distinction, Knight considers some of the inherent philosophical issues, but these are not his primary concern. Consequently, there are philosophical inconsistencies in Knight's position. *Risk, Uncertainty and Profit* is first and foremost a contribution to the theory of the firm and the theory of market competition. Knight came to probability indirectly after recognising that uncertainty is the ultimate source of entrepreneurial profits and losses (Hoppe 2007). Knight (1921, pp. 199–201 fn.) directs that 'the text must not be taken as expressing any view whatever as to the ultimate nature of reality or any other philosophic position', describing himself as a 'radical empiricist in logic' and 'an agnostic on all questions beyond the fairly immediate facts of experience'.

Knight Signpost 2: The Scope of Probability

Probability is concerned with partial knowledge when rigorous demonstration is not applicable

Knight distinguishes between exact knowledge and partial knowledge. Knight (1921, p. 199) defines partial knowledge as 'opinion, of greater or less foundation and value, neither entire ignorance nor complete and perfect information, but partial knowledge'. In contrast, exact knowledge is characterised by 'rigorous demonstration' (Knight 1921, p. 209). Probability is concerned with the problem of partial knowledge, which Knight (1921, p. 199) considered to be the essence of the 'problems of life' — 'We live only by knowing something about the future; while the problems of life, of conduct at least, arise from the fact that we know so little'.

Knight Signpost 3: The Nature of Probability

Probability is dualistic in nature, encompassing both the aleatory conception and the epistemic conception

Hacking (1975) identifies two basic conceptions of probability — the epistemic conception of probability as a degree of belief, and the aleatory conception of probability as relative frequencies (i.e., a physical attribute of reality). Knight uses both conceptions, treating probability-as-risk as aleatory but initially treating probability-as-uncertainty as epistemic, defining uncertainty as 'our imperfect knowledge of the future, a consequence of change, not change as such' (Knight 1921, p. 198). But subsequently, Knight distinguishes between situations of ignorance and real probability. Uncertainty due to ignorance occurs when statistical probabilities are unknown but are potentially knowable, which he illustrates with the example of an urn containing red and black balls

(Knight 1921, pp. 218–219). The use of an urn (or bag) of coloured balls (or pebbles) is a longstanding device for the exposition of probability theory that originated in the work of Jacob Bernoulli in the early 18th century and has been used by many writers on probability subsequently including Keynes (1921, pp. 75–76) in *A Treatise on Probability*. Ellsberg (1961) has termed the situation of unknown but knowable probabilities as 'ambiguity'. Knight (1921, p. 219). contrasts this type of ignorance with 'the doctrine of real probability, [which,] if it is valid, must, it seems, rest upon inherent unknowability in the factors' This definition of real probability suggests an aleatory conception of probability. Lawson (1988) recognises this tension in Knight's treatment of probability, initially classifying Knight as adopting a realist (i.e., aleatory) view of probability but subsequently viewing Knight (like Keynes) as an interactionist realist, conceptualising probability as a two-way interaction of theoretical belief (i.e., epistemic) and experience (i.e., aleatory). Lawson's interactionist realist interpretation of Knight is echoed by Packard, Clark, and Klein (2017), who distinguish between 'true uncertainty' and 'perceived uncertainty', arguing that Knight treats uncertainty as an interaction between (aleatory) objective environmental conditions (i.e., true uncertainty) and the entrepreneur's (epistemic) subjective perceptions and actions (i.e., perceived uncertainty).

Knight Signpost 4: *A Priori* Probability

A priori probability is a type of probability-as-risk applicable when outcomes are equiprobable and mutually exclusive

Knight (1921, pp. 214–217, 224–225) identifies mathematical or *a priori* probability as one type of probability-as-risk. *A priori* probability can be deduced mathematically from general principles when there is perfect homogeneity of instances as, for example, in the case of games of chance and throwing a perfect dice. Knight (1921, p. 220) considers that 'the whole mathematical theory of probability is a simple application of the principles of permutations and combinations for finding out the number of alternatives', which, in turn, requires that 'the alternatives themselves must be equally probable'. But Knight (1921, p. 219) recognises that the law of indifference (i.e., equiprobabilities) cannot be assumed 'out of hand' but 'we must always consult the empirical facts'.

Runde (1998, p. 539) argues that there is 'an elementary confusion' in Knight's definition of *a priori* probability, arising from a lack of clarity that the perfect homogeneity of instances is a necessary but not sufficient condition for calculating *a priori* probability that also requires the assumption of equiprobability. Runde illustrates the problem using the example of a weighted dice in which there is perfect homogeneity of instances, but the outcomes are not equiprobable, so that the probabilities cannot be derived *a priori* and can only be determined via empirical observation of repeated trials of the weighted dice. Runde concludes that Knight's definition of *a priori* probability should be restated to clarify the necessity of both equally probable and mutually exclusive outcomes.

Knight Signpost 5: Frequency Theory

Frequency theory is applicable only to statistical probability, the type of probability-as-risk relevant for repeatable and relatively homogeneous events

Statistical probability is identified by Knight as the most common type of probability-as-risk in business situations. Statistical probabilities are an empirical generalisation based on the observed relative frequencies of alternative outcomes in a specified group of similar events characterised by repeatable and relatively homogeneous events such as, for example, the fire hazards of buildings. From Knight's perspective, frequency theory is a special case applicable to probability-as-risk and relevant to the theory of the firm only as an explanation of the costs arising from insurable risks.

Knight Signpost 6: The Measurability of Probability

A priori and statistical probability are measurable, but probability estimates applicable in situations of true uncertainty are not susceptible to measurement

Knight designates probability-as-uncertainty as 'estimates'. Probability estimates are applicable in unique situations of 'true uncertainty' that 'accounts for the peculiar income of the entrepreneur' (Knight 1921, p. 232). Knight uses the decision of a firm to expand production capacity as an example of probability-as-uncertainty. In contrast to *a priori* and statistical probability which are objective and measurable, probability estimates are subjective and cannot be quantified since 'there is no valid basis of any kind for classifying instances' (Knight 1921, p. 225). Estimates represent 'that higher form of uncertainty not susceptible to measurement' (Knight 1921, p. 232).

Knight Signpost 7: The Justification of Induction

Inductive knowledge is justified by the law of regularity that the world consists of objects which, under the same circumstances, always behave in the same way

According to Knight (1921, p. 201), uncertainty arises in the context of forward-looking conscious behaviour to change the future situation. Such conscious behaviour requires knowledge of a 'dependable relation' (Knight 1921, p. 203) between the present and future possibilities. This inductive knowledge depends on the law of regularity: 'the presupposition that the world is made up of things, which, under the same circumstances, always behave in the same way' (Knight 1921, p. 204). It is this presupposition that underpins his concept of classification (Knight 1921, p. 205) as the grouping of the same kind of things that always behave in the same way. Classification requires a high (but not absolute) degree of homogeneity within groupings. Absolute homogeneity would, paradoxically, imply uniformity (Knight 1921, p. 218) with every outcome being exactly the same, whereas probability allows for similar objects behaving in similar ways but with the possibility of different outcomes. Knight allows the membership of groupings to shift depending on the specific purpose of the classification and describes these purpose-based groupings from which statistical probabilities can be derived as 'modes of resemblance' (Knight 1921, p. 206).

Knight Signpost 8: The Nature of Evidence

The evidence underlying the different types of probability constitutes a continuum from perfect homogeneity of classes of instances to extreme heterogeneity of unique singular instance

As Runde (1998) argues, Knight conceives of the evidence underlying the different types of probability as constituting a continuum based on the degree of heterogeneity of the instances considered. Statistical probability represents the intermediate range that lies between the two polar cases of *a priori* probability (i.e., perfect homogeneity of instances) and estimates (i.e., extreme heterogeneity of unique instances). 'There are all gradations from a perfectly homogeneous group of life or fire hazards at one extreme to an absolute exercise of judgment at the other' (Knight 1921, pp. 225–226). The difference between statistical probability and probability estimates is 'a matter of degree only' (Knight 1921, p. 225).

Knight Signpost 9: Confidence and Conduct

Under conditions of uncertainty, conduct depends on the best estimate and the degree of confidence in that estimate as measured by probability

Under conditions of (true) uncertainty, Knight argues that action depends on both the best estimate of the future outcome and the degree of certainty (or confidence) in that best estimate as measured by its probability.

> The business man himself not merely forms the best estimate he can of the outcome of his actions, but he is likely also to estimate the probability that his estimate is correct. The 'degree' of certainty or of confidence felt in the conclusion after it is reached cannot be ignored, for it is of the greatest practical significance. The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favourableness of the opinion itself. (Knight 1921, pp. 226–227)

Knight Signpost 10: The Correctness of Probability

The correctness of the entrepreneur's judgment (i.e., probability estimate) of the future relative to that of their competitors is determined *ex post* by the profit earned

There is a tension throughout *Risk, Uncertainty and Profit* stemming from the treatment of uncertainty as both a belief about the future (i.e., an *ex-ante* concept) and as a profit outcome (i.e., an *ex-post* concept). It is the latter *ex-post* conception of the correctness of probability that takes priority for Knight since his principal objective is to explain profit as the reward to entrepreneur for successfully bearing uncertainty. The profit earned by the entrepreneur 'depends on his success in producing the anticipated excess, and in this sense is a matter of the correctness of his judgment' but 'his success is equally a matter of (a) the failure of the judgment, or (b) an inferiority in capacity, on the part of his competitors' (Knight 1921, p. 281).

2.2. Knight's Fellow Travellers (1): Schumpeter, the von Mises Brothers and the Austrian School

There has been a growing recognition in recent years of the links between Knight's approach and that of Ludwig von Mises and the Austrian school. Yu (2002, p. 1) comments that 'Knight's contributions are essentially Austrian'. Yu argues that, although

Knight is regarded as the founder of the Chicago school, his core contributions to economic methodology, the theory of human agency, and the theory of uncertainty and entrepreneurship have been lost to the modern Chicago school but are still alive and well in the modern Austrian school. Yu shows that Knight was familiar with the work of Menger, Hayek, von Wieser and von Mises, and that Knight's own work has been extensively cited by Austrian economists. Yu (2002, p. 17) concludes that 'several key areas in modern Austrian economics are anticipated by Knight' and, indeed, goes as far as saying that Knight should be seen as the 'missing link between Menger and Mises'. Yu's characterisation of Knight as anticipating 'several key areas in modern Austrian economics' is uncontentious, but the claim that Knight is the 'missing link between Menger and Mises' is less so. Schumpeter could lay greater claim to that accolade, preceding Knight in the recognition of the entrepreneur as operating under conditions of uncertainty. For example, as Lachmann observes,

> As early as 1912 Schumpeter drew the distinction between the 'entrepreneur', the man who has the mental power to imagine that tomorrow will be different from today and who is able to act accordingly, and the 'static individual' who lacks the power and can only adapt himself to existing circumstances. Professor Knight, by a different route, reached virtually the same conclusion, viz., that in an uncertain world, uncertainty-bearing becomes a function of specialists. (1977, p. 141)

Strangely in *Risk, Uncertainty and Profit* Knight only cites Schumpeter's static analysis and makes no mention of his dynamic analysis and conceptualisation of entrepreneurship as the capability of projecting the future.

Hoppe (2007, p. 1) also notes a 'systematic yet rarely noted similarity in the works of Knight and Mises'. Furthermore, the link between Knight and von Mises is a fundamental building block of an important strand in strategic entrepreneurship theory in which entrepreneurship is framed as judgment under uncertainty (see below).

Ludwig von Mises's major work is *Human Action: A Treatise on Economics* (1949). There are close parallels between *Human Action* and *Risk, Uncertainty and Profit*. Von Mises cites Knight on four occasions but three of these citations concern capital theory and only one of these citations (von Mises 1949, p. 189 fn. 18) refers to Knight's distinction between risk and uncertainty, specifically relating Knight's approach to risk to what von Mises terms 'accidents' as distinct from 'human action' which von Mises considers the domain of uncertainty. (It should be noted that von Mises also cites Keynes six times, but in every case, the reference is to Keynes's macroeconomic theory, not his work on probability.)

Unlike Knight, von Mises is concerned with the philosophical and methodological foundations of his approach. In *Human Action*, von Mises sets out praxeology as the study of human action. The objective of praxeology is to understand what is going on in the minds of people when they make decisions to undertake an action. 'Action is to make choices and to cope with an uncertain future' (von Mises 1949, p. 249). The basic method of praxeology is to produce an understanding of human action using the method of imaginary constructions consisting of a conceptual image of a sequence of logically evolving events (von Mises 1949, p. 237).

Von Mises distinguishes between two types of probability — class probability and case probability. Class probability refers to knowledge of the behaviour of a whole class of

events in respect of the frequency of occurrence of phenomenon but with no knowledge of actual singular events or phenomena beyond that they belong to the class. Class probability coincides with Knight's concept of statistical probability. von Mises does not recognise the possibility of *a priori* probability, taking the general position that 'aprioristic reasoning is purely conceptual and deductive. It cannot produce anything else but tautologies and analytic judgments' (von Mises 1949, p. 38).

Case probability concerns unique events for which the notion of frequency is inappropriate (von Mises 1949, p. 111). Case probability coincides with Knight's concept of uncertainty (i.e., probability as estimates). von Mises (1949, p. 105) argues that case probability is only amenable to understanding using the methods of praxeology, not the methods of the natural sciences, which cannot determine how someone will act.

von Mises (1949, pp. 256–257). follows Schumpeter and Knight in emphasising the importance of the role of the entrepreneur, particularly the entrepreneur-promotor who as an entrepreneur reacts to changes in market data and as a promotor is a pacemaker rather than an imitator. It is the entrepreneur-promotor seeking large profits who is the driving force of the market and the dominant force in innovation and improvement.

Hoppe (2007) argues that Knight and Ludwig von Mises both adopt the frequentist interpretation of probability and share similar views on the limits of probability in economics and the social sciences in general. Hoppe links Knight and Ludwig von Mises to the frequentist approach of Ludwig's younger brother, the statistician, Richard von Mises. Richard's principal work, *Probability, Statistics and Truth*, was originally published in German in 1928, with the first English translation published in 1939. Although Ludwig knew his brother's work, it is not referenced in *Human Action*, which Hoppe suggests was due to their long estrangement. Hoppe argues that Ludwig's approach to probability was intended in part as a refinement of Richard's frequency theory, particularly in circumventing a perceived problem with the practical application of probability when conceptualised as the limiting values of relative frequencies in an infinite series of observed repeated trials.

Richard argues that the rational concept of probability only has meaning in the context of a well-defined collective (von Mises 1957, p. 11). 'We shall not speak of probability until a collective has been defined' (von Mises 1957, p. 18). The collective is defined as 'a sequence of uniform events or processes which differ by certain observable attributes' (von Mises 1957, p. 12). Provided that the collective satisfies the randomness (or what Richard terms the 'place selection') condition, it follows that probability is concerned with 'the probability of encountering a certain attribute in a given collective' (von Mises 1957, p. 12). It further follows that probability is the limiting value of the relative frequency in an unlimited sequence of observations of the same event or process that constitute a well-defined collective (von Mises 1957, p. 14). He dismisses as 'utter nonsense' (von Mises 1957, p. 17) any attempt to apply probability to a specific event that could be assigned to multiple collectives.

Richard rejects Knight's three-way categorisation of probabilities, considering only statistical probability to be properly defined as a probability. Richard provides three examples of unlimited repetition in which probability is relevant — games of chance, problems of insurance, and molecular processes (von Mises 1957, p. 10). By way of contrast, he uses the case of a war between Germany and the Republic of Liberia as a

situation that does not frequently repeat itself and, hence, represents a situation to which probability cannot be applied. In addition, Richard rejects the concept of *a priori* probability and explicitly distinguishes his frequentist approach to probability from the subjective conception of probability that he associates primarily with Keynes in *A Treatise on Probability* (von Mises 1957, p. 75). From Richard's perspective, Knight's theory of probability represents a confused amalgam of non-empirical (i.e., *a priori*) probability, objectivist frequency (i.e., statistical) probability and subjectivist probability (i.e., estimates).

Hoppe's contention that both Knight and Ludwig von Mises are frequentists in the same mould as Richard von Mises involves rather Procrustean methods of interpretation. Hoppe (2007, p. 5) is correct that Knight's 'definition of "empirical-statistical probability" as "insurable" contingency or "risk," … is in complete accordance with Richard von Mises's frequency interpretation, as is Ludwig von Mises's concept of class probability.' But much less convincing is Hoppe's dismissal of Knight's concept of *a priori* probability as a 'minor if unfortunate slip' (Hoppe 2007, p. 5). Similarly, Hoppe largely ignores Ludwig von Mises's concept of case probability, concentrating instead on Ludwig's definition of a class which assumes both randomness and full knowledge of the class as a whole and so avoids having to treat probability as a limiting value, and thereby represents an advance on Richard's concept of a collective. Hoppe suggests that the non-frequentist aspects of Knight and Ludwig von Mises's approaches may be due to both having 'come upon the subject of probability indirectly, in conjunction with the question concerning the source of entrepreneurial profits and losses' (Hoppe 2007, p. 4).

2.3. Knight's Fellow Travellers (2): Strategic Entrepreneurship Theory

Strategic entrepreneurship theory is 'the integration of entrepreneurial (i.e., opportunity-seeking behaviour) and strategic (i.e., advantage-seeking) perspectives in developing and taking actions designed to create wealth' (Hitt et al. 2001, p. 491). Uncertainty is seen as playing a key role in entrepreneurship and has been particularly associated with the work of Foss and Klein (see, for example, Klein 2008; Foss and Klein 2012; Packard, Clark, and Klein 2017).

Klein (2008) proposes a theory of entrepreneurship grounded in the 'Cantillon-Knight-Mises' understanding of entrepreneurship as judgment combined with the Austrian school's subjectivist account of capital heterogeneity. Klein sees Cantillon's distinction between entrepreneurs and wage earners based on the degree of certainty attached to their income as the origin of the conception of the entrepreneur as bearing uncertainty. It is a conception of the entrepreneur that is developed by both Knight and Ludwig von Mises in their respective theories of profit as the reward to bearing uncertainty successfully, although neither make any reference to Cantillon's work. Klein follows Knight in trying to integrate the analysis of the entrepreneur into the theory of the firm, by focusing on decisions under uncertainty rather than insurable risks. 'In a world of Knightian uncertainty, all profit opportunities involve decisions for which no well-specified profit maximisation problem is available' (Klein 2008, p. 179). Further, 'entrepreneurship is conceived as the act of putting resources at risk, with profit as the reward for anticipating future market conditions correctly, or at least more correctly than other entrepreneurs' (Klein 2008, p. 186). Klein (2008) and Foss and Klein (2012) conceptualise

entrepreneurial action as judgment where judgment is defined as 'dealing successfully with resource allocation decisions under uncertainty' (Foss and Klein 2012, p. 79).

Foss and Klein follow Hoppe (2007) in linking both Knight's concept of statistical probability and Ludwig von Mises's concept of class probability to the frequentist approach of Richard von Mises. Foss and Klein also follow Hoppe in arguing that purposeful human action cannot meet the randomness condition required by the frequentist approach so that class probability cannot be applied to action-events. Foss and Klein suggest that in this respect, Ludwig von Mises goes further than Knight in implying that only case probability is applicable to economic behaviour. Indeed, they consider that the term 'case probability' 'is misleading; what Mises really means is "case non-probabilities", or perhaps "case judgments without probabilities"' (Foss and Klein 2012, p. 87 fn. 9). Foss and Klein accept that Knight went beyond the frequentist approach in treating uncertainty as probabilistic: 'We think that Knight clearly accepted the idea of subjective probability and that he did not claim that it is meaningless to try to assign probabilities under uncertainty' (Foss and Klein 2012, p. 85). But Foss and Klein (2012, p. 85) find Knight's subjectivist position as 'surprising' given that Richard von Mises rejected the subjectivist approach as 'peculiar' (as quoted by Foss and Klein 2012, p. 85 fn. 5). Foss and Klein find it 'more surprising than the association of Knight with subjective probability theory, is the lumping of Knight and Keynes as like-minded proponents of "genuine", "radical", or "deep" uncertainty' (Foss and Klein 2012, p. 85). Foss and Klein (2012, p. 86) suggest that Keynes applied uncertainty to 'unique, rare (investor) situations' whereas Knight saw uncertainty 'as characterising many, reasonably mundane decisions'. Foss and Klein, like Hoppe, seem guilty of rather Procrustean methods of interpretation in regards to both Knight and Keynes (as well as Ludwig von Mises). The defining argument of Knight and von Mises is the subjectivity of behaviour in unique decision situations of uncertainty facing entrepreneurs in which probability cannot be reduced to an objective frequency distribution. Likewise, as discussed below, Keynes's conceives of uncertainty, not as a rare occurrence, but as a characteristic of all investment decisions.

Subsequently, Klein in Packard, Clark, and Klein (2017) adopt a more pluralist interpretation of Knight's approach as including both a subjective dimension (i.e., perceived uncertainty) and an objective dimension (i.e., true uncertainty), as discussed above. Using Shackle's theory of potential surprise (Shackle, 1949), Packard et al. differentiate between options and outcomes sets, as well as differentiating between closed (i.e., finite) and open (i.e., infinite) sets. This leads to four types of uncertainty: (i) risk and ambiguity (defined as closed options set and closed outcomes set); (ii) environmental uncertainty (defined as closed options set but open outcomes set); (iii) creative uncertainty (defined as open options set but closed outcomes set); and (iv) absolute uncertainty (defined as open options set and option outcomes set). Packard et al. define Knight's probability-as-risk as an ergodic stochastic context in which both the options and outcomes sets are closed, and a known probability can be attached to each option-outcome relationship. In contrast, Knight's probability-as-uncertainty is a nonergodic situation in which the options set and/or the outcomes set are open.

In summary, the characteristic focus of the fellow travellers on the Knightian branch of 'The Road Less Travelled' has been on entrepreneurial action under uncertainty, as exemplified by the contributions of Ludwig von Mises and strategic entrepreneurship theory.

Ludwig von Mises parallels the risk/uncertainty distinction in his own distinction between class and case probability but provides a more explicitly philosophically and methodologically grounded approach than Knight, arguing for praxeology as the appropriate method of analysis. A key strand in strategic entrepreneurship theory has been judgment under uncertainty which has led to further sub-classifications of Knightian uncertainty.

3. Keynes on Probability and Uncertainty

3.1. *Keynes's* A Treatise on Probability

Keynes Signpost 1: The Context of Probability and Uncertainty

A Treatise on Probability is primarily a contribution to probability theory, philosophy and statistical theory

Keynes's *A Treatise on Probability* although published in 1921 had a long gestation with Keynes's interest in probability dating back to his undergraduate days and his subsequent fellowship dissertation and originally motivated by a concern with the probabilistic underpinnings of Moore's *Principia Ethica*. In contrast to Knight's *Risk, Uncertainty and Profit* which is primarily a contribution to economic theory, Keynes's *A Treatise on Probability* is primarily a contribution to probability theory, philosophy and statistical theory. Keynes begins with the fundamentals of his logical approach to probability (Part I), which he uses to reformulate the basic theorems of probability (Part II), and then applies this approach to induction and analogy (Part III), other philosophical applications, including conduct (Part IV), and the foundations of statistical inference (Part V).

Keynes Signpost 2: The Scope of Probability

Probability theory is a branch of formal logic dealing with non-demonstrable, indirect knowledge when knowledge is partial

Keynes's principal objective is to generalise logic beyond the study of conclusive arguments characterised by demonstrative certainty to encompass arguments in metaphysics, science and conduct where 'most of the arguments, upon which we habitually base our rational beliefs, are admitted to be inconclusive in a greater or lesser degree' (Keynes 1921, p. 3). For Keynes, the theory of probability is a branch of formal logic concerned with indirect knowledge constituted by non-demonstrable arguments from propositions to conclusions when knowledge is partial. Formally, Keynes defines the probability-relation (P) as representing the rational degree of belief (α) to hold in a proposition (a) given the available evidence (h), and can be written as

$$P(a|h) = \alpha$$

Keynes Signpost 3: The Nature of Probability

Probability is an objective-epistemic concept representing rational degrees of belief in propositions not events; probabilities are subjective relative to individual's available evidence

Keynes conceptualises probability as an epistemic concept pertaining to beliefs rather than an aleatory concept corresponding to real events. Keynes dispenses with the term 'event' altogether and clarifies that his approach is 'to discuss the truth and the probability of propositions instead of the occurrence and the probability of events' (Keynes 1921, p. 5). Keynes's logical theory of probability is an objectivist theory in which probability statements have a similar rule-bound status as valid inferences in deductive logic. However, Keynes recognises that probabilities are subjective to the extent that individuals can differ in their degrees of belief in a given proposition because of differences in their available evidence and/or capabilities to reason logically.

Keynes Signpost 4: *A Priori* Probability

A priori probability is either direct knowledge known by direct acquaintance or derived logically by the principle of indifference

Much of the formal analysis of *A Treatise on Probability* in Part II 'Fundamental Theorems' is concerned with deriving the basic laws of probability that govern the consistency of the relationships between probability relations. But this naturally begs the question of how knowledge of the probability relations is acquired in the first place. Keynes treats *a priori* probability as either direct knowledge known by direct acquaintance or derived logically by the principle of indifference which provides the basis for establishing equiprobability. Keynes recognises that the principle of indifference often leads to paradoxical or contradictory conclusions. He devotes considerable attention to the problems involved in the proper application of the principle of indifference. Keynes establishes that *a priori* equiprobabilities based on indifference require significant contextual knowledge to identify what he calls 'indivisible alternatives' (Keynes 1921, p. 65) which form an exhaustive and mutually exclusive set of possibilities. In addition, the principle of indifference also requires that the relevant evidence must be 'symmetrical with regard to the alternatives, and must be applicable to each in the same manner' (Keynes 1921, p. 60). In other words, indifference requires the absence of any evidence that favours one or more alternatives over others.

Keynes Signpost 5: The Status of Frequency Theory

Frequency theory is based on invalid logical foundations, but there is a fundamental sympathy with its deep underlying conceptions of statistical theory; a generalised frequency theory needs to be developed

Keynes characterises the frequency theory of probability as concerned with a group, class or series of events in which probability equals the proportional frequency with which events tend to recur in a long run of trials. Keynes recognises that frequency probabilities are defined relative to a group with empirical knowledge of the group as a necessary prerequisite. Keynes identifies two main difficulties with frequency theory — the theoretical and practical difficulty of measuring and comparing probabilities and the difficulty of determining *a priori* probability. Keynes considers much of statistics to be based 'upon an inconsistent logical scheme, which, avowedly founded upon a theory of frequency, introduces principles which this theory has no power to justify' (Keynes 1921, p. 120). But, despite his concerns with the 'want of logic' (Keynes 1921, p. 468) of statistical

theory, Keynes remained 'in fundamental sympathy with the deep underlying conceptions of the statistical theory of the day' (Keynes 1921, p. 468). Rather than rejecting frequency theory altogether, Keynes proposes instead to develop a 'generalised frequency theory' (Keynes 1921, p. 109) that 'does not regard probability as being identical with statistical frequency, although it holds that all probabilities must be based on statements of frequency' and accepts that 'propositions rather than events should be taken as the subject-matter of probability' (Keynes 1921, p. 110).

Keynes Signpost 6: The Measurability of Probability

Numerical probabilities are a special case given undue attention; probabilities can be (i) numerical; (ii) non-numerical but comparable; (iii) non-comparable; or (iv) unknown

Keynes is particularly critical of the assumption that all probabilities are numerical, contending that numerical probabilities are a special case. 'The attention, out of proportion to their real importance, which has been paid to, on account of the opportunities for mathematical manipulation which they afford, to the limited class of numerical probabilities, seems to a part explanation of the belief, which it is the principal object of this chapter to prove erroneous, that all probabilities must belong to it' (Keynes 1921, p. 40). Keynes differentiates between four types of probabilities as regards their measurability and comparability: (i) numerical; (ii) non-numerical but comparable; (iii) non-comparable; and (iv) unknown. Keynes demonstrates that it is possible to construct ordered series (or paths) of probabilities which he illustrates diagrammatically and shows that these paths may intersect and cross depending on the degree of comparability of individual probabilities.

Keynes Signpost 7: The Justification of Induction

There are two general requirements for the valid application of the inductive method: (i) limited variety; and (ii) the atomic assumption

Keynes considers induction as a vital habit of mind that we use whenever we learn from experience (Keynes 1921, p. 241). But experience can only justify the probability of inductive arguments *a posteriori* not *a priori*. 'The inductive hypothesis stands in a peculiar position in that it seems to be neither a self-evident logical axiom nor an object of direct acquaintance' (Keynes 1921, pp. 293–294). Rather 'the probability we are right, when we make predictions on the basis of past experience depends not so much on the number of past experiences upon which we rely, as on the degree to which the circumstances of these experiences resemble the known circumstances in which the prediction is to take effect' (Keynes 1921, p. 267). Keynes rejects the proposition that material laws such as the law of uniformity of nature provide a justification for the inductive method. Instead, Keynes identifies the two general requirements for the valid application of the inductive method: (i) the world is a finite system in which variety is limited; and (ii) the world is atomic rather than organic. The validity of both assumptions, and, in turn, the validity of the inductive hypothesis depend on the specific context of its application.

Keynes Signpost 8: The Nature of Evidence

The weight of argument provides a measure of relevant evidence comprising pure induction, analogy and negative analogy

A common theme throughout *A Treatise on Probability* is that probabilities are always evidence-based, but this dependency is often overlooked. Keynes's basic definition of probability highlights the conditionality of probability on the available evidence. Keynes introduces the concept of the weight of a probability argument (V) to represent the amount of available relevant evidence (h). If additional relevant evidence (h_1) becomes available, then it follows that:

$$V(a|hh_1) > V(a|h)$$

Keynes identifies three broad types of evidence: pure induction, analogy and negative analogy. Pure induction represents evidence from identical instances. Analogy is evidence of similarity across instances. Negative analogy is evidence of variability in non-essential conditions across instances. Keynes highlights the crucial role of negative analogy in strengthening empirical generalisations and criticises Hume's account of induction for the lack of any explicit discussion of the role of negative analogy.

Keynes Signpost 9: Confidence and Conduct

Conduct cannot be reduced to the calculation of mathematical expectation; conduct depends not only on probability but also the weight of argument, the risk of loss and an intuitive judgment of the situation as a whole

Keynes is critical of the use of mathematical expectation as a guide to conduct. He argues that conduct depends not only on probability but also the weight of argument, the risk of loss and an intuitive judgment of the situation as a whole. Keynes considers combining probability, weight and risk in what he calls a 'conventional coefficient' (Keynes 1921, p. 348) but concludes that the problems with mathematical expectation are unlikely to be resolved 'in the discovery of some more complicated function of the probability wherewith to compound the proposed good' (Keynes 1921, p. 348).

Keynes Signpost 10: The Correctness of Probability

The correctness of probability is determined *ex ante* as a matter of logic based on the available evidence, not *ex post* based on the outcomes of actions

Keynes argues that the correctness of probability is determined *ex ante* as a matter of logic based on the available evidence, not *ex post* based on the outcomes of actions. Keynes quotes Herodotus on the point: 'There is nothing more profitable for a man than to take good counsel with himself; for even if the event turns out contrary to one's hope, still one's decision was right, even though fortune has made it of no effect' (Keynes 1921, p. 339). Probability and induction are questions of logic, not experience, that are not upset if truth turns out otherwise. 'Induction tells us that, on the basis of certain evidence, a certain conclusion is reasonable, not that it is true' (Keynes 1921, p. 273).

3.2. Keynes's Later Philosophical Thought

After the publication of *A Treatise on Probability* in 1921, Keynes's priority was economics, culminating in the publication of the *General Theory* in 1936. But, as Coates (1996, 1997) shows, Keynes's philosophical thought continued to develop during the formative years of the *General Theory* particularly through frequent and lengthy conversations with Wittgenstein and, until his tragically early death in January 1930, Ramsey. Wittgenstein was at the forefront of the anti-formalist developments in Cambridge in the interwar years as he moved from the analytical philosophy of his *Tractatus Logico-Philosophicus* to the ordinary language philosophy of *Philosophical Investigations*. Just as Wittgenstein moved towards a thicker, more context-based understanding of language, so Keynes moved to a thicker, more context-based perspective on probability and uncertainty focused on the economic behaviour of the capitalist economy.

Misak (2016) argues that the anti-formalist developments in Cambridge, England paralleled similar but earlier anti-formalist developments originating in Cambridge, Massachusetts, in the early 1870s led by the American pragmatists, C. S. Peirce and William James. Misak identifies Ramsey as the crucial figure who was influenced primarily by the 'getting-it-right' pragmatism of Peirce, stressing the importance of both truth and effectiveness as criteria for reasonable belief to guide action in contrast to the instrumentalist 'what-works' pragmatism associated with James. Crucially, as regards his impact on Keynes, Ramsey cites Peirce as the source of the distinction between formal logic and human logic that he uses in his critique of Keynes's logical theory of probability (see below). Bateman (2021) has also recently recognised the possible influence of Ramsey's pragmatism on Keynes. But this remains controversial with O'Donnell (2021) rejecting any substantive influence of Ramsey on Keynes.

Direct textual evidence of Keynes's later philosophical thought is fragmentary with four principal sources: (i) the 1926 biographical essay of Edgeworth; (ii) the 1926 letter to Urban, the German translator of the *A Treatise on Probability*; (iii) the 1931 review of Ramsey's *The Foundations of Mathematics and Other Logical Essays*; and (iv) Keynes's autobiographical essay, 'My Early Beliefs', read to the Bloomsbury Group in 1938.

Keynes's more context-based conception of human nature is evidenced by 'My Early Beliefs' in which he recognises his early beliefs as being based on a too rational view of human nature, so much so that 'we were not aware that civilisation was a thin and precarious crust … We completely misunderstood human nature, including our own. The rationality which we attributed to it led to a superficiality, not only of judgment, but also of feeling' (Keynes 1972, pp. 447–448).

One important consequence of the fuller contextualisation of Keynes's thought was the greater significance attached to his criticism of the relevance of the atomic hypothesis for the understanding of human behaviour as evidenced in the biographical essay on Edgeworth in which Keynes states that 'the atomic hypothesis which has worked so splendidly in physics breaks down in psychics' (Keynes 1972, p. 262). The limited applicability of the atomic hypothesis in the study of human behaviour had been a long-standing theme in Keynes's writings as, for example, in his early discussions of the problems of complexity and incommensurability in economics (Carabelli 2021). It also underpins his criticisms in *A Treatise on Probability* of the method of

mathematical expectation, associated with Edgeworth amongst others, as a basis for reasonable behaviour. Given that Keynes had identified the atomic hypothesis as a necessary requirement for the inductive method, the rejection of its relevance for human behaviour has profound implications for the methodology of economics. Keynes's biographical essay on Edgeworth provides a clear statement that the rejection of the atomic hypothesis had become a key methodological aspect of Keynes's break from orthodox economic theory.

In his letter to Urban, Keynes expresses dis-satisfaction with *A Treatise on Probability* but still considers that 'the problems as I have posed them may be the right starting point for further research' even although he now believes that 'the ultimate theory may differ very considerably from mine', specifically, 'some kind of frequency theory will be found in the end to be more fundamental to the whole conception of Probability than I have yet allowed' (as quoted in O'Donnell 1989, pp. 144–145). These comments are consistent with Keynes's view in the *A Treatise on Probability* that the logical theory of probability is a move towards a generalised frequency theory and re-iterate that Keynes considered his approach to be indicative of the way forward rather than definitive, particularly as regards the justification of induction. As Keynes (1972, p. 339) states in his review of Ramsey's *Foundations of Mathematics*, 'It is not getting to the bottom of the principle of induction merely to say that it is a useful mental habit'.

Keynes used the review of Ramsey's *Foundations of Mathematics* to respond to Ramsey's critique of the logical theory of probability in the essay, 'Truth and Probability', originally read in part to the Moral Sciences Club in Cambridge in 1926 but published in full only posthumously in *Foundations of Mathematics* (Ramsey 1931, pp. 156–198). Ramsey's principal criticism of the logical theory is the concept of *a priori* probability, arguing that the basis for degrees of belief lies outside formal logical analysis. Keynes accepts Ramsey's view that 'the calculus of probabilities belongs to formal logic' and 'simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a consistent system' in contrast to what Ramsey terms 'human logic' concerned with 'the basis of our degrees of belief' and as such distinct from both formal logic and descriptive psychology (Keynes 1972, pp. 338–339).

3.3. Keynes on Probability and Uncertainty in the General Theory

Whereas Knight's approach to probability and uncertainty was from the very outset contextualised in the economic behaviour of entrepreneurs, Keynes's approach to probability and uncertainty was only finally fully contextualised in the economic behaviour of the capitalist economy in the *General Theory*, the QJE 1937 summary of the key arguments, the 1938 correspondence with Townshend on risk and liquidity premia, and his 1939 critique of Tinbergen's econometric methods, some 15 years and more after the publication of *A Treatise on Probability*. That is not to say that Keynes did not contextualise some aspects of his approach to probability and uncertainty in his early economic writings. For example, as Carabelli (2002, 2021) and Carabelli and Cedrini (2013) demonstrate, there are clear influences of Keynes's approach to probability and uncertainty in his 1910 lectures on speculation particularly the distinction between calculable and non-calculable risk, as well as the distinction between speculation and gambling related to the degree of (partial) knowledge with speculation viewed as knowledge-based reasonable

behaviour and gambling as behaviour when there is insufficient knowledge to determine a reasonable course of action.

Keynes only made one explicit reference in the *General Theory* to *A Treatise on Probability* when he states: 'By "very uncertain" I do not mean the same thing as "very improbable", Cf. my *Treatise on Probability,* chap. 6, on "The Weight of Arguments"' (Keynes 1936, p. 148, fn. 1). But this reference is critical since it clarifies that for Keynes, the degree of uncertainty is not a matter of belief (i.e., probability) but depends on the amount of available relevant evidence (i.e., weight of argument). Uncertainty represents a situation in which 'there is no scientific basis on which to form any calculable probability' (Keynes 1937, p. 214).

Keynes's evidence-based approach to uncertainty is fundamental to his definition of the state of long-term expectations, which consists of two components: the most probable forecast and the state of confidence. Keynes defines confidence as 'how highly we rate the likelihood of our best forecast turning out quite wrong. If we expect large changes but are very uncertain as to what precise form these changes will take, then our confidence will be weak' (Keynes 1936, p. 148). It is at this point that Keynes refers to the weight of argument. This dualistic conception of the state of long-term expectations parallels the distinction in *A Treatise on Probability* between the rational degree of belief (i.e., probability) and the weight of argument.

It is the impact of uncertainty on the investment decision through the state of long-term expectations that results in the capitalist economy having the knife-edge property of extreme precariousness yet still displaying a high degree of stability. Both the precariousness and stability are rooted in the epistemic aspects of the investment decision. 'The outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of prospective yield have to be made' (Keynes 1936, p. 149). But despite its inherent precariousness, the capitalist economy remains highly stable because of the convention of 'assuming that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change' (Keynes 1936, p. 152). We are aware that uncertainty renders our knowledge precarious and subject to change but our convention of assuming that the immediate future will be like the past until we have specific evidence of change results in stability, albeit a fragile and precarious stability.

Keynes's critique of the doctrine of mathematical expectation and his recognition in 'My Early Beliefs' of a need for a less rationalistic conception of human behaviour are also contextualised in the *General Theory* when he argues that decisions to act are not a matter of calculation but ultimately depend on animal spirits: 'our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits — of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities' (Keynes 1936, p. 161).

In summary, Keynes's own journey on 'The Road Less Travelled' after 1921 focused mostly on the contextualisation of his logical theory of probability in macroeconomic theory. Two of the most crucial features of his post-1921 journey are the recognition of (i) the need for an organicist ontology to understand human behaviour; and (ii) the importance of the concept of weight of argument for the understanding of uncertainty

4. Keynes and Knight on Probability and Uncertainty: A Way Forward?

The ten signposts identified in Knight's *Risk, Uncertainty and Profit* and Keynes's *A Treatise on Probability* are summarised in Table 1. The basic contention of this paper is that, despite the significant degree of commonality in these signposts, particularly the first six signposts, the fundamental differences between Knight and Keynes are such as to preclude a meaningful synthesis on probability and uncertainty that could point the way

Table 1. Signposts for 'The Road Less Travelled': Knight and Keynes compared.

Signpost	Issue	Knight, *Risk, Uncertainty and Profit*	Keynes, *A Treatise on Probability*
Signpost 1	The Context of Probability and Uncertainty	Probability and uncertainty are contextualised in the economic theory of profit as the return to entrepreneurs for bearing uncertainty successfully	*A Treatise on Probability* is primarily a contribution to probability theory, philosophy and statistical theory
Signpost 2	The Scope of Probability	Probability is concerned with partial knowledge when rigorous demonstration is not applicable	Probability theory is a branch of formal logic dealing with non-demonstrable, indirect knowledge when knowledge is partial
Signpost 3	The Nature of Probability	Probability is dualistic in nature, encompassing both the aleatory conception and the epistemic conception	Probability is an objective-epistemic concept representing rational degrees of belief in propositions not events; probabilities are subjective relative to individual's available evidence
Signpost 4	*A Priori* Probability	*A priori* probability is a type of probability-as-risk applicable when outcomes are equiprobable and mutually exclusive	*A priori* probability is either direct knowledge known by direct acquaintance or derived logically by the principle of indifference
Signpost 5	The Status of Frequency Theory	Frequency theory is applicable only to statistical probability which the type of probability-as-risk relevant for repeatable and relatively homogeneous events	Frequency theory is based on invalid logical foundations, but there is a fundamental sympathy with its deep underlying conceptions of statistical theory; a generalised frequency theory needs to be developed
Signpost 6	The Measurability of Probability	*A priori* and statistical probability are measurable, but probability estimates applicable in situations of true uncertainty are not susceptible to measurement	Numerical probabilities are a special case given undue attention; probabilities can be (i) numerical; (ii) non-numerical but comparable; (iii) non-comparable; or (iv) unknown
Signpost 7	The Justification of Induction	Inductive knowledge is justified by the law of regularity that the world consists of objects which, under the same circumstances, always behave in the same way	There are two requirements for the valid application of the inductive method: (i) limited variety; and (ii) the atomic assumption
Signpost 8	The Nature of Evidence	The evidence underlying the different types of probability constitutes a continuum from perfect homogeneity of classes of instances to extreme heterogeneity of unique singular instance	The weight of argument provides a measure of relevant evidence comprising pure induction, analogy and negative analogy
Signpost 9	Confidence and Conduct	Under conditions of uncertainty, conduct depends on the best estimate and the degree of confidence in that estimate as measured by probability	Conduct cannot be reduced to the calculation of mathematical expectation; conduct depends not only on probability but also the weight of argument, the risk of loss and an intuitive judgment of the situation as a whole
Signpost 10	The Correctness of Probability	The correctness of the entrepreneur's judgment (i.e., probability estimate) of the future relative to that of their competitors is determined *ex post* by the profit earned	The correctness of probability is determined *ex ante* as a matter of logic based on the available evidence, not *ex post* based on the outcomes of actions

forward on 'The Road Less Travelled' irrespective of whether the point of departure is Keynesian or Knightian.

The possibility of a Keynesian-Knightian synthesis has been a common theme in radical/post-Keynesian theory with Keynes's approach to probability and uncertainty often presented in terms of Knight's risk/uncertainty distinction as, for example, in Faulkner, Feduzi, and Runde (2017) who locate Keynes's various real-world examples on a risk/uncertainty spectrum. Both Fontana and Gerrard (1999) and Dow (2016) use an augmented risk/uncertainty distinction, including situations of ambiguity to characterise the differences between mainstream and Keynesian approaches. But a detailed comparison of the ten signposts indicates that, despite the initial appearance of considerable congruence between the two approaches, there are fundamental differences between the Keynesian and Knightian approaches so much so that there seems little to be gained by radical/post-Keynesian from any attempt to integrate the two approaches.

4.1. Signpost 1: The Context of Probability and Uncertainty

In considering a possible Keynesian-Knightian synthesis, it is important to recognise that Knight and Keynes followed vastly different routes in developing their understanding of probability and uncertainty. Knight's approach from the outset was fully contextualised in economic behaviour, avoiding the philosophical issues involved in probability. In contrast, Keynes's *A Treatise on Probability* is principally concerned with developing propositional logic and its real-world implications (Roncaglia 2009), although it was not until the *General Theory* that Keynes more fully contextualised his approach to probability as an economic theory of the impact of uncertainty on the behaviour of the capitalist economy.

Uncovering the links between *A Treatise on Probability* and Keynes's later economic writings has been the core objective of the 'new Keynesian fundamentalism' (Gerrard 2012). This research, initially centred in Cambridge, began in the mid-1970s with the circulation of the first draft of Gay Meeks's paper on the philosophical foundations of Keynes's theorising on the investment decision (Meeks 1991) which Runde and Mizuhara (2003, p. 3) recognise as 'setting the tone and questions for much of the literature that followed'. The other ground-breaking Cambridge-based contributions are Lawson (1985, 1988), Carabelli (1988) and O'Donnell (1989).

Keynes's intellectual route meant that the philosophical foundations of his logical theory of probability are much more detailed and the issues to be confronted in contextualising probability as a theory of economic behaviour under uncertainty are more apparent. Hence, Keynes's contextualised conception of uncertainty provides a more appropriate basis for developing a radical alternative to mainstream theories.

4.2. Signpost 2: The Scope of Probability

Both Keynes and Knight consider the scope of probability as the logic of non-demonstrable reasoning under conditions of partial knowledge. On this point, they differ only with respect to the degree to which they formalise the logical aspects of probability. Again Keynes's more detailed treatment provides a firmer foundation for developing the formal logic of probability theory.

4.3. Signpost 3: The Nature of Probability

Both Keynes and Knight treat probability as a general concept relevant to all situations of partial knowledge, but they differ fundamentally over whether probability is epistemic or aleatory in nature. Knight adopts a dualistic approach to probability, aleatory in situations of risk (i.e., *a priori* and statistical probability) and epistemic in situations of uncertainty (i.e., probability estimates). Ludwig von Mises adopts a similar dualistic approach with class probability (i.e., risk) as aleatory and case probability (i.e., uncertainty) as epistemic. Keynes's logical theory of probability is also a general theory but unitary in nature, treating probability as purely epistemic. The all-encompassing approaches of Keynes and Knight are in stark contrast to that of Richard von Mises who adopts a restricted definition of probability, reserving the term 'probability' to situations in which a collective consisting of a sequence of uniform events or processes can be defined which, in Knight's terminology, would constitute situations of risk.

Keynes's position is the more consistent approach and is capable of encompassing Knight's approach. The fundamental difference is that Knight treats probability as concerned with a series of repeatable events in the case of risk but with propositions about a unique event in the case of uncertainty, whereas Keynes's logical theory of probability treats probability as only concerned with propositions. Knight's approach can be encompassed within Keynes's approach by treating *a priori* and statistical probability as propositions about a series of repeatable events and hence epistemic in nature. This reinterpretation of *a priori* and statistical probability changes nothing fundamentally in Knight's theory of profit.

4.4. Signpost 4: A Priori Probability

Both Keynes and Knight recognise the concept of *a priori* probability (which is rejected by both von Mises brothers and Ramsey) but there are important differences in their approach. For Knight, *a priori* probability plays a rather restricted role as a polar case of risk when the stochastic structure is known and alternative outcomes are mutually exclusive and equiprobable (Runde 1998). In most situations of risk, Knight treats probability as statistical, derived empirically. However, in Keynes's logical theory, *a priori* probability is more foundational since all probabilities are ultimately dependent on an initial probability known *a priori* either directly via direct acquaintance or indirectly via the law of indifference. Ramsey's critique of Keynes's concept of *a priori* probability is significant because it undermines Keynes's central claim that probability represents the rational (or objective) degree of belief given the available evidence. As noted above, Keynes conceded to Ramsey that the formal logical analysis of probability should be restricted to ensuring that probability statements form a consistent logical system with initial probabilities seen as a matter of human, not formal, logic. But as Keynes pointed out in the context of induction, there is more to human logic than merely being a useful mental habit, suggesting that further formal analysis is possible and necessary (O'Donnell 1989, 2021). Hence, it does not necessarily follow that conceding that *a priori* probability cannot be known directly implies acceptance of a personal/subjective theory of probability as advocated, for example, by De Finetti (1964) and Savage (1954). Furthermore, Ramsey's critique does not rule out *a priori* probability based on

the law of indifference in appropriate contexts, on which both Keynes and Knight agree. Ultimately the significance of *a priori* probability depends on the specific context.

4.5. Signpost 5: The Status of Frequency Theory

Relating back to Signpost 3 on the nature of probability, Keynes and Knight adopt different but not necessarily fundamentally inconsistent views on frequency theory. Knight, like Ludwig von Mises, treats frequency theory as typified by Richard von Mises's approach, as a special case applicable only to statistical/class probability that needs to be generalised by the development of a theory of estimates/case probability. Keynes also argues that frequency theory needs to be generalised, but he goes further than Knight in not only criticising the limitations in its applicability but also arguing that it is based on invalid logical foundations despite providing deep underlying conceptions for statistical theory. Hence Keynes argues for a generalised frequency theory involving a thorough recasting of probability in terms of propositions not events *per se*.

4.6. Signpost 6: The Measurability of Probability

Both Keynes and Knight agree that probabilities are not necessarily measurable. Their differences on this point are a matter of degree that, in Knight's terminology, comes down to where the line is drawn between measurable risk and non-measurable uncertainty. There is a sense that Keynes goes further than Knight in limiting the scope of numerical probabilities, but ultimately, this is an issue that depends on the specific context.

4.7. Signpost 7: The Justification of Induction

The remaining signposts involve fundamental differences between the approaches of Keynes and Knight largely due to Keynes's *A Treatise on Probability* representing a much more fully developed philosophical treatment of probability. The justification of induction is a case in point. Whereas Knight accepts that induction is justified by the law of regularity, Keynes goes much deeper in his analysis to show that induction ultimately depends on the twin assumptions of limited variety and atomism. Knight does not recognise the importance of the atomic hypothesis, although Ludwig von Mises does so indirectly in his argument that the study of human behaviour requires an alternative methodology, praxeology, to that of the physical sciences. The status of the atomic assumption is particularly crucial for Keynes when he comes to contextualise his approach in the study of economic behaviour. As his biographical essay on Edgeworth makes clear, Keynes restricts the relevance of the atomic hypothesis (i.e., closed systems) to the realm of physics and argues for an organicist hypothesis in the realm of psychics (i.e., human behaviour). This is the route followed by Tony Lawson whose initial work on Keynes's theories of probability and uncertainty has led to the articulation of a fully-fledged social ontology of an open-systems approach to the understanding of human behaviour in its social context (Lawson 1997, 2003, 2019).

4.8. Signpost 8: The Nature of Evidence

For both Keynes and Knight, the nature of evidence is fundamental to their respective approaches to probability with again the greater depth of the philosophical analysis in Keynes's *A Treatise on Probability* leading to a critical difference. For Knight, the degree of heterogeneity in the evidence is the fundamental distinguishing feature in his tripartite classification of probability, conceiving of a continuum from the perfect homogeneity of classes in *a priori* probability through the increasing heterogeneity of statistical probability to the extreme heterogeneity of unique singular instances in probability estimates. Keynes goes much deeper, distinguishing between pure induction, analogy and negative analogy. Keynes stresses the role of negative analogy (i.e., heterogeneity) in strengthening the evidential basis of a generalisation, and in this respect differs markedly from Knight whose risk/uncertainty distinction treats greater heterogeneity as restricting the relevant evidential base. The crucial importance of the evidential basis of probability in Keynes's approach is highlighted by both his oft-repeated stricture that probability is always relative to the available evidence, and the development of the concept of the weight of argument as a measure of the evidential basis of a probability statement. Keynes encountered several problems in producing an operational definition of weight of argument (Runde 1990) with the evaluation of the amount of analogy and negative analogy as the root cause of these difficulties. Operationalising the weight of argument lies at the core of Keynes's proposed generalised frequency theory.

4.9. Signpost 9: Confidence and Conduct

Another fundamental difference between Keynes and Knight that follows from the differences in their treatment of evidence is the role of confidence in conduct. Both Keynes and Knight adopt a dualistic conception of how probability affects conduct. Knight argues that entrepreneurial action depends on both the 'best estimate' and 'confidence' which appears to parallel Keynes's conception of the state of long-term expectations as comprising the most probable forecast and the state of confidence. However, Knight views confidence as the degree of certainty, which he defines as the probability that the estimate is correct. Crucially Keynes views confidence, as the footnote on page 148 of the *General Theory* clearly states, in terms of weight of argument and uncertainty rather than probability. When Keynes discusses confidence and the degree of uncertainty, he is concerned with the available evidence on which the most probable forecast is based (i.e., the empirical basis of the proposition), whereas Knight treats confidence and the degree of uncertainty as the probability that the best estimate is correct. In addition, Keynes goes further than Knight in stressing the non-rational aspects of conduct under uncertainty, criticising the doctrine of mathematical expectation and highlighting the role of the intuitive judgment in *A Treatise on Probability* and animal spirits in the *General Theory*.

There is a growing consensus in recent radical/post-Keynesian contributions on the importance of the weight of argument in Keynes's analysis. Dow (2016) links the Keynesian concept of fundamental uncertainty to weight of argument in her non-dualistic framework of risk, ambiguity and fundamental uncertainty. Zappia (2016) identifies weight of argument as one of the three fundamental aspects of *A Treatise of Probability*, alongside non-numerical probabilities and the rejection of the doctrine of mathematical

expectation. Faulkner, Feduzi, and Runde (2017) also recognise the importance of Keynes's notion of evidential weight when arguing for the need to expand Keynes's concept of uncertainty to include unimagined possibilities.

4.10. Signpost 10: The Correctness of Probability

A final fundamental difference between Keynes and Knight is their treatment of the correctness of probability. For Keynes, the correctness of probability is an *ex-ante* matter of logic, whereas for Knight, it is determined *ex post* by the profit outcome of the action justified by the probability. In part, this difference reflects the difference in perspective, with Keynes focusing on the formal logic of probability and Knight focusing on the explanation of profit outcomes. It also reflects Knight's avoidance of any detailed consideration of philosophical issues. But, as recognised by Faulkner, Feduzi, and Runde (2017), any consideration of probability and uncertainty inevitably raises epistemological issues on the relationship between truth, knowledge and belief. Knight, at least implicitly, appears to be adopting a correspondence theory of truth. Keynes's position remains more open and can be interpreted as consistent with the realist Peirce-Ramsey 'getting-it-right' strand of pragmatism.

The Keynesian way forward on 'The Road Less Travelled' is summarised in Table 2.

5. Summary and Conclusions

Keynes and Knight both grasped the essential difference between probability-as-risk and probability-as-uncertainty, but they travelled along vastly different roads to get there. Knight contextualised risk and uncertainty in the economic theory of profit as the reward for successful entrepreneurial action under uncertainty. The consequence of Knight's emphasis on context is that the philosophical foundations of his approach are

Table 2. The Keynesian way forward on 'The Road Less Travelled'.

Signpost	Issue	The Keynesian way forward
Signpost 1	The Context of Probability and Uncertainty	Probability and uncertainty contextualised in economic theory
Signpost 2	The Scope of Probability	Probability as the logic of non-demonstrable reasoning with partial knowledge
Signpost 3	The Nature of Probability	Probability as the rational degree of belief in a proposition given the available evidence
Signpost 4	*A Priori* Probability	*A priori* probability as a matter of human logic but can be derived logically when the law of indifference is applicable
Signpost 5	The Status of Frequency Theory	Frequency theory needs to be recast as a special case dealing with propositions about repeatable ergodic events
Signpost 6	The Measurability of Probability	Probability is not necessarily measurable
Signpost 7	The Justification of Induction	Induction requires further formal analysis; the inductive method has restricted applicability in the domain of economic/social behaviour due to the need to adopt an organicist ontology
Signpost 8	The Nature of Evidence	Evidence needs to be analysed in terms of pure induction, analogy and negative analogy
Signpost 9	Confidence and Conduct	Confidence as the degree of uncertainty associated with the amount of available relevant evidence (i.e., weight of argument), not probability *per se*; conduct depends on confidence, intuition and animal spirits, not just calculation
Signpost 10	The Correctness of Probability	Correctness of probability as an *ex-ante* matter of logic

less developed. Keynes's road was much longer, more circuitous and initially primarily concerned with the philosophical foundations, culminating in *A Treatise on Probability* before more fully contextualising his logical theory of probability in the behaviour of the economic system as a whole.

The different roads followed by Keynes and Knight have had one crucial consequence. Keynes's greater emphasis on the philosophical issues led him ultimately to treat uncertainty as relating to the weight of argument (i.e., the evidential base), not probability *per se*, whereas Knight defined uncertainty in terms of probability (i.e., the degree of belief), not the evidential base that determined the degree of belief. This fundamental difference is one of the main reasons that preclude a meaningful Keynesian-Knightian synthesis as a road for others to follow.

The most notable fellow travellers on the Knightian branch of this road less travelled have been Ludwig von Mises and strategic entrepreneurship theory. The Keynesian branch has been followed by the radical/post-Keynesian school with considerable scholarship on *A Treatise on Probability* and the associated emergence of the Cambridge social ontology project associated primarily with the work of Lawson. The challenge issued by Keynes in response to the critique of his work by Ramsey remains. That challenge is to develop the study of human logic which goes beyond mere descriptive psychology and gets to the bottom of the principle of induction, and in so doing provides the basis for a generalised frequency theory of probability. As evident in several recent radical/post-Keynesian contributions, there is a growing awareness that Keynes's conception of uncertainty as the weight of argument is a key step on 'The Road Less Travelled'. But to date, unfortunately, this road remains much less travelled due to the deep-rooted attachment in mainstream economics to de-contextualised theory that provides universality and mathematical precision at the cost of practical relevance.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

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