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The zero helicity and chirality of optical vortices

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ABSTRACT

We show that any uniformly linearly-polarised paraxial vortex mode carrying orbital angular momentum (OAM) has zero spin angular momentum (SAM) density, but exhibits non-zero helicity density distributions. Such a mode then possesses chirality as confirmed by experiment and so can engage with chiral matter. We show that confining the treatment for the general paraxial fields only to leading order leads directly to agreement of our theory with the experimental results, provided we ensure that crucially the paraxial fields obey duality. We find that the space integral of the helicity and chirality densities vanish identically for all such optical vortex modes without specifying the kind of mode. These generally applicable properties of optical vortex modes carrying orbital angular momentum thus assert that without optical spin due to elliptical wave polarisation of index σ, an optical vortex alone cannot possess total helicity, even though it always exhibits non-zero helicity density distributions.

The property of optical helicity is one of four fundamental properties of light; the other three are energy, momentum and angular momentum. The latter three, when considered in the contexts of optical vortex light have been reasonably well understood, but further work still remains to be done as regards knowledge of optical helicity and chirality. Our main goal in this article is to highlight novel features of optical helicity and chirality which, we emphasise, are the preserve of all paraxial optical vortex modes with uniform linear polarization.

The origins of the concept of helicity are traceable to the area of fluid dynamics which highlights helicity as an invariant topological property [1]. The helicity density is defined as the dot product of the fluid velocity $\mathbf{u}$ and the vorticity $\boldsymbol{\Omega} = \nabla \times \mathbf{u}$. In order to define the helicity density in the optical physics context one assigns the roles of the fluid velocity $\mathbf{u}$ and the vorticity $\boldsymbol{\Omega}$ to the vector potential $\mathbf{A}$ and the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$, respectively [2–4]. In addition to the areas of optical physics and fluid dynamics, helicity plays a role in diverse areas, as, for example, in plasma physics and astrophysics [5]. Furthermore, it is known to involve knots and links [6–11] and has led to applications in a number of inter-disciplinary contexts such as molecular biology [12] and particle physics [13]. It is, in fact, the optical part of chiral light-matter interactions that is responsible for all natural forms of optical activity.

In the recent literature (see, for example,[14,15]), the general expression for the helicity density of an electromagnetic field in free space is given in the Coulomb gauge and has the following form

$$\eta = \frac{1}{2}\left(Z_0^{-1}\mathbf{A}\cdot\mathbf{B} - Z_0\mathbf{C}\cdot\mathbf{D}\right) \tag{1}$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is the impedance of free space. The corresponding expression for the chirality density is

$$\chi = \frac{1}{2}\left(-\epsilon_0\mathbf{E}\cdot\dot{\mathbf{B}} + \mathbf{B}\cdot\dot{\mathbf{D}}\right) \tag{2}$$

where $\mathbf{D} = \epsilon_0\mathbf{E}$ is the displacement field and $\mathbf{C}$ is the dual vector potential such that $\nabla\times\mathbf{C} = -\mathbf{D}$ and $\dot{C} = -\mathbf{B}/\mu_0$. Here we will be dealing with cycle-averaged properties of monochromatic fields in which case the cycle-averaged spin angular momentum (SAM) density is defined as

$$\bar{\mathbf{s}} = \frac{\epsilon_0}{\omega}\Im[\mathbf{E}^*\times\mathbf{E}] \tag{3}$$

while the cycle-averaged helicity density $\bar{\eta}$ and chirality density $\bar{\chi}$ are given by

$$\begin{aligned}\bar{\eta} &= -\frac{\epsilon_0 c}{2\omega}\Im[\mathbf{E}^*\cdot\mathbf{B}] \\ &= \frac{c}{\omega^2}\bar{\chi}\end{aligned} \tag{4}$$

The evaluation of the above cycle-averaged densities of any optical vortex mode requires the correct specification of both the electric field $\mathbf{E}$ and the corresponding magnetic field $\mathbf{B}$. All electromagnetic fields, including the fields of optical vortex beams invariably have a longitudinal electric field component in addition to the transverse electric field components, as in the case of a Laguerre–Gaussian mode [16].

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A major property of the optical vortex mode is the cycle-averaged total helicity $\bar{\mathcal{H}}$ which is defined as the space integral of the cycle-averaged helicity density $\bar{\eta}$. We have

$$\bar{\mathcal{H}} = \int d^3\mathbf{r}\ \bar{\eta} \tag{5}$$

A form of the total helicity was first put forward by Woltjer [17] who called it the magnetic helicity and interpreted it to be a topological invariant representing the extent of the magnetic field linkage. As in the case of fluid dynamics the total optical helicity is conserved in a given light beam since the vortex lines are frozen-in [1]. The total optical helicity is related to the Hopf index $\mathcal{N}$ by the relation [3]

$$\mathcal{N} = \frac{1}{Q}\bar{\mathcal{H}} \tag{6}$$

where Q is an action constant which is characteristic of the vortex light mode. Similarly the total spin angular momentum $\bar{\mathcal{S}}$ is the space integral of the density $\bar{s}$.

Our aim here is to derive the above cycle-average properties which are common to any of the paraxial optical vortex modes, without saying which. As regards helicity, we make use of the property of topological invariance to deduce the Hopf index $\mathcal{N}$ and derive a general expression for the action constant Q. We also show that in the case we are considering of a uniformly linearly-polarised optical vortex, the helicity and chirality densities do not vanish, but their space-integrals both vanish. These general properties, as far as we know, have not been previously highlighted as applicable in general for any optical vortex without specifying which.

The current activity and interest in optical helicity and chirality have been invigorated by the report of the experiment by Wozniak et al. [18] which has shown that linearly polarised twisted light in the form of Laguerre–Gaussian light exhibits a chiral behaviour. Displaying their experimental helicity density distributions due to two otherwise identical doughnut modes for which the winding numbers were $\ell = 1$ and $\ell = -1$, Wozniak et al. [18] thus confirmed the typical chirality feature: that the density distributions of such a linearly-polarised light show lack of mirror symmetry since the two densities could not be superimposed on each other.

Prior to the advent of twisted light the property of optical helicity has been the preserve of optical spin angular momentum (SAM) arising from wave polarisation, in general elliptical polarisation, with spin angular momentum $\hbar\sigma$. The experimental finding by Wozniak et al. [18] indicates that optical vortex modes carrying orbital angular momentum (OAM), but which have no optical spin angular momentum, can engage with chiral matter via the helicity density distributions. This seems consistent with the generally-held belief that twisted light has spin-like properties as for example the confirmation that it can rotate matter.

In this communication we provide an analytical treatment for the helicity and chirality of paraxial uniformly linearly-polarised twisted light, confirming the chiral nature of the helicity density. Earlier theories put forward to explain the experimental results by Wozniak et al. include the work by Koksal et al. [19] who considered the case of tightly-focused Laguerre–Gaussian light based on the non-paraxial formalism of Barnett and Allen [20]. Forbes and Jones [21] showed how the experimental results could be explained by carefully considering the leading as well as higher orders of the expansion of the densities in powers of $1/k_z^2 w_0^2$, where k_z is the axial wavenumber and w_0 is the waist.

Here we show that confining the treatment for the general paraxial fields only to leading order leads directly to agreement of our theory with the experimental results, provided we ensure that crucially the paraxial fields obey duality.

Our theory here is specifically concerned with linearly-polarised vortex light of an arbitrary kind, so in addition to showing that the dual fields we start with lead to results confirming the experimental result by Wozniak et al. [18], we aim to explore whether or not this type of vortex light has optical spin angular momentum and whether the chirality results show that it has a Hopf index and an action constant. We find that, in general, uniformly linearly-polarised paraxial optical vortex modes carrying orbital angular momentum $\hbar\ell$ where ℓ is the winding number all have zero optical spin angular momentum, but display non-zero distributions of the helicity and the chirality densities and can interact with chiral matter. However, the total (space-integrated) densities vanish identically when the modes are uniformly linearly-polarised for all winding numbers ℓ. Thus we find that all linearly-polarised paraxial twisted light modes have zero total helicity and chirality as well as zero total optical spin angular momentum and they all possess a zero Hopf index.

The evaluation of the above cycle-averaged densities of any optical vortex mode requires the formal knowledge of both the electric field $\mathbf{E}$ and the corresponding magnetic field $\mathbf{B}$. Any optical vortex beam invariably has a longitudinal electric field component in addition to the transverse electric field components, as in the case of a Laguerre–Gaussian mode [16]. The importance of the longitudinal component in the context of optical vortex light was first pointed out by Rosales Guzman et al. [22] who were first to realise that longitudinal fields of optical vortices contribute unique terms to the helicity/chirality density.

Our plan here is to present an analytical treatment of optical helicity and chirality as well as optical spin angular momentum which addresses all uniformly linearly polarised paraxial optical vortex modes, without saying which, then deduce the Hopf index $\mathcal{N}$ and derive a general expression for the action constant Q.

We begin by considering the fields for the general uniformly linearly-polarised paraxial optical vortex field of frequency ω and axial wavevector component k_z propagating along the z-axis. Such a field is derivable from a vector potential $\mathbf{A}_{\ell m}(\mathbf{r},t)$ given in cylindrical polar coordinates in the form

$$\mathbf{A}_{\ell m}(\mathbf{r},t) = \hat{\mathbf{x}}\,\mathcal{U}_{\ell m}(\rho,\phi)e^{(ik_z z - i\omega t)} \tag{7}$$

where, without loss of generality, we have assumed that the mode is uniformly linearly-polarised along $\hat{\mathbf{x}}$. The indices ℓ and m are such that ℓ is the winding number of the vortex mode, while m may be a radial number, as in Laguerre–Gaussian modes, but could be redundant as in the case of Bessel modes. Note that the mode function $\mathcal{U}_{\ell m}$ depends only on (ρ,ϕ) and conforms to the paraxial regime. The magnetic field follows as $\mathbf{B}_{\ell m} = \boldsymbol{\nabla}\times\mathbf{A}_{\ell m}$

$$\mathbf{B}_{\ell m} = ik_z\hat{\mathbf{y}}\,\mathcal{U}_{\ell m}e^{ik_z z} - \hat{\mathbf{z}}(\partial_y\mathcal{U}_{\ell m})e^{ik_z z} \tag{8}$$

where we have dropped the time exponential $\exp(-i\omega t)$ for ease of notation. It is straightforward to check that the magnetic field satisfies $\boldsymbol{\nabla}\cdot\mathbf{B}_{\ell m} = 0$. The corresponding electric field must follow as a consequence of duality which requires that $\mathbf{E}_{\ell m}$ is related to $\mathbf{B}_{\ell m}$ by the Maxwell equation for a monochromatic field, which is $\mathbf{E}_{\ell m} = (ic^2/\omega)\boldsymbol{\nabla}\times\mathbf{B}_{\ell m}$. Thus the electric field must be of the form

$$\mathbf{E}_{\ell m} = ick_z\hat{\mathbf{x}}\,\mathcal{U}_{\ell m}e^{ik_z z} - c\hat{\mathbf{z}}(\partial_x\mathcal{U}_{\ell m})e^{ik_z z} \tag{9}$$

This electric field also conforms with the transversality condition $\boldsymbol{\nabla}\cdot\mathbf{E}_{\ell m} = 0$. Furthermore, we have to confirm that the paraxial magnetic field expression $\mathbf{B}_{\ell m}$, as displayed in Eq. (8) (and recalling that this has emerged as $\mathbf{B}_{\ell m} = \boldsymbol{\nabla}\times\mathbf{A}_{\ell m}$) must also emerge from $\mathbf{E}_{\ell m}$ in Eq. (9) using the second Maxwell equation for a monochromatic field, namely $\mathbf{B}_{\ell m} = (1/i\omega)\boldsymbol{\nabla}\times\mathbf{E}_{\ell m}$. The expressions in Eqs. (8) and (9) indeed conform to this, ensuring that the set of paraxial electromagnetic optical vortex modes satisfy duality. It is clear that both $\mathbf{E}_{\ell m}$ and $\mathbf{B}_{\ell m}$ have z- (longitudinal) components in addition to the transverse x– and y–components.

The amplitude function of the general vortex mode bearing the phase factor $e^{i\ell\phi}$ is as follows

$$\mathcal{U}_{\ell m}(\rho,\phi) \equiv \mathcal{F}_{\ell m}(\rho)e^{i\ell\phi} \tag{10}$$

The vortex feature of $\mathcal{F}_{\ell m}$ is such that this function vanishes at $\rho = 0$ at the vortex core and must also vanish at $\rho \to \infty$

$$\mathcal{F}_{\ell m}(\rho) = 0 \text{ for } \rho = 0 \text{ and } \rho \to \infty \tag{11}$$

It is also clear from Eq. (10) that such a vortex mode is an eigenfunction of the z-component of the orbital angular momentum operator $\hat{L}_z = -i\hbar\partial/\partial\phi$ with eigenvalue $\hbar\ell$.

As briefly pointed out earlier one of the effects that become manifest due to the increase in the significance of the longitudinal field of an optical vortex is that this type of light is endowed with the property whereby two beams which are identical except for the sign of the winding number ℓ are distinguishable. This is because one beam is a phase-inverted mirror image of the other and so the two modes possess different helicity and chirality [3,4,8,9,23–25]. More recent accounts have revived interest in these beam properties and their relation to optical spin [14,15,22,26–34].

With the dual electric and the magnetic fields of our general optical vortex mode as detailed in Eqs. (7) to (9), correct to leading paraxial order, we can now proceed to evaluate the cycle-averaged spin angular momentum density and the helicity density $\bar{\eta}$ and this suffices as regards consideration of the chirality since they are related by a proportionality constant.

First it is straightforward to check that the linearly-polarised optical vortex mode carries no total optical spin angular momentum for which the cycle-averaged density is given by Eq. (3). Using Eq. (9) we obtain for the spin density components

$$\bar{s}_x = 0; \quad \bar{s}_y = 0; \quad \bar{s}_z = 0, \quad so \ \bar{\mathbf{S}} = \mathbf{0} \tag{12}$$

This null (SAM) property is applicable to all linearly-polarised optical vortices in general.

Next we evaluate the cycle-averaged helicity density $\bar{\eta}$ and this suffices as regards consideration of the chirality since they are related by a proportionality constant. Substituting for the electric and magnetic fields using Eqs. (8) and (9) we have for the dot product $\mathbf{E}^* \cdot \mathbf{B}$

$$\mathbf{E}^* \cdot \mathbf{B} = c\left\{(\partial_x \mathcal{U})^*\right\}\left\{(\partial_y \mathcal{U})\right\} \tag{13}$$

where for ease of notation, we do not show the labels ℓm and the argument ρ, ϕ in the field function $\mathcal{U}$, as defined in Eq. (10). In Eq. (13) we identify the derivative terms as contributions to the helicity density due to the z-components (longitudinal components).

Since $\mathcal{U}$ is a function only of (ρ, ϕ), it is straightforward to evaluate the x- and y- derivatives. We obtain,

$$\partial_x \mathcal{U} = \mathcal{U}' \cos\phi - i\frac{\ell}{\rho}\mathcal{U}\sin\phi \tag{14}$$

and

$$\partial_y \mathcal{U} = \mathcal{U}' \sin\phi + i\frac{\ell}{\rho}\mathcal{U}\cos\phi \tag{15}$$

where $\mathcal{U}' = \partial\mathcal{U}/\partial\rho$.

Continuing with the evaluation of the general helicity density, we obtain for the dot product $\mathbf{E}^* \cdot \mathbf{B}$ after some algebra

$$\mathbf{E}^* \cdot \mathbf{B} = ic\frac{\ell}{\rho}\mathcal{F}'\mathcal{F} \tag{16}$$

where we have used Eqs. (14) and (15), together with Eq. (10) enabling the expression to be written in terms of $\mathcal{F}$. Thus we find for the cycle-averaged helicity density, as defined in Eq. (4)

$$\bar{\eta} = -\ell\frac{\epsilon_0 c^2}{2\omega}\left\{\frac{1}{\rho}\mathcal{F}'\mathcal{F}\right\} \tag{17}$$

Characteristically, this contribution is directly proportional to ℓ. Note that the helicity density distribution is in general non zero. To illustrate the chirality nature we consider a Laguerre–Gaussian mode for which $\mathcal{F}$ is given by

$$\mathcal{F}(\rho) = \mathcal{E}_0\sqrt{\frac{p!}{(p+|\ell|)!}}e^{-\frac{\rho^2}{w_0^2}}\left(\frac{\sqrt{2}\rho}{w_0}\right)^{|\ell|}L_p^{|\ell|}\left(\frac{2\rho^2}{w_0^2}\right) \tag{18}$$

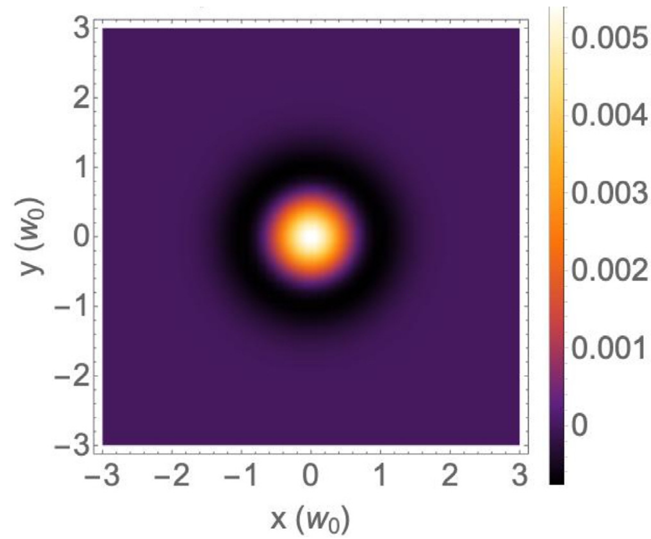

Fig. 1. The spatial variations within a mode cross section of the helicity density (arbitrary units) for a Laguerre Gaussian (doughnut) mode for which $\ell = +1$. Here $w_0 = 5\lambda$.

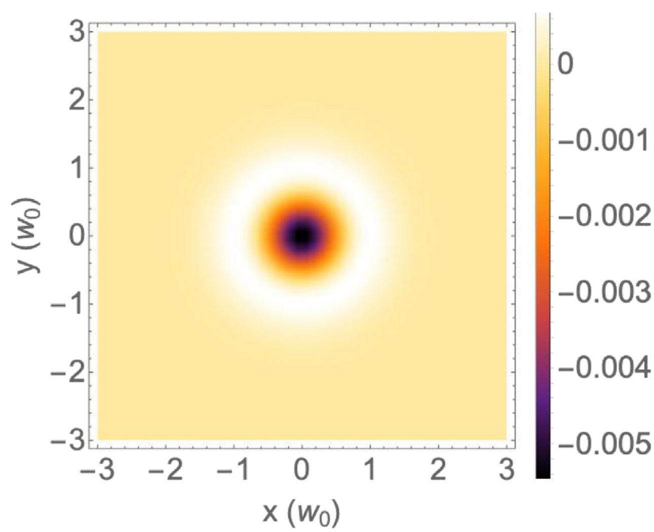

Fig. 2. The spatial variations within a mode cross section of the helicity density (arbitrary units) of Laguerre Gaussian (doughnut) mode for which $\ell = -1$. Here $w_0 = 5\lambda$.

where $\mathcal{E}_0$ is a normalisation constant, w_0 is the beam waist and the subscript m is now identified as p, the radial number. We show in Figs. 1 and 2 the variations of the helicity density Eq. (17) for the linearly-polarised doughnut modes $\ell = 1$ and $\ell = -1$ with $p = 0$ for both. These figures show clearly the chirality feature which was demonstrated experimentally [18], namely that the two density distributions cannot be superimposed on each other.

However, we now show that the total (space-integrated) helicity per unit length of the general linearly polarised mode is identically zero. We have

$$\begin{aligned}
\bar{\mathcal{H}} &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho\, d\rho\, \bar{\eta}(\rho) \\
&= -\ell\frac{\epsilon_0 c^2}{2\omega}\int_0^{2\pi} d\phi \int_0^{\infty}\left(\frac{\mathcal{F}'(\rho)\mathcal{F}(\rho)}{\rho}\right)\rho\, d\rho \\
&= -\ell\frac{\epsilon_0 \pi c^2}{4\omega}\int_0^{\infty}\left(\frac{d}{d\rho}\mathcal{F}^2\right)d\rho = -\ell\frac{\epsilon_0\pi c^2}{4\omega}\left[\mathcal{F}^2(\rho)\right]_0^{\infty} \\
&= 0
\end{aligned} \tag{19}$$

where the last equality follows from the fact that $\mathcal{F}^2(\rho = 0, \infty) = 0$, as defined in Eq. (11). Thus we have discovered the general result that, although the helicity density of a linearly-polarised vortex mode has non-zero variations which exhibit chirality, the space integral $\bar{\mathcal{H}}$ of the density vanishes identically for such an arbitrary mode. The statement $\bar{\mathcal{H}} = 0$, as far as we know, has not been emphasised before as a general property of uniformly linearly-polarised vortex modes carrying orbital angular momentum $\hbar\ell$

The significant point here is that without optical spin due to, for example, elliptical wave polarisation, an optical vortex alone cannot possess total helicity, even though it exhibits helicity density distributions, which in turn indicates that on radial integration the different regions of the helicity density distribution are cancelled out by other regions. As we have seen at the outset, the chirality density is proportional to the helicity density, so we have the same conclusion of a vanishing total cycle-averaged chirality for a linearly-polarised general vortex mode.

The situation changes drastically once the optical mode has wave polarisation different from linear, as for example elliptical polarisation, with which is associated optical spin $\sigma = \pm 1$. Recently, we considered the particular case in which vortex mode is elliptically polarised Laguerre–Gaussian light for which $\sigma \neq 0$ [32] for which $\mathcal{F}$ is as given by Eq. (18). We have now carried out explicit evaluations for a general optical vortex without specifying the form $\mathcal{F}$ and so confirmed that the total helicity $\bar{\mathcal{H}}_\sigma$ of the general polarised optical vortex mode is proportional to σ and can be written succinctly as follows

$$\bar{\mathcal{H}}_\sigma = \sigma \mathcal{Q} \tag{20}$$

Since $\sigma = \pm 1$, this follows the familiar pattern of optical spin helicity due to wave polarisation, but note that here it is modified by the presence of the optical vortex from which we deduce that the Hopf index as $\mathcal{N} = \sigma$ and the action constant $\mathcal{Q}$ is given by the general expression

$$\mathcal{Q} = \frac{\pi\epsilon_0 c^2}{2\omega}\int_0^\infty \left[2k_z^2\mathcal{F}^2(\rho) + \mathcal{F}'^2(\rho) + \frac{\ell^2}{\rho^2}\mathcal{F}^2(\rho)\right]\rho\, d\rho \tag{21}$$

The action constant $\mathcal{Q}$ represents the helicity strength per unit length. Unlike the Hopf index, which is common to all vortex modes, and equal to σ, the value of the action constant for an arbitrary paraxial vortex mode, as given by the integral in Eq. (21), clearly depends on what kind of optical vortex one is dealing with and can be evaluated once $\mathcal{F}$ is specified. Note that besides the first term in the integrand, which stems directly from the transverse field components, the second and the third terms depend on ℓ and so constitute spin–orbit coupling terms. These terms could increase or decrease the spin chirality, but we should note however that these terms must vanish in the far field leaving only the first term which leads to the spin chirality [28,33]. This behaviour involving a general $\mathcal{F}$ in Eq. (21) has been demonstrated in the case of a specific $\mathcal{F}$ corresponding to a Laguerre–Gaussian mode [32].

In conclusion, we have focused here on the optical chirality and helicity as well as optical spin angular momentum of an arbitrary linearly-polarised paraxial optical vortex mode. We have shown that such a general vortex mode displays has zero spin angular momentum, but non-zero cycle-averaged helicity and chirality density distributions. However, the total helicity and chirality of the mode arising from the space integrals of the densities all vanish identically. Interestingly a very recent article by Forbes [34] extended the consideration to the case of unpolarised Laguerre–Gaussian light and showed that such a light possesses chirality and helicity densities. We have identified the source of the densities when the general vortex mode is linearly-polarised as due entirely to the presence of the longitudinal field components. When the general mode is elliptically-polarised in the chirality density only the σ-dependent contribution results in a non-zero total spin helicity. Note that, besides the illustrations focusing on Laguerre–Gaussian modes leading to Figs. 1 and 2, we have a theory which is applicable to a general optical vortex.

Finally, we comment on the interaction of vortex light with chiral matter. We have shown that for a linearly-polarised mode the helicity density does not vanish even if optical spin is zero. As helicity density for monochromatic light is proportional to the chirality density, and chiral density is related to the chiral response in linear light-matter interactions [23], this scenario leads to the suggestion that it is always possible to excite a chiral response even with a linearly-polarized vortex light beam whose density presents space variations with which a chiral object interacts differently when located in different parts of the beam. Moreover, the variations of the chirality across the beam are proportional to the derivatives of the transverse field components. This suggests the possibility of optimizing the characteristics of the vortex mode in order to achieve superchirality [23] with twisted light and so improve the optical sensitivity to chiral matter.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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