Proceedings Paper:

Wan, Z., Liu, W. orcid.org/0000-0003-2968-2888 and Willett, P. (2022) Non-coherent source localization with distributed sensor array networks. In: 2022 IEEE 12th Sensor Array and Multichannel Signal Processing Workshop (SAM). 2022 IEEE 12th Sensor Array and Multichannel Signal Processing Workshop (SAM), 20-23 Jun 2022, Trondheim, Norway. Institute of Electrical and Electronics Engineers , pp. 86-90. ISBN 9781665406345

https://doi.org/10.1109/sam53842.2022.9827843

Non-Coherent Source Localization with Distributed Sensor Array Networks

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***Abstract*—The non-coherent source localization problem based on distributed sensor arrays can be formulated into a group sparsity based phase retrieval problem where only the magnitude (absolute value) of the received signals is available. Under such a framework, a two-dimensional localization method is proposed. Unlike traditional source localization methods, random phase errors at sensors of the distributed array will not affect estimation results by the proposed method. Simulation results indicate that the proposed non-coherent source localization method outperforms the traditional one in the presence of large phase errors, while still maintains an acceptable accuracy in the absence of phase errors.**

***Index Terms*—Target localization, distributed sensor array, magnitude-only measurements, group sparsity, non-coherent detection.**

I. Introduction

Source localization is a very important problem in sensor array signal processing and many methods have been proposed such as those based on received signal strength (RSS) [1], time of arrival (TOA) [2], time difference of arrival (TDOA) [3], [4], direct position determination (DPD) [5], [6] and angle of arriving (AOA) [7], [8].

For AOA based methods, a distributed sensor array structure is employed with multiple sensor arrays distributed in a two-dimensional (2-D) space, where synchronization among all distributed sensor arrays is not required. There are normally two steps: the first is applying existing direction of arrival methods such as those proposed in [9]–[11] to estimate AOAs at all distributed sensor arrays, while the second is to find intersections of those estimated AOAs in order to localize the sources, which can be realised by a maximum likelihood estimator based on the least squares formulation [7], [8]. Since information at different observers is processed separately to obtain the individual AOAs, these AOA based methods are sensitive to estimation accuracy at each array. Recently, in [12], [13], with the distributed array network, information across all sensor arrays is jointly exploited and the source localization problem was re-formulated into a sparsity maximization problem, where the area of interest in a 2-D Cartesian system is divided into grids along the x-axis and y-axis; under such a framework, a common spatial sparsity support corresponding to all distributed sensor arrays is enforced, leading to a better estimation performance, which also avoids the possible pairing and ambiguity problems associated with a two-step AOA based solution [12].

All the above AOA based methods and the sparsity based one have assumed that there are no phase errors in the sensor array model. In real applications, however, the phase information may not be reliable due to various reasons, and in the extreme case, the phase information may be lost completely, which unavoidably leads to an inaccurate estimation result. On the other hand, non-coherent (i.e. magnitude-only measurements) direction of arrival (DOA) estimation has been studied recently, where only magnitude information is captured at all sensors [14]–[18].

In [14], the problem was formulated into a sparse phase retrieval problem, where the inherent ambiguity issue of non-coherent measurements was resolved using a reference signal when only one unknown source impinges upon the array. With more unknown signals, more reference signals are required. To reduce the number of required reference signals to one for multiple incident signals, a method was proposed to firstly estimate the frequency component of non-coherent measurements, and then a high gain reference signal is employed to identify the DOA of unknown signals [15], [16], but its estimation accuracy relies on its frequency resolution, which requires a large number of measurements. Besides, this method fails to utilize the information of multiple snapshots jointly to improve its performance. To jointly exploit multiple snapshots, the non-coherent measurements can also be formulated into a compressive sensing based form, but a high gain reference signal still has to be applied [17]. Alternatively, with a normal gain reference signal, a dual-array structure was proposed to reduce the number of required reference signals to one in [16]. A new group sparsity based algorithm called ToyBar was proposed in [18], which can also be applied to the dual-array structure to remove the required reference signal. Most recently, it has been shown that with a uniform circular array (UCA), reference signal is not required either if there are more than two sources [19].

In this paper, the source localization problem with magnitude-only measurements based on a distributed sensor array structure is studied, where UCA is employed at each sensor array in order to be reference-signal free. Similar to

This work is supported by the UK Engineering and Physical Sciences Research Council (EPSRC) under grants EP/T517215/1 and EP/V009419/1.

[12], by dividing the area into 2-D grids, the non-coherent source localization problem is formulated into a sparsity based framework, and magnitude-only measurements at all observers can be exploited jointly, since the pre-defined grids provide a common spatial sparsity associated with true source positions for all the sensor arrays. As a result, this problem can be regarded as a group sparsity based phase retrieval problem, which can be solved by existing algorithms and obtain source locations directly. In addition, unlike existing AOA based methods, the performance of the proposed method will not be affected by phase errors at array sensors.

The remaining part is structured as follows. The signal model with distributed sensor arrays is described in Sec. II. The proposed non-coherent localisation method is presented in Sec. III. Simulation results are provided in Sec. IV and conclusions are drawn in Sec. V.

II. Signal model with distributed sensor arrays

Assume that there are K narrowband sources $s_k(p)$ located at Cartesian coordinates $L_k(x_k, y_k)$, $k = 1, 2, ..., K$, impinging on D deployed sensor arrays with coordinates $C_d(x_d, y_d)$, as shown in Fig. 1.

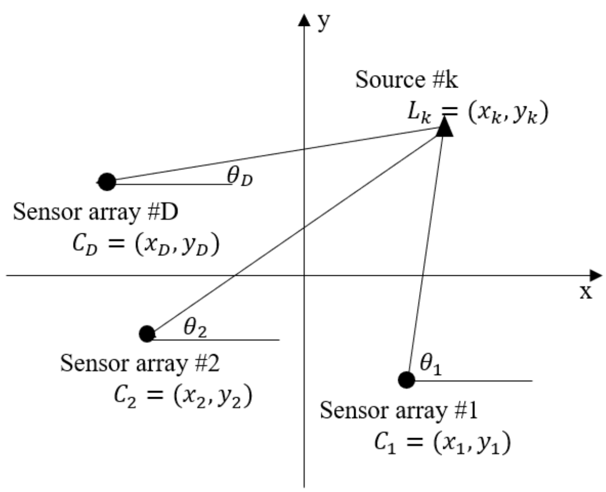


Fig. 1. Source localization geometry.

The number of sensors of the d-th sensor array is M_d, and the corresponding non-coherent measurements at the d-th sensor array is expressed as

$$\mathbf{z}_d[p] = |\mathbf{A}_d \mathbf{s}_d[p]| + \mathbf{n}_d, \tag{1}$$

with $\mathbf{s}_d[p] = [s_{d,1}[p], ..., s_{d,K}[p]]^T$, where $s_{d,k}$ represents the p-th snapshot of the k-th signal, $\mathbf{n}_d$ is the $M_d \times 1$ random Gaussian noise vector at the d-th sensor array, $|\cdot|$, and $[\cdot]^T$ are the element-wise absolute value operator and matrix transpose operator, separately, and

$$\mathbf{A}_d = [\mathbf{a}_d(\theta_1), ..., \mathbf{a}_d(\theta_K)]^T \tag{2}$$

is the steering matrix with its columns $\mathbf{a}(\theta_k)$, $k = 1, ..., K$, being the corresponding steering vectors. When employing a uniform circular array [19], $\mathbf{a}_{d,k}$ is given by

$$\begin{aligned} \mathbf{a}_d(\theta_k) &= [e^{-j\frac{2\pi r}{\lambda}\cos(\theta_{d,k}-\gamma_1)}, ..., e^{-j\frac{2\pi r}{\lambda}\cos(\theta_{d,k}-\gamma_{M_d})}], \\ \gamma_m &= 2\pi m / M_d, \end{aligned} \tag{3}$$

where λ is wavelength of the signals, r is radius of the circular array, and $\theta_{d,k}$ denotes the arriving angle between the k-th source and d-th sensor array, expressed as

$$\begin{aligned} \theta_{d,k} &= \operatorname{arctan2}(\Delta y_{d,k}, \Delta x_{d,k}), \\ \Delta y_{d,k} &= y_{l_k} - y_d, \\ \Delta x_{d,k} &= x_{l_k} - x_d, \end{aligned} \tag{4}$$

with $\operatorname{arctan2}(\cdot)$ being the inverse four-quadrant tangent operator.

Collecting P snapshots to form $\mathbf{Z}_d = \big[\mathbf{z}_d[1], ..., \mathbf{z}_d[P]\big]$, one has

$$\begin{aligned} \mathbf{Z}_d &= |\mathbf{A}_d \mathbf{S}_d| + \mathbf{N}_d, \\ \mathbf{S}_d &= \big[\mathbf{s}_d[1], ..., \mathbf{s}_d[P]\big]^T, \\ \mathbf{N}_d &= \big[\mathbf{n}_d[1], ..., \mathbf{n}_d[P]\big]. \end{aligned} \tag{5}$$

Note that, magnitude-only measurements suffer from some ambiguities, and two of them have effect on the AOA estimation results: one is mirroring and the other is spatial shift [14]–[16], [18]. For mirroring ambiguity, it refers to the phenomenon that signals arriving from $-\theta_{d,k}$ will generate the measurements with the same magnitude, while for spatial shift ambiguity, it refers to that all estimated arriving angles at the array are phased shifted by a specific amount. However, as shown in [19], while applying a UCA structure, those ambiguities would not appear if the valid DOA range is limited within $\theta_k \in [\theta - \pi/2, \theta + \pi/2]$, i.e. when $\theta = 0°$, the valid DOA range is within $[-90°, 90°]$, and for $-\pi/2 \le \theta_k \le \pi/2$, $\theta_k \pm \pi$ will exceed the limit. Therefore, by applying UCA, estimation results would not be affected by these ambiguity issues.

In the scenario of uncalibrated sensor array, each sensor may suffer from independent phase errors and (5) would be changed to

$$\mathbf{Z}_d = |\mathbf{E}_d \mathbf{A}_d \mathbf{S}_d| + \mathbf{N}_d, \tag{6}$$

where $\mathbf{E}_d$ is an $M_d \times M_d$ diagonal matrix with random phase terms, representing the phase errors at the d-th array. Since the measurement is magnitude only and $\mathbf{E}_d$ is diagonal, we have

$$|\mathbf{E}_d \mathbf{A}_d \mathbf{S}_d| = |\mathbf{A}_d \mathbf{S}_d|, \tag{7}$$

which indicates that, unlike those traditional methods, phase error at sensors of an array have no effect on magnitude-only measurements [14], and hence, in the remaining part of this paper, phase error matrix $\mathbf{E}_d$ is dropped for convenience.

III. Proposed method

A. Sparsity based non-coherent DOA estimation

If the admissible DOA range is divided into G grid points with $G \gg M_d$, an overcomplete steering matrix at the d-th sensor array

$$\tilde{\mathbf{A}}_d = [\mathbf{a}_d(\theta_1), ..., \mathbf{a}_d(\theta_G)] \tag{8}$$

can be formed with each column representing a potential incident angle. Accordingly, the source vector $\mathbf{s}_d[p]$ is extended to a $G \times 1$ sparse vector

$$\tilde{\mathbf{s}}_d[p] = [s_{d,1}[p], ..., s_{d,G}[p]]^T, \tag{9}$$

where only K entries at the corresponding incident angles are supposed to be non-zero.

For the multiple-snapshot case, where measurements are expressed as $\mathbf{Z}_d = [\mathbf{z}_d[1], \cdots, \mathbf{z}_d[P]]$, and source matrices are defined as $\tilde{\mathbf{S}}_d = [\tilde{\mathbf{s}}_d[1], \cdots, \tilde{\mathbf{s}}_d[P]]$, the DOA at the d-th array can be estimated by solving the following minimization problem

$$\min_{\tilde{\mathbf{S}}_d} \|\mathbf{Z}_d - |\tilde{\mathbf{A}}_d\tilde{\mathbf{S}}_d|\|_F^2 + \rho\|\tilde{\mathbf{S}}_d\|_{2,1}, \tag{10}$$

where $\|\cdot\|_{2,1}$ and $\|\cdot\|_F$ represent $l_{2,1}$ norm and Frobenius norm, respectively. The $l_{2,1}$ norm $\|\cdot\|_{2,1}$ is defined as

$$\|\tilde{\mathbf{S}}_d\|_{2,1} := \sum_{g=1}^{G} \|\mathbf{s}_{d,g}\|_2, \tag{11}$$

with $\mathbf{s}_{d,g}$ is the g-th row vector of $\tilde{\mathbf{S}}_d$.

B. Sparsity based non-coherent source localization

By dividing the admissible area of interest into G_x and G_y grids along the x-axis and y-axis in the Cartesian coordinate system, separately, the overcomplete steering matrix of the d-th sensor array can be expressed as

$$\begin{aligned}\tilde{\mathbf{A}}_d =&[\mathbf{a}_d(\theta_{11}), ..., \mathbf{a}_d(\theta_{1G_y},\\ &\mathbf{a}_d(\theta_{g_x1}), ..., \mathbf{a}_d(\theta_{g_xG_y}),\\ &......\\ &\mathbf{a}_d(\theta_{G_x1}), ..., \mathbf{a}_d(\theta_{G_xG_y})],\end{aligned} \tag{12}$$

where $\theta_{g_xg_y}$ is the angle between location (g_x, g_y) and the $d-th$ sensor array, obtained by

$$\begin{aligned}\theta_{d,k} &= \arctan2(\Delta y_{d,g}, \Delta x_{d,g}),\\ \Delta y_{d,g} &= y_{l_g} - y_d,\\ \Delta x_{d,g} &= x_{l_g} - x_d.\end{aligned} \tag{13}$$

It is noted that, incident sources from an arbitrary grid point would share the same spatial support of $\tilde{\mathbf{A}}_d$, $d = 1, ..., D$, although the arriving angles with respect to different arrays are different. Thus, a $(\sum_{d=1}^{D} M_d) \times G_xG_y$ steering matrix covering all D sensor arrays can be constructed as

$$\tilde{\mathbf{A}} = \text{blkdiag}\{\tilde{\mathbf{A}}_1, ..., \tilde{\mathbf{A}}_d\}, \tag{14}$$

where $\text{blkdiag}\{\cdot\}$ generates a block diagonal matrix from its entries. Therefore, the source localization problem can be formulated as a joint group sparsity based optimization problem, given by

$$\begin{aligned}&\min_{\tilde{\mathbf{S}}} \|\mathbf{Z} - |\tilde{\mathbf{A}}\tilde{\mathbf{S}}|\|_F^2 + \rho\|\tilde{\mathbf{S}}\|_{2,1},\\ &\mathbf{Z} = [\mathbf{Z}_1^T, ..., \mathbf{Z}_D^T]^T,\\ &\tilde{\mathbf{S}} = [\tilde{\mathbf{S}}_1^T, ..., \tilde{\mathbf{S}}_D^T]^T.\end{aligned} \tag{15}$$

In addition to the support shared in the temporal domain given in (10), groups of $\tilde{\mathbf{S}}$ also share the same support in spatial domain. As a result, $\tilde{\mathbf{S}}$ contains $G = G_xG_y$ groups and the g-th group, $g \in \{1, ..., G\}$, of $\tilde{\mathbf{S}}$ is a $1 \times DP$ vector, consisting of g-th row vectors of all $\tilde{\mathbf{S}}_d$, $d \in \{1, .., D\}$ in $\tilde{\mathbf{S}}$.

The problems in both (10) and (15) can be considered as a group sparsity based phase retrieval problem, which can be solved by existing algorithms such as the modified Gespar [14] and ToyBar [18].

C. Grid refinement

Similar to other sparsity based methods, the estimation results are dependent on the grid size. A denser grid usually leads to a more accurate location estimation result, but with a much higher computational complexity [11].

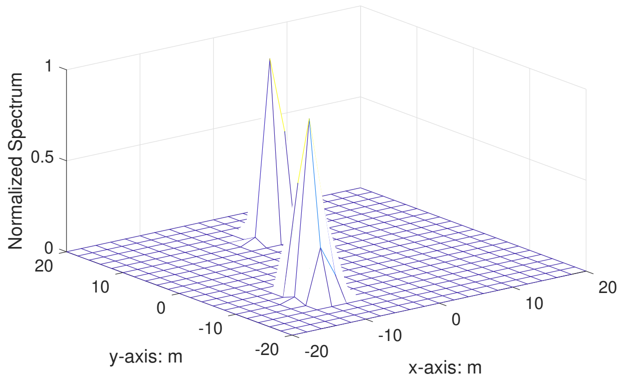
Therefore, instead of creating a dense grid initially, a coarse grid is firstly made; based on the localization results, a denser steering matrix is then built around the estimated locations of incident sources, and the algorithm is employed again to find a more accurate result.

IV. Simulations

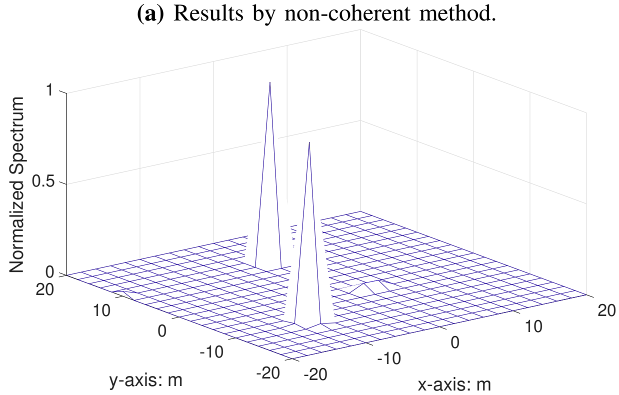
In this section, simulation results are provided to show the performance of the proposed non-coherent source localization method in comparison with the existing sparsity based coherent method in [12]. A recently proposed sparse phase retrieval algorithm called ToyBar in [18] is applied in the non-coherent scenario, and the number of iterations before stop is set to 500, with 20 random initialisations used in order to find the global minimum of the phase retrieval problem.

The area of interest is set as $[-20, 20]$m (metres) along both x-axis and y-axis. In the initial step, 2m is used as the stepsize for constructing the overcomplete steering matrix $\tilde{\mathbf{A}}$. In the refinement step, a new grid with stepsize 0.2m is formed around a distance of 2m to either side of the estimated location from the initial step. There are $D = 4$ distributed sensor arrays placed at $C_1 = (10, 40)$m, $C_2 = (30, 10)$m, $C_3 = (-80, 90)$m and $C_4 = (-20, 40)$m, while the locations for $K = 2$ sources are $L_1 = (-10, -10)$m and $L_2 = (0, 10)$m. The number of sensors at each distributed sensor array M_d is set as 20, while the radius r of the UCAs is set as $r = \frac{M_d \frac{\lambda}{2}}{2\pi}$, and $P = 50$ snapshots are collected in all simulations.

For the first set of simulations, the signal to noise ratio (SNR) is 15 dB, with phase error matrix $\mathbf{E}$ being an identity matrix (i.e. no phase error). The spatial spectrum of estimation results is shown in Fig. 2, where Fig. 2a provides the result of non-coherent measurements, while Fig. 2b is for coherent measurements. It can be seen that the two sources have been identified successfully by both methods. However, the resolution using coherent measurements (i.e. without the absolute value operation in (1)) is higher than that of the magnitude-only measurements as it provides sharper peaks.



(a) Results by non-coherent method.



(b) Results by coherent method.

Fig. 2. Spectrum of both non-coherent and coherent methods.

Then, the performances of the two methods are evaluated with different SNR values ranging from 0 dB to 20 dB in terms of the root mean square error (RMSE) in the absence of phase error. The results are shown in Fig. 3, with each point obtained by averaging over 100 trials and the peaks of spectrum are regarded as the true source locations. It can be observed that, although both methods achieve more accurate results with increasing SNR, the method with full measurements consistently outperforms that with magnitude-only measurements, especially when the noise level is high. However, it is not surprising since only magnitude information is used in the non-coherent scenario with magnitude-only measurements.

Finally, we examine the performance of both non-coherent and coherent methods in the presence of sensor phase errors. RMSE results are obtained with an average of 100 trials. The SNR is fixed at 15 dB, while the entries of the phase error matrix $\mathbf{E}_d$ follow the Gaussian distribution with standard deviation σ. As shown, the proposed non-coherent method is not affected by phase errors, with a steady performance, while the performance of the coherent method declines as the intensity of phase errors increases.

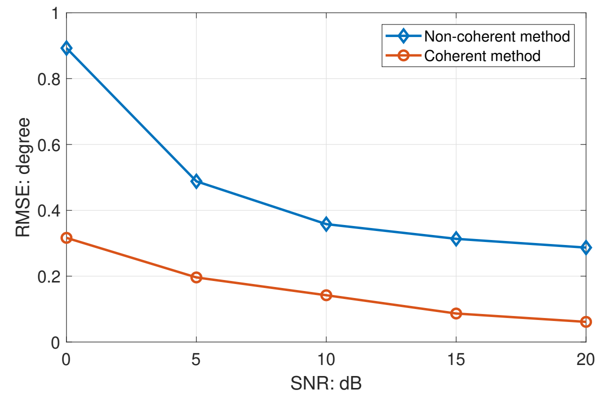


Fig. 3. RMSEs versus SNRs.

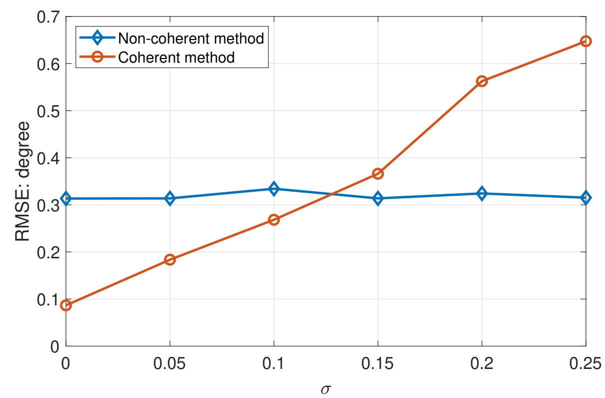


Fig. 4. RMSEs versus sensor phase error.

V. Conclusions

A source localization method with magnitude-only measurements based on distributed sensor arrays has been proposed. The non-coherent source localization problem was formulated in a joint sparse phase retrieval form, and the $l_{2,1}$ norm is employed to enforce spatial sparsity. Unlike those existing AOA based methods, phase error at sensor arrays has no effect on the proposed non-coherent one, which means that phase calibration is no longer required. Simulation results show that the proposed method outperforms the traditional method in terms of RMSE when the phase error occurs at the sensors, but at a cost of worse performance in the absence of phase errors.

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