Validating S-parameter measurements of RF integrated circuits at milli-Kelvin temperatures

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Techniques to precisely characterise RF components at milli-Kelvin temperatures support the development of quantum computing systems utilising these components. In this work, an S-parameter measurement setup to characterise RF integrated circuits at milli-Kelvin temperatures has been proposed and for the first time, the S-parameter measurements at milli-Kelvin temperatures have been validated using two independent calibration techniques, thereby providing more confidence in measurements. The techniques are demonstrated experimentally by comparing and validating calibrated S-parameter measurements of a cryogenic attenuator integrated circuits at milli-Kelvin temperatures.

Introduction: Radio frequency (RF) components and cables are used in quantum computing to transmit signals that manipulate the state of qubits at milli-Kelvin (mK) temperatures [1, 2]. RF components are usually designed to operate at temperatures ranging from −40°C to 80°C for most applications. Although some of these RF components are assumed to operate at mK temperatures, their S-parameters may significantly deviate from their room temperature specifications.

In order to precisely characterise the S-parameters of RF components at mK temperatures, a calibration scheme that shifts the reference planes of the measurement to the ends of the device under test (DUT) needs to be implemented [3], thus de-embedding the intervening components up to the ports of the vector network analyser (VNA), operating at room temperature. Some previous work has been demonstrated for calibrated measurements of two-port RF components at mK temperatures [4–8]. When developing new metrology capabilities, it is important to validate the measurements by comparing and verifying the measurement results obtained using independent measurement methods, through interlaboratory comparisons and through comparison of measurement results obtained using different calibration techniques. The measurements results discussed in [4–8] relied on using a single calibration technique and comparisons of the measurement results were not made using multiple independent calibration techniques, which is important in order to validate the measurements, when developing quality assured metrology capabilities at mK temperatures, to provide confidence in the measurements [9].

This letter focuses on validating calibrated S-parameter measurements of RF Integrated Circuits (RFIC) at mK temperatures by using independent calibration techniques for the first time. The letter introduces an S-parameter measurement system for characterising 2-port non-connectorised RFICs at mK temperatures and explains the working principle of independent Thru-Reflect-Line calibration techniques which are implemented to characterise a commercial 2 dB attenuator IC below 20 mK. The calibrated S-parameter measurements of the device using an independent calibration technique are used to validate the measurements.

Measurement setup: The S-parameter measurement set-up containing the microwave calibration unit (MCU) inside the coldest stage (<20 mK) of the dilution refrigerator is shown in Figure 1(a). The MCU (Figure 1(b)) consists of calibration standards and a DUT which are selected by two software controlled SP6T RF switches, driven at mK temperatures via super-conducting DC wiring. In this set-up, all four uncalibrated S-parameters are obtained by activating and measuring the respective input and output coaxial paths, connecting the standards and DUT in the MCU to the VNA test ports through two SPDT switches operating at room temperature. Attenuators are added in the input path

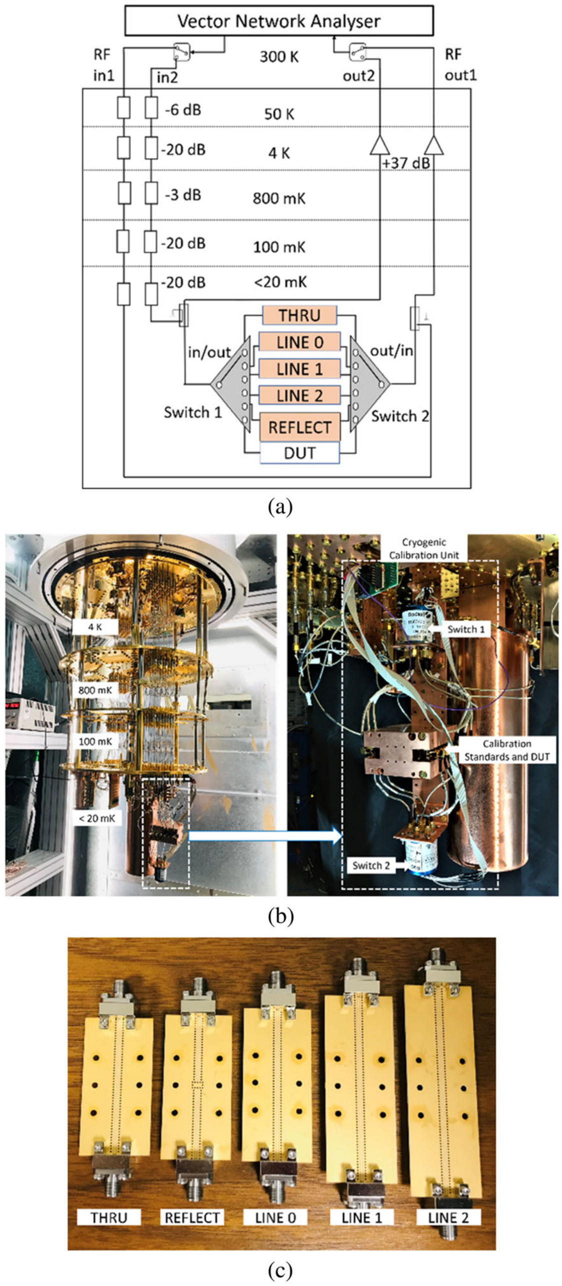

Fig. 1 *(a) Block diagram of cryogenic S-parameter measurement set-up. (b) Implementation of cryogenic S-parameter measurement setup. (c) cryogenic calibration standards*

at different stages to thermally stabilise the cables and reduce incoming thermal radiation, which also makes the setup compatible with measurements of quantum integrated circuits. Low noise amplifiers are added in the output paths to bring the signal levels to an acceptable level for VNA measurements. The MCU developed in this work can perform calibrated S-parameter measurements in the frequency range, 2–14 GHz.

Calibration standards and techniques: The TRL technique can be used to calibrate non-connectorised devices such as RFICs [10] without needing to precisely characterise the standards, compared to other techniques such as SOLT [3], making it suitable for use at mK temperatures. The Thru, Reflect and Line standards are designed and fabricated using separate PCBs as grounded co-planar waveguide (GCPW) transmission line structures, as shown in Figure 1(c). Rogers RO4350B with relative permittivity 3.48 and loss tangent 0.0037 at 10 GHz at room temperature is used as the PCB substrate. The design dimensions of the GCPW structure are estimated by solving the analytical equations in [11] for a characteristic impedance of GCPW transmission line of 50 Ω and optimised using CST Microwave Studio simulations.

The middle of the Thru PCB standard is set as the calibration reference plane. The Reflect standard is realised as a short-circuit, with two nominally identical short-circuits implemented on a single PCB, providing the two Reflect standards. The 8-term error model used in the TRL algorithm [10] requires the Reflect standards to be identical to solve the error terms and hence the reflection coefficient of both these short-circuit standards needs to be identical for a good quality calibration. Therefore, any differences between the two short-circuit standards will introduce residual errors in the calibration. The Line standard consists of an extra section of transmission line that provides a change in the transmission phase, with respect to the Thru connection. The generalised TRL calibration scheme works optimally when the difference in phase between the Thru and the Line standard is $(2n + 1)\lambda/4$ and fails completely when this phase difference is $(2n)\lambda/4$ (where, $n = 0, 1, 2, \ldots$) [12]. Therefore, line lengths are chosen to prevent phase differences that are close to these calibration failure points.

In a conventional $\frac{1}{4}$-wave TRL calibration, the Line standard is designed to provide a change in the transmission phase, with respect to the Thru connection, of approximately 90° – that is, $\frac{1}{4}$-wavelength – at frequencies around the middle of the frequency range. The calibration failure points occur at 0° and 180°. Therefore, when implementing a $\frac{1}{4}$-wave TRL calibration procedure, phase changes are designed to be within 20° and 160°, that is, at least 20° greater than 0° and at least 20° less than 180°. In this work, a single line standard is designed to be 6 mm ($\frac{1}{4}$ of the GCPW wavelength at 8 GHz) longer than the Thru standard to cover the frequency range of 2–14 GHz.

As an independent calibration technique, we implemented the $\frac{3}{4}$-wave TRL calibration, which has been demonstrated at millimetre-wave frequencies in waveguide media [12]. The $\frac{3}{4}$-wave TRL calibration has optimum performance at 270°, that is, $\frac{3}{4}$-wavelength, with failure points at 180° and 360°. Therefore, when implementing a $\frac{3}{4}$-wave TRL calibration procedure, phase changes are designed to be within 200° and 340°, that is, at least 20° greater than 180° and at least 20° less than 360°. Two different line standards are needed to cover the frequency range from 4 to 10.1 GHz. The equations in [12] are adapted to design the lengths of the line standards to achieve $\frac{3}{4}$-wave TRL calibration for a given frequency range:

Step 1: determine the length of the first line, l_1, which gives the minimum phase change, φ_{min} (=200°), at the lowest required frequency, f_{min}.

$$l_1 = \frac{\lambda_{eff(max)} \times \phi_{min}}{360}, \quad (1)$$

$$\lambda_{eff(max)} = \frac{c}{f_{min} \times \sqrt{\varepsilon_{eff}}}, \quad (2)$$

where $\lambda_{eff(max)}$ is the wavelength at the lowest required frequency, ε_{eff} is the effective permittivity of the substrate material of the line standard that relates to the relative permittivity and design dimensions of the GCPW structure [13], and c is the speed of light in vacuum. The measurements indicate that there is no appreciable change in the ε_{eff} values at room temperature and mK temperatures, for this material, and is estimated to be 2.425 at 8 GHz.

Step 2: establish the useable upper frequency limit, f_{Ul_1}, for the first line, l_1, that is, the frequency at which the maximum phase change, φ_{max} (=340°), occurs.

$$f_{Ul_1} = \frac{c \times \phi_{max}}{360 \times l_1 \sqrt{\varepsilon_{eff}}}. \quad (3)$$

Step 3: determine the length of the second line, l_2, which gives the maximum phase change, φ_{max} (=340°), at the maximum possible frequency.

$$l_2 = \frac{\lambda_{eff(min)} \times \phi_{max}}{360}, \quad (4)$$

where $\lambda_{eff(min)}$ is the wavelength at the highest recommended frequency f_{max}.

Step 4: establish the useable lower frequency limit, f_{Ll_2}, for l_2, that is, the frequency at which φ_{min} (=200°) occurs.

$$f_{Ll_2} = \frac{c \times \phi_{min}}{360 \times l_2 \sqrt{\varepsilon_{eff}}}. \quad (5)$$

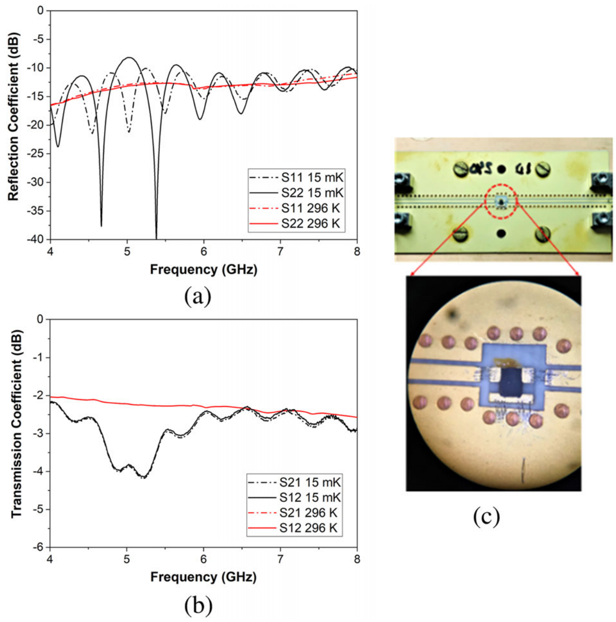

Fig. 2 *(a) Calibrated S-parameters of the 2 dB attenuator IC at 15 mK and 296 K using $\frac{1}{4}$-wave TRL: (a) reflection coefficient; (b) transmission coefficient; (c) close-up of DUT PCB and IC*

In order for the line standards to have continuous frequency coverage from f_{min} to f_{max}, avoiding any calibration failure points in between these frequencies, line lengths l_1 and l_2 must be chosen such that f_{Ul_1} must be greater than f_{Ll_2}.

Using the above equations, l_1 was designed to have a length of 26.8 mm with respect to the Thru standard, covering the frequency range 4–6.7 GHz, and l_2 was designed to have a length of 18 mm with respect to the Thru standard, covering the frequency range 6–10.1 GHz, with both lines covering the overall frequency range from 4 to 10.1 GHz.

Measurement results: The MCU was deployed inside the dilution refrigerator along with the fabricated calibration standards and DUT for operation at mK temperatures. A commercial 2 dB attenuator IC was used as the DUT to demonstrate the operation of the MCU. The calibrated S-parameters of the DUT at room temperature and mK temperature using $\frac{1}{4}$-wave TRL technique are shown in Figure 2.

The measurement results at room temperature demonstrate a good impedance match (with reflection coefficients less than -10 dB) for both DUT ports, from 4 to 8 GHz, as shown in Figure 2(a). A relatively flat 2 dB attenuation level can be seen in the transmission coefficient curves in Figure 2(b) with variation up to 0.4 dB across the frequency range. However, at mK temperatures, there is poorer impedance matching particularly around 5 GHz. This is likely to be due to the low temperature effects on bond wires on one side of the DUT. A corresponding degradation is seen in the transmission response around 5 GHz, increasing the DUT attenuation to around 4 dB. The calibrated amplitude and phase response of transmission lines l_1 (26.8 mm) and l_2 (18 mm) have been investigated as test devices after applying $\frac{1}{4}$-wave TRL technique and no unexpected resonances have been observed as was seen in Figure 2(a) for the 2 dB attenuator IC. This ensures that the resonances introduced in Figure 2(a) are not due to any residual errors in the calibration process. The calibrated measurements have enabled these subtle changes to be observed (e.g. in the bond wire performance at 15 mK).

The calibrated S-parameters of the DUT at mK temperature using the $\frac{3}{4}$-wave TRL calibration technique are shown in Figure 3. The calibrated measurements using $\frac{3}{4}$-wave and $\frac{1}{4}$-wave TRL technique show mostly good agreement, by comparison between Figures 2 and 3. However, it can be observed in the S22 response, at the frequency at which the line changes from line 1 to line 2, a step-change is introduced in the measurement results similar to the one reported in the $\frac{3}{4}$-wave TRL technique in [14]. Such step changes are likely due to contact impedance variations of the solder free edge-launch 3.5 mm connectors installed on the PCB boards, creating a different mismatch. To avoid this step change, when switching between different calibration line standards, a weighting

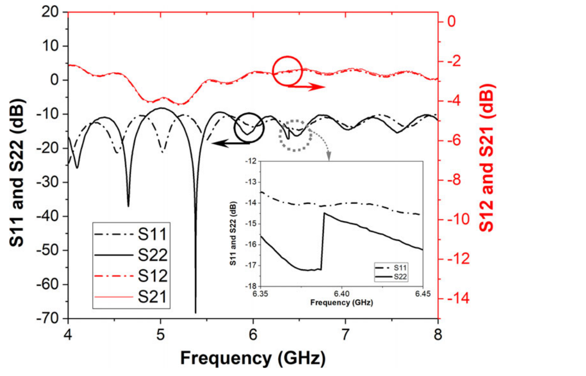

Fig. 3 *Calibrated S-parameters of the 2 dB attenuator IC at mK temperature using $\frac{3}{4}$-wave TRL technique*

function is introduced as a post-processing step. This effectively combines data from both line standards [14]. A sine squared weighting function is used to ensure that the weights are always positive, and that the weights decay rapidly away from the region where the $\frac{3}{4}$-wave calibration gives optimum performance – that is at the frequency corresponding to a $\frac{3}{4}$-wavelength ($\phi = 270°$). Therefore, at each measurement frequency, the data is weighted according to how well-suited the $\frac{3}{4}$-wave Line standards are for providing data for the calibration.

We assign weights, w_i ($0 < w_i < 1$), which change with the phase, φ_i, of Line standard i ($i = 1$ or 2) with respect to the Thru:

$$w_i = \sin^2 \phi_i. \tag{6}$$

Then a weighted mean, $\hat{x}$, of data, x_i from the two lines, at each frequency is used:

$$\hat{x} = \frac{\sum_{i=1}^{2} x_i w_i}{\sum_{i=1}^{2} w_i}, \tag{7}$$

where x_i is either the real or imaginary component, respectively, of each of the four calibrated S-parameters. The weighing technique is demonstrated in Figure 4 using the imaginary component of S22, where in Figure 4(a) a step change at 6.4 GHz can be observed, when the chosen line standard is changed from Line 1 to Line 2. The weighted mean in Equation (7) is now applied to obtain Figure 4(b), thus smoothing the transition at 6.4 GHz. By using independent calibration techniques, the S-parameter measurements have been validated at mK temperatures.

Conclusion: An S-parameter measurement setup to characterise RFICs at mK temperatures has been designed for operation from 2 to 14 GHz and tested at mK temperatures. Two independent calibration techniques have been proposed to validate S-parameter measurements at mK temperatures and to provide confidence in, and demonstrate the reliability of, such measurements. The calibration techniques have been demonstrated by characterising and validating the S-parameters of a cryogenic attenuator IC at mK temperatures. These techniques can be applied to characterise other RFICs, including Quantum Integrated Circuits.

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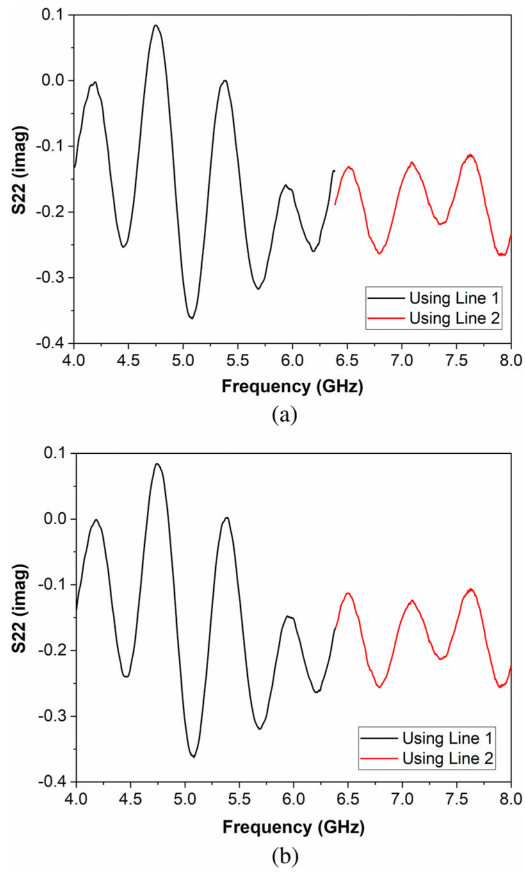

Fig. 4 *Imaginary part of S22 of DUT: (a) no weighting; (b) weighting applied*

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