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Prefaces, Sorites and Guides to Reasoning

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1. An Analogy

Is there an interesting relation between the Preface paradox and the Sorites paradox that might be used to illuminate either or both of those paradoxes and the phenomena of rationality and vagueness with which they, respectively, are bound up? In particular, if we consider the analogy alongside a familiar response to the Preface Paradox that employs degrees of belief, does this give any support to the thought that we should adopt some kind of degree-theoretic treatment of vagueness and the Sorites? I will argue that not only does it not give such support, but that some of the disanalogies count against such a treatment of vagueness more generally. Dorothy Edgington's work will be at the centre of my discussion. She considers analogies between the Sorites paradox and both the Preface and Lottery Paradoxes in her 1996 as part of the case for her important and original degree-theoretic account of vagueness.

I start by presenting the scenario within each paradox in such a way that brings out an analogy. First, a careful author believes each sentence in her book, but asserts in the Preface (acknowledging her fallibility) that some statement in her book is false.[1] Consider the argument from each of the accepted premises – the individual statements in the book – to their conjunction, or the quantification over them all. Although it is valid, this argument yields a conclusion the author explicitly rejects. She believes each of these premises, but the negation of their conclusion; moreover, she seems quite rational to do so. For the Sorites, consider a series of men, the x_i, starting with a 7 foot man and such that each is one hundredth of an inch shorter than the previous one. Consider x_k; x_{k+1} is only one hundredth of an inch shorter, so surely if x_k is tall, then so is x_{k+1}, so we have a series of compelling

[1] See Makinson 1965 for the classic presentation of the Preface Paradox. Since I am primarily interested in the analogy with the Sorites paradox, I will not attempt to contribute to debates directly tackling the Preface Paradox, and will sometimes make simplifications in my discussion of it.

conditionals, (C_1) “if x_1 is tall then x_2 is tall”, (C_2) “if x_2 is tall then x_3 is tall” ... (C_i) “if x_i is tall then x_{i+1} is tall” ... . But the argument from the claim that x_1 is tall and these conditionals yields the absurd conclusion that x_{4000} is tall, though he is shorter than 4 foot. Such an argument can typically be constructed for any vague F on the basis of a suitable Sorites series of items. Both many-premised arguments concerning the Preface and Sorites start with many individually compelling statements and end up with a conclusion we reject (i.e. regarding the truth of all sentences in the author’s book or the classification of the last, clearly non-tall member of the Sorites series).

In both cases the only rule needed is an utterly compelling, classically valid rule – conjunction introduction or modus ponens. And, in both cases, a natural informal first reaction to what has gone wrong has to do with the sheer quantity of premises. Whereas a move from two things that you believe to their conjunction is harmless, a move from thousands of individual things to the claim that they are *all* right is not safe. And whereas the move down a couple of steps of the Sorites series will not lead you astray, following hundreds of steps *will* do.

Both the Preface paradox and the Sorites paradox highlight an inconsistent set of claims all of which a (normal, apparently rational) subject believes. In itself, this situation is not paradoxical as the subject can simply have got some things wrong and thus have some false beliefs. With the Preface, the *paradox* arises because we seem to be committed to certain principles about *rationality*, e.g. that it’s never rational to believe inconsistent statements, or that it’s always rational to believe the consequences of our beliefs. The key tension, then, is between the subject’s apparently sensible beliefs and compelling principles about rationality. On the other hand, the Sorites is a paradox because we aren’t interested just in a subject’s *beliefs* but with the compelling hypothesis that each of the statements in the problematic set – the conditionals in the series, the claim that the first member of the series is F and the claim that the last is not-F – is *true*. This is an important difference between the two cases, but not yet a reason to discard the analogy.

There is another disanalogy that will, I argue, turn out to be particularly significant. In the Preface case, our author believes *each* of the statements but not *all* of them conjoined (i.e. the generalisation saying they are all true). But in the Sorites case, the subject typically believes each of the conditionals *and* believes all of them, or the generalisation over them taken together, namely "for all i, if x_i is tall then x_{i+1} is tall". Indeed, one of the most popular formulations of the Sorites paradox starts from a variant on that very generalisation, e.g. with the premise

(C_g) "If x is tall and y is one hundredth of an inch shorter than x, then y is also tall".

Again, adding the premise that x_1 is tall, we can derive the absurd conclusion that x_{4000} is tall.

By contrast, there is no way of compressing the Preface into a short, two-premise argument whose premises are still plausible. Any appeal to the analogy will have to downplay this popular version of the Sorites paradox and perhaps explain away the intuition that the generalised premise is true. Edgington, for example, claims that the "long Sorites" (the one with many individual conditional premises) is "basic", in contrast with the "short Sorites" (containing a single generalised or inductive Sorites premise) (1996, p.311). We will come back to this below.

2. Degrees of belief and the Preface

An appealing response to the Preface Paradox involves recognising degrees of belief: our author should (and does) believe each of the individual statements in the book to a degree just less than certainty and the slight doubts can add up, explaining her rational disbelief in the large conjunction of them all.[2] The degrees of belief can be modelled probabilistically, and compelling principles of rationality can then be formulated to replace a non-probabilistic requirement such as "believe the conclusion of a valid argument if you believe the premises".[3] A familiar result from Ernest Adams shows how the probability of a conclusion of a valid argument relates to the probabilities of its premises. If we call 1 minus the probability of p the improbability of p, we can say that the improbability of the conclusion cannot exceed the sum of the improbabilities of the premises.[4] This allows a valid argument with 10 premises of probability 0.9 to have a conclusion with probability 0.

[2] See, e.g., Foley 1992. My initial description of the paradox made no reference to degrees of belief, but we can use the above thought to explain our author's rationality, at least if we assume a correspondence between (non-quantitative) rational belief and rational belief above some threshold of degree of belief; cf Foley's "Lockean thesis". See, e.g., Hawthorne and Bovens (1999) on locating the threshold, which can change between people or across contexts.

[3] Or, perhaps, "don't believe both the premises and the negation of the conclusion of a valid argument": the differences between different options here will not matter for our purposes. See Field 2009 (p.259) for one option of such a probabilistic principle (where P(A) is the subject's degree of belief in A): "(D*) If it's obvious that A_1, ..., A_n together entail B, then one ought to impose the constraint that P(B) is to be at least $P(A_1) + ...+ P(A_n) - (n-1)$ in any circumstance where A_1 ... A_n and B are in question."

[4] Adams 1966. See also Edgington 1996.

Rational degrees of belief can be assumed to follow this structure, so that your degree of disbelief in the conclusion of a valid argument should not exceed the sum of the degrees of disbelief you have in the premises. Our author can then remain rational: there are so many statements in the book that the slight uncertainties (i.e. degrees of disbelief) in each add up to at least 1.

Acknowledging degrees of belief in a Preface-type case can help guide us with how to reason in the face of uncertainty: by recognising the extent to which our beliefs fall short of certainty and the number of premises, we can exercise the appropriate level of caution in drawing our conclusion, in line with the Adams principle. So with a short argument with just a couple of premises, we can remain confident in the conclusion, whereas lots more premises prompts lots more caution. Could appeal to a similar degree-theoretic framework in a theory of vagueness provide a similar story about how to reason in the face of vagueness?

Many theorists have sought to approach vagueness through a degree-theoretic framework whereby, for example, tallness can come in degrees, and the degree of tallness can drop through a Sorites series from the degree 1 definite cases to the degree 0 definitely not-tall cases, with different borderline cases taking different degrees in between. Many such theories employ degrees of truth: they regard truth as coming in degrees between 0 and 1 and the degree assigned to "x is tall" is taken to gradually drop from 1 (or very nearly 1) to 0 (or very nearly 0) through our Sorites series of men of decreasing height. Can some such theory provide the best guidance for our reasoning when vagueness is involved?

In Edgington's presentation of reasoning with uncertainty and vagueness, she discusses two analogous "scare stories" (1996, p.300 and p.302): the Preface suggests that we should only trust reasoning that starts from premises of which we are certain and the Sorites suggests that we can only trust valid arguments when our premises are completely clearly true. Given the vast extent of both uncertainty and vagueness that we deal with, this threatens to leave us very frequently without trustworthy arguments to use. Her approach builds on the thought that an appeal to degrees gets us out of these two problems and allows us to avoid the Scare Story. I will argue that it does not help with vagueness.

3. Degree-theoretic accounts of vagueness: Edgington's option

On Edgington's powerful degree theory, the structure of degrees of belief discussed above in relation to the Preface paradox is mirrored by another degree theoretic structure that she labels "verities", or "degrees of closeness to clear truth" (rather than the degrees of *truth* that other theorists employ).[5] The degrees assigned to complex statements thus exhibit a probabilistic structure. This, again, is unlike other degree theoretic accounts of vagueness according to which the degree of truth of a compound sentence is a function of the degrees of its components – for example, where the degree assigned to a conjunction is the minimum of the degrees of the conjuncts. On a probabilistic model, the degree of the conjunction depends on the relations between the conjuncts, just as the *probability* of a conjunction is not just determined by the probability of the conjuncts. More specifically, Edgington employs the idea of "conditional verity", where "the conditional verity of B given A is the value to be assigned to B on the hypothetical decision to count A as definitely true." (1996, p.306). For example, if Tim and Tek are both borderline tall but Tim is slightly taller than Tek, then the conditional verity of "Tim is tall given that Tek is tall" should be 1. For if we were to take it as true than Tek was tall then we would have to count Tim as tall too, since he is taller. But if we consider "Tek is tall given that Tim is tall", the verity will be less than one: deciding that Tim is tall doesn't yet settle that the shorter Tek is also tall. Rather, Edgington maintains, the decision induces a rescaling of the borderline cases shorter than Tim, rendering Tek near the top of the resulting borderline cases, so that "Tek is tall" would then have value close to 1 and so this is the value to be assigned to "Tek is tall given that Tim is tall".[6]

The value of compound sentences is then calculated using this notion of conditional verities, with the exception of negation, where the verity of not-A is 1 minus the verity of A. The verity of "A&B" is the value of A multiplied by the conditional verity of B given A. So, for example, "Tek is tall and Tim is tall" has the same verity as "Tek is tall" (since the conditional verity of "Tim is tall given Tek is tall" is 1). But "Tek is tall and Tim is *not* tall" appropriately gets verity 0, since "Tim is not tall given Tek is tall" gets assigned 0 (if we were to decide that Tek was tall, the taller Tim would also count as tall and "Tim is not tall"

[5] See Edgington 1996 for her theory. Cook (2002 p.239) interprets Edgington's verities as degrees of truth, but she avoids this commitment to degrees of truth (1996, p.299). I will not focus on this issue here; see my 2012, p.462 for some further discussion.
[6] See Keefe 2000, pp.99-100 for worries about the notion of conditional verity, at least for cases other than straightforward ones like the first described above.

would get verity 0).[7] The natural interpretation of the conditional "If A then B" takes it as equal to the conditional verity of B given A.[8] The individual conditional premises of the Sorites – the (C_i) – will then each have a verity close to 1, for the verity of Fx_{i+1} given Fx_i will be close to 1 (as we saw with "Tek is tall given Tim is tall").

As Edgington demonstrates, Adams's result has a parallel within her structure of verities: the unverity (1 minus the verity) of the conclusion of a valid argument cannot exceed the sum of the unverities of the premises. She calls this the "verity constraining property of valid arguments" (1996, p.307). The application to the Sorites then shows how the many premises that are each highly plausible, and indeed have a verity close to one, can yield a false conclusion, despite the validity of the argument. Verity seeps away, given the many premises falling short of verity 1, resulting in a false conclusion despite the validity of the argument.[9]

Just as the appeal to degrees of belief can show how an author can rationally believe all the sentences in their book and yet believe that there is something false in it, appeal to verities can show why we find each of the Sorites conditionals – the (C_i) – compelling, because they have verity of nearly 1, even though they entail something false. The analogy may seem to help understand or support the theory of vagueness in question here, but I will argue that the position is not as appealing as it may seem.

Let us return to Edgington's "Scare Story" and ask how far it helps to introduce verities and the verity constraining property of valid arguments. The worry with the Preface was that it shows that we can be justifiably highly confident of each of a group of claims (those in the body of the book), without that giving us grounds for confidence in a consequence of them

[7] This illustrates an advantage of Edgington's model over the kind of degree-theoretic alternatives discussed in the next section. According to the latter, "Tek is tall and Tim is not-tall" will get the minimum value of "Tek is tall" and "Tim is not tall" which will be around 0.5, even though we are strongly inclined to regard that statement as false. I won't examine this aspect of the debate here, however, focusing instead on issues more directly related to the analogy between the Preface paradox and the Sorites.

[8] See Edgington 1992, p.202. Edgington 1996, p.307 resists commitment to this identification (which is problematic in combination with some of her other views on conditionals), setting aside the question of whether it is correct. We will continue with the simplification of assuming it is correct for the purposes of considering the Sorites paradox. An alternative – which would follow her 1996 more closely – would have been to concentrate on the Sorites formulated with the material conditional, $A \supset B$ defined as $\neg(A \& \neg B)$, e.g. with the series of compelling premises $\neg(Fx_i \& \neg Fx_{i+1})$.

[9] In my 2012 I express a worry about this perspective on the Sorites given Edgington's denial that verities are degrees of truth: "if it isn't truth that seeps away (as on the more standard interpretation of degrees of truth), why does the conclusion end up false?" (p.462). I put that aside here.

(the claim that they are all true), suggesting that we should only trust reasoning that starts from premises of which we are certain. The story about degrees of belief, coupled with the probability-constraining property, ensures that we have guidance beyond the cases when our premises are certain. The parallel Scare Story in the vagueness case would suggest that we can only trust valid arguments when our premises have verity 1; the verity-constraining property promises a way out. The conclusions of our valid arguments cannot drop below certain levels, though the more premises there are with verities less than 1, the lower the conclusion can drop.

This may, then, help with many-premise Sorites where the reasoning goes wrong because it uses too many premises of verity close to but less than 1. But the "short Sorites" cannot be solved the same way: (C_g), "if x is tall then anyone one hundredth of an inch shorter is also tall" is assigned verity 0 and thus counts as clearly false (see Edgington 1996, p.311). It cannot be that this seems compelling because it is nearly true, so this counts against the theory. Edgington sees the "long Sorites" as "the basic form": "the universally quantified statement in the short version has to be understood via the relatively basic, relatively less complex statements which are its grounds" (1996, p.312), but she does not say much to support this claim. If this commits us to a corresponding explanation of our beliefs about the respective premises, then this prompts various psychological questions that we cannot address here. But, the account of our beliefs it suggests is not appealing. It is not that we survey all the instances of (C_i) and believe the generalisation over them all on that basis: we have not formed beliefs about most of the instances of (C_i) independently (I have never considered Fx_{237}, for example). Rather, for any arbitrary instance, our reasons for believing it are general ones, e.g. that one hundredth of an inch is too little difference in height to make the difference to whether someone is tall. And that kind of reason surely supports (C_g) directly. It is more plausible to think we believe that generalisation and (perhaps implicitly) believe each instance on those grounds.

Does Edgington's framework provide a guide to reasoning and one that answers the Scare Story? More specifically, insofar as it provides a guide, is it successful in such a way as to lend support to Edgington's theory and the employment of a degree-theoretic structure in the semantics of vagueness? A semantic theory can always yield *a* guide to reasoning in the sense that it tells you the status of a conclusion given the status of premises and thus we should be guided by the expectations about the status of the conclusion that this delivers, given the status of the premise. We want to know if it delivers a *good* guide: what reasons do we have for thinking that its verdicts on what counts as good reasoning are right? And considering the guide in this way will only help one's actual reasoning if we have access to what the semantic statuses of our premises are.

In the case of the framework surrounding the Preface, introspection (in judging one's own level of confidence in something) is a really good guide to one's degree of belief: even if we can't assign an *exact* degree this way, we are reliably accurate to a great extent in such judgements. This allows us to follow the instructions on what attitude to take to our conclusion given those degrees of belief in the premises. Things are not so straightforward with verities, however. In some cases, our degree of belief in a proposition is a good guide to its verity. In particular, it may be typically roughly right with atomic predications: the verity of "x_i is F" drops through the Sorites series in tandem with our inclination to classify x_i as F. But that is not enough to help with the arguments and reasoning we are interested in, for they will rarely involve atomic cases alone: the kinds of reasoning we are interested in all involve compound sentences. With non-atomic sentences, the extent to which we are inclined to believe something can be a bad indication of its verity, even when we're in possession of the key facts and would judge the verity of the relevant atomic sentences roughly correctly. For example, as we've seen, (C_g), the Sorites inductive premise, has verity 0, despite our strong inclination to believe it.

This might suggest that you should always calculate the appropriate degree-theoretic status for the premises by using the semantics: consider what the framework assigns to a sentence of that structure given the verities of the components and certain relations between them. But that would require you to know the status of the components, and we can't always assume that when we reason from a complex sentence, we do have access to that information. For example, this would be no good for standard employment of elimination rules (where the idea is to reason from complex sentences to the components, so shouldn't depend on prior knowledge of those components). Contrast this with the uncertainty case: our author's low degree of belief in "every statement in my book is true" is not based on a calculation of what that degree must be given various degrees of beliefs in the instances.

Note, that this isn't simply saying that the problem is that we're sometimes wrong in our assessment of the premises and that we can find a premise compelling when it is in fact a long way from nearly clearly true and quickly leads to a false conclusion. Any guide to reasoning must allow for this: it tells you how to extend your beliefs and/or whether to draw a given conclusion *given* what you believe and can't guard against error in your beliefs about the premises. In the treatment of the Preface, we must, of course, acknowledge that the author has a false belief in one of the claims in her book. But the degree-theoretic treatment of uncertainty doesn't just tell her that, which would provide no guidance when she has no reason to reject any one rather than another. It tells her, more helpfully, to lower her degree of belief in all the premises. Similarly, responding to the contradiction in the Sorites set-up by simply telling the subject that a premise is false provides little guidance. Edgington does not do this for the long Sorites (mirroring the probabilistic uncertainty case), but does do that for the short Sorites.

With Edgington's theory we could maintain that we *should* have a low degree of belief in the Sorites inductive premise because the semantics dictates that. But if there's no independent support of the pattern of degrees it yields, then the availability of the picture gives no support for the use of degrees in the theory of vagueness. When degrees are employed in the Preface case, this shows how the typical (compelling) combination of degrees of belief in the Preface propositions is rational after all. It predicts the degree of belief in the claim that everything in the book is true and that makes it a story that fits well with our beliefs and so a plausible story about them. The story with verities isn't like that. A story that assigns degrees that don't correspond to our actual degrees of belief needn't be any better off than a view like supervaluationism, according to which (C_g) is false.

The degree theorist may say that we *have* to recognise that the main premise of the short Sorites is false – so has verity 0 – given the truth of the other premise and the falsity of the conclusion. But the degrees would then be doing no work in that story. Note that we have no reason to introduce degrees into our semantics if they are just used in an explanation that would be available to us by considering uncertainty alone. For example, it might be said that in advising how one should respond to the long Sorites, we should tell the subject to have a lower degree of belief in each of those conditionals, rather than presenting them with a specific premise that they should reject. But this advice can be combined with any theory (e.g. an epistemic theory of vagueness) without assuming a role for verities or degrees of truth in the semantics or in the treatment of vagueness: we can just appeal to the account of reasoning with uncertainty that we've already accepted. There's no reason to think that the degrees of belief assigned on this basis correspond to anything relevant to the *semantics* of vagueness and so this constitutes no support for a degree theory of vagueness.

The analogy between the Preface Paradox and the Sorites Paradox thus does not support theories that require degrees in their semantics. And we cannot sustain the appealing degree-theoretic thought that something seeming true indicates that it is at least nearly true when, e.g., a sentence like (C_g) seems true but has verity 0. This is unlike in the Preface-type case where if we think a premise is true but we recognise we are less than certain of it, we can modify our confidence in the conclusion to an extent determined by the number of uncertain premises. That was how we avoided the original version of the Scare Story. We have seen that Edgington's story about vagueness does not deliver an equally satisfying solution to the corresponding Scare Story. Cases like the short Sorites sustain the threat that we should abandon reasoning with vague premises that seem true, because their seeming true is not a good indication that they are *nearly* true (or that we should think they are) and they can thus lead to a false conclusion in very few steps.

4. Non-probabilistic alternatives and the Scare Story again

The probabilistic structure of Edgington's degree theoretic account of vagueness is atypical of such accounts of vagueness. Many other accounts have been proposed and defended according to which the degree assigned to a complex sentence is a function of the degrees assigned to the components.[10] Different options are available corresponding to different choices of the function taken to capture each of the connectives. We will express "the degree of A" as $|A|$. The usual definition of conjunction has $|A\&B| = \text{Min}\{|A|, |B|\}$, though on some accounts it is defined as the product of $|A|$ and $|B|$. Most commonly, $|A \vee B| = \text{Max}\{|A|, |B|\}$ and $|{\sim}A| = 1 - |A|$. There is more variation over the definition of the conditional, which can affect the evaluation of the key premises of the long Sorites, the (C_i), where often the value of the conditional will just drop below 1 to the degree that the consequent is less true than the antecedent.[11]

[10] See e.g. Keefe 2000, Chapter 4 for a summary and discussion of this range of theories and Smith 2008 for a recent book-length defence of one option.

[11] E.g. according to Machina 1976, $|A \supset B| = 1$ when $|A| \leq |B|$ and $= (1 - |A|) + |B|$ otherwise.

An account of the quantifiers will also be needed to evaluate the short Sorites, with a popular option for the existential quantifier following on from the most common definition of disjunction so such that the existential generalisation is as true as its most true instance. This may seem to offer us a way to preserve the thought that sentences (like the premises of the Sorites) can seem true because they are nearly true, reinstating the parallel with the Preface paradox. For (C_g) "if x is tall then anyone one hundredth of an inch shorter is also tall" can come out nearly true because the generalisation takes the value of the least true instance and each of the (C_i) are nearly true.[12] One worry, though, is that such a degree theory of vagueness is not compatible with a good answer to Edgington's Scare Story and, in particular, it cannot provide an explanation of why we don't just need to abandon all our usual reasoning once we acknowledge uncertainty/vagueness. To explore this in more detail, we need to consider the various different definitions of validity that can be (and have been) adopted in degree-theoretic frameworks. For, starting from the standard classical definition of validity as necessary preservation of truth, there are several different ways to generalise this to accommodate degrees of truth.

On one popular degree-theoretic story about validity, an argument is valid if and only if its conclusion cannot be less true than its least true premise.[13] This is a sense in which degree of truth can count as being preserved in valid arguments. On such an account, all Sorites arguments will be invalid. For example, the least true of the (C_i) (the individual conditional premises) will be nearly true on most accounts and yet the conclusion will be (at least almost completely) false. Moreover, modus ponens will be invalid too, as, for example, if $|A| = 0.5$ and $|B| = 0.2$, then on the most common account of the conditional, $|A \supset B| = 0.7$ and the other premise, A, has value 0.5, yet the conclusion, B, drops to value 0.2 (i.e. below the value of either premise). The Scare Story is then particularly pressing if we are obliged to give up such fundamental rules of inference on acknowledging the possibility of vagueness (and sentences less than degree 1 true). Edgington says that this treatment of modus ponens (and other classically valid reasoning) "licenses modus ponens on clearly true premises, and licenses nothing on not-clearly-true premises. … it leaves us inferentially impotent in the presence of vagueness" (1996, p. 303).

[12] This, for example, will be the verdict on the account in Machina 1976, which is also the one Williamson argues that the degree theorist should accept (Williamson 1994, p.114-118).
[13] E.g. Machina 1976, Forbes 1983.

Even on an alternative account of the conditional according to which some of the (C_i) are not nearly true, those conditionals will never be as false as the conclusion. For example, Smith's account of the conditional (2008, p.268) employs the classical equivalence between ($A \supset B$) and ($\sim A \vee B$), thereby allowing some of the (C_i) to drop to around 0.5 (e.g. for the cases around the middle of the borderline cases when both $\sim Fx_i$ and Fx_{i+1} have values around 0.5). But none of those conditionals will drop much *below* 0.5, and so not as low as the conclusion.[14] So, this does not allow us to preserve validity within the Sorites according to the popular definition of validity above. As I'll reiterate below, we also lose the explanation that the conditionals seem true because they are nearly true.

An alternative definition of validity in a degree-theoretic framework identifies it with preservation of degree 1.[15] This can be seen to generalise the classical definition of validity (necessary preservation of truth) in a different way, regarding degree 1 as truth and only requiring preservation of that. On this account, the Sorites argument comes out valid. But we get no answer to the Scare Story, for the account does not help us reason with premises of values less than 1. Again, we threaten to be left "inferentially impotent in the presence of vagueness" (Edgington 1996, p.303).

Nicholas J.J. Smith advocates an account of validity that is different again and which he claims does preserve classical logic. According to that account, an argument is valid if and only if, whenever the premises are >0.5 degrees true, the conclusion is ≥0.5.[16] It might then be hoped that, unlike with the earlier definitions of validity, we get guidance for premises that have degrees <1 without abandoning the likes of modus ponens. Is *this* kind of account better equipped to deal with the Scare Story? I argue not. For one, the guidance it gives you as to how to reason will need to assume that what you care about is whether the conclusion is ≥ 0.5. That does not seem to reflect our reasoning practices. One kind of scenario we may well want to rule out in our reasoning is the kind where we start from a couple of highly compelling premises that are very nearly true and draw a conclusion that is barely over half true. Yet the Smith definition of validity allows some such reasoning to be valid.

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[14] If Fx_{i+1} is much below 0.5, so will Fx_i be, in which case $\sim Fx_i$, and so the conditional (C_i), will have a value higher than 0.5.
[15] E.g. Peacocke 1981.
[16] Smith 2008, p.222.

How does Smith deal with Sorites arguments: how can they deliver a false conclusion, despite compelling premises? On Smith's account of conditionals, one of the premises (C_i), is just less than 0.5 degrees, for the pair Fx_i and Fx_{i+1} whose values straddle the 0.5 boundary, will be such that $\sim Fx_i$ and Fx_{i+1} are both <0.5. So, the argument can be valid despite the conclusion being ≤0.5, since one of the premises is <0.5. Similarly, the generalisation (C_g) comes out less than 0.5 true. These cases help show that Smith's account is another account on which we cannot explain that the main premise of the Sorites is so compelling by saying that this is because it is nearly true. Indeed, this explanation cannot even be adopted for some of the individual conditionals (so the situation is even more serious than for Edgington). As with Edgington's account, the semantics does not have the same practical use for our reasoning, given that we cannot directly assess the right degree to be assigned to our premises in the way we can judge uncertainty of complex sentences. The subject's appeal to her degrees of belief (roughly, the extent to which she find something compelling) corresponds to degrees of truth at best only in atomic sentences, such as with the individual predications Fx_i through the Sorites series. But our *reasoning* requires complex sentences such as the conditionals in the Sorites argument. This is unlike the case where we introduce degrees of uncertainty to confront the Preface Paradox, where our uncertainty in simple or complex premises can typically be taken at face value and fed into the guidance of how uncertain we should be in our conclusion.

So the analogy with the appealing degree-theoretic approach to the Preface Paradox breaks down in the vagueness case. None of the various choices on the definition of validity or the account of the connectives will give us what we need. One short-coming has built on the initial observation of the disanalogy between the short Sorites and the corresponding compressed version of the Preface paradox. Once we recognise that the short Sorites is compelling, a tension is generated. On the one hand we want to acknowledge that its main premise is highly compelling, so should come out with a high degree value, otherwise we cannot maintain that "seems true is nearly true". And if we abandon that, we fall foul of Edgington's Scare Story again, since the vagueness of compelling premises does, after all, seem to give us a reason to abandon reasoning with them; this is unlike in the uncertainty case where apparent near certainty in premises does allow us to carry out short inferences and where there is a clear story about how our confidence should decline in longer cases. On the other hand, if the generalised premise of the short Sorites does get a high degree value, the shortness of the short Sorites makes it a case where premises of high values can lead you dramatically astray, so yet again we face the Scare Story, this time because it shows we can't even trust short arguments with premises that are nearly true.

This undermines the analogy between the Sorites and the Preface and undercuts what might have looked like a good reason to pursue a degree theory of vagueness, while also serving as a good illustration of why such theories can't fulfil the roles hoped of them. This should not, of course, be taken as an attempted refutation of degree theories of vagueness. It is still open to degree theorists to accept the way degrees of truth or verities diverge from degrees of belief and maintain that some choice of definitions of the connectives and of validity provides the right story about truth-values and logical consequence and can guide our reasoning. We need, they may go on, to accept those definitions and work with them rather than following our intuitions about, for example, the degrees of complex sentences. At the very least, some of the apparent advantages of this kind of approach will be lost, however, including the appealing analogy with reasoning with uncertainty.

In giving a semantics, Edgington and other degree theorists provide some kind of guide to reasoning: we should reason as the semantics dictates, e.g. assuming the conclusions of an argument to take values as dictated by the semantics and the values of the premises. A guide of this form can be good or bad (according to whether the semantics is good or bad), so its mere availability says nothing in favour of the semantics in question. One hope for the analogy with the Preface could have been that this analogy provides support for the chosen semantics for vagueness. But this is undermined by the fact that those semantics for vagueness do not have the same support from the agreement with our intuitions as that displayed in the uncertainty case, as demonstrated by considering generalisations like (C_g).

In the next section I turn to a theory of vagueness that I claim provides us a better guide to reasoning. The analogy with the Preface Paradox plays no role here, but I claim that this, in fact, works to the theory's advantage. We do not need to employ a degree theoretic semantics to reason with vague expressions; we can have a better guide by considering our premises and how to reason in the light of the supervaluationist's semantics.

5. Supervaluationism: a better guide to reasoning?

According to the supervaluationist theory of vagueness, a vague sentence is true (false) iff it is true (false) on all ways of making it precise (on all "precisifications").[17] If Tek is borderline tall, "Tek is tall" will be neither true nor false, because if we were to make "tall" precise, we could draw a boundary at a height above Tek's or at one below his height, so it is neither true that he is tall on all ways of making "tall" precise nor false on all those ways. The main premise of the Sorites paradox, (C_g), comes out false because on each way of making "tall" precise, there is a sharp boundary, so a consecutive pair in the series where one counts as "tall" and the next one doesn't. But since it is a different pair on different precisifications, there is no pair of which it is true to say the first is tall and the second isn't. The parallel premise of an unproblematic reversed argument, (R_g) "if *y* is one hundredth of an inch *taller* than *x*, and x is tall, then y is tall" does come out true, however, as it is true however we make "tall" precise.

To use this theory to guide our reasoning, we should consider our premises by asking whether they are true on all ways of making them precise and only go on to reason with them if so. We can then use classical logic in our reasoning as this is the logical system delivered by the supervaluationist theory of vagueness.[18] Appropriately, this story about how to reason helps us assess our premises, in addition to telling us what arguments are valid. We will do well, in general, to be cautious with our reasoning in certain cases where vagueness is present, and the supervaluationist guidance on reasoning provides a clear, plausible story about how to deal with various arguments. For example, when confronted with the Sorites argument, we are told not to go ahead and reason from the premises, because (C_g) is false on all precisifications. On the other hand, though still a vague sentence, the reversed premise (R_g) can be used as a premise of an argument since it is true on all precisifications.

Since supervaluationists declare the generalised Sorites premise (C_g) false, they need to explain away our intuition that it is true. This is typically done by, for example, pointing out that although it is false, it does not have a false instance, and by offering statements (e.g. ones containing the definitely operator) that do come out true according to supervaluationism which are suitably similar to the premise as to render it plausible that

[17] Supervaluationism is the theory of vagueness I defend in Keefe 2000. The locus classicus of the theory is Fine 1975.

[18] See Keefe 2000, p.174-81. There are some complications when it comes to arguments involving the "definitely" operator, but I put aside those issues for now.

there is some confusion between them, and yet which do not yield paradox.[19] It is true that degree theorists can and sometimes do endorse similar such explanations (see, e.g., Edgington 1996, p.311). That is no point in favour of the degree theorist's framework though, which is redundant in that explanation. There may appear to be a tension between my acceptance of the falsity of the generalised premise here, on my theory, and my earlier objection to the way that that premise came out with a low degree of truth on various degree theories. But my point is that the falsity of that premise on the degree theories undermined the relatively appealing thought that vague sentences seem true when they are merely nearly true, which was at the root of the motivation for the degree-theoretic framework and the response to the Scare Story, as considered above.

Could we get the best of all worlds by introducing a degree-theoretic structure within the supervaluationist framework? Such a structure can be provided by assigning a degree to a sentence according to the proportion of precisifications on which it is true.[20] This structure will then allow us to capture a gradual decline in degree of "x is tall" through the Sorites series, with the early borderline cases counting as tall on most, but not all, precisifications and the late borderline cases counting as not tall on most precisifications. As with other degree-theoretic pictures, though, the degrees thereby assigned to complex sentences will not typically reflect the extent to which we are inclined to believe them (for example, the main Sorites premise will, of course, come out true to degree 0). These degrees will not help with guiding our reasoning or with answering Edgington's Scare Story. When I come to reason from some premises, I cannot, for example, trust a vague sentence that I am more inclined to believe more than one than I find less compelling as the former may be one of those many complex cases which seems true without being nearly true. To see to what extent I should trust it, I need to look to the framework of precisifications. But then, I can follow the guidance that I don't trust that premise unless it is true on all precisifications. The degrees play no role there in cases like the Sorites with its false premise or the non-paradoxical parallel argument involving the true premise (R_g).

If a premise is false on all precisifications, I should not trust it and if it is true on all, I should. What if a premise is true on some and false on others, so is neither true nor false? Could

[19] See e.g. Keefe 2000, pp.184-86.

[20] For discussion of this idea, see Kamp 1975, Lewis 1970 and Keefe 2000, p.171-72. Edgington 1997 (pp.315-16) also suggests degrees treated in this way as a heuristic device in understanding "verities", noting that the relevant proportions of precisifications have the same probabilistic structure.

supervaluationist's degree play a role here? Consider, for example, the argument that extracts one step from the long Sorites, involving a nearly true predication of F to x_j and the individual conditional (C_j) deriving the conclusion that the next member of the series, x_{j+1}, is F. Perhaps the addition of degrees to the supervaluationist framework appropriately allows us to trust the conclusion of this reasoning to just a slight degree less than the conclusion. But, to see whether this is a significant addition to the general supervaluationist picture, we need to say more about what the supervaluationist who has not introduced degrees should say about reasoning with premises that are neither true nor false.

I maintain that advice on how to evaluate reasoning with premises that are neither true nor false is a complex affair. Clearly we should be cautious of accepting the conclusion. If there is no precisification on which the premises are all true together (so the conjunction of the premises is false), then those premises provide no grounds at all for accepting the conclusion. This is the situation with the long Sorites with the many conditional premises and in this case (if not always in others) we can see this incompatibility when considering the premises together. If the premises are all true together on some precisifications and not on others, the advice is less clear-cut. But, I suggest, it is both less clear what our supervaluationist story should say about these cases and what we should *want* a good story to say about them. The latter may vary depending on the kind of situation in which an argument is offered (even when one of its premises is known to be neither true nor false). Sometimes such an argument is advanced because we want the premise to be treated *as* true – a kind of stipulation. For example, if I offer the premise "x_m is tall" when we would normally call x_m borderline tall, that might be because I want this to be treated as true in the context. Now, we can make sense of this kind of situation in the supervaluationist framework by restricting the relevant precisifications in the context. We should then trust the conclusion in that context (i.e. given the stipulation) if the other premises are true on all those precisifications. For example, if I start from "x_m is tall" and use (R_g) ("if *y* is one hundredth of an inch taller than *x*, and x is tall, then y is tall" to work up the series), my conclusion that the later, taller, members of the series are tall should be accepted in the context. If, on the other hand, other premises are neither true nor false (true in some of the relevant precisifications, false in others), we will not be similarly entitled to draw the conclusion. For example, if I continue down the series using (C_g) instead, then the reasoning is questionable.

So, if faced with an argument with a premise that is neither true nor false, we need to decide whether or not to treat that premise as stipulated as true, and that will require us to look to the context. Care will need to be taken, especially as with certain complex sentences we can't consistently stipulate that they are true even if we want to – the Sorites premise might be an example of this phenomenon.

This only gives the beginnings of a story to guide our reasoning and the evaluation of reasoning in the presence of vagueness, even if supervaluationism is accepted. But, again, I take this to be partly a reflection on pragmatic complexities. This does not rule out the possibility that in occasional cases, the consideration of degrees – as captured by proportions of precisifications – could have a role in guiding our reasoning or the evaluation of other reasoning. But that would not undermine the merits of the supervaluationist story, and, indeed, might be thought to build on it. Moreover, for the purposes of this paper, it should be noted that reasoning within a Sorites argument is not the kind of case where considering degrees is of any help and so we do not have a reason to pursue the analogy between the Sorites Paradox and the Preface Paradox.

6. Conclusion

In our reasoning, we need to be alert to both uncertainty and vagueness: the Preface and Sorites paradoxes bring out problems relating to these phenomena. Modelling our uncertainty in terms of degrees of belief gives a picture that illuminates the Preface Paradox, and can guide us in assessing different arguments where uncertainty is present. But, I have argued, a degree-theoretic account of vagueness and the Sorites paradox cannot succeed in the same way. It may initially seem to promise a similar account whereby we are told what caution to exhibit in our reasoning with vague but compelling premises (as with compelling but uncertain premises) because we can assume they should be assigned a high but non-maximal degree. But, in fact, it is obliged to say that the things that seem true or close to true are often actually not to be assigned a high degree (and in many cases are completely false). The Scare Story – that uncertainty/vagueness may render us inferentially impotent – has not been avoided just by allowing some *simple* predications to seem true while being merely nearly true if the *compound* premises we reason with do not fit this pattern.

After exploring a range of degree-theoretic approaches here, I suggested abandoning the analogy between the two paradoxes and the degree-theoretic modelling that helps in one case. I then briefly argued that in dealing with vagueness, we should employ the supervaluationist theory in assessing both our premises and our forms of reasoning. There is no need to introduce degrees of truth or Edgington's verities and without them we have a better guide to reasoning.[21]

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