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RANS simulation of bubble coalescence and break-up in bubbly two-phase flows

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Abstract

In bubbly flows, the bubble size distribution dictates the interfacial area available for the interphase transfer processes and, therefore, understanding the behaviour and the average features of the bubble population is crucial for the prediction of these kinds of flows. In this work, by means of the STAR-CCM+ code, the S_γ population balance model is coupled with an Eulerian-Eulerian two-fluid approach and tested against data on upward bubbly pipe flows. The S_γ model, based on the moments of the bubble size distribution, tracks the evolution of the bubble sizes due to bubble break-up and bubble coalescence. Good accuracy for the average bubble diameter, the velocity and the void fraction radial profiles is achieved with a modified coalescence source. Numerical results show that better predictions are obtained when these flows are considered to be coalescence dominated, but, nevertheless, additional knowledge is required to progress in the development of coalescence and break-up models that include all the possible responsible mechanisms. In this regard, there is a requirement for experimental data that will allow validation of both the predicted bubble diameter distribution and the intensity of the turbulence in the continuous phase which has a significant impact on coalescence and break-up models. An advanced version of the model described, that includes a Reynolds stress turbulence formulation and two groups of bubbles to account for the opposite behaviour of spherical bubbles, which accumulate close to the pipe wall, and cap bubbles, that migrate towards the pipe centre, is proposed. The Reynolds stress model is found to better handle the interactions between the turbulence and the interphase forces, and the use of only two bubble groups seems sufficient to describe the whole bubble spectrum and the bubbly flow regime up to the transition to slug flow.

Keywords: Bubbly flow; RANS modelling; population balance; method of moments; bubble diameter distribution.

1. Introduction

Gas-liquid bubbly flows are common to a variety of processes encountered in numerous industrial sectors, including the nuclear sector as well as chemical and petro-chemical, oil and gas, mining, pharmaceutical and refrigeration industries, amongst others. In the nuclear industry, knowledge of the hydrodynamics of the two-phase flow is essential for the design and operation of boiling water reactors and natural circulation systems, and in the prediction of accident scenarios for pressurized water reactors as well as for other types of reactor. In chemical reactors, such as bubble columns and stirred tanks, gas bubbles are dispersed in the liquid phase to increase phase mixing and enhance heat and mass transfer processes.

In these flows, the exchange of mass, momentum and energy between the phases depends on the flow conditions, and on the interfacial area concentration in particular. This, in bubbly flows, is determined by the number and the size of the bubbles that are dispersed in the continuous liquid. Often, bubbles are not monodispersed and their distribution is far from steady, and evolves continuously in space and time, following interactions between the bubbles and the continuous phase and collisions between neighbouring bubbles (Lucas et al., 2005; 2010). These interactions induce bubble shrinkage and growth due to the pressure field and bubble break-up and coalescence, and, in boiling or reacting flows, also wall boiling, evaporation and mass transfer. The bubble distribution is therefore governed by these phenomena that, with bubble behaviour strongly related to bubble size and shape (Tomiyama et al., 1998), determine the local flow field, which, at the same time, affect the ratios of mass transfer, break-up and coalescence. In view of this strong coupling, understanding the evolution of the local bubble size distribution in these kinds of flows still represents a rather complex task which, nevertheless, is necessary if we are to be able to predict them with any degree of accuracy.

The use of computational fluid dynamic (CFD) techniques, applied today in design and as well as a development tool in most of the engineering disciplines, has the potential to significantly improve our ability to predict the mentioned processes. At the present time, application of multiphase CFD to industrial and system-scale calculations has been mainly limited to two-fluid Eulerian-Eulerian, Reynolds-averaged Navier-Stokes (RANS) based models (Prosperetti and Tryggvason, 2009; Tryggvason and Buongiorno, 2010). The use of more advanced techniques, such as direct numerical simulation and large eddy simulation with interface tracking methods (Toutant et al., 2008; Dabiri and Tryggvason, 2015), or Lagrangian tracking techniques (Molin et al., 2012), recently coupled with immersed boundary methods (Santarelli et al., 2015), is mostly constrained to very simple flow conditions in view of the required computational resources (Tryggvason and Buongiorno, 2010).

In two-fluid Eulerian-Eulerian RANS models, the conservation equations for each phase are derived from averaging procedures. Therefore, the details of the interphase structure are not resolved and interface exchange terms require explicit modelling (Fox, 2012; Prosperetti and Tryggvason, 2009). In these models, the bubble diameter is often needed as an input parameter that, therefore, becomes vital to properly predict the fluid dynamic behaviour of the system. Here, possible limitations can be avoided by coupling the CFD model with the population balance equation (PBE) approach which tracks the behaviour of the bubble size distribution in both physical and internal (e.g. bubble diameter or bubble volume) coordinate spaces (Buffo et al., 2013; Marchisio and Fox, 2005). The use of a PBE combined with CFD has been identified as a crucial development for the accurate prediction of bubbly flows, and significant advances have been achieved in recent years using this approach (Buffo et al., 2013; Cheung et al., 2009, 2013; Lehr et al., 2002; Liao et al., 2015; Lo and Zhang, 2009; Marchisio and Fox, 2005, 2007; Nguyen et al., 2013; Yao and Morel, 2004).

Many approaches have been considered for the solution of the PBE within a CFD code (Buffo et al., 2013). In class methods, the internal coordinate space, which is usually the bubble size spectrum, is discretized into numerous size classes and the PBE is integrated over each class to give a finite set of discrete PBEs (Kumar and Ramkrishna, 1996; Liao et al., 2015; Lo, 1996; Nandanwar and Kumar, 2008; Wang et al., 2005). In each class, bubbles may be considered as all having the same size (zero-order methods) or a specified distribution (higher-order methods), often a low-order polynomial (Vanni, 2000). In Monte Carlo methods, stochastic differential equations are solved for a finite number of artificial realizations of the dispersed phase population (Lee and Matsoukas, 2000; Lin et al., 2002; Zhao et al., 2007). For both the class and Monte Carlo methods, the drawback is the high computational cost involved. Respectively, the solution of at least one conservation equation for each class, with all the relevant source and sink terms, is required, or a very high number of realizations is necessary. In the last two decades, many authors have focused their efforts on the development of the interfacial area transport equation, in the context of both two-fluid CFD models and one-dimensional, advanced thermal hydraulic system codes (Hibiki and Ishii, 2000; Nguyen et al., 2013; Smith et al., 2012; Sun et al., 2004; Wu et al., 1998; Yao and Morel, 2004). Being derived from averaging over the whole bubble diameter spectrum, no bubble size distribution is retained and simplifying assumptions are often made, such as the use of constant or simple linear distributions (Ishii and Hibiki, 2006; Smith et al., 2012). Recently, promising results were achieved with progressively more advanced approaches based on the method of moments, originally introduced by Hulburt and Katz (1964). This method is based on the solution of a set of transport equations for the lower-order moments of the dispersed phase distribution (Marchisio and Fox, 2005). Progressively, more advanced methods have been developed, in particular in the category of quadrature-based methods of moments, such as the direct quadrature method (Marchisio and Fox, 2005) and the conditional quadrature method (Yuan and Fox, 2011). Overall, these methods are reported to provide good predictive accuracy without excessive computational cost (Buffo et al., 2013; Marchisio and Fox, 2005). The S_γ model, proposed by Lo and Rao (2007) for droplet two-phase flows, involves a limited number of moments of the bubble size probability distribution, which is assumed to follow a log-normal shape. The model was later extended to bubbly flows by Lo and Zhang (2009) and its ability to predict with a reasonable accuracy a number of different flows was demonstrated.

Alongside the method of solution, the other key aspect in regards to population balance based approaches is the availability of reliable closure models for the coalescence and break-up mechanisms. This issue has recently been the subject of numerous researches (Liao et al., 2015; Luo and Svendsen, 1996; Mukin, 2014; Prince and Blanch, 1990; Wang et al., 2005; Yao and Morel, 2004), and thorough reviews have been provided by Liao and Lucas (2009) for the break-up mechanism and by Liao and Lucas (2010) for the coalescence mechanism. Despite this, however, commonly accepted and reliable models have not yet emerged in view of the intrinsic complexity encountered when modelling coalescence and break-up in turbulent bubbly flows. Amongst others, the strong mutual interactions with the two-phase turbulence, for which a general and mature model is not yet available, and the coupling and relative importance of the different competitive mechanisms (e.g. turbulent collision, wake entrainment, shearing-off) prevent substantial progresses on the subject being achieved and, therefore, further understanding is required. The ongoing modelling effort is supported by the experimental data available from a number of studies (Grossetete, 1995; Hibiki and Ishii, 1999; Hibiki et al., 2001; Liu, 1993; Lucas et al., 2005, 2010; Prasser et al., 2007; Sanyal et al., 1999). In particular, detailed measurements of the average bubble size and the bubble size distribution have been obtained using the wire-mesh sensor technique (Lucas et al., 2005, 2010; Prasser et al., 2007).

In this paper, the S_γ model, implemented in the STAR-CCM+ code (CD-adapco, 2014), is combined with an Eulerian-Eulerian two fluid model and tested against data on air-water bubbly flows in pipes. With the aim to improve our ability to predict these flows and the evolution of the bubble diameter distribution, a different coalescence model is introduced and optimized. By means of sensitivity studies, the relative impact of bubble break-up and coalescence, and the influence of the continuous phase turbulence and the bubble-induced turbulence, are investigated. In terms of the turbulent flow field, and in view of the influence it has on the accuracy of the predictions, a Reynolds stress turbulence model is also included with the aim of extending the model's applicability to more complex flows, affected by known shortcomings of two-equation turbulence models. In bubbly flows, which are polydisperse by nature, the size determines the behaviour of the bubble, with small spherical bubbles flowing near the pipe wall and larger, deformed cap bubbles, migrating towards the pipe centre (Tomiyama et al., 2002b). Clearly, predicting this behaviour is mandatory if a general model capable of handling the entire bubble size spectrum is to be developed. In this regard, two bubble classes, each one with its own behaviour, are introduced in the final section of the paper. The ability of such a model, limited to only two bubble classes, to predict the whole bubble spectrum and the transition between wall-peaked and core-peaked void profiles, is then tested.

2. Experimental data

For any CFD technique to be applied with confidence, it is mandatory that the model has been previously validated against relevant experimental data. In this work, seven experiments from Liu (1993), Hibiki and Ishii (1999), Hibiki et al. (2001) and Lucas et al. (2005) were considered. The experimental conditions considered are summarized in Table 1.

Table 1: Experimental database used for validation.

Case	Source	j_w [m s^{-1}]	j_a [m s^{-1}]	α_{avg} [-]	$d_{B,avg}$ [mm]	Re_L [-]
Hi1	Hibiki et al. (2001)	0.986	0.242	0.191	3.4	49989
Hi2	Hibiki et al. (2001)	2.01	0.471	0.230	3.7	101903
HI1	Hibiki and Ishii (1999)	0.262	0.0549	0.245	3.4	6641
HI2	Hibiki and Ishii (1999)	1.75	0.399	0.253	3.8	44361
L1	Liu (1993)	1.0	0.2	0.160	4.2	57086
L2	Liu (1993)	3.0	0.2	0.062	3.4	171257
Lu1	Lucas et al. (2005)	0.255	0.0368	0.072	-	13030

Liu (1993) conducted experiments in a vertical pipe of 0.0572 m i.d. to study the bubble diameter and entrance length effects on the void fraction distribution in upward air-water bubbly flows. Bubble velocity, void fraction and average bubble diameter radial profiles were obtained from measurements at different axial locations. Hibiki and Ishii (1999), and Hibiki et al. (2001), measured water and air velocity, turbulence intensity, void fraction, bubble diameter and interfacial area concentration radial profiles at three consecutive axial locations and for an air-water bubbly flows in vertical pipes of diameter 0.0254 m and 0.0508 m. Lucas et al. (2005) used a wire-mesh sensor to study air-water upward flows inside a 0.0512 m diameter pipe. High-resolution measurements of the void fraction and the bubble diameter distribution were obtained. The experiments extended over a wide range of the bubble diameter spectrum, including some mixed radial void profiles where both spherical and cap bubbles were present, one of which was specifically included in the database to validate the model with two bubble classes. Over the whole database, the water superficial velocity considered is in the range 0.262 m s^{-1} < j_w < 3.0 m s^{-1} and the air superficial velocity is in the range 0.0368 m s^{-1} < j_a < 0.471 m s^{-1}. Average void fraction α_{avg} and average bubble diameters $d_{B,avg}$ reported in Table 1 were calculated by means of integration of the experimental profiles at the last measurement station. Table 1 also includes values of the Reynolds number of the flows, based on the characteristic dimension along the pipe.

3. Mathematical model

In a two-fluid Eulerian-Eulerian model, each phase is described by a set of averaged conservation equations. As the cases considered in this paper are limited to adiabatic air-water flows, only the continuity and momentum equations are solved, with the phases treated as incompressible with constant properties:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k) + \frac{\partial}{\partial x_i}\left(\alpha_k \rho_k U_{i,k}\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\alpha_k\rho_k U_{i,k}\right)+\frac{\partial}{\partial x_j}\left(\alpha_k\rho_k U_{i,k}U_{j,k}\right)=-\alpha_k\frac{\partial}{\partial x_i}p_k+\frac{\partial}{\partial x_j}\left[\alpha_k\left(\tau_{ij,k}+\tau_{ij,k}^{Re}\right)\right]+\alpha_k\rho_k g_i+M_{i,k} \tag{2}$$

In the above equations, α_k represents the volume fraction of phase k, whereas in the following, only α will be used to specify the void fraction of air. ρ is the density, $\boldsymbol{U}$ the velocity, p the pressure and $\boldsymbol{g}$ the gravitational acceleration. $\boldsymbol{\tau}$ and $\boldsymbol{\tau}^{\boldsymbol{Re}}$ are the laminar and turbulent stress tensors, respectively, and M_k accounts for the momentum exchanges between the phases. In the interfacial term, the drag force, lift force, wall force and turbulent dispersion force are included:

$$\boldsymbol{M}_k=\boldsymbol{F}_d+\boldsymbol{F}_l+\boldsymbol{F}_w+\boldsymbol{F}_{td} \tag{3}$$

The drag force represents the resistance opposed to bubble motion relative to the surrounding liquid and is expressed as:

$$\boldsymbol{F}_d=\frac{3}{4}\frac{C_D}{d_B}\alpha\rho_c|\boldsymbol{U}_r|\boldsymbol{U}_r \tag{4}$$

Here, $\boldsymbol{U_r}$ is the relative velocity between the phases and the subscript c identifies the continuous phase, which is water for all the experiments in Table 1. The drag coefficient, C_D, was calculated using the model of Tomiyama et al. (2002a), where the effect of the bubble aspect ratio on the drag was also accounted for (Hosokawa and Tomiyama, 2009) using:

$$C_D=\frac{8}{3}\frac{Eo}{E^{2/3}(1-E^2)^{-1}Eo+16E^{4/3}}F^{-2} \tag{5}$$

Here, F is a function of the bubble aspect ratio E. The bubble aspect ratio was derived from the following correlation and as a function of the distance from the wall y_w (Colombo et al., 2015):

$$E = \max\left[1.0 - 0.35\frac{y_w}{d_B}, E_0\right] \tag{6}$$

E_0 is calculated from the expression given by Welleck et al. (1966), where Eo is the Eötvös number:

$$E_0 = \frac{1}{1 + 0.163Eo^{0.757}} \tag{7}$$

A lift force, perpendicular to the direction of motion, is experienced by bubbles moving in a shear flow (Auton, 1987), according to:

$$\boldsymbol{F}_l = C_L \alpha \rho_c \boldsymbol{U}_r \times (\nabla \times \boldsymbol{U}_c) \tag{8}$$

In a pipe, the lift force has a strong influence on the radial movement of the bubbles and therefore on the void fraction radial distribution. Generally, a positive value of the lift coefficient C_L characterizes spherical bubbles, which are pushed towards the pipe wall by the lift force. In contrast, larger bubbles, deformed by the inertia of the surrounding liquid, experience a negative lift force and move towards the centre of the pipe (Tomiyama et al., 2002b). In air-water flows, a critical bubble diameter range for the change of sign in the lift coefficient between 5.0 mm and 6.0 mm was given by Tomiyama et al. (2002b). These authors also expressed the lift coefficient as a function of the Eötvös number, an approach adopted in other investigations (e.g. Krepper et al., 2008; Rzehak and Krepper, 2013). In this work, however, and in view of previously observed discrepancies between calculations and experimental data when using such an approach, constant values were chosen. More specifically, C_L = 0.1 was used for wall-peaked (Lahey and Drew, 2001; Lopez de Bertodano et al., 1994), and C_L = -0.05 for core-peaked, void profiles.

The presence of a solid wall modifies the flow field around the bubbles and the asymmetry in the flow distribution generates a hydrodynamic pressure difference on the bubble surface that keeps bubbles away from the wall (Antal et al., 1991):

$$\boldsymbol{F}_w = \max\left(0, C_{w,1} + C_{w,2}\frac{d_B}{y_w}\right)\alpha\rho_c\frac{|\boldsymbol{U_r}|^2}{d_B}\boldsymbol{n_w} \tag{9}$$

In this equation, $\boldsymbol{n}_w$ is the normal to the wall and C_{w1} and C_{w2} are constants that modulate the strength and the region of influence of the wall force. Here, values of C_{w1} = -0.4 and C_{w2} = 0.3 were used (Colombo et al., 2015). Finally, the turbulent dispersion force was modelled as (Burns et al., 2004):

$$\boldsymbol{F}_{td} = \frac{3}{4}\frac{C_D\alpha\rho_c|\boldsymbol{U}_r|}{d_B}\frac{\nu_{t,c}}{\sigma_\alpha}\left(\frac{1}{\alpha} + \frac{1}{(1-\alpha)}\right)\nabla\alpha \tag{10}$$

where $\nu_{t,c}$ is the turbulent kinematic viscosity of the continuous phase, obtained from the turbulent viscosity $\mu_{t,c}$, calculated from the single-phase relation (more details can be found in the following Section 3.1, where the turbulence model is presented), divided by the continuous phase density ρ_c. σ_α is the turbulent Prandtl number for the void fraction, assumed equal to 1.0 (Burns et al., 2004).

3.1. Multiphase turbulence modelling

Turbulence was solved only in the continuous phase, with a multiphase formulation (CD-adapco, 2014) of the standard k-ε turbulence model (Jones and Launder, 1972):

$$\begin{aligned}\frac{\partial}{\partial t}\big((1-\alpha)\rho_c k_c\big) + \frac{\partial}{\partial x_i}\big((1-\alpha)\rho_c U_{i,c} k_c\big) \\ = \frac{\partial}{\partial x_i}\left[(1-\alpha)\left(\mu_c + \frac{\mu_{t,c}}{\sigma_k}\right)\frac{\partial k_c}{\partial x_i}\right] + (1-\alpha)\big(P_{k,c} - \rho_c\varepsilon_c\big) + (1-\alpha)S_k^{BI}\end{aligned} \tag{11}$$

$$\frac{\partial}{\partial t}\left((1-\alpha)\rho_c\varepsilon_c\right)+\frac{\partial}{\partial x_i}\left((1-\alpha)\rho_c U_{i,c}\varepsilon_c\right)$$
$$=\frac{\partial}{\partial x_i}\left[(1-\alpha)\left(\mu_c+\frac{\mu_{t,c}}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon_c}{\partial x_i}\right]+(1-\alpha)\frac{\varepsilon_c}{k_c}\left(C_{\varepsilon,1}P_{k,c}-C_{\varepsilon,2}\rho_c\varepsilon_c\right)+(1-\alpha)S_\varepsilon^{BI} \tag{12}$$

In the equations above, $P_{k,c}$ is the production term due to shear and S_k^{BI} and S_ε^{BI} the source terms due to bubble-induced turbulence. The turbulent viscosity $\mu_{t,c}$ was evaluated from the single-phase relation:

$$\mu_{t,c}=C_\mu\rho_c\frac{k_c^2}{\varepsilon_c} \tag{13}$$

Turbulence was not resolved in the dispersed phase, but was obtained from the continuous phase. More specifically, it was directly related to the turbulence of the continuous phase by means of a response coefficient C_t, assumed equal to unity (Gosman et al., 1992; Troshko and Hassan, 2001). Experimental measurements do in fact suggest that a value of unity is approached starting from void fractions as small as 6 % (Behzadi et al., 2004).

In bubby flows, the generation of turbulence by the bubbles often modifies significantly the turbulence in the continuous phase, with respect to the single-phase flow (Lance and Bataille, 1991; Shawkat et al., 2007; Wang et al., 1987). The bubble contribution to turbulence was accounted for with bubble-induced source terms in Eq. (12) and Eq. (13). In particular, the drag force was considered as the only source of turbulence generation due to the bubbles and all the energy lost by the bubbles to drag was assumed to be converted into turbulence kinetic energy inside the bubble wakes (Kataoka and Serizawa, 1989; Rzehak and Krepper, 2013; Troshko and Hassan, 2001):

$$S_k^{BI} = K_{BI}\boldsymbol{F_d}\boldsymbol{U_r} \tag{14}$$

The corresponding turbulence dissipation rate source is equal to the turbulence kinetic energy source divided by the timescale of the bubble-induced turbulence τ_{BI}. In this work, the mixed timescale introduced by Rzehak and Krepper (2013) was chosen, derived from the velocity scale of the turbulence and the length scale of the bubbles:

$$S_\varepsilon^{BI} = C_{\varepsilon,BI}\frac{S_k^{BI}}{\tau_{BI}} = 1.0\frac{k^{0.5}}{d_B}S_k^{BI} \tag{15}$$

The mixed timescale is expected to mimic the split of eddies which move past the bubbles (Rzehak and Krepper, 2013) and the shift of the energy of turbulence to smaller length scales observed in experiments (Lance and Bataille, 1991; Shawkat et al., 2007). The mixed timescale, used in combination with the coefficient K_{BI} = 0.25 in Eq. (14), has been found to provide accurate predictions over a wide range of bubbly pipe flows (Colombo and Fairweather, 2015).

A multiphase Reynolds stress turbulence model (RSM) was also included in the overall model and, based on the single-phase formulation, the Reynolds stresses ($R_{ij} = \tau_{i,j}^{Re}/\rho_c$) are given by (CD-adapco, 2014):

$$\begin{aligned}\frac{\partial}{\partial t}\Big((1-\alpha)\rho_c R_{ij}\Big) + \frac{\partial}{\partial x_j}\Big((1-\alpha)\rho_c U_{i,c} R_{ij}\Big)\\ = \frac{\partial}{\partial x_j}\big[(1-\alpha)D_{ij}\big] + (1-\alpha)\big(P_{ij} + \Phi_{ij} - \varepsilon_{ij}\big) + (1-\alpha)S_{ij}^{BI}\end{aligned} \tag{16}$$

Here, P_{ij} is the turbulence production. The Reynolds stress diffusion D_{ij} was modelled accordingly to Daly and Harlow (1970), whilst the isotropic hypothesis was used for the turbulence dissipation rate term ε_{ij}. Φ_{ij} is the pressure-strain correlation, accounting for pressure fluctuations that redistribute the turbulence kinetic energy amongst the Reynolds stress components. This was modelled using the "SSG model" which is quadratically non-linear in the anisotropy tensor (Speziale et al., 1991):

$$\Phi_{ij} = -[C_{1a}\varepsilon + C_{1b}tr(P)]a_{ij} + C_2\varepsilon\left(a_{ik}a_{kj} - \frac{1}{3}a_{mn}a_{mn}\delta_{ij}\right) + \left[C_{3a} - C_{3b}\left(a_{ij}a_{ij}\right)^{0.5}\right]kS_{ij} + C_4k\left(a_{ik}S_{jk} + a_{jk}S_{ik} - \frac{2}{3}a_{mn}S_{mn}\delta_{ij}\right) + C_5\left(a_{ik}W_{jk} + a_{jk}W_{ik}\right) \tag{17}$$

In Eq. (17), a_{ij}, S_{ij} and W_{ij} are components of the anisotropy, strain rate and rotation rate tensors, respectively. The bubble-induced turbulence source term was calculated using Eq. (14) and then split amongst the normal Reynolds stress components following Colombo et al. (2015):

$$S_{ij}^{BI} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} S_k^{BI} \tag{18}$$

Values of the coefficients used for the k-ε model and the RSM can be found in Table 2.

Table 2. Coefficients of the turbulence models.

***k*-ε**	$\sigma_k = 1.0$; $\sigma_\varepsilon = 1.3$; $C_{1\varepsilon} = 1.44$; $C_{2\varepsilon} = 1.92$; $C_\mu = 0.09$
RSM SSG	$C_{1a} = 1.7$; $C_{1b} = 0.9$; $C_2 = 1.05$; $C_{3a} = 0.8$; $C_{3b} = 0.65$; $C_4 = 0.625$; $C_5 = 0.2$

3.2. The S_γ model

The S_γ model (Lo and Rao, 2007; Lo and Zhang, 2009) was used to model the evolution of the bubble population following break-up and coalescence events. In the S_γ model, the bubble size distribution is assumed to obey to a pre-defined log-normal probability distribution $P(d_B)$. Therefore, it is not necessary to divide the bubble size spectrum into a large number of bubble classes, but the bubble population can be characterized from a limited number of parameters, S_γ, related to the moments of the bubble size distribution M_γ:

$$S_\gamma = nM_\gamma = n\int_0^\infty d_B^\gamma P(d_B)\, d(d_B) \tag{19}$$

where n is the bubble number density. The zero[th] order moment is equal to the bubble number density n, whereas S_2 and S_3 are closely related to the interfacial area concentration a_i and to the void fraction:

$$S_0 = n;\; S_2 = n\int_0^\infty d_B^2 P(d_B)\, d(d_B) = \frac{a_i}{\pi};\; S_3 = n\int_0^\infty d_B^3 P(d_B)\, d(d_B) = \frac{6\alpha}{\pi} \tag{20}$$

From a knowledge of S_2 and S_3, the average bubble diameter can be determined by using the definition of the Sauter mean diameter (SMD):

$$d_{SM} = d_{32} = \frac{S_3}{S_2} = \frac{6\alpha}{a_i} \tag{21}$$

In addition, the variance of the distribution can also be calculated:

$$\sigma^2 = \ln\left(\frac{d_{32}}{d_{30}}\right) = \ln\left[\frac{(S_3/S_2)}{(S_3/S_0)^{1/3}}\right] \tag{22}$$

The two average diameters, d_{32} and d_{30}, are equal only in presence of a homogeneous distribution. Once the model is combined with a two-fluid Eulerian–Eulerian model that solves for the void fraction, S_3 is known, and only two additional moments, namely S_0 and S_2, are sufficient to characterize the bubble size distribution. For each moment, a transport equation of the following type needs to be solved:

$$\frac{\partial S_\gamma}{\partial t} + \nabla \cdot \left(S_\gamma \boldsymbol{U}_a \right) = S_{br}^{\gamma} + S_{cl}^{\gamma} \tag{23}$$

In this equation, the velocity of the air $\boldsymbol{U_a}$ is given by the two-fluid model and S^{γ}_{br} and S^{γ}_{cl} are source terms that account for bubble break-up and coalescence in the γ^{th} moment equation. Amongst the different mechanisms, interactions induced by turbulence were assumed to be dominant (Lo and Zhang, 2009; Yao and Morel, 2004) and the only sources of break-up and coalescence in Eq. (23).

The source term for bubble break-up is expressed as:

$$S_{br}^{\gamma} = \int_0^\infty K_{br} \Delta S_\gamma^{br} n P(d_B) d(d_B) \tag{24}$$

Here, K_{br} is the break-up rate, which is the reciprocal of the break-up time τ_{br}. ΔS_γ^{br} is the change in S_γ due to a single break-up event, which, from conservation of volume, is:

$$\Delta S_\gamma^{br} = d_B^\gamma \left(N_f^{\frac{3-\gamma}{\gamma}} - 1 \right) \tag{25}$$

The number of daughter bubbles N_f was assumed equal to 2 (Lo and Zhang, 2009; Luo and Svendsen, 1996; Yao and Morel, 2004). The break-up source term then becomes:

$$S_{br}^{\gamma} = \int_0^\infty \frac{d_B^\gamma \left(N_f^{3-\gamma/3} - 1 \right)}{\tau_{br}} n P(d_B) d(d_B) \tag{26}$$

The break-up timescale follows from the frequency of the second oscillation mode of a droplet (Lo and Zhang, 2009):

$$\tau_{br} = 2\pi k_{br}\sqrt{\frac{3\rho_d + 2\rho_c}{192\sigma} d_B^3} \tag{27}$$

where k_{br}=0.2, the subscript d identifies the dispersed phase and σ is the surface tension. The break-up criterion was expressed as a function of a critical Weber number We_{cr}, therefore a bubble breaks when the Weber number is higher than the critical value:

$$d_{cr} = (1 + C_\alpha)\left(\frac{2\sigma We_{cr}}{\rho_c}\right)^{3/5} \varepsilon^{-2/5} \tag{28}$$

C_α, equal to 4.6, is a correction factor that accounts for nearby bubbles that disrupt the influence of the surrounding inertial forces. In Lo and Zhang (2009), We_{cr} = 0.31, whilst in Yao and Morel (2004), We_{cr} = 1.24.

The general source term for bubble coalescence is:

$$S_{cl}^{\gamma} = \int_0^\infty \int_0^\infty K_{cl}^{d,d\prime} \Delta S_{\gamma,cl}^{d_B,d_B'} n^2 P(d_B') d(d_B') P(d_B) d(d_B) \tag{29}$$

Here, $K_{cl}^{d,d\prime}$ is the coalescence rate between two bubbles with diameters d_B and d_B', and $\Delta S_{\gamma,cl}^{d_B,d_B'}$ is the change in S_γ due to a single coalescence event. To avoid excessive complication, a uniform bubble distribution with an equivalent mean diameter d_{eq} was assumed when computing the change in S_γ due to a single coalescence event (Lo and Zhang, 2009):

$$\Delta S_{\gamma,cl}^{d,d\prime} = d_{eq}^{\gamma}\left(2^{\gamma/3} - 2\right) \tag{30}$$

The coalescence rate is expressed as:

$$K_{cl}^{d,d\prime} = F_{cl} k_{cl} d_{eq}^{2} u_r P_{cl} \tag{31}$$

Following Chester (1991), Lo and Zhang (2009) considered two different coalescence mechanisms resulting from viscous and inertial collisions. For viscous coalescence, the film drainage model was applied for the coalescence probability (Prince and Blanch, 1990). When two bubbles collide, they trap a thin liquid film between them that prevents coalescence. If the interaction time in the turbulent flow is sufficient for the film to drain out until rupture of the film occurs, then the bubbles coalesce, otherwise the bubbles are separated and coalescence does not occur. The drainage time was calculated from a model for a partially mobile interface and a quasi-steady flow in the film (Lo and Zhang, 2009):

$$t_d = \frac{\pi \mu_d \sqrt{F_i}}{2 h_{cr}} \left(\frac{d_{eq}}{4\pi\sigma}\right) \tag{32}$$

Here, F_i is the interaction force during collision and h_{cr} the critical film thickness (Lo and Zhang, 2009). The coalescence probability is then expressed from the interaction time t_i and the drainage time t_d:

$$P_{cl} = exp(-t_d/t_i) = exp(t_d\dot{\gamma}) \tag{33}$$

where the interaction time is the inverse of the Kolmogorov shear rate:

$$\dot{\gamma} = \sqrt{\frac{\varepsilon\rho_c}{\mu_c}} \tag{34}$$

Finally, in Eq. (31), k_{cl} = $(8\pi/3)^{0.5}$ and the relative velocity between the bubbles $u_r = \dot{\gamma}d_{eq}$. Alternatively, for inertial collision, k_{cl} = $(2\pi/15)^{0.5}$ and $u_r = (\varepsilon d_{eq})^{1/3}$. With regard to the probability of coalescence, the major role is played by bubble shape oscillations and, therefore, the coalescence probability was expressed following Chester (1988):

$$P_{cl} = \frac{\Phi_{max}}{\pi}\left[1 - \frac{k_{cl,2}^2(We - We_0)^2}{\Phi_{max}^2}\right]^{1/2} \tag{35}$$

where Φ_{max} is the maximum phase difference (Lo and Zhang, 2009), $k_{cl,2}$ = 12.7, $We_0 = 0.8We_{cr}$ and $h_0 = 8.3h_{cr}$.

A different coalescence model, proposed by Yao and Morel (2004), was also considered in this work. When using the Yao and Morel (2004) approach, the break-up model described above was retained, except for the value of We_{crit} which was modified to 1.24, following the authors' proposal. In Yao and Morel (2004), the number of coalescence events per unit volume and unit time, which is assumed to be mainly due to the collisions induced by turbulence, is expressed as:

$$K_{cl}^{d,d'}n^2 = -C_1\frac{\varepsilon^{1/3}\alpha^2}{d_{SM}^{11/3}}\frac{1}{g(\alpha) + C_2\sqrt{We/We_{crit}}}exp\left(-C_3\sqrt{We/We_{crit}}\right) \tag{36}$$

The first part of this equation represents the collision rate between the bubbles, whilst the exponential function describes the probability of coalescence following a collision event. The function *g(α)* accounts for the effect of the packing of the bubbles when the void fraction is higher than a certain value. From Yao and Morel (2004), C_1 = 2.86, C_2 = 1.922, C_3 = 1.017 and We_{crit} = 1.24.

When two groups of bubbles were included, additional source terms were added to the mass and momentum conservation equations to account for the exchanges between the groups. In a similar manner as above, the conservation equation for the moment of the bubble size distribution becomes:

$$\frac{\partial S_{\gamma,n}}{\partial t} + \nabla \cdot \left(S_{\gamma,n} \boldsymbol{U}_{a,n}\right) = S^{\gamma}_{br,n} + S^{\gamma}_{cl,n} + D^{\gamma}_{br,n} + B^{\gamma}_{cl,n} + B^{\gamma}_{br,n} + D^{\gamma}_{cl,n} \tag{37}$$

In this equation, the subscript *n* identifies the bubble group and assumes the values *s* for spherical bubbles and *c* for cap bubbles. D^{γ}_{br} and D^{γ}_{cl} are source terms for the death of bubbles by break-up to the previous group and by coalescence to the following group. Conversely, B^{γ}_{br} and B^{γ}_{cl} are due to the birth of bubbles by coalescence from the previous group and by break-up from the following group. Obviously, when only two groups are considered, Eq. (37) simplifies and the only source terms to be considered are the death of cap bubbles which gives rise to the birth of spherical bubbles by break-up, and the death of spherical bubbles with the birth of cap bubbles by coalescence.

In this work, break-up of cap bubbles into spherical bubbles has been neglected, with this assumption explained and justified in detail in the results section. To calculate the additional sources accounting for exchanges between groups, using Eq. (29), Eq. (30) and the hypothesis of a uniform bubble distribution for the coalescence source, the source terms for the death of spherical bubbles by coalescence are obtained as:

$$D^0_{cl,s} = -2 \cdot \left(K^{d,d'}_{cl,s} n^2_s\right) f(d_B) \tag{38}$$

$$D^2_{cl,s} = -2d^2_{eq} \left(K^{d,d'}_{cl,s} n^2_s\right) f(d_B) \tag{39}$$

$f(d_B)$ is a function that expresses the probability that a coalescence event between two spherical bubbles leads to the birth of a cap bubble. Therefore, it is the ratio of the number of coalescence events that generate a cap bubble to the total number of coalescence events amongst the spherical bubble population. The coefficients -2 and $-2d^2_{eq}$ are calculated from the second contribution to Eq. (30) and reflect the fact that, in these events, the results is not a net change in the value of S_γ for the spherical bubbles, but a loss of two bubbles and their interfacial area to the cap bubbles. Accordingly, from the first contribution to Eq. (30), the gain in S_γ in the cap bubble group due to coalescence events in the spherical bubble group is obtained as:

$$B^0_{cl,c} = \left(K^{d,d'}_{cl,c} n^2_c\right) f(d_B) \tag{40}$$

$$B^2_{cl,c} = 1.59 \cdot d^2_{eq} \left(K^{d,d'}_{cl,c} n^2_c\right) f(d_B) \tag{41}$$

From Eq. (38), the mass source from spherical to cap bubbles can be obtained, using the volume average bubble diameter:

$$\Gamma_{sc} = -\Gamma_{cs} = -D^0_{cl,s} \frac{\pi d^3_{30,s}}{6} \rho_a \tag{42}$$

Finally, for simplicity, the function $f(d_B)$ was assumed equal to ratio of the SMD to the critical diameter:

$$f(d_B) = \frac{d_{SM}}{d_c} \tag{43}$$

In the previous equation, d_c is the critical diameter at which bubble behaviour changes from a spherical bubble to a cap bubble.

The overall model, implemented in the STAR-CCM+ CFD code (CD-adapco, 2014), was solved in a two-dimensional axisymmetric geometry. At the inlet, fully-developed phase velocities and void fraction boundary conditions were imposed, together with an imposed pressure at the outlet and the no-slip condition at the wall. Experimental measurements of average bubble diameter at the first measurement station were used for the bubble diameter inlet boundary condition. Therefore, experimental measurements at the last station were compared against predictions at a distance from the inlet equal to that between the first and the last measurement stations. Strict convergence of residuals was ensured, together with a mass balance error lower than 0.01 % for both phases. Experiment HI2 was selected for a mesh sensitivity study, the results of which are presented in Figure 1 in terms of the radial profiles of water velocity, turbulence kinetic energy, void fraction and SMD. The radial profiles are shown as a function of the normalized radial position r/R, which is equal to 0 at the pipe centre and to 1 at the pipe wall. Four grids were tested with a progressively increasing number of grid nodes (10 × 100, 15 × 150, 20 × 200, and 25 × 250). The water velocity and void fraction distributions are rather insensitive to the number of nodes, but some differences between the various grids are apparent for the turbulence kinetic energy and the SMD. From the results in Figure 1, the grid with 20 × 200 nodes was chosen for other simulations. All grids had a first grid node higher than, but close, to $y^{+} = 30$, which is the lower limit for the use of wall functions.

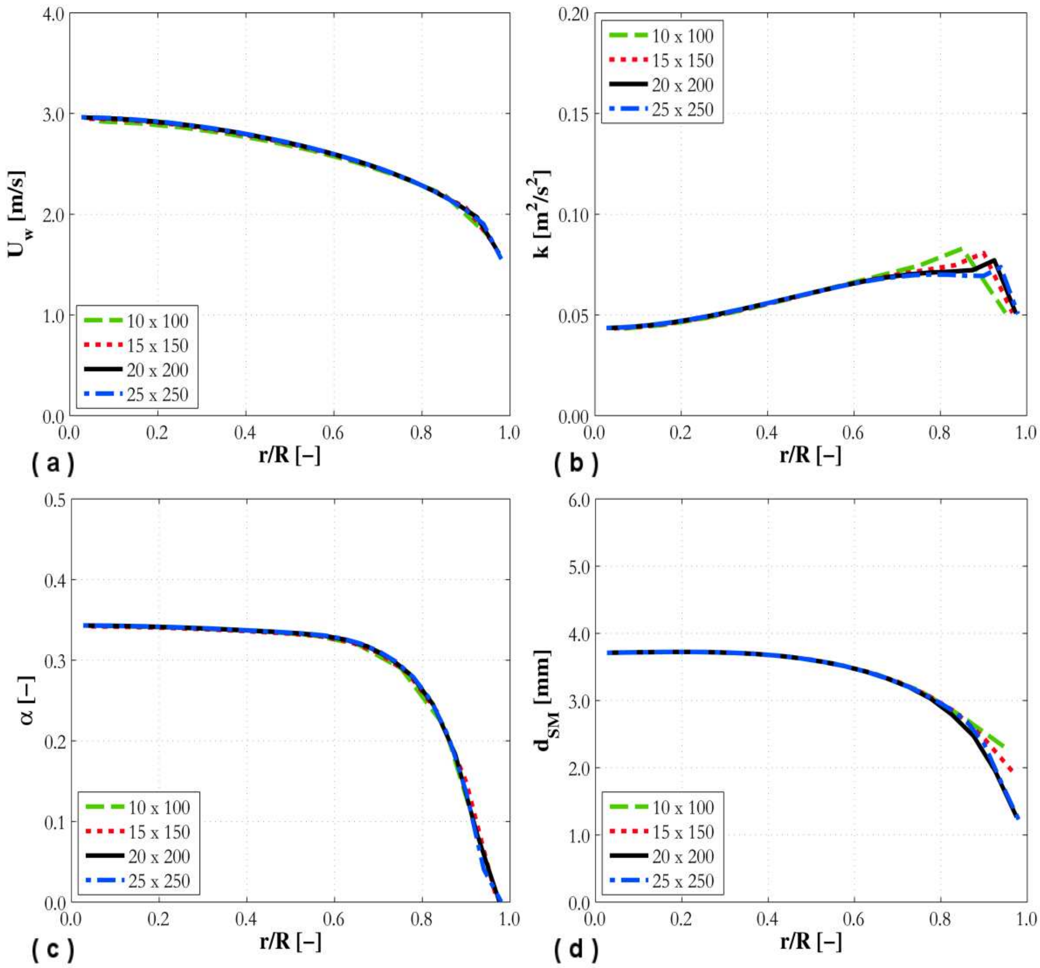

Figure 1. Mesh sensitivity study in terms of radial and axial node numbers for experiment HI2. Water velocity (a), turbulence kinetic energy (b), void fraction (c) and SMD (d) radial profiles are presented.

4. Results and discussion

This section describes and discusses the simulation results and comparisons against experimental data. First, the experiments of Liu (1993), Hibiki and Ishii (1999) and Hibiki et al. (2001) were simulated with the YM model (Yao and Morel, 2004) and the results are presented in Figure 2 and Figure 3. As can be seen, the YM model generally overestimates the SMD. In particular, marked overestimations were obtained at the lowest liquid velocities (Hi1, HI1 and L1), whereas, at higher velocities (Hi2, HI2 and L2), the overestimation is reduced and, for experiment HI2 (Figure 3a) only, good agreement with data is found. The tendency of the YM model to over-predict the bubble diameter has already been noted by Cheung et al. (2007) and Nguyen et al. (2013). To serve as a benchmark, predictions from the LZ model (Lo and Zhang, 2009) are also included in Figure 2 and Figure 3. Overall, the LZ model provides better accuracy when predicting the SMD. Nevertheless, and similar to YM, a strong dependency on the liquid velocity is apparent. At low velocity, good agreement, or limited overestimation of the SMD, was obtained (with respect to YM) but, at higher velocities, LZ under predicts the experiments. In addition, as already reported in Lo and Zhang (2009), the bubble diameter is generally under predicted in the near wall region, probably as a consequence of the excessively strong bubble break-up rate there.

The availability of experimental data allowed a further optimization of the YM model to be made. As the over prediction of the bubble diameter is possibly due to an excessive amount of bubble coalescence in the flow, this was limited by modifying the value of We_{crit} in Eq. (36), where it mainly impacts the coalescence probability. Therefore, a lower We_{crit} reduces the coalescence probability or, from a different perspective, it reduces the interaction time available to the liquid film trapped between the two colliding bubbles to drain out. Calibration of the model was limited to the coalescence model (the model for break-up was not changed from that of Lo and Zhang (2009), except for the value of We_{crit}, equal to 1.24 for YM). Even if the SMD is still overestimated at low liquid velocity and underestimated at high liquid velocity, acceptable agreement was achieved in all the tested conditions with We_{crit} = 0.10 (YM opt. lines in Figure 2 and Figure 3). Overall, the improvement in the accuracy with respect to the original YM and LZ models is significant. In the near wall region, where LZ significantly under predicts the experimental data, the value of the bubble diameter is well predicted, with the exception of experiment HI1 (Figure 2g) in which the flow rate is particularly low. In addition, for the LZ model, optimization on a case-by-case basis has been found necessary to reach a comparable accuracy (Lo and Zhang, 2009), whereas, in this work, the same value of We_{crit} was maintained for all flow conditions considered. In view of this finding, additional research work is required to develop more general and accurate models of bubble break-up and coalescence.

Figure 2 and Figure 3 also show radial profiles of the mean water velocity and void fraction (for L1 and L2, Figure 3e and Figure 3h, the air velocity is also provided). Overall, simulation results are in good agreement with the experiments. The mean velocity is under predicted for L2 and, but only in the pipe core region, for Hi1. With regards to the void fraction, the best agreement is found for the wall-peaked void profiles (Figure 2c, Figure 3f and Figure 3i). In contrast, the core-peaked void profiles were more difficult to predict. As it is possible to see from Figure 2 and Figure 3, a larger bubble size spectrum characterizes the core-peaked void profiles (Hi2, HI1 and HI2) with respect to the wall-peaked profiles, where the average bubble diameter radial distribution is generally flatter. This complicates the simulation of the momentum transfer at the interphase, even with the use of a population balance model. As shown in Figure 2f, Figure 2i and Figure 3c, a sharp increase in the near wall region, followed by an almost flat profile, is usually predicted. The experiments, however, show a more gentle but continuous increase of the void fraction towards the pipe centre. Predictions are similar amongst the three different models considered. This suggests that it is the interphase momentum forces (lift and wall forces in particular) that mostly determine the radial void fraction and mean velocity profiles. In this regard, the use of constant lift force coefficients, not dependent on the bubble diameter, may significantly inhibit changes in the lift force induced by changes in the latter diameter.

The role of the critical Weber number in the YM model is the focus of the results given in Figure 4, where the SMD profile is shown for three different values of We_{crit}. It has already been mentioned how We_{crit} mainly affects the coalescence probability. Specifically, a lower We_{crit} reduces the coalescence probability and, therefore, the average bubble diameter. This effect is equivalent to reducing the interaction time available for the liquid film trapped between two colliding bubbles to drain out, or, equivalently, to increasing the time required by this liquid film to drain out. Figure 4 includes two different experimental datasets. It is observed that the reduction in coalescence with We_{crit} is higher at the low flow rate (Figure 4a), while the effect of a lower We_{crit} is reduced at the higher flow rate (Figure 4b). At high flow rates, therefore, the interaction time is low given the high level of turbulence, and hence the coalescence probability has a correspondingly low value. As a consequence, the amount of decrease achievable by tuning We_{crit} is also low. At low flow rates, in contrast, the coalescence probability is higher due to the longer interaction times that occur in a low level turbulence field, and hence this probability can be significantly affected by a change in the value of We_{crit}.

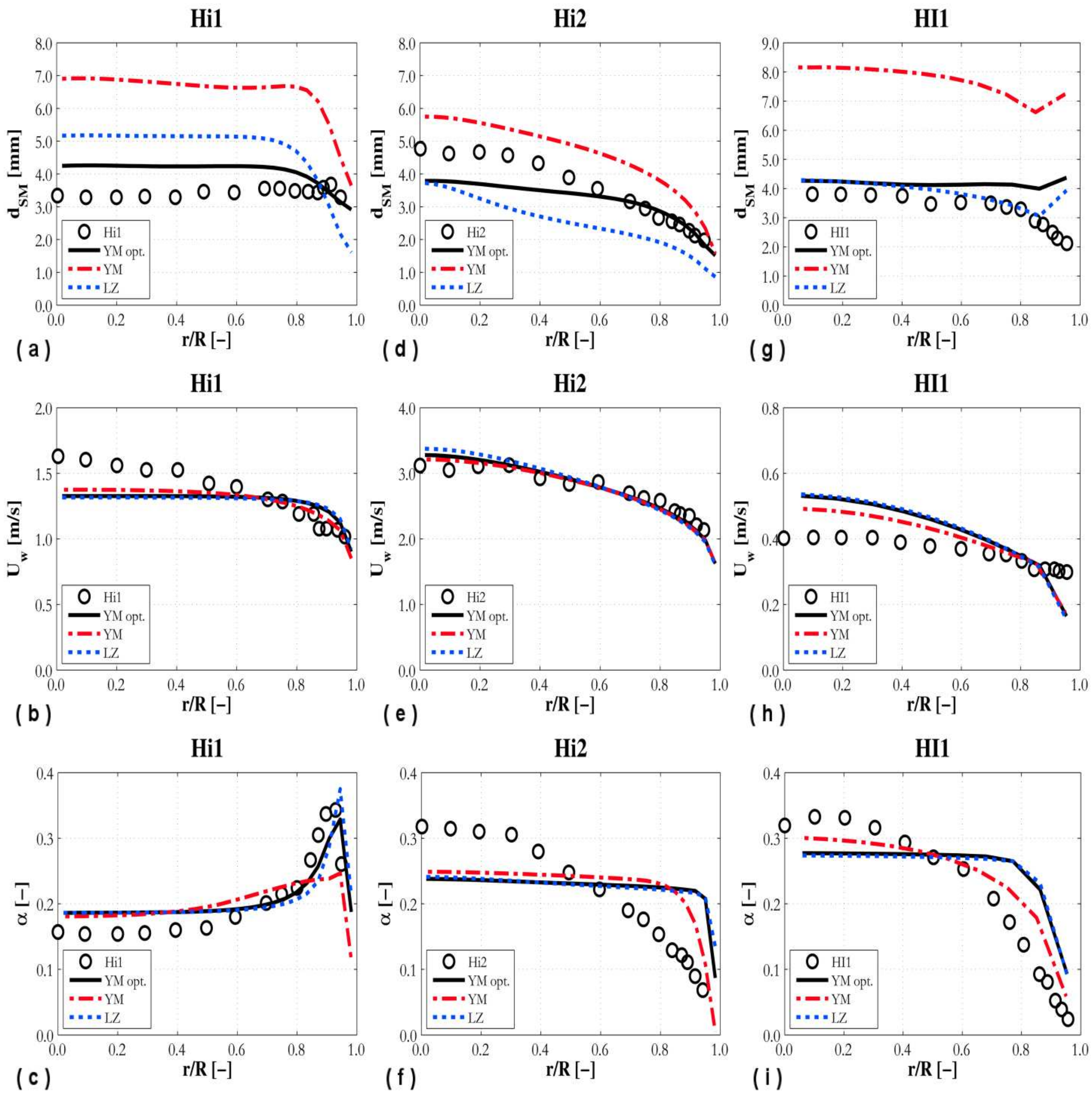

Figure 2. SMD, mean water velocity and void fraction radial profiles compared against experiments Hi1 (a-c), Hi2 (d-f) and HI1 (g-i). Simulation results are shown for LZ (---), YM (-·-) in its standard form (Eq. 36) and after optimization (YM opt., —).

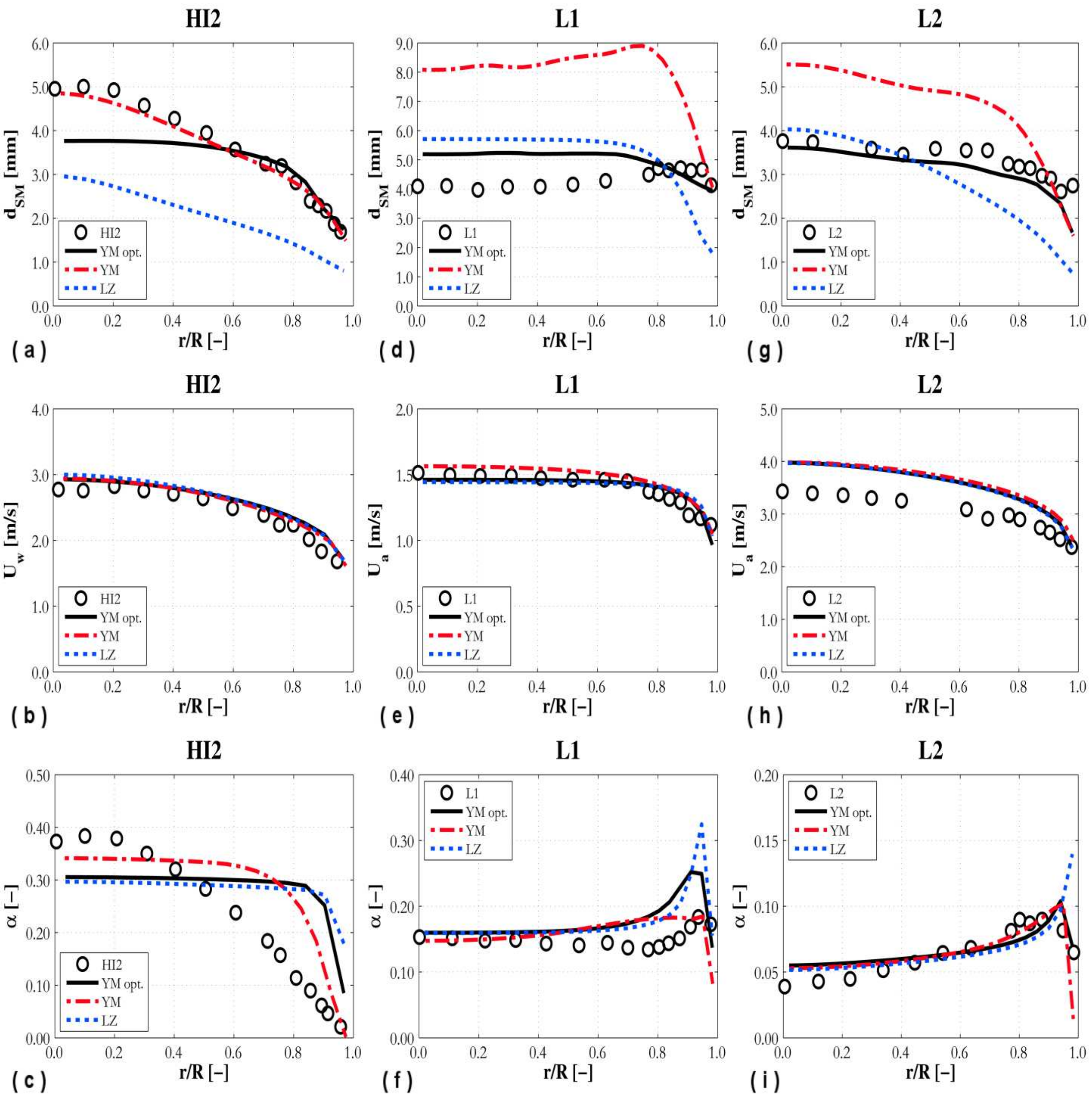

Figure 3. SMD, mean velocity and void fraction radial profiles compared against experiments HI2 (a-c), L1 (d-f) and L2 (g-i). Simulation results are shown for LZ (---), YM (-··-) in its standard form (Eq. 36) and after optimization (YM opt., —).

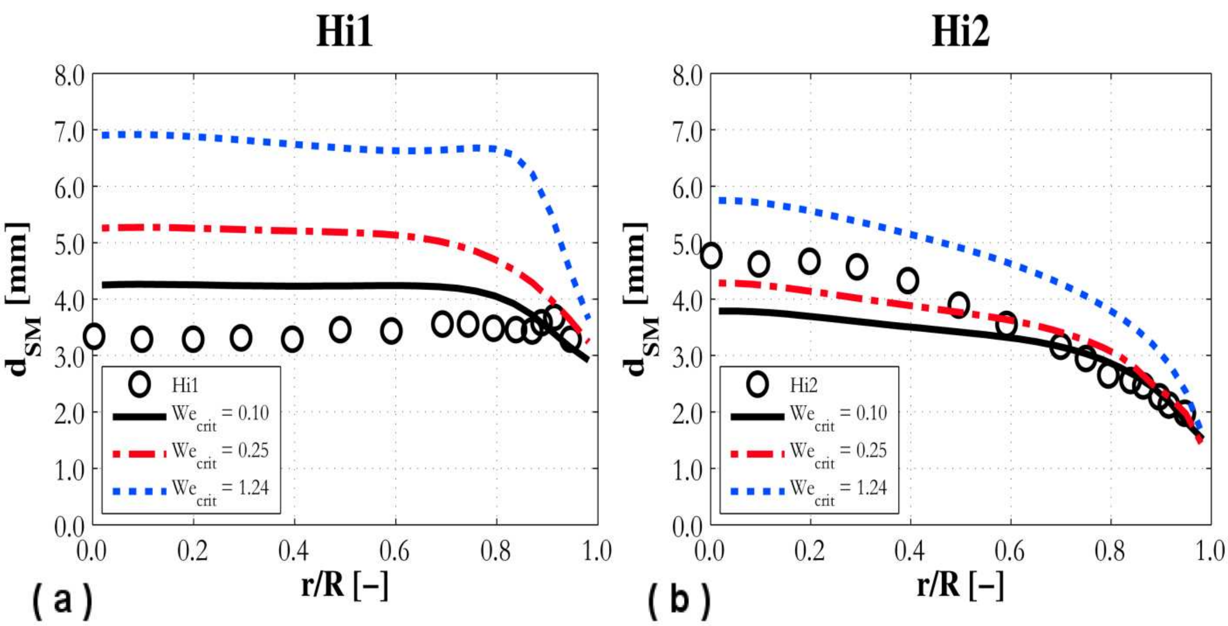

Figure 4. SMD radial profiles obtained with YM and We_{crit} = 0.1 (—), We_{crit} = 0.25 (-··-) and We_{crit} = 1.24 (---). Predictions are compared against experiments Hi1 (a) and Hi2 (b).

4.1. Effect of the break-up model

As mentioned, no changes were introduced in the break-up model, except for the value of the We_{crit}, which, for YM, was increased to 1.24 following the authors' proposal (Yao and Morel, 2004). Since no clear indications of the amount of bubble break-up occurring are available for the flows studied in this work, additional simulations neglecting break-up were made to evaluate the impact of the break-up model on the predictions. In Figure 5, four of the experiments were predicted with and without accounting for break-up. For the majority of the pipe cross-section, the effect of break-up on the bubble diameter distribution is seen to be negligible. In the near wall region, break-up is effective in reducing the SMD, but only at the highest liquid velocities (Figure 5b and Figure 5d). At low velocities, break-up is negligible even in the region close to the wall (Figure 5a and Figure 5c). Overall, and in view of the agreement obtained with these experiments, these results suggest that coalescence is the dominant mechanism in these flows.

Since only the net result of the combined action of both break-up and coalescence is available in terms of the experimental data, this being the SMD, additional sensitivity studies were made, increasing the impact of both. The same We_{crit} value of 0.25 was adopted in both the break-up and the coalescence models. The increase in the rate of coalescence with a higher critical Weber number was already addressed in Figure 4. A lower We_{crit} in the break-up model increases the break-up rate since a lower energy is required to break-up the bubble. The value of We_{crit} adopted is now close to that used in the LZ model and, therefore, a comparable amount of break-up is to be expected. The results are presented in Figure 6. Even if some improvement is obtained for a number of flows (Figure 6a, Figure 6c and Figure 6e), excessive break-up causes an under prediction of bubble diameter at high liquid velocities (Figure 6b, Figure 6d and Figure 6f). In addition, and except for experiment HI1 (Figure 6c), the bubble diameter is always underestimated in the near wall region, where, in view of the higher levels of turbulence, break-up is expected to be more significant. Again, these results are similar to those obtained with the LZ model (Figure 2 and Figure 3), for which an excessive amount of break-up, in particular in the near wall region, has already been reported (Lo and Zhang, 2009). This further supports the case for these flows being coalescence dominated.

Overall, and despite the previous results, it remains difficult to precisely evaluate the accuracy of the model with regard to the competitive action of coalescence and break-up, and the mechanisms involved. As mentioned, only the net result is available through data on the bubble diameter. Therefore, additional knowledge is required on the physics of these flows, and on the interaction between bubbles and with the continuous phase in particular. The lack of information on these processes is a significant constraint on the further development of these models that needs to be overcome if more accurate modelling is to be achieved. As an example, the recent tendency has been to include all possible mechanisms of bubble break-up and coalescence (e.g. turbulent collision, wake entrainment, shearing-off) (Liao et al., 2015; Smith et al., 2012; Sun et al., 2004). Even if this may benefit the generality of the developed models, the relative influence of each mechanism has been generally optimized with additional constants tuned against average bubble diameter measurements, which, at the present time, is the only real option available to modellers.

Without a clear knowledge of the effective impact of each mechanism as a function of the flow conditions, however, accurate prediction of the average bubble diameter does not guarantee the accuracy of each individual model, and possibly increases the uncertainty in the results and limits the applicability of the model itself. In view of this, advances must rely on the availability of more detailed experimental measurements or, perhaps, accurate direct numerical simulations of bubble behaviour.

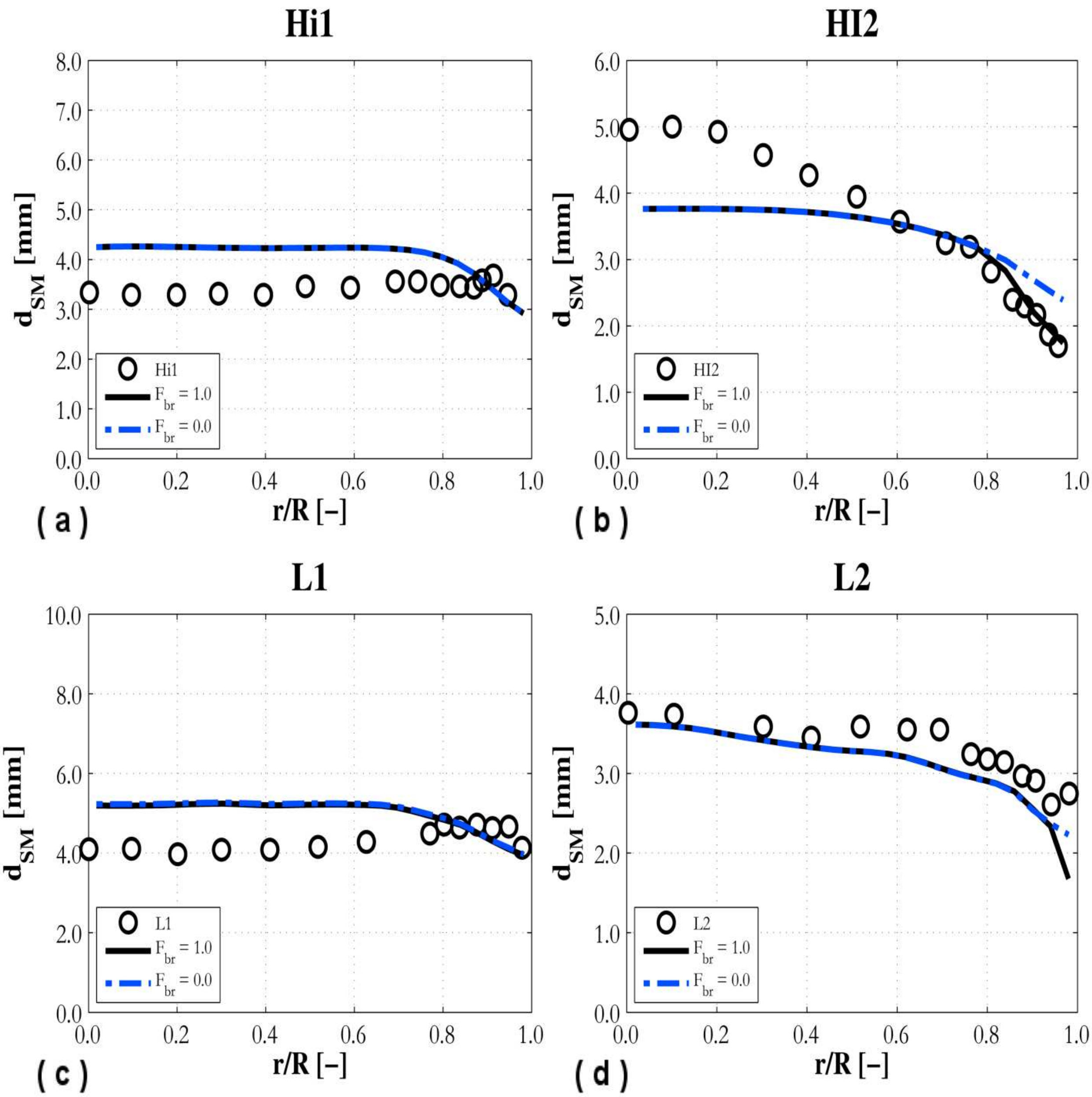

Figure 5. SMD radial profiles with (—) and without (-··-) considering the effect of bubble break-up in the flow. Predictions are compared against experiments Hi1 (a), Hi2 (b), L1 (c) and L2 (d).

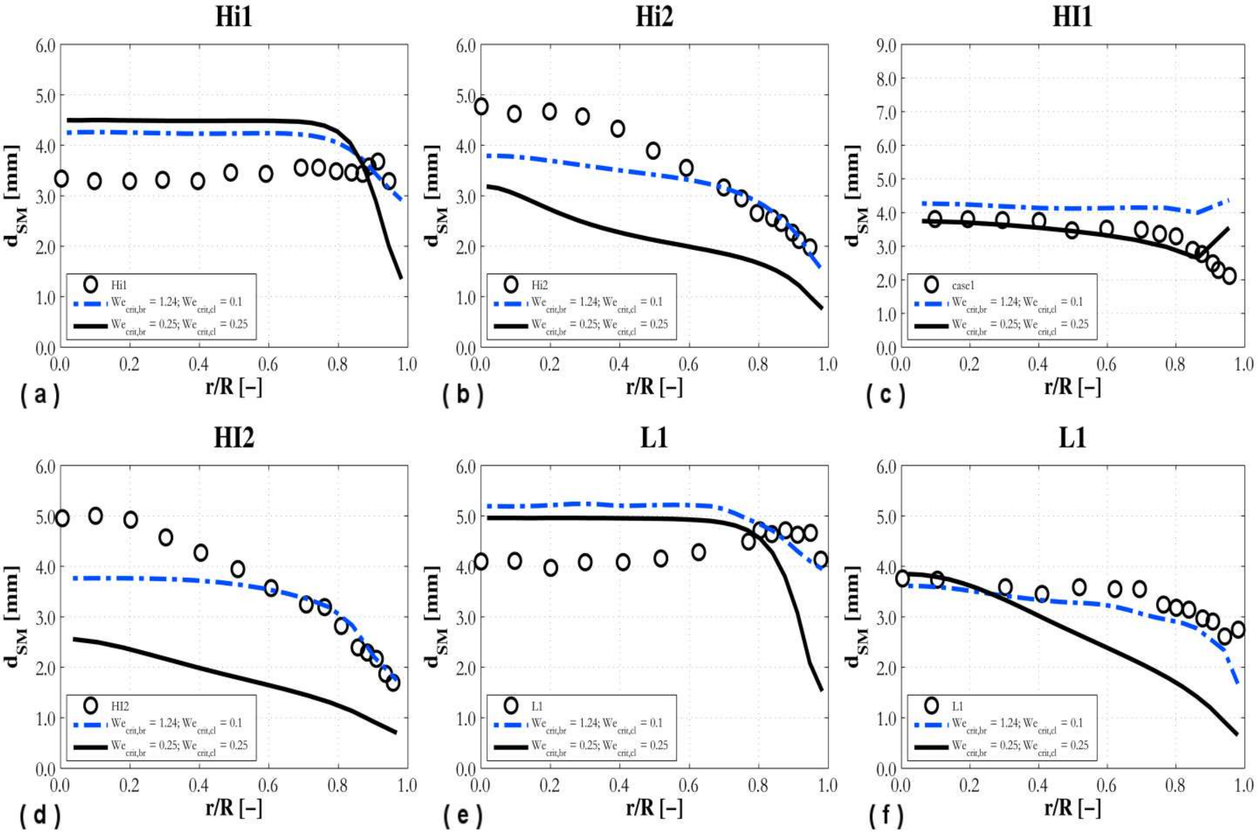

Figure 6. SMD radial profiles at different rates of coalescence and break-up of bubbles in the flow ($We_{crit,br}$ = 1.24 and $We_{crit,cl}$ = 0.1 (—); $We_{crit,br}$ = 0.25 and $We_{crit,cl}$ = 0.25 (-··-)). Predictions are compared against the experiments in Table 1.

4.2. Continuous phase turbulence sensitivity

Turbulence parameters affect in different ways the models for coalescence and break-up, and, as the latter models are based on the collision of bubbles due to turbulence, they are expected to have a significant impact on results. The sensitivity to the turbulence model predictions has already been investigated in some literature studies (Nguyen et al., 2013; Yao and Morel, 2004), but, in many more, the assessment and optimization of the coalescence and break-up models was carried out without considering the accuracy of the turbulence predictions. The aim of this section, therefore, is to address the dependency of results on the continuous phase turbulence.

In bubbly flows, the contribution of the bubbles to the continuous phase turbulence is accounted for, in the *k*-*ε* turbulence model, by source terms in the equations of that model (Eq. (11) and Eq. (12), Section 3.1).

Figure 7 shows radial profiles of the predicted SMD as a function of the amount of bubble-induced turbulence, together with the continuous phase streamwise turbulence intensities *I*. Turbulence measurements are available only from Hibiki and Ishii (1999) and Hibiki et al. (2001), where turbulence intensity was calculated by dividing the streamwise r.m.s of the velocity fluctuations by the maximum liquid velocity. Three different cases are considered: no bubble-induced turbulence, and Eq. (14) with K_{BI} = 0.25 and K_{BI} = 1.0. At low flow rates (HI1, Figure 7i), or for wall-peaked void profiles (Hi1, Figure 7g, and L1, Figure 7k), where the presence of the bubbles induces a flat mean velocity profile and a strong reduction of the shear-induced turbulence production in the pipe centre, the contribution of the bubble-induced turbulence is significant. For the high flow rate wall-peaked case (L2, Figure 7l), where the turbulence level is already high and the void fraction in the pipe centre is low, and the core-peaked void profiles (Hi2, Figure 7h, and HI2, Figure 7j), where the shear-induced production remains significant, the impact of the bubble-induced contribution is less. In the first case scenario, significant differences in the turbulence level cause bubble diameter profiles to be very different from one another (Figure 7a, Figure 7c and Figure 7e). This means that these results are dependent on the continuous phase turbulence and, for some flows, on the bubble-induced turbulence model as well. Therefore, for a proper model validation, both the average bubble diameter and the continuous phase turbulence predictions need to be compared against experiments. Conversely, the results may be dependent not only on the flows used for validation, but also on the specific bubble-induced turbulence model. Unfortunately, turbulence measurements are not available for all the experiments considered. Moreover, for the data of Hibiki et al. (2001), turbulence levels were always under predicted, even when considering all the drag force to be converted to turbulence kinetic energy. It must be pointed out that the turbulence intensities in these data appear significantly higher than for other experiments in the literature having comparable geometry and flow conditions (Liu, 1998; Serizawa et al., 1975; Wang et al., 1987). For HI1 and HI2, instead, satisfactory predictions were obtained. In view of the limited number of simultaneous measurements of both the bubble diameter distribution and the flow turbulence, some additional comparisons are shown in Figure 8, taking advantage of a previous validation of the bubble-induced turbulence model (Eq. (14) and Eq. (15)), which showed satisfactory accuracy over a wide range of conditions (Colombo and Fairweather, 2015). In Figure 8, radial profiles of the r.m.s. of streamwise velocity fluctuations are compared against different bubbly flow data in vertical pipes. For these validations, the bubble diameter was fixed and assumed equal to experimental observations, even if only rough averaged values were available for the majority of the experiments. Even if some discrepancies are still apparent, the overall agreement can be considered satisfactory. This additional validation, although useful, did not allow a comparison of bubble diameter and turbulence for the same experiment and, therefore, concerns related to data availability still remain. Recently, the development of advanced experimental techniques has allowed detailed measurements of the average bubble diameter and the bubble diameter distribution (Lucas et al., 2005, 2010; Prasser et al., 2007). However, in view of the previous results and to better support the modelling effort, experimental measurements need to allow not only the validation of the bubble diameter distribution, but also of the continuous phase turbulence level.

In Figure 7, YM predicts a higher SMD, therefore a higher coalescence ratio, with a decrease in the continuous phase turbulence. Collision rate increases with turbulence, while coalescence probability reduces, with the latter being the dominant effect. This qualitatively behaviour needs further examination. In Figure 9, the same sensitivity study is made for the LZ model, for experiments Hi1, Hi2 and L1. The turbulence intensity behaviour remains the same, but the bubble diameter predictions are changed. At low liquid velocity (Hi1 and L1) and without the bubble-induced turbulence model, bubble diameter is high at the wall, where the turbulence remains high, whereas it is low in the centre of the pipe due to the reduced turbulence in this region. When the turbulence level is increased, the coalescence is also increased, and, consequently, the SMD. With a further increase of the turbulence, the bubble diameter is reduced by a decrease of the coalescence or, more likely, by an increase of bubble break-up, which is higher for this model (Section 4.2). At high velocity (Hi2), the break-up is already high even without including bubble-induced turbulence. Therefore, with an increase of the turbulence level, the break-up is further increased and a decrease of the SMD is observed. For YM, even if a reduction in the coalescence following an increase of the turbulence, at already high turbulence levels, cannot be excluded, in the limit of zero turbulence, an increase of the coalescence is expected following an increase in the turbulence. Therefore, despite the good accuracy shown, the qualitative behaviour of YM with the turbulence level, which is different from that of LZ, suggests the need for additional future verification of these models.

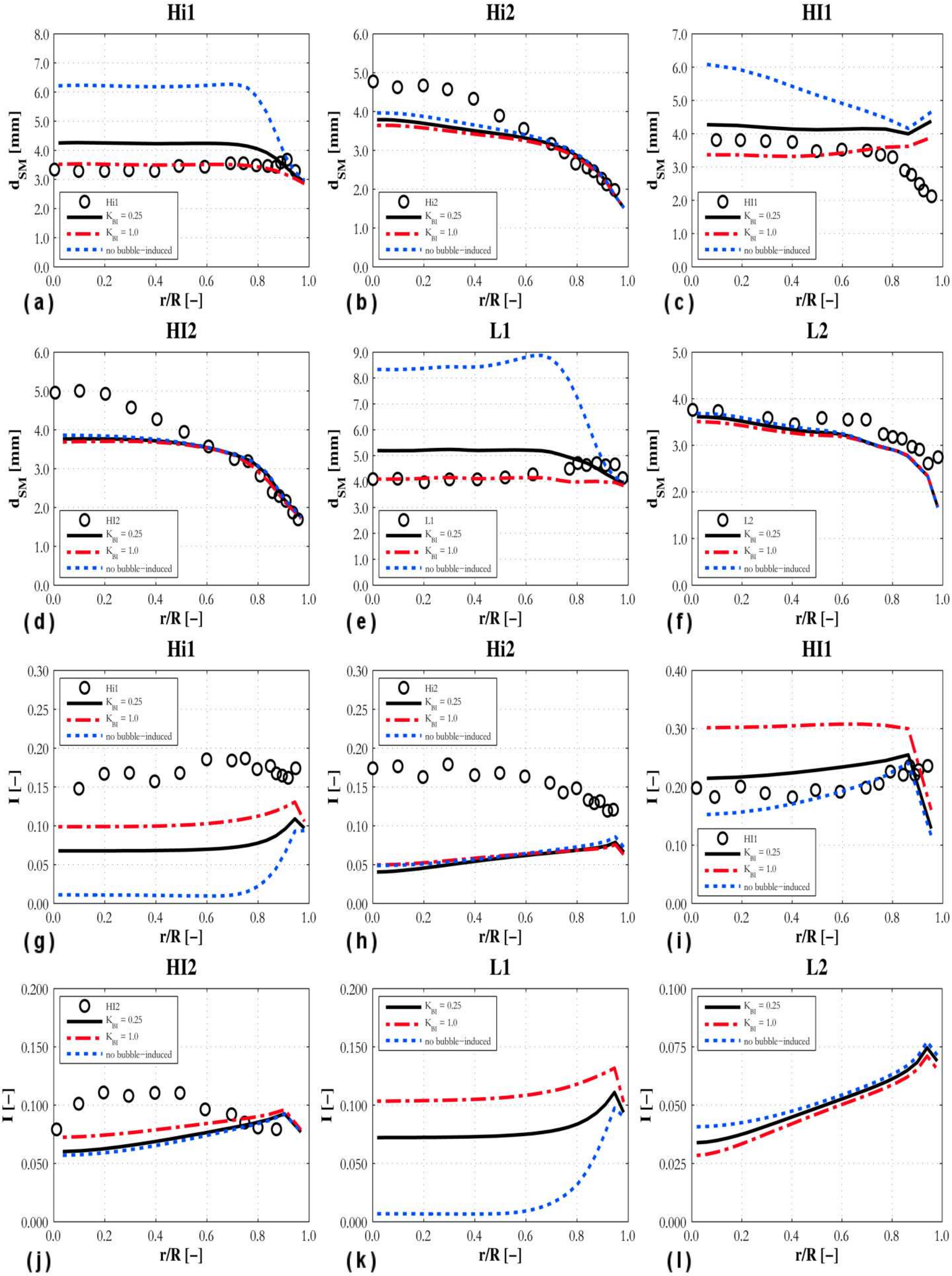

Figure 7. SMD (a-f) and turbulence intensity (g-l) radial profiles without bubble-induced turbulence (---), and with bubble-induced turbulence, and for K_{BI} = 0.25 (—) and K_{BI} = 1.0 (-··-). Predictions, obtained with YM and We_{crit} = 0.1, are compared against experiments in Table 1.

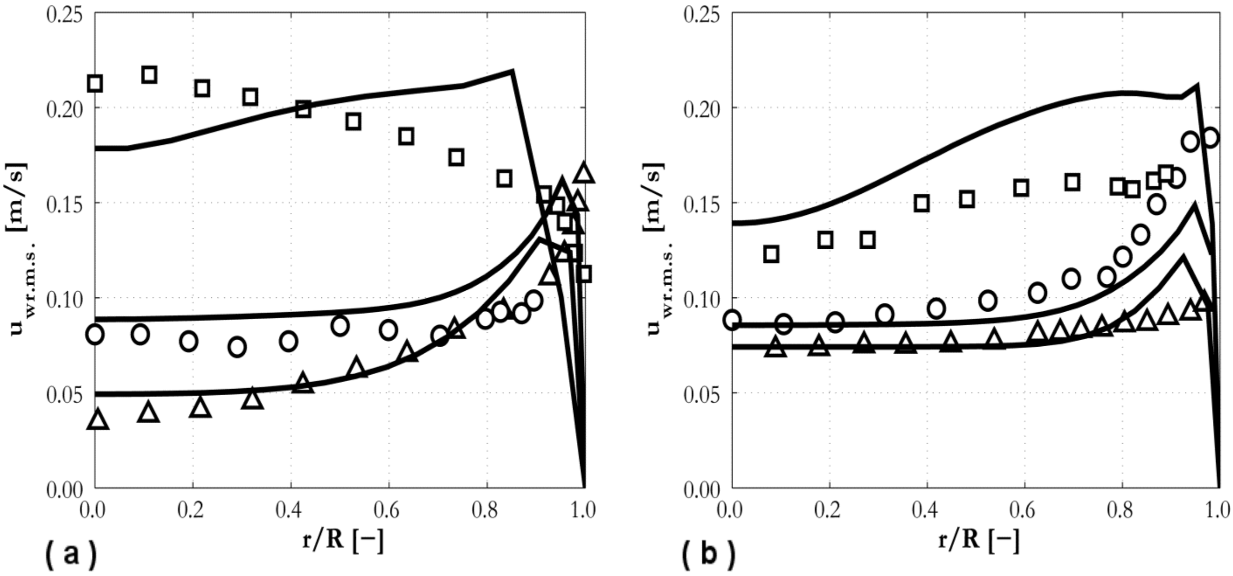

Figure 8. Radial profiles of r.m.s. of streamwise velocity fluctuations compared against experiments in bubbly pipe flows (Colombo and Fairweather, 2015). (a) Liu and Bankoff (1993), j_w = 1.087 m/s, j_a = 0.112 m/s (Δ); Serizawa et al. (1975), j_w = 1.03 m/s, j_a = 0.291 m/s (○); Liu and Bankoff (1993), j_w = 0.376 m/s, j_a = 0.347 m/s (□). (b) Wang et al. (1987), j_w = 0.71 m/s, j_a = 0.1 m/s (Δ); Liu (1998), j_w = 1.0 m/s, j_a = 0.22 m/s (○); Serizawa et al. (1975), j_w = 1.03 m/s, j_a = 0.436 m/s (□).

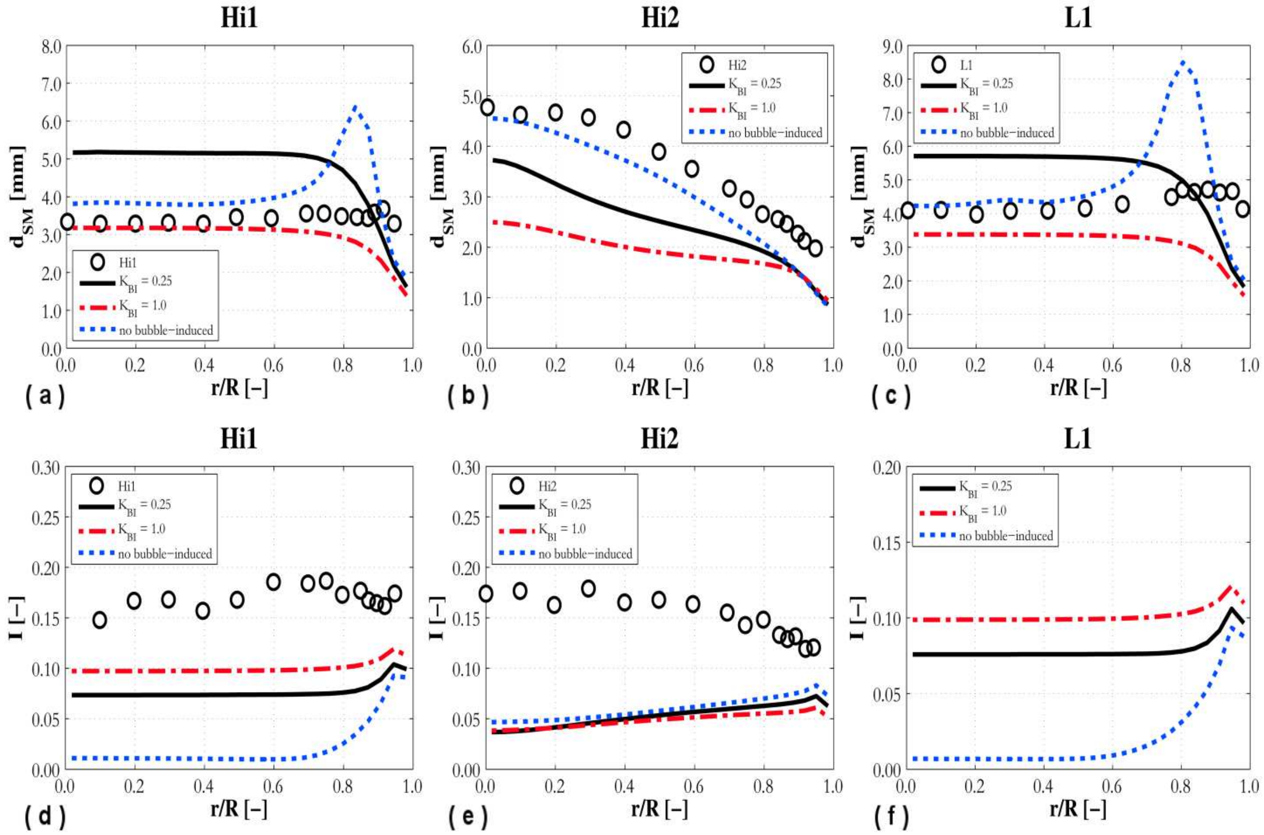

Figure 9. SMD (a-c) and turbulence intensity (d-f) radial profiles without bubble-induced turbulence (---), and with bubble induced turbulence, and for K_{BI} = 0.25 (—) and K_{BI} = 1.0 (-·-). Predictions, obtained with LZ, are compared against experiments Hi1 (a,d), Hi2 (b,e) and L1 (c,f).

4.3. Reynolds stress turbulence model

Using the YM model, the same tests were repeated with a Reynolds stress turbulence model and the results are presented in Figure 10 and Figure 11. A comparable level of agreement with data is found using both turbulence models for the SMD profiles (Figure 10 a-c and Figure 11 a-c), and similar velocity profiles were obtained (Figure 10 d-f and Figure 11 d-f). Similar void fraction profiles were also obtained for the wall-peaked cases (Figure 10g, Figure 11h and Figure 11i), although for the core-peaked profiles, the behaviour of the void fraction is reproduced better by the RSM (Figure 10h, Figure 10i and Figure 11g). More specifically, in such cases the void fraction gently increases from the wall towards the pipe centre. However, for the k-ε model, the increase is sharper near the wall, and the profile is then flatter towards the pipe centre. In a turbulent bubbly flow, the turbulence may interact with the interphase forces, inducing a radial pressure gradient in the flow that impacts upon the distribution of the dispersed phase (Ullrich et al., 2014). Generally, since the turbulence is higher near the wall, the pressure accordingly increases towards the pipe centre. It is this pressure gradient that is likely responsible for the over predicted void fraction peak for experiment L2 (Figure 11i).

In bubbly pipe flows, the turbulence is anisotropic, and this anisotropy can be reproduced using a Reynolds stress model (Colombo and Fairweather, 2015). Therefore, different results should be expected when using a k-ε model, because of the different turbulent stresses, or if the turbulence kinetic energy is added to the pressure. It must be noted, however, that differences between the two turbulence modelling approaches might be obscured by the influence of the interfacial momentum forces, which have been the object of a significant amount of optimization and refinement in the past. It is the opinion of the authors, however, that even when a similar accuracy is obtained (wall-peaked profiles), the use of a Reynolds stress formulation provides more insight into the distinctive features of the flow and should assist the development of models of more general applicability. In this regard, Ullrich et al. (2014) predicted some wall-peaked void fraction profiles with an RSM, whilst neglecting lift and wall reflection forces.

Differences between the turbulence model predictions are also apparent in the turbulence intensity profiles (Figure 10 j-l and Figure 11 j-l). These, even if small for the majority of cases, induce differences in the coalescence rates which, as discussed in the previous section, are strongly dependent on the turbulence in the continuous phase. The different coalescence rates, together with differences in the void fraction profiles, can be considered the reason for the slight disparity in the bubble diameter and the mean velocity profiles between the k-ε model and the RSM.

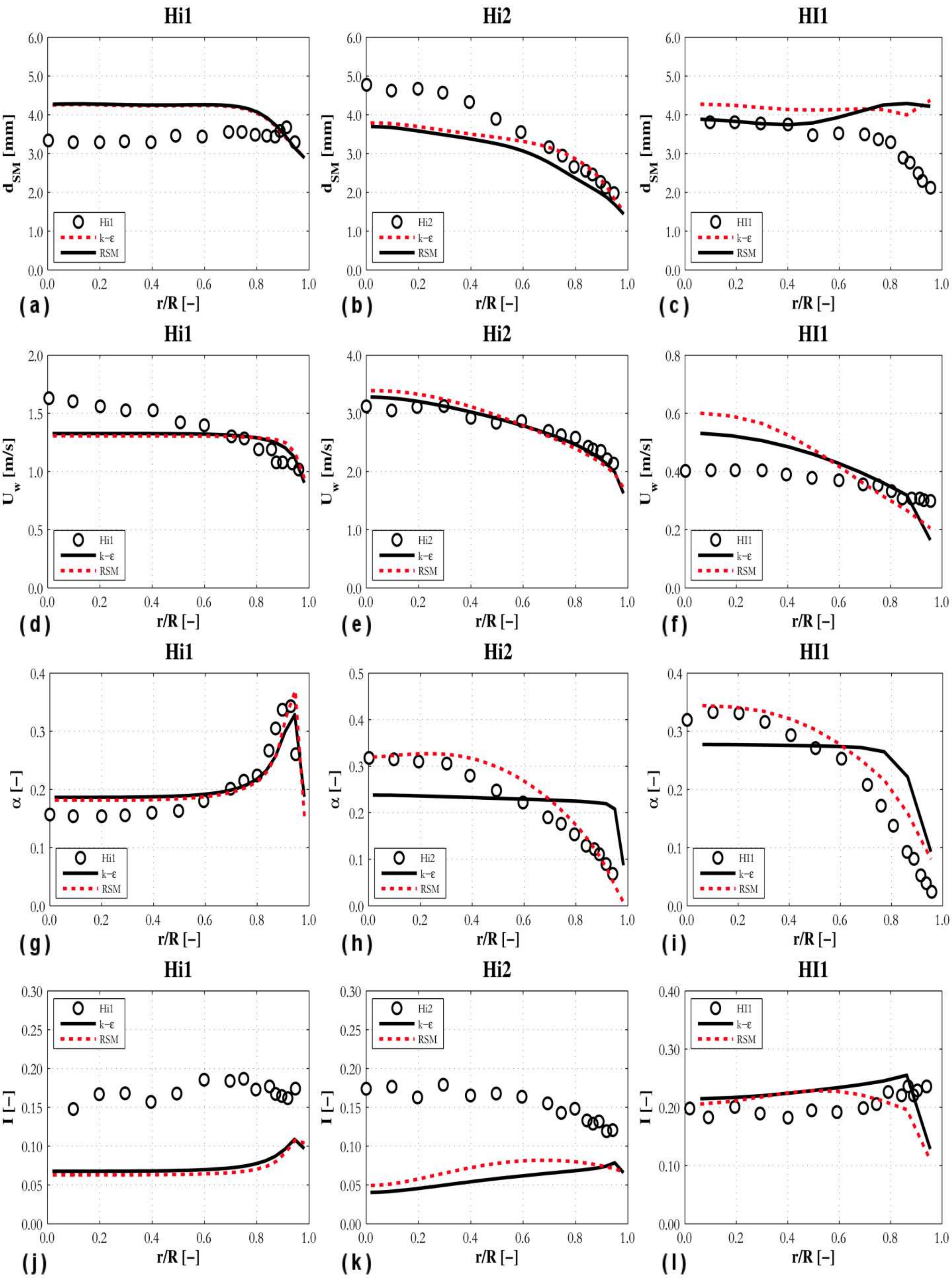

Figure 10. SMD (a-c), mean velocity (d-f), void fraction (g-i) and turbulence intensity (j-l) radial profiles compared against experiments Hi1, Hi2 and HI1. Predictions were obtained with a *k-ε* (—) and a Reynolds stress (---) turbulence formulation.

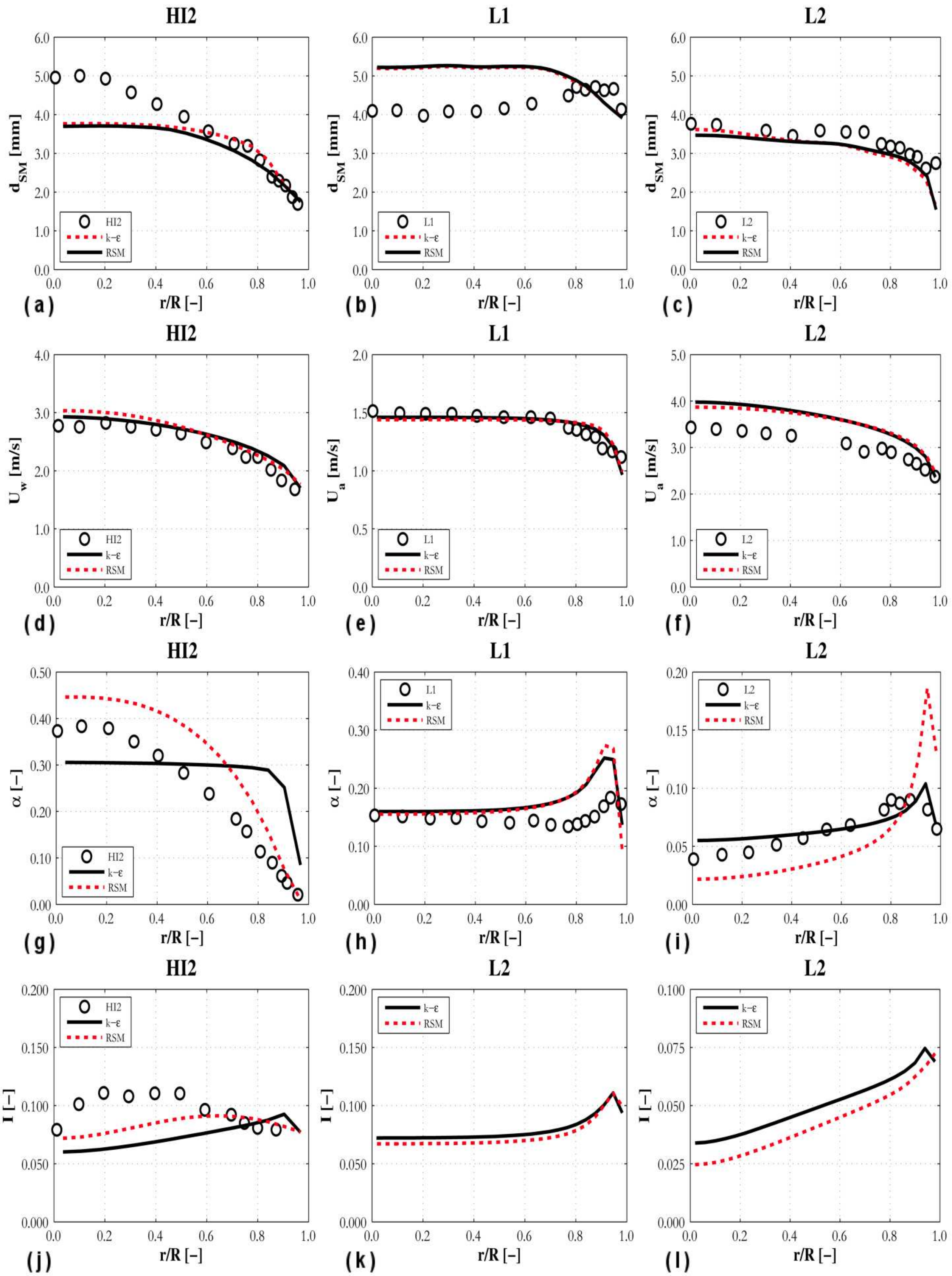

Figure 11. SMD (a-c), mean velocity (d-f), void fraction (g-i) and turbulence intensity (j-l) radial profiles compared against experiments HI2, L1 and L2. Predictions were obtained with a *k-ε* (—) and a Reynolds stress (---) turbulence formulation.

4.4. Two-group model

It was mentioned in the introduction how bubbly flows are generally characterized by polidispersity and by an extended range of bubble sizes. The comparisons in the previous sections demonstrated the different behaviour of spherical and larger cap bubbles, showing wall-peaked or core-peaked void fraction profiles induced by the value of the average bubble diameter. When both types of bubble are present in a comparable amount, the void fraction profile may exhibit both wall- and core-peaked features, as is the case for the experiment L1, depicted in Figure 12 (Lucas et al., 2005). These experiments are particularly difficult to predict because the distinctive features of both bubble types must be reproduced. Therefore, an advanced model with two different bubble classes was specifically implemented to predict these kinds of flows. In view of the results from the previous sections, and the in general negligible impact of break-up, only the additional sources due to the coalescence of two spherical bubbles into a cap bubble were considered. For this case, the value of the critical diameter d_c was assumed equal to 5 mm. Comparison against experimental data is provided in Figure 12, based on the RSM predictions. As shown in the figure, the void fraction radial profile and the behaviour of both the spherical and the cap bubbles are well predicted. Near the wall, the void fraction profile increases rapidly because of the presence there of the majority of the spherical bubbles. After a region where it remains almost flat, the void fraction increases again towards the pipe centre where the cap bubbles accumulate, pushed there by the negative lift force. In a similar manner, close to the wall, the SMD is close to the SMD of the spherical bubbles, whereas it tends to the SMD of the cap bubbles towards the pipe centre.

The bubble size distribution, which is tracked by the S_y model, is shown at three different axial locations in Figure 13. The plots display h_{dB}, which is, following the work of Lucas et al. (2005), the contribution of each bubble size to the total void fraction:

$$h_{d_B} = \frac{d(\alpha)}{d(d_B)} \tag{44}$$

In this way, the contribution of larger bubbles, which are few in number but carry a significant amount of the total air volume, is properly accounted for (Lucas et al., 2005). Experimental data were obtained by averaging over the whole pipe cross-section. For the predictions, the bubble distribution was extracted from the simulation at each node and is shown in Figure 13 for the near-wall region (Figure 13a) and for the pipe centre (Figure 13b). At the first axial location (*L/D* = 8.4), two distinct peaks are shown in both the experimental and the numerical results. Starting from the inlet, the predominance of coalescence events leads to the formation of larger bubbles, as is demonstrated by the second peak in the profile at around 6 mm. Obviously, being still close to the inlet, large bubbles represent only a small fraction of the total void fraction. At this location, the total void fraction is overestimated, as can be seen from the higher peak values predicted. This is due to the fact that it was not possible to match the inlet conditions of the experiment exactly due to lack of data, in particular for the velocity of the phases. Therefore, some distance from the inlet is required for the flow to establish. Predicted values of the void fraction at the two other locations are indeed significantly closer to the experimental values. At the second axial location (*L/D* = 29.9), the bubble population evolves and, since coalescence remains predominant, the number of larger bubbles increases. Two distinctive peaks are still present, but the larger diameter peak is now the greatest. This shift of the bubble diameter spectrum to larger values is well reproduced by the simulation, with the main difference with experiment being a larger number of bubbles in the region between the two peaks. At the final location (*L/D* = 59.2), the larger bubbles are in the majority, with the first peak at around 4 mm now being very small. The same evolution is found in the simulation, with a more diffuse distribution and an extended spectrum of diameters. It should be noted that the variance of the distribution is lower and the first peak still present near the wall where the majority of the spherical bubbles are present. In contrast, near the pipe centre, where the majority of the larger bubbles accumulate, the averaged experimental spectrum is overestimated and the bubble population extends to even higher values of the bubble diameter. The experimental profile, therefore, can be qualitatively considered an average of these two behaviours. In view of these results, the evolution of the bubble diameter distribution is predicted with a satisfactory accuracy, even with the rather simple model adopted which could be subject to numerous further improvements. Therefore, the challenge of predicting the whole bubble size spectrum from small spherical to large cap bubbles seems to be manageable with the use of only two bubble groups.

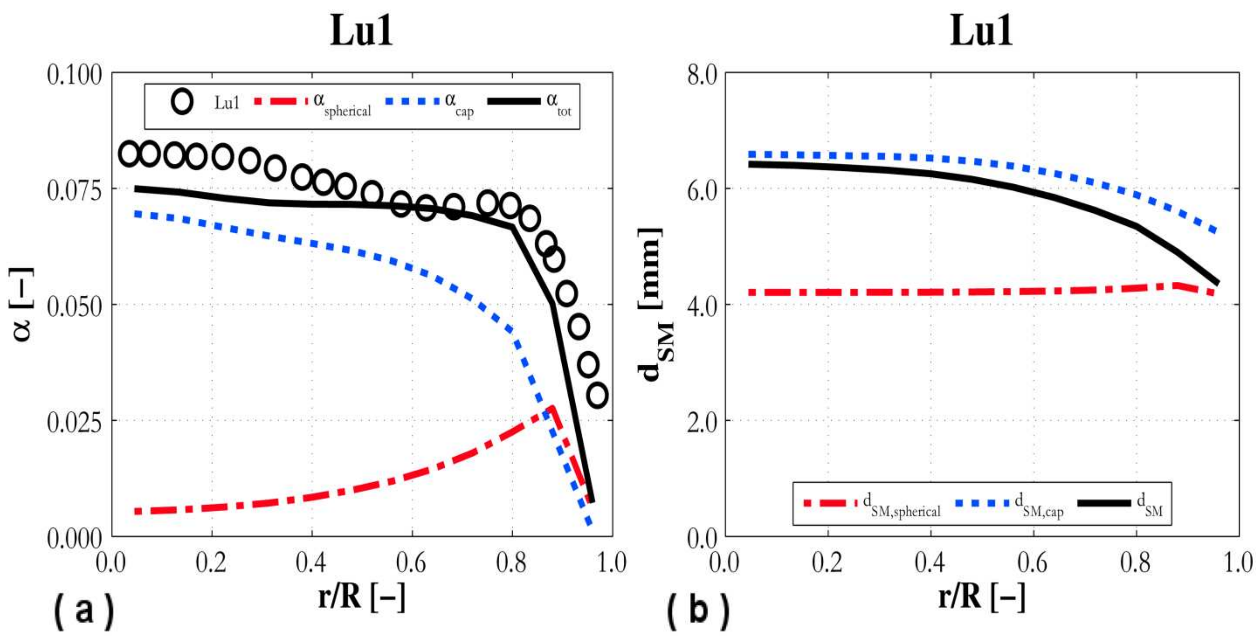

Figure 12. Void fraction (a) and SMD (b) radial profiles considering two bubble classes. Along with total values (—), which are compared against Lu1 experiment, predictions for spherical (-··-) and cap bubbles (---) are also shown.

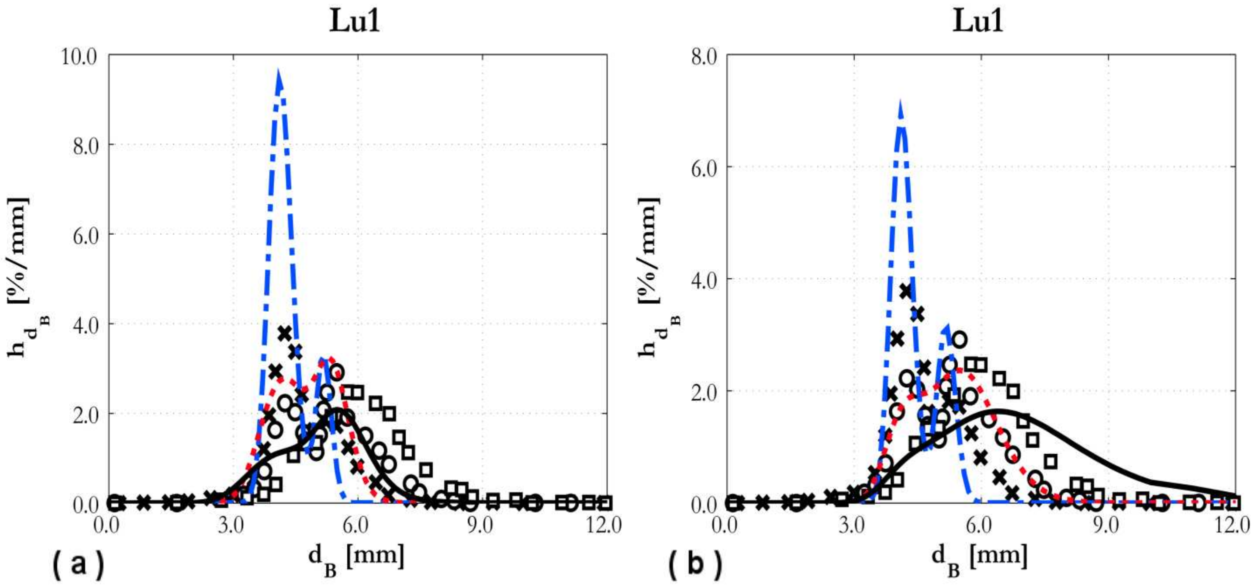

Figure 13. Bubble diameter distribution extracted from the simulations (lines) compared against the experiments (markers) at three axial locations: *L/D* = 8.4 (x, -·-); *L/D* = 29.9 (○, --); *L/D* = 59.2 (□, —). Simulation results are displayed in two different locations: (a) pipe wall; (b) pipe centre.

5. Conclusions

In this work, the S_γ model (Lo and Zhang, 2009), based on the moments of the bubble size distribution, was coupled with an Eulerian-Eulerian two-fluid model with the STAR-CCM+ code, and tested against the data from seven upward bubbly flow experiments in pipes. Through the S_γ model, the evolution of the bubble size distribution was followed through the flows, so that the average SMD and the interfacial area concentration, which are crucial for the prediction of the phase interactions, could be tracked. Being based on the method of moments, the S_γ model also has the advantage that the required computational resources are limited. The addition of a different coalescence model (Yao and Morel, 2004), based on the collision of bubbles in turbulence and on the film drainage model, and further optimized against the experiments, allowed reproduction of the experimental radial profiles of the SMD. More specifically, a constant critical Weber number value of 0.10 in the coalescence model was sufficient to obtain a satisfactory predictive accuracy.

A sensitivity study suggested a negligible effect of the bubble break-up model and the best results were achieved by considering these flows to be dominated by bubble coalescence. However, the lack of availability of experimental data, limited to the average bubble diameter alone, constrains research work in the field. In particular, it is extremely difficult to evaluate the competitive contributions of break-up and coalescence, and to extend the modelling to cover all possible mechanisms involved. Therefore, additional knowledge is required, by means of experiments or direct numerical simulations. Continuous phase turbulence was noted to significantly influence the predictions of the model. In this regard, validation of turbulence models needs to be carried out in conjunction with that for the bubble diameter evolution, and requires the availability of additional complete datasets. In addition, different coalescence models were found to display different qualitative behaviour following changes in the flow field turbulence level, and this requires further investigation.

Lastly, an advanced version of the overall model described was tested. This included a Reynolds stress turbulence formulation and two groups of bubbles, accounting for spherical bubbles accumulating close to the wall and cap bubbles migrating towards the pipe centre. The RSM, in addition to performing better in flows where known shortcomings of two-equation turbulence models are present, provides better accuracy in predicting core-peaked void fraction profiles and properly accounts for the interaction between the turbulence and the interphase forces. Comparison with a complex void fraction profile suggested that extension of the model to only two bubble groups is sufficient to describe the whole bubble spectrum, and the bubbly flow regime up to the transition to slug flow, even though additional comparisons with data are necessary.

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