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Sparsity-Aware Robust Normalized Subband Adaptive Filtering algorithms with Alternating Optimization of Parameters

Yi Yu, *Member, IEEE*, Zongxin Huang, Hongsen He, *Member, IEEE*, Yuriy Zakharov, *Senior Member, IEEE*,
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Abstract—This paper proposes a unified sparsity-aware robust normalized subband adaptive filtering (SA-RNSAF) algorithm for identification of sparse systems under impulsive noise. The proposed SA-RNSAF algorithm generalizes different algorithms by defining the robust criterion and sparsity-aware penalty. Furthermore, by alternating optimization of the parameters (AOP) of the algorithm, including the step-size and the sparsity penalty weight, we develop the AOP-SA-RNSAF algorithm, which not only exhibits fast convergence but also obtains low steady-state misadjustment for sparse systems. Simulations in various noise scenarios have verified that the proposed AOP-SA-RNSAF algorithm outperforms existing techniques.

Index Terms—Impulsive noises, subband adaptive filters, sparse systems, time-varying parameters.

I. INTRODUCTION

FOR highly correlated input signals, the normalized subband adaptive filtering (NSAF) [1] algorithm provides faster convergence than the normalized least mean square (NLMS) algorithm and retains comparable complexity. The NSAF algorithm was proposed based on the multiband structure of subband filters [2], which adjusts the fullband filter's coefficients to remove the aliasing and band edge effects of the conventional subband structure [2]. However, in practice the non-Gaussian noise with impulsive samples could commonly happen such as in echo cancellation, underwater acoustics, audio processing, and communications [3], [4], and in this scenario, the NSAF performance degrades. To deal with impulsive noises, several robust subband algorithms based on different robust criteria were proposed, see [5]–[10] and references therein, and most of them can be unified as the NSAF update with a specific scaling factor.

Furthermore, it is interesting to improve the adaptive filter performance by exploiting the system sparsity. For example,

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the impulse responses of propagation channels in underwater acoustic and radio communications are usually sparse [11], [12], only a few coefficients of which are non-zero. Aiming at sparse systems, existing examples are classified into the proportionate type and sparsity-aware type. The family of proportionate NSAF (PNSAF) algorithms [13] assigns an individual gain to each filter coefficient, which has faster convergence than the NSAF algorithm. Later, robust PNSAF algorithms were also presented [10], [14] to deal with impulsive noises. On the other hand, the family of sparsity-aware algorithms incorporates the sparsity-aware penalty into the original NSAF's and PNSAF's cost functions; as a result, sparsity-aware NSAF (SA-NSAF) [15], [16] and sparsity-aware PNSAF [17] algorithms were developed. In sparse system identification, these sparsity-aware algorithms can obtain better convergence and steady-state performance than their original counterparts.

However, the superiority of sparsity-aware algorithms depends mainly on the sparsity-penalty parameter, which is often chosen in an exploratory way thus reducing the practicality of the algorithms. Besides, they encounter the problem of choosing the step-size, which controls the tradeoff between convergence rate and steady-state misadjustment. Hence, adaptation techniques for the sparsity-penalty and the step-size parameters are necessary. In the literature, they are rarely discussed simultaneously regardless of the Gaussian noise or impulsive noise scenarios. In [18], the variable parameter SA-NSAF (VP-SA-NSAF) algorithm was proposed for the Gaussian noise, in which these two parameters are jointly adapted based on a model-driven method, but it requires knowledge of variances of the subband noises. In [19], by optimizing the parameters in the sparsity-aware individual-weighting-factors-based sign subband adaptive filter (S-IWF-SSAF) algorithm with the robustness in the impulsive noise, the variable parameters S-IWF-SSAF (VP-S-IWF-SSAF) algorithm was presented, while it lacks the generality in sparsity-aware subband algorithms.

In this paper, we propose a unified sparsity-aware robust NSAF (SA-RNSAF) framework to handle impulsive noises, which can result in different algorithms by only changing the robustness criterion and the sparsity-aware penalty. We then devise adaptive schemes for adjusting the step-size and the sparsity-aware penalty weight, and develop the alternating optimization of the parameters based SA-RNSAF (AOP-SA-RNSAF) algorithm, with fast convergence and low steady-state

misadjustment for sparse systems.

II. STATEMENT OF PROBLEM AND SA-RNSAF ALGORITHM

Consider a system identification problem. The relationship between the input signal $u(n)$ and desired output signal $d(n)$ at time n is given by

$$d(n) = \mathbf{u}^T(n)\mathbf{w}^o + \nu(n), \quad (1)$$

where the $M \times 1$ vector \mathbf{w}^o is the impulse response of the sparse system that we want to identify, $\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-M+1)]^T$ is the $M \times 1$ input vector, and $\nu(n)$ is the additive noise independent of $u(n)$. For estimating \mathbf{w}^o , the SAF with a coefficient vector $\mathbf{w}(k)$ is used, shown in Fig. 1 with N subbands, where k denotes the sample index in the decimated domain. The input signal $u(n)$ and the desired output signal $d(n)$ are decomposed into multiple subband signals $u_i(n)$ and $d_i(n)$ via the analysis filters $\{\mathbf{h}_i\}_{i=1}^N$, respectively. For each subband input signal $u_i(n)$, the corresponding output of the fullband filter $\mathbf{w}(k)$ is $y_i(n)$. Then, both $d_i(n)$ and $y_i(n)$ are critically decimated to yield signals $d_{i,D}(k)$ and $y_{i,D}(k)$, respectively, with lower sampling rate, namely, $d_{i,D}(k) = d_i(kN)$ and $y_{i,D}(k) = \mathbf{u}_i^T(k)\mathbf{w}(k)$, where $\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \dots, u_i(kN-M+1)]^T$. By subtracting $y_{i,D}(k)$ from $d_{i,D}(k)$, the decimated subband error signals are obtained:

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{u}_i^T(k)\mathbf{w}(k), \quad i = 1, 2, \dots, N, \quad (2)$$

which are used to adjust the coefficient vector $\mathbf{w}(k)$ ¹.

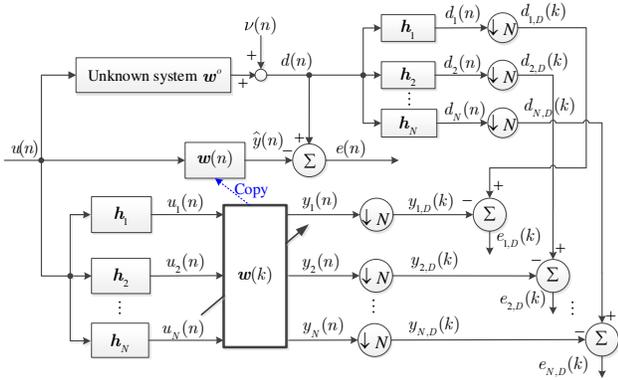


Fig. 1. Multiband structure of subband adaptive filter.

In practice, the additive noise $\nu(n)$ can be non-Gaussian consisting of Gaussian and impulsive components. Hence, for the identification of a sparse vector \mathbf{w}^o in the presence of impulsive noise, we define the following minimization problem:

$$\arg \min_{\mathbf{w}(k+1)} [\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2 + \rho f(\mathbf{w}(k+1))], \quad (3)$$

¹In some applications, we could also eventually need the output error $e(n)$ in the original time domain. To this end, we obtain $\mathbf{w}(n)$ by copying $\mathbf{w}(k)$ for every N input samples, and then compute the output error by $e(n) = d(n) - \mathbf{u}^T(n)\mathbf{w}(n)$.

subject to

$$e_{p,i}(k) = [1 - \mu\phi_i(k)]e_{i,D}(k), \quad (4a)$$

$$\phi_i(k) = \frac{\varphi'(e_{i,D}(k))}{e_{i,D}(k)}, \quad (4b)$$

for subbands $i = 1, \dots, N$, where $e_{p,i}(k) = d_{i,D}(k) - \mathbf{u}_i^T(k)\mathbf{w}(k+1)$ denotes the *a posteriori* decimated subband error, $\mu > 0$ will be called the step-size in the sequel, and $\phi_i(k)$ is called the scaling factor of the i -th subband. In (3), $f(\mathbf{w})$ is a sparsity-aware penalty function and $\rho > 0$ is the weight of this penalty term. In (4b), $\varphi'(e) \triangleq \frac{\partial \varphi(e)}{\partial e}$, where $\varphi(e) \geq 0$ is an even function of variable e , defining the robustness to impulsive noise.

By using the Lagrange multiplier method, we obtain the solution of (3) subject to (4a) as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=1}^N \phi_i(k) \frac{e_{i,D}(k)\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|_2^2} - \rho \left[f'(\mathbf{w}(k+1)) - \sum_{i=1}^N \frac{\mathbf{u}_i(k)\mathbf{u}_i^T(k)}{\|\mathbf{u}_i(k)\|_2^2} f'(\mathbf{w}(k+1)) \right]. \quad (5)$$

Note that the derivation of (5) also uses an approximation in the SAF, that is $\mathbf{u}_i^T(k)\mathbf{u}_j(k) \approx 0$ for $i \neq j$ [1]. Then, by introducing an intermediate estimate $\boldsymbol{\psi}(k)$, we propose to implement (5) in two steps:

$$\boldsymbol{\psi}(k) = \mathbf{w}(k) + \mu \sum_{i=1}^N \phi_i(k) \frac{e_{i,D}(k)\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|_2^2}, \quad (6a)$$

$$\mathbf{w}(k+1) = \boldsymbol{\psi}(k) - \rho \mathbf{P}(k), \quad (6b)$$

where

$$\mathbf{P}(k) = f'(\boldsymbol{\psi}(k)) - \sum_{i=1}^N \frac{\mathbf{u}_i(k)\mathbf{u}_i^T(k)}{\|\mathbf{u}_i(k)\|_2^2} f'(\boldsymbol{\psi}(k)). \quad (7)$$

This completes the derivation of the update for the SA-RNSAF algorithm. In this algorithm, the steps (6a) and (6b) have their own roles. The former behaves like the RNSAF algorithm to obtain a coarse estimate $\boldsymbol{\psi}(k)$ of the sparse vector \mathbf{w}^o in impulsive noise. Subsequently, the step (6b) forces the inactive coefficients in $\boldsymbol{\psi}(k)$ to zero, thus obtaining a more accurate sparse estimate $\mathbf{w}(k+1)$.

It is noteworthy that the parameters μ and ρ control the SA-RNSAF's performance. Specifically, the step-size μ controls the convergence rate and steady-state misadjustment of the algorithm. Moreover, the SA-RNSAF algorithm can be superior to the RNSAF algorithm when dealing with sparse systems, but ρ must be chosen within a theoretically existed range while this range is unpredictable actually (see Remark 1 below). As such, we will derive adaptive recursions for adjusting μ and ρ . However, it is challenging to solve the global optimization problem on μ and ρ , as (6a) and (6b) depend on each other. Interestingly, μ and ρ mainly affect the steps (6a) and (6b), respectively, thus we can use alternating optimization [20] to solve this global optimization problem. Accordingly, the adaptations of μ and ρ will be designed independently according to (6a) and (6b), respectively.

III. PROPOSED AOP-SA-RNSAF ALGORITHM

By using the band-dependent variable step-size (VSS) $\mu_i(k)$ and $\rho(k)$ instead of some fixed values, we rearrange (6a) and (6b) as follows:

$$\boldsymbol{\psi}(k) = \boldsymbol{w}(k) + \sum_{i=1}^N \mu_i(k) \phi_i(k) \frac{e_{i,D}(k) \mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|_2^2}, \quad (8a)$$

$$\boldsymbol{w}(k+1) = \boldsymbol{\psi}(k) - \rho(k) \mathbf{P}(k). \quad (8b)$$

A. Adaptation of the step-size

By subtracting (8a) from \boldsymbol{w}^o , we obtain

$$\tilde{\boldsymbol{\psi}}(k) = \tilde{\boldsymbol{w}}(k) - \sum_{i=1}^N \mu_i(k) \phi_i(k) \frac{e_{i,D}(k) \mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|_2^2}, \quad (9)$$

where $\tilde{\boldsymbol{w}}(k) = \boldsymbol{w}^o - \boldsymbol{w}(k)$ and $\tilde{\boldsymbol{\psi}}(k) = \boldsymbol{w}^o - \boldsymbol{\psi}(k)$ define the deviation vectors for the final estimate $\boldsymbol{w}(k)$ and the intermediate estimate $\boldsymbol{\psi}(k)$ with respect to the true value. By pre-multiplying $\mathbf{u}_i^T(k)$ on both sides of (9) and applying the approximation $\mathbf{u}_i^T(k) \mathbf{u}_j(k) \approx 0$ for $i \neq j$ again, it is established that

$$e_{\varepsilon,i}(k) = [1 - \mu_i(k) \phi_i(k)] e_{i,D}(k) \quad (10)$$

for $i = 1, 2, \dots, N$, where $e_{\varepsilon,i}(k) = d_{i,D}(k) - \mathbf{u}_i^T(k) \boldsymbol{\psi}(k)$ defines the intermediate *a posteriori* error at the i -th subband resulting from the step (6a). By squaring both sides of (10) and taking the expectations over all the terms, the following relation is obtained:

$$\mathbb{E}\{e_{\varepsilon,i}^2(k)\} = [1 - \mu_i(k) \phi_i(k)]^2 \mathbb{E}\{e_{i,D}^2(k)\}, \quad (11)$$

where $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation. In (11), a common assumption is used that the step-size $\mu_i(k)$ and the scaling factors $\{\phi_i(k)\}_{i=1}^N$ are deterministic at iteration k in contrast with the randomness of $e_{i,D}(k)$ [7], [21].

Motivated by [21], we wish to compute the subband step-sizes in such a way that $\mathbb{E}\{e_{\varepsilon,i}^2(k)\} = \sigma_{\nu,i}^2$, $i = 1, 2, \dots, N$, which means that the powers of the intermediate *a posteriori* subband errors always equal those of the subband noises, where $\sigma_{\nu,i}^2 \triangleq \mathbb{E}\{e_{i,D}^2(k)\}$ denotes the power of the i -th subband noise excluding impulsive interferences. In this requirement, then from (11) we can obtain the following equation:

$$\mu_i(k) \phi_i(k) = 1 - \sqrt{\frac{\sigma_{\nu,i}^2}{\sigma_{e_{i,D}}^2(k)}}, \quad (12)$$

where $\sigma_{e_{i,D}}^2(k) \triangleq \mathbb{E}\{e_{i,D}^2(k)\}$ indicates the power of $e_{i,D}(k)$ without impulsive noises. For robust adaptive algorithms with the scaling factors, there is a common property [6], [7] that when impulsive noises happen, the scaling factors $\phi_i(k)$ will become very small (close to zero), thereby preventing the adaptation (8a) from the interference caused by impulsive noises. If the impulsive noise is absent, $\phi_i(k)$ will approximately equal one to ensure fast convergence. As such, we can change (12) to

$$\mu_i(k) = 1 - \sqrt{\frac{\sigma_{\nu,i}^2}{\sigma_{e_{i,D}}^2(k)}}. \quad (13)$$

To implement (13), the statistical quantities $\sigma_{e_{i,D}}^2(k)$ and $\sigma_{\nu,i}^2$ are replaced with their estimates $\hat{\sigma}_{e_{i,D}}^2(k)$ and $\hat{\sigma}_{\nu,i}^2(k)$, respectively. Specifically, $\hat{\sigma}_{e_{i,D}}^2(k)$ is calculated in an exponential window way as

$$\hat{\sigma}_{e_{i,D}}^2(k) = \zeta \hat{\sigma}_{e_{i,D}}^2(k-1) + (1-\zeta) \phi_i^2(k) e_{i,D}^2(k), \quad (14)$$

where ζ is a weighting factor often chosen as $\zeta = 1 - 1/(\kappa M)$ with $\kappa \geq 1$. Similar to [21], $\hat{\sigma}_{\nu,i}^2(k)$ is calculated by the following equations:

$$\hat{\sigma}_{u_i}^2(k) = \zeta \hat{\sigma}_{u_i}^2(k-1) + (1-\zeta) u_i^2(kN), \quad (15a)$$

$$\hat{\boldsymbol{r}}_{ue_i}(k) = \zeta \hat{\boldsymbol{r}}_{ue_i}(k-1) + (1-\zeta) \phi_i(k) e_{i,D}(k) \mathbf{u}_i(k), \quad (15b)$$

$$\hat{\sigma}_{\nu,i}^2(k) = \hat{\sigma}_{e_{i,D}}^2(k) - \frac{\|\hat{\boldsymbol{r}}_{ue_i}(k)\|_2^2}{\hat{\sigma}_{u_i}^2(k) + \epsilon_1}, \quad (15c)$$

where ϵ_1 is a small positive number (e.g., 10^{-5}). Note that, we introduce the scaling factor $\phi_i(k)$ in (14) and (15b) for each subband to suppress impulsive noises.

Accordingly, (13) can be rewritten as

$$\mu_i(k) = 1 - \sqrt{\frac{\hat{\sigma}_{\nu,i}^2(k)}{\hat{\sigma}_{e_{i,D}}^2(k) + \epsilon_2}}, \quad (16)$$

where ϵ_2 is also a small positive number. It is stressed that the estimated values of multiple statistical quantities are used in (15c), and thus $\hat{\sigma}_{\nu,i}^2(k)$ could be negative at some iterations. To avoid this, we add the step $\hat{\sigma}_{\nu,i}^2(k) \leftarrow \hat{\sigma}_{\nu,i}^2(k-1)$ after (15c).

B. Adaptation of the sparsity penalty weight

By subtracting (8b) from \boldsymbol{w}^o , we obtain

$$\tilde{\boldsymbol{w}}(k+1) = \tilde{\boldsymbol{\psi}}(k) + \rho(k) \mathbf{P}(k). \quad (17)$$

By pre-multiplying both sides of (17) by their transpose, we obtain

$$\|\tilde{\boldsymbol{w}}(k+1)\|_2^2 = \|\tilde{\boldsymbol{\psi}}(k)\|_2^2 + \Delta(k), \quad (18)$$

where

$$\Delta(k) = 2\rho(k) \tilde{\boldsymbol{\psi}}^T(k) \mathbf{P}(k) + \rho^2(k) \|\mathbf{P}(k)\|_2^2. \quad (19)$$

Remark 1: (18) clearly reveals that the proposed SA-RNSAF algorithm will outperform the RNSAF algorithm for identifying sparse systems, if and only if $\Delta(k) < 0$ holds². It follows that $\rho(k)$ should satisfy the inequality

$$0 < \rho(k) < 2 \frac{[\boldsymbol{\psi}(k) - \boldsymbol{w}^o]^T \mathbf{P}(k)}{\|\mathbf{P}(k)\|_2^2}. \quad (20)$$

Moreover, since $\Delta(k)$ is the quadratic function of $\rho(k)$, there is an optimal $\rho(k)$ such that $\Delta(k)$ arrives at the negative maximum value. Consequently, the optimal $\rho(k)$ is given as

$$\rho_{\text{opt}}(k) = \frac{[\boldsymbol{\psi}(k) - \boldsymbol{w}^o]^T \mathbf{P}(k)}{\|\mathbf{P}(k)\|_2^2}. \quad (21)$$

Although Remark 1 states that the relations (20) and (21) are existing in sparse systems, they are incalculable due to the fact that the sparse vector \boldsymbol{w}^o is unknown. To solve this

²Following a derivation similar to that in Appendix D in [19], $\Delta(k) < 0$ is likely to be true as long as \boldsymbol{w}^o is sparse.

problem, we use the previous estimate $\mathbf{w}(k)$ to approximate \mathbf{w}^o , then (21) can be reformulated as

$$\hat{\rho}_{\text{opt}}(k) = \max \left\{ \frac{[\boldsymbol{\psi}(k) - \mathbf{w}(k)]^T \mathbf{P}(k)}{\|\mathbf{P}(k)\|_2^2}, 0 \right\}, \quad (22)$$

where $\hat{\rho}_{\text{opt}}(k)$ is set to zero at $k = 0$.

The recursion (8) equipped with $\mu_i(k)$ in (16) and $\hat{\rho}_{\text{opt}}(k)$ in (22) constitutes the proposed AOP-SA-RNSAF algorithm.

Remark 2: The proposed AOP-SA-RNSAF update generalizes different algorithms, depending on the choice of $\varphi(e)$ in (4b) and $f(\mathbf{w})$ in (3). In the literature, several robust criteria against impulsive noises [6], [7], [9], [10], [14] defined by $\varphi(e)$ and sparsity-aware penalties [15], [16], [18], [19], [22] defined by $f(\mathbf{w})$ have been studied, which can be applied in the AOP-SA-RNSAF. Nevertheless, this paper does not consider the effect of different choices of $\varphi(e)$ and/or $f(\mathbf{w})$, which is worth studying in future work. Note that, when setting $\varphi(e) = \frac{1}{2}e^2$, the proposed algorithm is called the alternating optimization of parameters based SA-NSAF (AOP-SA-NSAF) suited for Gaussian noise environments, which is a sparsity-aware variant of the VSS-NSAF algorithm presented in [21].

Remark 3: By firstly computing the inner product $\mathbf{u}_i^T(k)f'(\boldsymbol{\psi}(k))$ in $\mathbf{P}(k)$, and then calculating $\mathbf{P}(k)$ only requires $2M$ multiplications, $2M - M/N$ additions, and 1 division. Therefore, the complexity of the proposed AOP-SRNSAF algorithm is still low with $\mathcal{O}(M)$ arithmetic operations per input sample.

IV. SIMULATION RESULTS

To evaluate the proposed AOP-SA-RNSAF algorithm, simulations are conducted to identify the acoustic echo paths with $M = 512$ taps. The sparsenesses, defined as $\chi(\mathbf{w}^o) = \frac{M}{M-\sqrt{M}} \left(1 - \frac{\|\mathbf{w}^o\|_1}{\sqrt{M}\|\mathbf{w}^o\|_2}\right)$, of two echo paths are $\chi(\mathbf{w}_1^o) = 0.9357$ (sparse) [19] and $\chi(\mathbf{w}_2^o) = 0.3663$ (dispersive or non-sparse) [14], respectively. The length of the adaptive filters is the same as that of \mathbf{w}^o . The correlated input signal $u(n)$ is a first-order autoregressive (AR) process with the pole at 0.9, generated by filtering a white Gaussian noise with zero-mean and unit variance. The analysis filters $\{\mathbf{h}_i\}_{i=0}^{N-1}$ for decomposing signals $d(n)$ and $u(n)$ are obtained by cosine-modulated filter banks, where the length of the prototype filter for $N = 4$ subbands is 33 to obtain 60 dB stopband attenuation. The high stopband attenuation is to guarantee that adjacent analysis filters have almost no overlap and the cross-correlation of nonadjacent subbands is negligible [2]. The normalized mean square deviation (NMSD), defined as $E\{\|\mathbf{w}(n) - \mathbf{w}^o\|_2^2 / \|\mathbf{w}^o\|_2^2\}$, is the performance measure. All the results are the average over 50 independent runs.

For the AOP-SA-RNSAF algorithm, we use the modified Huber (MH) function for $\varphi(e)$ and the log-penalty for $f(\boldsymbol{\psi}(k))$. The MH function is formulated as $\varphi(e_{i,D}(k)) = e_{i,D}^2(k)/2$ if $|e_{i,D}(k)| < \xi_i$ and $\varphi(e_{i,D}(k)) = 0$ if $|e_{i,D}(k)| \geq \xi_i$ [10], where ξ_i is the threshold. Accordingly, when $|e_{i,D}(k)| \geq \xi_i$ (usually impulsive noises occur), then the scaling factor in (4b) is $\phi_i(k) = 0$, which makes the adaptation step (8a) freeze to suppress impulsive interferences; otherwise, $\phi_i(k) = 1$, which retains fast convergence. Note that, the threshold ξ_i for each subband i is

set to $\xi_i = 2.576\hat{\sigma}_{\varepsilon,i}(k)$, where $\hat{\sigma}_{\varepsilon,i}^2(k)$ is the variance of $e_{i,D}(k)$ excluding impulsive samples. $\hat{\sigma}_{\varepsilon,i}^2(k)$ is computed by $\hat{\sigma}_{\varepsilon,i}^2(k) = \lambda\hat{\sigma}_{\varepsilon,i}^2(k-1) + c_\sigma(1-\lambda)\text{med}(\mathbf{a}_{\varepsilon,i})$, where $\lambda \in (0.9, 1)$ is the forgetting factor (but $\lambda = 0$ at $k = 0$), $\text{med}(\cdot)$ denotes the median operator to remove outliers in the data window $\mathbf{a}_{\varepsilon,i} = [e_{i,D}^2(k), e_{i,D}^2(k-1), \dots, e_{i,D}^2(k-N_w+1)]$ with a length of N_w , and $c_\sigma = 1.483(1 + 5/(N_w - 1))$ is the correction factor. The log-penalty is given as $f(\boldsymbol{\psi}_k) = \sum_{m=1}^M \ln(1 + |\psi_{m,k}|/\theta)$ [19] which characterizes the sparsity of systems, where $\psi_{m,k}$ is the m -th element of $\boldsymbol{\psi}_k$, and the shrinkage factor $\theta > 0$ cuts apart inactive and active entries in $\boldsymbol{\psi}_k$. Thus, $f'(\boldsymbol{\psi}_{m,k})$ in (7) is computed element-wise as $f'(\boldsymbol{\psi}_{m,k}) = \frac{\text{sgn}(\psi_{m,k})}{\theta + |\psi_{m,k}|}$, $m = 1, \dots, M$. In our simulations, the additive noise $\nu(n)$ is described by the symmetric α -stable process, also called the α -stable noise, whose characteristic function is formulated as $\phi(t) = \exp(-\gamma|t|^\alpha)$ [3]. The parameter $\alpha \in (0, 2]$ represents the impulsiveness of the noise that for smaller α leads to stronger impulsive noises, and $\gamma > 0$ behaves like the variance of the Gaussian density. In particular, it reduces to the Gaussian noise for the case of $\alpha = 2$. In the following simulations, we set $\gamma = 0.02$.

Example 1: the impulsive noise is absent, i.e., $\alpha = 2$. The proposed AOP-SA-NSAF algorithm in Remark 2 is compared with the NSAF, VP-SA-NSAF [18], VSS-NSAF, and VSS-PNSAF algorithms in Fig. 2, where both VSS-NSAF and VSS-PNSAF are obtained from [10] but in the Gaussian noise we reset $\varphi(e) = \frac{1}{2}e^2$ instead of using the MH function. For a fair evaluation, we select the log-penalty parameter $\theta = 0.005$ for all the sparsity-aware algorithms. As can be seen, the VSS-NSAF algorithm obtains fast convergence and low steady-state misadjustment, which overcomes the trade-off problem in the NSAF algorithm. By considering the sparsity of the underlying system, both VP-SA-NSAF and VSS-PNSAF algorithms further improve the convergence rate. As compared to the VSS-PNSAF algorithm, the proposed AOP-SNSAF algorithm shows slower initial convergence, but it achieves higher reduction in the steady-state misadjustment.

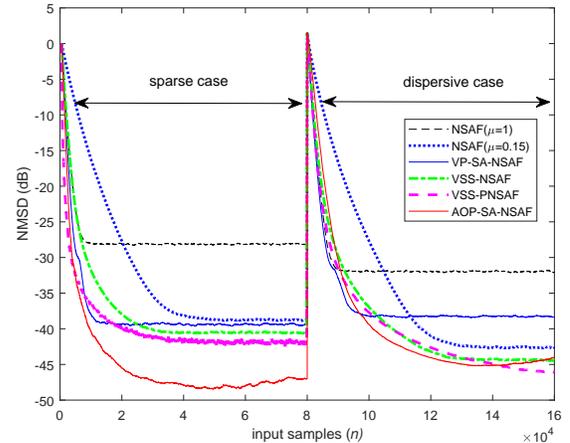


Fig. 2. NMSD performance of NSAF-type algorithms in the Gaussian noise. The parameters of algorithms are listed as follows: $\eta = 0.99$, $\lambda = 0.95$, and $\mu_{\max} = 1$ for VP-SA-NSAF; $\tau = 3$ for VSS-NSAF; $\tau = 5$ and $\zeta = 0$ for VSS-PNSAF; $\kappa = 6$ for AOP-SA-NSAF.

Example 2: $\alpha = 1.6$ displays the presence of impulsive noises. Fig. 3 depicts the NMSD performance of the NSAF,

M-NSAF [10], VSS-M-NSAF [10], VSS-M-PNSAF [10], VP-IWF-SSAF with RA [19], and the proposed AOP-SA-RNSAF algorithms³. For the M-estimate based algorithms, we choose the common M-estimate parameters $\lambda = 0.99$ and $N_w = 20$. It is seen that the NSAF algorithm shows poor misadjustment in the α -stable noise, and other algorithms exhibit robust convergence. Among these robust algorithms, the proposed AOP-SA-RNSAF algorithm is the best choice for identifying sparse systems, due to the fact that it has lower steady-state misadjustment than the VSS-M-PNSAF and VP-S-IWF-SSAF with RA algorithms, even if it has a slightly slower initial convergence than the VSS-M-PNSAF algorithm.

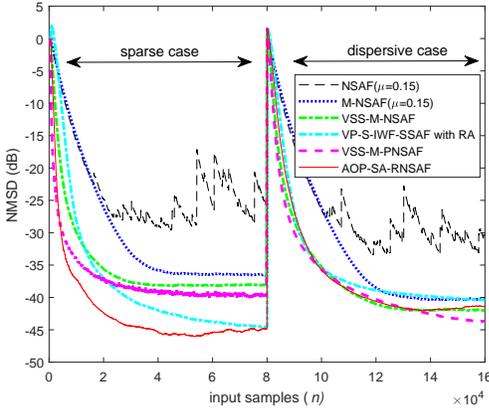


Fig. 3. NMSD performance of NSAF-type algorithms in the α -stable noise. The parameters of algorithms are listed as follows: $\mu_{\min} = 10^{-5}$, $\tau = 2$, and $\chi = 1$ for VP-S-IWF-SSAF; $\tau = 3$ for VSS-M-NSAF; $\tau = 5$ and $\zeta = 0$ for VSS-M-PNSAF; $\kappa = 6$ for AOP-SA-RNSAF.

It can be seen in Figs. 2 and 3 that, after w^o becomes dispersive at the middle of input samples, the proportionate-type (i.e., VSS-PNSAF, VSS-PNSAF) and sparsity-aware type (i.e., VP-SA-NSAF, AOP-SA-NSAF, AOP-SA-RNSAF) algorithms still show almost the same performance as the competing algorithms (i.e., VSS-NSAF and VSS-M-NSAF) in both Gaussian and α -stable noise scenarios. In addition, as α decreases from 2 to 1.6, the steady-state misadjustment of the proposed AOP-SA-RNSAF algorithm increases, but this algorithm is still convergent.

V. CONCLUSION

In this paper, a unified SA-RNSAF framework for algorithms was developed for identifying sparse systems in impulsive noise environments. By replacing directly the specified robustness criterion and sparsity-aware penalty, it can yield different SA-RNSAF algorithms. We then developed adaptive techniques for the step-size and the sparsity penalty weight in the SA-RNSAF algorithm, thus arriving at the AOP-SA-RNSAF algorithm with a further performance improvement in terms of the convergence rate and steady-state misadjustment. Simulations for the sparse system identification have demonstrated the effectiveness of the proposed algorithms.

³Since the variance of the α -stable noise is nonexistent, here we do not show the performance of the VP-SA-NSAF algorithm.

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