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Can the Stochastic Discount Factor Explain Unemployment Fluctuations?

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Can the Stochastic Discount Factor Explain Unemployment Fluctuations?

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Abstract

   Recent developments in Macro-labor show that discount rates may play an important role in unemployment fluctuations. This paper examines this hypothesis by using a standard search model of equilibrium unemployment with the canonical consumption-based stochastic discount factor. When the discount rate is inferred from data on real consumption in the U.S., little fluctuations in unemployment are generated from the model. Moreover, a counterfactual positive correlation between consumption growth and unemployment emerges from the model. This contradicts the post-war U.S. data. Those results hold even if the model contains habit formation in consumption and the wage is assumed to be invariant to discounts. The paper also studies the role of other factors in amplifying the impact of the discount rate shock, including endogenous job separation, variations in firms' profit per worker and the risk premium.

   Keywords: search frictions; discount rates; unemployment fluctuations
   (*JEL* E23, E24, E32, J24, J31, J41, J63)

   Recent developments in Macro-labor show that discount rates may play an important role in unemployment fluctuations. In a recession, risk premiums rise so discount rates rise. As a consequence, the expected present value of the flow of claims from a job match falls. This discourages job creation, therefore unemployment rises. This line of thought is promising as it not only has the potential to explain large unemployment fluctuations observed in the data but also shows how labor market dynamics are connected to the financial market.

   Modern macroeconomic theory ties discount rates to consumption. Thus, the intertemporal substitution motives lie at the heart of the consumption-based stochastic discount factor (henceforth, SDF). The (in)ability of such motives to explain risk premium and its volatility has been extensively discussed in the literature. It is less clear whether such motives are able to explain the variation in vacancy creation and the resulting unemployment fluctuations. This paper aims to answer that question. Specifically, the paper aims to study whether the fluctuations in discount rate inferred from U.S. consumption data can generate

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a sizable proportion of unemployment fluctuations, and whether the resultant unemployment dynamics can track the patterns of unemployment observed in the data and match other key empirical regularities, such as the persistence of high unemployment during recessions and the negative correlation of unemployment with consumption growth.

To answer those questions, the paper uses a standard Diamond-Mortensen-Pissarides search model of equilibrium unemployment (henceforth, DMP model). In the main part of the paper, the discount rate is the sole driving force of labor market dynamics. It affects hiring decisions through two channels: variations in the discount rate itself and the risk premium rising from the negative co-movement between the discount rate and returns from hiring a worker. Following the seminal work by Hall (2017), this paper largely focuses on the first channel [1]. Variations in the discount rate depend on the state of the economy. In this regard, I use the data on the U.S. real personal consumption expenditure per capita from 1947 to 2019 to infer the discount rates in the same period. By feeding the model actual data, the paper examines how closely the unemployment from the model can track the patterns of the actual U.S. unemployment rate. This approach meets an important criteria set by Hall (2017), that is, if the discount rate is a main force driving unemployment, the simulated unemployment driven by variations in discounts should match not only the volatility but also the patterns of the actual unemployment data.

Aside from the fact that the simulated unemployment loosely tracks the patterns of the actual U.S. unemployment rate, several key findings in the paper raise questions about the importance of discount rates in understanding unemployment fluctuations. First, the model generates a counterfactual positive correlation between consumption growth and unemployment. This is because, under the intertemporal substitution motive, economic agents discount the future flow of claims based on their expected consumption. Higher expected consumption growth means a larger decline in the expected marginal utility. This leads to a lower discount and therefore a lower present value of flow of claims from a job match. In other words, if times are good and expected consumption levels are high, any expected returns from hiring a worker are less desirable than those that are paid off when times are bad and additional consumption is more highly valued. As a consequence, when times are good, firms post fewer vacancies and unemployment rises. Thus, a positive correlation between consumption growth and unemployment emerges from the model. This contradicts the post-war U.S. data. The counterfactual positive correlation between the two variables raises doubts about the importance of discount rates in driving unemployment.

Second, despite the fact that the discount rates inferred from consumption data have similar volatility to the U.S. unemployment rate, the DMP model generates very small fluctuations in unemployment. This result holds, even if the wage is assumed to be invariant to discounts and labor market tightness. The

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[1]Literature often assumes that the risk premium rising from the negative co-movement between the discount rate and returns from hiring a worker is negligible. Section VI.F calculates the risk premium using data on after-tax corporate profits per worker and confirms this hypothesis.

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size of the resources available to the firm for vacancy creation, which is often seen as a key factor to the unemployment volatility, plays no role in generating unemployment fluctuations when the discount rate is the driving force. The wage elasticity with respect to discount rates, on the other hand, is an important factor to the amplification mechanism in the model. The paper considers two leading alternative wage formations: Nash bargaining and credible bargaining. The two bargaining protocols bring different levels of wage elasticity into the model. Under Nash bargaining, the discount rate affects the wage through labor market tightness. Under credible bargaining, the discount rate has a direct impact on the wage. In both cases, even a mild elasticity of the wage with respect to discounts can significantly dampen the amplification mechanism in the DMP model, creating a small response of unemployment to a change in discounts.

Third, the paper finds that adding habit formation in consumption into the DMP model is unable to improve the model's performance. Adding external habit into the utility function can significantly increase the volatility of discount rates. But it also reduces the persistence of discount rates. They pose opposing impacts on unemployment volatility with similar magnitude. As a consequence, the volatility of unemployment is hardly affected by habit formation. This result is in contrast to the previous results from macro-finance literature where habit formation is used to explain the volatility of price/dividend ratio in the stock market (e.g., Campbell and Cochrane, 1999). This discrepancy casts doubt on discount rates being a link between the financial and labor markets.

The baseline model follows a large strand of literature by posting a constant job separation rate. Thus, the discount rate only affects job creation and therefore the outflows from unemployment. One extension of the baseline model contains endogenous job separation in the style of Mortensen and Pissarides (1994). This opens a channel through which the discount rate affects firms' layoff decisions. A higher discount rate implies a lower present value of keeping a job match, thus the layoff rate would be higher. The paper shows this channel can have a non-trivial impact on layoff rate and inflows to unemployment, if keeping a job match requires a high reservation productivity. Nonetheless, the model's overall ability to generate unemployment fluctuations is still weak. With endogenous job separation and the wage invariant to discounts, the model can explain up to one third of observed unemployment fluctuations.

The rest of the paper is organized as follows. After reviewing some related research, section I lays out a standard DMP model. Section II calibrates the model. Section III feeds the model actual consumption data and shows how the discount rate affects unemployment dynamics. Section IV considers habit formation in consumption, and studies its impact on unemployment dynamics. Section V studies how the wage elasticity affects the amplification mechanism. Section VI shows how discounts affect job separations and inflows to unemployment. Section VII studies the role of other factors in generating unemployment fluctuations, including variations in profit flow from a job match and the risk premium rising from the co-movement between profit flow and discounts. Section VIII offers concluding remarks.

Related Research

This paper contributes to a growing body of literature that examines the role of the discount rate in driving labor market dynamics. An early contributor to this topic is Mukoyama (2009), who argues that discount rates can explain the observed unemployment volatility if there are extreme variations. Hall (2017) rationalizes extreme variations in discount rates by connecting discount rates to stock market volatility. In his paper, discount rates are artificially constructed in order that the model in his paper can loosely track both the vacancy/unemployment ratio and the price/dividend ratio. This paper is built upon Hall (2017) but differs from it in three key aspects. First, discount rates in this paper are inferred from the real consumption data. This paper shows that discount rates in Hall (2017), which connect the stock market and the labor market, are not supported by the real data. Discount rates inferred from real consumption data display much less persistence, especially when habit formation is considered. Moreover, low persistence of discounts is the key reason the DMP model fails to generate large unemployment fluctuations. Second, this paper incorporates habit formation in the DMP model and studies its implications for unemployment. Third, this paper studies the impact of discount rates on job firing and inflows to unemployment.

Martellini, Menzio and Visschers (2020) examine the hypothesis that unemployment fluctuations are driven by fluctuations in the discount rate in the context of Menzio and Shi (2011). Their key critique to this hypothesis is that a change in the discount rate leads to a counterfactual positive co-movement between outflows from unemployment and inflows to unemployment. In their model, an increase in the discount rate will lower the reservation quality of a job match, thus reducing inflows to unemployment. The main reason they obtain this result is that, in the spirit of Menzio and Shi (2011), workers decide at which point to quit from a job match. This assumption per se implies the reservation quality of a job match is defined to equate the value of unemployment and the value of a job match. If an increase in the discount rate leads to a larger decline in the value of unemployment than the value of a job match, workers are more reluctant to quit their jobs. This reduces inflows to unemployment. A key difference between their paper and this one is that this paper shows, when the layoff decisions are in the hands of the firms, an increase in the discount rate will increase the inflows to unemployment, even if it does not optimize workers' benefits. Thus, a change in the discount rate can generate a negative co-movement between the Ins and Outs of unemployment.

Kehoe, Lopez, Midrigan and Pastorino (2020) propose an important mechanism through which the discount rate may amplify the impact of a productivity shock on unemployment fluctuations. In a recession, when the economy is in a low state, the expected present value of claims from a job match falls. Due to the intertemporal substitution motive, the present value of a claim far into the future decreases more than the value of a claim in the near future. This channel, once interacting with human capital accumulation on the job, can have a strong impact on the present value of a job match, thus have a strong impact

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on vacancy creation and unemployment.

Fernandez-Villaverde, Mandelman, Yu and Zanetti (2020) explore the amplification mechanism rising from search complementarities between firms. The search effort exerted by one party will affect the other party's search effort. This inter-firm linkage generates multiple equilibria. Changes in fundamentals, which are subject to discount rate shocks or productivity shocks, move the economy between different equilibria, generating large and persistent business cycle fluctuations.

Borovicka and Borovickova (2018) study the risk premium rising from the co-movement between the discount rate and the return on hiring a worker, and its role in unemployment fluctuations. When the path of the discount rate is inferred from the financial market data, they assert that the risk premium only plays a limited role in unemployment fluctuations. Their key finding is to provide a non-parametric bound on two moments of firms' profit flows which any profit flow must satisfy in order to meet the optimal hiring decision and to generate large unemployment fluctuations. Empirical estimates of the profit flow do not meet the bound. The potential causes of the discrepancy between those estimates and the theoretical object are discussed in their paper. One important difference between their paper and this one is that their paper focuses on the variations in the risk premium. By contrast, this paper focuses on the variations in the discount rate and the implications for unemployment.

Hall (2017) surveys other recent developments on this topic.

I. The Model

The theoretical framework this paper builds upon is a standard DMP model of equilibrium unemployment. The labor market is characterised by search frictions. Aggregate hiring is determined by the matching function

$$h_t = m u_t^\alpha v_t^{1-\alpha} \tag{1}$$

The matching function shows hiring (h_t) is determined by the number of vacancies (v_t) and the number of unemployed workers who are actively searching for jobs (u_t). α is the matching elasticity with respect to unemployment. Using the matching function, the vacancy filling rate q_t can be defined as $q_t = h_t/v_t = m\theta_t^{-\alpha}$, where θ_t, the ratio of vacancies to unemployment, measures the tightness of the labor market.

Firms recruit unemployed workers by posting a vacancy at the flow cost of γ. The value of an unfilled job vacancy is

$$V_t = -\gamma + \frac{1}{1+r_t} E_t[q(\theta_t)J_{t+1} + (1 - q(\theta_t))V_{t+1}]. \tag{2}$$

With the probability $q(\theta_t)$, a vacancy turns into a productive job match J_t. The firm's value of a job match is defined as

$$J_t = y_t - w_t + \frac{1}{1+r_t} E_t[(1-\tau)J_{t+1} + \tau V_{t+1}] \tag{3}$$

Once a job match is established, the employed worker produces y_t units of output and receives a wage payment w_t. The job match dissolves at the end of period with exogenous probability τ [2]. The driving force of the model, the SDF, is defined as

$$\frac{1}{1+r_t} = E_t \beta \frac{U'(C_{t+1})}{U'(C_t)}. \tag{4}$$

The utility function takes the form $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, where σ measures constant relative risk-aversion.

Households supply a measure one of infinitely lived workers. Each worker is either in a job match or actively searching for a job. The value function for a job-seeker is

$$U_t = z + \frac{1}{1+r_t} E_t[f(\theta_t)L_{t+1} + (1 - f(\theta_t))U_{t+1}]. \tag{5}$$

During the job search, a job-seeker receives a flow value z per period. $f(\theta_t)$ measures the probability of finding a job, defined as $f_t = h_t/u_t = m\theta_t^{1-\alpha}$. Once employed, a worker receives a value L_t from a job match. L_t is defined as

$$L_t = w_t + \frac{1}{1+r_t} E_t[(1-\tau)L_{t+1} + \tau U_{t+1}]. \tag{6}$$

I assume that the total labor force is normalized to unity and inelastic. So the employment can be written as $n_t = 1 - u_t$. The law of motion for employment is

$$n_{t+1} = (1-\tau)n_t + h(u_t, v_t). \tag{7}$$

The model assumes free entry to the labor market for the firms, so $V_t = 0$. Thus the vacancy cost equals the firm's share of a match surplus, $\gamma = \frac{1}{1+r_t} E_t q(\theta_t) J_{t+1}$. Substituting this into (3) yields the job creation condition

$$y_t = w_t + \gamma[(1+r_t)\frac{1}{q(\theta_t)} - (1-\tau)E_t\frac{1}{q(\theta_{t+1})}]. \tag{8}$$

II. Specification and Parameters

The baseline model assumes that the wage is rigid, in the sense that the wage is invariant to discount rates and labor market tightness. As shown in section V, this assumption maximizes the amplification mechanism in the standard DMP model when the discount rate is the driving force of unemployment. It further assumes that the wage responds one-to-one to any variation in labor productivity, i.e. $y_t - w_t \equiv \pi$. Thus, the productivity shock plays no part in driving unemployment dynamics. Although those wage specifications are ad hoc, it is easy to prove that any wage contract that satisfies those two specifications lies in the bargaining set once π is properly calibrated. Those ad hoc specifications are

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[2]In section VI, I extend the baseline model to incorporate endogenous job separation.

adopted for two reasons. First, they allow a focus on the implication of discounts for the dynamics of unemployment. Second, the model is able to estimate the upper limit of this implication. In section VI, the paper considers two leading alternative wage formations, i.e. Nash bargaining and credible bargaining, and studies how the amplification mechanism is muted under those wage regimes. In section VII, the paper examines how the variations in the firm's profit flow, due to other exogenous shocks, interact with the variations in discounts and propagate unemployment fluctuations.

In parameter calibration, where possible, I follow earlier studies. A time period is normalized to be one month. The calibrated parameter values are outlined in Table 1. For the matching function, I set $\alpha = 0.5$; this lies in the plausible range of estimates surveyed by Petrongolo and Pissarides (2001). The discount factor is set as $\beta = 0.996$, so the annual discount rate is 5%. The average monthly job separation rate is $\tau = 0.0345$, a value taken from Hall (2017). For the coefficient of relative risk aversion, I follow most studies and set $\delta = 2$.

Table 1: Calibrated Parameter Values

Parameter	Definition	Value
α	Matching Elasticity w.r.t Unemployment	0.5
β	Discount Factor	0.996
m	Matching Efficiency	1
τ	Job Separation Rate	0.0345
γ	Flow Cost of Posting a Vacancy	0.537
δ	Relative Risk Aversion	2
π	Firm's Profits	0.015

The firm's profit is set at $\pi = 0.015$. Unlike a large strand of literature in which the productivity shock drives unemployment, the size of the firm's profit is irrelevant to the quantitative results in this paper. This is proved in section VI. The only criterion required for this calibration is to verify that the wage lies within the bargaining set. The lower and the upper bounds of the bargaining set are the worker's and firm's reservation wage, denoted as w^l and w^h respectively.

The worker's reservation wage, w^l, equates the unemployment value, U_t, to the employment value, L_t, so

$$w^l = z + \frac{1}{1 + r_t} E_t[f(\theta_t) + \tau - 1](L_{t+1} - U_{t+1}) \tag{9}$$

The firm's reservation wage must satisfy $J_t = 0$, so

$$w^h = y_t + \frac{1}{1 + r_t} E_t[(1 - \tau)J_{t+1} + \tau V_{t+1}] \tag{10}$$

As pointed out by Hall (2005), any wage in the bargaining set $[w^l, w^h]$ will result in the formation of a job match, as both worker and firm will benefit from the match. Under my calibration, $f(\theta_t) + \tau - 1 \leq 0$ and $L_t - U_t \geq 0$, so (9) implies $w^l \leq z$. Chodorow-Reich and Karabarbounis (2016) present some evidence on the value of the opportunity cost of employment z. The upper bound of their estimate is 0.96. So, for any reasonable value of z, the lower end of the bargaining

set will not rise above the wage $w = 0.985$. (10) implies $w^h \geq y$. As $w < y$ in my calibration, the wage also lies below the upper end of the bargaining set.

The flow cost of posting a vacancy γ is calibrated to match the average unemployment rate between 1947 and 2019, $u = 0.057$. This gives $\gamma = 0.537$; this value lies in the range of calibrated values of γ in the literature [3].

III. Results

This section examines the stochastic discount factor when it is fed actual consumption data and studies the implications of discount movements for unemployment dynamics. Using U.S. real personal consumption expenditure per capita from 1947 to 2019, the first step is to calculate the discount factor. Low consumption in a current state leads to high marginal utility, so the discount factor is below β and the future claims are heavily discounted. When the economy oscillates between a series of low consumption and high consumption states, discount factors fluctuate around β. Figure 1 shows the SDF inferred from the U.S. consumption data.
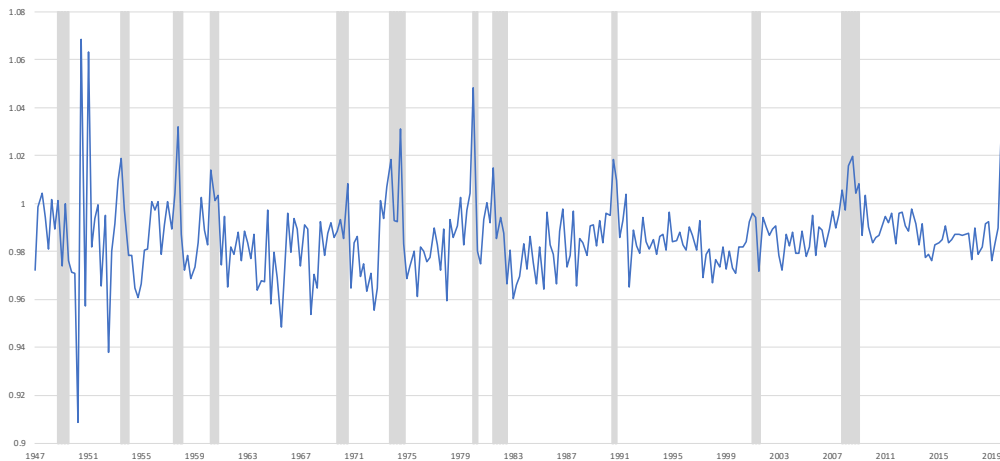

Figure 1: The SDF inferred from the U.S. Consumption Data

Three observations from Figure 1 are worth noting. First, the discount factor becomes more volatile during the recessions. This is not surprising since recessions are often associated with large fluctuations in consumption. Second, in 9 out of 11 recessions between 1947 and 2019, discount factors are above β, meaning future payoffs tend to be more highly valued. In those recessions, households experience dramatic declines in consumption. Thus, any payoff that provides additional consumption in the periods with low expected consumption is more desirable. Third, a related observation is that falls in consumption in those recessions are associated with a sharp rise in the discount factor. Falls in consumption reduce expected consumption. This pushes up the discount factor.

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[3]The value of γ is contentious. Shimer (2005) uses a quarterly vacancy cost of 0.213. Hagedorn and Manovskii (2008) use a weekly vacancy cost of 0.584. Hall (2005) assumes a monthly cost of 0.986 while Pissarides (2009) assumes 0.356.

The discount factor reaches its peak when the expected consumption decreases to the lowest level. Once the economy is in a state that consumption is at its local minimum, the discount factor starts to fall because expected consumption starts to rise.
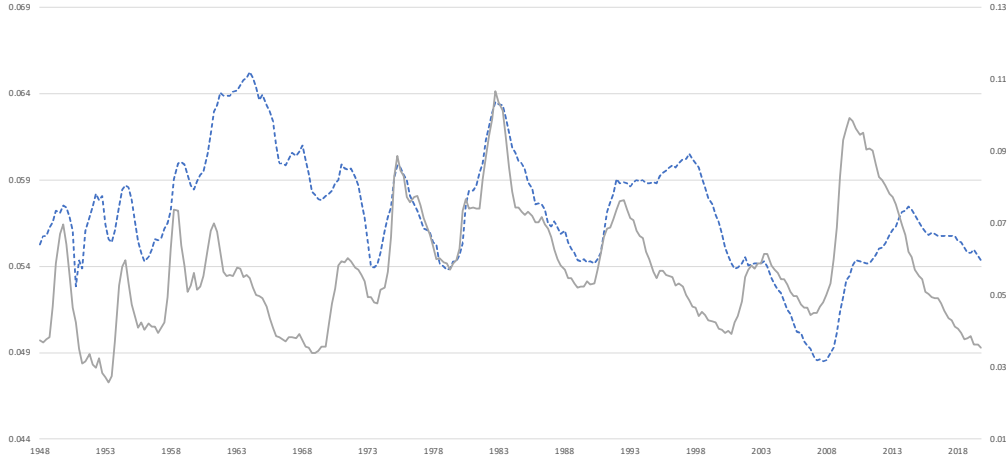

Figure 2: The Unemployment Rate from the Model (Blue Line, Left Scale) and The Actual U.S. Unemployment Rate (Grey Line, Right Scale)

Next, I use the values of the discount factor inferred from consumption to calculate the implied labor market tightness, job finding rate and unemployment rate. Figure 4 shows the model's prediction for the unemployment rate, along with the actual U.S. unemployment rate. The unemployment rate from the model displays patterns similar to the actual unemployment rate. The model is able to account for some of the cyclical and long-term fluctuations in unemployment. In particular, it tracks the booms and recessions between 1960s and 1990s reasonably well. However, the model is unable to match the magnitude of those fluctuations. The standard deviation of unemployment from the model is just above one fifth of its empirical counterpart. Moreover, the model fails to account for the negative correlation between consumption growth and the unemployment rate. Table 2 reports those statistics.

Table 2: Standard Deviations and Correlations of Simulated and Historical Data

	Cons Growth	SDF	Unrate	Actual Unrate
Standard Deviation (%)	1.980	1.609	0.339	1.641

	Corr (Cons Growth, Unrate)	Corr (Cons Growth, Actual Unrate)
1948-2019	0.254	-0.230
1959-1989	0.169	-0.190
1989-2019	0.333	-0.563

A. Explaining the Mechanism

Despite the counterfactual positive correlation between consumption growth and unemployment, the unemployment rate from the model shares similar patterns

with the actual U.S. unemployment rate. This subsection explains the transmission mechanism behind this success. In short, the good match with the data is due to the time lag between the movement of consumption and unemployment observed in the data.

In the data, consumption growth moves ahead of unemployment in a couple of quarters. Consumption growth normally starts to decrease several quarters before the recession and bounces back to positive soon after the recession ends (see Figure 3) [4]. Therefore, consumption growth reaches its lowest rate either during or just after a recession. This echoes the observation that the discount factor reaches its peak in the middle of a recession and experiences a subsequent decline before the recession ends. The recovery of consumption, which results in a decline in the discount factor, leads to a rise in unemployment in the model.
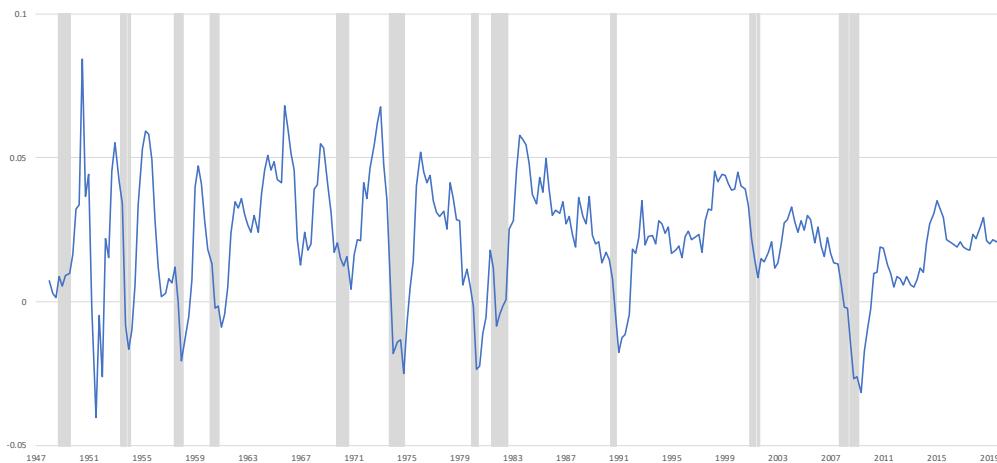

Figure 3: The Annualized Real Consumer Consumption Growth, Quarterly U.S. Data

Also in the data, due to the time lag, the actual unemployment rate normally reaches its peak when consumption growth has already bounced back to the pre-recession level or is about to bounce back (see Figure 4). This explains why the actual unemployment rate and the unemployment rate from the model appear to reach their peak at about the same time. This coincidence does not justify the causal chain that links consumption growth and unemployment in the model. An increase in unemployment from the model is driven by the increasing consumption growth, whereas, in the data, an increase in unemployment is associated with the decrease in the consumption growth in the past [5].

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[4] A quick recovery of consumption growth does not mean a quick recovery of consumption. Consumption still takes years to move back to its long-run trend. But in the stochastic discount factor, what matters is the growth rate of consumption.

[5] For the post-war U.S. data 1948-2019, the contemporaneous correlation between consumption growth and unemployment is -0.23. The correlation between consumption growth and unemployment with two quarters' lag, i.e. corr (c_{t-2}, u_t), is -0.41. For the last three decades, those two correlations are -0.56 and -0.72 respectively.
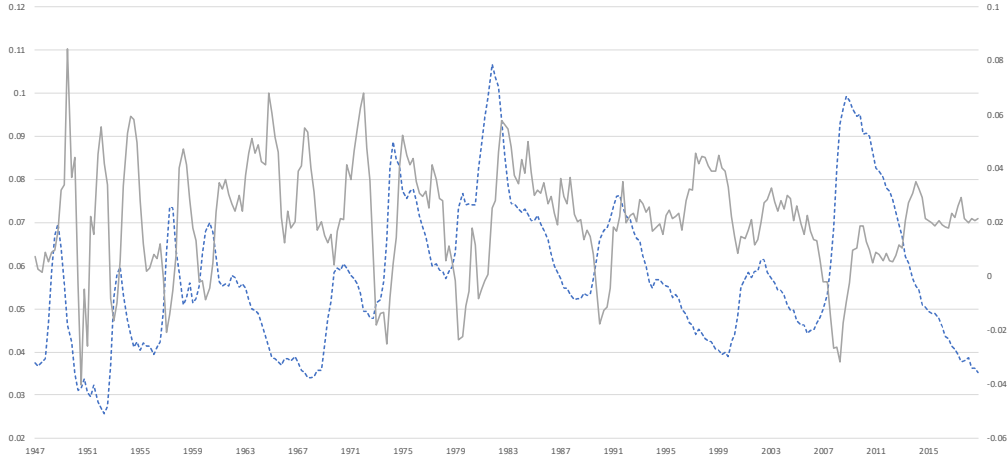
Figure 4: The U.S. Unemployment Rate (Blue Line, Left Scale) and The Annualized Consumption Growth (Grey Line, Right Scale), Quarterly U.S. Data

B. Unemployment Volatility

Table 2 shows the discount factor is almost as volatile as the actual unemployment rate, yet little volatility of unemployment is generated in the model. To explain this result, I examine the elasticity of unemployment with respect to the discount rate. Using the steady-state version of eq.(1), (2), and (4), this elasticity can be written as

$$\eta_{u,r} = -\frac{1-\alpha}{\alpha}\frac{r}{\tau+r}\frac{f(\theta)}{\tau+f(\theta)}. \tag{11}$$

To account for the observed unemployment volatility, $\eta_{u,r}$ needs to be -1. The estimated value of the matching elasticity with respect to unemployment normally lies in the range 0.5-0.7, so the first factor $\frac{1-\alpha}{\alpha}$ is bounded by 1. The third factor $\frac{f(\theta)}{\tau+f(\theta)}$ is also bounded by 1. Thus, the second factor $\frac{r}{\tau+r}$ needs to be no less than 1. Given that the job separation rate is positive, this implies the discount rate needs to be close to infinite. This is clearly not plausible. For any reasonable value of the discount rate, the second factor would be much smaller than 1, causing the elasticity $\eta_{u,r}$ to be much smaller than 1.

The intuition behind the small elasticity $\eta_{u,r}$ can be illustrated by free-entry condition. The free-entry condition, $V = 0$, implies $\gamma = E_t \frac{1}{1+r_t}q(\theta_t)J_{t+1}$. Suppose that the discount factor decreases by 1%, the present value of a job match $\frac{1}{1+r_t}J_{t+1}$ will decrease by approximately 1%. To restore the equilibrium, the vacancy filling rate needs to increase by roughly 1%, so the expected return from posting a vacancy still equals the vacancy cost. This means firms will post fewer vacancies. But because, under any reasonable calibration, the vacancy filling rate is quite sensitive to labor market tightness, the decrease in labor market tightness would be smaller than 1%. The increase in unemployment would be even smaller.

One way to generate large unemployment volatility is to increase the volatility of the discount factor. This can be done by increasing the risk aversion

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coefficient σ. Under my calibration, the power utility function needs a risk aversion coefficient $\sigma \geq 13$. Campbell and Cochrane (1999) point out two issues with assuming a high risk aversion coefficient. One is that it implies an implausiblely high risk-free interest rate. [6] With $\sigma = 13$, the second issue is that it implies the risk-free interest rate varies across time by 13 times the variation in consumption growth. This is against the empirical evidence, e.g. Campbell (1999). Moreover, by increasing the risk aversion coefficient, the model still fails to account for the negative correlation between the consumption growth and unemployment.

C. The Correlation of Consumption Growth with Unemployment

Table 2 shows the consumption based SDF implies a counterfactual positive correlation between the consumption growth and unemployment. This is in sharp contrast with the actual data. The U.S. data in Table 2 shows this correlation is negative between 1948 and 2019 and has become stronger in the last three decades. Thus, if the discount rate plays a key role in unemployment fluctuations, the model should predict a negative correlation. To explore why a counterfactual positive correlation appears in the model, it is necessary to calculate the correlations of the discount factor with the consumption growth and unemployment respectively. The results are reported in Table 3.

Table 3: Correlations of Simulated Data from the Model

	Corr (SDF, Cons Growth)	Corr (SDF, Unemployment Rate)
1948-2019	-0.173	-0.345
1959-1989	-0.277	-0.366
1989-2019	-0.444	-0.408

Although there is no direct data to test the correlation between the discount factor and unemployment, what is in line with the expectation is that the model predicts a negative correlation between the two variables. This negative correlation varies little across different periods, showing a stable relationship between the discount and unemployment implied by the DMP model.

Throughout the whole sample period, the model also predicts a negative correlation between the discount factor and consumption growth. This negative correlation accounts for the model predicting a counterfactual positive correlation between consumption growth and unemployment. The economic agent discounts the flows of future claims based on intertemporal substitution motives. High expected consumption means low expected marginal utility. Therefore, the discount factor is small, meaning the flow of claims will be more heavily

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[6]With the power utility function $\beta(C_{t+1}/C_t)^{-\sigma}$, the log risk-free interest rate is written as

$$r_t^f = -ln(\beta) + \sigma g - \sigma^2 \frac{\omega^2}{2} \tag{12}$$

where g and ω^2 are the mean and variance of the consumption growth. Data on U.S. real personal consumption per capita between 1947 and 2019 suggests the annual growth rate of consumption $g = 2.13$ percent and $\omega = 1.98$ percent. With $\beta = 0.96$ and $\sigma = 13$, the annual risk-free interest rate is 28.38 percent.

discounted. This logic is perfectly fine when it is applied to asset pricing. However, it becomes problematic when it is applied to firms' other decisions, such as hiring. When the expected consumption is high, any returns from hiring a worker are less desirable and therefore are more heavily discounted than those returns that are received when the consumption is low. So, higher expected consumption growth leads the firm to post fewer vacancies. This results in higher unemployment and a positive correlation between the consumption growth and unemployment.

The implication of the counterfactual positive correlation between consumption growth and unemployment is twofold. If one believes the canonical consumption-based stochastic discount factor is the correct way to model the discount for the firm, then the results in this section, and also in the following sections, indicate that the discount rate might not play an important role in unemployment fluctuations. Alternatively, one might use this counterfactual positive correlation to argue the unfitness of the consumption-based stochastic discount factor for understanding firms' behavior. Any argument along this line of thought will ultimately spark a debate on how discounts should be modeled in macroeconomic models.

D. Comparison with Hall (2017)

The two key findings in this section, i.e., small unemployment fluctuations in response to variations in discounts, and the model-predicted counterfactual positive correlation between the consumption growth and unemployment, challenge the view in Hall (2017) that variations in discounts play an important role in unemployment fluctuations. The modeling environment in this paper is in line with Hall (2017) and shares the same mechanism. It differs from Hall (2017) in two aspects. First, this paper so far assumes the wage is completely rigid, in the sense that the wage is invariant to the discount rate and labor market tightness. By contrast, the wage in Hall (2017) resulting from credible bargaining displays a milder stickiness. This difference does not undermine the first key finding in this section. Rather, it strengthens the finding because the interaction between variations in discounts and wage rigidity is key to the amplification mechanism. Section V discusses this in more detail.

Second, fluctuations in discounts in this paper are inferred from the data on U.S. real personal consumption, as modern macroeconomic theory ties discount rates to consumption. By contrast, discount rates in Hall (2017) are constructed based on five artificially identified states so that the model in his paper can loosely track the vacancy/unemployment ratio and price/dividend ratio observed in the U.S. data. This results in a stark difference in the behavior of discount rates. The discount rate in Hall (2017) displays high volatility and implausiblely high persistence. The standard deviation of discounts in his paper is 0.048, versus 0.016 in this section. The autocorrelation of discounts in his paper is 0.89, versus 0.08 in this section. The high volatility of discounts in Hall (2017) can be rationalized by incorporating habit formation into the model, as shown in the next section. However, the high persistence of discounts in his paper is

not supported by the consumption data. Further evidence includes Borovicka and Borovickova (2018) who construct the paths of the SDF using both macroeconomic and financial data. Their Figure 2 (top panel, p. 16) is quite similar to Figure 1 in this paper. The two approaches agree on the low persistence of discounts and high volatility during NBER recessions.

IV Results with Habit Formation

Habit formation has been adopted in macro-finance as a possible solution to the equity premium puzzle. Its success lies in the fact that adding habit formation can increase the curvature of the utility function. Thus, small movements in consumption lead to large variations in discounts, even if economic agents are low risk averse. In this section, I incorporate habit formation into the standard DMP model in section I and study the implication of consumption habit for unemployment dynamics.

With habit formation and constant relative risk aversion, the utility function is now written as

$$U(C_t, h_t) = \frac{(C_t - \varphi C_{t-1})^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0, \varphi > 0. \tag{13}$$

The parameter φ measures the intensity of habit formation and introduces non-separability of preferences over time. By introducing habit formation, the marginal utility of consumption becomes more sensitive to a change in consumption. To see this point, the elasticity of the marginal utility with respect to consumption is derived:

$$\eta_{U'_t, C_t} = \frac{dU'_t}{dC_t} \frac{C_t}{U'_t} = -\sigma \frac{C_t}{C_t - \varphi C_{t-1}}. \tag{14}$$

Note that the same elasticity when the habit is not present is simply $-\sigma$. The difference between the two elasticities can be significant when the economy is in a bad state as, in this case, $C_t - \varphi C_{t-1}$ can be very small.

Adding habit formation into the model does not affect the steady state values of unemployment and vacancies. Thus, the calibration of the existing parameters is the same as in Table 1. The only new parameter is the intensity of consumption habit, φ. Havranek, Rusnak and Sokolova (2017) recently studied 597 estimates of habit formation, reported in 81 published papers. According to their study, the mean value of φ using U.S. macro data is close to 0.6. Figure 5 shows the simulated unemployment rate when φ is 0.6, together with the actual U.S. unemployment rate. Some key statistics from the simulations are reported in Table 4.

Under habit formation, the standard deviation of the discount factor is almost three times as big as its counterpart from the model without habit formation; see Table 4. This is consistent with what (14) predicts. The average consumption surplus ratio, $C_t/(C_t - \varphi C_{t-1})$, when $\varphi = 0.6$, is 2.48. Thus, the marginal utility becomes much more sensitive to a change in consumption. This leads to larger variations in discounts.
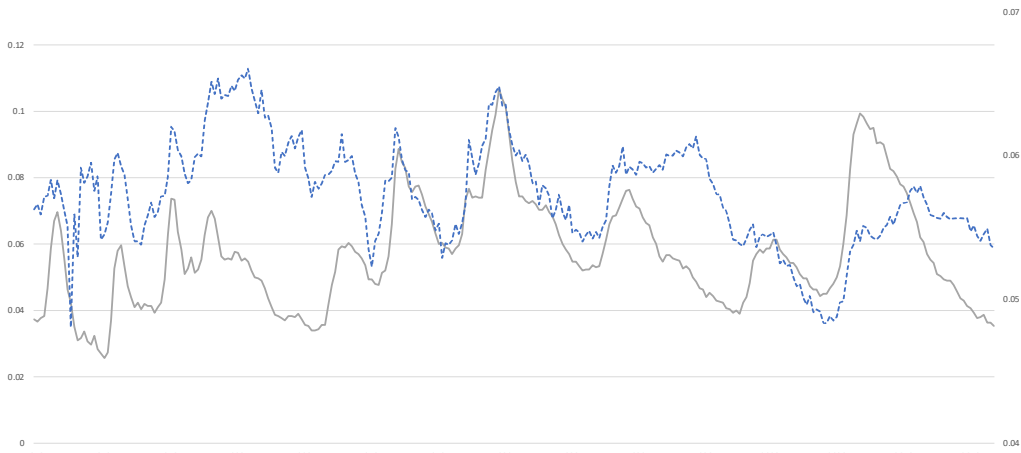
Figure 5: Unemployment Rate from the Model (Blue Line) with Habit Formation ($\varphi = 0.6$)

However, the high volatility of discounts is not transmitted to the volatility of unemployment. The standard deviation of the unemployment rate from the model is still far below its empirical counterpart, and is only marginally bigger than its counterpart from the model without habit formation. The failure to generate higher unemployment volatility is likely due to the fact that adding habit formation reduces the persistence of discount rates. Table 4 shows that the autocorrelation of discount factors turns negative under habit formation. This is in sharp contrast with the large positive autocorrelation of unemployment. The low persistence of discounts dampens the amplification mechanism, and therefore offsets the impact of larger variations in discounts on unemployment.

In terms of matching the patterns of the U.S. unemployment rate, adding habit formation into the model does not improve the model's performance. The patterns of the simulated unemployment rate in Figure 5 loosely track the business cycle fluctuations in the U.S. unemployment rate, similar to the simulated unemployment rate in Figure 2. The model still generates a counterfactual positive correlation between the consumption growth and unemployment, albeit by a smaller magnitude.

Table 4: Key Statistics of Simulated and Historical Data

Standard Deviation (%)	Without Habit Formation	With Habit Formation	U.S. Data
The SDF	1.610	4.617	—
Unemployment Rate	0.339	0.363	1.641
Qtly Autocorr of SDF	0.080	-0.501	—
Qtly Autocorr of Unrate	0.975	0.934	0.967
Corr (Unrate, Cons)	0.254	0.055	-0.230

To summarise, this section studies the implications of habit formation for unemployment dynamics. Under the standard calibration of structural parameters of the labor market, and the reasonable calibrated value for φ, the model with habit formation is unable to generate observed unemployment fluctuations. Under habit formation, discounts inferred from U.S. consumption data are more

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volatile but less persistent. Their opposing impacts on unemployment almost cancel each other out, leading to only a mild increase in the volatility of unemployment.

V. Wage Flexibility

Wage flexibility has been regarded as one of the key factors affecting labor market volatility. In the baseline model (with and without habit formation), the wage is completely rigid in the sense that it is invariant to discounts and labor market tightness. In this section, I consider two leading alternative wage formations, i.e., Nash bargaining and credible bargaining, and study their impact on unemployment volatility when the discount rate is the driving force. I show that, under reasonable calibrations, both bargaining protocols significantly reduce the amplification mechanism in the model, resulting in even smaller elasticity of unemployment with respect to discounts.

To proceed, I use comparative steady state analysis to discuss how different wage formations affect labor market volatility. The elasticity of unemployment with respect to discounts is used to measure the labor market volatility implied by each model. The unemployment elasticity is determined by the firms' free-entry condition and the wage-setting arrangement. It is not affected by the way of modeling discounts. Thus, the model with and without habit formation has the same unemployment elasticity. The focus here is how wage flexibility affects the unemployment elasticity.

Under *Nash bargaining*, the wage is set to maximise $(L_t - U_t)^\phi J_t^{1-\phi}$, where ϕ is the worker's relative bargaining power. The steady state version of the resultant wage is

$$w = (1 - \phi)z + \phi(y + \theta c). \tag{15}$$

Under *credible bargaining*, a firm and a worker take turns making wage offers. In each bargaining round, each party either accepts the counterparty's offer or rejects and proposes a counteroffer in the next bargaining round. After a delay, the firm incurs a cost of delay $\zeta > 0$ while the worker enjoys the value of leisure z. For simplicity, I follow Ljungqvist and Sargent (2017) by assuming that the probability that job terminates is the same before and after the wage bargain. The steady state version of the wage is [7]

$$w = \frac{1}{2 + r - \tau}[(1 + r)z + (1 - \tau)(y + \zeta)]. \tag{16}$$

Table 5 reports the elasticity of unemployment with respect to discounts under each bargaining protocol. Under both bargaining regimes, the wage is responsive to a change in discount. Under Nash bargaining, the discount rate affects the wage through labor market tightness. Consider an increase in the discount rate. This decreases the value of a job match and therefore discourages

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[7]See pp. 2647-2648 in Ljungqvist and Sargent (2017) for the details of the derivation for the wage equation.

the posting of new vacancies. The wage will decrease due to a slacker labor market. The lower wage partly reverses the negative impact of a higher discount rate on job creation. Under credible bargaining, the discount rate has a direct impact on the wage. A higher discount rate lowers the wage due to a lower present value of future productivities. By assuming the probability that a job terminates is the same before and after the wage bargain, the wage is unaffected by labor market tightness. This leads to a milder wage flexibility, compared to Nash bargaining. Despite this, the quantitative analysis below shows that, under both bargaining protocols, the elasticity of unemployment with respect to discounts is much smaller than when the wage is completely rigid.

Table 5: Elasticity of Unemployment w.r.t the Discount Rate I

	Elasticity of Unemployment w.r.t. the Discount Rate
Sticky Wage	$\frac{(1-\alpha)r}{\alpha(\tau+r)}\frac{f(\theta)}{\tau+f(\theta)}$
Nash Bargaining	$\frac{(1-\alpha)r}{\alpha(\tau+r)+\phi\theta q(\theta)}\frac{f(\theta)}{\tau+f(\theta)}$
Credible Bargaining	$\frac{(1-\alpha)r}{\alpha(\tau+r)}\frac{2(1+r)\zeta-q(\theta)(y-z)}{(2+r-\tau)\zeta}\frac{f(\theta)}{\tau+f(\theta)}$

To measure the implication of wage elasticity for unemployment volatility, I carry out a simple exercise. I calibrate three new parameters, ϕ (the worker's bargaining power), z (the opportunity cost of employment), and ζ (the cost to the firm of delay), to let the wages across different bargaining regimes have the same steady-state value as in the baseline model (with sticky wage). Thus, the only difference between the three models is the wage elasticity. Then I compare the unemployment elasticity implied by those models.

The existing parameters are calibrated as in section I. The cost to the firm of delaying bargaining is assumed to be equal to the cost of maintaining vacancy, so $\zeta = 0.537$. Given the values of the existing parameters set in section I and the value of ζ, to let the steady-state value of the wage under credible bargaining meet the target $w = 0.985$, the opportunity cost of employment z needs to be 0.454. The assigned value of ζ and z are close to Hall (2017). Given the value for z and the steady-state labor market tightness, I set $\phi = 0.696$, so the wage under Nash bargaining can meet the target $w = 0.985$. With those calibrations, I calculate the elasticity of unemployment with respect to the discount rate under each wage formation; results are reported in Table 6.

Table 6: Elasticity of Unemployment w.r.t the Discount Rate II

Parameter	Value
The Opportunity Cost of Employment	0.454
The Cost to the Firm of Delay	0.537
The Worker's Bargaining Power	0.696
Wage Formation	Elasticity of Unemployment w.r.t the Discount Rate
Sticky Wage	0.201
Nash Bargaining	0.010
Credible Bargaining	0.043

In the baseline model, where the wage is invariant to the discount rate, the

elasticity of unemployment with respect to the discount rate is 0.201. [8] Under both bargaining protocols, the elasticities of unemployment are much smaller. With the worker's bargaining power set as $\phi = 0.696$ under Nash bargaining, a change in discount hardly has any impact on unemployment.

One might argue that the calibrated value of the worker's bargaining power is too high and therefore the wage is too sensitive to labour market conditions. Indeed, this sensitivity plays a key role in dampening the amplification mechanism. Note that, even if the worker's bargaining power is set as $\phi = 0.052$, following Hagedorn and Mankovskii (2008), in which case I set $z = 0.972$ in order to meet the target $w = 0.985$, the elasticity of unemployment is 0.08, still much smaller than 0.201 in the baseline model.

VI. Endogenous Job Separation

The baseline model follows a large strand of literature by posting a constant job separation rate. Thus, the job firings, and the inflows to unemployment, are immune from changes in discounts. In this section, I consider a DMP model with endogenous job separation and use it to examine how the inflows to unemployment are affected by the discount rate.

To introduce endogenous job separation, two changes are made to the baseline model. First, the output of a job match is changed to xy. As in the rest of the paper, y denotes the aggregate labor productivity. The new variable x is the idiosyncratic productivity. Following Mortensen and Pissarides (1994), x is hit by an idiosyncratic productivity shock at Poisson rate s. When an idiosyncratic shock arrives, x moves from its current value to some new value x', drawing from a general distribution $G(x)$ with support in the range $0 \leq x \leq 1$. Any newly established job match has a value of x equal to 1.

Second, the job separation is no longer an exogenous process. The firm chooses a reservation productivity R, defined as the productivity level below which keeping a job match is no longer profitable. Whenever $x < R$, the firm will choose to terminate the job match. Thus, the job separation rate now becomes $sG(R)$.

With those two changes, the job creation condition is now written as

$$\frac{1-R}{r+s} = \frac{\gamma}{q(\theta)}. \tag{17}$$

This modified job creation condition shows that the expected gain from a new job match is equal to the expected hiring costs that the firm has to pay.

The job separation is governed by the following condition:

$$R - \frac{w}{y} + \frac{s}{r+s} \int_R^1 (\chi - R)dG(\chi) = 0. \tag{18}$$

This condition implies that the reservation productivity is the productivity level at which the benefit of keeping a job match $(Ry + \frac{s}{r+s}y \int_R^1 (\chi - R)dG(\chi))$ is

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equal to the wage (w). It also implies the reservation productivity R increases with the discount rate. A higher discount rate leads to a lower option value of a job match (which is measured by the integral expression in (18)) [9]. This requires a higher reservation productivity. Thus, a higher discount rate leads to larger inflows to unemployment.

Using (17), (18) and the Beveridge curve, I derive the elasticity of unemployment with respect to the discount rate. The result is reported in Table 7, together with the same elasticity when the job separation is exogenous.

Table 7: Elasticity of Unemployment w.r.t the Discount Rate III

Job Separation	Elasticity of Unemployment w.r.t the Discount Rate
Exogenous Job Separation	$\frac{(1-\alpha)r}{\alpha(\tau+r)}\frac{f(\theta)}{\tau+f(\theta)}$
Endogenous Job Separation	$\{\frac{(1-\alpha)r}{\alpha(s+r)} + \frac{r(w/y-R)}{\alpha(1-R)[sG(R)+r]}\}\frac{f(\theta)}{sG(R)+f(\theta)}$

Notes: In both cases, I assume that the wage is invariant to the labor market tightness.

Table 8: Elasticity of Unemployment w.r.t the Discount Rate IV

Job Separation	Elasticity of Unemployment w.r.t the Discount Rate
Exogenous Job Separation	0.201
Endogenous Job Separation	0.210 - 0.405

To facilitate the comparison, I assume $\tau = sG(R)$, meaning that the two models have the same steady state job separation rate. Endogenizing job separation can potentially increase the response of unemployment to a change in discount. For example, assuming that $s = 1$ and $G(R) = 0.0345$, every month a firm would receive an idiosyncratic productivity shock; the probability that a job match becomes unprofitable after the shock is 3.45%. Given those assumptions, and the calibration of existing parameters in section I, the estimated elasticity of unemployment lies in the range of 0.210 to 0.405, depending on the value of the reservation productivity [10]. The median of this estimate, the case when $R = 0.94$, indicates that adding endogenous job separation into the model can increase unemployment volatility by approximately 50 percent. So, the model with endogenous job separation can explain roughly 30% of the observed unemployment fluctuations. The increase in the unemployment volatility would be smaller if the wage responded to a change in discounts. Nonetheless, the DMP model with endogenous job separation shows a change in the discount rate can have a non-negligible impact on job firing and the inflows to unemployment if the reservation productivity is high. This implies that discount rates might play an important role in a recession.

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[9]Because of the possibility that a job productivity might change, the option value captures the expected gain by keeping a job match.

[10]It is reasonable to assume that the mean value of x is closer to 1 than 0. Thus $\frac{s}{r+s}\int_R^1(\chi - R)dG(\chi) \geq \frac{1-R}{2}$. With this condition, (18) implies $R \leq \frac{2w}{y} - 1$. So under the calibration in section I, $R \in (0, 0.97]$.

VII. Further Topics

A. Layoffs, Quits and the Discount Rate

If the discount rate has a non-negligible impact on the inflows to unemployment, can the discount rate inferred from consumption capture this relationship? This subsection intends to answer this question by looking at U.S. data on the total nonfarm layoffs and quits rate, and by checking their relationship with the discount rate inferred from consumption.

According to the DMP model with endogenous job separation, there should be a positive correlation between the discount rate and job separations. By combining the total nonfarm layoffs rate and total nonfarm quits rate, both from JOLTS, figure 6 shows the total separations rate, together with the discount rate inferred from U.S. real personal consumption expenditure per capita. There is a mild co-movement between the total separations rate and the discount rate in the data. The correlation between the two series is just below 0.04. However, this co-movement is largely due to the positive correlation between the total quits rate and the discount rate [11]. The correlation between the total layoffs rate and the discount rate is -0.31. This negative correlation is particularly strong in the second half of the sample (-0.44). Figure 7 shows the total layoffs rate and the discount rate.
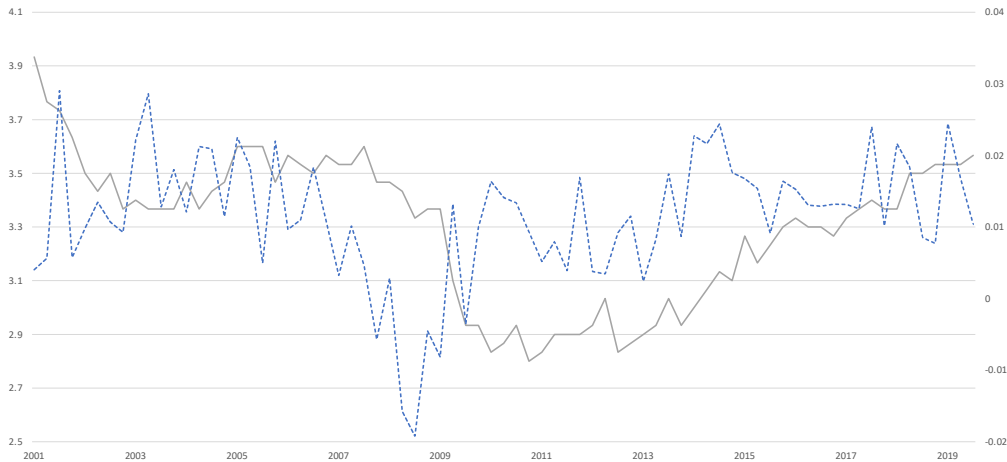

Figure 6: Total Job Separations Rate (Grey Line, Left Scale) and the Discount Rate (Blue Line, Right Scale)

In a DMP model in which there is no elasticity in labor supply and no on-the-job search, the job separation rate is essentially the layoffs rate. Thus, as both the model and intuition would justify, there should be a co-movement between the total layoffs rate and the discount rate. Figure 7 shows that the total layoffs rate, which captures firm-initiated job separations, is strongly counter-cyclical. The discount rate, which is inferred from consumption data, displays a pro-cyclical

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[11]Figure 6 shows that the total job separations rate is strongly pro-cyclical. This is largely due to the pro-cyclicality of the total quits rate. Fewer people would voluntarily quit their jobs in a recession as the chance of finding a new job is lower.
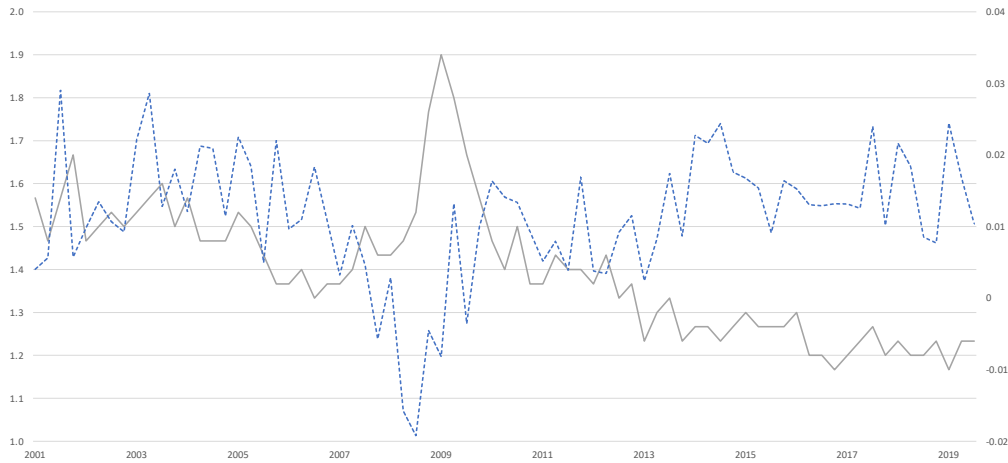
Figure 7: Total Layoffs Rate (Grey Line, Left Scale) and the Discount Rate (Blue Line, Right Scale)

pattern. The pro-cyclicality of the discount rate is caused by the pro-cyclicality of consumption. During the Great Recession, consumption falls sharply, the marginal utility of consumption rises, and the discount rate falls. The question here is, should the discount rate fall in a recession? The answer to this question depends on the nature of the claims that are discounted. Hall (2017) shows, if the claims have a similar risk as they have financial assets, financial market indicators are used to infer the discount rate, such as the ratio of the stock price to its dividends, then the discount rate would rise. From the consumption point of view, the key risk of the claims from a job match is whether the flow of the claims co-moves with consumption. This risk is assessed in subsection D.

B. Job Openings

The discount rate affects the outflows from unemployment through vacancy creation. A lower discount rate leads to a higher expected present value of a job match, encouraging firms to post more vacancies. Using the data from JOLTS (2001-2019) and Barnichon Help-Wanted Index (1951-2001), Figure 8 shows the quarterly U.S. Job Openings between 1951-2019, together with vacancies from the baseline model.

Vacancies from the model display patterns similar to the U.S. job openings rate. The model is able to account for some of the cyclical and long-term fluctuations in the job openings rate, especially in the first half of the sample period. The model tracks the booms and recessions between the 1960s and 1990s reasonably well. The match becomes generally worse from the early 2000s up until the eve of the Great Recession. This is similar to Figure 2 for unemployment. However, the model fails to match the magnitude of the fluctuations in vacancies. The standard deviation of vacancies from the model is below one fifth of its empirical counterpart. Moreover, the model is unable to account for the strong positive correlation between consumption growth and job openings. The correlation between the U.S. consumption growth and the U.S. job openings rate is
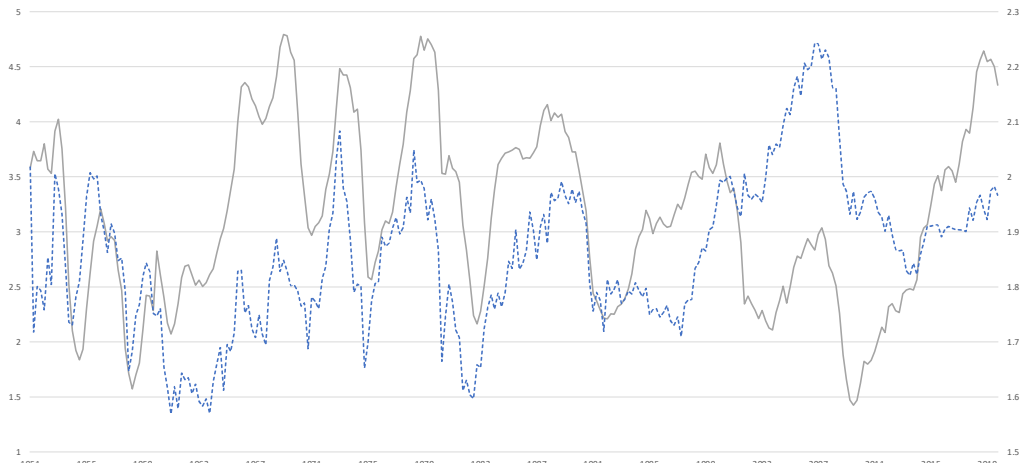
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Figure 8: U.S Job Openings (Grey Line, Left Scale) and Vacancies (Blue Line, Right Scale) from the Model

0.36 for the whole sample period and 0.55 for the last three decades, whereas the model predictions are 0.15 and -0.005 respectively.

C. Variations in the Profit Flow

In this subsection, I study the impact of the variations in the profit flow on unemployment dynamics. I use data to set a path for the profit per worker, in place of the fixed profit in the baseline model. Following Borovicka and Borovickova (2018), the profit per worker is constructed using after-tax corporate profits, divided by the GDP deflator and employment. The data is detrended using the filter proposed by Hamilton (2018). Then the data is linearly transformed to satisfy the following two conditions: the average profit per worker should be 0.015, in line with the calibration for the fixed profit; and the profit should be non-negative (see the results in Figure 9). In this setup, the variations in the discount rate and in the profit flow jointly drive unemployment. The model's prediction for unemployment is reported in Figure 10, along with the actual unemployment rate.

By allowing for variations in the profit flow, the model is able to account for 40% of the fluctuations in unemployment. Recall that the model generates 20% of the observed fluctuations in unemployment when the profit per worker is fixed. However, the model's performance of matching the patterns of the U.S. unemployment rate has declined. This can be seen by comparing Figure 10 with Figure 2. This is also reflected in the correlation between the unemployment rate from the model with the actual unemployment rate. When the discount rate is the sole driving force of unemployment, this correlation is 0.15. By allowing for variations in the profit flow, this correlation decreases to 0.07. This is largely because unemployment does not track the movements of the estimated profit per worker. When profit flow is the sole driving force of unemployment, the correlation between the actual and simulated unemployment becomes negative (see Table 8).
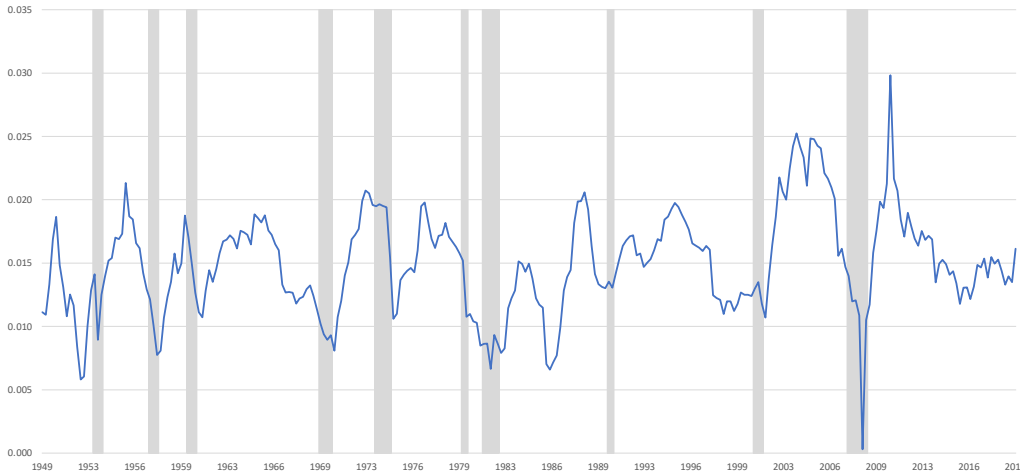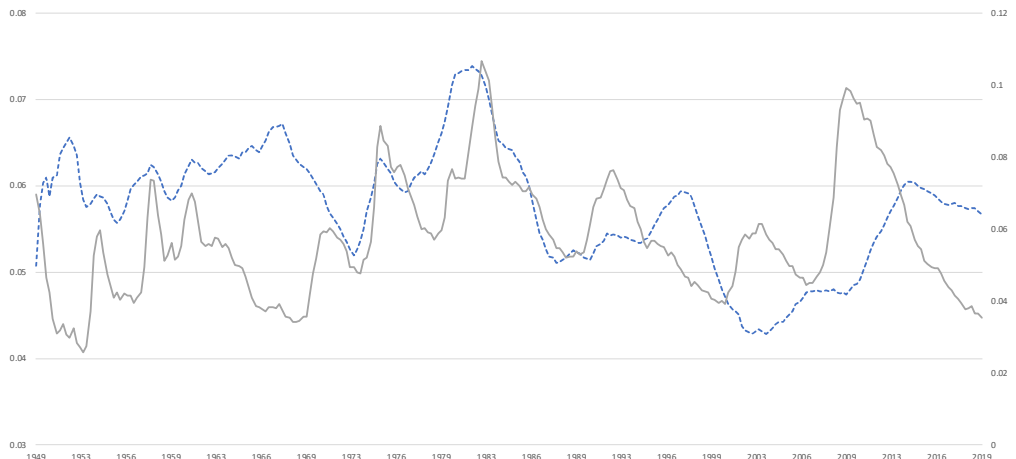
Figure 9: The Estimated Profit per Worker

Figure 10: Unemployment Rate from the Model (Blue Line, Left Scale) and U.S Unemployment Rate (Grey Line, Right Scale)

Table 9: Key Statistics of the Simulated Unemployment Rate

The Driving Force	Standard Deviation (%)	Correlation with the U.S Data
SDF	0.339	0.146
Profit Flow	0.437	-0.027
SDF+Profit Flow	0.685	0.074

D. Risk Premium

This paper follows Hall (2017) by assuming the risk premium rising from the co-movement between the discount factor and the returns from hiring a worker is negligible. Borovicka and Borovickova (2018) show that the risk premium can play a quantitatively meaningful role in unemployment fluctuations if the profit flow exhibits a large conditional variance. They also note that, when the discount factor is inferred from the financial market and the profit flow is approximated by using after-tax corporate profits per worker, the risk premium only explains a small part of unemployment fluctuations. This subsection aims to quantify the

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risk premium when the discount factor is inferred from consumption.

The job creation condition (8) can be rewritten as

$$\frac{\gamma}{q(\theta_t)} = E_t[\frac{1}{1+r_t}(y_t - w_t + (1-\tau)\frac{\gamma}{q(\theta_{t+1})})] \tag{19}$$

Using the definition of covariance, I obtain the following equation:

$$1 = E_t[\frac{1}{1+r_t}]E_t[\frac{y_t - w_t + (1-\tau)\gamma/q(\theta_{t+1})}{\gamma/q(\theta_t)}]+Cov_t[\frac{1}{1+r_t}, \frac{y_t - w_t + (1-\tau)\gamma/q(\theta_{t+1})}{\gamma/q(\theta_t)}] \tag{20}$$

The covariance term captures the risk premium rising from a negative correlation between the discount factor and the return on hiring a worker. Using the profit flow constructed in the previous subsection, the model implied vacancy filling rates, and the path of the discount factor, the resultant covariance is -2.11×10^{-5}. The negligible covariance is due to the small volatility of returns from hiring a worker and a weak correlation (-0.08) between those returns and the discount factor. It justifies the assumption in this paper that the risk premium is negligible.

VIII. Concluding Remarks

Modern macroeconomic theories tie discount rates to consumption. Under the hypothesis that discount rates drive labor market dynamics, firms discount the flows of future claims from a job match based on the intertemporal substitution motives. When times are good and consumption levels are high, any returns from hiring a worker are less desirable than the returns that pay off when times are bad and additional consumption is more highly valued. Thus, fewer vacancies are posted and a positive correlation between the consumption growth and unemployment emerges. This contradicts the post-war U.S. data. This counterfactual positive correlation casts doubt on the importance of discounts in driving labour market dynamics.

Using U.S. real personal consumption expenditure to infer the discount rates, the paper also finds that small fluctuations in unemployment are generated by a standard DMP model when the discount rate is the sole driving force. This result holds even if habit formation is considered and the wage is assumed to be invariant to discounts and to the labor market condition. Under habit formation, discount rates are more volatile but much less persistent. Those opposing impacts on unemployment almost cancel each other out. As a result, the volatility of unemployment is hardly affected by habit formation. The small volatility of unemployment provides another piece of evidence against the importance of discounts.

The paper considers two leading alternative wage formations, Nash bargaining and credible bargaining, and shows that even a mild elasticity of the wage with respect to discounts can significantly dampen the amplification mechanism in the DMP model. On a positive note, endogenizing job separations enables the

inflows to unemployment to vary with discounts. The model with endogenous job separation can potentially explain around 30% of unemployment fluctuations observed in the data if the wage is completely rigid.

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