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ARTICLE TYPE

Iterative feedback tuning for optimal repetitive constraint-following control of uncertain mechanical systems using Udwadia-Kalaba theory

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Summary

This paper presents a novel optimal constraint-following controller for uncertain mechanical systems (MSs). The MS's uncertainty is unknown (possibly time-varying) and bounded, but the bound is unspecified. Employing the frame of Udwadia-Kalaba theory, we design a robust controller with two tunable control gains for MSs, which guarantees some deterministic performances. Assuming the constraints to be holonomic and periodic, we transform the controlled MSs and introduce an iterative feedback tuning (IFT) method for optimizing the proposed controller. After optimization, the final control scheme can achieve the equilibrium between the system performance and control cost for an arbitrary single constraint period. Simulations on a two-link rotational manipulator are provided at last to demonstrate the proposed approach.

KEYWORDS:
Udwadia-Kalaba theory, iterative feedback tuning, uncertain mechanical systems, constraint following, optimal robust control

1 | INTRODUCTION

Constraint following, in connecting to industrial practices, is one of the most critical tasks for the motion control of mechanical systems(MSs)[1]. While in practical engineering, most MSs perform repetitive tasks. In other words, the constraints a real MS subjected to are usually periodic. Although various uncertainties like measurement noises, model simplifications, and vibration noises, etc., may seriously affect the MS's performance, the repetitive tasks provide an excellent resource for the self-learning of the controller. This paper combines a model-based controller and a data-based learning scheme to design an efficient constraint-following control approach for MSs considering uncertainties.

The constraints imposed on an MS generally fall into two categories: passive and servo constraints. The former are usually satisfied by forces from the environment or the structure of the MS. While to meet servo constraints, a control engineer typically needs to work out a control scheme and input some extra energy. Research on the passive-constraint following problem has made significant contributions. Readers could see Reference 1 for a survey. As control engineers usually address, the focus of this paper is the servo constraint-following problem. There are already brilliant control strategies for the constraint following of mechanical systems with or without uncertainties. In Reference 2, an adaptive tracking controller was proposed for Euler-Lagrange systems. Addressing a planar underactuated vehicle's trajectory-tracking problem, Reference 3 used nonholonomic

[0]**Abbreviations:** MS,mechanical system; IFT, iterative feedback tuning; ARC, adaptive robust controller; RHS, right hand side;MPC, model predictive control; UDS, uncertain dynamic system; EL, Euler-Lagrange; UMS, uncertain mobile robot

constraints to obtain feasible state trajectories first. The authors then adopted a transitional trajectory to build reduced-order error dynamics. The trajectory tracking problem thus becomes a dynamic stabilization problem, and a slide mode control is employed for the dynamical stabilization. As for uncertain underactuated Euler-Lagrange (EL) systems, References 4 presented a robust adaptive control. Unlike other control strategies using segregated actuated or non-actuated dynamics, the method in References 4 is structure-independent making it a more general control scheme for various EL systems[5]. A performance-based iterative learning algorithm is proposed in Reference 6 for a bilateral upper limb robot, which quickly finds optimal patient-orientated parameters for given training trajectories. And thus increase the efficient training time. By modeling aircraft engines as an uncertain dynamic system (UDS), Reference 7 proposed a robust controller for the UDS considering both matched and unmatched uncertainties. Using fuzzy sets to describe the uncertain parameters of the UDS, researchers then optimized a design parameter by taking the control cost and the performance threshold into consideration[7]. Reference 8 combined a tube-based model predictive control (MPC) and adaptive control for the trajectory tracking control of mobile robots, where the tube-based MPC is designed for kinematic constraints, and the adaptive control is used to handle dynamic constraints. However, the above control methods somehow use linearizations or nonlinear cancellations for the constrained dynamics of MSs.

Noticeably, a different control strategy named Udwadia-control was provided by Udwadia[9] based on the Udwadia-Kalaba theory[10]. Compared with extant control methods, the most significant advantage of Udwadia's approach is that it can control general, nonlinear, structural MSs without linearization or nonlinear cancellation. Based on the Udwadia-control strategy, many significant contributions have been made. In Reference 11, the authors adopted the Udwadia–Kalaba theory to study the dynamics of flexible multibody systems and compared the numerical efficiency of their approach with the classical coordinate partitioning scheme. Reference 13 used the Udwadia-control frame to build their trajectory-tracking controller for a nonholonomic mobile robot. Combining the sliding mode control and Udwadia-control, Reference 14 designed a controller for nonlinear multibody systems. Considering both equality and inequality constraints, Reference 15 proposed a constraint-following control for active suspension systems. In Reference 16, the control of artificial swarm MSs is addressed by taking both agents' behaviors and the desired trajectory-tracking of the swarm MSs into consideration. Then a swarm tracking controller based on the Udwadia-Kalaba theory is proposed, which guarantees the controlled swarm system to obey the required motion. Among all the applications and extensions using the Udwadia-control scheme, Chen proposed an adaptive robust controller (ARC) for approximate constraint following of uncertain MSs[17]. Chen's ARC takes advantage of the Udwadia-control for nonlinear MSs and can guarantee deterministic control performances. Thus Chen's controller is widely applied[18−21]. Reference 22 proposed a robust control scheme for a 2-DOF lower-limb rehabilitation device, and a cooperative-game theory was used for the controller optimization. In Reference 23, the motion control of the uncertain mobile robot (UMS) is explored. An adaptive robust controller based on the Udwadia-Kalaba theory is then designed, with the trajectory being regarded as servo constraints and the prescribed performance being transformed into inequality constraints. Both the Udwadia-control and Chen's ARC are using pure rigid math analysis for uncertain MSs[24]. While an explicit math model for uncertainties is functional, the experimental data of controlled MSs also contains essential information for improving control performances. That is the reason for the rapid development of data mining and machine learning. As far as the authors know, no research has been reported on introducing data-learning theory into the Udwadia-control to design optimal constraint-following controllers for uncertain MSs.

Related work to this paper is References 25-28. In Reference 25, the authors presented an adaptive robust control strategy for constraint following of underactuated MSs. The two tunable control parameters are optimized using a nash-game approach. Reference 26 proposed an adaptive robust controller for the underactuated MSs. Modeling the uncertainty of the underactuated MSs in a fuzzy method, the authors then optimize the tunable control gains using the nash-game theory. A high-order robust control for uncertain MSs was designed in Reference 27. The tunable controller can guarantee some deterministic performances. A Stackelberg game theory is then adopted for the optimization tuning of the controller. Researches above seldom consider the following control of periodic constraints, and the 0 order constraint-following errors are hardly involved in the control design when addressing holonomic constraints. Noticeably, Reference 28 presented a robust controller for a permanent magnet linear motor. The robust controller is designed using the Udwadia-Kalaba theory and considers both 0 and 1st order constraint-following errors. However, the tunable control gains in Reference 28 are not optimized. Despite brilliant controllers and optimization algorithms that have been designed for the constraint-following control of uncertain MSs, there are still some limitations to be further explored. First, former controllers based on the Udwadia-Kalaba theory are efficient to eliminate the 1st order holonomic constraint-following errors. While they show low efficiency when addressing 0 order holonomic constraint-following errors. Second, extant control optimization strategies like the fuzzy optimization[18], the cooperative game theory[22], the nash game theory[26], and the Stackelberg game theory[27] are all model-based. When dealing with periodic constraints, the repetitive tasks provide valuable input-output data for the self-learning of the controller, and thus a more practical perspective to optimize controllers. However, few works have been done on controlling uncertain MSs considering periodic constraints, and using the repetitive input-output data for optimizing constraint-following controllers. Addressing the above two problems, the main contributions of this paper are twofold. First, we design a robust constraint-following controller based on the Udwadia-Kalaba theory, guaranteeing some deterministic performances for uncertain MSs. Second, the robust controller involves two tunable control gains, and we use an IFT approach to optimize the controller after transforming the controlled systems.

The rest of this paper is as follows. Section 2 reviews some preliminaries on the Udwadia-Kalaba theory. In section 3, a robust constraint-following controller for an uncertain MS is designed. A Lyapunov proof is provided to demonstrate the validity of the proposed controller. Assuming the constraint to be periodic, section 4 transforms the controlled uncertain mechanical system and presents an IFT-based algorithm for the control optimization. Section 5 provides a two-link rotational manipulator as a simulation example to testify the robust control scheme and the IFT optimization strategy. Finally, in section 6, we conclude this paper.

2 | PRELIMINARIES ON UDWADIA-KALABA THEORY

Consider an MS whose dynamics is given by Lagrange or Newton-Euler method as:

$$M(q(t),t)\ddot{q}(t) = Q(q(t),\dot{q}(t),t) + B\tau(t) \tag{1}$$

where $t \in \mathrm{R}$ is the independent variable. $q(t) \in \mathrm{R}^n$ is the generalized coordinate describing the system, and $\dot{q}(t) \in \mathrm{R}^n$, $\ddot{q}(t) \in \mathrm{R}^n$ are the corresponding velocity and acceleration. $M(q(t),t)$ is the inertia matrix, $Q(q(t),\dot{q}(t),t)$ are the known forces acting on the system (Coriolis/centrifugal force, gravitational force, friction force, etc.) whose constraints are released. B is the coefficient matrix between actuators and the coordinate. $\tau(t)$ is the control input from actuators. We assume that the functions $M(\cdot)$, $Q(\cdot)$, and B are continuous (this can be generalized to be Lebesgue measurable in t) and are of appropriate dimensions. For the ease of notation, we may rewrite $M(q(t),t)$ as $M(\cdot)$ or M and make similar simplifications for the rest symbols.

Suppose the MS subjects to h holonomic constraints:

$$f_l(q,t) = 0, \quad l = 1,\cdots,h \tag{2}$$

Equation (2) is the 0 order form of costraints. Differentiating (2) to t once results

$$\sum_{i=1}^{n} A_{li}(q,t)\dot{q}_i = c_l(q,t), \quad l = 1,\cdots,h \tag{3}$$

where $\dot{q}_i$ is the ith component of $\dot{q}$, $A_{li}(\cdot)$ and $c_l(\cdot)$ are continuously differentiable in q and t with

$$\begin{aligned} A_{li}(q,t) &= \frac{\partial f_l(q,t)}{\partial q_i} \\ c_l(q,t) &= \frac{\partial f_l(q,t)}{\partial t} \end{aligned} \tag{4}$$

Rewrite (3) in matrix form as

$$A(q,t)\dot{q} = c(q,t) \tag{5}$$

where $A = [A_{li}]^{h\times n}$, $c = [c_1 \ \cdots \ c_h]^{\mathrm{T}}$. Equation (5) is called the 1st order form of constraints.
Differentiate (3) to t once and write the final results in matrix form, we get

$$A(q,t)\ddot{q} = b(q,\dot{q},t) \tag{6}$$

where $b = [b_1 \ \cdots \ b_h]^{\mathrm{T}}$. And $\forall l = 1,\cdots,h$,

$$b_l(q,\dot{q},t) = \frac{d}{dt}c_l(q,t) - \sum_{i=1}^{n}\frac{d}{dt}A_{li}(q,t)\dot{q}_i \tag{7}$$

where

$$\frac{d}{dt}c_l(q,t) = \sum_{k=1}^{n}\frac{\partial c_l(q,t)}{\partial q_k}\dot{q}_k + \frac{\partial c_l(q,t)}{\partial t} \tag{8}$$

$$\frac{d}{dt}A_{li}(q,t) = \sum_{k=1}^{n}\frac{\partial A_{li}(q,t)}{\partial q_k}\dot{q}_k + \frac{\partial A_{li}(q,t)}{\partial t} \tag{9}$$

Equation (6) is called the 2nd order form of constraints. The target of this paper is designing a controller τ that renders the MS to follow constraints (2), (5) and (6).

Assumption 1. For any $(q,t) \in \mathrm{R}^n \times \mathrm{R}$, $M(q,t) > 0$.

Remark 1. The inertia matrix $M(q,t)$ is always believed to be positive definite, however, there are counter examples where q is not the generalized coordinate[29].

Satisfying assumption 1, and according to the Udwadia-Kalaba theory[10], the control force

$$Q^c(q,\dot{q},t) = M^{1/2}(q,t)\left(A(q,t)M^{-1/2}(q,t)\right)^{+}\left(b(q,\dot{q},t) - A(q,t)M^{-1}(q,t)Q(q,\dot{q},t)\right) \tag{10}$$

can make the system (1) to meet constraints (6) and minimize the control cost. Here, "+" stands for the Moore–Penrose generalized inverse.[30]

Equation (10) provides an analytical method for calculating the constraint-following force of MSs. It shows a significant advantage in simplicity compared with the classical Newton-Euler method or Lagrange's multiplier approach. However, Q^c is determined based on precise models of MSs, which are practically infeasible. Given the dynamic model as (1), we can generally get a reference value for each parameter while the actual value is unknown. The following section will present a robust control scheme invoking (10) for the uncertain MS.

3 | ROBUST CONTROL DESIGN

Assuming the uncertainty in model (1) to be bounded, while the bound is unknown. Let's do a decomposition first

$$M(q,t) = \overline{M}(q,t) + \Delta M(q,t) \tag{11}$$

$$Q(q,\dot{q},t) = \overline{Q}(q,\dot{q},t) + \Delta Q(q,\dot{q},t) \tag{12}$$

where $\overline{M}(\cdot)$ and $\overline{Q}(\cdot)$ are the nominal parameters, and $\Delta M(\cdot)$, $\Delta Q(\cdot)$ are the corresponding uncertain portions.

Let

$$\beta_1(q,t) = \left[\, f_1(q,t) \ \cdots \ f_h(q,t) \,\right]^{\mathrm{T}} \tag{13}$$

$$\beta_2(q,\dot{q},t) = A(q,t)\dot{q} - c(q,t) \tag{14}$$

According to equation (2) and (5), $\beta_1(q,t)$ and $\beta_2(q,\dot{q},t)$ can be interpreted as the 0 and 1st order constraint-following errors.

Assumption 2. For any $(q,t) \in \mathrm{R}^n \times \mathrm{R}$, and a given $H \in \mathrm{R}^{n\times n}, H > 0$ and $H^{\mathrm{T}} = H$, there is $A\overline{M}^{-1}BB^{+} = A\overline{M}^{-1}$ and $HA\overline{M}^{-1/2}(A\overline{M}^{-1/2})^{+} = H$.

Therefore, this paper proposes the controller (15) for the constraint following of uncertain MSs:

$$\tau = \mu_1(q,\dot{q},t) + \mu_2(q,\dot{q},t) \tag{15}$$

where

$$\mu_1(q,\dot{q},t) = B^{+}\overline{M}^{1/2}(q,t)\left(A(q,t)\overline{M}^{-1/2}(q,t)\right)^{+}\left(b(q,\dot{q},t) - A(q,t)\overline{M}^{-1}(q,t)\overline{Q}(q,\dot{q},t)\right) \tag{16}$$

$$\mu_2(q,\dot{q},t) = -B^{+}\overline{M}^{1/2}(q,t)\left(A(q,t)\overline{M}^{-1/2}(q,t)\right)^{+}\left(k_P\beta_1(q,t) + k_D\beta_2(q,\dot{q},t)\right) \tag{17}$$

and $k_P, k_D > 0$ are tunable control gains. The controller (15) consists of two parts with μ_1 being used for the constraint following of the nominal MS. μ_2 is designed to eliminate the constraint-following error due to the uncertainty.

Theorem 1. Let $\delta(t) := \left[\beta_1^{\mathrm{T}}(q,t) \ \beta_2^{\mathrm{T}}(q,\dot{q},t)\right]^{\mathrm{T}}$, consider the system (1), the control (15) renders the following performance:

(i). Uniform stability: For each $\varpi > 0$, there exists a $\varepsilon > 0$ such that for any solution $\delta(\cdot)$ with $\|\delta(t_0)\| < \varepsilon$, then $\|\delta(t)\| < \varpi$ for all $t \geq t_0$.

(ii). Convergence to 0: For any given constraint $\delta(\cdot)$,

$$\lim_{t\to\infty} \beta_1(q,t) = 0 \tag{18}$$

Proof of Theorem 1. Consider a Lyapunov function candidate

$$V(\beta_1,\beta_2) = k_P \beta_1^{\mathrm{T}} H \beta_1 + \beta_2^{\mathrm{T}} H \beta_2 \tag{19}$$

The first derivative of V with respect to t is

$$\dot{V} = 2k_P \beta_1^{\mathrm{T}} H \dot{\beta}_1 + 2\beta_2^{\mathrm{T}} H \dot{\beta}_2 \tag{20}$$

Recalling $\dot{\beta}_1 = \beta_2$, $\dot{\beta}_2 = A\ddot{q} - b$, then

$$\dot{V} = 2k_P \beta_1^{\mathrm{T}} H \beta_2 + 2\beta_2^{\mathrm{T}} H (A\ddot{q} - b) \tag{21}$$

For the second term on the (RHS) of equation (21), there is

$$\begin{aligned} 2\beta_2^{\mathrm{T}} H (A\ddot{q} - b) &= 2\beta_2^{\mathrm{T}} H \left[A\overline{M}^{-1} \left(\overline{Q} + B\mu_1 + B\mu_2 \right) - b \right] \\ &= 2\beta_2^{\mathrm{T}} H \left[\overbrace{A\overline{M}^{-1} \left(\overline{Q} + B\mu_1 \right) - b}^{\tilde{A}} + \overbrace{A\overline{M}^{-1} B\mu_2}^{\tilde{B}} \right] \end{aligned} \tag{22}$$

Combining equation (16) and recalling assumption 2, we have

$$\begin{aligned} 2\beta_2^{\mathrm{T}} H \tilde{A} &= 2\beta_2^{\mathrm{T}} H \left[A\overline{M}^{-1}\overline{Q} + A\overline{M}^{-1} BB^{+}\overline{M}^{1/2} \left(A\overline{M}^{-1/2} \right)^{+} \left(b - A\overline{M}^{-1}\overline{Q} \right) - b \right] \\ &= 2\beta_2^{\mathrm{T}} H \left(A\overline{M}^{-1}\overline{Q} - A\overline{M}^{-1}\overline{Q} + b - b \right) \\ &= 0 \end{aligned} \tag{23}$$

Furthermore, according to equation (17),

$$\begin{aligned} 2\beta_2^{\mathrm{T}} H \tilde{B} &= -2\beta_2^{\mathrm{T}} H A\overline{M}^{-1} BB^{+}\overline{M}^{1/2} \left(A\overline{M}^{-1/2} \right)^{+} \left(k_P \beta_1 + k_D \beta_2 \right) \\ &= -2k_P \beta_2^{\mathrm{T}} H \beta_1 - 2k_D \beta_2^{\mathrm{T}} H \beta_2 \end{aligned} \tag{24}$$

Therefore,

$$\begin{aligned} \dot{V} &= \overbrace{2k_P \beta_1^{\mathrm{T}} H \beta_2 - 2k_P \beta_2^{\mathrm{T}} H \beta_1}^{=0} - 2k_D \beta_2^{\mathrm{T}} H \beta_2 \\ &= -2k_D \beta_2^{\mathrm{T}} H \beta_2 \end{aligned} \tag{25}$$

Adopting the Rayleigh's principle[30], we have

$$\lambda_{\min}(H) \left\| \beta_2 \right\|^2 \leq \beta_2^{\mathrm{T}} H \beta_2 \leq \lambda_{\max}(H) \left\| \beta_2 \right\|^2 \tag{26}$$

where $\lambda_{\min}(H)$, $\lambda_{\max}(H) > 0$ are the minimum and maximum eigenvalues of H. As $k_D > 0$, then

$$\dot{V} \leq -2k_D \lambda_{\min}(H) \left\| \beta_2 \right\|^2 \tag{27}$$

By (27), the Lyapunov derivative is non-positive and we can conclude the uniform stability. Besides, according to Barbalat's lemma[31], we have the convergence of $\beta_1 \to 0$ as $t \to \infty$. □

4 | CONTROL GAIN OPTIMIZATION BASED ON ITERATIVE FEEDBACK TUNING

4.1 | Optimization procedure

The controller (15) involves two tunable control gains, and they directly affect the constraint-following performance and the control cost. From an economic perspective, one may be interested in finding the optimal combination of $[k_P \quad k_D]^{\mathrm{T}}$ that can reach the equilibrium between the system performance and control cost. This section will adopt an IFT approach for optimization.

For uncertain MSs under repetitive constraints, the constraint-following torque μ_1 is also periodic and shares the same period with constraints (2). Furthermore, μ_1 is determined given the nominal parameters of (1) and 2nd order constraint (6), the varying torque μ_2 is what should be optimized during the repetitive constraint-following experiment. Notice μ_2 is equivalent to a proportional-differential(PD) control, of which k_P is the proportional gain and k_D is the differential gain. Therefore, we can transform the system (1) under control (15) as Figure 1.

In Figure 1, $\tilde{P}$ is the uncertain plant (1) under the Udwadia-control μ_1. The input to plant system is the constraint-following

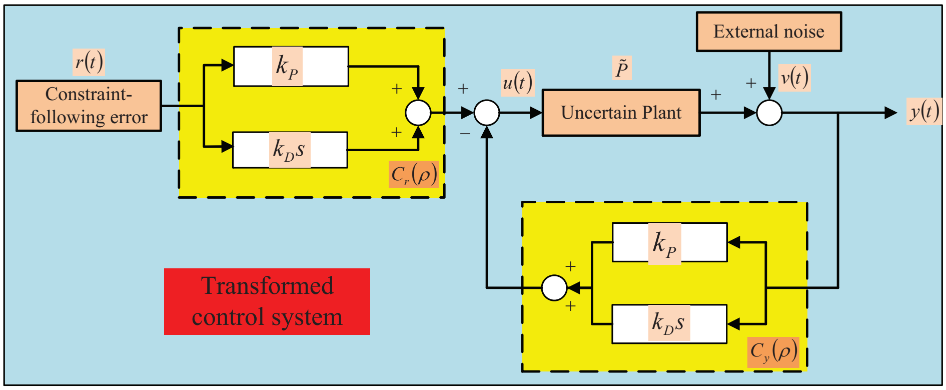

FIGURE 1 Transformed control system.

error $r(t)$, $u(t)$ is the control force μ_2 handling the uncertainty, $v(t)$ is stochastic external noises, $y(t)$ is the system output. According to Reference 32, the PD control must be consider as a 2-DOF controller with common parameters $C_r(\rho) = C_y(\rho)$, $\rho = [k_P \quad k_D]^T$.

From the control diagram in Figure 1, the system output can be elicited as

$$y_t(\rho) = \frac{C_r(\rho)\tilde{P}}{1 + C_y(\rho)\tilde{P}} r_t + \frac{1}{1 + C_y(\rho)\tilde{P}} v_t := T_0 r_t + S_0 v_t \tag{28}$$

The target for the IFT optimization is to make the output $y_{i+1}(t)$ of the $i+1$ iteration closer to the nominal constraint $y_d(t)$ compared to the i iteration.

In order to reach the target, let's define a tuning criterion first

$$J(\rho) = \frac{1}{2N}\left(\sum_{i=1}^{N} \tilde{y}_t(\rho)^2 + \lambda \sum_{i=1}^{N} u_t(\rho)^2\right) \tag{29}$$

where N is the number of samples for a single period, $\tilde{y}_t = y_t(\rho) - y_d(t)$ is the constraint-following error, λ is a predefined weight between the system performance and control cost. The optimization of the controller (15) is now equivalent to solve the following minimization problem:

$$\rho^* = \arg\min_{\rho} J(\rho) \tag{30}$$

As for (30), take the partial derivative of $J(\rho)$ with respect to ρ results

$$\frac{\partial J}{\partial \rho}(\rho) = \frac{1}{N}\left(\sum_{t=1}^{N} \tilde{y}_t(\rho)\frac{\partial \tilde{y}_t}{\partial \rho}(\rho) + \lambda \sum_{t=1}^{N} u_t(\rho)\frac{\partial u_t}{\partial \rho}(\rho)\right) \tag{31}$$

where $\partial \tilde{y}_t/\partial \rho = \partial y_t/\partial \rho$. And

$$\frac{\partial y_t}{\partial \rho}(\rho) = \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial \rho}(\rho)T_0 r_t - \frac{\partial C_y}{\partial \rho}(\rho)T_0 y_t\right] \tag{32}$$

$$\frac{\partial u_t}{\partial \rho}(\rho) = S_0\left[\frac{\partial C_r}{\partial \rho}(\rho)r_t - \frac{\partial C_y}{\partial \rho}(\rho)y_t\right] \tag{33}$$

Equation (32) and (33) can be approximately calculated using only the experimental data. Normally, three sets of experimental data are necessary for an iteration[32]. The math forms for the input and output of experiments in the i-th iteration are

$$\begin{cases} \text{Input}: r_i^1 = r & \text{Torque}: u_t^1 = S_0(C_r r - C_y v_i^1) & \text{Output}: y_t^1 = T_0 r + S_0 v_i^1 \longrightarrow \text{1st experiment} \\ \text{Input}: r_i^2 = y^1 & \text{Torque}: u_t^2 = S_0(C_r y^1 - C_y v_i^2) & \text{Output}: y_t^2 = T_0 y^1 + S_0 v_i^2 \longrightarrow \text{2nd experiment} \\ \text{Input}: r_i^3 = r & \text{Torque}: u_t^3 = S_0(C_r r - C_y v_i^3) & \text{Output}: y_t^3 = T_0 r + S_0 v_i^3 \longrightarrow \text{3rd experiment} \end{cases} \tag{34}$$

where for r_i^j, i represents the i th iteration, $j = 1, 2, 3$ represents the j th set of experiment, such notations also apply to u, y, and v throughout this paper. The symbol u_t^j represents the control force at time t, the notation also applies to y.

By (34), a reference input is applied to the plant in the first experiment, and the output is recorded. The second experiment takes the output of the first experiment as its reference input, and a third experiment sharing the same reference input of the first experiment is designed as a reference. In addition, the last experiment is necessary for real devices, while it can be omitted in simulations where the external noise v_i does not exist.

The external noise v_i^j, $j = 1, 2, 3$ are from different experiments of the same plant, and are thus mutually independent. Therefore, we can approximately calculate $\partial y_t/\partial\rho$ and $\partial u_t/\partial\rho$ as

$$\text{est}\left[\frac{\partial y_t}{\partial\rho}(\rho)\right] = \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)y_t^3 - \frac{\partial C_y}{\partial\rho}(\rho)y_t^2\right] \tag{35}$$

$$\text{est}\left[\frac{\partial u_t}{\partial\rho}(\rho)\right] = \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)u_t^3 - \frac{\partial C_y}{\partial\rho}(\rho)u_t^2\right] \tag{36}$$

Recalling $C_r(\rho)$ and $C_y(\rho)$ are PD controllers sharing the same parameter vector $\rho = [k_P \quad k_D]^{\mathrm{T}}$, and the transfer function of the PD controller is $k_P + k_D s$. As for the discrete system, we have

$$C_r(\rho) = C_y(\rho) = \frac{\left(k_P T + k_D\right) z^2 - \left(k_P T + 2k_D\right) z + k_D}{Tz^2 - Tz} \tag{37}$$

where T and z denote the sampling time and unit delay operator, respectively. Then,

$$\frac{\partial C_r}{\partial k_P} = \frac{\partial C_y}{\partial k_P} = \frac{Tz^2 - Tz}{Tz^2 - Tz} \tag{38}$$

$$\frac{\partial C_r}{\partial k_D} = \frac{\partial C_y}{\partial k_D} = \frac{z^2 - 2z + 1}{Tz^2 - Tz} \tag{39}$$

Therefore, equation (35) and (36) can be rewritten as

$$\text{est}\left[\frac{\partial y_t}{\partial\rho}(\rho)\right] = \begin{bmatrix} \dfrac{\partial y_t}{\partial k_P} \\ \dfrac{\partial y_t}{\partial k_D} \end{bmatrix} = \frac{1}{C_r(\rho)}\begin{bmatrix} \dfrac{\partial C_r}{\partial k_P}(\rho)(y_t^3 - y_t^2) \\ -\dfrac{\partial C_r}{\partial k_D}(\rho)y_t^2 \end{bmatrix} \tag{40}$$

$$\text{est}\left[\frac{\partial u_t}{\partial\rho}(\rho)\right] = \begin{bmatrix} \dfrac{\partial u_t}{\partial k_P} \\ \dfrac{\partial u_t}{\partial k_D} \end{bmatrix} = \frac{1}{C_r(\rho)}\begin{bmatrix} \dfrac{\partial C_r}{\partial k_P}(\rho)(u_t^3 - u_t^2) \\ -\dfrac{\partial C_r}{\partial k_D}(\rho)u_t^2 \end{bmatrix} \tag{41}$$

For the i th iteration, $\partial J(\rho_i)/\partial\rho$ is approximately obtained as

$$\text{est}\left[\frac{\partial J(\rho_i)}{\partial\rho}\right] = \frac{1}{N}\left(\sum_{t=1}^{N}\tilde{y}_t(\rho)\text{est}\left[\frac{\partial y_t}{\partial\rho}(\rho)\right] + \lambda\sum_{t=1}^{N}u_t(\rho)\text{est}\left[\frac{\partial u_t}{\partial\rho}(\rho)\right]\right) \tag{42}$$

thus the new control gains $\rho_{i+1} = \left[k_P^{i+1} \quad k_D^{i+1}\right]^{\mathrm{T}}$ for $i+1$ iteration is

$$\rho_{i+1} = \rho_i - \eta_i R_i^{-1}\text{est}\left[\frac{\partial J(\rho_i)}{\partial\rho}\right] \tag{43}$$

where $\eta_i > 0$ is the step size, R_i is the Hessian matrix and can be calculated by

$$R_i = \frac{1}{N}\left(\sum_{t=1}^{N}\text{est}\left[\frac{\partial y_t}{\partial\rho}(\rho)\right]\text{est}\left[\frac{\partial y_t}{\partial\rho}(\rho)\right]^{\mathrm{T}} + \lambda\sum_{t=1}^{N}\text{est}\left[\frac{\partial u_t}{\partial\rho}(\rho)\right]\text{est}\left[\frac{\partial u_t}{\partial\rho}(\rho)\right]^{\mathrm{T}}\right) \tag{44}$$

(43) gives the final step of an iteration, the key to the IFT method is obtaining est $\left[\partial J(\rho_i)/\partial\rho\right]$ through experiments (34). To summarize, Figure 2 shows the general procedure of the IFT-based control optimization.

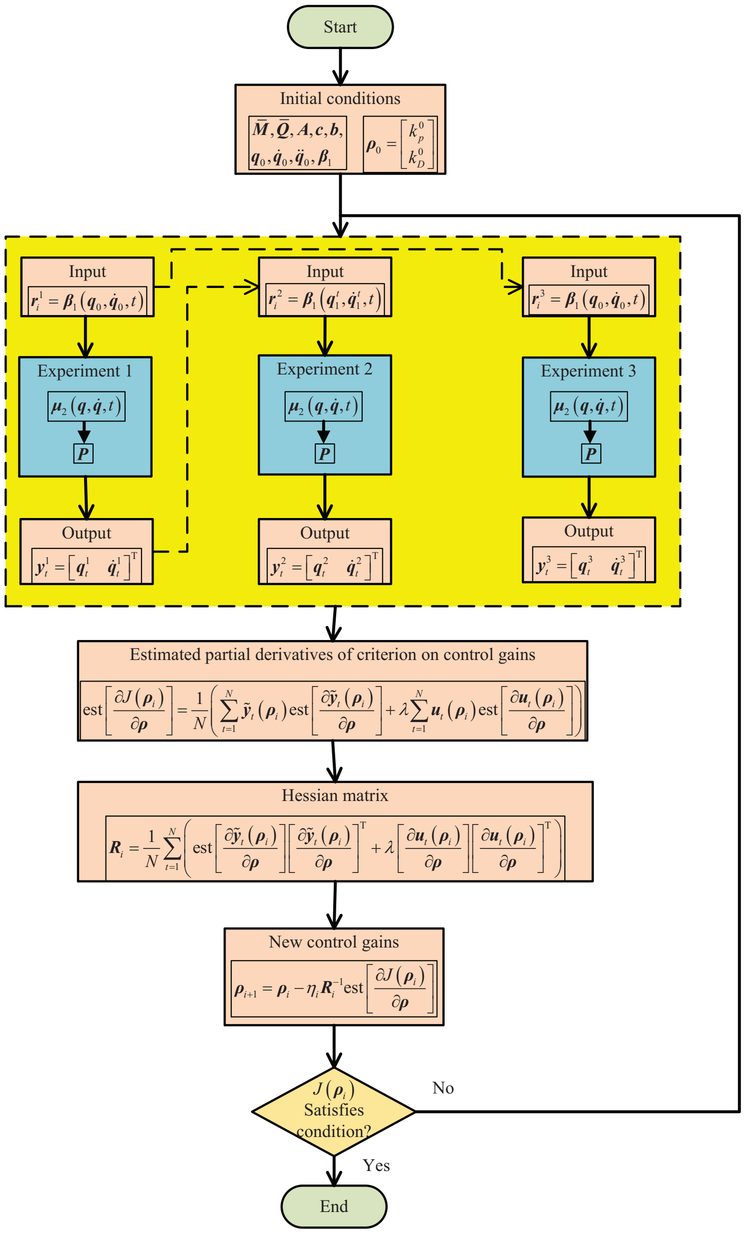

FIGURE 2 IFT-based control optimization.

4.2 | Convergence of the IFT algorithm

In order to reach the convergence of the IFT algorithm, two conditions must be satisfied[32] : 1) est $\left[\partial J(\rho_i)/\partial\rho\right]$ is unbiased; 2) The step size γ_i converges to 0 with $i \to \infty$ but not too fast.

According to equation (32), est $\left[\partial J(\rho)/\partial\rho\right]$ is unbiased only if both est $\left[\partial y_t(\rho)/\partial\rho\right]$ and est $\left[\partial u_t(\rho)/\partial\rho\right]$ are unbiased. As

$$\frac{\partial y_t}{\partial\rho}(\rho) = \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)y_t^3 - \frac{\partial C_y}{\partial\rho}(\rho)y_t^2\right] + \frac{S_0}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)v_i^3 - \frac{\partial C_y}{\partial\rho}(\rho)v_i^2\right] \tag{45}$$

$$\frac{\partial u_t}{\partial\rho}(\rho) = \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)u_t^3 - \frac{\partial C_y}{\partial\rho}(\rho)u_t^2\right] + \frac{S_0 C_y(\rho)}{C_r(\rho)}\left[\frac{\partial C_r}{\partial\rho}(\rho)v_i^3 - \frac{\partial C_y}{\partial\rho}(\rho)v_i^2\right] \tag{46}$$

and v_i^2, v_i^3 are independent stochastic noise of the same plant, thus (35) and (36) are unbiased estimation of $\partial y_t(\rho)/\partial\rho$ and $\partial u_t(\rho)/\partial\rho$, so is est $\left[\partial J(\rho_i)/\partial\rho\right]$. Thus we conclude the satisfaction of condition 1.

As for the condition 2, if γ_i satisfy

$$\sum_{i=1}^{\infty}\gamma_i = \infty, \qquad \sum_{i=1}^{\infty}\gamma_i^2 < \infty \tag{47}$$

then the control gain ρ_i will converge to a steady vector. This paper chooses $\gamma_i = a/i^2$, $a > 0$ as the step size, then (47) is satisfied. Besides, we also choose the Newton-Gauss matrix $\boldsymbol{R}_i$ to speed up the convergence rate[33]. Thus condition 2 is satisfied. By condition 1 and 2, we conclude the convergence of the IFT algorithm.

5 | NUMERICAL EXAMPLE

This section provides a two-link manipulator as a simulation example to testify the proposed robust controller (15) and the IFT optimization algorithm[34]. Figure 3 shows the model of the manipulator, and Table 1 lists the corresponding parameters.

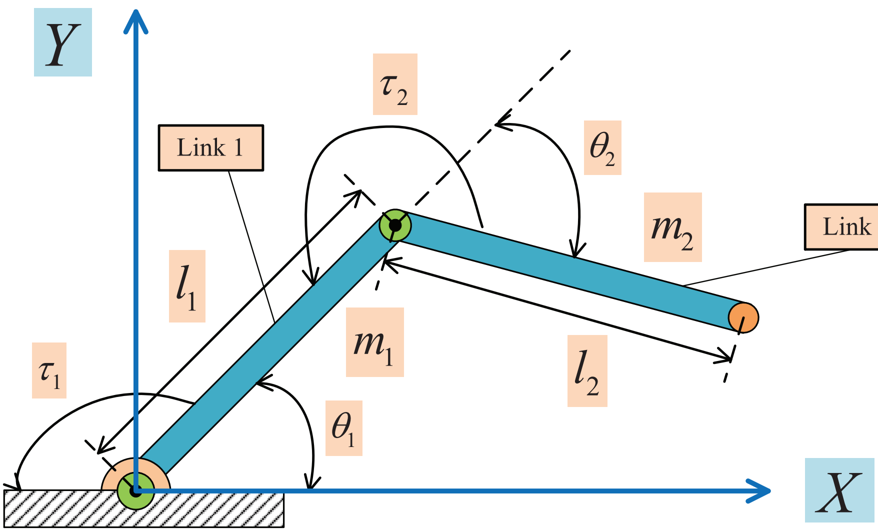

FIGURE 3 2-link rotational manipulator.

The dynamic model of the manipulator can be described in the form of (1), where

$$M(q,t) = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2\cos\theta_2 + l_1^2\left(m_1+m_2\right) & l_2^2 m_2 + l_1 l_2 m_2\cos\theta_2 \\ l_2^2 m_2 + l_1 l_2 m_2\cos\theta_2 & l_2^2 m_2 \end{bmatrix}, \quad q = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}, \quad \tau = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

$$Q(q,\dot{q},t) = \begin{bmatrix} m_2 l_1 l_2\dot{\theta}_2^2\sin\theta_2 + 2m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 - m_2 l_2 g\cos\left(\theta_1+\theta_2\right) - \left(m_1+m_2\right) l_1 g\cos\theta_1 \\ -m_2 l_1 l_2\dot{\theta}_1^2\sin\theta_2 - m_2 l_2 g\cos\left(\theta_1+\theta_2\right) \end{bmatrix}, \quad B = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

TABLE 1 Parameters of 2-link manipulator.

Physical meaning and unit	Parameter symbol	Values
Mass of link 1 (kg)	m_1	$\overline{m}_1 = 1$
Mass of link 2 (kg)	m_2	$\overline{m}_2 = 0.5$
Length of link 1 (m)	l_1	$l_1 = 1$
Length of link 2 (m)	l_2	$l_2 = 2$
Joint angle of link 1 (rad)	θ_1	
Joint angle of link 2 (rad)	θ_2	
Actuating torque of joint 1 (N.m)	τ_1	
Actuating torque of joint 2 (N.m)	τ_2	
Gravity acceleration (m.s^{-2})	g	$g = 9.8$

As for simulations, we impose a simple periodic constraint for the manipulator as

$$\theta_1 + \theta_2 = 0 \tag{48}$$

and the period is set to be 4π seconds. Constraint (48) can be transformed in the form of (5) and (6) with

$$A = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}}, \quad c = 0, \quad b = 0$$

As q is the generalized coordinate in our case, assumption 1 is satisfied. Let $H = 1$, by direct algebra, assumption 2 can be easily verified. Then the robust controller (15) is determined.

Assuming the masses of link 1 and link 2 are uncertain, and denote $m_1 = \overline{m}_1 + \Delta m_1$, $m_2 = \overline{m}_2 + \Delta m_2$. We execute simulations using MATLAB, and the initial conditions are $\theta_1(0) = 0.15$, $\theta_2(0) = -0.2$, $\dot{\theta}_1(0) = 0$, $\dot{\theta}_2(0) = 0$. The uncertainties are set as $\Delta m_1 = 0.1\sin(t/2)$, $\Delta m_2 = 0.05\cos(t/2)$, and the initial control gains are $k_P^0 = 40$, $k_D^0 = 0.1$. The first simulation consists of 8 iterations. Figure 4(a) shows the tuning criterion $J(\rho)$ during these iterations. $J(\rho)$ decreases quickly from around 2.52 to about 0.75 in 8 iterations. Then the curve keeps steady. Therefore, the controller optimization is reached by eight iterations. More experiments will not bring significant improvements on the control performance, only take more energy cost. Figure 4(b) presents the proportional gain k_P in 8 iterations, and Figure 4(c) shows the differential gain k_D in 8 iterations. k_P increases from 40 to 40.32 after iterations. Then its variation begins to slow down. In Figure 4(c), k_D increases from 0.1 to 1.2 after eight iterations. The variation curve then comes to steady. From Figure 4, We can conclude the effectiveness of the IFT algorithm on our robust controller in 8 iterations. To further demonstrate the proposed controller and the IFT optimization strategy, Figure 5 shows the system performance comparison before and after eight iterations. Despite the same initial constraint-following error, the tuned controller can approximately eliminate errors in less than one period. However, the original controller shows a quite slow speed in eliminating the error.

The curves in Figure 4 are not steady enough, we provide more iterations to depict variations of the tuning control gains and the criterion. Figure 6(a) shows the tuning criterion during 50 iterations. No distinct variations on $J(\rho)$ have emerged after the first eight iterations. The proportional gain for 50 iterations is shown in Figure 6(b). From the 8th iteration to the 50th iteration, k_P increases about 0.04, while k_P increases nearly 0.32 in the first eight iterations. The magnitude of variations for k_P for the first eight iterations is 8 times of it for the last 42 iterations. The differential gain for 50 iterations is shown in Figure 6(c). From the 8th iteration to the 50th iteration, k_D increases about 0.1, while k_D increases nearly 1.1 in the first eight iterations. The magnitude of variations for k_D for the first eight iterations is 11 times of it for the last 42 iterations. Therefore, we conclude the validity of 8 iterations of the IFT algorithm for our robust controller.

For comparison, the ARC method in Reference 17 is simulated, and the results are shown in Figures 7-9. Figure 7 compares the constraint-following results among the proposed robust controller (after $i = 8$ iterations), the ARC, and the nominal trajectory. The magnitude of the constraint-following error under our tuned robust controller is much smaller than the one under the ARC. The tuned robust controller can eliminate the constraint-following error in about one period, while the ARC can not eliminate the error in one period. The explanation for the inefficiency of the ARC in our case is that only 1st order constraint-following error is considered when designing the ARC[17]. While both 0 and 1st order constraint-following errors are involved in our robust controller as given in equation (17), we also considered the optimization of tunable control gains k_P and k_D. Figure 7 shows the superiority of the robust controller and its IFT optimization algorithm over the ARC. For further comparison, Figure 8-9

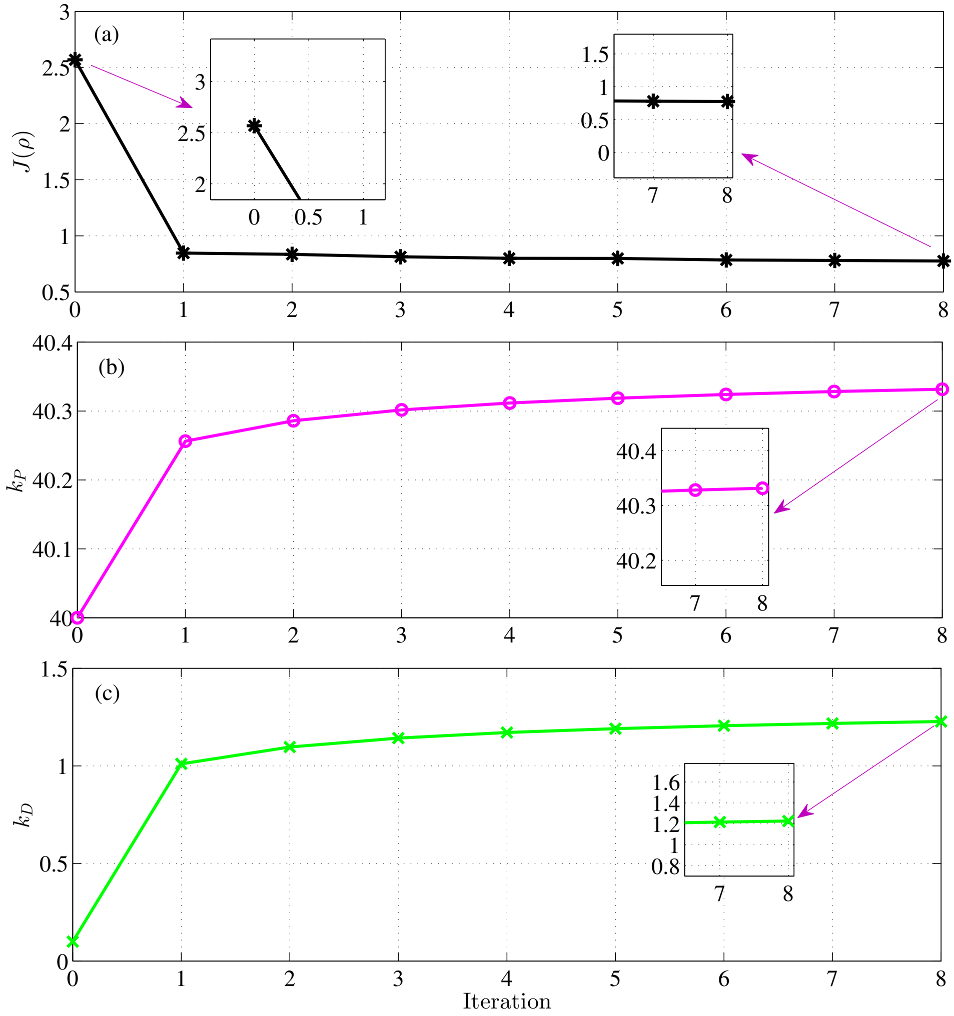

FIGURE 4 Parameter variations during 8 iterations.

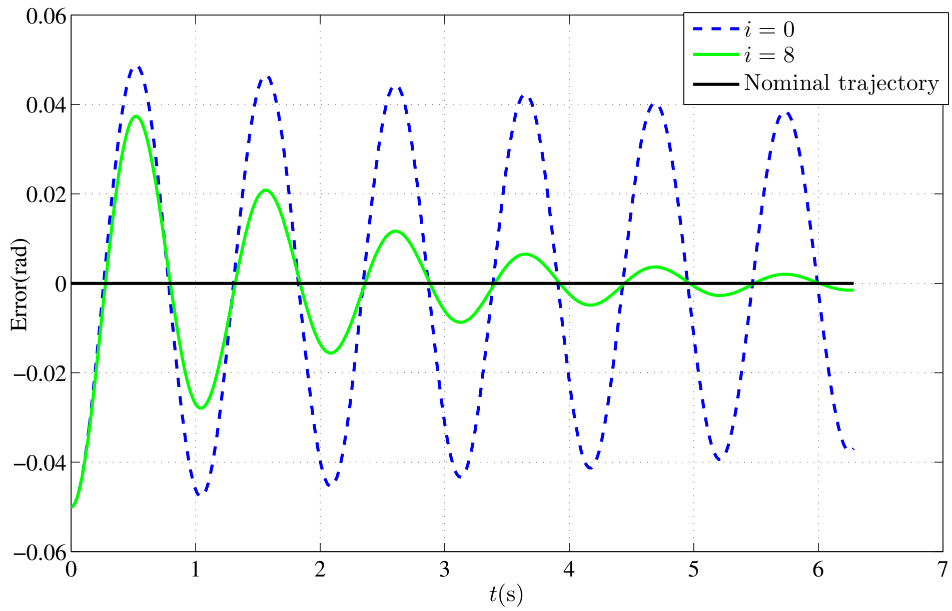

FIGURE 5 Trajectory tracking comparison: $i = 0$ vs $i = 8$ vs Nominal trajectory.

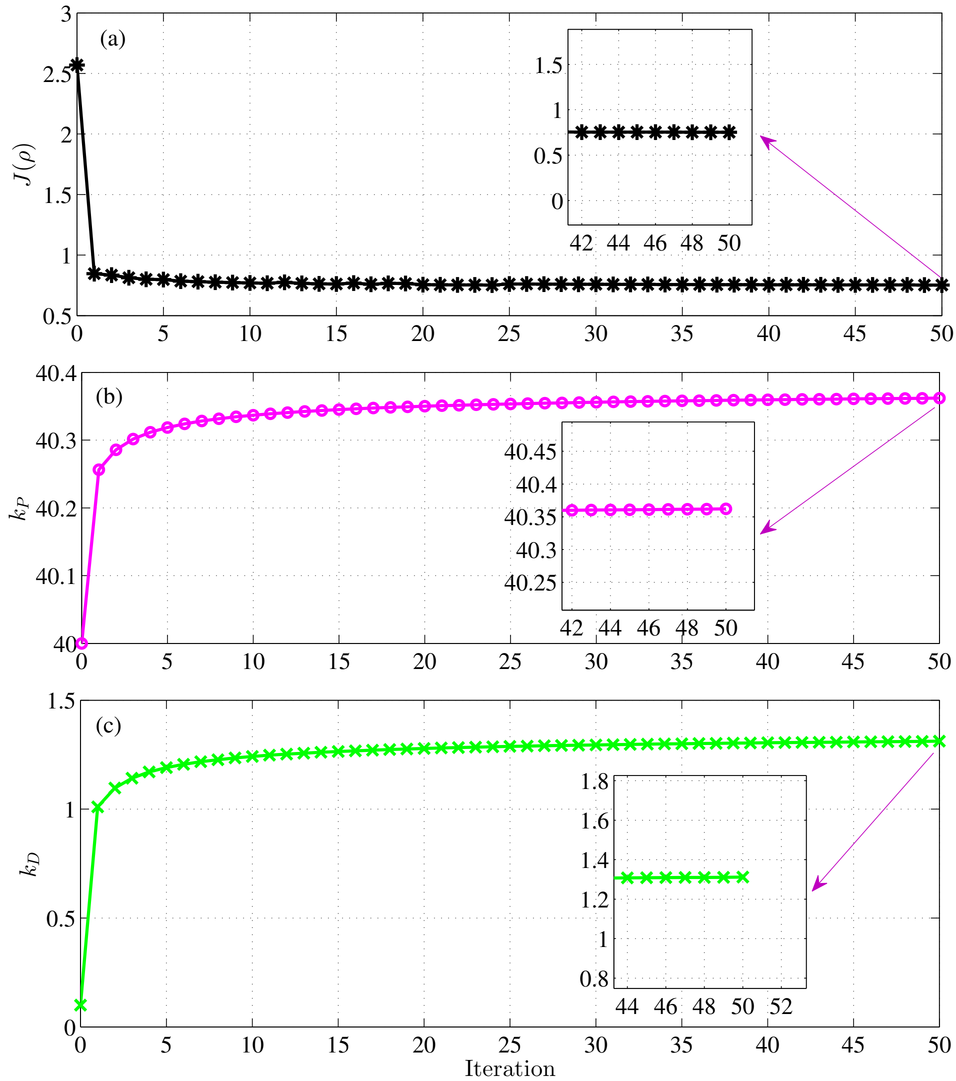

FIGURE 6 Parameter variations during 50 iterations..

show the input torques for joint 1 and joint 2 among the robust controller without tuning ($i = 0$), the tuned robust controller (after $i = 8$ iterations), and the ARC. The input torques for both joints under the ARC are larger than those under the robust controller. Besides, larger peak values of the input torques can be seen for the robust controller without tuning than the one after 8 iterations. Therefore, we conclude the validity of the IFT optimization algorithm on the robust controller and the superiority of the tuned robust controller over the ARC.

6 | CONCLUSIONS

This paper presents a novel optimal robust constraint-following control scheme for uncertain mechanical systems (MSs) subjected to periodic constraints. The robust controller is based on the Udwadia-Kalaba theory, which makes no linearization for nonlinear systems. Under the proposed controller, the uncertain MS can guarantee some deterministic performances. Furthermore, an iterative feedback tuning approach is introduced to optimize the robust controller. After optimization, the uncertain MS can reach the equilibrium between the system performance and the control cost. Simulation results on a classical 2-link rotational manipulator demonstrated the optimal robust controller. This paper considers only the optimal control of holonomic periodic constraints. Future work could focus on MSs with both holonomic and non-holonomic periodic constraints.

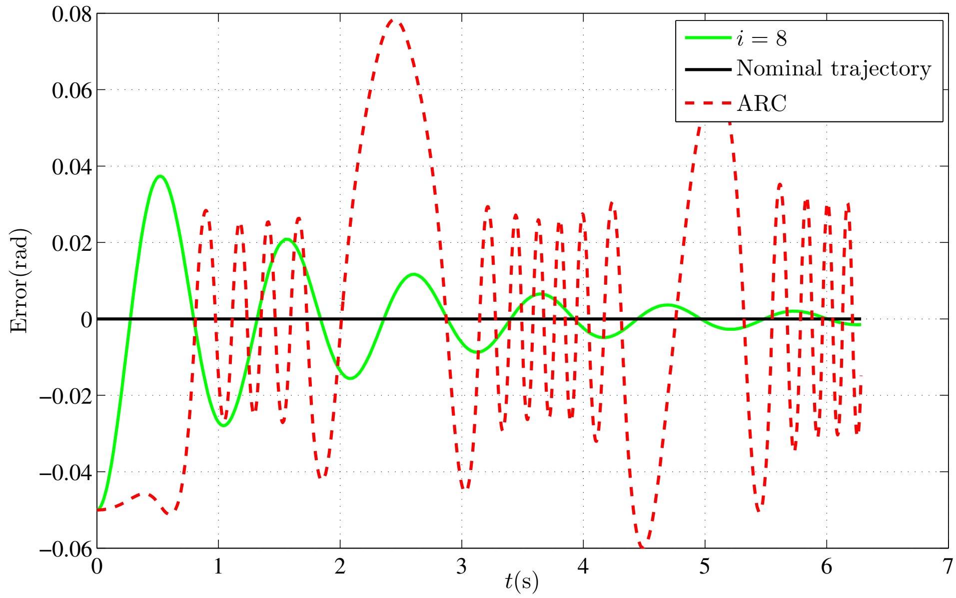

FIGURE 7 Trajectory tracking comparison: $i = 8$ vs ARC vs Nominal trajectory.

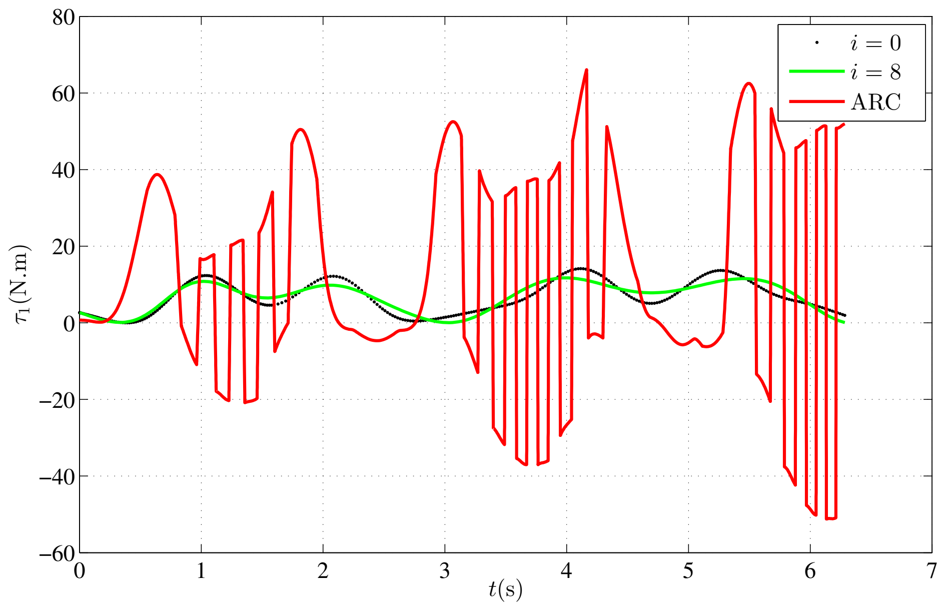

FIGURE 8 Input torque τ_1 comparison: $i = 0$ vs $i = 8$ vs ARC.

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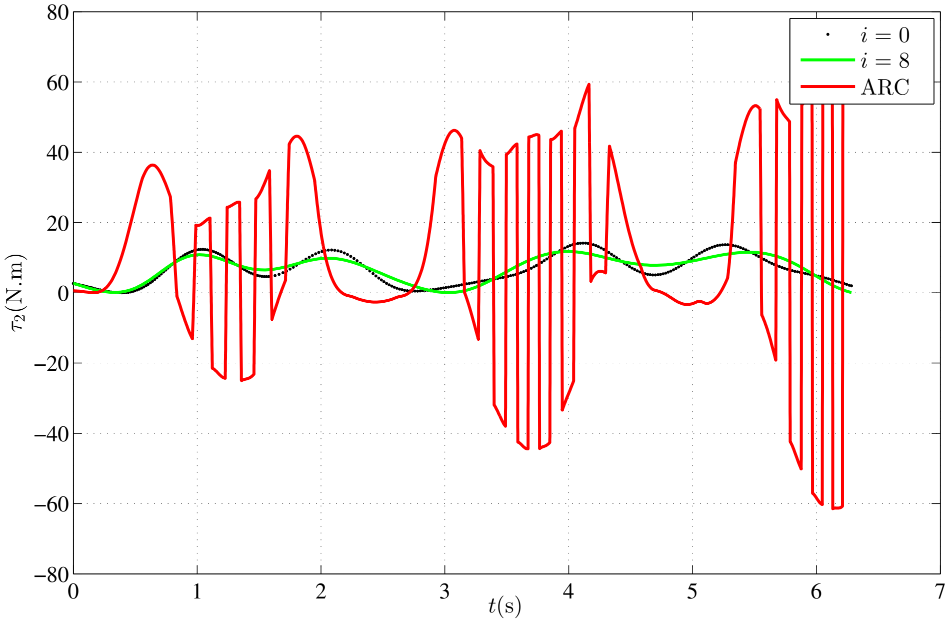

FIGURE 9 Input torque τ_2 comparison: $i = 0$ vs $i = 8$ vs ARC.

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