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The second European Conference on the Structural Integrity of Additively Manufactured Materials

# Nominal stresses to assess damage in notched additively manufactured steel subjected to constant and variable amplitude multiaxial fatigue loading

Luca Susmel\*

*Department of Civil and Structural Engineering, the University of Sheffield, Sheffield S1 3JD, UK*

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## Abstract

The present paper reports on the use of the Modified Wöhler Curve Method (MWCM) applied along with nominal stresses to estimate lifetime of notched additively manufactured metallic materials subjected to constant and variable amplitude multiaxial fatigue loading. The MWCM is applied by defining the critical plane via the direction that experiences the maximum variance of the resolved shear stress. In this setting, owing to the fact that the shear stress resolved along this direction is a monodimensional stress quantity, fatigue cycles are counted directly by using the standard Rain-Flow method. Based on a large data set taken from the literature, the validation exercise being performed strongly supports the idea that the MWCM used in conjunction with nominal stresses is successful in estimating fatigue damage also in notched 3D-printed metallic materials subjected to constant/variable amplitude multiaxial load histories.

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## 1. Introduction

Whether as a replacement for existing production methods on economic grounds, or because of the ability to produce components that have up until now been impossible, it is clear that additive manufacturing (AM) will certainly

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\* Corresponding author. Tel.: +44 (0) 114 222 5073; fax: +44 (0) 114 222 5700.

*E-mail address:* [l.susmel@sheffield.ac.uk](mailto:l.susmel@sheffield.ac.uk)

have an impact on the future of manufacturing. As far as fatigue design is concerned, since the pioneering work done both by Neuber (1958) and Peterson (1959), fatigue strength reduction factors,  $K_f$ , have been widely used by structural engineers to perform the fatigue assessment of notched components subjected to either pure axial, pure bending, or pure torsional loading. Even if, in such circumstances, this approach has proven to allow components to be designed by always reaching an adequate level of safety, examination of the state of the art shows that the scientific community has not agreed yet on a universally accepted strategy to be used to estimate fatigue damage in notched 3D-printed components subjected to constant/variable amplitude multiaxial fatigue loading.

In this challenging scenario, by using a large data set taken from the literature (Wang et al., 2021), the aim of this paper is to evaluate the reliability of the Modified Wöhler Curve Method (MWCM) (Susmel, 2009) when it is applied along with nominal net stresses to perform the multiaxial fatigue assessment of notched components of additively manufactured (AM) AISI 316L.

## 2. Fundamentals of the Modified Wöhler Curve Method

Consider a notched component subjected to a complex system of time-variable forces resulting in a multiaxial stress state at the critical section (Fig. 1a). According to the classical approach due to Neuber [1] and Peterson [2], the above stress state has to be determined in terms of nominal quantities calculated with respect to the reference section (Fig. 1a). After defining the necessary nominal stresses, the orientation of the critical plane at point O (Fig. 1a) can directly be determined by locating that plane containing the direction experiencing the maximum variance of the resolved shear stress - direction MV in Figure 1b (Susmel, 2010).

The MWCM takes as its starting point the idea that the plane of maximum shear stress amplitude is the plane where the probability of having the micro/meso-crack initiation process reaches its maximum value. Its application requires the calculation of the shear stress amplitude and the normal stress components (both amplitude and mean value) relative to this plane (Susmel & Lazzarin, 2002; Lazzarin & Susmel 2003). Under constant amplitude (CA) loading, fatigue lifetime is estimated via bi-parametric modified Wöhler curves (Fig. 2). The fatigue damage extent is quantified via the crack initiation plane stress ratio, which is defined as (Susmel, 2008; Susmel, 2009):

$$\rho_{eff} = \frac{m \cdot \sigma_{n,m} + \sigma_{n,a}}{\tau_a} \quad (1)$$

In definition (1)  $\tau_a$  is the shear stress amplitude relative to the critical plane, whereas  $\sigma_{n,m}$  and  $\sigma_{n,a}$  are the mean value and the amplitude of the stress normal to this plane, respectively. Material constant  $m$  is the so-called mean stress sensitivity index and can be determined by running appropriate experiments (Susmel, 2009). Stress ratio (1) is sensitive to the presence of non-zero mean stresses as well as to the degree of multiaxiality and non-proportionality of the load history being assessed (Susmel, 2008).

Assuming that both the inverse slope of the modified Wöhler curves,  $k_\tau(\rho_{eff})$ , and the reference shear stresses,  $\tau_{A,Ref}(\rho_{eff})$ , at  $N_A$  cycles to failure are linear functions of the  $\rho_{eff}$  ratio, the number of cycles to failure under CA multiaxial fatigue loading can be calculated as follows – see Fig. 2 (Lazzarin & Susmel, 2003):

$$N_{f,e} = N_A \left[ \frac{\tau_{A,Ref}(\rho_{eff})}{\tau_a} \right]^{k_\tau(\rho_{eff})}, \quad (2)$$

where (Susmel, 2009):

$$k_\tau(\rho_{eff}) = (k_n - k_{0n}) \cdot \rho_{eff} + k_{0n} \quad (3)$$

$$\tau_{A,Ref}(\rho_{eff}) = \left( \frac{\sigma_{An}}{2} - \tau_{An} \right) \cdot \rho_{eff} + \tau_{An} \quad (4)$$

In relationships (3) and (4),  $k_n$  and  $\sigma_{An}$  are the slope and the nominal stress amplitude extrapolated at  $N_A$  cycles to failure characterising the fully-reversed uniaxial fatigue curve. Constants  $k_{0n}$  and  $\tau_{An}$  are the corresponding quantities

associated with the fully-reversed torsional fatigue curve. Nominal stresses  $\sigma_{An}$  and  $\tau_{An}$  are referred to the reference section.

Assume now that the notched component seen in Fig. 1a is subjected to a variable amplitude (VA) load history. As done for the CA case, the stress analysis is again performed in terms of nominal stresses at point O (Fig. 1a). Under VA loading, the shear stress amplitude relative to the critical plane,  $\tau_a$ , is calculated from the variance of stress signal  $\tau_{MV}(t)$ , i.e. (Susmel, 2010; Susmel & Tovo, 2011):

$$Var[\tau_{MV}(t)] = \frac{1}{T} \int_0^T [\tau_{MV}(t) - \tau_m]^2 \cdot dt \Rightarrow \tau_a = \sqrt{2 \cdot Var[\tau_{MV}(t)]}, \tag{5}$$

where  $\tau_{MV}(t)$  is the shear stress resolved along direction MV (Fig. 1b),  $\tau_m$  is the average value of  $\tau_{MV}(t)$  and T is the time interval over which the assessed load history is defined (Fig. 1c). In a similar way (Susmel, 2010; Susmel & Tovo, 2011), the mean value,  $\sigma_{n,m}$ , and the amplitude,  $\sigma_{n,a}$ , of the stress,  $\sigma_n(t)$ , normal to the critical plane take on the following values (Fig. 1d):

$$\sigma_{n,m} = \frac{1}{T} \int_0^T \sigma_n(t) \cdot dt; Var[\sigma_n(t)] = \frac{1}{T} \int_0^T [\sigma_n(t) - \sigma_{n,m}]^2 \cdot dt \Rightarrow \sigma_{n,a} = \sqrt{2 \cdot Var[\sigma_n(t)]} \tag{6}$$

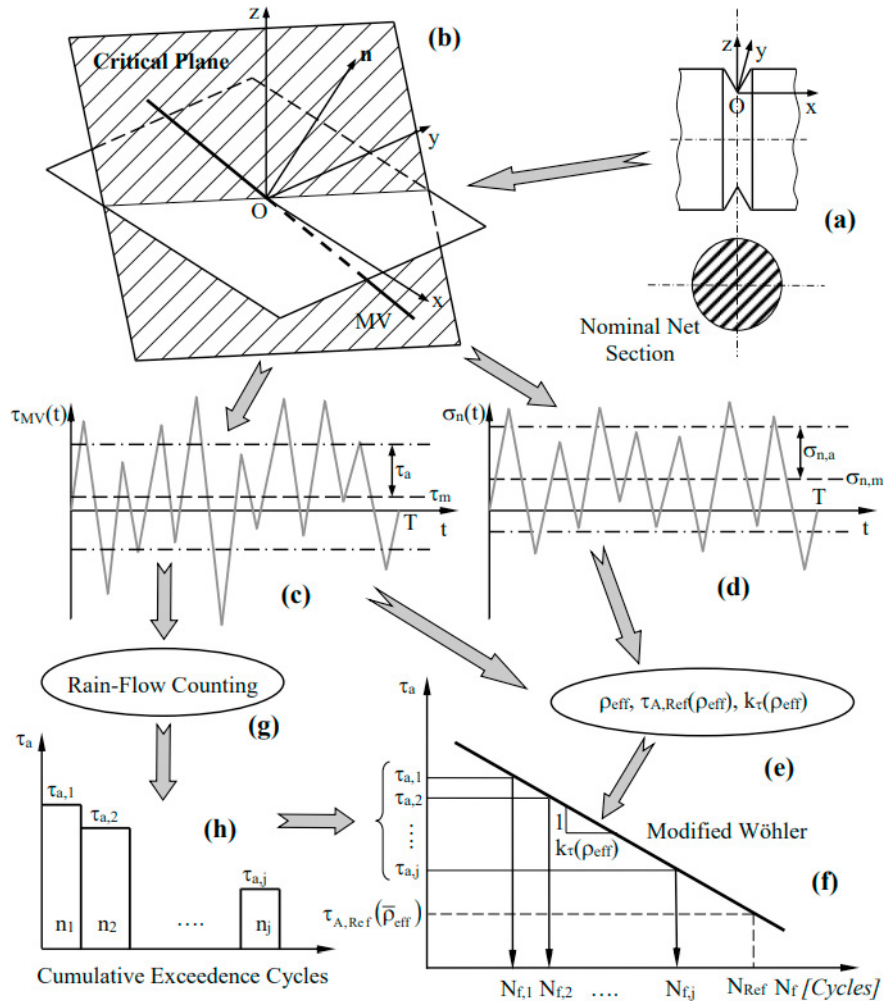


Fig. 1. In-field use of the MWCM to estimate lifetime under variable amplitude multiaxial fatigue loading.

As soon as  $\tau_a$ ,  $\sigma_{n,m}$  and  $\sigma_{n,a}$  are known, the value of  $\rho_{\text{eff}}$  associated with the VA load history under investigation is directly estimated according to definition (1). Stress ratio  $\rho_{\text{eff}}$  is then used to estimate the position of the corresponding modified Wöhler curve via relationships (3) and (4) (Figs 1e and 1f). Subsequently, by taking full advantage of the standard Rain-Flow method, the resolved shear stress cycles can now be counted (Figs 1c and 1g) to build the corresponding load spectrum (Fig. 1h). Finally, the calculated load spectrum has to be used to estimate the damage content associated with any counted shear stress level (Figs. 1h and 1f), the estimated number of cycles to failure being equal to:

$$D = \sum_{i=1}^j \frac{n_i}{N_{f,i}} \Rightarrow N_{f,e} = \frac{D_{cr}}{D} \sum_{i=1}^j n_i \quad (7)$$

To conclude, it is worth observing that according to the classical rule due to Palmgren and Miner, the critical value of the damage sum,  $D_{cr}$ , can directly be taken equal to unity.

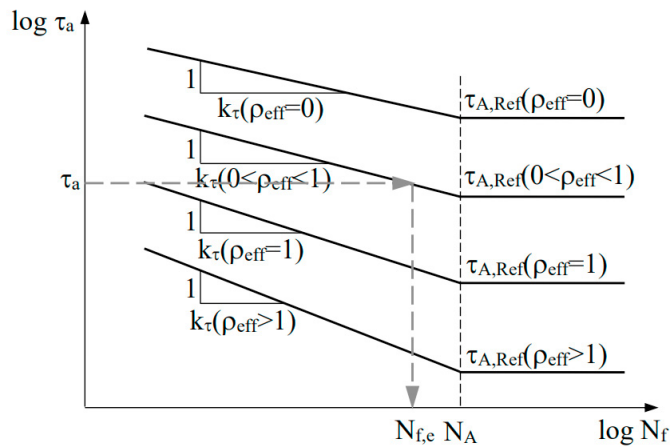


Fig. 2. Modified Wöhler diagram and modified Wöhler curves.

### 3. Experimental validation

The results considered in the present paper were generated by testing notched cylindrical samples of AM AISI 316L having gross diameter equal to 12 mm and net diameter equal to 8 mm (Wang et al., 2021). Three geometrical configurations were investigated:

- sharply V-notched samples having root radius,  $r_n$ , equal to 0.07 mm, resulting in a net stress concentration factor equal to 7.2 under axial loading,  $K_t$ , and to 3.1 under torsion,  $K_{tt}$ ;
- intermediate U-notched specimens having  $r_n=2$  mm ( $K_t=1.8$ ,  $K_{tt}=1.3$ );
- bluntly U-notched cylindrical bars having  $r_n=5$  mm ( $K_t=1.4$ ,  $K_{tt}=1.1$ ).

Both the CA and the VA results summarised in Figs 4 and 5 were generated under combined tension and torsion by exploring in-phase (IPh) and  $90^\circ$  out-of-phase (OoPh) situations, with zero ( $R=-1$ ) and non-zero mean stress ( $R=0$ ). The tests under VA loading were run by adopting the load spectrum shown in Fig. 3. In this chart  $\Sigma_{a,i}$  and  $T_{a,i}$  are the amplitudes of the axial and torsional nominal stresses at the  $i$ -th stress level, respectively, and  $\Sigma_{a,\max}$  and  $T_{a,\max}$  are the maximum values of the amplitudes in the spectrum itself.

The fully-reversed axial and torsional fatigue curves (expressed in terms of nominal stress quantities) needed to calibrate the MWCM were derived from the corresponding plain fatigue curves via the following classic relationships (Lee et al., 2005):

$$K_f - 1 = q(K_t - 1); K_{ft} - 1 = q(K_{tt} - 1) \quad (8)$$

In Eqs (8)  $K_f$  and  $K_{ft}$  are the fatigue strength reduction factors under axial and torsional fatigue loading, respectively, whereas  $q$  is the notch sensitivity factor, where, by definition (Lee et al., 2005):

$$K_f = \frac{\text{unnotched endurance (fatigue) limit}}{\text{notched endurance (fatigue) limit}} \tag{9}$$

$$K_{ft} = \frac{\text{unnotched torsional endurance (fatigue) limit}}{\text{notched torsional endurance (fatigue) limit}} \tag{10}$$

For the AM stainless steel under consideration,  $q$  was estimated to be equal to 0.082. This value for  $q$  was calculated via the endurance limit of the uniaxial fatigue curve generated by testing the sharply V-notched specimens ( $K_t=7.2$ ).

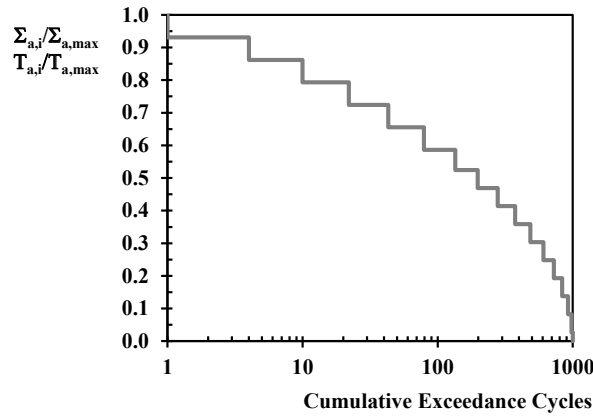


Fig. 3. Adopted VA load spectrum.

The fully-reversed plain uniaxial fatigue curve had negative inverse slope equal to 15.3 and reference stress amplitude at  $2 \cdot 10^6$  cycles to failures equal to 249 MPa. The fully-reversed plain torsional fatigue curve was instead characterised by a slope of 32.7 and a reference shear stress amplitude at  $2 \cdot 10^6$  cycles to failures of 216.1 MPa. A mean stress sensitivity index,  $m$ , of 0.53 was directly determined from the plain uniaxial fatigue curve generated under  $R=0$  (Susmel, 2009).

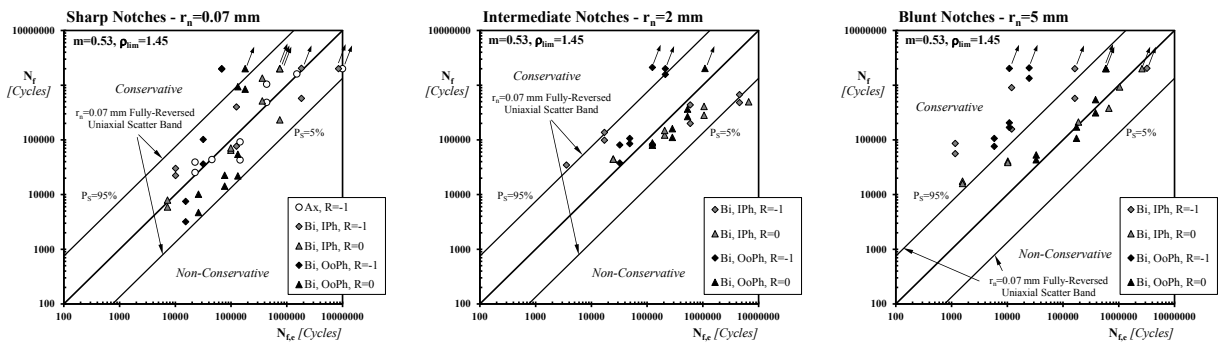


Fig. 4. Accuracy of the MWCM applied in terms of nominal stresses in estimating lifetime of notched AM 316 L subjected to CA loading.

The experimental,  $N_f$ , vs. estimated,  $N_{fe}$ , lifetime diagrams reported in Figs 4 and 5 make it evident that the proposed methodology proved to be capable of estimates falling within the scatter band associated with the notch

curve used to estimate  $q$ . In Figs 4 and 5 the target error bands are delimited by the two straight lines calculated for a probability of survival,  $P_s$ , equal to 5% and 95%, respectively.

#### 4. Conclusions

The MWCM applied in terms of nominal stresses is seen to be successful in estimating fatigue lifetime of notched components of AM AISI316 L subjected to CA/VA multiaxial fatigue loading. Such a high level of accuracy was obtained independently of tested geometrical feature as well as of profile of the CA/VA loading path being investigated. These encouraging results strongly support the idea that the MWCM is an effective tool that allows 3D-printed metallic components to be designed against multiaxial fatigue by always reaching an adequate level of safety.

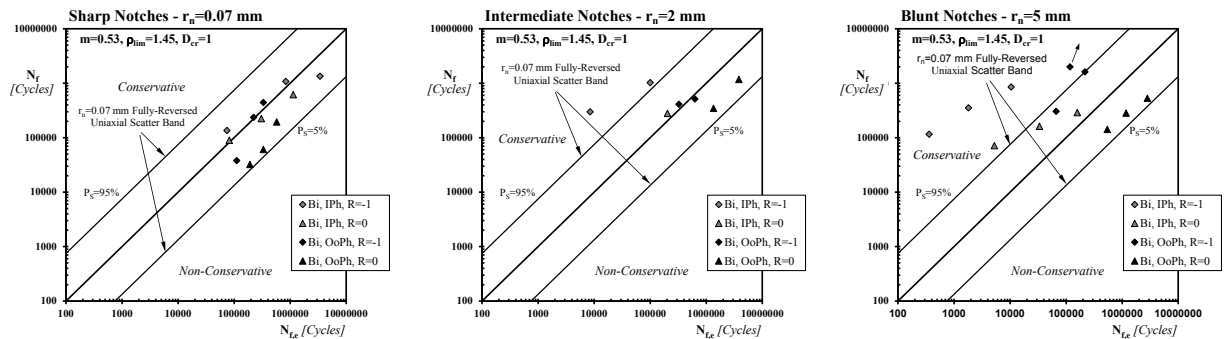


Fig. 5. Accuracy of the MWCM applied in terms of nominal stresses in estimating lifetime of notched AM 316 L subjected to VA loading.

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