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Following the Curve? Reviewing the physical basis of the SCS curve number method for estimating storm runoff.

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Abstract

Less attention has been paid to runoff generation from semi-arid than from humid-temperate catchments. The SCS curve number approach is simple to apply and widely used, but lacks physical underpinning. Here output from a runoff generation models is compared with data from field measurements, making use of 11 years data from rainfall and runoff events at the Sierra de Enguera Soil Erosion Experimental Station in Eastern Spain. Runoff from natural rainfall events was monitored for ten years on bare plots of 1-16 metre length. The largest storm event was of 142 mm, generating runoff of up to 115 mm on the smallest plots. The model presented simulates overland storm flow on a sloping rough and unvegetated surface, representing an area of 320x320 m. Green-Ampt infiltration constants are randomly assigned to each cell in a 128x128 grid, and rectangular storms applied at a range of total amounts and intensities to simulate runoff at each transect across the area. A simple algebraic expression is developed to estimate total runoff and storage in terms of storm size and duration, and plot length, with parameters that reflect infiltration behaviour, and this expression is compared with the SCS curve number approach. For the very largest storms, both expressions converge asymptotically towards 100% runoff, but the revised expression greatly improves estimates of runoff from smaller events. Output of these simulations is compared with measured storm runoff data on bare runoff plots at the Sierra de Enguera experimental Station in SE Spain and gives further support to the proposed expression for storm runoff.

Keywords: storm runoff, runoff plot, simulation model, SCS curve number

INTRODUCTION

There have been a number of papers suggesting improvements to storm runoff forecasting, many of them through modifying the SCS Curve Number method, both for stand-alone use and for incorporation into other models. The simplicity of the curve number approach recommends it, although it is recognized that the method is largely empirical and that reliable runoff estimates depend on many other factors besides total storm precipitation.

Estimation of total storm runoff requires the partition of the rainfall between storage (in the soil) and overland flow runoff. In addition, it is hypothesized that some storage is filled before runoff begins. However, little attention in formulating and applying the SCS method has been paid to the physical processes involved, and, in particular, how runoff responds to the size of the area modelled. Although a substantial amount of work has been done in experimental watershed (e.g. Parsons et al., 2006; Wainwright et al., 2008; Turnbull et al. 2013; Kampf et al., 2018), this has not generally been applied to modify the curve number method.

In its most widely used form (Soil Conservation Service, 1972; Hawkins et al, 2008), the curve number method estimates storm runoff as

$$Q = \frac{(R-\alpha S)^2}{R+S(1-\alpha)} \tag{1}$$

Where R is the storm rainfall in mm

S is the soil storage in mm

and the αS term represents the portion of storage that is abstracted prior to the commencement of runoff, with α commonly taken as 0.2.

The Storage constant S is related to the Curve number CN by the relationship

$S = 25.4(1000/CN - 10)$.

For small storms, there is zero runoff for $R<\alpha S$, and for large storms runoff tends towards $Q=R-S(1+\alpha)$.

As well as its application as a stand-alone model for small catchments (Hawkins et al, 2008), the SCS method has been incorporated into other models including the Soil-Water Assessment Tool (SWAT, Nietsh et al, 2011) and EPIC (Sharpley & Williams, 1990), and extended for larger areas (Williams & LaSeur, 1976). A large number of relevant factors may be included for estimating or modifying the Curve Number. These include relatively static factors such as vegetation/crop cover, tillage and residue management, gradient

(Sharply & Williams, 1990; Huang et al, 2005) and dynamic factors including antecedent moisture (Castillo et al., 2003), storm duration and accumulated evapotranspiration (Shi & Wang, 2020; Ajmal et al, 2020). These controlling factors have generally been used to improve estimates of the curve number, and in some cases of the proportion α.

The SCS Curve number has been the most widely used model to forecast total storm runoff from total storm rainfall and is embedded within a number of other models that are used to forecast erosion and solute behaviour at range of scales. However, despite its widespread application, the curve number method has little theoretical underpinning, and is not explicitly related to spatial scale or topography (Hawkins et al., 2008). Various attempts to partially remedy these deficiencies (Bartlett et al, 2016, Williams et al, 2012) have been proposed, and this paper proposes an alternative expression for total storm runoff that is designed to overcome some of these shortcomings The simulation model supporting the proposed model allows explicit representation of both topography, storm intensity and spatial variability in infiltration parameters, helping to provide an improved physical basis for modifying and transferring model parameters between sites and storm conditions.

Lateral near-surface flow has been envisaged as generated, either where rainfall intensity exceeds the infiltration capacity of the soil surface (infiltration excess overland flow: Horton, 1931), or where the soil becomes saturated so that additional rainfall is diverted laterally (saturation excess overland flow: Dunne and Black, 1970). It is now recognised (Cammeraat, 2002; McDonell, 2003) that rainfall intensity may exceed the capacity of the soil to percolate downwards at either the surface or within the soil (Figure 1), and that significant spatial variations in infiltration rate can lead to a ‘fill and spill’ pattern of saturated patches and upstanding roughness elements which guide the inter-connection of runoff during storms (Chu et al., 2013, Penuela et al., 2016) to drive surface and/or subsurface lateral flow. In this paper these spatial patterns are simulated for infiltration excess overland flow with the saturated patches at the surface, although there may be strong similarities between surface and subsurface patterns (McDonnell, 2013).

[Insert Figure 1]

Rainstorms are rarely simple in profile or in antecedent conditions, so that any simple rainfall-runoff relationship is likely to show wide variations. The bar for acceptability is therefore low (Wendt et al., 1986), and the approach is only justifiable if kept simple, facilitating inclusion in other models.

Contributions to improving runoff estimation can be made through conceptual modelling or through analysis of experimental data. Here a conceptual model has been implemented to generate runoff for simple block storms of different sizes, and for plots of different length and gradient, to provide an alternative algebraic expression for storm runoff to replace the SCS curve number approach. Model results have been compared with field runoff data for plots of different length over ten years of natural rainfall. The advantages of this approach are seen as providing an expression for total storm runoff that rivals the simplicity of the SCS curve number method, provides improved forecasts of runoff, particularly for smaller storms, and has a more explicit relationship with the underlying physical processes and controls.

This paper explores the internal structure and implicit connectivity within a modelled hillslope, in order to further generalise and improve the rainfall-runoff model previously presented and examine its internal structure. The model output that underlies this relationship is illustrated through examples. The storms have been generated on a roughened surface draped across a uniformly sloping rectangular plot. Figure2(a) shows the contours on an example surface used here. Figure 2(b) shows total storm discharge at every point on this surface, after applying a 120mm storm at an intensity of 30 mm per hour. Discharge is in units of mm x cell length. The random convergences are sufficient to create local concentrations of catchment area and discharge towards the base of the slope, with discharge generally increasing with area, both downslope and laterally, in convergent areas.

[Insert Figure 2]

The model outputs and the proposed expression for storm runoff have been compared with measurements of rainfall and runoff made over ten years in the El Teularet experimental site in Southeast Spain.

2. METHODS

The modelling approach taken here has been applied for conditions of infiltration excess overland flow, although comparisons may be drawn with sub-surface fill-and-spill configurations. Flow patterns and slope-base output from a square sloping plot have been simulated for isolated storms of constant intensity. The plot has a roughened surface and randomly distributed infiltration parameters. During a storm, areas of low infiltration create patches of saturation that generate overland flow which may connect with other saturated patches or re-infiltrate downslope in patches of higher infiltration. In the early part of a storm, saturated patches appear to be random and poorly connected. Saturated areas close to the slope base provide some outflow, even in brief and/or low intensity storms. In progressively larger storms, overland flow connections with the slope base are established farther and farther up the slope as infiltration rates fall and flows converge. After rainfall stops, there is no more flow contribution from the top of the slope, but connected flow persists downslope in areas of flow concentration and shapes the recession limb of the outlet hydrograph. In principle, every cell has a film of flowing water that receives rainfall, loses infiltration, and from which overland flow is redistributed to downslope cells. There are no closed depressions as in the puddle model developed by Chu et al. (2013), and every cell therefore has the potential to become part of a connected flow path. The pattern of saturation is most strongly guided by the differences in infiltration parameters and how this is expressed as infiltration accumulates over time in each cell. Connected cells are also affected by the surface topography, which creates local convergence of flow, and these differences are most evident towards the end of and after rain, allowing flow to persist longest in downslope areas of convergence.

2.1 Model description

The detailed model that has been used (Kirkby, 2014) generates infiltration excess overland flow on a grid representing either a fractally roughened uniform slope or a more topographically structured surface with distinct valleys, but without closed depressions. An updated and revised version of this model is used here. Green Ampt (1911) infiltration parameters are randomly assigned for each cell (nominally of 2.5m in the 128x128 cell grid), with values drawn from a specified distribution based on field measurements in S.E. Spain. Overland flow is routed downslope across the surface, according to a probability distribution of overland flow 'droplets' based on D8 flow directions, a method that has also been applied for saturation excess overland flow (Gao et al 2016,2017). Overland flow is generated in a source cell wherever inflow and rainfall exceed infiltration. 50 replicated instances of this overland flow are each treated as independent droplets. Each 'droplet' travels towards a neighbouring cell randomly chosen from the probabilities assigned to each of the D8 downslope directions. In travelling to the neighbouring cell, droplet mean velocity is calculated from the local gradient and overland flow depth, and interpreted as the probability of stopping in the receiving cell within the current time step. If the droplet does not stop this process repeated until the droplet comes to rest. This process of droplet routing is performed for the 50 replicate droplets and their mean is used to define the redistribution of the overland flow generated in the source cell in each time step.

Previous work (Kirkby, 2014) simulated total storm runoff, Q at the slope base from simple storms of constant intensity and total storm rainfall R, on initially dry surfaces.

The expressions derived are well behaved at extreme values in the following ways.

First $Q = 0$ when $R = 0$. This is a self-evident requirement. Second, there is very low runoff for small storms, for which the expression adopted behaves like $Q \sim R^{n+1}$. This seems to be a more appropriate response than the sharp lower threshold for runoff in the SCS curve number method, since both model and field data (Luk & Morgan, 1981; Cammeraat, 2002) show that, in even small storms, patches of low infiltration near the outlet boundary are able to deliver small amounts of runoff before their flow contribution is absorbed in higher infiltration areas. Thirdly, at high storm amounts, total storm runoff asymptotically approaches total storm rainfall, a behaviour in common with the SCS approach. However, the expression previously proposed seemed to suggest that the volume of infiltrated water stored in the soil, together with the volume of water in detention upon the surface decreased as total storm rainfall was increased, appearing to violate the requirements of mass balance, and leading to the revised formulations proposed here, that differ appreciably in the forecast volume of runoff for the largest storms.

2.2 Erosion plot data

The model described above is compared with data for runoff from the site of El Teularet in the Sierra de Enguera, SSE of Valencia, Spain (Cerda et al., 2005; Bagarello et al, 2018). The Sierra de Enguera range within the Massís del Caroig in Eastern Spain (750 m.a.s.l., 38° 55′ N, 00° 50′ W) was selected to establish the Sierra de Enguera Soil Erosion and Degradation   Research Station (Figure 3). This is a rainfed and rangeland use region in the Eastern part of the Iberian Peninsula. The climate is typical Mediterranean with a mean annual temperature of 12.7 °C as registered in the nearby meteorological station of Las Arenas Enguera (5 km from the study area) and mean annual rainfall of 540 mm. The soil is a Typic Xerorthent (personal communication, Soil Survey Staff, 2014) with a clay loam texture.. Glyphosate herbicide was applied following the strategy of local farmers who apply herbicide to suppress weeds, and used here to maintain a bare soil surface.

[Insert Figure 3]

A set of five plots under different agriculture and forest managements were established between 2002 and 2003, and the first measurements took place in January 2004. The data used in this investigation were collected from 2004 to 2014 from the bare soil plots. Plots were bounded with aluminium sheets, 1 mm thick and 50 mm high, to achieve plots of different sizes (1×1; 1×2; 1×4; 2×8 and 3×16 m^2) (Figure 3). Plots having different areas were obtained varying both plot lengths and widths and were established in an area having a gradient of 5%. Runoff (mm), sediment concentration (g L^{-1}) and soil loss (g m^{-2}) were measured after each rainfall event. More than 6 hours without rainfall was used as the threshold to distinguish rainfall events. Runoff was collected from each plot by a 0.15 m wide and 0.15 m deep gutter. The collected runoff was conveyed, by a 0.4 m diameter pipe, into containers with storage capacities of 125, 250, 375, 600 and 1000 litres for the 1, 2, 4, 16 and 48 m^2 plots, respectively. Runoff volume was recorded after each major rainfall event.

The storm runoff model has been fitted to these runoff data, to demonstrate the potential applicability of the model as an effective forecasting tool.

The measurements of rainfall for 2004-2014 from the El Teularet runoff plots provided over 450 rain days. Runoff measurements were made on 300 occasions, so that there was some missing data, almost exclusively for the smallest rain events.. When it was not possible to measure runoff after every rainfall event, events between successive runoff measurements have been combined into a single ‘effective rainfall’. Thus was done by correlating runoff from single rain events with its associated rainfall, and giving greatest weight to the smaller events..It was found that runoff was proportional to the third power of rainfall, and it will be seen below that this relationship is consistent with the final expression proposed below. Effective rainfall over successive rainfall events where some runoff collection was missing was therefore calculated as $[\Sigma(r^{3)}]^{(1/3)}$, giving appropriately greater weight to the largest rainfall in the sequence. With this consolidation, measurable runoff was compared with the effective rainfall for 220 runoff measurements.

3. RESULTS

3.2 Modelled storm runoff

The relationship between discharge and area is illustrated explicitly in figure 4, for two contrasting storm sizes, and with higher infiltration rates in (b). Values for every cell across the grid are plotted for three cross-slope transects near the top, middle and bottom of the slope. The horizontal axis shows the areas drained to each individual cell across the transect, with their average equal to the transect value, x. It can be seen that the roughening of the surface produces a wide range of areas across the transect. In each case there are strong overall relationships, and the trend within individual transects differs slightly from the overall trend. It is also clear from the regression lines that discharge increases less than linearly with area, and more strongly so for the smaller storm and higher infiltration (in figure 4b), so that runoff (discharge per unit area) is decreasing with area drained. In a simulated storm, four stages of response can be distinguished. Figure 5 shows two example hydrographs that illustrate these stages.

[Insert Figure 4]

[Insert Figure 5]

1. At the very start of a storm, infiltration capacity is theoretically very large, following the Green-|Ampt expression

   $$f = A + B/S \qquad (2)$$

   where f is the instantaneous infiltration rate (mm. hr^{-1}),

   S is the conceptual near-surface storage (mm: initially zero)

   and A, B are the parameter values that are randomly and independently distributed across grid cells. A is the steady infiltration rate that conceptually leaks from the near-surface store until exhausted, and represents the steady final long-term infiltration rate. B controls the initial rapid infiltration onto the near-surface store, S.

   In this first stage, almost all rain infiltrates into the near surface store, and there is only very limited runoff from saturated patches close to the outlet.

2. Quite soon near-surface storage increases, and, in the second stage, infiltration rate is controlled by equation (2) over an increasing proportion of the area. Average detention depths increase but slope-base runoff increases only slowly, since much of the ponded water is not connected to the slope base. In large storms, runoff may approach an almost

steady state, in which rainfall intensity is partitioned between infiltration and runoff (figure 5b), whereas in smaller storms (figure 5a) runoff continues to increase.

3. A third stage begins when storm rainfall ends. Existing detention continues to support infiltration, though over a shrinking area as there is no further contribution from the top of the slope. This allows further addition to the near-surface store for a while from the shrinking ponded area. Average detention and runoff both decrease sharply, with losses due to the runoff itself and the continuing infiltration.
4. In the final stage, all remaining water in the slope has infiltrated, and the near-surface stores gradually drain into the soil beneath.

[Insert Figure 6]

Figure 6 helps to further illustrate these stages of runoff and storage for storms of different total storm size (8 – 480 mm) and intensity. In (a) and (b) storms all have a duration of two hours, and so widely varying intensities. In (c) and (d) the storms are at constant intensity of 60 mm.hour^{-1} and differing duration. In each case, infiltration initially absorbs almost all rainfall, and the small volume of runoff behaves as a power function of rainfall. For the fixed duration storms in (a) and (b) the final storage increases only very slightly with storm size and almost all additional rainfall is converted into runoff. With the storms of fixed intensity in (c) and (d), final storage rises significantly with increasing storm size., and not all of the additional rainfall contributes to runoff. These differences are primarily due to the different durations of infiltration during rainfall and post-rainfall saturation. Slope length is also shown as a controlling variable. The effect of increasing slope length is seen in an increase in the total available average storage depth for large storms, and in the exponent of rainfall for small storms.

The relationships seen here may be described as showing two asymptotic behaviours. For small storms infiltration approaches 100% of rainfall. For large storms, total infiltrated storage approaches an upper limit that increases primarily with storm duration, but also with slope length, through its effect on the duration of runoff after the storm ends. These two extreme behaviours are described by the relationships:

$$S = R \quad \text{for } R \ll \Theta \tag{3}$$

$$S = \Theta \quad \text{for } R \gg \Theta \tag{4}$$

Where R = storm rainfall (mm),

S = Storm cumulative infiltration (mm)

and Θ = Storage threshold for cumulative infiltration (mm)

Empirically, the storage threshold may be expressed as

$$\Theta = b + aT + c \log_2(L/L_0) \quad (5)$$

Where T is storm duration (hours),

and L is slope length (m)

For the simulation shown the constants a, b, c, L_0 take the values

a =10 mm.hr $^{-1}$; b = 10 mm; c= 2 mm; L_0 = 2.5 m.

Repeated runs suggest that the constant $[b - c \log_2(L_0)]$ reflects the initially declining infiltration rate [B in equation (2)]; the constant a reflects the long-term final infiltration rate [A in equation (2)]. The constant c reflects the duration of runoff after the end of rainfall, perhaps also reflecting long term infiltration rate.

Combining the asymptotic expression of equation (3) and (4), It is proposed to use the Michaelis-Mentem (Michaelis and Menten, 1913)/ Budyko (Budyko and Gerasimov, 1961) family of expressions, which take the form

$$\frac{1}{S^m} = \frac{1}{R^m} + \frac{1}{\Theta^m} \quad (6)$$

$$Q = R - S \quad (7)$$

where Q = storm total runoff (mm), for some exponent $m > 1$.

For runoff. this expression behaves asymptotically like

$$Q = \frac{R^{m+1}}{m\,\Theta^m} \quad \text{for } R \ll \Theta \quad (8)$$

$$Q = R - \Theta + \frac{\Theta^{m+1}}{m\,R^m} \text{ for } R \gg \Theta \quad (9)$$

and, at the cross-over point (R= Θ),

$$Q = R.(1 - 2^{-1/m}) \quad \text{for } R = \Theta \quad (10)$$

These expression [equations (5)- (7)] provide an adequate description of the runoff response across the range of storms. There is a power law response for small storms, with the exponent m = 2 -5, and the runoff coefficient (Q/R) approaches 100% for the largest storms. Figure 7 compares values of (a) total storm storage with standard error of estimate (SEE) = 2.4 mm and (b) total storm runoff, with SEE of 0.24 for the base 10 logarithm of runoff (equivalent to

1.7 x), obtained by comparing the full simulation with equations (5) to (7) above. It can be seen that there is a high level of agreement in both storage and runoff between the full model results and the simplified expressions of equations (6) and (7). Figure 7(c) compares the full model storage with the SCS method for Curve Numbers of 80 and 90, always showing much greater divergences from the simulated storage (SEE=13.2 mm, with the best CN=79 for 60 mm/hour storms, and SEE = 3.3, with the best CN = 85 for 2-hour storms). Substantial improvements in forecast runoff are also evident, particularly for smaller storms, although, to provide a useful forecast of storm runoff, the effect for large storms is, naturally, seen as the more important. These expressions in equations (5) to (7) are proposed as an enhanced replacement for the SCS curve number method.

[Insert Figure 7]

The expression is relatively insensitive to the topography of the sloping surface. If similar storms are applied to the roughened surface of figure 2 and to a more strongly valleyed surface, estimated runoff values lie within the confidence bands, perhaps because runoff generation is a near-linear process. However, the three-dimensional shape of the surface has a profound influence on sediment transport. If, as a first approximation, sediment transport is estimated as proportional to discharge squared multiplied by gradient the pattern of sediment transport strongly reflects the structure of ridges and valleys and is then strongly influenced by the differences in area drained.

3.3 Comparison with erosion plot data.

The measurements of daily rainfall and runoff provided 220 runoff measurements from the 450 observed rain days. When it was not possible to measure runoff after every rainfall event, events between successive runoff measurements have been combined into a single 'effective rainfall' as described above, providing measured runoff corresponding to effective rainfall for 220 events.

Following equations (6) and (7) proposed above, the plot runoff and storage were estimated for all events and for the four plot lengths (1, 2, 4, 8 and 16m). With these data, it was found that the best fit between observed and estimated runoff was obtained when the exponent m in equation (6) took the value of 2.0. A value for the storage threshold, Θ was then fitted for each of the plot lengths. With these values, Figure 8(a) shows the level of agreement between observed and estimated runoff for the 220 events and four plot lengths. 90% of the data points lie within a factor of 5x around the 1:1 line. Figure 8 (b) shows the non-linear relationship found between the storage threshold, Θ and plot length. With no data on storm duration, it is difficult to compare directly with equation (2) above, though both show a diminishing increase in threshold with increasing plot length.

[Insert Figure 8]

Values for the storm threshold have been selected to optimise estimates of runoff, and Figure 8(c) shows their impact on estimates of storage. Here the solid curves indicate the estimated storage [from equation (6)]. The plotted points are binned values, each the average for ten sequential values of ranked storm rainfall. The upper grey line is the 1:1 line, which has been seen to be the asymptotic state for small rainfalls [equation (3) above]. These curves should be visually compared with the forms of figures 6(b) and 6(d) above, suggesting that the field data lies closer to the constant duration (with the exponent in equation 6, $m = 2.8$) than to the constant intensity storm model ($m = 4.7$). The consistent behaviour of the proposed storm runoff model provides some confidence in proposing equations (6) and (7) as a viable alternative to the widely used runoff model encapsulated in the SCS curve number approach.

4. DISCUSSION

4.1 Comparison with other Curve Number revisions

Some modifications to the SCS method allow the Curve Number value to respond to storm rainfall depth (Zhang et al, 2019). Others (Bartlett at al, 2016) distinguish storage before and after the initiation of runoff, summarizing their results in the modified SCS-CNx form which can be written as

$$Q = \frac{R^2 + R\frac{Sp}{1-p}}{R + \frac{S}{1-p}} \qquad (11)$$

For a constant p that is a pre-threshold runoff index.
Assymptotic behaviour for small storms is $Q=pR$, and for large storms $Q=R-S/(1-p)$.

All of these methods focus primarily on selecting the appropriate value for the curve number, and, from it, the asymptotic storage in large storms. However, the runoff response in storms of less than 20% of the final storage, S, varies widely between the various runoff models, and estimates from these smaller storms may severely underestimate the final storage, so that only a long period of record can give a reliable storage amount. It is therefore argued that it is important to improve estimation of runoff from smaller storms, even though their combined contribution to the water balance is small, and to the erosion balance even smaller.

One way of improving runoff estimation is through a more explicit logging of infiltration through a storm, and this has been proposed as an alternative method within SWAT, applying the Green-Ampt Mein-Larson method (Neitsch et al, 2011), although comparative tests suggest only modest improvements in catchment runoff estimates (King et al, 1999). This approach has also led to modification that allow continuous runoff simulation (Williams et al., 2012)

4.2 Spatial patterns of runoff generation

As illustrated in figure 4 above, the storage evolves, both temporally and spatially during the storm, and for as long as overland flow persists after rainfall has ended. The relevant final storage is, therefore, not a fixed property of the soil but also the result of the temporal and spatial evolution of the storm and its runoff. It is proposed here that one important missing component that is still absent in most revisions of the Curve Number approach is a consideration of spatial behaviour in generating runoff, with evolving patterns of downslope connectivity (Cammeraat, 2002; Hopp & McDonnell, 2009) in response to spatially variable infiltration rates. The existence of these connected patches of saturation allows some runoff generation, even in small storms, from areas of low infiltration close to the outlet. The saturated patches also maintain connected flow paths that allow runoff to persist after the end of rainfall. In both these cases, and more generally, the effective storage supporting runoff exhibited may be less than the spatially averaged value. Non-functional surface storage lies in the disconnected patches of surface detention, which is lost to infiltration without ever reaching the lower margin of a slope.

There has been considerable discussion about how plot length influences storm runoff (Wainwright et al. , 2008; Kinnell, 2008), with general agreement that longer plots/ larger areas generate less runoff (per unit area). The relationships shown in figure 6 may help to explain some of the observed differences. They show that small storms show much greater responsiveness to plot length and that both storm runoff and the final storage of infiltrated water are sensitive to storm duration, so that plot length is only one factor controlling storm runoff. This set of inter-relationships is explicit in equations (5) – (7) above.

4.3 Proposed storm runoff model

It is clear that the model proposed in equations (6) to (7) above, for storm runoff under conditions of infiltration excess overland flow, behaves very similarly to the original and modified versions of the SCS curve number model with storm runoff equal to rainfall minus a final storage depth. However behaviour differs increasingly for the smaller and more frequent storms that necessarily provide much of the data for calibrating storm runoff models.

The proposed model behaves as a power law expression for storms significantly smaller than the final storage. This behaviour is well represented in both the computational model and the runoff plot data presented. The experimental data, continuously collected over a decade, included no storm of more than 140 mm, and this was equal to the estimated storage threshold for a 20m long plot. Extrapolating the values shown in figure 8, thresholds for field-sized plots of, say 100m length, are estimated as 370 mm (with RI~100y). This comparison emphasises both the need to estimate final storage (Θ in equation 6) using evidence from smaller storms, and the value of using small runoff plots to support these estimates.

The simulation model supporting the proposed model allows explicit representation of both topography, storm intensity and spatial variability in infiltration parameters, helping to provide an improved physical basis for modifying and transferring model parameters between sites and storm conditions. Figure 8b, for instance, shows the clear dependence of the final storage value (Θ in equation 6) on slope length, and Figure 6 shows the dependence of the exponent m on storm intensity.

4.4 Alignment between modified CN method and current proposal

Although the proposed runoff estimator inevitably diverges from the SCS-CN method and its derivatives for small storms, it is possible to improve the degree of convergence for larger storms by modifying the SCS expression to give agreement both for asymptotically large storms and at the cross-over point defined as the storm size for which rainfall is equal to the final storage.

Agreement for arbitrarily large storms is achieved by modifying the denominator of equation 11 to the form:

$$Q = \frac{(R-\alpha S)^2}{R+S(1-2\alpha)} \qquad (12).$$

In this form, the final storage is *S,* agreeing with the final storage of *Θ* obtained from equation (6) above.

At the cross-over point ($R=S$) the runoff coefficient for the two expressions is given by:

from equation (12), $Q/S=(1-\alpha)/2$

and from equation (10), $Q/\Theta = 1-2^{-1/m}$

For the experimental value of m=2, the corresponding value of α in equation (12) is then 0.42

4.5 Wider implications

Figures 9 and 10 illustrate the implications of the model when extrapolated to longer slopes and more extreme rainfalls. Using the same parameter as in the field data, runoff coefficients are estimated as a function of storm rainfall for slopes of 5 to 50m in figure 9, where they are compared with SCS curve number relationships for the same final storages. Over the storm sizes seen in the field data, the greatest relative divergences between the two approaches are found for the large number of storms of less than 30 mm and with less than 10% runoff, for which the SCS method (as in equation 9) consistently underpredicts the small volumes of runoff. For larger storms, particularly with longer slope lengths, the SCS method seems to over-predict storm runoff.

[Insert Figure 9]

Figure 10 shows how, with the parameter values fitted to the Enguera field site, the runoff coefficient, declines with distance, particularly for smaller storms. Discharge continues to increase downslope, but approaches an upper limit in which additional rainfall is almost balanced by infiltration. This analysis is in accord with the field observation that, for semi-arid areas, intense rainfall appears to generate runoff almost everywhere, while little is recorded in streamflow.

[Insert Figure 10]

The importance of the proposed alternative runoff estimate is not, however, seen to lie in the quality of fit to individual data sets, all of which show wide variations that can be contained within either the curve number expression or the alternative form proposed here. What seems much more important and useful is that the proposed expression explicitly includes scaling for both rainfall intensity and slope length, providing a model with much greater possibilities for transference across scales and between sites and climates. Experimentation within the model environment also shows that the parameters in equation (5) also respond rationally to changes in infiltration parameters and their spatial variability, to gradient and, to a small extent, to micro-topography expressed through the potential for locally divergent flow. The potential to apply a consistent model at different spatial scales within a catchment is of value, not only in support of field experiments but also to distribute runoff and sediment transport within a field area or within a landscape evolution model.

It should be emphasised that the proposed model is based on the assumption that overland flow as generated by infiltration excess mechanisms, whether at the surface or within the soil, and should not be applied where this assumption is not met.

5. CONCLUSIONS

The expression proposed here is presented as an enhancement to the widely used SCS curve number method for estimating storm runoff from small catchments. Its particular strength is seen in better estimating the runoff from smaller storms. Although this is of less urgent interest than the response to major storms, the form of this relationship can be used to provide an estimate of the threshold storage for large events, as can be seen from the dependence on the threshold in equation (8) above.

Both the model and the field data in this paper refer to soil surfaces with no vegetation cover. Variations in infiltration rates and the tendency for perennial shrubs to form and grow on low mounds are thought to increase the importance of patchiness in overland flow (Rossi & Ares, 2012, 2016), if generated at the soil surface.

Additional benefits of the proposed formulation lie in its simplicity. Like the curve number method, it has only two parameters, Θ and m (in equations 5 -7 above). This simplicity allows ready incorporation into larger models, for erosion or solute transport, with the potential to apply the same model at every point within an area.

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DATA AVAILABILITY

The runoff plot data used in the is study are available from the authors on reasonable request.

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FIGURE LEGENDS

Figure 1: Conceptual regimes of lateral flow in the soil, defined by the depth below which rainfall or percolation intensity exceeds vertical hydraulic conductivity. Conditions are expected to change over time between and during storms, and to exhibit spatial variability.

Figure 2:
(a)Topography for example run.
'Smoothish' surface: 320 x 320 m (128 x 128 cells) area. 2m contours. 5% gradient
Divide at top: rolling lateral margins.
(b) Total storm discharge for example run on surface of (a). Total storm discharge (mm x cells) from a 120 mm storm at 30 mm/hour

Figure3: View of the study plot at the Sierra de Enguera experimental station. Bare plots of width x length 1x1, 1x2, 1x4, 2x8, 4x16 m. Runoff collectors in the foreground. Inset location map of SE Spain.

Figure 4.
(a)Total storm discharge as a function of drainage area for 3 lateral transects (x = 20, 64, 118) across the area.
120 mm storm at 30mm/hr. Discharge increases less than linearly, so that runoff is declining downslope ($Q\sim A^{-.4}$).
(b) Total storm runoff as a function of drainage area for 3 lateral transects across the area. 30 mm storm at 60mm/hr., with 4x higher infiltration rates. Note the divergence of individual transect trends.

Figure 6: Total storm slope base runoff and total storage for uniform storms of different total rainfall, and over different plot lengths.
(a) and (b): For storms of 2-hour duration
(c) & (d): for storms at 60 mm/hr

Note log scales for rainfall & runoff. Arithmetic scale for storage.
For small storms, almost all rain infiltrates, so that storage =rainfall, and total runoff is small, generated by low infiltration patches close to the base of the slope. This behaves as a power law of storm rainfall (with exponent 2-5) . For large rainfalls, runoff approaches 100%, and ultimate storage depends on duration, scaled to steady long-term infiltration rate. Total runoff is controlled by the limiting storage, which depends on storm and runoffduration.
The curves also show the relationship for slopes of lengths of 30, 40, 80 and 320 m. For low rainfalls, storage lies close to the 1:1 line, at which all rainfall infiltrates.. Note log scales for runoff (a & b), and differing scales for storage (b & d).

Figure 7. Comparison between Total storm (a) Storage and (b) Runoff estimated from full model simulation in 128x128 cell grid, and regression model of equations (4) to (6) with the following parameter values: a = 10 mm.hr^{-1}; b = 10 mm; c = 2 mm; L_0 =2.5 m; m=4
In graph legends, x = length in metres
Input values for slope length, L = 20-320 m,
Storm rainfall R = 8 – 480 mm, Storm Duration, T = 8 min – 8 hr.
(c) Full model storage vs. SCS model for CN = 80 & 90 for 320 m plot length

Figure 8: Regression model estimate of runoff compared with measured values for Sierra de Enguera runoff plots:

(a) Data for 220 events with measured runoff, 2005-2014.
    Maximum event = 230 mm rainfall with 115 mm runoff.
    Estimated storage, $S = \Theta/[1+(\Theta/R)^2]^{0.5}$
    Estimated runoff, Q = R-S

Where R = storm rainfall, S = storm storage, Θ =storage threshold.
Lines bracket 90% of the data points around the 1:1 line

(b) Storage threshold, Θ as a function of plot length, L.

$$\Theta = 26.5\ L^{0.57}$$

(c) Event storage, S as a function of R & Θ. Data points are binned values, each the average for 10 events in rank order.

Figure 9: Response of runoff coefficient to storm size, storm intensity and slope length, following equation (5). Note non-zero responses to small storms. Parameter values as for Sierra de Enguera site. Dotted curves are SCS curve number estimates, with the same final storage (shown beside each pair of curves).

Figure 10: Extrapolated response of runoff coefficient to slope length for a range of storm rainfalls (R)following equation (5), with parameter values as for Sierra de Enguera site. Shorter slopes imply higher drainage densities.