Simulating quantum gravity with optical lattices

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Unlike the fundamental forces of the Standard Model, such as electromagnetic, weak and strong forces, the quantum effects of gravity are still experimentally inaccessible. The weak coupling of gravity with matter makes it significant only for large masses where quantum effects are too subtle to be measured with current technology. Nevertheless, insight into quantum aspects of gravity is key to understanding unification theories, cosmology or the physics of black holes. Here we propose the simulation of quantum gravity with optical lattices which allows us to arbitrarily control coupling strengths. More concretely, we consider (2+1)-dimensional Dirac fermions, simulated by ultra-cold fermionic atoms arranged in a honeycomb lattice, coupled to massive quantum gravity, simulated by bosonic atoms positioned at the links of the lattice. The quantum effects of gravity induce interactions between the Dirac fermions that can be witnessed, for example, through the violation of Wick's theorem. The similarity of our approach to current experimental simulations of gauge theories suggests that quantum gravity models can be simulated in the laboratory in the near future.

Introduction:- An important open question in physics is that of observing quantum aspects of gravity. The coupling of gravity with matter is so weak that large, macroscopic masses are needed in order to generate an effect. Nevertheless, quantum effects are dominant in microscopic scales where gravity is negligible, thus, making quantum effects of gravity to be well beyond the reach of our current technology. This lack of any experimental evidence impedes our understanding of gravity at a fundamental level. For example, several often conflicting proposals exist for the quantisation Meanwhile, the (3 + 1)-dimensional of spacetime. graviton, the quantum particle that mediates gravity, is unrenormalisable, suffering from infinities in the theoretical level that cannot be removed. A possible way to resolve this problem is to employ quantum simulations, e.g. with optical lattices, where couplings can be arbitrarily tuned. Such a simulation will make it possible to indirectly observe the behaviour of quantum gravity and thus establish experimental verification in this frontier of physics.

Ultra-cold atoms in optical lattices have proven to be an ideal system for probing interacting high-energy theories or condensed matter physics that are otherwise inaccessible. From realising many-body localisation [1– 4] and non-ergodicity in strongly correlated systems [5, 6] to simulating lattice gauge theories [7] or probing topological phases [8–10], optical lattice technology is an invaluable component in advances of modern physics [11]. Recently, a shift in the paradigm of quantum simulations with cold atoms has happened with the possibility of realising the coupling between Dirac fermions and quantum bosonic fields [12]. Such simulations keep the promise of realising lattice gauge theories that can be used to illustrate quark confinement, a fundamental phenomenon that still remains a mystery [13].

Here, we propose an optical lattice simulation of a particular (2 + 1)-dimensional quantum gravity theory coupled to Dirac fermions. In recent years 2 + 1dimensions have attracted great attention since they provide tractable models for quantum gravity. Unlike (3 + 1)-dimensional gravity, pure (2 + 1)-dimensional Einstein gravity has no local propagating degrees of freedom and can be reformulated as a Chern-Simons theory [14, 15] with boundary degrees of freedom given in terms of two-dimensional conformal field theories [16– 19]. In order to introduce local gravitational degrees of freedom that imprint their effects on the local properties of the Dirac fermions we add a mass term in the Einstein-Hilbert action. In higher dimensions, adding a mass for the graviton has been considered as a possibility to resolve the cosmological constant problem, as well as in the construction of theories for dark matter [20]. In four dimensions, the first consistent theory of massive gravity free of Boulware-Deser ghost was found by de Rham, Gabadaze and Tolley [21]. Subsequently, massive generalisations of the Einstein-Hilbert action in 2+1 dimensions have been explored such as the Topologically Massive Gravity, New Massive Gravity and Zwei-Dreibein Gravity [22–24]. Massive gravity theories also have applications in condensed matter physics. The long wavelength limit of the Girvin-MacDonald-Platzman mode of the fractional quantum Hall effect, also known as the *magnetoroton*, is properly described as a massive graviton excitation [25-27].

To identify the right architecture for the simulation of the gravitational theory we first consider the realisation of the Dirac field. We employ a two-dimensional optical lattice configuration that has been shown experimentally to simulate (2 + 1)-dimensional Dirac fermions [9, 10]. We effectively encode an arbitrary gravitational metric in the couplings of the optical lattice by making them position dependent [28]. Fluctuations in the gravitational field can be encoded in the optical lattice by employing bosonic atoms positioned at the links of the lattice [12]. These atoms are coupled to the fermionic ones and are subject to self-interactions in a particular way that gives rise to a semiclassical expansion of massive quantum gravity coupled to Dirac fermions. The fluctuation of gravity causes the fermions to be effectively interacting [29], an effect that can be experimentally witnessed in the fermionic correlators through the violation of Wick's theorem [30]. The components necessary for this simulation, such as 2D Dirac fermions, mixtures of bosonic and fermions optical lattices and for controlling atomic intra and inter-species interactions, are routinely implemented with current experiments. Hence, optical lattices offer an invaluable tool towards realising fundamental properties of quantum gravity in the laboratory.

Dirac fermions coupled to gravitational fluctuations:- We first present the (2+1)-dimensional gravitational theory that we want to simulate. As we are interested in obtaining spacetime geometries that can be realised with optical lattices we take the time-components of the effective metric to be fixed. This condition corresponds to gravitational metrics $g_{\mu\nu}$ in Gaussian form [31],

$$ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + \left(l^{2}\delta_{ij} + 8\pi Gh_{ij}\right)dx^{i}dx^{j},$$
(1)

where G is Newton's constant, l is a constant with spatial dimensions and $\mu = (t, i)$ denotes space-time indices with i = (x, y). To simplify the required optical lattice architecture we consider a gravity model where the components, h_{ij} , are diagonal. The coupling of the gravitation field with the Dirac fermions is best described in terms of the dreibein field e^A_{μ} , given by $g_{\mu\nu} = \eta_{AB} e^A_{\mu} e^B_{\nu}$, where A = (0, a = 1, 2) denotes Lorentz indices and $\eta = \text{diag}(-, +, +)$ is the metric of the local tangent Minkowski space. Parallel transport is then defined by the spin connection ω_{μ}^{A} satisfying $\partial_{\mu}e_{\nu}^{A}$ – $\Gamma^{\rho}_{\mu\nu}e^A_{\rho} + \epsilon^A_{\ BC}\omega^B_{\mu}e^C_{\nu} = 0$, where $\Gamma^{\rho}_{\mu\nu}$ is the affine Christoffel connection. Without loss of generality we consider a fluctuating gravitational field in a flat background, \bar{e}_{μ}^{A} . In this case the gravitational fluctuations translate into fluctuations of dreibein, ξ^A_{μ} , and the spin connection, v^a_{μ} , of a flat background, i.e.

$$e^{A}_{\mu} = \bar{e}^{A}_{\mu} + 8\pi G \xi^{A}_{\mu}, \qquad \omega^{A}_{\mu} = 8\pi G v^{A}_{\mu}.$$
 (2)

In the case of vanishing torsion, the components of v^A_{μ} can be expressed in terms of the dreibein perturbations and its derivatives (see Appendix A). Space-time geometries of the form given in (1), with h_{ij} diagonal, are described by

$$\bar{e}^{A}_{\mu} = \begin{pmatrix} 1 & 0\\ 0 & l \,\delta^{a}_{i} \end{pmatrix}, \quad \xi^{A}_{\mu} = \begin{pmatrix} 0 & 0\\ 0 & \xi^{a}_{i} \end{pmatrix}, \quad \xi^{a}_{i} = \begin{pmatrix} \xi^{1}_{x} & 0\\ 0 & \xi^{2}_{y} \end{pmatrix}, \tag{3}$$

where $\xi_i^{a'}$'s are the spatial dreibein fluctuations. The metric fluctuations are then given by $h_{ij} = l\left(\delta_{ai}\xi_j^a + \delta_{aj}\xi_i^a\right)$. Furthermore, we consider torsionless geometries, for which the spin connection perturbation v_{μ}^A can be expressed in terms of derivatives of ξ_{μ}^A (see Appendix A).

In the following we consider a gravitational model described by the action

$$S[\psi,\xi] = S_{\text{Dirac}}[\psi,\xi] + S_{\text{gr}}[\xi], \qquad (4)$$

where S_{Dirac} is the action for a Dirac spinor ψ that includes the coupling of the geometry ξ_i^a to the fermionic current, whereas S_{gr} is a purely gravitational action that describes the spatial dreibein fluctuations ξ_i^a in a flat background geometry.

The fermion action describes a massless Dirac field on curved space

$$S_{\text{Dirac}} = \frac{i}{2} \int d^3 x |e| \left(\bar{\psi} e^{\mu}_A \gamma^A \overrightarrow{D}_{\mu} \psi - \bar{\psi} \overleftarrow{D}_{\mu} e^{\mu}_A \gamma^A \psi \right), \quad (5)$$

where $\bar{\psi} = \psi^{\dagger} \gamma^{0}$ and the covariant derivatives acting on fermion fields are defined as $\vec{D}_{\mu} = \vec{\partial}_{\mu} + \omega_{\mu}$ and $\vec{D}_{\mu} = \vec{\partial}_{\mu} - \omega_{\mu}$ with $\omega_{\mu} = \frac{1}{4} \epsilon_{ABC} \omega_{\mu}^{A} \gamma^{B} \gamma^{C}$. In order to compare the fermionic action, S_{Dirac} , with the one from the optical lattice simulation we rescale the corresponding spinor ψ as $\psi \longrightarrow \psi/\sqrt{|e|}$, so that they both satisfy the flat anti-commutation relations [32]. Next we split the temporal and spatial indices and implement the semiclassical expansion (2). For small Newton's constant the resulting action S_{Dirac} to linear order in G is given by

$$S_{\text{Dirac}}[\psi,\xi] = i \int d^3x \left[\bar{\psi}\gamma^0 \dot{\psi} + \bar{\psi}\gamma^i \partial_i \psi \right] - \frac{8i\pi G}{l^2} \int d^3x \left[\xi^i_a \bar{\psi}\gamma^a \partial_i \psi + \frac{1}{2} \partial_i \xi^i_a \bar{\psi}\gamma^a \psi \right],$$
⁽⁶⁾

where we have defined the inverse field $\xi_a^i = \delta_b^i \delta_i^a \xi_j^b$.

In order to describe the dynamics of the gravitational field we start with the Palatini action for e^a and ω^a given by

$$S_{\rm gr} = \frac{1}{8\pi G} \int d^3 x \epsilon^{\mu\nu\rho} e^A_\mu \left(\partial_\nu \omega_{A\rho} + \frac{1}{2} \epsilon_{ABC} \,\omega^B_\nu \omega^C_\rho \right).$$
(7)

As before, we consider the semiclassical expansion of $S_{\rm gr}$ with zero torsion and background curvature. The dominant order in the *G* expansion is then given by the massless Fierz-Pauli action for $h_{\mu\nu} = \bar{e}_{A\mu}\xi_{\nu}^{A} + \bar{e}_{A\nu}\xi_{\mu}^{A}$ (see Appendix B). The particular action and geometries are

suitably chosen so that the resulting Hamiltonian can be directly modelled with ultra-cold atoms in optical lattices. Moreover, considering a metric in Gaussian coordinates (1) is convenient at the quantum level, since it allows to eliminate the so-called *conformal divergences* in the graviton path integral [31].

Action (7) describes a topological theory with no propagating degrees of freedom. In order to have local degrees of freedom, akin to the (3 + 1)-dimensional gravity, we introduce a mass term for the gravitational field. Here, we consider massive gravitons ξ^A_{μ} described by the Fierz-Pauli theory [33] obtained by adding the mass term

$$-4\pi G \mu^2 \epsilon^{\mu\nu\rho} \epsilon_{ABC} \bar{e}^A_\mu \xi^B_\nu \xi^C_\rho , \qquad (8)$$

in the Lagrangian (7). Combining (7) and (8) finally gives (see Appendix B)

$$S_{\rm gr}[\xi] = -4\pi G \int d^3x \epsilon^{ij} \epsilon_{ab} \left(\dot{\xi}^a_i \dot{\xi}^b_j + \mu^2 \xi^a_i \xi^b_j \right), \qquad (9)$$

which is the effective massive gravitational action, with ξ_i^a given by (3). Note that we have introduced the mass term (8) by hand. It is known that the Fierz-Pauli action (B7) can be obtained from a full gravitational theory with quadratic-in-curvature terms, called New Massive Gravity [23, 34], upon linearisation. Alternatively, a different gravitational theory, whose weak field limit leads to the same massive Fierz-Pauli action has been proposed in [35].

To determine the optical lattice configuration required to simulate this gravity model we need first to obtain its Hamiltonian. The Hamiltonian density corresponding to the action (4) is obtained with a Legendre transformation (see Appendix C)

$$\mathcal{H} = \psi^{\dagger} h(\boldsymbol{p}) \psi + \mathcal{H}_{\rm gr}, \qquad (10)$$

where the single particle Hamiltonian $h(\mathbf{p})$ is given by

$$h(\boldsymbol{p}) = \frac{\gamma^0}{l} \left(\delta^i_a \gamma^a - \frac{8\pi G}{l} \xi^i_a \gamma^a \right) (-i\partial_i) + \frac{4i\pi G}{l^2} \partial_i \xi^i_a \gamma^0 \gamma^a,$$
(11)

and the gravitational Hamiltonian \mathcal{H}_{gr} equals

$$\mathcal{H}_{\rm gr} = -\frac{1}{16\pi G} \epsilon_{ij} \epsilon^{ab} \pi^i_a \pi^j_b + 4\pi G \mu^2 \epsilon^{ij} \epsilon_{ab} \xi^a_i \xi^b_j.$$
(12)

Here $\pi_a^i = \text{dial}(\pi_1^x, \pi_2^y)$ is the canonical momentum conjugate to ξ_i^a given in (3). The geometric fluctuations and their conjugate momenta can be expressed as

$$\xi_x^1 = \frac{1}{\sqrt{2}} (q_1^{\dagger} + q_1), \qquad \pi_1^x = -\frac{i}{\sqrt{2}} (q_1^{\dagger} - q_1), \xi_y^2 = \frac{1}{\sqrt{2}} (q_2^{\dagger} + q_2), \qquad \pi_2^y = -\frac{i}{\sqrt{2}} (q_2^{\dagger} - q_2),$$
(13)

where the operators q_a . a = 1, 2, satisfy the bosonic commutation relations $[q_a(x), q_b^{\dagger}(y)] = \delta_{ab} \delta^{(2)}(x-y)$ and



Fig. 1. (Left) The optical lattice configuration with fermionic and bosonic atoms. The unit cell of the honeycomb lattice (dashed box) comprises two fermionic modes a and b. The fermionic atoms tunnel along the three different directions of the trivalence lattice with couplings J_x , J_x and J_z . The vectors $n_1 = (\sqrt{3}/2, 3/2)$ and $n_2 = (-\sqrt{3}/2, 3/2)$ transport between different unit cells. (Right) In the low energy limit the dispersion relation $E(\mathbf{p})$ of the fermions is given by Dirac cones. Non-equal tunnelling couplings, J_x and J_z , cause the Dirac cone to be deformed, with its geometry encoded in the dreibein components e_x^1 and e_y^2 , effectively describing background gravity. Bosonic modes α_x and α_z , describing Bose-Einstein condensates, are inserted that control the fermionic tunnelling couplings with their Fluctuations of the condensates simulate populations. fluctuations in e_x^1 and e_y^2 , i.e. gravity fluctuations.

 $[q_a(x), q_b(y)] = 0 = [q_a^{\dagger}(x), q_b^{\dagger}(y)]$. It is in terms of these quantum operators that we can establish a map between (10) and an optical lattice Hamiltonian.

Optical lattice simulator:- We now design an optical lattice configuration that simulates the quantum gravity model given by (10) in its low energy limit. We first present a configuration that gives rise to Dirac fermions in a fixed background geometry encoded in the tunnelling couplings of the lattice [28]. Then we introduce link quantum variables d_j that fluctuate this geometry. Consider a two-dimensional optical lattice with honeycomb configuration where fermionic atoms, a and b, live at its vertices, as shown in Fig. 1(Left). The fermions are subject to the tunnelling Hamiltonian

$$H_{\text{latt}} = \sum_{\boldsymbol{i}} \left[J_x(\boldsymbol{i}) a_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}+\boldsymbol{n}_1} + J_x(\boldsymbol{i}) a_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}+\boldsymbol{n}_2} + J_z(\boldsymbol{i}) a_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}} \right] + \text{h.c.}$$
(14)

where $\mathbf{i} = (i_x, i_y)$ gives the position of unit cells on the lattice, the couplings of the first two terms are equal, and both J_x and J_z are in general position dependent. At half-filling, the low energy sector of the model is described by the Dirac Hamiltonian $H_{\text{latt}} \approx \int d^2x \ \psi^{\dagger} \ \tilde{h}(\mathbf{p}) \ \psi$ [28] with (see Appendix D)

$$\tilde{h}(\boldsymbol{p}) = -\frac{\sqrt{3}i}{2}\sqrt{4J_x^2 - J_z^2} \gamma^0 \gamma^1 \partial_x - \frac{3i}{2}J_z \gamma^0 \gamma^2 \partial_y + \frac{\sqrt{3}i}{2}\partial_x \left(\sqrt{4J_x^2 - J_z^2}\right) \gamma^0 \gamma^1 + \frac{3i}{2}\partial_y J_z \gamma^0 \gamma^2.$$
(15)

The coefficients of $\gamma^0 \gamma^1 (-i\partial_x)$ and $\gamma^0 \gamma^2 (-i\partial_y)$ play the role of the diagonal space components of the dreibein, e_x^1 and e_y^2 , respectively, as shown in Fig. 1(Right). To introduce quantum fluctuations in the *J* couplings, and thus in the corresponding dreibeins, we insert bosonic modes α_x and α_z at the edges of the lattice, as shown in Fig. 1(Right). The modes control the tunnelling of fermions from site *i* to site *k* in the m = x, z direction, through the interaction $\Delta_m \alpha_m^{\dagger} \alpha_m a_i^{\dagger} b_k$ [12]. We take the α_j modes to correspond to a bosonic condensation with particle density D_m and quantum fluctuations d_m , i.e.

$$\alpha_m = D_m + d_m,\tag{16}$$

where $[d_m(\mathbf{i}), d_m^{\dagger}(\mathbf{j})] = \delta_{\mathbf{ij}}$, for m = x, z. In the weak fluctuations limit $\langle d_m^{\dagger} d_m \rangle \ll D_m^2$, the interactions between bosons and fermions give rise to tunnelling couplings of the form $J_m = \Delta_m D_m (D_m + d_m^{\dagger} + d_m)$. If we choose the optical lattice parameters as

$$D_x = \sqrt{2}D_z = -\frac{l}{4\pi G}, \qquad \Delta_x = \frac{\Delta_z}{2} = \frac{32\pi^2 G^2}{3l^3}, \quad (17)$$

with the operator redefinition $q_1 = 2\sqrt{2}d_x/3 - d_z/3$ and $q_2 = d_z$ that preserves bosonic commutation relations, we find that, to linear order in G, the optical lattice Hamiltonian (14) is mapped to the field theory one (11) (for details, see Appendices B and D). Constant tunnelling terms can be added in (14) to arbitrarily tune the values of the densities D_m .

The self-interaction terms (12) of the gravitational Hamiltonian can be obtained from the following purely bosonic interactions

$$\mathcal{H}_{\text{boson}} = \frac{1}{24\pi G} (\alpha_z^{\dagger} - \alpha_z) \left(\sqrt{2} (\alpha_x^{\dagger} - \alpha_x) - \frac{1}{2} (\alpha_z^{\dagger} - \alpha_z) \right) + \frac{8\pi G \mu^2}{3} (\alpha_z^{\dagger} \alpha_z + \alpha_x^{\dagger} \alpha_x) - \frac{256\pi^3 G^3 \mu^2}{3l^2} \alpha_z^{\dagger} \alpha_z \left(\alpha_x^{\dagger} \alpha_x - \frac{1}{2} \alpha_z^{\dagger} \alpha_z \right),$$
(18)

restricted to the weak fluctuation regime of the α_j 's. This Hamiltonian includes tunnelling, pairing and interacting terms between the α_j bosonic atoms that can be realised by employing Feshbach resonances [36–38]. Hence, it is possible to simulate the quantum gravity model coupled to Dirac fermions given in (10) with a mixture of bosonic and fermionic ultra-cold atoms in optical lattices.

Expected signatures of quantum gravity:- The optical lattice system given by (14) and (18) in the low energy limit simulates Dirac fermions coupled to quantum gravity (10). We present now how to identify the presence of a fluctuating gravitational field from the behaviour of the fermionic quantum correlations. The partition function of the system at temperature T is given by

$$Z = \int \mathcal{D}\xi \,\mathcal{D}\pi \,\mathcal{D}\psi \mathcal{D}\bar{\psi} \,\exp\left(-\frac{1}{k_B T} \int d^2 x \,\mathcal{H}\right), \quad (19)$$

where k_B is the Boltzmann constant. To determine the behaviour of the fermions due to their interactions mediated by gravitational fluctuations we can integrate the bosonic part ξ and π of (19) and derive the effective fermionic Hamiltonian. Integrating out the momenta, π , leads to an irrelevant global factor in Z. We can subsequently integrate out the bosonic field ξ_i^a , which up to an overall constant yields

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-\frac{1}{k_B T} \int d^2 x \ \mathcal{H}_{\text{eff}}\right), \qquad (20)$$

with the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -i\bar{\psi}\gamma^i\partial_i\psi - \frac{4\pi G}{l^2\mu^2}\epsilon_{ab}\epsilon^{ij}\mathcal{J}^a_i\mathcal{J}^b_j,\qquad(21)$$

where $\mathcal{J}_i^a = \frac{i}{2l} \left(\bar{\psi} \gamma^a \partial_i \psi - \partial_i \bar{\psi} \gamma^a \psi \right)$ is the fermionic current. As a result the partition function of the system is effectively described in terms of interacting fermions with a coupling proportional to the gravitational constant G.

It is possible to detect the presence of interactions (21) mediated by the gravitational field by measuring the fermionic correlations and testing the applicability of Wick's theorem. In the absence of gravitational fluctuations, i.e. for Newton's constant $G \rightarrow 0$ in (2) or for fluctuations $\xi_i^a \to 0$, the effective partition function corresponds to free fermions, as seen by (21). For this case, Wick's theorem states that all four-point quantum correlators of the ground state can be exactly decomposed in terms of two-point correlators. Such fourpoint correlators of fermions can be expressed in terms of fermionic densities and two-point correlations and thus can be directly measured in cold atom experiments [11]. When the interactions induced by the gravitational field are present, i.e. for $G \neq 0$, Wick's decomposition does not apply, leaving a difference that can be determined by measurements of fermionic correlations [30]. In the perturbative regime considered here this difference gives a measure of the coupling G between the Dirac fermions and the gravitational field.

Conclusions: – Among the forces of nature gravity keeps its quantum aspects well hidden. This lack of experiential evidence hinders the theoretical understanding of quantum gravity and its unification with the rest of the fundamental forces within the Standard Model. Here we propose a way of simulating quantum gravity coupled to Dirac fermions in the laboratory. By employing tools similar to the ones used to simulate scalar or gauge fields coupled to Dirac fermions, we are able to introduce a quantum gravity model coupled to Dirac fermions. The interaction effects of quantum gravity can be witnessed by quantum correlations of the fermions via a violation of Wick's decomposition.

Our work opens up a host of various applications. The components used in our proposal can be realised in the laboratory with current or near-future technology thus opening a new direction towards the investigation of quantum gravity. Beyond the experimental front it is intriguing to consider the behaviour of various quantum gravity and cosmology theories in 1 + 1, 2 + 1or 3 + 1 dimensions with or without fermions. We envision that our proposal will initiate a new line of investigations where interacting models that simulate gravity can be realised in the laboratory and guide theoretical investigations towards the understanding of quantum aspects of gravity in nature.

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Appendix A: Spin connection

We consider a three-dimensional geometry whose metric tensor is defined by a set of dreibeins e^A_μ

$$g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu. \tag{A1}$$

where here A = (0, a = 1, 2) denotes Lorentz indices, $\mu = (t, i = x, y)$ stand for manifold indices, and $\eta = \text{diag}(-, +, +)$ is the Minkowski metric. Parallel transport is defined by the spin connection ω_{μ}^{A} , which can be obtained from the *vielbein postulate*

$$\partial_{\mu}e^{A}_{\nu} - \Gamma^{\rho}_{\mu\nu}e^{A}_{\rho} + \epsilon^{A}_{\ BC}\omega^{B}_{\mu}e^{C}_{\nu} = 0, \qquad (A2)$$

where $\Gamma^{\rho}_{\mu\nu}$ is the affine Christoffel connection and ϵ^{ABC} the Levi-Civita symbol ($\epsilon^{012} = 1$). In this case, the gravitational fluctuations translate into fluctuations of dreibein and the spin connection

$$e^{A}_{\mu} = \bar{e}^{A}_{\mu} + \frac{1}{8\pi G}\xi^{A}_{\mu}, \qquad \omega^{A}_{\mu} = \bar{\omega}^{A}_{\mu} + \frac{1}{8\pi G}v^{a}_{\mu}.$$
 (A3)

We restrict the analysis to flat backgrounds with constant dreibein and vanishing spin connection. Therefore we set

$$\bar{e}^A_\mu = constant \,, \qquad \bar{\omega}^A_\mu = 0.$$
 (A4)

Furthermore, we consider torsionless geometries. In this case taking the antisymmetric part of (A2) allows one to express ω_{μ}^{A} in terms the dreibein and its derivatives. For the corresponding perturbations (A3) one finds the relation

$$\epsilon^{\mu\nu\rho} \left(\partial_{\nu} \xi^{A}_{\rho} + \epsilon^{A}_{BC} \bar{e}^{B}_{\nu} v^{C}_{\rho} \right) = 0, \qquad (A5)$$

which can be used to find the form of the spin connection perturbations v^A_μ

$$v^{A}_{\mu} = M^{AB}_{\mu\nu} \epsilon^{\nu\alpha\beta} \partial_{\alpha} \xi_{B\beta} , \quad M^{AB}_{\mu\nu} \equiv \frac{1}{\bar{e}} \left(\frac{1}{2} \bar{e}^{A}_{\mu} \bar{e}^{B}_{\nu} - \bar{e}^{A}_{\nu} \bar{e}^{B}_{\mu} \right),$$
(A6)

where $\bar{e} = \det e^A_{\mu}$. When restricted to metrics of the form (1) and thus we impose

$$\bar{e}^A_\mu = \begin{pmatrix} 1 & 0\\ 0 & \bar{e}^a_i \end{pmatrix}, \qquad \xi^a_\mu = \begin{pmatrix} 0 & 0\\ 0 & \xi^a_i \end{pmatrix}$$
(A7)

and thus, the components of the spin connection reduce to

$$\begin{aligned} v_t^0 &= -\frac{4\pi G}{\bar{e}} \epsilon^{ij} \bar{e}_i^a \dot{\xi}_{aj} , \qquad v_i^0 &= -\frac{8\pi G}{\bar{e}} \epsilon^{jk} \bar{e}_i^a \partial_j \xi_{ak} , \\ v_t^a &= 0 , \qquad v_i^a &= 8\pi G M_{ij}^{ab} \epsilon^{jk} \dot{\xi}_{bk} . \end{aligned}$$
(A8)

Appendix B: Gravity action

In order to describe the gravitational fluctuations, we start with the Palatini action for e^a_{μ} and ω^a_{μ}

$$S_{\rm gr} = \frac{1}{8\pi G} \int d^3x \,\epsilon^{\mu\nu\rho} e^A_\mu \left(\partial_\nu \omega_{A\rho} + \frac{1}{2} \epsilon_{ABC} \,\,\omega^B_\nu \,\omega^C_\nu \right). \tag{B1}$$

Using (A3), the action can be expanded in powers of the (2 + 1)-dimensional reduced Planck mass $1/8\pi G$ as

$$S_{\rm gr}[\xi] = \frac{1}{8\pi G} S_{\rm gr}^{(0)} + S_{\rm gr}^{(1)} + 8\pi G S_{\rm gr}^{(2)}.$$
 (B2)

Defining the background curvature and torsion,

$$\bar{R}^{A}_{\mu\nu} = \partial_{[\mu}\bar{\omega}^{A}_{\nu]} + \frac{1}{2}\epsilon^{A}_{BC}\bar{\omega}^{B}_{[\mu}\bar{\omega}^{C}_{\nu]} = 0,$$

$$\bar{T}^{A}_{\mu\nu} = \partial_{[\mu}\bar{e}^{A}_{\nu]} + \epsilon^{A}_{BC}\bar{\omega}^{B}_{[\mu}\bar{e}^{C}_{\nu]} = 0$$
(B3)

the different terms in the action (B2) can be written as

$$S_{\rm gr}^{(0)} = \frac{1}{2} \int d^3 x \, \epsilon^{\mu\nu\rho} \bar{e}^A_\mu \bar{R}_{A\nu\rho},$$

$$S_{\rm gr}^{(1)} = \frac{1}{2} \int d^3 x \, \epsilon^{\mu\nu\rho} \left(\xi^A_\mu \bar{R}_{A\nu\rho}, + \bar{v}^A_\mu \bar{T}_{A\nu\rho} \right),$$

$$S_{\rm gr}^{(2)} = \int d^3 x \, \epsilon^{\mu\nu\rho} \left(\xi^A_\mu \bar{D}_\nu v_{A\rho} + \frac{1}{2} \epsilon_{ABC} \, \bar{e}^A_\mu v^B_\nu v^C_\rho \right).$$

(B4)

Considering a flat torsionless background implies that $\bar{R}^A_{\mu\nu} = 0 = \bar{T}A_{\mu\nu}$. Replacing (A6) in (B4) and using tensor notation then yields

$$S_{\rm gr}[\xi] = -4\pi G \int d^3x M^{AB}_{\mu\nu} \epsilon^{\mu\alpha\beta} \epsilon^{\nu\gamma\delta} \partial_\alpha \xi_{A\beta} \partial_\gamma \xi_{B\delta}.$$
(B5)

One can check that the action (B5) boils down to the massless Fierz-Pauli action for $h_{\mu\nu} = \xi^A_{\mu} \bar{e}_{A\nu} + \xi^A_{\nu} \bar{e}_{A\mu}$.

We are interesting in adding a mass μ to the geometry fluctuations ξ^A_{μ} . We do so by means of the Fierz-Pauli mass term

$$-4\pi G\mu^2 \epsilon^{\mu\nu\rho} \epsilon_{ABC} \bar{e}^A_\mu \xi^B_\nu \xi^C_\rho \,. \tag{B6}$$

Thus, implementing (A7) and (A8) in (B5) and adding the term (B6) restricted to those conditions, we find the following effective massive gravitational action

$$S_{\rm gr}[\xi] = -4\pi G \int d^3x \epsilon^{ij} \epsilon_{ab} \left(\dot{\xi}^a_i \dot{\xi}^b_j + \mu^2 \xi^a_i \xi^b_j \right). \tag{B7}$$

where we have defined $\epsilon^{ij} \equiv \epsilon^{0ij}$.

Appendix C: Field theory Hamiltonian

The Hamiltonian density associated to the Lagrangian in (4) is obtained after a straightforward Legendre transformation.

$$\mathcal{H} = \Pi^{\dagger} \dot{\psi} + \dot{\psi} \Pi + \pi_a^i \dot{\xi}_i^a - \mathcal{L}, \qquad (C1)$$

where the canonical momenta read

$$\Pi^{\dagger} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\psi^{\dagger} , \qquad \Pi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}^{\dagger}} = 0 ,$$

$$\pi^{i}_{a} = \frac{\partial \mathcal{L}}{\partial \dot{\xi}^{a}_{i}} = -8\pi G \epsilon^{ij} \epsilon_{ab} \dot{\xi}^{b}_{j}, \qquad (C2)$$

and satisfy the Poisson brackets

$$\left\{ \psi_{\alpha}(x), \Pi_{\beta}^{\dagger}(y) \right\} = \left\{ \psi_{\alpha}^{\dagger}(x), \Pi_{\beta}(y) \right\} = -\delta_{\alpha\beta}\delta^{(2)}(x-y),$$

$$\left\{ \xi_{i}^{a}(x), \pi_{b}^{j}(y) \right\} = \delta_{b}^{a}\delta_{i}^{j}\delta^{(2)}(x-y).$$
(C3)

The Hamiltonian density reduces to

$$\mathcal{H} = \psi^{\dagger} h(\boldsymbol{p}) \psi + \mathcal{H}_{\rm gr}, \qquad (C4)$$

with the single particle hamiltonian $h(\mathbf{p})$

$$h(\boldsymbol{p}) = \frac{\gamma^0}{l} \left(\delta^i_a \gamma^a - \frac{8\pi G}{l} \xi^i_a \gamma^a \right) (-i\partial_i) + \frac{4i\pi G}{l^2} \partial_i \xi^i_a \gamma^0 \gamma^a,$$
(C5)

and the gravitational Hamiltonian $\mathcal{H}_{\rm gr}$

$$\mathcal{H}_{\rm gr} = -\frac{1}{16\pi G} \epsilon_{ij} \epsilon^{ab} \pi^i_a \pi^j_b + 4\pi G \mu^2 \epsilon^{ij} \epsilon_{ab} \xi^a_i \xi^b_j.$$
(C6)

Since our analysis considers geometry fluctiations ξ^a_i that are diagonal, we define quantum operators of the form

$$\xi_i^a = \begin{pmatrix} \xi_x^1 & 0\\ 0 & \xi_y^2 \end{pmatrix}, \qquad \begin{aligned} \xi_x^1 &= \frac{1}{\sqrt{2}} (q_1^{\dagger} + q_1), \\ \xi_y^2 &= \frac{1}{\sqrt{2}} (q_2^{\dagger} + q_2), \end{aligned}$$
(C7)

where the operators q_a , a = 1, 2 satisfy the commutation relations

$$[q_a(x), q_b^{\dagger}(y)] = \delta_{ab} \delta^{(2)}(x - y),$$

$$[q_a(x), q_b(y)] = 0 = [q_a^{\dagger}(x), q_b^{\dagger}(y)].$$
(C8)

Thus, the canonical Poisson brackets (C3) are promoted commutators $\{ , \} \rightarrow -i[]$, which then fixes the form of the momenta ξ_a^i to be

$$\pi_a^i = \begin{pmatrix} \pi_1^x & 0\\ 0 & \pi_2^y \end{pmatrix}, \qquad \begin{array}{l} \pi_1^x = -\frac{i}{\sqrt{2}}(q_1^{\dagger} - q_1),\\ \pi_2^y = -\frac{i}{\sqrt{2}}(q_2^{\dagger} - q_2). \end{array}$$
(C9)

The flat background spatial metric is taken as $g_{ij} = l^2 \delta_{ij}$ with l an arbitrary constant, which implies

$$\bar{e}_i^a = l\delta_i^a, \qquad \bar{e}_a^i = \frac{1}{l}\delta_a^i, \qquad (C10)$$

and thus the gamma matrices on this background geometry reduce to

$$\gamma^t = \gamma^0, \quad \gamma^i = \frac{1}{l} \delta^i_a \gamma^a$$
 (C11)

From this expressions we see that the single particle Hamitlonian (C5) can be written as

$$h(\mathbf{p}) = \left(\frac{1}{l} - \frac{4\sqrt{2}\pi G}{l^2}(q_1^{\dagger} + q_1)\right)\gamma^1(-i\partial_x) + \left(\frac{1}{l} - \frac{4\sqrt{2}\pi G}{l^2}(q_2^{\dagger} + q_2)\right)\gamma^2(-i\partial_y) + \frac{2\sqrt{2}i\pi G}{l^2}\left(\partial_x(q_1^{\dagger} + q_1)\gamma^1 + \partial_y(q_2^{\dagger} + q_2)\gamma^2\right),$$
(C12)

whereas the gravitational Hamiltonian and $({\rm C6})$ takes the form

$$\mathcal{H}_{\rm gr} = \frac{1}{16\pi G} (q_1^{\dagger} - q_1)(q_2^{\dagger} - q_2) - 4\pi G \mu^2 (q_1^{\dagger} + q_1)(q_2^{\dagger} + q_2).$$
(C13)

In the following, we show how to translate (C4) into an optical lattice Hamiltonian.

Appendix D: Optical Lattice Hamiltonian

We start considering graphene lattice with unequal tunneling couplings

$$H_{\text{latt}} = \sum_{i} \left(J_x a_i^{\dagger} b_{i+n_1} + J_y a_i^{\dagger} b_{i+n_2} + J_z a_i^{\dagger} b_i \right) + \text{h.c.},$$
(D1)

where $\mathbf{i} = (i_x, i_y)$ denotes the position of the unit cells. Expanding the operators in Fourier modes and defining

$$\psi_{\mathbf{k}} = \begin{pmatrix} a_{\mathbf{k}} \\ b_{\mathbf{k}} \end{pmatrix}, \quad \tilde{h}(\mathbf{k}) = \begin{pmatrix} 0 & f(\mathbf{k}) \\ f^*(\mathbf{k}) & 0 \end{pmatrix}, \qquad (D2)$$

where $f(\mathbf{k}) = J_x e^{-i\mathbf{k}\cdot\mathbf{n}_1} + J_y e^{-i\mathbf{k}\cdot\mathbf{n}_2} + J_z$, we find

$$H_{\text{latt}} = \sum_{\boldsymbol{k}} \psi_{\boldsymbol{k}}^{\dagger} \tilde{h}(\boldsymbol{k}) \psi_{\boldsymbol{k}}.$$
 (D3)

We consider the special case $J_x = J_y$. The Fermi points, P_{\pm} , are defined by

$$f(\boldsymbol{P}_{\pm}) = 0 \Rightarrow \boldsymbol{P}_{\pm} = \pm \begin{pmatrix} \frac{2}{\sqrt{3}} \arccos(-J_z/J_x) \\ 0 \end{pmatrix}. \quad (D4)$$

Now, we expand $f(\mathbf{k})$ around the Fermi points

$$f(\boldsymbol{P}_{\pm} + \boldsymbol{p}) = \boldsymbol{p} \cdot \nabla f(\boldsymbol{P}_{\pm}) = A_{\pm} p_x + B_{\pm} p_y, \quad (D5)$$

where we have defined

$$A_{\pm} = \mp \frac{\sqrt{3}}{2} \sqrt{4J_x^2 - J_z^2}, \quad B_{\pm} = -\frac{3}{2}J_z.$$
 (D6)

Expanding the Hamiltonian H_{latt} around the Fermi point yields

$$H_{\text{latt}} = \sum_{\boldsymbol{p}} \begin{pmatrix} a_{+}^{\dagger} & b_{+}^{\dagger} \end{pmatrix} (A_{+}\sigma^{1}p_{x} - B_{+}\sigma^{2}p_{y}) \begin{pmatrix} a_{+} \\ b_{+} \end{pmatrix} + \sum_{\boldsymbol{p}} \begin{pmatrix} a_{-}^{\dagger} & b_{-}^{\dagger} \end{pmatrix} (A_{-}\sigma^{1}p_{x} - B_{-}\sigma^{2}p_{y}) \begin{pmatrix} a_{-} \\ b_{-} \end{pmatrix}.$$
(D7)

Defining the four spinor and the gamma matrices

$$\psi_{\mathbf{p}} = \begin{pmatrix} a_+\\b_+\\b_-\\a_- \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & -\mathbf{1}\\\mathbf{1} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i\\\sigma^i & 0 \end{pmatrix},$$
(D8)

with $\sigma^i = (\sigma^1, \sigma^2)$ Pauli matrices. Then we find $H_{\text{latt}} = \int d^2x \ \psi^{\dagger} \ \tilde{h}(\boldsymbol{p}) \ \psi$ with the single particle Hamiltonian

$$\tilde{h}(\boldsymbol{p}) = \frac{\sqrt{3}}{2} \sqrt{4J_x^2 - J_z^2} \,\gamma^0 \gamma^1 p_x + \frac{3}{2} J_z \gamma^0 \gamma^2 p_y.$$
(D9)

where, for convenience, we have flipped the orientation of the y axis, i.e. $p_y \rightarrow -p_y$. In order to ensure hermiticity of the full Hamiltonian the momentum takes the form $p_i = -i(\overrightarrow{\partial}_i - \overrightarrow{\partial}_i)/2$. We can now generalise this Hamiltonian by considering that the couplings J_x and J_z are position dependent, varying slowly compared to the lattice spacing. Next, we add a bosonic self-interacting Hamiltonian $\mathcal{H}_{\text{boson}}$,

$$\mathcal{H}_{\text{boson}} = \frac{1}{24\pi G} (\alpha_z^{\dagger} - \alpha_z) \left(\sqrt{2} (\alpha_x^{\dagger} - \alpha_x) - \frac{1}{2} (\alpha_z^{\dagger} - \alpha_z) \right) + \frac{8\pi G \mu^2}{3} \left(\alpha_z^{\dagger} \alpha_z + \alpha_x^{\dagger} \alpha_x \right) - \frac{256\pi^3 G^3 \mu^2}{3l^2} \alpha_z^{\dagger} \alpha_z \left(\alpha_x^{\dagger} \alpha_x - \frac{1}{2} \alpha_z^{\dagger} \alpha_z \right).$$
(D10)

The optical lattice Hamiltonian that we will consider is then

$$H_{\text{latt}} = \int d^2 x \left[\psi^{\dagger} \tilde{h}(\boldsymbol{p}) \psi + \mathcal{H}_{\text{boson}} \right], \qquad (D11)$$

where

$$\tilde{h}(\boldsymbol{p}) = \frac{\sqrt{3}}{2} \sqrt{4J_x^2 - J_z^2} \gamma^0 \gamma^1 (-i\partial_x) + \frac{3}{2} J_z \gamma^0 \gamma^2 (-i\partial_y) + \frac{\sqrt{3}i}{2} \partial_x \left(\sqrt{4J_x^2 - J_z^2} \right) \gamma^0 \gamma^1 + \frac{3i}{2} \partial_y J_z \gamma^0 \gamma^2.$$
(D12)

In order to show that (D11) can be mapped to (C4), first we consider the following form of the couplings

$$J_m = \Delta_m D_m \left(D_m + d_m^{\dagger} + d_m \right) , \quad m = x, z.$$
 (D13)

where the operators d_m satisfy the commutation relations

$$[d_m(x), d_n^{\dagger}(y)] = \delta_{mn} \delta^{(2)}(x - y),$$

$$[d_m(x), d_n(y)] = 0 = [d_m^{\dagger}(x), d_n^{\dagger}(y)],$$
(D14)

Note that this is a continuum version of the commutation relations given below (16). One can show that, by considering

$$D_{x} = -\frac{l}{4\pi G}, \qquad \Delta_{x} = \frac{32\pi^{2}G^{2}}{3l^{3}},$$

$$D_{z} = -\frac{\sqrt{2}l}{8\pi G}, \qquad \Delta_{z} = \frac{64\pi^{2}G^{2}}{3l^{3}},$$
(D15)

we find that the different terms in $\tilde{h}(\mathbf{p})$ given in (D12) can be written in the form

$$\frac{3}{2}J_z = \frac{1}{l} - \frac{4\sqrt{2\pi}G}{l^2} \left(d_z^{\dagger} + d_z\right),$$

$$\frac{\sqrt{3}}{2}\sqrt{4J_x^2 - J_z^2} \approx \frac{1}{l} - \frac{16\pi G}{3l^2} \left(d_x^{\dagger} + d_x\right) \qquad (D16)$$

$$+ \frac{4\sqrt{2\pi}G}{3l^2} \left(d_z^{\dagger} + d_z\right),$$

where in the last equality we have used the approximation

$$\left|\left\langle d_m^{\dagger} + d_m \right\rangle\right| \ll \left|D_m\right|, \qquad (D17)$$

valid for bosonic states in the weak fluctuation regime. Finally, we consider the operator redefinition

$$q_1 = \frac{2\sqrt{2}}{3}d_x - \frac{1}{3}d_z, \qquad q_2 = d_z,$$
 (D18)

which satisfy the canonical commutation relations given in (C8), and takes the single particle Hamiltonian $\tilde{h}(\boldsymbol{p})$ given (D12) into the corresponding Hamiltonian $h(\boldsymbol{p})$ describing fermions coupled to dreibein fluctuations given in (C12).

Now we turn our attention to the bosonic Hamiltonian (D10), where the operators α_m are defined as

$$\alpha_m = D_m + d_m. \tag{D19}$$

In this case we reverse the procedure and show that, up to an irrelevant additive constant, the gravitational Hamiltonian \mathcal{H}_{gr} given in (C13) can be mapped to (D10). This can be directly shown by noticing that

$$(d_m^{\dagger} - d_m)(d_n^{\dagger} - d_n) = (\alpha_m^{\dagger} - \alpha_m)(\alpha_n^{\dagger} - \alpha_n),$$

$$(d_m^{\dagger} + d_m)(d_m^{\dagger} + d_m) \approx \frac{\alpha_m^{\dagger} \alpha_m \alpha_n^{\dagger} \alpha_n}{D_m D_n} - \frac{D_m}{D_m} \alpha_n^{\dagger} \alpha_n$$

$$- \frac{D_n}{D_m} \alpha_m^{\dagger} \alpha_m + D_m D_n,$$

(D20)

and using (D15) and (D18). Note that in the last relation we have used $\alpha_m^{\dagger} \alpha_m \approx D_m^2 + D_m (d_m^{\dagger} + d_m)$, which holds in the approximation (D17).

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