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# Bottom-up dust nucleation theory in oxygen-rich evolved stars

## I. Aluminum oxide clusters

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### ABSTRACT

**Context.** Aluminum oxide (alumina,  $\text{Al}_2\text{O}_3$ ) is a promising candidate as a primary dust condensate in the atmospheres of oxygen-rich evolved stars. Therefore, alumina *seed* particles might trigger the onset of stellar dust formation and of stellar mass loss in the wind. However, the formation of alumina dust grains is not well understood.

**Aims.** To shed light on the initial steps of cosmic dust formation (i.e. nucleation) in oxygen-rich environments by a quantum-chemical bottom-up approach.

**Methods.** Starting with an elemental gas-phase composition, we construct a detailed chemical-kinetic network describing the formation and destruction of aluminium-bearing molecules and dust-forming  $(\text{Al}_2\text{O}_3)_n$  clusters up to the size of dimers ( $n=2$ ) coagulating to tetramers ( $n=4$ ). Intermediary species include the prevalent gas-phase molecules  $\text{AlO}$  and  $\text{AlOH}$ , and  $\text{Al}_x\text{O}_y$  clusters with  $x=1-5$ ,  $y=1-6$ . The resulting extensive network is applied to two model stars, representing a semi-regular variable and a Mira-type, and to different circumstellar gas trajectories including a non-pulsating outflow and a pulsating model. The growth of larger-sized  $(\text{Al}_2\text{O}_3)_n$  clusters with  $n=4-10$  is described by the temperature-dependent Gibbs free energies of the most favourable structures (i.e. the global minima clusters) as derived from global optimisation techniques and calculated by density functional theory. We provide energies, bond characteristics, electrostatic properties and vibrational spectra of the clusters as a function of size  $n$  and compare these to corundum corresponding to the crystalline bulk limit ( $n \rightarrow \infty$ ).

**Results.** The circumstellar aluminium gas-phase chemistry in oxygen-rich giants is primarily controlled by  $\text{AlOH}$  and  $\text{AlO}$ , which are tightly coupled by the reactions  $\text{AlO}+\text{H}_2$ ,  $\text{AlO}+\text{H}_2\text{O}$ , and their reverse. Models of semi-regular variables show comparatively higher  $\text{AlO}$  abundances, and a later onset and a lower efficiency of alumina cluster formation when compared to Mira-like models. The Mira-like models exhibit an efficient cluster production accounting for more than 90% of the available aluminium content, which is in agreement with the most recent ALMA observations. Chemical equilibrium calculations fail to predict the alumina cluster formation, as well as the abundance trends of  $\text{AlO}$  and  $\text{AlOH}$  in the AGB dust formation zone. Furthermore, we report the discovery of hitherto unreported global minima candidates and low-energy isomers for cluster sizes  $n=7, 9$ , and  $10$ . A homogeneous nucleation scenario, where  $\text{Al}_2\text{O}_3$  monomers are successively added, is energetically viable. However, the formation of the  $\text{Al}_2\text{O}_3$  monomer itself represents an energetic bottleneck. Therefore, we provide a bottom-up interpolation of the cluster characteristics towards the bulk limit by excluding the monomer, approximately following a  $n^{-1/3}$  dependence.

**Key words.** Physical data and processes: Astrochemistry, Molecular data, Molecular processes – Solid state: refractory – Stars: abundances, AGB and post-AGB, atmospheres, mass-loss, winds, outflows – dust

## 1. Introduction

Asymptotic Giant Branch (AGB) stars are a major contributor to the global dust budget in galaxies (Höfner & Olofsson 2018). Owing to their refractory nature, alumina (stoichiometric formula  $\text{Al}_2\text{O}_3$ ) is a promising candidate to represent the first dust condensate in oxygen-rich AGB stars. Related alumina clusters are thought to initiate dust formation in these environments and are often referred to as *seed particles* (Gail & Sedlmayr 2013). However, the sizes and compositions of these aluminium oxide clusters are not well characterised. In this study, we investigate a range of Al:O stoichiometries in order to review these predictions and to construct realistic models of these initial dust

seeds. The emergence of a specific condensate is predicted by its condensation temperature (Tielens 2005) and depends on the thermal stability of the solid, as well as the gas density and its composition. Usually, the evaluation of the stability of the likely condensates is based on macroscopic bulk properties such as the vapour pressure, which is a measure of the volatility of a substance. Hence, the most refractory condensate is expected to have the lowest vapour pressure. Corundum ( $\alpha$ -alumina), corresponding to the most stable crystalline bulk form of alumina, fulfills this condition (Gail et al. 2013). The growth and size distribution of dust grains is commonly described by Classical Nucleation Theory (CNT). However, the applicability of CNT in an expanding circumstellar envelope has been questioned (Donn & Nuth 1985; Goumans & Bromley 2012; Bromley et al. 2016; Gobrecht et al. 2017). In particular, the concept of vapor

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pressures and the universal assumption of thermodynamic equilibrium are in contradiction with the synthesis and growth of dust grains in highly dynamical AGB atmospheres. Moreover, in CNT, the properties of small solids are derived from the (crystalline) bulk material. However, the properties of nano-sized clusters often differ significantly from bulk analogues. The constraints associated with extreme small size leads to clusters with non-crystalline structures, whose characteristics (e.g. energy, geometry, bond lengths and angles, atomic coordination) differ substantially from the bulk material (Bromley & Zwijnenburg 2016). In particular, the energetic stability of such nano-clusters are typically higher than that of clusters with structures directly obtained from "top down" cuts from the parent bulk crystalline material, which represent meta-stable, or even unstable, configurations (Lamiel-Garcia et al. 2017). In addition, the concept of surface free energy (or tension), which is fundamental in CNT, is not applicable to small clusters, where it is difficult to differentiate between surface and bulk. Surface energies can only be applied to clusters with fairly large sizes (e.g. faceted bulk cut clusters). We understand nucleation as the formation and growth of stable seed nuclei (i.e., clusters) from prevalent gas-phase molecules, whose abundance varies in time (i.e. is often not in equilibrium). Therefore, a cluster is intermediate in size between a molecule and a bulk solid.

Pure oxygen in the gas phase has no other stable forms than atomic O, molecular O<sub>2</sub> and O<sub>3</sub>. Though solid O<sub>2</sub> ice exists in the interstellar medium, its condensation temperature is far too low to instigate circumstellar dust nucleation. A homo-atomic monomeric nucleation, as occurs in the case of carbon (see e.g. (Gail et al. 1984)), is thus not applicable. Therefore, the nucleation likely proceeds via several chemical elements (i.e. via a hetero-atomic scenario). Inorganic metal oxides are promising nucleation candidates, as they are particularly thermally and structurally stable. In fact, the major part of oxygen-rich stardust is in the form of silicates (Henning 2010), which are composed of oxygen, silicon and at least one other metal (usually Mg or Fe). However, they do not represent the first dust species emerging in the atmospheres of oxygen-rich AGB stars. The thermal stability of solid enstatite (MgSiO<sub>3</sub>) and forsterite (Mg<sub>2</sub>SiO<sub>4</sub>), corresponding to Mg-rich members of the pyroxene and olivine silicate family, is lower than that of alumina, gehlenite (Ca<sub>2</sub>Al[AlSiO<sub>7</sub>]) and spinel (MgAl<sub>2</sub>O<sub>4</sub>) (Wetzel et al. 2012; Gail et al. 2013). Despite their refractory nature, the latter dust species are limited by the availability of the elements Ca and Al, both being approximately one order of magnitude less abundant than Si. Therefore, refractory Al-bearing condensates could represent seed nuclei in oxygen-rich circumstellar envelopes. Less refractory, but more abundant materials like Mg- and Fe-silicates can condense on the seeds at later stages of the wind acceleration, further radially outwards in the circumstellar envelope. Iron-rich silicates are unlikely to be condensation seeds, as their large opacity to stellar radiation would lead to the subsequent heating up and evaporation of the dust grains (Woitke 2006). Consequently, the inclusion of iron in silicates allows them to act as a thermostat and tends to occur at later stages of the wind acceleration, further radially outwards in the stellar wind of AGB stars. Furthermore, nano-sized (Mg-rich) silicates are thought to become important in the interstellar medium (Escatllar et al. 2020).

Some oxygen-rich AGB stars show a spectral emission feature around 13  $\mu\text{m}$  (Little-Marenin & Little 1990), which is commonly attributed to Al-O vibrational stretching and bending modes (Begemann et al. 1997). The carrier of this dust feature has been hypothesised to be spinel (Posch et al. 1999;

Fabian et al. 2001) or alumina (Sloan et al. 2003). The strength of the 13  $\mu\text{m}$  feature correlates with some CO<sub>2</sub> emission lines in the range of 3.3–16.3  $\mu\text{m}$  (Justtanont et al. 1998). Moreover, in many stars the 13  $\mu\text{m}$  feature is accompanied by emission features around 11, 20, 28, and 32  $\mu\text{m}$ , respectively (Sloan et al. 2003). As potential carriers for these additional emissions, different polymorphs (i.e. crystal structures) of alumina have been suggested (Sargent 2019). A recently conducted microgravity experiment in a sounding-rocket has shown that solid Al<sub>2</sub>O<sub>3</sub> exhibits broad emission in the 11–12  $\mu\text{m}$  wavelength range (Ishizuka et al. 2018). Other dust features often seen in oxygen-rich AGB stars are located at around 10  $\mu\text{m}$  and 18  $\mu\text{m}$  and they are attributed to Si–O stretching and Si–O–Si bending modes, respectively (Hackwell et al. 1970). Observational studies have shown that the features of silicate and alumina can appear together, but also separately (Karovicova et al. 2013; Decin et al. 2017; Takigawa et al. 2017). Stellar sources, showing the 13  $\mu\text{m}$  feature only, include S Ori and RCnC, which exhibit low mass-loss rates of the order of 10<sup>−8</sup> M<sub>⊙</sub>/yr to 10<sup>−7</sup> M<sub>⊙</sub>/yr. There are a number of M-type AGB stars that show both, the Si–O and the Al–O vibration modes, including GX Mon, W Hya and R Dor. Stars showing only the silicate feature might also bear alumina, but the emission at 13  $\mu\text{m}$  could be blended with a mantle of silicate material on the grains. This could, for example, be the case in the high mass-loss rate AGB star IK Tau.

The dust shell, that is associated with the 13  $\mu\text{m}$  feature, is located at 1.4–3 stellar radii (R<sub>★</sub>), whereas the silicate shell associated with the 10  $\mu\text{m}$  and 18  $\mu\text{m}$  features is located further out at distances of about 5 R<sub>★</sub> (Karovicova et al. 2013; Ohnaka et al. 2017; Takigawa et al. 2017). These findings observationally confirm the higher thermal stability of alumina in comparison with Mg-rich silicates. Begemann et al. (1997) derived infrared optical constants for amorphous types of alumina from laboratory experiments and found an emission peaking at 11.5–11.8  $\mu\text{m}$ . Albeit the amorphous laboratory-synthesized grains reflect the non-crystalline character of the clusters investigated in this study, we note a substantial difference in size. The clusters we consider are (sub)nanometer sized, whereas amorphous alumina particles produced by the sol-gel technique are micron-sized (factor of 1000 larger). Demyk et al. (2004) investigated the vibrational properties of (Al<sub>2</sub>O<sub>3</sub>)<sub>n</sub> by cluster beam experiments and found that the band positions depend on the cluster size *n*. Small clusters (*n* ≤ 8) exhibit vibrational bands around 11  $\mu\text{m}$  and the larger sized clusters around 15  $\mu\text{m}$ , pointing towards similarities with the spectra of crystalline  $\gamma$ -alumina, but not with  $\alpha$ -alumina.

The evolutionary progression of a star in the AGB phase is reflected by its increasing mass-loss rate that will eventually end in a relatively short superwind phase with a high mass-loss rate (Lagadec & Zijlstra 2008). The AGB mass-loss rate is also correlated with the regularity and in particular the period of the stellar pulsations, showing smaller rates for semi-regular AGB stars and larger rates for Mira-type stars (McDonald & Zijlstra 2016). A strikingly large fraction of the stars showing the 13  $\mu\text{m}$  emission feature are semi-regularly variable AGB stars, suggesting that these stars have not yet reached the tip of the AGB and therefore evolutionarily precede Mira-type AGB stars exhibiting regular long-period pulsations. (Sloan et al. 1996).

The metal Aluminium (Al) is the 11th most abundant element in the solar system and has an abundance of  $\sim 3 \times 10^{-6}$  with respect to the total gas (Asplund et al. 2009). Hence, the overall amount of aluminium-bearing molecules, alumina clusters and dust is limited by the availability of aluminium. In the past decades, several Al-bearing molecules have been found in

circumstellar environments including AlF (Ziurys et al. 1994), AlCl (Cernicharo & Guelin 1987), AlO (Tenenbaum & Ziurys 2009) and AlOH (Tenenbaum & Ziurys 2010). AlF has the highest bond energy (681 kJ mol<sup>-1</sup>) of all Al-containing diatomic molecules, followed by AlCl (515 kJ mol<sup>-1</sup>) and AlO (499 kJ mol<sup>-1</sup>). These bond energies have been computed in the present study and are in agreement with the compilation of bond energies provided by Luo (2007); Gail & Sedlmayr (2013). AlCl (and tentatively AlF) were detected in the envelope of the carbon-rich AGB star IRC 102+16 (Cernicharo & Guelin 1987). AlCl is also found in two oxygen-rich AGB stars with different mass-loss rates (Decin et al. 2017). The aluminium-bearing molecules AlO and AlOH were first detected in the envelopes of the red supergiant VY Canis Majoris Tenenbaum & Ziurys (2009). In subsequent studies, AlO (and AlOH) were identified by their rotational transitions in the circumstellar envelopes of several of low-mass oxygen-rich AGB stars with different mass-loss rates (Kamiński et al. 2016; De Beck et al. 2017; Decin et al. 2017). Moreover, a visible AlO transition from an electronically excited state has been observed in absorption and emission in the spectra of the prototypical star Mira (o Ceti) (Kamiński et al. 2016). Related transition dipole moments and radiative lifetimes of the excited states have been the subject of recent experimental studies, see e.g. Launila & Berg (2011); Bai & Steimle (2020). The photon absorption cross section at 4823 Å, corresponding to a strong electronic transition (B<sup>2</sup>Σ<sup>+</sup> → X<sup>2</sup>Σ<sup>+</sup>) in AlO, was recently experimentally determined by Gómez Martín et al. (2017).

In a previous study, four different species (TiO<sub>2</sub>, MgO, SiO and Al<sub>2</sub>O<sub>3</sub>) and their role as nucleation candidates in oxygen-rich circumstellar envelopes were examined (Boulangier et al. 2019). The authors assumed a quasi-stationary circumstellar envelope and applied a chemical-kinetic network to a model grid with constant pressures and temperatures. Furthermore, homogeneous and homo-molecular cluster growth was adopted for each of the nucleation candidates. Gobrecht et al. (2016) studied the kinetic nucleation and subsequent coagulation of two dust components, alumina and forsterite. The authors assumed a two-step process, where alumina nucleates first homogeneously, and subsequently, forsterite condenses heterogeneously on the surface of the alumina seeds. In this study, we focus on Al<sub>2</sub>O<sub>3</sub> as a nucleation candidate (Al<sub>2</sub>O<sub>3</sub>) by developing an extensive chemical-kinetic network that makes use of benchmark quantum calculations at a high level of theory (CBS-QB3, Montgomery et al. (2000)), includes the reaction rate estimates derived from recent experiments and statistical rate theory, and goes beyond the formation of the smallest formula units (monomers). We then apply the chemical network to two model stars, a semi-regular variable AGB star and a Mira-type AGB star. The circumstellar gas trajectories include a non-pulsating outflow model, described by a β-velocity law, and a pulsating model, described by Lagrangian flows corresponding to the pulsationally-induced excursions of a circumstellar post-shock gas.

This paper is organized as follows. In Section 2, we describe the methods used to derive the lowest-energy candidate structures and their refinement with quantum-chemical density functional theory (DFT) methods. In Section 3, we present the results of the cluster calculations including energy, structure, and vibrational spectra as well as the kinetic networks applied to circumstellar gas trajectories. We summarise our findings in Section 5.

**Table 1.** The parameter ranges used in this study to compute the interionic Buckingham pair potential are listed as (1) charges of aluminium, q(Al), (2) charges of oxygen q(O), given in atomic units, (3) A in eV, (4) B in Å, (5) C in eV Å<sup>-6</sup>.

q(Al)	q(O)	A(Al-O)	B(Al-O)	C(Al-O)
		A(O-O)	B(O-O)	C(O-O)
+3	-2	4534.2	0.2649	0.0
		25.410	0.6937	32.32

## 2. Methods

### 2.1. Global optimisation searches

The computational cost for a geometry optimisation of an aluminium oxide cluster increases with its size (or with the number of atoms or electrons), typically following a power-law. However, our ability to explore the potential energy landscapes is more limited by the number of possible isomers, which exponentially increases with cluster size (Stillinger & Weber 1983; Arslan & Gven 2005). To reduce the number of possible structural configurations to explore (and hence also the computational effort), global optimisation searches for low-energy aluminium-oxide clusters are performed. We employ the Monte-Carlo Basin-Hopping (MC-BH) global optimisation technique (Wales & Doye 1998) with interionic pair potentials of an Al-O system to find candidate low-energy clusters. For our purposes, we used an in-house, modified version of the GMIN programme (Bromley & Flikkema 2005). The general form of the interionic Buckingham pair potential (including the Coulomb potential) is

$$U(r_{ij}) = \frac{q_i q_j}{r_{ij}} + A \exp\left(-\frac{r_{ij}}{B}\right) - \frac{C}{r_{ij}^6}, \quad (1)$$

where  $r_{ij}$  is the relative distance between two atoms,  $q_i$  and  $q_j$  the charges of atom  $i$  and  $j$ , respectively, and  $A$ ,  $B$  and  $C$  the Buckingham parameters. The first term in Eq. 1 describes the interionic electrostatic interactions, the second term the short-range, steric repulsion term due to the Pauli exclusion principle, and the last term describes the attractive van der Waals interaction. The potential describes the repulsion and attraction of charged particles, in this case, of aluminium and oxygen ions within an Al–O containing cluster. To reduce the probability of missing stable configurations in our searches, we used a large number of structurally diverse initial geometries. Moreover, we performed test calculations by swapping the Al and O atoms in the most stable configurations accounting for atomic segregation (i.e. covalent bonds between identical atoms). We applied the parameter set listed in Table 1, commonly used for structure optimisation of Al–O systems (Woodley et al. 1999). The searches cover diverse structural families including compact geometries, void cages and open-cage-like clusters by choosing various seed structures (i.e. initial geometries). In systems like aluminium oxides, electronic polarisation should also play a role. Therefore, we re-optimised our candidate isomers using a potential that describes polarisation via a core-shell model according to the parameters of Bush et al. (1994) using the General Utility Lattice Program (GULP) developed by Gale (1997). We did not find any additional structural isomers, but achieved a more realistic energetic ordering of different cluster isomers with respect to more accurate quantum calculations (see next subsection). Although the use of interionic potentials is an approximation, it enabled us to perform tractable yet thorough searches. With our approach we aimed to minimise the probability of missing stable alumina

cluster configurations. For a comparison of the performance of different alumina interionic potentials (i.e. force fields), we refer to Laurens et al. (2020).

## 2.2. Quantum chemical calculations

For the smallest molecular systems (i.e. up to 12 atoms, slightly more than the size of an alumina dimer), we performed quantum-chemical compound method calculations. Compound methods combine a high level of theory and a small basis set with methods that employ lower levels of theory with larger basis sets. We use the benchmark Complete Basis Set (CBS-QB3) method that extrapolates several single-point energies to a more accurate CBS-QB3 energy in the basis set limit (Montgomery et al. 2000). We note that the CBS-QB3 method is prohibitive for large clusters ( $> 15$  atoms). CBS-QB3 calculations were performed for the following species: Al, O, H, Cl, F, H<sub>2</sub>, H<sub>2</sub>O, OH, AlO, AlOH, SiO, AlCl, AlF, Al<sub>2</sub>O, AlO<sub>2</sub>, OAlOH, Al(OH)<sub>2</sub>, Al<sub>2</sub>O<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, Al<sub>2</sub>O<sub>4</sub>, Al<sub>3</sub>O<sub>2</sub>, Al<sub>3</sub>O<sub>3</sub>, Al<sub>3</sub>O<sub>4</sub>, Al<sub>3</sub>O<sub>5</sub>, Al<sub>4</sub>O<sub>3</sub>, Al<sub>4</sub>O<sub>4</sub>, Al<sub>4</sub>O<sub>5</sub>, Al<sub>4</sub>O<sub>6</sub>, Al<sub>4</sub>O<sub>7</sub>, Al<sub>5</sub>O<sub>4</sub>, Al<sub>5</sub>O<sub>5</sub>, Al<sub>5</sub>O<sub>6</sub>, Al<sub>5</sub>O<sub>7</sub>, AlSiO<sub>3</sub>, and HAlSiO<sub>3</sub>. Once multiple sets of candidate structures with different initial geometries, temperatures and parameters were found, we refined the  $\sim 50$ -100 most favourable candidate structures for each size in subsequent optimisations at a DFT level of theory using two different hybrid density functionals, B3LYP (Becke 1993) and PBE0 (Perdew et al. 1996), in combination with the 6-311+G(d) basis set. We performed these calculations using the computational chemistry software package Gaussian09 (Frisch et al. 2010). We used f-type orbitals (7F) for the basis functions and an ultrafine grid corresponding to the standard input. The DFT calculations were performed at 0 K and a pressure of 0 atm. In the Born-Oppenheimer approximation used here, the Potential Energy Surface (PES) does not depend on temperature. Hence, the optimised cluster geometry is also temperature-independent. However, the vibrational population and the computation of the thermodynamic quantities (i.e. enthalpy, entropy, and Gibbs free energy) depend on temperature. Moreover, the entropy and the Gibbs free energy are also pressure-dependent. We include a vibrational analysis to calculate the vibrational zero-point energy as well as appropriate partition functions for any other conditions. The partition functions are composed of electronic, translational, rotational and vibrational contributions and are used to compute the enthalpy, the entropy and eventually the Gibbs free energy. Moreover, a vibrational analysis helps to identify and exclude possible transition states characterised by an imaginary frequency. The predicted vibrational spectra of the clusters can then be compared with astronomical observations and laboratory experiments.

## 2.3. Transition State Theory and RRKM

Rate coefficients for reactions with intermediate local minima on their potential energy surfaces were calculated with Rice-Ramsperger-Kassel-Markus (RRKM) theory, using the Master Equation Solver for Multi-Energy well Reactions (MESMER) program (Glowacki et al. 2012). The geometries of the Al-containing molecules (reactants, products and intermediates) were first optimized at the B3LYP/6-311+g(2d,p) level of theory within the Gaussian 16 suite of programs (Frisch et al. 2010), and the resulting rotational constants and vibrational frequencies used for the MESMER calculations. The CBS-QB3 method was

used to obtain more accurate relative energies of these stationary points. Each intermediate species formed during the reaction was assumed to dissociate back to the reactants or forward to the products, or be stabilised by collision with H<sub>2</sub> as a third body (for the astrochemical environment). The internal energy of each intermediate was divided into a contiguous set of bins (typical width of 110 cm<sup>-1</sup>) containing a bundle of rovibrational states. The density of these bundles was calculated with the theoretical vibrational frequencies and rotational constants, without making a correction for anharmonicity and using a classical density of states treatment for the rotational modes. Each bin was then assigned a set of microcanonical rate coefficients for dissociation to reactants or products (as appropriate). These rate coefficients were determined using an inverse Laplace transformation to link them directly to the capture rate coefficient –calculated using long-range transition state theory (Georgievskii & Klippenstein 2005). The probability of collisional transfer between bins was estimated using the exponential-down model, where the average energy for downward transitions is designated  $\langle \Delta E \rangle_{down}$ , and the probabilities for upward transitions were determined by detailed balance  $\langle \Delta E \rangle_{down}$  down was treated as temperature-independent, with a value of 200 cm<sup>-1</sup> for H<sub>2</sub> (Gilbert & Smith 1990). The Master Equation, which describes the evolution with time of the adduct bin populations, was then expressed in matrix form and solved to yield the rate coefficient for bimolecular reactions and recombination at a specified pressure and temperature.

## 3. Results

### 3.1. Precursors of alumina dust

The molecular gas-phase precursors of stoichiometric (Al<sub>2</sub>O<sub>3</sub>)<sub>n</sub> clusters are likely AlO and AlOH, which are the most abundant and prevalent aluminium-bearing molecules in an oxygen-rich circumstellar gas. Furthermore, we include other related aluminum-oxygen-hydrogen species containing Al<sub>2</sub>O, AlO<sub>2</sub>, Al<sub>2</sub>O<sub>2</sub>, OAlOH, and Al(OH)<sub>2</sub>, as well as aluminum halides like AlF and AlCl. As a first step, we assess the accuracy of the employed electronic structure methods (CBS-QB3, B3LYP, PBE0) and the basis sets (6-311+G(d) and cc-pVTZ). For this purpose, we compare the calculated enthalpies of formation at 0 K ( $\Delta H_f^0$ ) with experimental data of the NIST-JANAF database<sup>1</sup> (see Table 2). Note, that in contrast to the binding energies,  $\Delta H_f^0(0)$  is scaled with respect to the atomic heats of formation. Overall, the CBS-QB3 method results in the most accurate energies for the considered aluminium-bearing molecules.

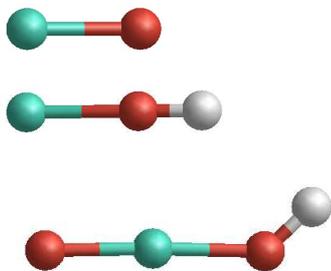
#### 3.1.1. Molecular dust precursors

In the following, we use the term *binding* energy corresponding to the CBS-QB3 energy with respect to the constituent atoms and which is normalised with respect to the number of atoms in the respective molecule or cluster. Note that the binding energies are not scaled to the atomic heats of formation, unlike the JANAF enthalpies of formation. *Relative* energies correspond to the energy difference between two isomers of a given composition and size, typically between the lowest-energy structure and a higher-lying isomer.

<sup>1</sup> <https://janaf.nist.gov/>

## AIO

The ground state of AIO (displayed in Figure 1 has an unpaired electron and has thus a spin multiplicity of 2 (doublet state). Its equilibrium bond length is 1.630 Å, which is in good agreement with the experimental value of 1.618 Å (Huber & Herzberg 1979). The AIO bond dissociation energy is 498 kJ mol<sup>-1</sup> (i.e., a binding energy of 249 kJ mol<sup>-1</sup> per atom), which is in very good agreement to the experimental dissociation energy of 501.9±10.6 (Luo 2007).



**Fig. 1.** Structures of the molecules AIO, AIOH and OAIOH. Al atoms are displayed in turquoise, O atoms in red, and H atoms in white.

## AIOH

The singlet state AIOH molecule (also displayed in Figure 1) can be present in linear or in bent form. At 0 K the linear isomer is more stable than the bent form by only 2.8 kJ mol<sup>-1</sup>. Although recent findings of Trabelsi & Francisco (2018) indicate a slightly bent form as the AIOH ground state, we find the linear form to be more stable for all temperatures considered in this study. The AIOH binding energy is 327 kJ mol<sup>-1</sup>. The AIOH dissociation leads either to the products Al+OH with a dissociation energy of 554 kJ mol<sup>-1</sup>, or to AIO+H with a dissociation energy of 482 kJ mol<sup>-1</sup>. The two AIOH dissociation channels are highly endothermic and therefore, the molecule is relatively stable once it has formed.

## OAIOH

The energetically most favourable geometry of OAIOH (once again illustrated in Figure 1) shows the terminal H atom bent at an angle of ~ 45° with respect to the O-Al-O inter-atomic axis. The binding energy of OAIOH is 352 kJ mol<sup>-1</sup>. OAIOH can be formed by the reaction AIO+H<sub>2</sub>O. This reaction was studied in detail by Mangan et al. (2021) showing that OAIOH is a minor product of the reaction.

The stability and structure of ground state OAIOH has been investigated by Cobos (2002). We consider three possible dissociation channels of OAIOH. With an energy barrier of 429 kJ mol<sup>-1</sup>, O + AIOH is the least endothermic reaction, followed by AIO+OH with 485 kJ mol<sup>-1</sup> and OAIO+H with 520 kJ mol<sup>-1</sup>.

The enthalpies of formation (i.e. binding energies), derived from our DFT calculations, seem to be systematically lower than those derived empirically in JANAF (2). However, we note that the JANAF values are often based on incomplete or extrapolated experimental data dating back to 1970's. Therefore, the tabulated JANAF energies may not be accurate. Nevertheless, it is a standard reference database, and we include those values for

comparison purposes. We find better agreement with JANAF by using a larger numerical basis set (cc-pVTZ) compared with a Gaussian type basis set (6-311+G(d)). In addition, we find that the B3LYP functional to provide values in better agreement with the JANAF values than PBE0. The enthalpies derived using the composite CBS-QB3 method shows the closest agreement with the JANAF values, particularly for the case of Al<sub>2</sub>O<sub>2</sub>. However, CBS-QB3 tend to be prohibitive for large systems with more than ~ 15 atoms. Therefore, we use the B3LYP/cc-pVTZ method as a compromise between accuracy and computational feasibility.

## 3.1.2. Al<sub>x</sub>O<sub>y</sub> (where $x=2-5$ and $y=1-6$ ) clusters

The choice of the stoichiometric range ( $x=2-5$ ,  $y=1-6$ ) of Al<sub>x</sub>O<sub>y</sub> clusters (displayed in Table 3) is based on stoichiometric reasoning (i.e. realistic Al:O ratios that are in the vicinity of 2:3) and on the abundances of aluminium and oxygen (and hydrogen). Primarily, the energetic and kinetic stability of the Al-O-H containing species determines their abundances. However, on shorter timescales, the species abundances can be set by the availability of their component (i.e., the elements). For example, despite its lower bond energy (~300 kJ/mol), AlH is temporarily more abundant than the strongly bound AIO molecule (~500 kJ/mol), due to the omnipresence of hydrogen. Therefore, we consider Al<sub>x</sub>O<sub>y</sub> clusters with no more than one Al atom in excess i.e.,  $x \leq y + 1$ . The molecule Al<sub>2</sub>O is a linear molecule with a CBS-QB3 binding energy of 355 kJ mol<sup>-1</sup>, which is relatively large. As a symmetric linear molecule, Al<sub>2</sub>O cannot be observed by pure rotational spectroscopy, owing to the lack of a permanent dipole moment. A non-zero quadrupole moment is present, but the related transitions are 'forbidden'. Other Al<sub>2</sub>O isomers with permanent dipole moments (including triplet states) lie ~ 300 kJ mol<sup>-1</sup> above the electronic ground state and are thus unlikely to be observed by rotational lines (Danilovich et al. 2020). However, we note that asymmetric and bending stretches in Al<sub>2</sub>O could be observed by (ro-)vibrational spectroscopy.

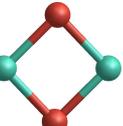
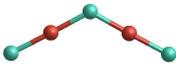
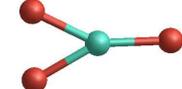
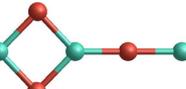
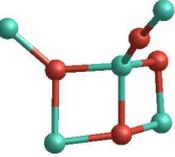
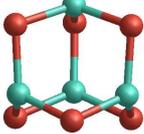
Also, the global minimum (GM) structure of AlO<sub>2</sub> is symmetric and linear and so is not observable by its pure rotational spectrum. Although AlO<sub>2</sub> isomers with relative energies below 200 kJ mol<sup>-1</sup> exist, their dipole moments are modest making them difficult to detect. The binding energy of AlO<sub>2</sub> is 296 kJ mol<sup>-1</sup>. The lowest-energy Al<sub>2</sub>O<sub>2</sub> configuration has a diamond-shaped, rhombic form and has a binding energy of 386 kJ mol<sup>-1</sup>. Owing to its symmetry this molecule has no permanent dipole moment. Hence, Al<sub>2</sub>O<sub>2</sub> is not observable by rotational transitions. Asymmetric and bending vibration modes of Al<sub>2</sub>O<sub>2</sub> can be observed, but the related intensities of these vibrations modes are comparatively rather low. The linear Al<sub>2</sub>O<sub>2</sub> isomer has a relative energy of 32 kJ mol<sup>-1</sup> above the GM. Moreover, we also find a triplet diamond-shaped Al<sub>2</sub>O<sub>2</sub> molecule with a relative energy of 232 kJ mol<sup>-1</sup> above the GM. We conclude that circumstellar Al<sub>2</sub>O<sub>2</sub> is predominantly in the form of the GM structure. Al<sub>2</sub>O<sub>3</sub> corresponds to the monomer of stoichiometric alumina and will be discussed separately in Section 3.1.3. The Al<sub>2</sub>O<sub>4</sub> GM candidate shows a flat D<sub>2h</sub> symmetric structure consisting of a diamond-shaped Al<sub>2</sub>O<sub>2</sub> ring with an extra oxygen atom attached to each of the Al atoms. The Al<sub>2</sub>O<sub>4</sub> binding energy is 414 kJ mol<sup>-1</sup>.

The lowest-energy Al<sub>3</sub>O<sub>2</sub> configuration has a binding energy of 384 kJ mol<sup>-1</sup>. It is a bent linear chain with a bond angle of 122.5° on the central Al atom. Similarly, the GM Al<sub>3</sub>O<sub>3</sub> structure is flat and kite-shaped. It can be seen as a

**Table 2.** Enthalpies of formation,  $\Delta H_f^0$ , at  $T = 0$  K in  $\text{kJ mol}^{-1}$  of Al-bearing molecules for density functionals/basis sets used in this study, compared with the JANAF thermochemical database.

	AlO	AlO <sub>2</sub>	Al <sub>2</sub> O	Al <sub>2</sub> O <sub>2</sub>	AlOH	OAlOH	AlH	AlCl	AlF	Al <sub>2</sub>
PBE0/6-311+G	+113.	+8.	-86.	-240.	-117.	-253.	+269.	-38.	-224.	+590.
B3LYP/6-311+G	+98.	-21.	-98.	-249.	-134.	-277.	+253.	-31.	-238.	+599.
PBE0/cc-pVTZ	+101.	-20.	-99.	-281.	-143.	-292.	+266.	-44.	-237.	+586.
B3LYP/cc-pVTZ	+85.	-50.	-113.	-292.	-159.	-316.	+250.	-38.	-251.	+595.
CBS-QB3	+76.	-70.	-116.	-395.	-190.	-372.	+242.	-68.	-277.	+597.
JANAF	+67.04	-85.01	-144.48	-391.31	-175.63	-454.51	+259.51	-51.66	-265.62	+486.28

**Table 3.** Global minimum (GM) candidate structures of Al<sub>x</sub>O<sub>y</sub>,  $x = 1-5$   $y = 1-6$  clusters including binding energies per atom (in  $\text{kJ mol}^{-1}$ ). The color-coding of the atoms follows that used in Figure 1 and holds for all Figures.

	AlO <sub>y</sub> ( $x = 1$ )	Al <sub>2</sub> O <sub>y</sub> ( $x = 2$ )	Al <sub>3</sub> O <sub>y</sub> ( $x = 3$ )	Al <sub>4</sub> O <sub>y</sub> ( $x = 4$ )	Al <sub>5</sub> O <sub>y</sub> ( $x = 5$ )
Al <sub>x</sub> O ( $y = 1$ )	 249.	 355.			
Al <sub>x</sub> O <sub>2</sub> ( $y = 2$ )	 296.	 386.	 384.		
Al <sub>x</sub> O <sub>3</sub> ( $y = 3$ )	 294.	 388.	 383.	 425.	
Al <sub>x</sub> O <sub>4</sub> ( $y = 4$ )		 414.	 437.	 452.	 426.
Al <sub>x</sub> O <sub>5</sub> ( $y = 5$ )			 437.	 465.	 460.
Al <sub>x</sub> O <sub>6</sub> ( $y = 6$ )			 429.	 484.	 471.

composite of Al<sub>2</sub>O<sub>2</sub> and AlO and has a binding energy of 383  $\text{kJ mol}^{-1}$ . The Al<sub>3</sub>O<sub>4</sub> GM has a C<sub>3v</sub> symmetric pyramidal form. It differs from the other Al<sub>x</sub>O<sub>y</sub> molecules in the sense that it is the smallest three dimensional molecule in this study and has a 3-coordinated O atom (top of the pyramid), as has been noticed previously (Patzner et al. 2005). The binding energy of this structure is 437  $\text{kJ mol}^{-1}$ .

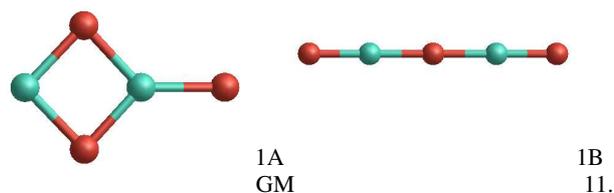
The GM Al<sub>3</sub>O<sub>5</sub> structure has a C<sub>s</sub> symmetry and was recently reported by Armstrong et al. (2019). The binding energy per atom is 438  $\text{kJ mol}^{-1}$ . The geometry as well as the bond energy is very close to that of Al<sub>3</sub>O<sub>4</sub>. The dangling oxygen atom is expected to be the most reactive site of the molecule. We also find another structurally similar isomer that is just 8.1  $\text{kJ mol}^{-1}$  higher in energy. It has a three-dimensional geometry with a C<sub>2v</sub> symmetry and was firstly found by Martinez et al. (2001). The most favourable Al<sub>3</sub>O<sub>6</sub> is a flat structure consisting of a

$\text{Al}_3\text{O}_3$  ring having a terminal oxygen atom on each of the three Al atoms (Gowtham et al. 2004). The lowest-energy  $\text{Al}_4\text{O}_3$  isomer is a flat arrangement around a central Al atom with a perfect three-fold symmetry (point group  $D_{3h}$ ). All other investigated geometries did not converge, or, are found to be transition states relaxing to the three-fold GM. The binding energy is  $426 \text{ kJ mol}^{-1}$ . The most stable  $\text{Al}_4\text{O}_4$  configuration is a chain with a rhombus in the center. The flat  $C_{2v}$ -symmetric geometry can be regarded as an  $\text{Al}_2\text{O}_2$  diamond with a terminal AlO group on each side. Structurally,  $\text{Al}_4\text{O}_4$  can be regarded as a compound of  $\text{AlO}+\text{Al}_2\text{O}_2+\text{AlO}$ . The binding energy at the CBS-QB3 level of theory is  $452 \text{ kJ mol}^{-1}$ . With respect to the AlO molecule, the  $\text{Al}_4\text{O}_4$  dissociation energy is  $406 \text{ kJ mol}^{-1}$ . The most favourable  $\text{Al}_4\text{O}_5$  cluster is a three-dimensional  $C_{2v}$  symmetric configuration consisting of two 6-member rings sharing two AlO bonds as previously reported by Zhong et al. (2011). In this configuration all O atoms are 2-coordinated and all Al atoms are 3-coordinated. Its binding energy per atom is quite high ( $465 \text{ kJ mol}^{-1}$ ), but still below the alumina dimer ( $484 \text{ kJ mol}^{-1}$ ). The isomer with the second lowest energy is still  $89 \text{ kJ mol}^{-1}$  above the GM candidate and has a  $C_s$  symmetry. Other  $\text{Al}_4\text{O}_5$  GM candidates reported in the literature show even larger relative energies and are far above our GM candidate. For example, the GM candidate reported by (Lam et al. 2015) is still  $97 \text{ kJ mol}^{-1}$  above our finding. The structure of the most stable  $\text{Al}_5\text{O}_4$  isomer has been predicted by Das & Raghavachari (2008). We find a CBS-QB3 binding energy of  $426 \text{ kJ mol}^{-1}$  per atom. Its negative ion ( $\text{Al}_5\text{O}_4^-$ ) is found to be a highly symmetric planar structure with strong electron affinity. A very similar quasi-planar structure has also been predicted as a GM candidate for neutral  $\text{Al}_5\text{O}_4$  (Armstrong et al. 2019). However, its CBS-QB3 energy is  $32 \text{ kJ mol}^{-1}$  above our non-planar GM candidate. The GM isomer candidate of  $\text{Al}_5\text{O}_5$  is a flat  $C_{2v}$ -symmetric structure consisting of a six-member ring with two *cis*-oriented terminal Al-O groups. Its CBS-QB3 binding energy is  $460 \text{ kJ mol}^{-1}$  per atom and  $422 \text{ kJ mol}^{-1}$  per AlO unit. Another GM candidate, reported by (Armstrong et al. 2019), consisting of a  $\text{Al}_4\text{O}_4$  cube with an O-Al chain on one of the Al edges, has a CBS-QB3 energy which is  $29 \text{ kJ mol}^{-1}$  above our GM candidate. The lowest-energy  $\text{Al}_5\text{O}_6$  isomer was also previously found by Armstrong et al. (2019). It has a binding energy of  $472 \text{ kJ mol}^{-1}$  and shows no symmetry. A quasi-planar ‘heart’-shaped isomer with a  $C_s$  symmetry lies  $14 \text{ kJ mol}^{-1}$  above the GM candidate.

### 3.1.3. Alumina ( $\text{Al}_2\text{O}_3$ )<sub>n</sub>, n = 1–10, clusters

#### Alumina monomer ( $\text{Al}_2\text{O}_3$ )

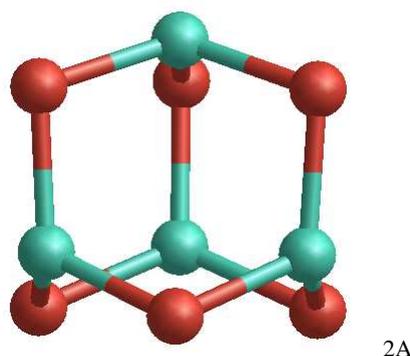
The most favourable alumina monomer structure (1A) has a kite-shaped form and is in a triplet state (see Figure 2). Its geometry is flat and obeys a  $C_{2v}$  point symmetry. The binding energy (per atom) on the CBS-QB3 level of theory is  $388 \text{ kJ mol}^{-1}$ , which is higher than for DFT calculations using the B3LYP functional ( $359 \text{ kJ mol}^{-1}$ ) and the PBE0 functional ( $360 \text{ kJ mol}^{-1}$ ). The linear singlet alumina monomer (1B) has a relative energy of  $15 \text{ kJ mol}^{-1}$ ,  $11 \text{ kJ mol}^{-1}$ , and  $44 \text{ kJ mol}^{-1}$  above 1A at the CBS-QB3, B3LYP and PBE0 levels of theory, respectively. All other isomers have significantly higher energies than 1A and 1B, and are metastable with respect to 1A and 1B.



**Fig. 2.** Left: GM structure of the alumina monomer  $\text{Al}_2\text{O}_3$  (1A) Right: Second lowest-energy isomer of the alumina monomer (1B).

#### Alumina dimer ( $\text{Al}_2\text{O}_3$ )<sub>2</sub>

The GM alumina dimer cluster (2A) is displayed in Figure 3. The geometry of the GM cluster is a cage composed of four 6-membered rings and shows a tetrahedral symmetry (point group  $T_d$ ). All Al atoms are 3-coordinated and all O atoms are 2-coordinated corresponding to the valence of the atoms. Moreover, owing to its symmetry, 2A is characterised by one single bond distance of  $1.744 \text{ \AA}$  for all 12 bonds of the clusters. Owing to its energetic stability, its symmetry and its approximate sphericity, 2A is a natural and logical link between a molecular regime controlled by chemical-kinetics and cluster coagulation (see also Gobrecht et al. (2016)). Unfortunately, as a consequence of the tetrahedral symmetry, 2A has no permanent dipole moment and observations in the IR are challenging. The energetically second-lowest dimer isomer have CBS-QB3, B3LYP and PBE0 energies that are  $42 \text{ kJ mol}^{-1}$ ,  $45 \text{ kJ mol}^{-1}$ , and  $32 \text{ kJ mol}^{-1}$  above 2A, respectively. Owing to this considerable energy difference, only 2A is considered to contribute to the dimer abundance. The CBS-QB3 binding energy of 2A is  $484 \text{ kJ mol}^{-1}$  corresponding to the largest binding energy in Table 3. As for the monomer, the CBS-QB3 binding energy is higher than the predictions of B3LYP ( $442 \text{ kJ mol}^{-1}$ ) and PBE0 ( $446 \text{ kJ mol}^{-1}$ ).

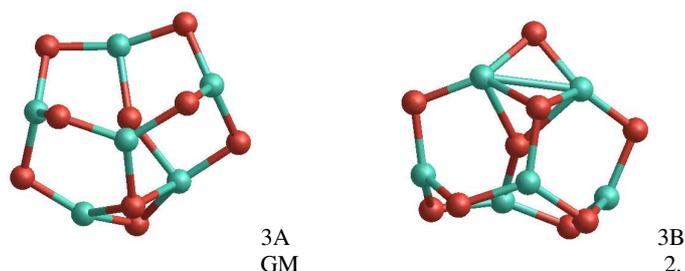


**Fig. 3.** GM structure of the alumina dimer ( $\text{Al}_2\text{O}_3$ )<sub>2</sub> (2A)

#### Alumina trimer ( $\text{Al}_2\text{O}_3$ )<sub>3</sub>

The most favourable alumina trimer (3A) is a ‘tea-cosy’-shaped structure (see Figure 4). The binding energy at the CBS-QB3, B3LYP and PBE0 levels of theory are  $515 \text{ kJ mol}^{-1}$ ,  $470 \text{ kJ mol}^{-1}$  and  $476 \text{ kJ mol}^{-1}$ , respectively. As Li & Cheng (2012) have shown, there are four energetically low-lying structural isomers that are close in energy (all within an energy range of  $5.5 \text{ kJ mol}^{-1}$  at the B3LYP/6-311+G(d) level of theory). Consequently,

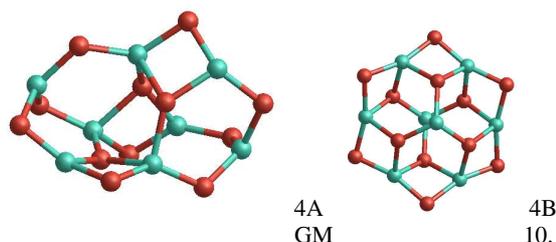
all four isomers are expected to contribute to the overall abundance of alumina trimers. We confirm the narrow spacing in energy of the two most stable configurations (3A and 3B) by more accurate CBS-QB3 calculations. However, we find that 3C, as predicted in Li & Cheng (2012), lies  $21 \text{ kJ mol}^{-1}$  (for B3LYP) and  $44 \text{ kJ mol}^{-1}$  (for PBE0) above 3A; for 3D we found an imaginary frequency indicating a transition state and not a real minimum. As pointed out, the second most stable trimer (3B) is energetically degenerate as its CBS-QB3, B3LYP and PBE0 energies lie only  $0.7 \text{ kJ mol}^{-1}$ ,  $2.1 \text{ kJ mol}^{-1}$  and  $0.2 \text{ kJ mol}^{-1}$  above 3A respectively.



**Fig. 4.** GM candidate structure of the alumina trimer ( $\text{Al}_2\text{O}_3$ )<sub>3</sub>. *Left: 3A ; Right: 3B.*

#### Alumina tetramer ( $\text{Al}_2\text{O}_3$ )<sub>4</sub>

The lowest energy configuration of the alumina tetramer (4A) found with the B3LYP functional has no particular symmetry (point group  $C_1$ ). Its B3LYP and PBE0 binding energy are  $484 \text{ kJ mol}^{-1}$  and  $488 \text{ kJ mol}^{-1}$ , respectively. With the PBE0 functional we find a different lowest-energy isomer (4B) showing a highly symmetric  $D_{3d}$  structure (see Figure 5). The binding energy of 4B is found to be  $483 \text{ kJ mol}^{-1}$  (B3LYP) and  $497 \text{ kJ mol}^{-1}$  (PBE0). Experiments have shown that 4A is the GM structure and indicate that the B3LYP method accurately describes  $\text{Al}_2\text{O}_3$  structures (Sierka et al. 2007).

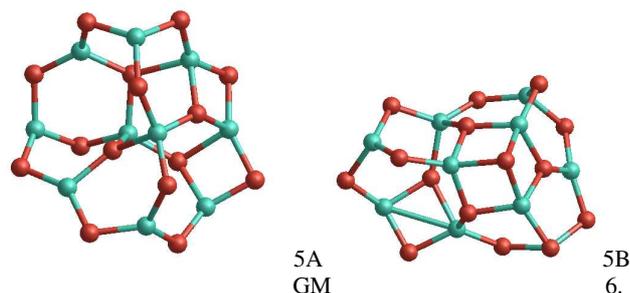


**Fig. 5.** Global minimum candidates of the alumina tetramer ( $\text{Al}_2\text{O}_3$ )<sub>4</sub>. *Left: 4A; Right: 4B*

#### Alumina pentamer ( $\text{Al}_2\text{O}_3$ )<sub>5</sub>

The lowest-energy alumina pentamer shows no symmetry ( $C_1$ ) and is displayed in Figure 6. The B3LYP and PBE0 binding energies of 5A are  $492 \text{ kJ mol}^{-1}$  and  $497 \text{ kJ mol}^{-1}$ , respectively. As for the tetramer ( $n=4$ ) we find a different GM candidate (5B) at the PBE0/6-311+G level of theory. 5B has no particular symmetry and lies  $5.5 \text{ kJ mol}^{-1}$  above 5A using B3LYP, but is  $3.4 \text{ kJ mol}^{-1}$  lower in energy using PBE0. As the energy differences between 5A and 5B are small in calculations using both functionals, B3LYP and PBE0, we conclude that these degenerate

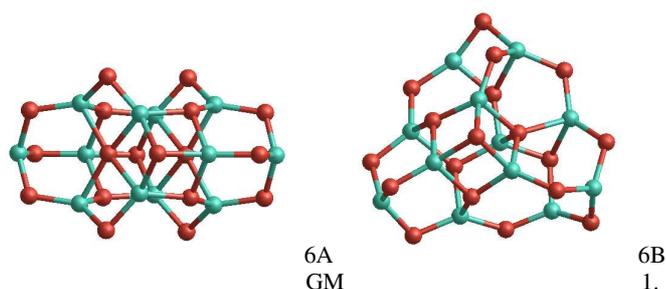
structures could both be considered as GM structures and they contribute equally to the pentamer abundance.



**Fig. 6.** Global minimum candidates of the alumina pentamer ( $\text{Al}_2\text{O}_3$ )<sub>5</sub>. *Left: 5A; Right: 5B .*

#### Alumina hexamer ( $\text{Al}_2\text{O}_3$ )<sub>6</sub>

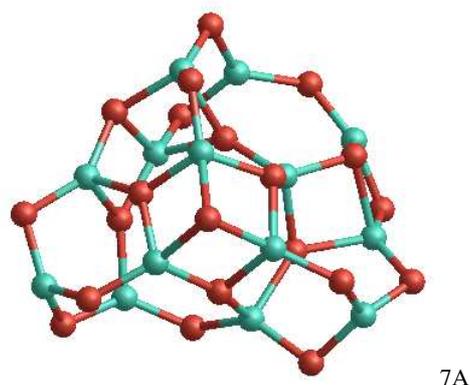
The most favourable alumina hexamer is shown in Figure 7. It exhibits a  $C_{2h}$  symmetric structure. The binding energy per atom is  $499 \text{ kJ mol}^{-1}$  (B3LYP) and  $505 \text{ kJ mol}^{-1}$  (PBE0). Due to its symmetry 6A has no net dipole moment. It should be noted that another hexamer (6B) structure is essentially degenerate with an energy of only  $0.5 \text{ kJ mol}^{-1}$  above 6A (B3LYP). With the PBE0 functional, the energy difference between 6A and 6B is larger ( $14.5 \text{ kJ mol}^{-1}$ ) though also not significantly different. This degeneracy has already been noted by Li & Cheng (2012). We applied a larger basis set (cc-pVTZ) to 6A and 6B to test the reliability of our results with the 6-311+G(d,p) basis set. We find that 6A is lower than 6B by  $5.9 \text{ kJ mol}^{-1}$  (B3LYP/cc-pVTZ) and  $21.0 \text{ kJ mol}^{-1}$  (PBE0/cc-pVTZ). These results indicate that the calculations with the 6-311+G(d,p) basis set are well-founded and that 6A is indeed the lowest-energy isomer for  $n=6$  though structure 6B is relatively close in energy. Structure 6B has no symmetry ( $C_1$ ). Its binding energy at the B3LYP and PBE0 level of theory are  $499 \text{ kJ mol}^{-1}$  and  $505 \text{ kJ mol}^{-1}$ , respectively.



**Fig. 7.** Global minimum candidates of the alumina hexamer ( $\text{Al}_2\text{O}_3$ )<sub>6</sub>. *Left: 6A; Right: 6B.*

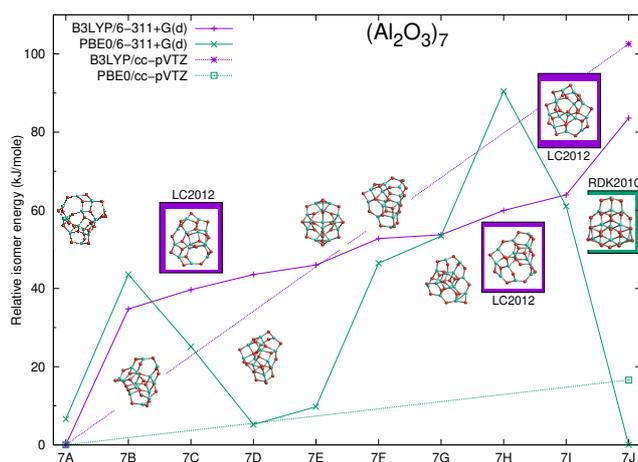
#### Alumina heptamer ( $\text{Al}_2\text{O}_3$ )<sub>7</sub>

We report the discovery of six energetically low-lying isomers (7A, 7B, 7D, 7E, 7F, 7G) including a GM candidate (7A), which are shown in Figures 8 and 9. To our knowledge, these six isomers were hitherto not reported. 7A does not show any symmetry and have B3LYP and PBE0 binding energies of  $504 \text{ kJ mol}^{-1}$  and  $509 \text{ kJ mol}^{-1}$  per atom, respectively.



**Fig. 8.** Global minimum candidate (7A) of the alumina heptamer ( $\text{Al}_2\text{O}_3$ )<sub>7</sub>.

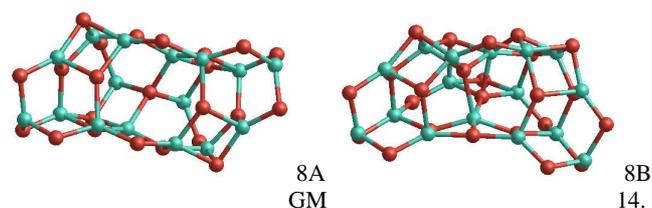
Using the PBE0 functional, we find a different lowest-energy isomer (7J) that was previously reported by Rahane et al. (2011). As for  $n = 6$ , we perform benchmark calculations of 7A and 7J with a larger, numerical basis set (cc-pVTZ) revealing that 7A is lower in energy (B3LYP/cc-pVTZ:  $102.6 \text{ kJ mol}^{-1}$ , PBE0/cc-pVTZ:  $16.6 \text{ kJ mol}^{-1}$ ) than 7J (see Figure 9). Therefore, we assume 7A to be the GM candidate.



**Fig. 9.** Relative energies of the lowest-energy alumina heptamer isomers ( $\text{Al}_2\text{O}_3$ )<sub>7</sub>. Structures reported by Li & Cheng (2012) are indicated in violet as LC2012 and the GM candidate reported by Rahane et al. (2011) in green as RDK2010.

Using the cc-pVTZ basis set, we note that the relative energies of 7A and 7J differ by more than  $80 \text{ kJ mol}^{-1}$ , comparing B3LYP with PBE0. Moreover, the energetic ordering of the structural isomers is very different for PBE0/6-311+G(d) (7J,7A) and B3LYP/6-311+G(d) (7A,7B,7C,7D,7E,7F,7G,7H,7I,7J). This is a rather surprising result, as the hybrid density functionals B3LYP and PBE0 differ primarily by the amount of Hartree-Fock (HF) exchange (PBE0: 25%, B3LYP: 20%), which is rather small. We conclude that the alumina  $n = 7$  isomer energies are very sensitive to the choice of the functional (amount of HF exchange). A thorough study on the sensitivity is beyond the scope of this paper, but the set of isomers 7A–7J could be useful as a test case for the performance of (hybrid) density functionals.

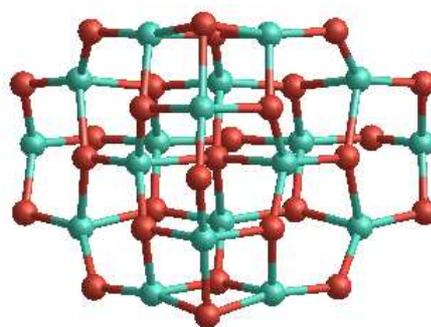
#### Alumina octamer ( $\text{Al}_2\text{O}_3$ )<sub>8</sub>



**Fig. 10.** Global minimum candidate structures of the alumina octamer ( $\text{Al}_2\text{O}_3$ )<sub>8</sub>.

The lowest-lying alumina octamer clusters are extensively discussed and reported in Gobrecht et al. (2018). Here we summarise our main findings. As for  $n = 4, 5$ , and  $7$  we also find for  $n = 8$  different GM structures depending on whether the PBE0 or the B3LYP functional is applied in combination with the 6-311+G(d) basis set. With the B3LYP functional, the lowest-energy isomer (8A) is a  $C_2$  symmetric structure (see Figure 10). Its B3LYP and PBE0 binding energies (per atom) are  $507 \text{ kJ mol}^{-1}$  and  $513 \text{ kJ mol}^{-1}$ , respectively. Isomer 8B has no special symmetry and B3LYP and PBE0 energies of  $507 \text{ kJ mol}^{-1}$  and  $514 \text{ kJ mol}^{-1}$ , respectively. Both structures, 8A and 8B, show geometries with large aspect ratios.

#### Alumina nonamer ( $\text{Al}_2\text{O}_3$ )<sub>9</sub>

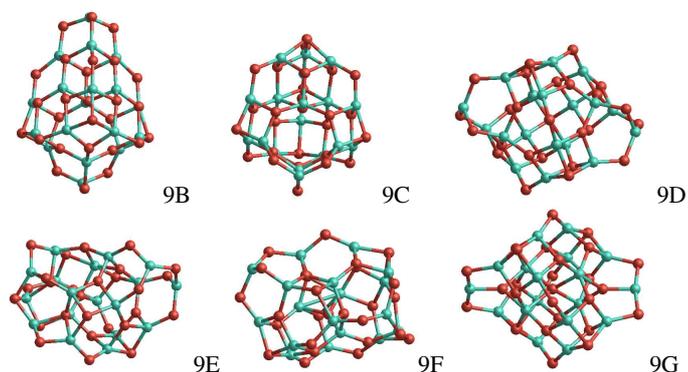


9A

**Fig. 11.** GM candidate (9A) of the alumina nonamer ( $\text{Al}_2\text{O}_3$ )<sub>9</sub>.

The most favourable isomer for  $n = 9$  is a  $C_s$  symmetric structure depicted in Figure 11. Its overall shape resembles a tetrahedron with a 4-coordinated oxygen atom in the center. This cluster structure strongly resembles a truncated block of  $\alpha$ -alumina also showing 4-coordinated oxygen atoms. With two 3-coordinated exceptions, the Al atoms are 4-coordinated and are located at the surface of the cluster. By the method of mirror images one could artificially increase the size of 9A resulting in 6-coordinated Al atoms and 4-coordinated Al atoms with similar bond angles as in  $\alpha$ -alumina. For 9A, we find binding energies of  $514 \text{ kJ mol}^{-1}$  (B3LYP) and  $520 \text{ kJ mol}^{-1}$  (PBE0), respectively. Furthermore, we find five other hitherto unreported, energetically-metastable structures (9B, 9C, 9D, 9E, 9F) with relative energies  $> 45 \text{ kJ mol}^{-1}$  above 9A (see Figure 12). It is unexpected that four out of the six newly discovered low-energy isomers show a high degree

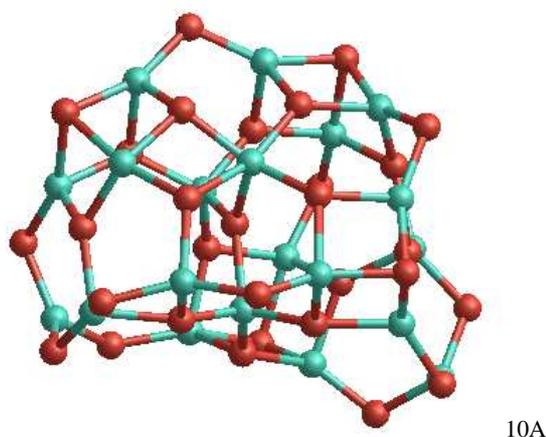
of symmetry. Structure 9G corresponds to the GM candidate reported by Rahane et al. (2011).



**Fig. 12.** Low-energy isomers of the alumina nonamer  $(\text{Al}_2\text{O}_3)_9$ .

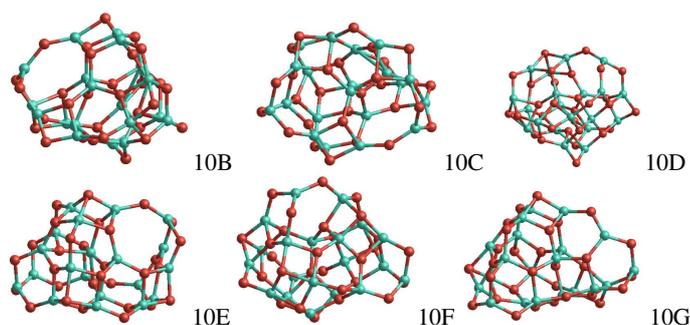
### Alumina decamer $(\text{Al}_2\text{O}_3)_{10}$

We present a new GM candidate isomer for  $n = 10$  without a particular symmetry (space group  $C_1$ , see Figure 13). On the B3LYP and PBE0 level of theory, the binding energy of 10A is  $515 \text{ kJ mol}^{-1}$  and  $521 \text{ kJ mol}^{-1}$ , respectively.



**Fig. 13.** Global minimum candidate (10A) of the alumina decamer  $(\text{Al}_2\text{O}_3)_{10}$ .

Our search also resulted in 22 further structures that lie below the GM candidate of Rahane et al. (2011) having a relative energy of  $18 \text{ kJ mol}^{-1}$  with respect to 10A. We show the next higher-lying decamer isomers (10B, 10C, 10D, 10E, 10F, 10G) in Figure 14 to give the reader an impression of the structural complexity, the narrow energy spacing, and the extent of our searches. The energetic ordering and the relative energies of the investigated alumina decamers are largely independent of the used functional (B3LYP und PBE0).



**Fig. 14.** Low-energy isomers (*Top*: 10B, 10C, 10D, *Bottom*: 10E, 10F, 10G) of the alumina decamer  $(\text{Al}_2\text{O}_3)_{10}$ .

### 3.2. Thermodynamic viability of the monomer formation

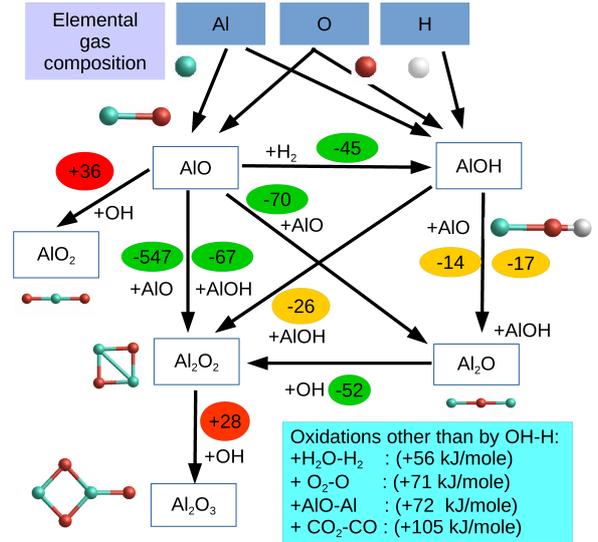
We investigate several chemical-kinetic formation routes towards the monomers  $(\text{Al}_2\text{O}_3)$  and dimers  $(\text{Al}_4\text{O}_6)$  alumina clusters. The dimers can coagulate to tetramers  $(\text{Al}_8\text{O}_{12})$  marking the end point of our chemical-kinetic description. The subsequent cluster nucleation and growth is treated homogeneously as function of cluster size  $n$  (see Section 3.8). A recent study investigated the kinetic formation of alumina tetramers via elementary reactions by means of TST and RRKM calculations (Saba et al. 2021). However, in contrast to our current study, the authors do not consider the nucleation at astrophysical sites like circumstellar envelopes. Consequently, Saba et al. (2021) do not include hydrogenated aluminium oxides, describe the oxidation reactions by  $\text{O}_2$  and  $\text{O}$  only, and uses the high pressure limit.

As a first step we assess the viability of a reaction according to its reaction enthalpy and its temperature-dependent Gibbs free reaction energy as derived from the CBS-QB3 calculations. In contrast to the previous subsections, the *reaction* energy or enthalpy corresponds to the sum of the total energies, which are not normalised to the atoms. The structural viability is subsequently investigated by reaction trajectory calculations as described in Section 2.3, accounting for geometrical rearrangements (i.e. breaking and formation of chemical bonds). In the end, it is the kinetics and related energy barriers controlling the chemistry. However, the energetic viability (i. e. the exogonicity) is a necessary but insufficient prerequisite for a chemical reaction to occur. These principal reaction viabilities are addressed in the following. The reaction scheme linking these molecular precursors to the  $\text{Al}_2\text{O}_3$  monomer is displayed in Figure 15. All considered  $\text{Al}_x\text{O}_y\text{H}_z$  molecules and clusters up to the size of a dimer are linked to each other by various chemical reactions that can proceed in both directions, forward and backward (see Figures 15 and 16).  $\text{AlO}$  can react with itself to form  $\text{Al}_2\text{O} + \text{O}$ ,  $\text{AlO}_2 + \text{Al}$ , or  $\text{Al}_2\text{O}_2$  via the termolecular channel ( $-548 \text{ kJ mol}^{-1}$ ). The first product channel towards  $\text{Al}_2\text{O} + \text{O}$  is exothermic by  $70 \text{ kJ mol}^{-1}$  at 0 K and becomes less favourable (endergonic) around 1800 K. The products  $\text{AlO}_2 + \text{Al}$  are suppressed for all temperatures by at least  $100 \text{ kJ mol}^{-1}$ .  $\text{AlOH}$  might also react with itself in order to form  $\text{Al}_2\text{O}_2$  and  $\text{H}_2$  which has a heat of reaction of  $-21 \text{ kJ mol}^{-1}$ . However, at temperatures above 300 K the latter process is suppressed as it becomes increasingly endergonic. Moreover, this latter reaction involves the breaking of two O–H bonds and the recombination of two H atoms to  $\text{H}_2$ . Instead, it is more likely that two  $\text{AlOH}$  molecules form  $\text{Al}_2\text{O} + \text{H}_2\text{O}$  ( $\Delta H_r(0\text{K}) = -26 \text{ kJ mol}^{-1}$ ). The latter process becomes endergonic above 700 K.  $\text{AlO}$  and

AlOH can also react with each other to form either  $\text{Al}_2\text{O}_2+\text{H}$  or  $\text{Al}_2\text{O}+\text{OH}$ . The channel  $\text{AlO}_2+\text{AlH}$  is not considered here as it has a large reaction endothermicity ( $\sim 300 \text{ kJ mol}^{-1}$ ) and it involves multiple bond breaking ( $\text{Al}-\text{O}$  and  $\text{O}-\text{H}$ ). Also, the possible products  $\text{OAlOH}$  and  $\text{Al}$  are suppressed by energy barriers ( $> 70 \text{ kJ mol}^{-1}$ ) and by structural hindrance. Therefore, the primary products of  $\text{AlO}$  and  $\text{AlOH}$  are  $\text{Al}_2\text{O}$  and  $\text{Al}_2\text{O}_2$ .

The oxidations in Figure 15 are described by reactions with the prevalent species  $\text{H}_2\text{O}$  and  $\text{OH}$ , but also with  $\text{CO}_2$ , and  $\text{O}_2$ . In terms of electronic energies (at  $T = 0 \text{ K}$ ), oxidations by  $\text{OH}$  are most favourable. If the oxidation reaction proceeds instead via  $\text{H}_2\text{O}$ , the reaction enthalpy is  $+56.5 \text{ kJ mol}^{-1}$  higher. Oxidations by  $\text{O}_2$  ( $+71.0 \text{ kJ mol}^{-1}$ ) and  $\text{CO}_2$  ( $+105.5 \text{ kJ mol}^{-1}$ ) are less energetically favourable than oxidations by  $\text{OH}$ .

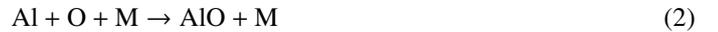
To form the monomer (i. e.  $\text{Al}_2\text{O}_3$ ),  $\text{Al}_2\text{O}$  or  $\text{Al}_2\text{O}_2$  need to be oxidised. The double oxidation of  $\text{Al}_2\text{O}$  by water has been suggested by Dell’Agli et al. (2014). Moreover, Gobrecht et al. (2016) described a kinetic formation of the monomer by  $\text{Al}_2\text{O}_2+\text{H}_2\text{O}$ . However, the oxidation of  $\text{Al}_2\text{O}_2$  to the alumina monomer,  $\text{Al}_2\text{O}_3$ , is hampered by substantial endothermicities. In the case of an oxidation by  $\text{H}_2\text{O}$ , the reaction is endothermic by  $\Delta H_r=+85 \text{ kJ mol}^{-1}$ . Though the oxidation by  $\text{OH}$  is less endothermic ( $+28 \text{ kJ mol}^{-1}$ ), the reverse reaction ( $\text{Al}_2\text{O}_3+\text{H}$ ) proceeds on much faster timescales owing to the large abundance of atomic hydrogen ( $\text{H}$ ). From an energetic perspective, the comparatively stable compounds  $\text{Al}_2\text{O}$  and  $\text{Al}_2\text{O}_2$  represent energetic bottlenecks in the synthesis of the alumina monomer. Hence, an efficient alumina formation route does not involve  $\text{Al}_2\text{O}$  and  $\text{Al}_2\text{O}_2$ , or, does not proceed via the monomer. We consider also the possibility that the monomer forms via the species  $\text{AlO}_2$ . The reaction enthalpy of  $\text{AlO}_2+\text{AlOH}$  to  $\text{Al}_2\text{O}_3$  and  $\text{H}$  is exothermic by  $72 \text{ kJ mol}^{-1}$  at  $0 \text{ K}$  and stays exothermic up to  $T = 2700 \text{ K}$ . However, the formation of  $\text{AlO}_2$  itself is hampered. The oxidation of  $\text{AlO}$  by  $\text{OH}$  is endothermic by  $36 \text{ kJ mol}^{-1}$  as discussed above.  $\text{AlOH}$  is even harder to oxidise and requires for  $\text{OH}$  an energy of at least  $80 \text{ kJ mol}^{-1}$ . Oxidations by other species such as  $\text{H}_2\text{O}$ ,  $\text{CO}_2$  and  $\text{O}_2$  have even larger endothermicities as indicated in Figure 15. Depending on the environment conditions,  $\text{H}_2\text{O}$ ,  $\text{CO}_2$ , and  $\text{O}_2$  might be more abundant than  $\text{OH}$ , and can thus increase their reaction fluxes and oxidation efficiencies. Furthermore, for temperatures  $T > 0$ , vibrational and rotational contributions of the reacting species can change the order of the favoured oxidiser. The oxidation of  $\text{Al}_2\text{O}_2$  to  $\text{Al}_2\text{O}_3$  is endothermic at  $0 \text{ K}$ , but at  $4000 \text{ K}$  the free energies of reaction would allow for oxidations by  $\text{OH}$  and  $\text{CO}_2$ , but not by  $\text{H}_2\text{O}$  and  $\text{O}_2$ . Oxidations by atomic  $\text{O}$  are energetically most favourable. However, for molecules and small clusters, such reactions require a third body  $\text{M}$  as catalyst to absorb the excess energy and stabilise the reaction product. This is a consequence of the low pressures prevailing in AGB circumstellar envelopes making an autocatalysis unlikely. Larger-sized clusters ( $n \geq 4$ ) can be oxidized by atomic  $\text{O}$  more easily, since their density of states is high allowing for ro-vibrational relaxations without the presence of a third body  $\text{M}$ . Thus, although these oxidations by atomic  $\text{O}$  are the most exothermic ones, they proceed rather slowly for the smallest species, representing the starting point of a bottom-up approach, and require sufficiently high gas densities. For a typical termolecular association rate these gas densities need to be of the order of  $> 10^{14} \text{ cm}^{-3}$  in order to compete with other bimolecular reactions.



**Fig. 15.** Reaction scheme: Formation routes towards the alumina monomer ( $\text{Al}_2\text{O}_3$ ). Reaction enthalpies are given in  $\text{kJ mol}^{-1}$  and colour-coded according to their values. Red corresponds to suppressed exothermic reactions ( $\Delta H > 0 \text{ kJ mol}^{-1}$ ), yellow to slightly exothermic reactions ( $\Delta H > -30 \text{ kJ mol}^{-1}$ ), and green to very exothermic reactions ( $\Delta H < -30 \text{ kJ mol}^{-1}$ ).

### 3.3. Chemical-kinetic network

Starting with the atoms  $\text{Al}$ ,  $\text{O}$ , and  $\text{H}$  and assuming a pure atomic gas phase mixture, the molecules  $\text{AlO}$  and  $\text{AlOH}$  can form via



where  $\text{M}$  denotes an inert molecule acting as a catalyst and removing the reaction excess energy. The reaction  $\text{Al}+\text{OH}+\text{M} \rightarrow \text{AlOH}+\text{M}$  is in principle also possible, once  $\text{OH}$  has formed and is available as a reagent. As an alternative to a purely atomic gas, thermodynamic (chemical) equilibrium abundances can also be used as a starting point. However, the choice of the initial conditions (atomic versus thermodynamic equilibrium (TE) abundances) has only minor effects on the final abundances, as will be shown in Section 3.5. From RRKM unimolecular decomposition calculations of  $\text{AlO}$  and  $\text{AlOH}$ , we fit and deduce kinetic rates for the reverse processes of reactions 2 and 3. We find activation barriers of  $477 \text{ kJ mol}^{-1}$  and  $466 \text{ kJ mol}^{-1}$  for the dissociation of  $\text{AlO}$  and  $\text{AlOH}$ , respectively. Atomic  $\text{Al}$  can also react with  $\text{OH}$  in a bimolecular reaction to form  $\text{AlO}$



A series of trajectories were run on reaction 4, using the Atom Centered Density Matrix Propagation (ADMP) molecular dynamics model (Schlegel et al. 2002). Because of the severe change in reduced mass of the system ( $\text{H}$  is one of the products), the reaction dynamics are constrained even though the reaction is exothermic by  $72 \text{ kJ mol}^{-1}$ . The reaction takes place on both singlet and triplet surfaces. Although there is an energy barrier on the triplet surface, the barrier is ‘late’ (i.e. between the intermediate  $\text{AlOH}$  and the products  $\text{AlO}+\text{H}$ ) and, importantly, the barrier height is  $50 \text{ kJ mol}^{-1}$  below the height of the reactants

Al + OH. There are two requirements for successful reaction: the OH must be vibrating with at least 2 quanta of vibrational energy (corresponding to a vibrational temperature of  $\sim 2 \times 5300$  K); and the collision energy needs to be modest (to allow angular momentum to be conserved), corresponding to a kinetic temperature of 500 K or less. These two requirements are counter to each other: high temperature is needed for a significant OH vibrational population, but low temperatures to favour the modest collision energy. The rate coefficient was then constructed by multiplying a typical collision frequency ( $5 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1}$ ) by the probability that the collision energy is less than  $1.5 k_B T$  where  $T=500$  K, and the probability that the OH has at least 2 vibrational quanta. This results in the Arrhenius expression of reaction 161 in Table B.1. The alternative product channel AlH+O is very endothermic ( $+125 \text{ kJ mol}^{-1}$ ) and thus not included in our network.

Once formed, the concentrations of AlO and AlOH are primarily regulated by the reaction



This reaction is exothermic by  $-45 \text{ kJ mol}^{-1}$ . Despite its exothermicity, experiments lead by Parnis et al. (1989) showed a negligible reactivity of AlO with  $\text{H}_2$  and hence, give an upper limit of the bimolecular rate constant of  $5 \times 10^{-14} \text{ cm}^3 \text{ s}^{-1}$  at room temperature. A more recent study (Mangan et al. 2021) experimentally determined the reaction rate of 5 including a detailed characterisation of the PES, showing good agreement with the theoretical study of Sharipov et al. (2011). The products AlOH + H form predominantly through H atom abstraction via a linear Al–O–H–H transition state lying  $45 \text{ kJ mol}^{-1}$  above the reagents AlO+ $\text{H}_2$ . Despite the reaction barrier of  $45 \text{ kJ mol}^{-1}$ , the best fit with respect to the experimental data points results in a lower barrier height of  $31 \text{ kJ mol}^{-1}$ . Another product of reaction 5 is Al+ $\text{H}_2\text{O}$ . However, its branching ratio is negligibly small (0.008%). The resulting kinetic rate for reaction 5 is temperature-dependent and has an activation energy corresponding to an equivalent temperature of 2030 K.

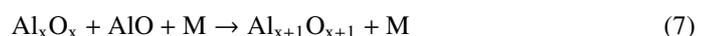
Another process impacting the AlO/AlOH balance is the reaction



having a reaction enthalpy of  $+11.6 \text{ kJ mol}^{-1}$ . Mangan et al. (2021) found that this reaction proceeds predominantly via an AlO– $\text{H}_2\text{O}$  adduct with a small submerged barrier re-arranging to Al(OH) $_2$ , which finally dissociates to AlOH and OH without a barrier. A direct pathway via a quasi-linear transition state involves a barrier of  $60 \text{ kJ mol}^{-1}$  and is not competitive. An alternative product channel of 6 is OAlOH + H showing also a slightly endothermic reaction enthalpy ( $+8.6 \text{ kJ mol}^{-1}$ ). Since a direct reaction route via a OAlOH $_2$  transition state involves a significant barrier of  $87 \text{ kJ mol}^{-1}$ , the dominant OAlOH production channel takes course via the dissociation of Al(OH) $_2$ , similar to in the AlOH production channel. The formation of AlO $_2$  and  $\text{H}_2$  is very endothermic.

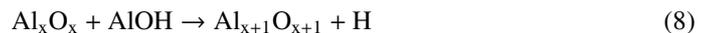
The reaction of AlO+OH can also form AlOH. However, although it is exothermic by  $56 \text{ kJ mol}^{-1}$ , it was found to proceed rather slowly. ACMP trajectories on the triplet surface (which connects with the products AlOH + O( $^3\text{P}$ ) in their electronic ground states) show that this reaction requires the AlO to have at least 1 quantum of vibrational excitation corresponding to a vibrational temperature of 1370 K. A low collision energy is also

required, corresponding to a kinetic temperature of 600 K or less, and the reaction cross section has a relatively small impact parameter ( $< 0.5 \text{ \AA}$ ). The resulting rate coefficient between 1000 and 2000 K is around  $3 \times 10^{-12} \text{ cm}^3 \text{ s}^{-1}$  with a small temperature dependence. The alternative reaction channel forming AlO $_2$  + H is endothermic by  $35 \text{ kJ mol}^{-1}$  and is characterised by a large decrease in reduced mass, and so is not competitive with the AlOH + O product channel. Finally, we examine alumina dimer (i.e. Al $_4\text{O}_6$ ) formation pathways that do not involve the unfavourable alumina monomer (see Section 3.2). We find an enhanced stability of (AlO) $_x$ ,  $x = 1 - 4$ , clusters with an Al:O stoichiometry of 1:1, compared to small Al $_x\text{O}_y$ ,  $x, y = 1 - 4$ ,  $x \neq y$  clusters with a different stoichiometry than 1:1 (see Table 3). These findings are consistent with the results of Patzer et al. (2005) and Lam et al. (2015). Consequently, it is instructive to consider reactions within the stability valley of the (AlO) $_x$ ,  $x = 1-4$ , clusters. The successive addition of AlO molecules



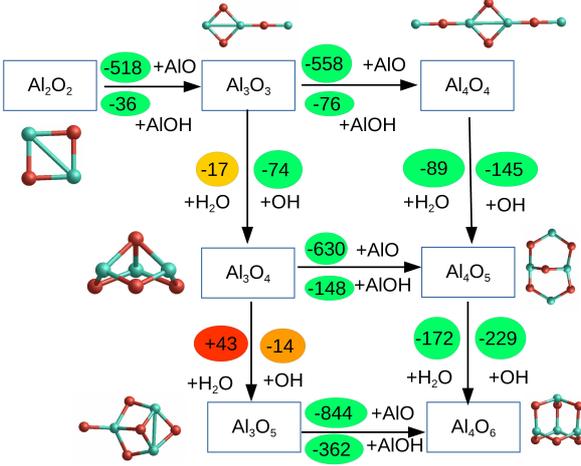
is energetically favourable as the number of strong Al–O bonds ( $\sim 500 \text{ kJ mol}^{-1}$  per bond) increases naturally with size  $x$ . However, this process requires a third body M and is therefore only effective in the densest circumstellar regimes (typically  $n_{\text{gas}} > 10^{14} \text{ cm}^{-3}$ ), or once  $x$  is large enough.

The successive addition of AlOH molecules



proceed as bimolecular reactions, but have significantly lower heat of reactions (lower by  $482 \text{ kJ mol}^{-1}$  at  $T = 0$  K), corresponding to the dissociation energy of AlOH  $\rightarrow$  AlO+H as compared to the AlO addition, since the ejection of an H atom is energetically expensive. Nevertheless, for  $x = 1-3$ , the reaction with AlOH is energetically viable with enthalpies of  $-65.9 \text{ kJ mol}^{-1}$  ( $x = 1$ ),  $-36.0 \text{ kJ mol}^{-1}$  ( $x = 2$ ), and  $-76.0 \text{ kJ mol}^{-1}$  ( $x = 3$ ). For  $x = 4$  (Al $_4\text{O}_4$ +AlOH  $\rightarrow$  Al $_5\text{O}_5$ +H) the enthalpy is still slightly exothermic by  $-3.9 \text{ kJ mol}^{-1}$ , but at higher temperatures ( $> 100$  K) the reaction becomes endothermic/endergonic. It is thus probable that a subsequent cluster growth reaction departs from an Al:O stoichiometry of 1:1. Therefore, starting with Al $_2\text{O}_2$  (see Figure 16) the formation of Al $_3\text{O}_3$  is energetically viable. The oxidations of Al $_3\text{O}_3$  and Al $_4\text{O}_4$  are both exothermic and can readily form Al $_3\text{O}_4$  and Al $_4\text{O}_5$ . However, the oxidations of Al $_3\text{O}_4$  represent an energetic bottleneck, since only the oxidation by OH is energetically viable.

Once the alumina dimer (Al $_4\text{O}_6$ ) is formed, we assume that it coagulates to the alumina tetramer (Al $_8\text{O}_{12}$ ) in a very exothermic reaction. The alumina dimer can also react with other Al-bearing species than itself to form products containing 5 to 7 Al atoms. Due to the exothermicity of many cluster growth reactions, these processes are likely to occur. A characterization of these intermediate Al–O–H cluster species and related kinetic pathways will be the subject of a future study. However, owing to the exponentially increasing number of possible reaction pathways, we do not explicitly take these intermediate species into account in this study, but subsume them by an association reaction. This allows us to neatly link the kinetically controlled regime with the larger-sized (Al $_2\text{O}_3$ ) $_n$  clusters with  $n \geq 4$ .



**Fig. 16.** Reaction scheme: Formation routes towards the alumina dimer. Reaction enthalpies are given in  $\text{kJ mol}^{-1}$  and the color-coding is the same as in Figure 15, where very exothermic reactions are colour-coded in green, slightly exothermic reactions in orange, and endothermic reactions in red.

### 3.3.1. Kinetic rate evaluation

For large reactive systems, it is very expensive to calculate the PES including all possible and unknown transition states. Therefore, we assess kinetic rates involving more than 6 atoms with collision and capture theory. Exothermic reactions without a barrier of the form  $A+B$  can proceed at the collision frequency, which is given as

$$k_{\text{coll}}(T) = \sigma_{AB} \sqrt{\frac{8k_B T}{\pi \mu_{AB}}} \quad (9)$$

where  $\sigma_{AB} = \pi(r_A + r_B)^2$  is the (geometrical) cross section,  $\mu_{AB} = \frac{m_A m_B}{m_A + m_B}$  is the reduced mass and  $k_B$  the Boltzmann constant. The radii  $r_A$  and  $r_B$  of species A and B are determined from the calculated structures. In Table D.1, we provide two sets of cluster volumes and radii. The first set is derived from atomic core coordinates and calculated by Delauney triangulation. The second set exhibits larger values, as it includes also “electron interaction” volumes derived from atomic van-der-Waals radii. Owing to its  $T^{0.5}$  temperature dependence, the collisional frequency is increased by a factor of  $\sqrt{10} \approx 3.16$  for a characteristic (circum-)stellar temperature of 3000 K, as compared to room temperature. At these elevated temperatures the collision rate can take large values in many cases and is rather simplistic. In the kinetic network shown in Table B.1, we included two collision rates proceeding via the formation of intermediates that decompose in either direction without a barrier (denoted as *collision*). However, the majority of the reactions considered in this study have a more complex PES and are not well described by simple collision theory.

In addition to a pure geometric collision also long-range interactions between the reactants can contribute to the cross section. These interactions can be taken into account by using van der Waals radii, which are larger than the geometric radii and account for the spatial occupancy of the bond atoms. Generally, the long-range interaction effect can also be included directly in

the rate expression. For example, in the reaction  $\text{AlO} + \text{H}_2$  dipole-induced dipole forces or Debye forces, denoted as  $k_{DiD}$ , and dispersive or London forces, denoted as  $k_{disp}$ , act. If both reagents are polar (which is not the case for  $\text{H}_2$ ) also dipole-dipole forces, denoted as  $k_{DD}$ , are present. To quantify and summarise the contribution of the long-range forces we use the form introduced by Georgievskii & Klippenstein (2005):

$$k_{\text{capt}}(T) = 1.3 \times \max_k \begin{cases} k_{DD} \\ k_{DiD} \\ k_{disp} \end{cases} \quad (10)$$

where  $k_{\text{capt}}$  is the so called capture rate and corresponds to the upper limit for the rate of an exothermic reaction. The detailed calculation of  $k_{DD}$ ,  $k_{DiD}$ , and  $k_{disp}$  including the factor of 1.3 can be found in Georgievskii & Klippenstein (2005). In addition to the mass of the reagents, the calculation of capture rates requires the knowledge of the dipole moments, approximate polarisabilities, and vertical ionisation potentials. In our network (in Table B.1), the capture rate is used for 46 exothermic reactions (denoted as *capture*), where the investigation of the PES is too expensive and TST- or RRKM-based rates are not obtainable. The majority of the capture rates are dominated by dispersive forces and show a  $T^{\frac{1}{6}}$  dependence. Dipole-dipole interactions, which follow a  $T^{-\frac{1}{6}}$  dependence, are predominant for just three rates in our kinetic network.

In order to calculate the rates for the reverse (endothermic) reactions, we apply the principle of detailed balance (for more details see Eq. A.1). By fitting the reverse rate, we find Arrhenius-parametrized rate expressions (denoted as *detailed balance* in Table B.1). In some cases the fitting results in unphysically high pre-exponential factors  $A$  of  $> 2 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$ . The large pre-exponential factors tend to occur in reactions where the reduced mass of the products is much smaller than the reactants (typically when one of the products is H or  $\text{H}_2$ ). In these cases the conservation of angular momentum constrains the reaction cross section. In such cases, we do not use the unphysically large detailed balance rates but apply a capture rate with energy barrier corresponding to the CBS-QB3 0K enthalpy (denoted as *reverse capture* in Table B.1).

The basic chemical network consists of 54 atomic, molecular and cluster species and 163 individual reactions that can be found in Table B.1 of the Appendix. We are aware that this network is not complete in terms of species (e.g. sulphur-containing species) and processes (e.g. ionisation) considered. However, we ran test calculations with an extensive kinetic network including 50 additional non-aluminum-bearing species and 286 additional reactions. Moreover, we carefully build up a chemical-kinetic network by using either rates from our extensive literature search, or, where these do not exist estimating rate coefficients using sound theoretical methods currently available. Thereby, we always respect the balance between forward and backward reaction, instead of adopting rates from (astro-)chemical kinetic rate databases. Note that some rate expressions differ from the simple Arrhenius parametrisation, as their complex temperature-dependence cannot be represented by an Arrhenius formulation. Furthermore, some rates are based on the Lindemann expression consisting of high- and low-pressure limiting rate terms. In circumstellar envelopes we can safely ignore the high pressure limit and use the low-pressure limit for the rate constant, as the prevailing pressures are orders of magnitude below 1 atm. Consequently, dissociations typically proceed via bimolecular channels as collisions with a body M. However, a photolysis or photodissociation, i.e. a decay induced by high-

energy photons, is also possible. Finally, we include the photodissociations of AlO, AlOH, and Al<sub>2</sub>O. The photolysis rates of AlO, AlOH, and Al<sub>2</sub>O were estimated using time-dependent density function theory (TD-DFT) (Bauernschmitt & Ahlrichs 1996). The vertical excitation energies and transition dipole moments were calculated for transitions from the ground state of each molecule up to the first 30 electronically excited states. The resulting absorption cross section for each molecule was then convolved up to its dissociation threshold with a model stellar irradiance flux from the MARCS data-base for an evolved star with  $T_{\star} = 2500$  K (Gustafsson et al. 2008). Oxygen-rich AGB stars typically show moderately lower effective temperatures ( $T = 2000\text{--}2400$  K), but there are no MARCS models with  $T < 2500$  K available. The photolysis thresholds were set to correspond to the bond dissociation energies, giving 252, 238 and 213 nm for AlO, AlOH and Al<sub>2</sub>O, respectively. Note that these are upper limits to the dissociation threshold wavelengths, because a photon with more than the bond energy may be required depending on the position of the upper dissociating electronic state of the molecule. The photolysis rate was then computed as a function of temperature by red-shifting the photolysis threshold to reflect the increasing internal energy of the molecule with temperature. The resulting photodissociation rates at 1 stellar radius are listed in Table B.1. For radial distances further out in the envelope, we assume a geometrical dilution ( $\propto R^{-2}$ ) of the stellar radiation field, and no attenuation by dust.

### 3.4. Equilibrium abundances

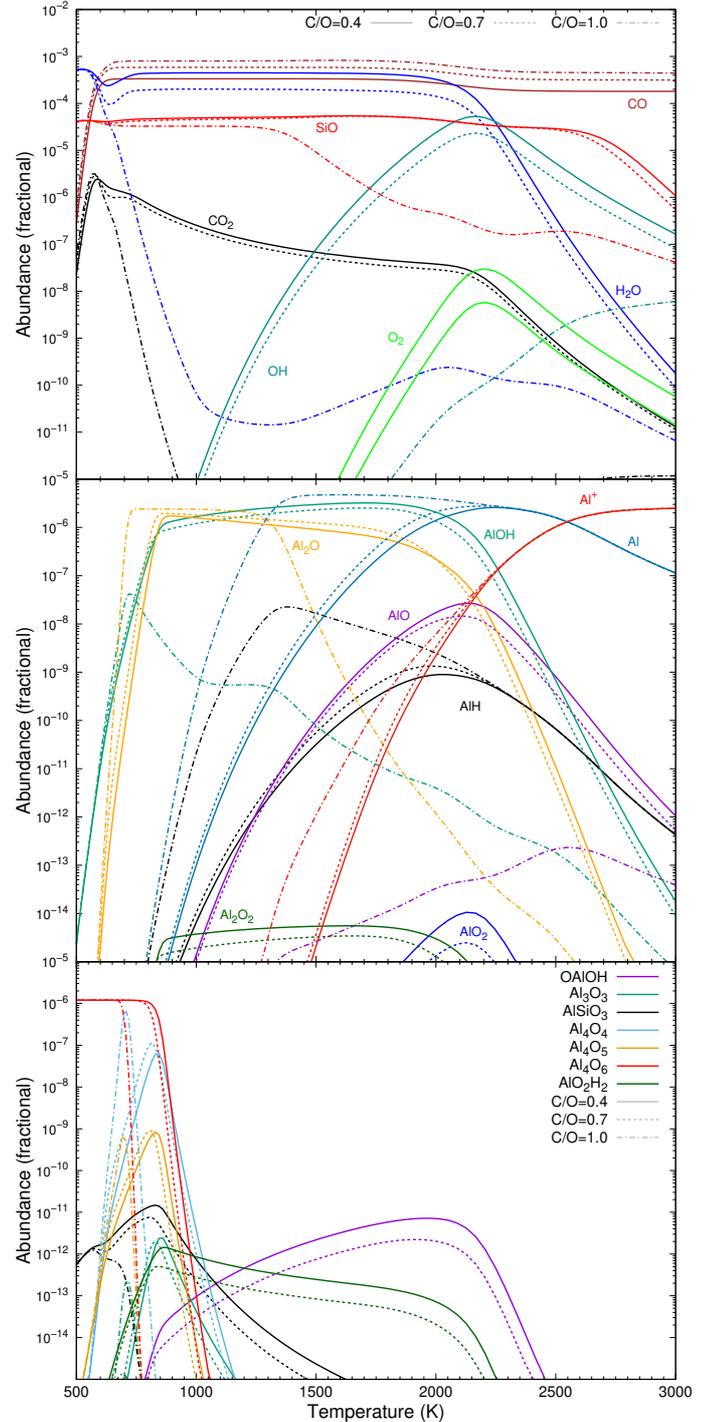
To compare our chemical-kinetic derived non-equilibrium abundances (see Section 3.5) with thermodynamic equilibrium (TE) abundances, we perform calculations with the chemical equilibrium software GGchem (Woitke et al. 2018). Moreover, we add the alumina-related cluster species presented in this study to the list of molecules.

For convenience, we provide the fitting parameters  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  in the form presented in Stock et al. (2018) for the term  $\frac{-\Delta_f G(T)^0}{RT}$ , i.e.

$$\frac{-\Delta_f G(T)^0}{RT} = \frac{a}{T} + b \ln(T) + cT + dT^2. \quad (11)$$

The fitting parameters  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  for all aluminium-bearing species used in this study are provided in Table C.1 of the Appendix. Note that  $\Delta_f G(T)^0 \neq \Delta_f G(T)$ , since  $\Delta_f G(T)$  includes, like the JANAF-NIST thermochemical tables, a scaling to the atomic (elemental) heats of formation with their corresponding stoichiometric factors. Thus,  $-\Delta_f G(0)^0$  corresponds to the real, unscaled free energies of formation at 0 K. We perform TE calculations for different C/O ratios of 0.4, 0.7, and 1.0, and for a typical atmospheric (photospheric) pressure of  $10^{-5}$  bar as a function of temperature pertaining to the dust formation zone in oxygen-rich AGB stars.

First, we present the abundances of the prevalent molecular species CO and SiO including the oxidation agents as OH, H<sub>2</sub>O, CO<sub>2</sub>, and O<sub>2</sub> (see upper panel of Figure 17). The most abundant molecules (apart from H<sub>2</sub>) in oxygen-rich conditions are CO and H<sub>2</sub>O showing fractional abundances above  $10^{-4}$ . At temperatures  $T \geq 2200$  K the OH and H abundances increase at the cost of H<sub>2</sub>O. SiO is also fairly abundant with a value of  $>10^{-5}$  up to  $T=2700$  K. CO<sub>2</sub> and O<sub>2</sub> play a comparatively minor role hardly exceeding  $10^{-6}$  on the abundance scale. CO<sub>2</sub> is moderately abundant at lower temperatures ( $T \leq 2200$ ), and O<sub>2</sub> only at higher temperatures. Note that CO<sub>2</sub> and O<sub>2</sub> not only show lower equilib-



**Fig. 17.** *Upper panel:* Thermodynamic equilibrium (TE) abundances of the abundant molecules CO, CO<sub>2</sub>, H<sub>2</sub>O (black), OH, and O<sub>2</sub> as a function of temperature for different C/O ratios. *Middle panel:* TE abundances of the aluminium-bearing molecules AlO, AlOH, Al<sub>2</sub>O, and AlO<sub>2</sub> as well as Al and Al<sup>+</sup>, as a function of temperature for different C/O ratios. *Lower panel:* TE abundances of the aluminium-bearing clusters OAlOH, Al(OH)<sub>2</sub>, Al<sub>3</sub>O<sub>3</sub>, Al<sub>4</sub>O<sub>4</sub>, Al<sub>4</sub>O<sub>5</sub>, Al<sub>4</sub>O<sub>6</sub>, and AlSiO<sub>3</sub> participating in the formation of the alumina dimer (Al<sub>4</sub>O<sub>6</sub>) as a function of temperature for different C/O ratios. Straight lines correspond to C/O=0.4, dashed lines to C/O=0.7, and dashed-dotted line to C/O=1.0, respectively. The pressure is kept constant at  $10^{-5}$  bar. .

rium abundances than H<sub>2</sub>O and OH, but also the corresponding kinetic oxidations are comparatively less exothermic in the kinetic models (see Figure 15 and related discussion). Therefore, H<sub>2</sub>O and OH should represent the primary oxidation species. The C/O=1.0 models (dot-dashed lines), characterising atmospheres of chemical transient S-type AGB stars, show generally lower abundances of the prevalent, oxygen-bearing molecules, except for CO, where the abundance is higher. This result is not unexpected in equilibrium conditions, as a C/O close to unity leaves little oxygen available to form other molecules.

The TE abundances of the aluminium-bearing molecules are shown in the middle panel of Figure 17. They can be roughly grouped in three temperature zones: at high temperatures above T≥2500 K, aluminium is predominantly in ionized form (Al<sup>+</sup>), in the intermediate temperature range (T=2000–2500 K) atomic Al is the primary aluminium carrier, and for lower temperatures (T≤2000 K) the molecules AlOH and Al<sub>2</sub>O are the most abundant gas phase species. The TE AlO abundance reaches its maximum (~10<sup>-8</sup>) around T=2130 K whereas AlO<sub>2</sub> has negligible abundances. In the C/O=1.0 case, AlOH is shifted towards lower temperatures, whereas AlO peaks at higher temperatures with respect to the C/O=0.4 and C/O=0.7 cases.

In the bottom panel of Figure 17, clusters acting as potential intermediates (OAlOH, Al<sub>3</sub>O<sub>3</sub>, AlSiO<sub>3</sub>, Al<sub>4</sub>O<sub>4</sub>, Al<sub>4</sub>O<sub>5</sub>, Al<sub>4</sub>O<sub>6</sub>, AlO<sub>2</sub>H<sub>2</sub>) in the cluster nucleation route are included. At temperatures below T<800 K the equilibrium alumina dimer (Al<sub>4</sub>O<sub>6</sub>) abundance dominates the aluminium content. The steep decrease beyond T=800 K is accompanied with a sharp rise of the species AlOH and Al<sub>2</sub>O (see middle panel). Moreover, except for OAlOH and AlO<sub>2</sub>H<sub>2</sub>, the abundances of the intermediates are negligible for temperatures above T=1000 K.

In summary, under the conditions of thermodynamic equilibrium, alumina dimers (and larger compounds with same stoichiometry) do not form in significant quantities at temperatures above T=800 K. This is inconsistent with observations of the higher temperatures found in the dust formation zone. Therefore, chemical equilibrium fails to predict the onset of alumina dust formation, assuming that the nucleation proceeds via the stable and promising dimer cluster.

### 3.5. Non-equilibrium chemical-kinetic models

For the non-equilibrium computations we use a chemical-kinetic reaction network (instead of assuming chemical equilibrium), which is given in the Appendix B.1. Furthermore, we expose the chemical species to different hydrodynamic trajectories characterized by varying thermodynamic conditions (i.e. density and temperature). The trajectories include a non-pulsating outflow model represented by a  $\beta$ -velocity law, and a pulsation model that will be discussed in more detail. Moreover, we distinguish between two model stars, SRV representing Semi-Regular Variable AGB stars and MIRA representing MIRA-type pulsating AGB stars. Apart from their pulsation properties (amplitude and period), which are generally shorter for SRVs than for MIRAs, other quantities also differ for these two types of AGB stars. They include temperature, density, and mass-loss rate. Following stellar evolution models, the differences in these quantities suggest that SRV and MIRA represent two successive evolutionary states of the same type of AGB star (Kerschbaum & Hron 1992).

For reasons of comparability and consistency, we choose a fixed C/O ratio of 0.75. We are aware that the C/O ratio is subject to changes during the evolution on the AGB. Recurrent Third Dredge-Up (TDU) and mixing episodes grad-

**Table 4.** Parameters for the model stars SRV and MIRA used to calculate hydrodynamic trajectories. Number densities  $n_0 \equiv n(1R_\star)$  are given in units of cm<sup>-3</sup>, temperatures  $T_0 \equiv T(1R_\star)$  in units of K, the terminal  $v_\infty$  and shock velocity  $v_s$  in units of km s<sup>-1</sup> and the pulsation period (pp) in units of days. Number densities and temperatures correspond to their values at the photosphere ( $1 R_\star$ ).

Model	$n_0$	$T_0$	$v_\infty$	$v_s$	pp
SRV	1.e14	2400	5.7	10	332
MIRA	4.e14	2000	17.7	20	450

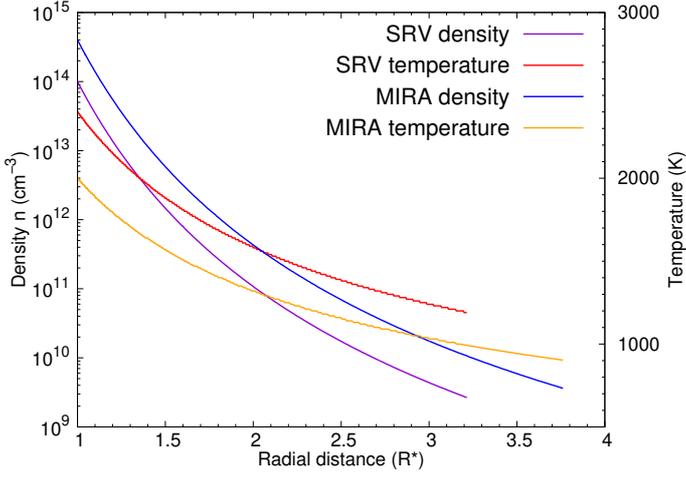
ually enhance the carbon content at the stellar surface leading to an evolution-induced increase of the C/O ratio (Cristallo et al. 2015). Therefore, SRV type stars are expected to have a (slightly) lower C/O ratio than MIRA type AGB stars. According to the FRUITY stellar evolution models, a star with an initial mass of 1.5 M<sub>⊙</sub> and solar metallicity (Z=0.014) experiences five TDU mixing episodes with an interpulse period (i.e. the time between two TDUs) lasting about 1–2×10<sup>5</sup> years (Cristallo et al. 2015). During the time span of an interpulse period, the C/O ratio is constant and takes values of 0.41 (TDU 1), 0.56 (TDU 2), 0.75 (TDU 3), 0.96 (TDU 4) and 1.00 (TDU 5). Therefore, our initial elemental composition corresponds to third of five TDU episodes of the FRUITY model described above. Moreover, our choice of C/O=0.75 lies in between the solar value of 0.55 (Asplund et al. 2009) and a C/O ratio of 0.95 characteristic for an MS type AGB star, and hence reflects a certain progression in stellar evolution. We performed test calculations for the two cases with solar C/O ratios (0.55) and for a stellar model resembling an AGB star of MS type (C/O=0.95). The lower C/O slightly shifts the chemistry towards higher abundances of O-bearing species, but qualitatively preserved the difference between SRV and MIRA models. In the stellar model resembling an MS star, no alumina clusters form and the aluminum chemistry is predominated by the halides AlF and AlCl instead of AlOH and AlO, as expected. Therefore, the main conclusions from our kinetic modelling study are not altered by these tests.

### 3.6. Non-pulsating outflow model

The non-pulsating outflow models in our study are represented by a  $\beta$ -velocity law. The  $\beta$ -velocity law represents a stellar outflow model, where the velocity increases monotonically with the distance from the star and accounts for the net acceleration of the wind. It should be kept in mind that the beta velocity approximation does not take into account stellar pulsations. Its general form is given by

$$v(r) = v_0 + (v_\infty - v_0) \times \left(1 - \frac{r_0}{r}\right)^\beta \quad (12)$$

where  $v_0$  and  $v_\infty$  are the initial and terminal velocities and  $r_0$  the initial radius, often referred to as dust condensation radius. Here, we assume that  $r_0=1 R_\star$ ,  $v_0=v(r_0)=1.5$  km s<sup>-1</sup>, and  $\beta=1.0$  for both model stars, SRV and MIRA. Typically,  $r_0$  is found to be often greater than  $1 R_\star$ . However, we intentionally choose the stellar surface as the starting point, since we do not impose a dust condensation radius, but aim to derive the onset of dust formation by the kinetic cluster growth. Therefore, we naturally account for a dust formation *zone* rather than a dust formation *radius*. Moreover, our choice of  $r_0$  allows a more direct comparison with the pulsating models, which is one of the main purposes of this study. We choose a value of  $\beta = 1.0$  for both model stars,



**Fig. 18.** Density and temperature profiles of the SRV and MIRA model star envelopes. These profiles apply to the non-pulsating models, as well as to the preshock conditions of the pulsating models

which is in accordance with Decin et al. (2010). We note that the value for  $\beta$  is often (slightly) larger for oxygen-rich AGB stars (see e.g. Khouri et al. (2014); Maercker et al. (2016)). A larger value of  $\beta$  implies a slower wind acceleration and consequently, a longer timescale for the chemistry to occur and for the alumina clusters / dust to form. Therefore,  $\beta=1$  corresponds to the conservative limit of a comparatively quickly accelerating oxygen-rich AGB wind.

The terminal wind speed  $v_\infty$  is dependent on the model star and has typical values of  $5\text{--}20\text{ km s}^{-1}$ . For the SRV model we assume  $v_\infty=5.7\text{ km s}^{-1}$  and for the MIRA model  $v_\infty=17.7\text{ km s}^{-1}$  based on the results of Maercker et al. (2016) for R Dor and of Decin et al. (2017) for IK Tau. The initial velocity  $v_0$  is consistent with a microturbulent velocity of  $1.1\text{--}1.5\text{ km s}^{-1}$  derived from the line broadening close to the stellar photosphere (Schöier et al. 2004). The gas temperature  $T$  is given as a function of radius and follows a power law with an exponent  $\alpha=0.6$  (Willacy & Millar 1997):

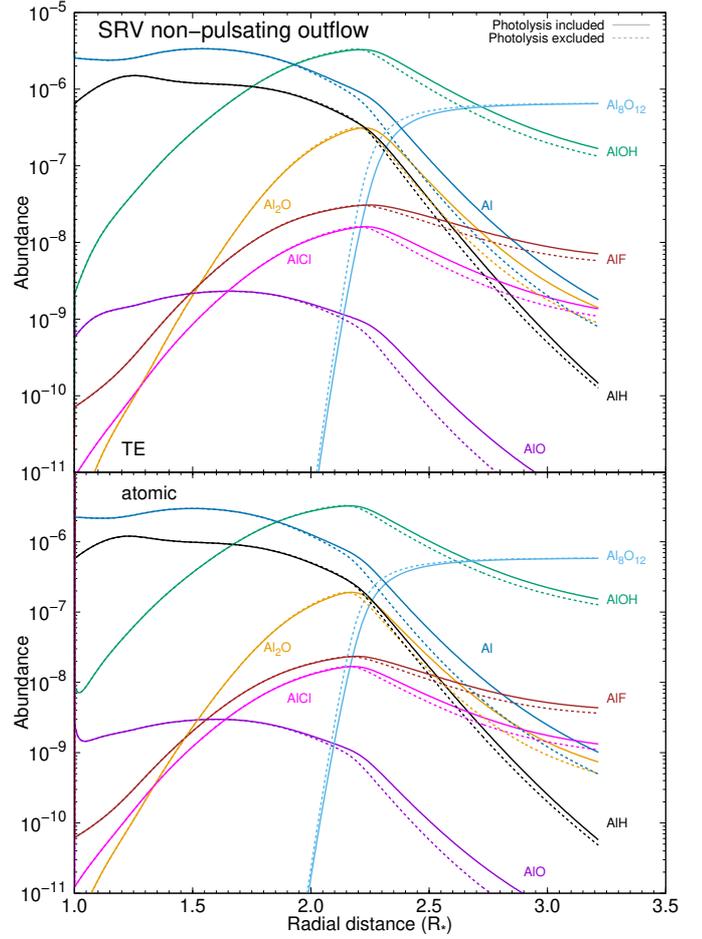
$$T(r) = T_\star \left( \frac{r}{R_\star} \right)^{-\alpha} \quad (13)$$

The radial density profile is derived from the scale height at the stellar surface,  $H_\star = R_{\text{gas}} T_\star R_\star^2 / \mu M_\star G$  of an ideal gas with a parameter  $\gamma=0.89$  (see e.g. Gobrecht et al. (2016) and reference therein) and is given as

$$n(r) = n_\star \exp \left( \frac{R_\star(1-\gamma^2)}{H_\star(1-\alpha)} \left( 1 - \frac{r}{R_\star} \right)^{\alpha-1} \right) \quad (14)$$

Since the kinetic rate network is formulated as a set of differential equations in time, we transform the monotonically decreasing temperature and density into functions of time by using a constant  $dt = dr/v(r)$ .

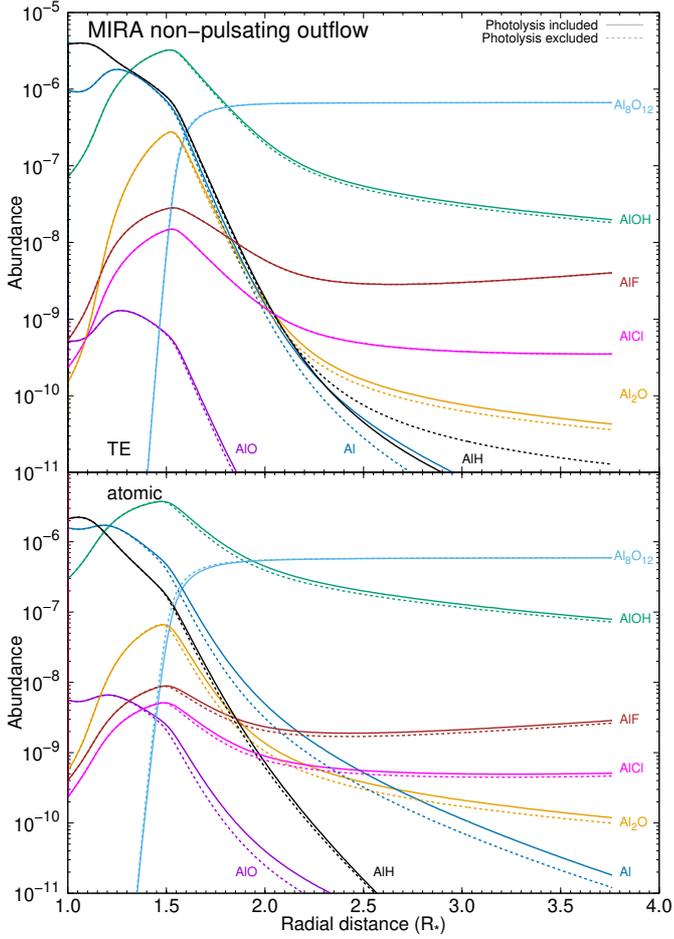
In the SRV non-pulsating outflow model (i.e. the SRV  $\beta$ -velocity law) the aluminum chemistry is dominated by atomic Al for  $r < 1.8 R_\star$ , by AIOH for  $1.8 R_\star < r < 2.7 R_\star$  and by alumina tetramers ( $\text{Al}_8\text{O}_{12}$ ) for  $r < 2.7 R_\star$  (see Figure 19). The exclusion of the photolysis reactions (shown in dashed lines) has a minor impact on the species abundances. Also the choice of the initial abundance distribution (TE versus atomic) does hardly affect the species abundances (top versus bottom panel). Alumina



**Fig. 19.** Abundances of the prevalent aluminium-bearing species in the SRV  $\beta$ -velocity model. Straight lines show abundances including the AIOH photolysis, dashed lines without it.

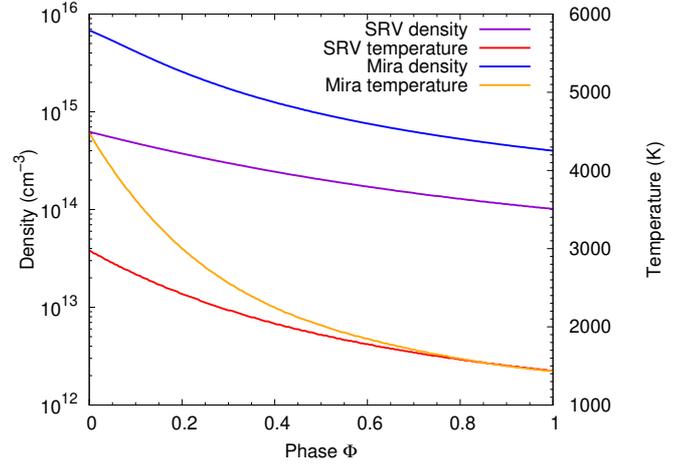
tetramer clusters start to form around  $2 R_\star$  thereby impacting the other aluminum-bearing species. At  $2.2 R_\star$  a significant amount of clusters have formed, which is accompanied by the decrease of the Al-containing molecules. AIO shows an abundance of a few times  $10^{-9}$  before the cluster formation becomes dominant. This is about an order of magnitude lower than predicted by recent ALMA observations (Decin et al. 2017). AIOH shows a strong increase in its abundance and reaches a maximum around  $2 R_\star$  and decreases again, when the alumina tetramers form. In comparison to observations, AIOH is overpredicted in our SRV non-pulsating model, except for the innermost region inside  $1.1 R_\star$ . AICI reaches its peak fractional abundance in the cluster forming zone, where it agrees well with the observation for the semi-regular variable R Doradus. In summary, the AIOH abundance is overpredicted and the AIO abundance is underpredicted, both by about an order of magnitude, as compared with observations. Moreover, closest agreement between the SRV non-pulsating model and observations is found before and at the cluster formation zone, but not afterwards.

The species abundances in the MIRA non-pulsating outflow model (i.e.  $\beta$ -velocity law) shown in Figure 20 have some common features with the SRV models, but also show some important variety. First, the molecular chemistry occurs closer to the star, which can be exemplified by the AIOH abundance peaking



**Fig. 20.** Abundances of the prevalent aluminium-bearing species in the MIRA non-pulsating outflow model. Straight lines show abundances including the photolysis, dashed lines without it.

at  $1.5 R_*$  and exceeding the atomic Al abundance for radii beyond  $1.2 R_*$ . This is closer to the star than in the SRV models and can be attributed to the lower temperatures and the higher gas densities in the MIRA models. Between  $1.4$  and  $1.7 R_*$  tetramer clusters form in significant amounts and dominate the Al content for  $r > 2 R_*$ , essentially independent of the photolysis. The cluster synthesis impacts the aluminium chemistry also in the MIRA non-pulsating outflow in decreasing their molecular abundances. The oxides AIO and  $Al_2O$ , as well as atomic Al and AlH are much stronger depleted than the aluminium halides AlF and AlCl, which even slightly increase for  $r > 2.5 R_*$ . Obviously, AIO,  $Al_2O$ , AlH, and atomic Al are molecular precursor to alumina clusters. AIO is only present in the innermost zones ( $r < 1.8 R_*$ ) and decreases to negligible amounts, once the cluster have formed. This is agreement with recent observation showing no AIO in the dust formation zone / inner wind in Mira-type AGB stars like IK Tau (Decin et al. 2017; Danilovich et al. 2020) We note overall larger abundances of AIO, AlOH,  $Al_2O$  and Al, when starting with a gas that is initially atomic (instead of assuming chemical equilibrium) for the MIRA non-pulsating models. In comparison with Decin et al. (2017), the AlOH abundance in the MIRA non-pulsating model is a bit high showing better agreement in the TE case (upper panel in Figure 20).



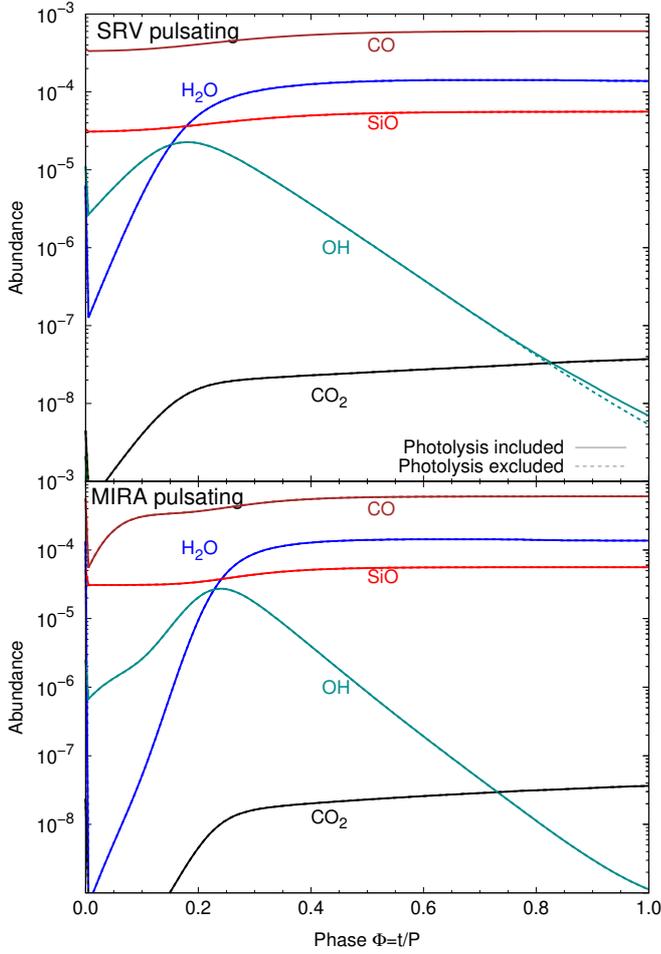
**Fig. 21.** The temperature and density profiles of the postshock gas at  $1 R_*$  in the two models, SRV and MIRA, as a function of pulsation phase  $\Phi = t/P$ , where  $t$  is the time and  $P$  is the pulsation period.

### 3.7. Pulsating model

In the pulsating model, we follow Lagrangian trajectories of an atmospheric gas parcel experiencing a pulsation-induced shock and a subsequent adiabatic expansion. The resulting trajectories correspond to ballistic and periodic excursions (Bertschinger & Chevalier 1985). This formalism has been widely used in combination with a chemical-kinetic network by Cherchneff et al. (1992); Willacy & Cherchneff (1998); Duari et al. (1999); Cherchneff (2006); Gobrecht et al. (2016) and applied to different circumstellar environments. We apply the pulsating model for both model stars, SRV and MIRA, starting at the stellar photosphere ( $1 R_*$ ). In both model stars, gas density and temperatures decrease monotonically with time (see Figure 21) during one pulsation cycle. After a complete pulsation cycle the gas densities have relaxed to their pre-shock values. Like in the non-pulsating  $\beta$ -velocity law, the SRV and MIRA model stars differ by their initial densities  $n_0$  and temperatures  $T_0$  (see Table 4). In addition to these quantities, shock velocities (defining the jumps in temperature and gas density) and pulsation periods differ for SRV and MIRA, and are required input quantities for the pulsating models. They are different for SRV and MIRA (see Table 4). The physical timescale is given by the pulsation period, which typically lasts of the order of hundreds of days. In the case of the SRV model it is 338 days, and in the MIRA model 470 days.

As a first step we follow the chemistry for one full pulsation cycle with a period  $P$ , starting at the immediate post-shock (phase  $\Phi = t/P = 0.0$ ) and ending just before the next shock arrives (phase  $\Phi = t/P = 1.0$ ). The abundances of the prevalent oxygen-bearing species CO,  $CO_2$ ,  $H_2O$ , OH, SiO for both models, SRV and MIRA, are displayed in Figures 22.

We find good agreement with observations and the temperature dependence of the main species also reflects the equilibrium calculations presented in Section 3.4. Moreover, the inclusion of the three photolysis rates has only a minor effect (OH at late phase) on the prevalent species. In both models (SRV and MIRA), similar trends with pulsation phase (time) are found.  $CO_2$ ,  $H_2O$ , and CO reform in the postshock gas, whereas OH is destroyed. In both pulsation models, SRV and MIRA, molecular oxygen ( $O_2$ ) plays a minor role with abundances not exceeding

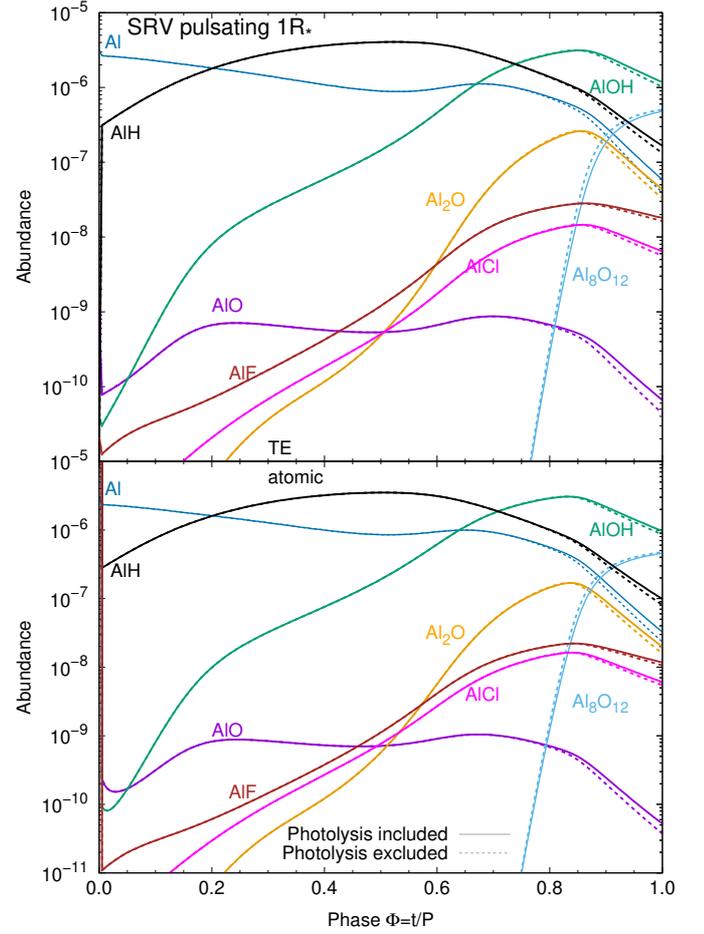


**Fig. 22.** The kinetic (non-equilibrium) abundances of the prevalent gas phase molecules in the postshock gas at  $1 R_*$  of the pulsating models as a function of pulsation phase. *Upper panel:* SRV *Lower panel:* MIRA

$1 \times 10^{-9}$ . On the one hand this result shows that the applied chemical-kinetic network produces good agreement with measured abundances. On the other hand, it shows that these main species are overall less affected by kinetic processes. However, this is not the case for the aluminium-bearing species including the (molecular) precursors of alumina dust, as will be shown below.

### 3.7.1. SRV pulsating model

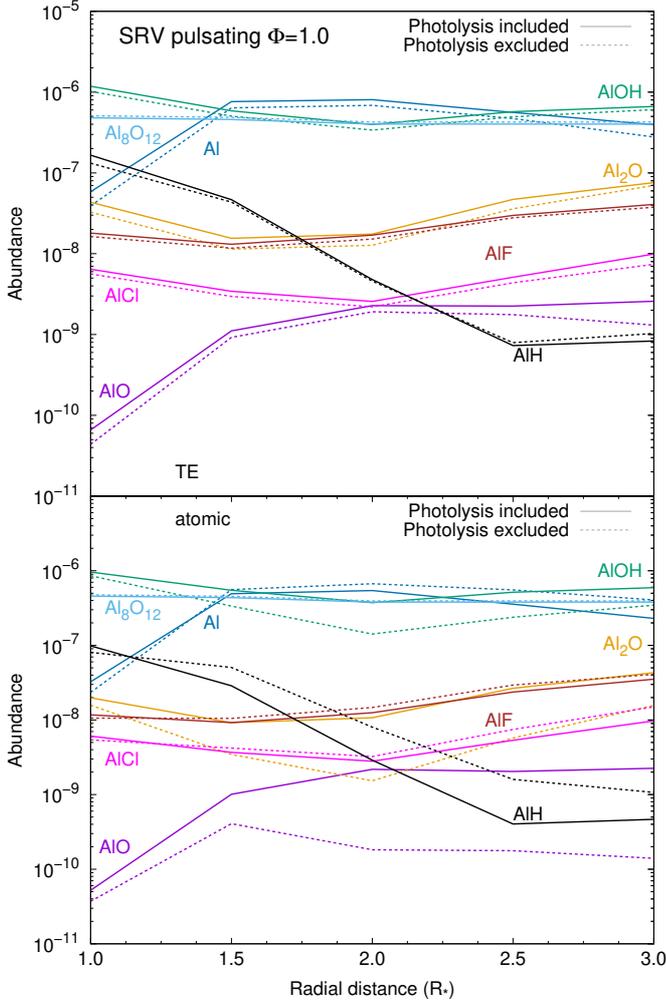
In the SRV pulsating model at  $1 R_*$  (see Figure 23), aluminium hydride (AlH) and atomic Al controls the aluminium content in the early post-shock gas ( $\Phi < 0.6$ ) and AlOH is the most abundant aluminium-bearing species at later phases  $\Phi > 0.6$ . A few dozen days later, at  $\Phi > 0.75$ , alumina tetramers start forming and account for 71.8 % of the aluminium content at  $\Phi = 1.0$  (AlOH accounts for 22.1 %). Note, that an alumina tetramer contains 8 Al atoms, whereas the molecules AlO and AlOH bear just one Al atom. These stoichiometries need to be taken into account, when inspecting the species contributions to the total aluminium content. Before the clusters are forming, the AlO abundance at  $1 R_*$  is of the order of few times  $10^{-10}$ , and decreases afterwards. This is still about two orders of magnitude lower than



**Fig. 23.** The kinetic abundances of aluminum-bearing species in the postshock gas at  $1 R_*$  of the pulsating SRV model as a function of pulsation phase  $\Phi$ . Dashed lines show the abundances, when the photodissociation is excluded. *Upper panel:* initial abundances are given by chemical equilibrium (TE). *Lower panel:* initial abundances correspond to a pure atomic gas composition.

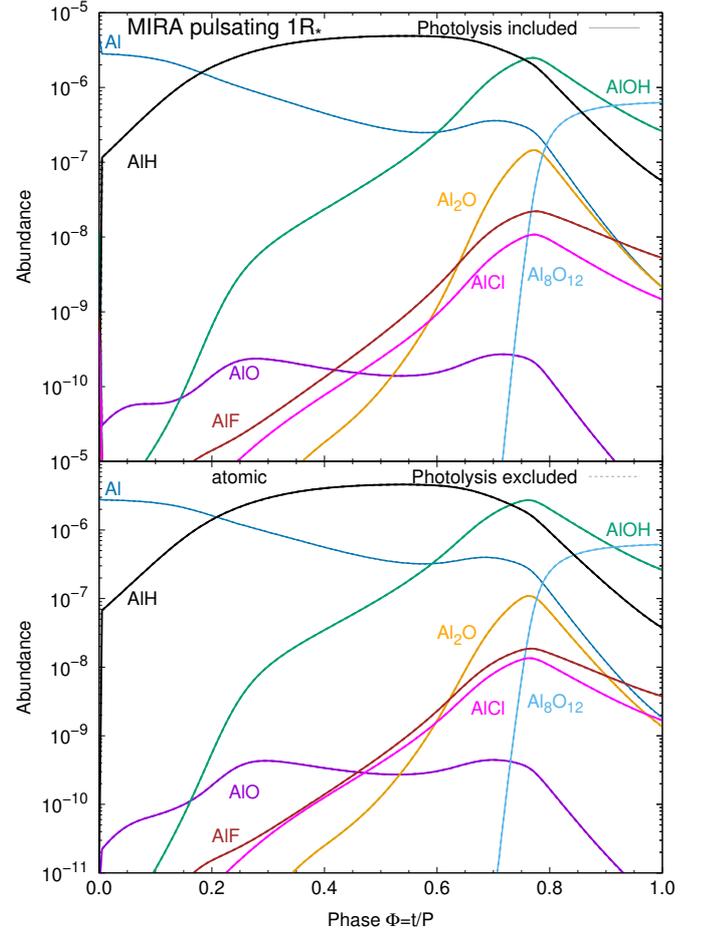
observed, but also still very close to the star. In contrast, AlOH shows large abundances at late phase ( $\Phi > 0.7$ ) exceeding the observational abundance by 2 orders of magnitude.

One of the main drawbacks of the applied pulsating model is that its trajectory is purely periodic, as it returns to its initial radial position after a complete pulsation period. Therefore, a consecutive (continuous) trajectory describing an outflow (i.e. mass loss) is not possible with the assumptions made of this pulsating model and a rescaling to larger radii is required. We are aware that pulsating models with a gradually outward directed flow exist (see e.g. Liljegren et al. (2018); Cristallo et al. (2021)), but they presuppose (imply) a certain degree of condensation. This assumption in turn uses pre-existing solids, whose formation is the subject of our present study. Therefore, a self-consistent coupling of these complementary methods is desirable, but it is beyond the scope of our current investigation. Nevertheless, bearing the approximative nature of our approach in mind, we follow the evolution of the pulsating model in steps of  $0.5 R_*$  up to a distance of  $3 R_*$  from the star (see Figure 24). Most of the species abundances ( $\text{Al}_2\text{O}$ , AlF,  $\text{Al}_8\text{O}_{12}$ )



**Fig. 24.** The kinetic (non-equilibrium) abundances of the aluminum-bearing species in the postshock gas of the SRV model after a complete pulsation cycle (Phase  $\Phi = 1.0$ ) as a function of radial position. Dashed lines show the abundances, when the AIOH photodissociation is excluded. *Upper panel:* initial abundances are given by chemical equilibrium. *Lower panel:* initial abundances correspond to a pure atomic gas composition.

are shaped and largely determined at the first pulsation at  $1 R_*$ , where the gas densities are highest and the pulsation is strongest, and show little variation for larger radii. However, we find an increasing trend for AIO and a decreasing trend of AIH with distance from the star in the case. The reaction  $\text{AIO} + \text{H}_2$  plays an important role. In the consecutive pulsation model, the  $\text{H}_2$  amount decreases gradually with the distance of the star, as a consequence of the decreasing gas density. In addition, the decreasing gas temperatures reduce also the efficiency of  $\text{AIO} + \text{H}_2$  reaction with a barrier of 2092 K. In other words, with increasing distance from the star, less and less AIO is consumed by the  $\text{AIO} + \text{H}_2 \rightarrow \text{AIOH} + \text{H}$  reaction, owing to the lack of molecular  $\text{H}_2$ , and leads to a moderate increase of AIO up to a fractional abundance of  $2 \times 10^{-9}$ .

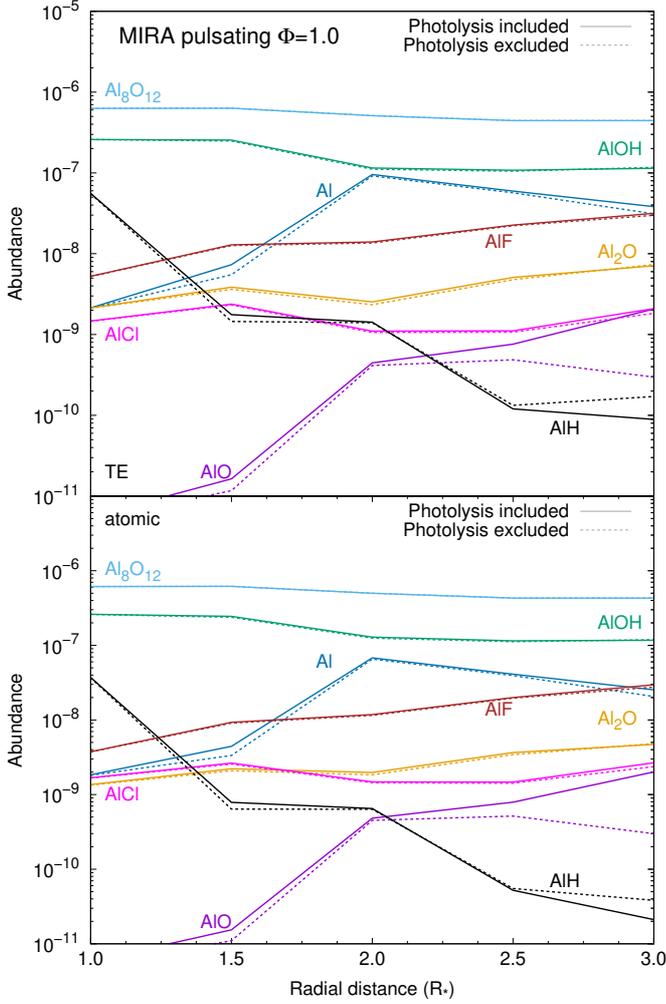


**Fig. 25.** The kinetic (non-equilibrium) abundances of the aluminum-bearing species in the postshock gas of the MIRA model as a function of pulsation phase  $\Phi$ . Dashed lines show the abundances, when the AIOH photodissociation is excluded. *Upper panel:* initial abundances are given by chemical equilibrium. *Lower panel:* initial abundances correspond to a pure atomic gas composition.

### 3.7.2. MIRA pulsating model

In Figure 25, the abundances of the main aluminium-bearing species in the MIRA pulsating model at  $1 R_*$  are shown. In comparison with the SRV pulsating model, the effect of the photolysis reactions is entirely negligible for the MIRA pulsation at  $1 R_*$ . The alumina tetramer clusters start to form in significant amounts at phase  $\Phi = 0.7$  and account for 93.8 % of the total aluminium content at phase  $\Phi = 1.0$ . The Al-containing molecules AIOH,  $\text{Al}_2\text{O}$ , AICI, and AIF show similar abundance profiles as in the SRV case and peak, when the clusters start to form. These peaks appear sharper and the increase of  $\text{Al}_8\text{O}_{12}$  steeper than in the SRV pulsating models. On the other hand, the pulsation period in the MIRA model (470 days) is longer than in the SRV model (338 days). Moreover, we note practically no difference, when starting with an pure atomic gas or molecular TE abundances.

In Figure 26, the MIRA model abundances of the consecutive pulsating model are shown. Clearly, the aluminium content is dominated by the alumina clusters and AIOH, independent of the inclusion of photolysis and of the initial gas mixture (TE or



**Fig. 26.** The non-equilibrium abundances of the aluminum-bearing species in the postshock gas of the MIRA model after a complete pulsation cycle (Phase  $\Phi = 1.0$ ) as a function of radial position. Dashed-dotted lines show the abundances, when the AIOH photodissociation is excluded. *Upper panel:* initial abundances are given by chemical equilibrium (TE). *Lower panel:* initial abundances correspond to a pure atomic composition.

atomic). We note that the photolysis processes start to play a role for  $r > 1.0 R_*$ , particularly in the case of an atomic initial gas. This can be explained, as the photolysis reactions decrease as  $r^{-2}$  and therefore decrease less rapidly than the total gas density showing an exponential decline with radial distance. At  $3 R_*$  the AIO abundance is similar in both pulsating models, SRV and MIRA. However, for  $r < 2R_*$ , we note that AIO is orders of magnitude less abundant in the MIRA model when compared to the SRV model, reflecting recent observations. The halides AIF and AICI show comparable amounts in the SRV and MIRA models.

### 3.8. Homogeneous nucleation to the bulk limit (corundum)

As indicated in the previous sections, the chemical-kinetic cluster growth is formulated up to the size of the alumina tetramer cluster,  $\text{Al}_8\text{O}_{12}$ . The subsequent nucleation is treated implicitly. The choice of the alumina tetramer as the size linking

the chemical-kinetic regime with the thermodynamically described larger  $(\text{Al}_2\text{O}_3)_n$  clusters appears arbitrary at first glance. However, by inspecting the reaction enthalpies, we find values of  $< -800$  kJ/mole (corresponding to equivalent temperatures  $\sim 10^5$  K) for the association of two  $\text{Al}_2\text{O}_3$  monomers and for the association of two  $\text{Al}_4\text{O}_6$  clusters. As pointed out in Section 3.2, the alumina monomer is comparatively unfavourable in terms of thermodynamics and kinetics. Therefore, the favourable association of two  $\text{Al}_4\text{O}_6$  clusters with a negligible reverse dissociation rate represents an ideal transition from a chemical-kinetic controlled regime to a cluster treatment based on thermodynamics. In addition, the density of rovibrational states, which increases with the number of atoms contained in the cluster, is large enough for  $n > 4$  to relax the necessity of a chemical-kinetic treatment. In the following, the cluster properties (energies, bond lengths, coordination, vibration modes) are analysed with increasing cluster size and compared with the bulk limit. In the case of alumina, the bulk limit corresponds to its most stable crystalline solid form, which is  $\alpha$ -alumina. The chemical-kinetic treatment beyond the tetramer is possible in principle and the choice of the tetramer as end point is somewhat arbitrary. However, in the chemically-rich gas mixture of circumstellar envelopes, the dust nucleation is likely to proceed via several species (i.e. heterogeneously). Therefore, we present the results for the homogeneous alumina nucleation in the form of size-dependent (up to  $n = 10$ ) cluster characteristics, their interpolation towards larger sizes ( $n > 10$ ), and the comparison to the crystalline bulk limit ( $n \rightarrow \infty$ ).

$\alpha$ -alumina, or corundum, is the thermodynamically most stable crystalline form (i.e. polymorph) of solid  $\text{Al}_2\text{O}_3$  and corresponds to our reference bulk limit. Other alumina polymorphs (like  $\theta$ ,  $\delta$  and  $\gamma$ ) are metastable or ‘transitional’ and transform to corundum at temperatures higher than  $\sim T = 1000 - 1200$  K (Levin & Brandon 1998). As alumina clusters grow in size, two phase transitions occur: from amorphous to the cubic  $\gamma$ -alumina form, and from  $\gamma$ -alumina to corundum ( $\alpha$ -alumina). Calorimetry experiments indicate that alumina becomes crystalline at a size of  $\sim 40\text{\AA}$  approximately (Tavakoli et al. 2013).

Many generic cluster properties (e.g. potential energies, melting temperatures),  $G(n)$ , can be approximated by a series expansion in the form of

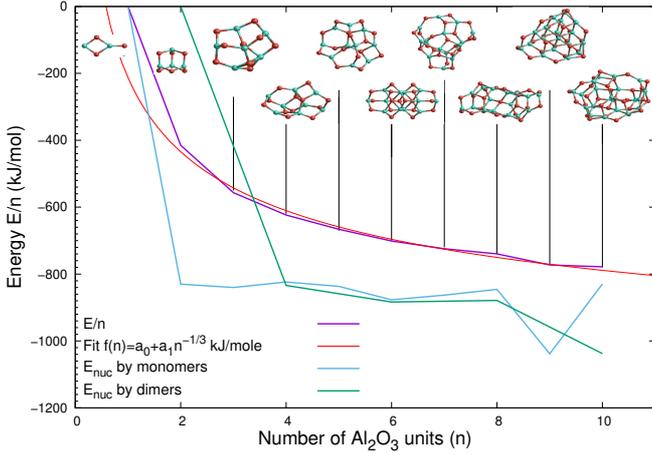
$$G(n) = a_0 + a_1 n^{-\frac{1}{3}} + a_2 n^{-\frac{2}{3}} + a_3 n^{-1} + \dots, \quad (15)$$

where  $n$  is the cluster size, or the number of  $\text{Al}_2\text{O}_3$  formula units,  $a_0 = G_{\text{bulk}}$  is a characteristic constant for the corresponding bulk phase of the cluster, and  $a_1$  is proportional to the fraction of surface atoms divided by ‘volume’ atoms (Johnston 2002). In the case of energies,  $a_1$  is often ascribed to the surface tension. In the case of simple, closely-packed clusters higher-order terms (second order and higher) can be neglected. Therefore, Eq. 15 is classically approximated to describe the energy of a cluster with a single and simple size dependence as

$$E(n) = a_0 + a_1 n^{-\frac{1}{3}}. \quad (16)$$

The electronic cluster energies (normalised to size  $n$ ) are shown in Figure 27. Note that the *cluster* energy, calculated as

$$E = E(n) - nE(\text{Al}_2\text{O}_3) \quad (17)$$



**Fig. 27.** Cluster energies (in  $\text{kJ mol}^{-1}$ ), normalised with respect to the  $\text{Al}_2\text{O}_3$  monomer (which is set to 0), are shown with purple straight lines. Nucleation energies (in  $\text{kJ mol}^{-1}$ ) are shown by monomer addition (in green) and by dimers (in blue). Fit with  $a_0 = -1287.9 \text{ kJ mol}^{-1}$  and  $a_1 = 1075.42 \text{ kJ mol}^{-1}$  of the normalised binding energies in red.

with a subsequent scaling of the alumina monomer ( $\text{Al}_2\text{O}_3$ ) energy to zero, is different from the electronic *binding* energy calculated as

$$E_b = E(n) - n(2E(\text{Al}) - 3E(\text{O})), \quad (18)$$

taking also into account the bonds inside a stoichiometric  $\text{Al}_2\text{O}_3$  unit.

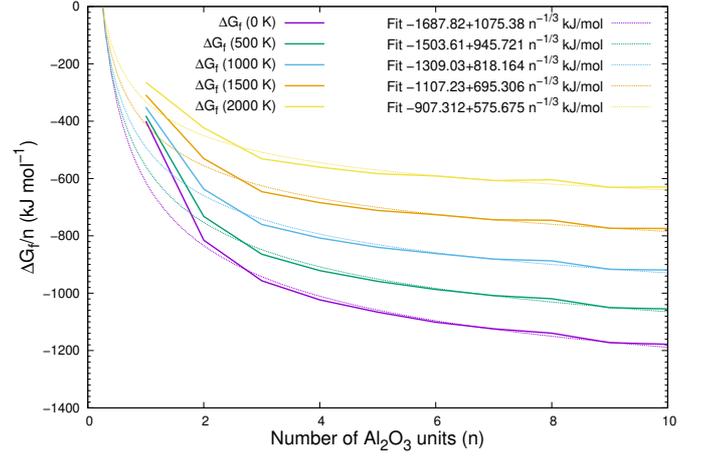
As a consequence of an increasing number of bonds with size, *cluster* and *binding* energies decrease monotonically with size  $n$ . By definition, these two energies differ only by a constant value and show the same relative tendencies. In Figure 27, the cluster energies at  $T = 0 \text{ K}$  are shown as a function of cluster size  $n$ . We find that the calculated cluster energies can be roughly approximated by a first-order fit (Eq. 16), when excluding the monomer ( $n = 1$ ) from the fitting procedure. A best-fit constant  $a_1$  of  $1075.42 \text{ kJ/mol}$  is found. Assuming a ‘monomeric radius’ in the range of  $r_{\text{mon}} = 2.08 - 2.60 \text{ \AA}$  corresponding to the range given by geometric and van-der-Waals dimensions (see Table D.1), surface tensions for different temperatures can be derived. Since the classical concept of surface tensions is not applicable to clusters on the (sub-)nano scale, we just provide the fit parameters  $a_0 = -1287.9 \text{ kJ mol}^{-1}$  and  $a_1 = 1075.42 \text{ kJ mol}^{-1}$ .

A closer inspection of the cluster energies reveals that some sizes ( $n = 3, 4, 5$ ) are more favourable, and some sizes ( $n = 8, 10$ ) are less favourable than predicted by the fit to Eq. 16. For a temperature of  $T = 0 \text{ K}$ , the fit for the binding energies of the clusters (see Table 5) with Eq. 16, we find a best fit with  $a_0 = -2906.48 \text{ kJ mol}^{-1}$  and  $a_1 = 699.504 \text{ kJ mol}^{-1}$ . Moreover, we plot the nucleation energies of the clusters. The nucleation energy is defined as the difference in energy between the nucleating particles and the newly-formed cluster. In the case of monomeric growth / nucleation, the nucleation energy is

$$E_{\text{nuc}} = E(n) - E(1) - E(n-1). \quad (19)$$

and in the case of dimer addition nucleation reaction it becomes

$$E_{\text{nuc}} = E(n) - E(2) - E(n-2). \quad (20)$$



**Fig. 28.** Gibbs free energies  $\Delta G_f(T)$  of the clusters (in  $\text{kJ mol}^{-1}$ ), normalised to the cluster size  $n$ , as a function of cluster size  $n$ , are shown for different temperatures.

The nucleation energies compare systems of the same size (number of atoms) and a normalisation is thus not required. Therefore, the monomer nucleation energy can be regarded as a finite-difference derivative of the cluster energy curve. The monomeric nucleation energies show that the GM clusters with sizes  $n = 4, 8$  and  $10$ , represent energetic bottlenecks, though the nucleation is still exothermic. In contrast, the GM clusters with sizes  $n = 2, 3, 6$ , and  $9$  have comparatively large nucleation energies and are expected to form quickly. As the monomer ( $n = 1$ ) is not an outstandingly stable and abundant compound as has been pointed out in Section 3.1, we also plot the energy for the nucleation by alumina dimers. The dimer nucleation scenario also indicates that the change in nucleation energy is largest for the formation of the tetramers ( $n = 4$ ) marking the end point of our chemical-kinetic description. Generally, the nucleation is not constrained by the addition of monomers or dimers, but can proceed via polymers of any kind (see e.g. Boulangier et al. (2019)). Moreover, as previously mentioned, a heterogeneous nucleation, involving more than one chemical dust species, seems more realistic.

In Figure 28, we plot the Gibbs free energy of formation  $\Delta G_f(T)$  (scaled to the elemental heats of formation) as a function of the cluster size  $n$  for temperatures  $T = 0, 500, 1000, 1500$  and  $2000 \text{ K}$ . For  $T = 0 \text{ K}$ , we find the overall lowest Gibbs free energies, which are also fitted according to Eq. 16. The corresponding fit parameters are  $a_0 = -1687.82$  and  $a_1 = 1075.38 \text{ kJ mol}^{-1}$ . On the one hand, this result is consistent with the fits to the cluster energies in Figure 27, showing an almost identical slope ( $a_1$ ). On the other hand, the bulk parameter ( $a_0$ ) is reasonably close to the standard free energy of formation of  $1643.7 \text{ kJ mol}^{-1}$  as derived from electrochemical cell experiments (Ghosh & Kay 1977) and to the energy of formation of  $1663.6 \text{ kJ mol}^{-1}$  reported in the JANAF tables. For larger temperatures, the bulk offsets  $a_0$  becomes less negative, but also the slopes ( $a_1$ ) become less steep. Furthermore, we note the same cluster sizes with comparatively enhanced ( $n = 3, 4$ ) and reduced ( $n = 8, 10$ ) favourability for temperatures  $> 0 \text{ K}$ .

**Table 5.** Point group (symmetry), average coordination numbers  $\bar{n}_{Al}$  and  $\bar{n}_O$ , the average bond distance  $\bar{d}(AlO)$  in Å (Ångström), and electronic binding energy  $E_0$  per  $Al_2O_3$  unit  $n$  of the GM clusters in  $\text{kJ mol}^{-1}$  for each cluster size  $n$

$n$	Symmetry	$\bar{n}_{Al}$	$\bar{n}_O$	$\bar{d}(AlO)$ (Å)	$E_0/n$ ( $\text{kJ mol}^{-1}$ )
1	$C_{2v}$	2.5	1.67	1.760	1794.95
2	$T_d$	3.0	2.00	1.744	2209.97
3	$C_1$	3.17	2.11	1.773	2351.68
4	$C_1$	3.38	2.25	1.771	2418.44
5	$C_1$	3.4	2.26	1.771	2461.09
6	$C_{2h}$	3.83	2.55	1.814	2496.35
7	$C_1$	3.64	2.43	1.790	2519.28
8	$C_2$	3.88	2.58	1.806	2534.45
9	$C_s$	3.89	2.59	1.809	2567.66
10	$C_1$	3.95	2.63	1.810	2573.33
$\alpha$	$D_{3d}$	6.0	4.0	1.91	3614.34

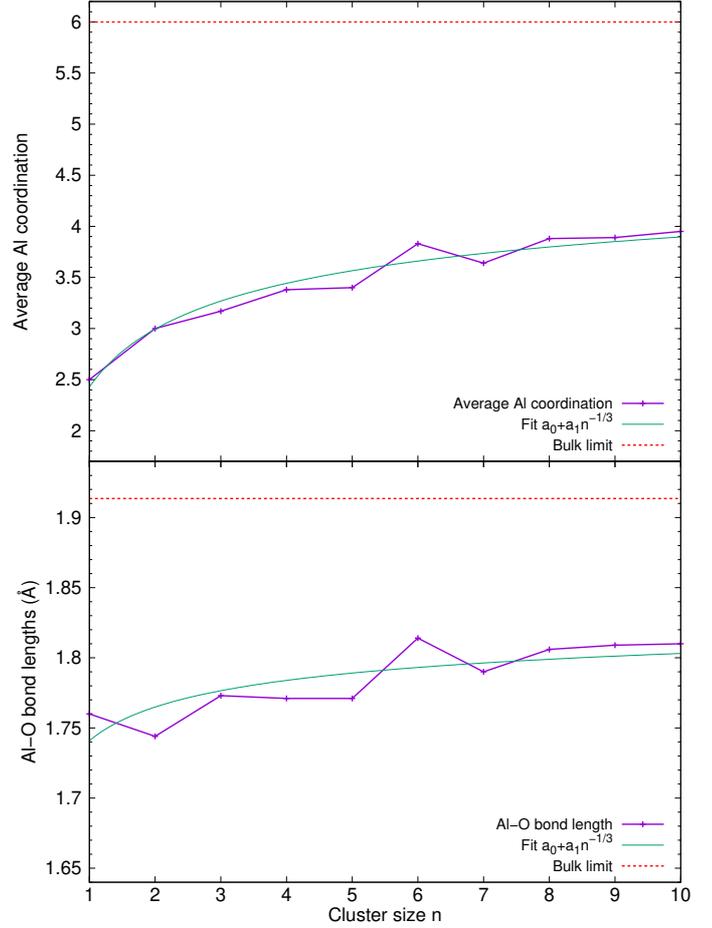
### 3.9. Geometric and electrostatic cluster properties

In this section, we will discuss geometric (bond coordination and bond lengths) and electrostatic (atomic charges, dipole moments and vibration modes) properties of the GM clusters and compare them with each other and with corundum (i.e. the bulk limit). The average atomic coordination is plotted in the upper panel of Figure 29 and increases monotonically with the cluster size  $n$ , except for  $n = 6$  representing an outlier. For the dimer ( $n = 2$ ) the coordination corresponds to the valence of the Al and O atoms. The atomic coordination inside the bulk (i.e.  $\alpha$ -alumina) is 6 for Al and 4 for O. Hence, the atomic coordination of the clusters considered in this study are well below the bulk values. The average Al–O bond distances of the GM clusters are shown in the lower panel of Figure 29. They show a less-regular pattern than the Al coordination, but also indicate an overall increasing trend. For sizes  $n = 3 - 5$  and for  $n = 8 - 10$  the average AIO distances are almost identical and sizes  $n = 1$  and  $n = 6$  represent outliers. Generally, it is not surprising that the average atomic coordination and AIO bond length increase with size  $n$ , as it reflects the decreasing surface-to-volume ratio with size (see Table 5). If we apply a fit of the form of Eq. 16 to the average Al coordination number  $n_{Al}$ , we find fitting parameters of  $a_0 = 5.16747$  and  $a_1 = -2.73878$ . For the average Al–O bond distance, we find fitting parameters of  $a_0 = 1.8557605 \text{ \AA}$  and  $a_1 = -0.11595 \text{ \AA}$ .

Since the fit values of  $a_0$  for both, average coordination and bond length, are below the value of  $\alpha$ -alumina ( $\bar{n}_{Al}=6$ ,  $\bar{d}(AlO) = 1.91 \text{ \AA}$ ), the bulk limits are not reached by these fits. We conclude that these fits are not suitable to find the approximate size, where the bulk limit is reached. However, they can serve as predictions for alumina clusters with size  $n > 10$ , which are computationally demanding to explore.

In Table 5, we summarise our findings on the geometry of the GM clusters, including also the point group symmetries (given in the Schönflies notation) and the binding energy per  $Al_2O_3$  unit, as a function of cluster size  $n$ .

In Table 6, we consider electrostatic properties of the GM clusters of by first examining the atomically partitioned Mulliken charges. In general, we find significantly lower average magnitudes of atomic charges for the PBE0-optimised structures than for the same clusters optimised with B3LYP. In both cases, the Mulliken average charges generally tend to decrease with cluster size  $n$ , with a slight increase between  $n = 6$  and



**Fig. 29.** Average Al coordination (top panel) and average AlO bond length in the GM clusters as a function of cluster size  $n$ . The values for the bulk limit ( $\alpha$ -alumina) are given in dashed red lines.

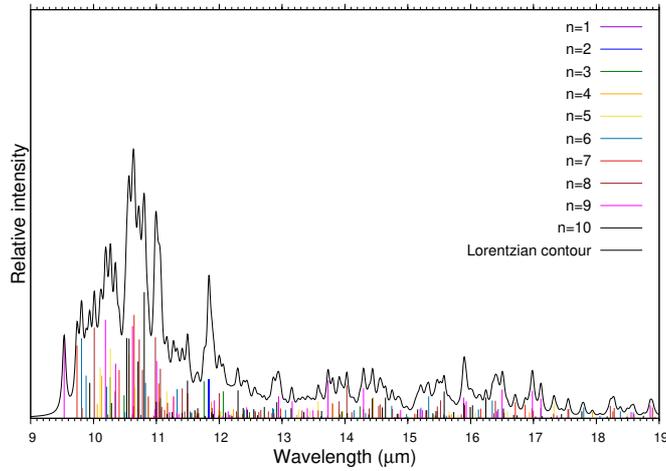
$n = 8$ . The more accurate CBS-QB3 calculations (for  $n = 1 - 3$ ) exhibit larger atomic charges. In general, the tendency for the (Mulliken) charges to decrease with size  $n$  indicates a transition from more ionic clusters to a more covalent solid, which is not unexpected.

The dipole moments  $D$  of the GM clusters are crucial for astronomical observations, since only clusters with a  $D \neq 0$  can be detected by rotational lines. Therefore, the dimer ( $n = 2$ ) and the hexamer ( $n = 6$ ) GM isomers with a  $D = 0$  are not observable by pure rotational spectroscopy, even though they might be present. Apart for  $n = 9$ , the PBE0- and B3LYP-derived dipole moments agree with each other. For the symmetric nonamer GM cluster the B3LYP dipole moment is very large and does not agree with value for the PBE0 optimisation. By closer inspection, we find tiny differences between the B3LYP- and PBE0-optimised geometries, which could be the reason for for the discrepancy of these two functionals.

Finally, we aim to address the radiative properties of the GM clusters by investigating their vibrations by using the rigid-rotor harmonic oscillator approximation and performing a harmonic vibrational analysis. Anharmonic contributions might play a role for large oscillation amplitudes and low-lying excited electronic states could contribute significantly, particularly at high tem-

**Table 6.** Average Mulliken charges of the Al atoms,  $\bar{q}_{Al}$  in the GM clusters. Owing to charge neutrality, atomic charges of O atoms can be computed as  $\bar{q}_O = -\frac{3}{2}\bar{q}_{Al}$ . Total dipole moments of the GM clusters are given in the units of Debye.

n	Mulliken charges			Dipole moments		
	B3LYP	PBE0	CBS	B3LYP	PBE0	CBS
1	0.60	0.58	1.21	2.59	2.46	2.42
2	1.01	0.89	1.21	0.00	0.00	0.00
3	0.67	0.59	1.61	1.23	1.01	1.49
4	0.61	0.51		1.46	1.56	
5	0.58	0.48		3.93	3.91	
6	0.37	0.24		0.00	0.00	
7	0.40	0.28		3.90	4.02	
8	0.42	0.30		3.57	3.64	
9	0.33	0.15		9.07	5.49	
10	0.30	0.20		5.67	5.60	



**Fig. 30.** Vibration modes of the  $(Al_2O_3)_n$ ,  $n = 1-10$ , GM clusters (color-coded) as a function of the wavelength (in  $\mu m$ ). The black contour corresponds to non-weighted (in size and metastable isomers) Lorentzian distribution.

peratures. Recent investigations of the vibrational IR spectra of nano silicates revealed that anharmonic as well as thermal effects can have significant impact on the spectra (Zamirri et al. 2019; Guiu et al. 2021). However, an investigation of these anharmonic effects and temperature dependence are challenging and beyond the scope of this paper. In this study, we primarily aim to show the positions of the vibrational bands necessary to calculate partition functions, to exclude transition states, and to explore whether the observed spectral alumina features at 11  $\mu m$  and 13  $\mu m$  could arise from (small) alumina clusters.

In Figure 30 the vibration modes of the  $(Al_2O_3)_n$ ,  $n = 1-10$ , GM clusters and their related intensities are shown as a function of wavelength. In order to transform the discrete vibration modes in a continuous spectrum and compare to an observed spectrum, line broadening in the form of a Lorentzian distribution with a half width  $\gamma = 0.033\mu m$ , corresponding to a typical ALMA setup resolution in submillimeter range, is applied. The contour (in black) comprises all unweighted  $(Al_2O_3)_n$ ,  $n = 1-10$ , GM clusters. In principle, metastable clusters could also contribute to the species abundance and spectra. However, like the anharmonic and temperature effects, they are not included in this study.

The 13  $\mu m$  feature seen in many M-type AGB stars principally could arise from small  $Al_2O_3$  clusters. However, our results indicate that the clusters under consideration here do not emit intensely in this wavelength region. The 13  $\mu m$  feature possibly originates from bulk-like alumina and heterogeneously mixed solids. This result is consistent with Begemann et al. (1997), who ruled out amorphous (non-crystalline) alumina as source of the 13  $\mu m$  feature. Moreover, the authors derived laboratory optical constants for compact and porous amorphous solid alumina, resulting in a very broad emission feature around 11–12  $\mu m$ , peaking at 11.5–11.8  $\mu m$ . In this wavelength range our calculated IR intensity is comparatively low, though it is still stronger than at 13  $\mu m$ . van Heijnsbergen et al. (2003) showed in an experimental study that large-sized aluminium oxide clusters (with sizes  $n \approx 15-40$ , but not necessarily with a Al:O stoichiometry of 2:3) exhibit two main vibration modes at 11.8  $\mu m$  and at 15  $\mu m$ . At 11.8  $\mu m$ , our calculated IR spectra shows a local maximum (see Figure 30), which predominantly arises from the dimers ( $n = 2$ ). However, at 15  $\mu m$  the overall IR intensity is rather low for the alumina clusters with  $n = 1-10$ . We also note that amorphous particles were considerably larger than our GM clusters. Owing to the approximate nature that does not take into account anharmonicities or temperature effects, we cannot rule out small (impure) alumina clusters as a source of both spectral features, at 11 and 13  $\mu m$ .

#### 4. Discussion

In Tables 7 and 8, we summarise our results for the TE calculations, the non-pulsating and the pulsating models for the two types of AGB stars, SRV and MIRA. The abundances derived from recent observations are listed in Table 9 for comparison. In the SRV models, atomic Al is found to be important for all cases. In comparison, the MIRA models show lower, but still considerable amounts of atomic Al. Strong and broad doublet resonance lines (around 395 nm) of the neutral Al atom, likely originating from the photosphere, are found in Mira-variable stars (Merrill et al. 1962; Kamiński et al. 2016). Reliable abundance estimates of atomic Al are however lacking. Therefore, we confirm the presence of atomic Al close to the photosphere of M-type AGB stars. AIO is by far the most studied aluminum-bearing molecule in oxygen-rich AGB stars. AIO is a diatomic molecule with known transitions and has a large dipole moment. It is thus not surprising that it has been found in a number of late-type stars, since its discovery in red supergiants (Tenenbaum & Ziurys 2009). In Table 9, we list recent detections of AIO and give abundances, where available. Kamiński et al. (2016) observed AIO in the archetypical star Mira instelf. The authors give a broad abundance range between  $10^{-9}$  and  $10^{-7}$ , owing to large uncertainties in the measurement of the column density of AIO. Their semi-analytical model results in an AIO abundance of  $5 \times 10^{-10}$  in Mira. We find generally lower AIO abundances in our kinetic MIRA models, except for in the consecutive pulsating model for  $r > 2 R_*$  (see Table 8). De Beck et al. (2017) conducted a search for AIO in 7 AGB stars, 6 of which are MIRA-type (R Aqr, TX Cam, o Cet (Mira), R Cas, W Hya, IK Tau) and one of SRV type (R Dor). Definitively, they found AIO transitions in two stars (o cet and R Aqr) and tentatively, in three further stellar sources. The authors derive a source-averaged AIO fractional abundance of  $4.5 \times 10^{-8}$ , based on a rotational diagram of IK Tau, also hinting at difficulties in deriving AIO abundances for individual stars. More recent ALMA observations (Decin et al. 2017; Danilovich et al. 2020) show the absence of AIO in the inner

envelope of IK Tau, a MIRA-type variable, but its presence in R Dor, a typical SRV type star. In R Dor, they derive AIO fractional abundance of  $(3.7-7.8) \times 10^{-8}$ . These values compare well with the SRV consecutive pulsating model. Moreover, we note higher AIO fractions in the SRV-type stars than in the MIRA-like AGB stars, which is consistently reproduced by our models for both types of trajectories, non-pulsating and pulsating models. AIOH represents the predominant Al-bearing molecule in both sets of models, SRV and MIRA. Its spectroscopic identification and a related abundance estimation by (radiative) transitions is challenging, owing to the following reasons. At present, there are no spectroscopic constants available in the literature for the AIOH electronic bands in the optical regime. Observations of rotational (and vibrational) transitions at longer wavelengths are feasible, but some rotational AIOH transitions are blended by rotational transitions of vibrationally excited TiO molecules and by rotational lines of a neutron-enriched isotopologue of SO<sub>2</sub> (Decin et al. 2020). In addition, it is debated that circumstellar AIOH also exists in a bent configuration (Trabelsi & Francisco 2018), exhibiting different spectroscopic constants. At high temperatures, AIOH will be a quasi-linear molecule, because the difference in energy between the bent and linear forms is very small. Therefore, an accurate observational abundance estimate is demanding and it tends to underestimate the AIOH content. This might explain the systematicallay larger AIOH model abundances in comparison with observations. Overall, the AIOH/AIO ratio in our circumstellar models is larger than the observationally derived ratio indicating that the kinetic conversion of AIOH in AIO in our models is too inefficient. Neither the photodissociations nor the bimolecular reactions AIOH+H and AIOH+OH (that produce AIO) are able to noticeably transform the stable singlet AIOH molecule into the doublet AIO, although we used the most elaborate kinetic data available. Therefore, possible resorts include that

- the AIOH destroying (and AIO forming) reactions are described with too low (too large) rate expressions.
- efficient, but unnoticed processes are missing in our present scheme.
- the observationally derived AIO and AIOH abundances are artifacts of a rather simplistic treatment assuming a static peak abundances that follow a Gaussian distribution with an e-fold or dissociation radius.

To test the first hypothesis, we varied the AIOH photodissociation rate. Best agreement with the observations of AIO and AIOH for all models (non-pulsating and pulsating, SRV and MIRA) is found, when we increased the photolysis rate by a factor of  $10^4$ . However, this artificial upscaling of the photodissociation leads to a vanishing alumina tetramers/cluster production in the SRV pulsation models. In the non-pulsating models, alumina clusters are still forming, but show a later onset of their synthesis.

The ground states of Al<sub>2</sub>O and AlO<sub>2</sub> are not observable by their rotational transitions. Our calculations indicate that Al<sub>2</sub>O is likely to be an intermediate in the alumina cluster formation and shows noticeable abundances, whereas AlO<sub>2</sub> is negligible in abundance and cluster formation. Therefore, there are no detections nor abundance estimates for these symmetric and linear molecules. For completeness, the aluminium-bearing halides AlF and AlCl are also listed in Tables 7, 8 and 9. These species become more important in S-type AGB stars and are not the main subject of this study examining oxygen-rich AGB stars. Nevertheless, the aluminium-bearing species AlF and AlCl

**Table 7.** Overview of the SRV model abundances for thermodynamic equilibrium (TE), the non-pulsating model ( $\beta$ -velocity), and the pulsating model. The fractional abundances (normalised to the total gas) are given as  $a(b) = a \times 10^b$

Species	TE	non-pulsating		pulsating	
	1R <sub>*</sub>	2R <sub>*</sub>	3R <sub>*</sub>	1 R <sub>*</sub>	2 R <sub>*</sub>
Al	3.2(-6)	1.7(-6)	4.7(-9)	5.8(-8)	8.1(-7)
AIO	9.6(-10)	1.6(-9)	7.7(-12)	6.6(-11)	2.3(-9)
AIOH	3.9(-11)	2.5(-6)	2.6(-7)	1.2(-6)	3.4(-7)
Al <sub>2</sub> O	1.2(-14)	1.6(-7)	3.0(-9)	4.3(-8)	1.8(-8)
AlO <sub>2</sub>	2.9(-16)	4.9(-16)	4.5(-20)	5.4(-18)	7.4(-18)
Al <sub>8</sub> O <sub>12</sub>	0	2.0(-12)	6.3(-7)	4.8(-7)	4.0(-7)
AlCl	2.8(-12)	1.1(-8)	1.9(-9)	6.4(-9)	2.6(-9)
AlF	2.1(-11)	2.3(-8)	8.8(-9)	1.8(-8)	1.7(-8)

**Table 8.** Overview of the MIRA model abundances for thermodynamic equilibrium (TE), the non-pulsating model ( $\beta$ -velocity), and the pulsating model. The fractional abundances (normalised to the total gas) are given as  $a(b) = a \times 10^b$ .

Species	TE	non-pulsating		pulsating	
	1 R <sub>*</sub>	2R <sub>*</sub>	3R <sub>*</sub>	1 R <sub>*</sub>	2 R <sub>*</sub>
Al	3.2(-6)	1.6(-9)	8.6(-12)	2.1(-9)	9.6(-8)
AIO	4.2(-9)	2.2(-12)	2.6(-14)	2.4(-12)	4.5(-10)
AIOH	1.2(-8)	2.1(-7)	3.2(-8)	2.6(-7)	1.2(-7)
Al <sub>2</sub> O	6.3(-11)	1.7(-9)	7.7(-11)	2.1(-9)	2.5(-9)
AlO <sub>2</sub>	1.2(-15)	2.3(-20)	3.1(-24)	7.3(-20)	6.1(-20)
Al <sub>8</sub> O <sub>12</sub>	0	6.4(-7)	6.7(-7)	6.3(-7)	5.1(-7)
AlCl	5.5(-10)	1.4(-9)	3.8(-10)	1.6(-9)	1.1(-9)
AlF	8.4(-10)	5.9(-9)	3.0(-9)	5.3(-9)	1.4(-8)

**Table 9.** Overview of recent observations of the species AIO, AIOH and AlCl in the inner winds of oxygen-rich AGB stars of SRV and MIRA type. Note that a conversion factor of  $\sim 0.46$  is applied to some molecules to translate from relative (with respect to H<sub>2</sub>) to fractional (with respect to the total gas including He) abundances. The fractional abundances (normalised to the total gas) are given as  $a(b)=a \times 10^b$

Species	SRV	MIRA	Reference
AIO	3.7(-8)	0	Decin et al. (2017)
	7.8(-8)	0	Danilovich et al. (2020)
		5.0(-10)	Kamiński et al. (2016)
AIOH	4.4(-8)	0	De Beck et al. (2017)
	6.9(-10)	2.1(-9)	Decin et al. (2017)
AlCl	9.2(-9)	4.1(-10)	Decin et al. (2017)

might hamper the formation of aluminium oxide molecules and clusters by locking up aluminium. Decin et al. (2017) found that AlCl is about 3 times more abundant in oxygen-rich SRV type stars. The  $\beta$ -velocity law at 3 R<sub>\*</sub> best fits these observations, but also the (single) pulsating models at 1 R<sub>\*</sub> reflect the magnitude and trends of these recent observations.

About 50 years ago Henderson (1970) suggested that the combustion of aluminium particles proceeds via the clustering of AIO molecules, followed by an evaporation of aluminum, to finally form alumina. Since then, Al combustion including related clustering and burning times have been extensively studied in the laboratory (see e.g. Desai et al. (1997); Beckstead (2005) and references therein). More recent studies have used aluminized fuels and modelled the (Al<sub>2</sub>O<sub>3</sub>)<sub>n</sub> cluster formation by numerical simulations (Starik et al. 2015; Savel'ev & Starik 2018). These studies were predominantly performed under conditions that are radically different from circumstellar envelopes:

in the combustion environment the pressure is at least 5 orders of magnitude higher, the bath gas is  $N_2$ , and there are high levels of oxidants ( $O_2$  and  $CO_2$ ). Bearing in mind these intrinsic differences, a comparison of the data from aluminium combustion experiments with our study is not applicable. An adaptation of our model and tests against experimental data could be done in a future study. Moreover, we note that the majority of the clustering rates used in this study (in particular reaction involving species with more than 4 atoms) is based on collision and capture theory. These theoretically derived rates could differ from currently lacking laboratory measurements. However, we aimed to minimize this effect by using the most recent and accurate experimental rate data available.

The stoichiometric  $(Al_2O_3)_n$  show a gradual monotonic trend with cluster size  $n$  in energy, atomic coordination, average bond length, and atomic charges. However, for all these quantities, we find outliers to these trends, and these outlier cannot be predicted by interpolations. Our derived fit parameter for the bulk limit ( $a_0 = -1687.82 \text{ kJ mol}^{-1}$ ) shows excellent agreement with the JANAF-NIST database and other experimental data (Ghosh & Kay 1977). Calorimetric experiments on the specific surface areas show that two size-dependent phase transition occur in alumina (McHale et al. 1997; Tavakoli et al. 2013). The smallest clusters reside in the amorphous regime characterized by sizes  $n < 800$ , or in terms of diameters, by  $d < 40 \text{ \AA}$ . For sizes  $800 < n < 1900$  (or diameters of  $d = 40 - 116 \text{ \AA}$ ), alumina is predominantly in cubic  $\gamma$ -alumina crystals. Only larger-sized alumina particles ( $n > 1900$ ,  $d > 116 \text{ \AA}$ ) are mainly present in the form of hexagonal corundum. Therefore, our cluster calculations up to a size of  $n = 10$  and their interpolation to larger sizes clearly pertain to the size-regime of the non-crystalline (amorphous) phase, and cannot be represented by top-down derived (crystalline) energies and structures.

## 5. Conclusions

We modeled the nucleation of alumina dust particles from a bottom-up perspective, starting with atomic and molecular precursors and using a chemical-kinetic approach applied to different circumstellar gas trajectories representing two model stars, a Semi-Regular Variable (SRV) and a regular variable (MIRA). In both sets of circumstellar models, SRV and MIRA, alumina cluster particles form efficiently, independent of their initial gas-phase compositions. The results of the non-pulsating monotonic outflow ( $\beta$ -velocity law) and the pulsating (ballistic trajectory) models show similar trends. The non-pulsating models predict a later onset of cluster formation at larger radii ( $2-2.5 R_\star$ ) in the SRV envelope, as compared with the MIRA model star, where the clusters form already at  $1.5 R_\star$ . The pulsating models show that alumina clusters form yet at  $1 R_\star$  in the late post-shock gas. In the SRV pulsating model, the amount of alumina clusters formed at  $1 R_\star$  is already substantial ( $\sim 72\%$  of the total aluminium content). However, in the MIRA pulsating envelope, the cluster synthesis is even more fruitful showing that more than  $90\%$  of the total Al content is in condensable clusters after one pulsation period. These results reflect the evolutionary sequence along the AGB showing an increase of the stellar mass-loss rate. Furthermore, the models consistently show larger AIO abundances in the SRV case, as compared with the MIRA case, reflecting the trends of the most recent ALMA observations. Still, the modelled AIOH abundances exceed those derived from observations by a about two orders of magnitude. A possible process, destroying the

stable singlet AIOH molecule, is the photolysis of AIOH. However, in AGB stars with rather low temperatures ( $< 2500 \text{ K}$ ) the AIOH photolysis is of secondary importance. Therefore, it is predominantly the bimolecular reaction  $AIO + H_2 \leftrightarrow AIOH + H$  determining the AIO/AIOH ratio. The effectiveness of the latter process depends on the concentration of molecular hydrogen, which is generally larger in the denser MIRA models. Therefore, the AIO/AIOH ratio is primarily controlled by the  $H/H_2$  in circumstellar envelopes.

Larger-sized  $(Al_2O_3)_n$  clusters ( $n = 4 - 10$ ) and their interpolation to the bulk-limit ( $n \rightarrow \infty$ ) are described by size-dependent properties, as a chemical-kinetic treatment becomes increasingly expensive and prohibitive. We find that the cluster energy can be approximated by a fit of the form  $E(n) = -1687.82 + 1075.38 n^{-1/3} \text{ kJ mol}^{-1}$ . The offset of  $-1687.82 \text{ kJ mol}^{-1}$  is reasonably close to experimental values for bulk alumina (corundum). Nevertheless, it is very likely that the nucleation in oxygen-rich AGB stars proceeds heterogeneously involving additional species, that are different from pure aluminium-oxygen clusters. Moreover, we note that the small size regime under consideration here is still deeply in the non-crystalline (amorphous) size regime and that phase transitions could affect our bottom-up nucleation fit.

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**Appendix A: Equilibrium constant and detailed balance:**

$$K_{eq} = \frac{k_f}{k_r} = \exp\left(-\frac{\Delta G_r(T)}{RT}\right) \quad (\text{A.1})$$

where  $K_{eq}$  is the dimensionless equilibrium constant,  $k_f$  and  $k_r$  are the forward and the reverse reaction rates, respectively.

**Appendix B: Chemical-kinetic network**

**Appendix C: Thermochemistry**

**Appendix D: Cluster geometries**

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<sup>1</sup> dipole-dipole up to 1700K

<sup>2</sup> dipole-dipole up to 300K

<sup>3</sup> dipole-dipole up to 400K

<sup>2</sup> Having a late barrier, a successful reaction on the triplet potential energy surface (PES) requires that AlO is vibrationally excited, and a translational collision energy less than 7.5 kJ mol<sup>-1</sup>. The resulting rate coefficient is estimated to be 3.0-3.7(-12) cm<sup>3</sup> s<sup>-1</sup> for T=1000-2000 K.

**Table B.1.** Kinetic rate network listing (1) the reaction number, (2) the reaction, (3) the CBS-QB3 heat of reaction (enthalpy) at  $T=0$  K, (4) the critical temperature  $T_c$ , where the free energy of reaction,  $\Delta G_r(T)$  changes its sign, (5) the reaction rate with the pre-exponential factor  $A$  given as  $a(-b) \equiv a \times 10^{-b}$  (for unimolecular reactions (photodissociations in units of  $s^{-1}$ , for bimolecular reactions in units of  $cm^3 s^{-1}$ , for termolecular reactions in units of  $cm^6 s^{-1}$ ), and (6) the reference or method of calculation.

Number	Reaction	$\Delta H_r(0K)$	$T_c$	Rate k	Reference / remark
1	$H + H + H_2 \rightarrow H_2 + H_2$	-437	3800	$8.85(-33)(T/300)^{-0.6}$	NIST
2	$H_2 + H_2 \rightarrow H + H + H_2$	+437	3800	$1.5(-9)(T/300)^{0.34} \exp(-48346.4/T)$	NIST
3	$H + H + H \rightarrow H_2 + H$	-437	3800	$8.82(-33)$	NIST
4	$H_2 + H \rightarrow H + H + H$	+437	3800	$2.54(-8)(T/300)^{-0.10} \exp(-52555.6/T)$	NIST
5	$H + H + He \rightarrow H_2 + He$	-437	3800	$4.96(-33)$	NIST
6	$H_2 + He \rightarrow H + H + He$	+437	3800	$8.85(-10)\exp(-48346.4/T)$	NIST
7	$OH + OH \rightarrow H_2O + O$	-67.4	4400	$1.65(-12)(T/300)^{1.1} \exp(-50.5/T)$	NIST
8	$H_2O + O \rightarrow OH + OH$	+67.4	4400	$1.84(-11)(T/300)^{0.95} \exp(-8573.7/T)$	NIST
9	$CO + OH \rightarrow CO_2 + H$	-105	3000	$3.52(-12)\exp(-2630.2/T)$	NIST
10	$CO_2 + H \rightarrow CO + OH$	+105	3000	$2.51(-10)\exp(-13229.1/T)$	NIST
11	$OH + H + H_2O \rightarrow H_2O + H_2O$	-494	4000	$1.19(-30)(T/300)^{-2.1}$	NIST
12	$H_2O + H_2O \rightarrow OH + H + H_2O$	+494	4000	$2.66(-7)\exp(-7500.0/T)$	NIST
13	$OH + H \rightarrow H_2 + O$	-10.9	1100	$6.86(-14)(T/300)^{2.8} \exp(-1949.5/T)$	NIST
14	$H_2 + O \rightarrow OH + H$	+10.9	1100	$3.44(-13)(T/300)^{2.67} \exp(-3159.3/T)$	NIST
15	$H_2 + OH \rightarrow H_2O + H$	-56.5	no	$1.55(-12)(T/300)^{1.6} \exp(-1659.7/T)$	NIST
16	$H_2O + H \rightarrow H_2 + OH$	56.5	no	$6.82(-12)(T/300)^{1.6} \exp(-9719.8/T)$	NIST
17	$C + O \rightarrow CO + h\nu$	-1073	no	$1.58(-17)(T/300)^{0.34} \exp(-1297.4/T)$	Fit to Dalgarno et al. (1990)
18	$C + O + M \rightarrow CO + M$	-1073	no	$2.00(-34)$	NIST
19	$CO + M \rightarrow C + O + M$	1073	no	$4.40(-10)\exp(-98600.0/T)$	Appleton et al. (1970), eq. 4.7
20	$H + O + M \rightarrow OH + M$	-426	4000	$4.36(-32)(T/300)^{-1.00}$	NIST
21	$OH + M \rightarrow H + O + M$	426	4000	$4.00(-9)\exp(-50000/T)$	NIST
22	$CO + O + M \rightarrow CO_2 + M$	-532	3800	$1.20(-32)\exp(-2160/T)$	NIST
23	$CO_2 + M \rightarrow CO + O + M$	532	3800	$8.02(-11)\exp(-26900/T)$	Willacy et al. (1998)
24	$OH + H + M \rightarrow H_2O + M$	-494	4000	$2.59(-31)(T/300)^{-2.00}$	NIST
25	$H_2O + M \rightarrow OH + H + M$	494	4000	$5.80(-9)\exp(-52920/T)$	NIST
26	$Al + O + M \rightarrow AlO + M$	-498	4900	$3.34(-32)(T/300)^{-1.79}$	RRKM fit
27	$AlO + M \rightarrow Al + O + M$	498	4900	$4.01(-10)\exp(-57391/T)$	Gómez Martín et al. (2017) RRKM fit
28	$AlO + H + M \rightarrow AlOH + M$	-482	4900	$1.32(-28)(T/300)^{-3.08}$	RRKM fit
29	$AlOH + M \rightarrow AlO + H + M$	482	4900	$1.47(-7)\exp(-55985/T)$	RRKM fit
30	$Al + CO_2 \rightarrow Al + CO$	33.6	800	$6.32(-10)\exp(-4522.0/T)$	RRKM fit
31	$AlO + CO \rightarrow Al + CO_2$	-33.6	800	$10^{-16.6318+3.43135 \log(T)-0.5786 \log(T)^2}$	Mangan et al. (2020) RRKM fit
32	$Al + O_2 \rightarrow AlO + O$	-0.9	no	$1.68(-10)(T/300)^{-0.26}$	Gómez Martín et al. (2017)
33	$AlO + O \rightarrow Al + O_2$	0.9	no	$7.15(-12)\exp(-945./T)$	RRKM fit
34	$Al + H_2O \rightarrow AlOH + H$	-60.4	no	$1.7(-12)\frac{-422}{T} + 1.45(-10)\exp(-2657./T)$	Mangan et al. (2021)
35	$AlOH + H \rightarrow Al + H_2O$	60.4	no	$4.31(-11)\exp(-9457./T)$	Mangan et al. (2021)
36	$AlO + H_2 \rightarrow AlOH + H$	-45.0	no	$5.37(-13)(T/300)^{2.77} \exp(-2190./T)$	Mangan et al. (2021)
37	$AlOH + H \rightarrow AlO + H_2$	45.0	no	$8.89(-11)\exp(-9092/T)$	Mangan et al. (2021)
38	$AlO + O + M \rightarrow AlO_2 + M$	-391	2900	$6.74(-30)(T/300)^{-2.53}$	Plane et al. (2021) RRKM fit
39	$AlO_2 + M \rightarrow AlO + O + M$	391	2900	$4.51(-7)\exp(-44166/T)$	Plane et al. (2021) RRKM fit
40	$AlO + CO_2 \rightarrow AlO_2 + CO$	140	no	$1.81(-10)(T/300)^{-0.81} \exp(-18138.2/T)$	detailed balance
41	$AlO_2 + CO \rightarrow AlO + CO_2$	-140	no	$2.55(-12)(T/300)^{0.17}$	Mangan et al. (2020) RRKM fit
42	$AlO + O_2 \rightarrow AlO_2 + O$	106	no	$3.54(-11)(T/300)^{0.19} \exp(-13140/T)$	detailed balance
43	$AlO_2 + O \rightarrow AlO + O_2$	-106	no	$1.90(-10)(T/300)^{0.17}$	Mangan et al. (2020) RRKM fit
44	$AlO + AlO \rightarrow Al_2O + O$	-70.2	1800	$1.42(-9)(T/300)^{0.17}$	capture
45	$Al_2O + O \rightarrow AlO + AlO$	70.2	1800	$1.42(-9)(T/300)^{0.17} \exp(-8447.4/T)$	reverse capt
46	$AlOH + AlOH \rightarrow Al_2O + H_2O$	-25.9	600	$9.85(-10)(T/300)^{0.17}$	capture
47	$Al_2O + H_2O \rightarrow AlOH + AlOH$	25.9	600	$9.85(-10)(T/300)^{0.17} \exp(-3593.2/T)$	reverse capt
48	$AlO + AlOH \rightarrow Al_2O + OH$	-14.3	500	$1.00(-9)(T/300)^{0.17}$	capture
49	$Al_2O + OH \rightarrow AlO + AlOH$	14.3	500	$1.00(-9)(T/300)^{0.17} \exp(-1727.3/T)$	reverse capture
50	$AlO + AlO + M \rightarrow Al_2O_2 + M$	-547	3500	$2.35(-33)$	capture
51	$Al_2O_2 + M \rightarrow AlO + AlO + M$	547	3500	$1.45(-10)(T/300)^{-0.63} \exp(-65887.9/T)$	detailed balance
52	$AlOH + AlOH \rightarrow Al_2O_2 + H_2$	-21.0	400	$9.8(-10)(T/300)^{0.17}$	capture
53	$Al_2O_2 + H_2 \rightarrow AlOH + AlOH$	21.0	400	$9.8(-10)(T/300)^{0.17} \exp(-2524.6/T)$	reverse capture
54	$AlO + AlOH \rightarrow Al_2O_2 + H$	-65.9	1000	$1.00(-9)(T/300)^{0.17}$	capture
55	$Al_2O_2 + H \rightarrow AlO + AlOH$	65.9	1000	$1.00(-9)(T/300)^{0.17} \exp(-7926.3/T)$	reverse capture
56	$Al_2O + O_2 \rightarrow Al_2O_2 + H$	19.5	no	$8.11(-12)(T/300)^{1.69} \exp(-1965.8/T)$	detailed balance
57	$Al_2O_2 + O \rightarrow Al_2O + O_2$	-19.5	no	$7.46(-10)(T/300)^{0.17}$	capture
58	$Al_2O + H_2O \rightarrow Al_2O_2 + H_2$	4.9	no	$5.90(-11)(T/300)^{1.37} \exp(-1391.5/T)$	detailed balance (capture)
59	$Al_2O_2 + H_2 \rightarrow Al_2O + H_2O$	-4.9	no	$1.20(-11)\exp(-17285/T)$	TST calculation

60	$\text{Al}_2\text{O} + \text{OH} \rightarrow \text{Al}_2\text{O}_2 + \text{H}$	-51.5	no	$5.69(-10)(T/300)^{0.17}$	capture
61	$\text{Al}_2\text{O}_2 + \text{H} \rightarrow \text{Al}_2\text{O} + \text{OH}$	51.5	no	$1.86(-07)(T/300)^{-1.55} \exp(-6310.9/T)$	detailed balance
62	$\text{Al}_2\text{O}_2 + \text{H}_2\text{O} \rightarrow \text{Al}_2\text{O}_3 + \text{H}_2$	85.0	no	$1.78(-09)(T/300)^{0.77} \exp(-11797.1/T)$	detailed balance
63	$\text{Al}_2\text{O}_3 + \text{H}_2 \rightarrow \text{Al}_2\text{O}_2 + \text{H}_2\text{O}$	-85.0	no	$1.52(-9)(T/300)^{0.17}$	capture
64	$\text{Al}_2\text{O}_2 + \text{OH} \rightarrow \text{Al}_2\text{O}_3 + \text{H}$	28.2	4200	$2.57(-11)(T/300)^{1.78} \exp(-1977.4/T)$	detailed balance
65	$\text{Al}_2\text{O}_3 + \text{H} \rightarrow \text{Al}_2\text{O}_2 + \text{OH}$	-28.2	4200	$1.13(-9)(T/300)^{0.17}$	capture
66	$\text{AlO}_2 + \text{AlOH} \rightarrow \text{Al}_2\text{O}_3 + \text{H}$	-72.5	2700	$1.24(-9)(T/300)^{0.17}$	capture
67	$\text{Al}_2\text{O}_3 + \text{H} \rightarrow \text{AlO}_2 + \text{AlOH}$	72.5	2700	$1.24(-9)(T/300)^{0.17} \exp(-8714.8/T)$	reverse capture
68	$\text{AlO} + \text{H}_2\text{O} \rightarrow \text{OAlOH} + \text{H}$	8.6	5400	$2.03(-11)\exp(-1360.0/T)$	Mangan RRKM fit
69	$\text{OAlOH} + \text{H} \rightarrow \text{AlO} + \text{H}_2\text{O}$	-8.6	5400	$3.91(-09)(T/300)^{-1.94} \exp(-725.6/T)$	detailed balance
70	$\text{AlO} + \text{H}_2\text{O} \rightarrow \text{AlOH} + \text{OH}$	11.5	1000	$3.89(-10)\exp(-1295.0/T)$	Mangan et al. (2021)
71	$\text{AlOH} + \text{OH} \rightarrow \text{AlO} + \text{H}_2\text{O}$	-11.5	1000	$6.05(-10)(T/300)^{0.17}$	capture
72	$\text{AlO} + \text{H}_2\text{O} + \text{M} \rightarrow \text{Al}(\text{OH})_2 + \text{M}$	-314	3100	$2.40(-26)(T/300)^{-3.87}$	Mangan et al. (2021)
73	$\text{Al}(\text{OH})_2 + \text{M} \rightarrow \text{AlO} + \text{H}_2\text{O} + \text{M}$	314	3100	$1.00(-10)\exp(-17205/T)$	Estimate
74	$\text{OAlOH} + \text{SiO} \rightarrow \text{AlSiO}_3 + \text{H}$	-6.0	100	$1.22(-9)(T/300)^{0.17}$	capture
75	$\text{AlSiO}_3 + \text{H} \rightarrow \text{OAlOH} + \text{SiO}$	6.0	100	$1.22(-9)(T/300)^{0.17} \exp(-724.1/T)$	reverse capture
76	$\text{AlSiO}_3 + \text{AlO} \rightarrow \text{Al}_2\text{O}_3 + \text{SiO}$	-28.3	no	$1.64(-9)(T/300)^{-0.171}$	capture
77	$\text{Al}_2\text{O}_3 + \text{SiO} \rightarrow \text{AlSiO}_3 + \text{AlO}$	28.3	no	$6.28(-10)(T/300)^{-0.18} \exp(-3110.2/T)$	detailed balance
78	$\text{OAlOH} + \text{SiO} + \text{M} \rightarrow \text{HAlSiO}_3 + \text{M}$	-356	1500	$2.03(-33)$	capture
79	$\text{HAlSiO}_3 + \text{M} \rightarrow \text{OAlOH} + \text{SiO} + \text{M}$	356	1500	$2.72(-10)(T/300)^{2.37} \exp(-35190.3/T)$	detailed balance
80	$\text{HAlSiO}_3 + \text{H} \rightarrow \text{AlSiO}_3 + \text{H}_2$	-87.3	no	$1.13(-9)(T/300)^{0.17}$	capture
81	$\text{AlSiO}_3 + \text{H}_2 \rightarrow \text{HAlSiO}_3 + \text{H}$	87.3	no	$5.07(-13)(T/300)^{-1.04} \exp(-11390.8/T)$	detailed balance
82	$\text{Al}_2\text{O}_3 + \text{SiO} + \text{M} \rightarrow \text{Al}_2\text{O}_3\text{SiO} + \text{M}$	-460	3000	$1.38(-33)$	capture
83	$\text{Al}_2\text{O}_3\text{SiO} + \text{M} \rightarrow \text{Al}_2\text{O}_3 + \text{SiO} + \text{M}$	460	3000	$4.10(-11)(T/300)^{-0.8} \exp(-53641.8/T)$	detailed balance
84	$\text{Al}_2\text{O}_3\text{SiO} + \text{AlO} \rightarrow \text{Al}_3\text{O}_4 + \text{SiO}$	-200	no	$1.13(-9)(T/300)^{0.17}$	capture
85	$\text{Al}_3\text{O}_4 + \text{SiO} \rightarrow \text{Al}_2\text{O}_3\text{SiO} + \text{AlO}$	200	no	$1.67(-8)(T/300)^{0.40} \exp(-19243/T)$	detailed balance
86	$\text{Al}_3\text{O}_4 + \text{AlOH} \rightarrow \text{Al}_4\text{O}_5 + \text{H}$	-148	3200	$1.22(-9)(T/300)^{0.17}$	capture
87	$\text{Al}_4\text{O}_5 + \text{H} \rightarrow \text{Al}_3\text{O}_4 + \text{AlOH}$	148	3200	$1.22(-9)(T/300)^{0.17} \exp(-17777.0/T)$	reverse capture
88	$\text{Al}_2\text{O}_2 + \text{AlO} + \text{M} \rightarrow \text{Al}_3\text{O}_3 + \text{M}$	-518	5000	$2.38(-33)$	capture
89	$\text{Al}_3\text{O}_3 + \text{M} \rightarrow \text{Al}_2\text{O}_2 + \text{AlO} + \text{M}$	518	5000	$1.65(-12)(T/300)^{-1.16} \exp(-62359.5/T)$	detailed balance
90	$\text{Al}_2\text{O}_2 + \text{AlOH} \rightarrow \text{Al}_3\text{O}_3 + \text{H}$	-36.5	no	$1.40(-9)(T/300)^{0.17}$	capture
91	$\text{Al}_3\text{O}_3 + \text{H} \rightarrow \text{Al}_2\text{O}_2 + \text{AlOH}$	36.5	no	$3.20(-11)\exp(-4713/T)$	TST calculation
92	$\text{Al}_2\text{O}_3 + \text{OH} \rightarrow \text{Al}_2\text{O}_4 + \text{H}$	-119	4200	$6.63(-10)(T/300)^{0.172}$	capture
93	$\text{Al}_2\text{O}_4 + \text{H} \rightarrow \text{Al}_2\text{O}_3 + \text{OH}$	119	4200	$3.18(-7)(T/300)^{-0.96} \exp(-13422.8/T)$	detailed balance
94	$\text{Al}_2\text{O}_3 + \text{H}_2\text{O} \rightarrow \text{Al}_2\text{O}_4 + \text{H}_2$	-62.2	2000	$7.84(-10)(T/300)^{0.173}$	capture
95	$\text{Al}_2\text{O}_4 + \text{H}_2 \rightarrow \text{Al}_2\text{O}_3 + \text{H}_2\text{O}$	62.2	2000	$7.84(-10)(T/300)^{0.17} \exp(-7478.2/T)$	reverse capture
96	$\text{Al}_2\text{O}_3 + \text{AlOH} \rightarrow \text{Al}_3\text{O}_4 + \text{H}$	-139	1600	$8.96(-10)(T/300)^{0.17}$	capture
97	$\text{Al}_3\text{O}_4 + \text{H} \rightarrow \text{Al}_2\text{O}_3 + \text{AlOH}$	139	1600	$8.96(-10)(T/300)^{0.17} \exp(-16655.4/T)$	reverse capture
98	$\text{Al}_3\text{O}_3 + \text{H}_2\text{O} \rightarrow \text{Al}_3\text{O}_4 + \text{H}_2$	-17.1	250	$7.83(-10)(T/300)^{0.17}$	capture
99	$\text{Al}_3\text{O}_4 + \text{H}_2 \rightarrow \text{Al}_3\text{O}_3 + \text{H}_2\text{O}$	17.1	250	$7.83(-10)(T/300)^{0.17} \exp(-2077.8/T)$	reverse capture
100	$\text{Al}_3\text{O}_3 + \text{OH} \rightarrow \text{Al}_3\text{O}_4 + \text{H}$	-73.6	1000	$7.01(-10)(T/300)^{0.17}$	capture
101	$\text{Al}_3\text{O}_4 + \text{H} \rightarrow \text{Al}_3\text{O}_3 + \text{OH}$	73.6	1000	$7.01(-10)(T/300)^{0.17} \exp(-8869.2/T)$	reverse capture
102	$\text{Al}_3\text{O}_3 + \text{AlO} + \text{M} \rightarrow \text{Al}_4\text{O}_4 + \text{M}$	-558	4400	$1.76(-33)$	capture
103	$\text{Al}_4\text{O}_4 + \text{M} \rightarrow \text{Al}_3\text{O}_3 + \text{AlO} + \text{M}$	558	4400	$1.88(-11)(T/300)^{-1.18} \exp(-67158.6/T)$	detailed balance
104	$\text{Al}_3\text{O}_3 + \text{AlOH} \rightarrow \text{Al}_4\text{O}_4 + \text{H}$	-76.4	2000	$1.03(-9)(T/300)^{0.17}$	capture
105	$\text{Al}_4\text{O}_4 + \text{H} \rightarrow \text{Al}_3\text{O}_3 + \text{AlOH}$	76.4	2000	$1.03(-9)(T/300)^{0.17} \exp(-9181.9/T)$	reverse capture
106	$\text{Al}_4\text{O}_4 + \text{H}_2\text{O} \rightarrow \text{Al}_4\text{O}_5 + \text{H}_2$	-88.8	1100	$8.46(-10)(T/300)^{0.17}$	capture
107	$\text{Al}_4\text{O}_5 + \text{H}_2 \rightarrow \text{Al}_4\text{O}_4 + \text{H}_2\text{O}$	88.8	1100	$8.46(-10)(T/300)^{0.17} \exp(-10673.0/T)$	reverse capture
108	$\text{Al}_4\text{O}_4 + \text{OH} \rightarrow \text{Al}_4\text{O}_5 + \text{H}$	-145	1700	$7.57(-10)(T/300)^{0.17}$	capture
109	$\text{Al}_4\text{O}_5 + \text{H} \rightarrow \text{Al}_4\text{O}_4 + \text{OH}$	145	1700	$7.57(-10)(T/300)^{0.17} \exp(-17464.4/T)$	reverse capture
110	$\text{Al}_3\text{O}_4 + \text{AlO} + \text{M} \rightarrow \text{Al}_4\text{O}_5 + \text{M}$	-630	4500	$2.07(-33)$	capture
111	$\text{Al}_4\text{O}_5 + \text{M} \rightarrow \text{Al}_3\text{O}_4 + \text{AlO} + \text{M}$	630	4500	$1.04(-10)(T/300)^{-1.15} \exp(-75773.8/T)$	detailed balance
112	$\text{Al}_4\text{O}_5 + \text{H}_2\text{O} \rightarrow \text{Al}_4\text{O}_6 + \text{H}_2$	-172	2000	$1.56(-9)(T/300)^{0.17}$	capture
113	$\text{Al}_4\text{O}_6 + \text{H}_2 \rightarrow \text{Al}_4\text{O}_5 + \text{H}_2\text{O}$	172	2000	$3.00(-9)\exp(-21435.0/T)$	detailed balance (truncated)
114	$\text{Al}_4\text{O}_5 + \text{OH} \rightarrow \text{Al}_4\text{O}_6 + \text{H}$	-229	2500	$1.40(-9)(T/300)^{0.17}$	capture
115	$\text{Al}_4\text{O}_6 + \text{H} \rightarrow \text{Al}_4\text{O}_5 + \text{OH}$	229	2500	$1.40(-9)(T/300)^{0.17} \exp(-27501.7/T)$	reverse capture
116	$\text{AlO} + \text{AlH} \rightarrow \text{Al}_2\text{O} + \text{H}$	-267	no	$1.12(-9)(T/300)^{0.17}$	capture
117	$\text{Al}_2\text{O} + \text{H} \rightarrow \text{AlO} + \text{AlH}$	267	no	$1.59(-7)(T/300)^{0.15} \exp(-31560.5/T)$	detailed balance
118	$\text{AlO}_2 + \text{AlH} \rightarrow \text{Al}_2\text{O}_2 + \text{H}$	-353	no	$1.40(-9)(T/300)^{0.17}$	capture
119	$\text{Al}_2\text{O}_2 + \text{H} \rightarrow \text{AlO}_2 + \text{AlH}$	353	no	$1.29(-6)(T/300)^{-1.35} \exp(-42708.5/T)$	detailed balance

120	$\text{Al}_2\text{O}_2 + \text{AlH} \rightarrow \text{Al}_3\text{O}_2 + \text{H}$	-73.0	no	$1.62(-9)(T/300)^{0.17}$	capture
121	$\text{Al}_3\text{O}_2 + \text{H} \rightarrow \text{Al}_2\text{O}_2 + \text{AlH}$	73.0	no	$1.68(-10)(T/300)^{-0.87} \exp(-7920.0/T)$	detailed balance
122	$\text{Al}_2\text{O}_3 + \text{AlH} \rightarrow \text{Al}_3\text{O}_3 + \text{H}$	-317	no	$1.04(-9)(T/300)^{0.17}$	capture
123	$\text{Al}_3\text{O}_3 + \text{H} \rightarrow \text{Al}_2\text{O}_3 + \text{AlH}$	317	no	$4.67(-8)(T/300)^{-0.85} \exp(-37825.9/T)$	detailed balance
124	$\text{Al}_2\text{O}_4 + \text{AlH} \rightarrow \text{Al}_3\text{O}_4 + \text{H}$	-272	no	$8.27(-10)(T/300)^{0.17}$	capture
125	$\text{Al}_3\text{O}_4 + \text{H} \rightarrow \text{Al}_2\text{O}_4 + \text{AlH}$	272	no	$3.02(-6)(T/300)^{-0.85} \exp(-33036.1/T)$	detailed balance
126	$\text{Al}_3\text{O}_3 + \text{AlH} \rightarrow \text{Al}_4\text{O}_3 + \text{H}$	-117	no	$1.22(-9)(T/300)^{0.17}$	capture
127	$\text{Al}_4\text{O}_3 + \text{H} \rightarrow \text{Al}_3\text{O}_3 + \text{AlH}$	117	no	$2.72(-9)(T/300)^{-0.86} \exp(-13171.8/T)$	detailed balance
128	$\text{Al}_3\text{O}_4 + \text{AlH} \rightarrow \text{Al}_4\text{O}_4 + \text{H}$	-256	no	$1.44(-9)(T/300)^{0.17}$	capture
129	$\text{Al}_4\text{O}_4 + \text{H} \rightarrow \text{Al}_3\text{O}_4 + \text{AlH}$	256	no	$1.77(-10)(T/300)^{-0.84} \exp(-29882.8/T)$	detailed balance
130	$\text{Al}_3\text{O}_5 + \text{AlH} \rightarrow \text{Al}_4\text{O}_5 + \text{H}$	-387	no	$1.15(-10)(T/300)^{0.17}$	capture
131	$\text{Al}_4\text{O}_5 + \text{H} \rightarrow \text{Al}_3\text{O}_5 + \text{AlH}$	387	no	$1.44(-07)(T/300)^{-0.84} \exp(-46031.3/T)$	detailed balance
132	$\text{Al}_3\text{O}_6 + \text{AlH} \rightarrow \text{Al}_4\text{O}_6 + \text{H}$	-682	5100	$1.15(-9)(T/300)^{0.17}$	capture
133	$\text{Al}_4\text{O}_6 + \text{H} \rightarrow \text{Al}_3\text{O}_6 + \text{AlH}$	682	5100	$1.00(-10)\exp(-81959.0/T)$	capture reverse
134	$\text{Al} + \text{F} + \text{M} \rightarrow \text{AlF} + \text{M}$	-682	no	$3.32(-32)(T/300)^{-1.66}$	RRKM fit
135	$\text{AlF} + \text{M} \rightarrow \text{Al} + \text{F} + \text{M}$	682	no	$5.76(-10)\exp(-80937.0/T)$	RRKM fit
136	$\text{Al} + \text{Cl} + \text{M} \rightarrow \text{AlCl} + \text{M}$	-515	5400	$3.31(-31)(T/300)^{0.2}$	RRKM fit
137	$\text{AlCl} + \text{M} \rightarrow \text{Al} + \text{Cl} + \text{M}$	515	5400	$1.00(-9)\exp(-619370/T)$	RRKM fit
138	$\text{Al} + \text{H} + \text{M} \rightarrow \text{AlH} + \text{M}$	-301	3200	$4.42(-31)(T/300)^{0.34}$	Swihart et al. (2003)
139	$\text{AlH} + \text{M} \rightarrow \text{Al} + \text{H} + \text{M}$	301	3200	$1.00(-9)\exp(-36206/T)$	RRKM fit
140	$\text{AlCl} + \text{H} \rightarrow \text{Al} + \text{HCl}$	82.8	no	$1.00(-10)\exp(-9959/T)$	RRKM fit
141	$\text{Al} + \text{HCl} \rightarrow \text{AlCl} + \text{H}$	-82.8	no	$1.52(-10)\exp(-803/T)$	Rogowski et al. (1989)
142	$\text{AlCl} + \text{H} \rightarrow \text{AlH} + \text{Cl}$	214	no	$8.20(-12)(T/300)^{0.36} \exp(-25738.0/T)$	detailed balance
143	$\text{AlH} + \text{Cl} \rightarrow \text{AlCl} + \text{H}$	-214	no	$7.19(-12)(T/300)^{0.5}$	collision
144	$\text{F} + \text{H}_2 \rightarrow \text{HF} + \text{H}$	-133	3200	$1.10(-10)\exp(-450/T)$	NIST
145	$\text{HF} + \text{H} \rightarrow \text{F} + \text{H}_2$	133	3200	$8.37(-11)(T/300)^{0.6} \exp(-16357/T)$	NIST
146	$\text{Cl} + \text{H}_2 \rightarrow \text{HCl} + \text{H}$	4.3	400	$3.87(-12)(T/300)^{1.58} \exp(-1610/T)$	NIST
147	$\text{HCl} + \text{H} \rightarrow \text{Cl} + \text{H}_2$	-4.3	400	$2.41(-12)(T/300)^{1.44} \exp(-1240/T)$	NIST
148	$\text{Al} + \text{HF} \rightarrow \text{AlH} + \text{F}$	269	no	$6.34(-10)(T/300)^{0.57} \exp(-31487.8/T)$	detailed balance
149	$\text{AlH} + \text{F} \rightarrow \text{Al} + \text{HF}$	-269	no	$3.92(-10)(T/300)^{0.17}$	capture
150	$\text{Al} + \text{HF} \rightarrow \text{AlF} + \text{H}$	-112	no	$4.96(-10)(T/300)^{0.17}$	capture
151	$\text{AlF} + \text{H} \rightarrow \text{Al} + \text{HF}$	112	no	$1.79(-10)(T/300)^{0.02} \exp(-12303.0/T)$	detailed balance
152	$\text{AlO} + \text{Cl} \rightarrow \text{AlCl} + \text{O}$	-17.4	no	$6.79(-12)(T/300)^{0.5}$	collision
153	$\text{AlCl} + \text{O} \rightarrow \text{AlO} + \text{Cl}$	17.4	no	$3.18(-12)(T/300)^{0.48} \exp(-1833.5/T)$	detailed balance
154	$\text{AlOH} + \text{Cl} \rightarrow \text{AlCl} + \text{OH}$	38.5	no	$6.90(-9)(T/300)^{-0.79} \exp(-5043.7/T)$	detailed balance
155	$\text{AlCl} + \text{OH} \rightarrow \text{AlOH} + \text{Cl}$	-38.5	no	$6.35(-9)(T/300)^{0.17}$	capture
156	$\text{AlOH} + \text{Cl} \rightarrow \text{AlO} + \text{HCl}$	49.3	no	$4.65(-9)(T/300)^{-0.74} \exp(-6143.9/T)$	detailed balance
157	$\text{AlO} + \text{HCl} \rightarrow \text{AlOH} + \text{Cl}$	-49.3	no	$7.45(-10)(T/300)^{0.17}$	capture
158	$\text{Al}_4\text{O}_6 + \text{Al}_4\text{O}_6 \rightarrow \text{Al}_8\text{O}_{12}$	-834	no	$2.00(-9)$	coagulation
159	$\text{Al}_8\text{O}_{12} \rightarrow \text{Al}_4\text{O}_6 + \text{Al}_4\text{O}_6$	834	no	$6.5(-3)\exp(-100000./T)$	detailed balance
160	$\text{AlO} + \text{OH} \rightarrow \text{AlOH} + \text{O}$	-55.9	no	$3.0(-12)$	Estimate from trajectory calculation <sup>2</sup>
161	$\text{AlOH} + \text{O} \rightarrow \text{AlO} + \text{OH}$	55.9	no	$1.50(-11)(T/300)^{-0.97} \exp(-6861.8/T)$	detailed balance
162	$\text{Al} + \text{OH} \rightarrow \text{AlO} + \text{H}$	-71.9	no	$1.40(-10)\exp(-9640./T)$	Estimate from trajectory calculation
163	$\text{AlO} + \text{H} \rightarrow \text{Al} + \text{OH}$	71.9	no	$7.35(-11)(T/300)^{-0.14} \exp(-17406.3/T)$	RRKM fit
164	$\text{Al}_2\text{O} + h\nu_{\lambda < 213\text{nm}} \rightarrow \text{AlO} + \text{Al}$	-568	4100	$8.20(-7)\exp(0.00273T)$	(Mangan et al. 2021)
165	$\text{AlO} + h\nu_{\lambda < 252\text{nm}} \rightarrow \text{Al} + \text{O}$	-498	4900	$9.68(-7)\exp(0.00102T)$	(Mangan et al. 2021)
166	$\text{AlOH} + h\nu_{\lambda < 238\text{nm}} \rightarrow \text{AlO} + \text{H}$	-482	4900	$1.53(-4)(\exp(0.0014T))$	(Mangan et al. 2021)

**Table C.1.** Fitting parameters a,b,c,d,e for the computation of  $\Delta G_f^0(T)$  used in equilibrium calculations.

Species	a	b	c	d	e
AlO	5.99300E+04	-1.37970E+00	-1.73737E+00	3.91773E-04	-2.51845E-08
AlOH	1.18027E+05	-2.05964E+00	-8.69160E+00	5.90024E-04	-3.04906E-08
OAlOH	1.69811E+05	-3.52430E+00	-1.43673E+01	1.25241E-03	-7.34479E-08
HAlOH	1.34662E+05	-4.57736E+00	-4.35980E+00	1.66011E-03	-9.78547E-08
AlO <sub>2</sub> H <sub>2</sub>	2.08760E+05	-5.10482E+00	-1.50931E+01	1.92278E-03	-1.11241E-07
AlO <sub>2</sub>	1.07130E+05	-2.61203E+00	-8.89728E+00	7.50030E-04	-4.93246E-08
Al <sub>2</sub> O	1.28411E+05	-2.40921E+00	-1.05621E+00	6.49789E-04	-4.25611E-08
Al <sub>2</sub> O <sub>2</sub>	1.86105E+05	-3.31380E+00	-1.99433E+01	1.58854E-03	-1.06058E-07
Al <sub>2</sub> O <sub>3</sub>	2.34139E+05	-3.95729E+00	-2.82239E+01	2.11009E-03	-1.40019E-07
Al <sub>2</sub> O <sub>4</sub>	2.99834E+05	-4.30886E+00	-4.27871E+01	2.51478E-03	-1.66932E-07
Al <sub>3</sub> O <sub>3</sub>	3.08682E+05	-4.20634E+00	-3.99769E+01	2.44693E-03	-1.61816E-07
Al <sub>3</sub> O <sub>4</sub>	3.69176E+05	-5.56346E+00	-5.20710E+01	3.40984E-03	-2.28468E-07
Al <sub>4</sub> O <sub>4</sub>	4.36049E+05	-4.97450E+00	-6.35660E+01	3.25225E-03	-2.14255E-07
Al <sub>4</sub> O <sub>5</sub>	5.05163E+05	-6.53645E+00	-7.58250E+01	4.29600E-03	-2.85685E-07
Al <sub>4</sub> O <sub>6</sub>	5.84165E+05	-8.82800E+00	-8.42817E+01	5.63090E-03	-3.74702E-07
AlSiO <sub>3</sub>	2.67025E+05	-4.60668E+00	-2.59855E+01	2.35614E-03	-1.54434E-07
HAlSiO <sub>3</sub>	2.75443E+05	-1.00583E+01	-7.73659E+00	6.85770E-03	-8.55013E-07
Al <sub>2</sub> O <sub>3</sub> SiO	3.80803E+05	-9.85869E+00	-2.79375E+01	9.23519E-03	-1.18218E-06
AlF	8.19970E+04	-1.31009E+00	-2.43594E+00	4.12562E-04	-3.03640E-08
AlCl	6.20720E+04	-9.43409E-01	-4.14453E+00	2.44222E-04	-1.87099E-08
Al <sub>2</sub>	1.09331E+04	-7.39015E-01	-5.44758E+00	1.35276E-04	-1.05029E-08
AlH	3.62088E+04	-1.68480E+00	1.07115E+00	4.62791E-04	-2.91925E-08
Al <sub>3</sub> O <sub>5</sub>	4.22272E+05	-6.47463E+00	-6.17019E+01	4.04645E-03	-2.69671E-07
Al <sub>3</sub> O <sub>6</sub>	4.65554E+05	-5.97223E+00	-7.53001E+01	3.95721E-03	-2.60889E-07
Al <sub>4</sub> O <sub>3</sub>	3.59063E+05	-3.47918E+00	-5.55172E+01	2.29771E-03	-1.50608E-07
Al <sub>3</sub> O <sub>2</sub>	2.31189E+05	-2.55393E+00	-3.25776E+01	1.41095E-03	-9.25233E-08
Al <sub>8</sub> O <sub>12</sub>	1.16740E+06	-1.57762E+01	-1.99568E+02	1.12760E-02	-7.49050E-07
Al <sub>10</sub> O <sub>15</sub>	1.48494E+06	-1.94214E+01	-2.57899E+02	1.41833E-02	-9.41950E-07
Al <sub>12</sub> O <sub>18</sub> (B)	1.80753E+06	-2.37453E+01	-3.17029E+02	1.75208E-02	-1.16698E-06
Al <sub>14</sub> O <sub>21</sub>	2.12820E+06	-2.75694E+01	-3.73757E+02	2.04905E-02	-1.36329E-06
Al <sub>16</sub> O <sub>24</sub>	2.44711E+06	-3.23660E+01	-4.30676E+02	2.40589E-02	-1.60386E-06
Al <sub>18</sub> O <sub>27</sub>	2.78902E+06	-3.60898E+01	-4.90629E+02	2.70106E-02	-1.79999E-06
Al <sub>20</sub> O <sub>30</sub>	3.10574E+06	-4.06379E+01	-5.45915E+02	3.04129E-02	-2.02790E-06

**Table D.1.** Cluster volumes (in  $\text{\AA}^3$ ) and radii (in  $\text{\AA}$ ) as derived from atomic core coordinates (Delauney triangulation) and from van-der-Waals interactions. The radii are calculated assuming spherical volumes.

$n$	Delauney		van der Waals	
	Volume	Radius	Volume	Radius
1	0.0	0.0	73.73	2.60
2	13.88	1.49	128.59	3.13
3	29.63	1.92	185.49	3.54
4	51.16	2.30	249.61	3.91
5	80.00	2.67	308.37	4.19
6	102.00	2.90	358.37	4.41
7	142.74	3.24	347.3	4.36
8	153.15	3.32	473.94	4.84
9	154.17	3.33	520.73	4.99
10	226.18	3.78	594.16	5.22