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HYBRID PRECODING FOR PARTIAL-FULL MIXED CONNECTION MMWAVE MIMO

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ABSTRACT

We propose hybrid precoding algorithm for less-investigated partial-full mixed connection (PFMC) architecture in millimeter wave (mmWave) multiple input multiple output (MIMO). The RF chains and the antennas are divided into various subgroups with full connection existing between the RF chains and the antennas of a particular subgroup. We cast the hybrid precoding problem for the PFMC architecture as a matrix factorization problem and propose an algorithm based on alternating minimization with a view to minimize the Euclidean distance between the hybrid precoding sub-matrix for each subgroup and the corresponding sub-matrix of the fully digital optimal precoder. The proposed precoder is able to produce energy efficiency better than the precoders for FC and PC architectures and the existing precoder for the PFMC architecture.

1 Introduction

The conventional digital precoding is replaced by hybrid precoding in millimeter wave (mmWave) multiple input multiple output (MIMO) systems in a bid to reduce the number of radio frequency (RF) chains which are costly and power consuming. The lower dimensional digital precoding achieved using a small number of RF chains which is followed by analog beamforming usually accomplished by phase shifters, constitute hybrid precoding. There are mainly two kinds of hybrid architectures, partially connected (PC) and fully connected (FC) architectures.

In the FC architecture, each RF chain is connected to all the antennas through phase shifters and the signals output from different RF chains are summed just before feeding the antenna. In the PC architecture, the antennas are divided into subgroups equal to the number of RF chains and each RF chain is connected to all the antennas of only one of the subgroups. Most of the works [1–5] on hybrid precoder consider FC architecture. The advantage of FC architecture is that it provides very good performance even with a smaller number of RF chains compared to the number of antennas. In fact, [2] shows that the minimum number of RF chains required in FC architecture to realize the full digital precoder is twice the number of transmitted data streams.

The FC architecture comes with the added complexities introduced by the complex network of the phase shifters. The PC architecture, on the other hand, has reduced complexity but offers reduced beamforming gain and as a result, provides limited performance. The power consumption in the PC architecture is also lower than the FC architecture as the number of phase shifters is lesser by a factor of number of RF chains. In [6], an iterative procedure built on the concept of successive interference cancelation (SIC) is proposed for PC MIMO for a case where number of RF chains is equal to data streams. [7] proposes an alternating minimization algorithm based on semi-definite programming (SDP) as solution to hybrid precoding problem for PC MIMO.

Apart from FC and PC architectures, a new hybrid architecture was proposed in [8] that combines both FC and PC structures which we call partial-full mixed connection (PFMC). In the PFMC architecture, the RF chains and the antennas are organized into different subgroups. All the RF chains of each subgroup are connected to all the antennas of that particular subgroup through phase shifters. This architecture is 'partially-connected' in the sense that each RF chain is connected to only a subset of antennas, and 'fully-connected' in regard to the connection of RF chains of a particular subgroup to the antennas. In general, the PFMC architecture lies in between the FC architecture and the PC architecture both in terms of spectral performance and complexity.

Two hybrid precoders are presented in [8] for the PFMC architecture, one based on SIC and another based on matrix factorization. In this paper, we model the hybrid precoding for PFMC architecture as matrix factorization problem for individual subgroups and propose a hybrid precoder based on alternating minimization approach. The proposed hybrid precoder for the PFMC structure not only produces better energy efficiency and spectral performance than the SIC based precoder in [8], but also exhibits better energy efficiency compared to the precoders for FC and PC architectures.

Apart from the usual notations, $\mathbf{X}^{(i,j)}$ represents the

$(i,j)^{th}$ element of matrix $\mathbf{X}$; $\exp(\mathbf{X})$ is a matrix whose $(i,j)^{th}$ entry is $\exp\left(\mathbf{X}^{(i,j)}\right)$, where $\exp(.)$ is the exponential operator; $\mathbf{X}^{\dagger}$ is the pseudoinverse of matrix $\mathbf{X}$.

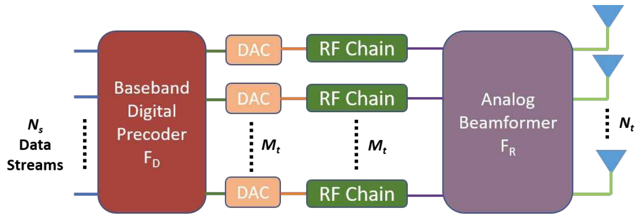

Fig. 1: System diagram showing transmitter side of mmWave single user MIMO system with hybrid precoding.

2 System Model

We consider a single user mmWave MIMO downlink system with the transmitter connected to N_t transmit antennas and the receiver connected to N_r receive antennas. The receiver side is similar to the transmitter shown in Fig. 1 with everything in reverse direction. The transmitter and the receiver consist of M_t and M_r RF chains respectively. The transmitter is able to transmit N_s data streams such that $N_s \leq M_t \leq N_t$ and $N_s \leq M_r \leq N_r$. We consider PFMC architecture at both the transmitter and the receiver. At the transmitter, as shown in Fig. 2, the RF chains and antennas are divided into S different sub-groups with each subgroup having C RF chains and N antennas. Thus, $N_t = SN$ and $M_t = SC$. We assume that $C \leq N_s$, which is realistic because we do not want high value of C which would mean high number of RF chains.

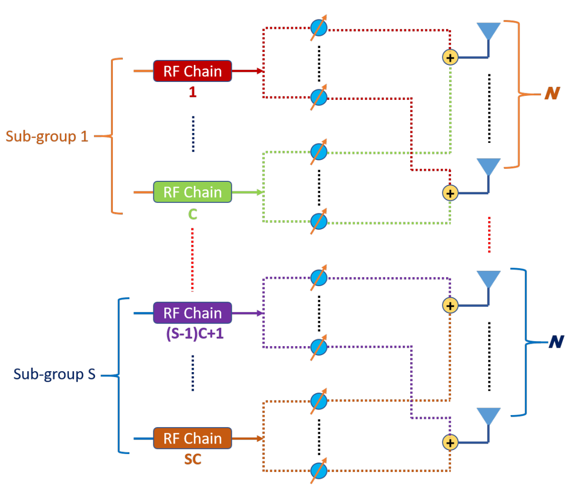

Fig. 2: System diagram showing partial-full mixed connection.

The transmit signal $\mathbf{s} \in \mathbb{C}^{N_s}$ is processed by the hybrid precoder $\mathbf{F} = \mathbf{F}_{\mathrm{R}}\mathbf{F}_{\mathrm{D}}$, where $\mathbf{F}_{\mathrm{R}} \in \mathbb{C}^{N_t \times M_t}$ is the analog precoder and $\mathbf{F}_{\mathrm{D}} \in \mathbb{C}^{M_t \times N_s}$ is the digital precoder. Similarly, the received signal at the receiver goes the hybrid combiner $\mathbf{W} = \mathbf{W}_{\mathrm{R}}\mathbf{W}_{\mathrm{D}}$, where the analog combiner $\mathbf{W}_{\mathrm{R}}$ is a $N_r \times M_r$ matrix and the digital combiner $\mathbf{W}_{\mathrm{D}}$ is a $M_r \times N_s$ matrix. We assume $\mathbb{E}\left[ss^H\right] = \frac{P}{N_s}\mathbf{I}_{N_s}$, where P is the total transmit power. The hybrid precoder $\mathbf{F}$ needs to satisfy the total power constraint. If we assume narrow-band block-fading channel model, the received signal, after combining, is given by

$$\mathbf{y} = \mathbf{W}_{\mathrm{D}}^H\mathbf{W}_{\mathrm{R}}^H\mathbf{H}\mathbf{F}_{\mathrm{R}}\mathbf{F}_{\mathrm{D}}\mathbf{s} + \mathbf{W}_{\mathrm{D}}^H\mathbf{W}_{\mathrm{R}}^H\mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is the channel from the transmitter to the receiver and $\mathbf{n} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}_{N_r}\right)$ is the $N_r \times 1$ complex noise vector. The mmWave channel $\mathbf{H}$ is modeled using the clustered channel model based on extended Saleh Valenzuela model [1] and given by

$$\mathbf{H} = \sqrt{\frac{N_t N_r}{N_c N_p}} \sum_{i=1}^{N_c}\sum_{\ell=1}^{N_p} \alpha_{i\ell}\mathbf{a}_r(\theta_{i\ell})\mathbf{a}_t{}^H(\phi_{i\ell}), \tag{2}$$

where N_c represents the number of clusters, N_p is the number of paths in each cluster and $\alpha_{i\ell}$ is the complex gain. $\mathbf{a}_r(\theta_{i\ell})$ and $\mathbf{a}_t(\phi_{i\ell})$ represent respectively the antenna array response vectors of the receiver and the transmitter, where $\theta_{i\ell}$ and $\phi_{i\ell}$ are the azimuth angles of arrival (AoAs) and the azimuth angles of departure (AoDs) respectively.

3 Problem Formulation

In this section, we will only model the hybrid precoder for the PFMC architecture. All the arguments about the hybrid precoder holds true for the hybrid combiner except the total power constraint. The analog precoding matrix $\mathbf{F}_{\mathrm{R}}$ has a block diagonal structure as

$$\mathbf{F}_{\mathrm{R}} = \begin{bmatrix} \mathbf{F}_{\mathrm{R}_1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\mathrm{R}_2} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{F}_{\mathrm{R}_S} \end{bmatrix}, \tag{3}$$

where $\mathbf{F}_{\mathrm{R}_i} \in \mathbb{C}^{N \times C}$ is analog precoding sub-matrix corresponding to the i^{th} sub-group, $1 \leq i \leq S$. The analog beamformer and analog combiner are constructed using the network of phase shifters which, as a result, forces the non-zero element of $\mathbf{F}_{\mathrm{R}}$ to have a constant unit amplitude, *i.e.*, $|\mathbf{F}_{\mathrm{R}_i}^{(m,n)}| = 1$ for all i. The optimal hybrid precoder can be constructed by minimizing the Euclidean distance between the optimal fully digital precoder and the hybrid precoder [1]. Hence, the hybrid precoding design problem can be stated as

$$\begin{aligned} (\mathbf{F}_{\mathrm{R}}^\star, \mathbf{F}_{\mathrm{D}}^\star) = \underset{\mathbf{F}_{\mathrm{R}},\mathbf{F}_{\mathrm{D}}}{\arg\min} \quad & \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\mathrm{R}}\mathbf{F}_{\mathrm{D}}\|_F^2 \\ \text{s.t.} \quad & \|\mathbf{F}_{\mathrm{R}}\mathbf{F}_{\mathrm{D}}\|_F^2 = N_s, \\ & \mathbf{F}_{\mathrm{R}} = \mathit{blkdiag}\left(\mathbf{F}_{\mathrm{R}_1}, \dots, \mathbf{F}_{\mathrm{R}_S}\right) \\ & |\mathbf{F}_{\mathrm{R}_i}^{(m,n)}| = 1, \quad 1 \leq i \leq S, \quad \forall m,n, \end{aligned} \tag{4}$$

where the optimal fully digital precoder $\mathbf{F}_{\text{opt}}$ is the matrix containing the leading N_s right singular vectors of the channel matrix and the optimal combining matrix contains the leading N_s left singular vectors. The objective function of (4) can be written as

$$\begin{aligned} \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}}\|_F^2 &= \sum_{i=1}^{S} \left\|\mathbf{F}_{\text{opt}_i} - (\mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}})_i\right\|_F^2 \\ &= \sum_{i=1}^{S} \left\|\mathbf{F}_{\text{opt}_i} - \mathbf{F}_{\text{R}_i}\mathbf{F}_{\text{D}_i}\right\|_F^2, \end{aligned} \tag{5}$$

where $\mathbf{F}_{\text{opt}_i}$ and $(\mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}})_i$ are the i^{th} sub-matrices corresponding to $(i-1)N+1$ to iN rows and all the columns of $\mathbf{F}_{\text{opt}}$ and $\mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}}$ respectively, whereas $\mathbf{F}_{\text{D}_i}$ corresponds to $(i-1)C+1$ to iC rows and all the columns of $\mathbf{F}_{\text{D}}$. Thus, the hybrid precoding problem can be restated as

$$\begin{aligned} (\mathbf{F}_{\text{R}}^{\star}, \mathbf{F}_{\text{D}}^{\star}) = &\arg\min_{\mathbf{F}_{\text{R}_i}, \mathbf{F}_{\text{D}_i}} \sum_{i=1}^{S} \left\|\mathbf{F}_{\text{opt}_i} - \mathbf{F}_{\text{R}_i}\mathbf{F}_{\text{D}_i}\right\|_F^2 \\ \text{s.t.} \quad & \|\mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}}\|_F^2 = N_s, \\ & \mathbf{F}_{\text{R}} = \mathit{blkdiag}\left(\mathbf{F}_{\text{R}_1}, \ldots, \mathbf{F}_{\text{R}_S}\right) \\ & |\mathbf{F}_{\text{R}_i}^{(m,n)}| = 1, \quad 1 \le i \le S, \quad \forall m, n. \end{aligned} \tag{6}$$

4 Proposed Method based on Alternating Minimization

We develop a simple algorithm based on alternating minimization to determine the hybrid precoder. In the first stage, we solve the problem,

$$\begin{aligned} \mathbf{F}_{\text{R}_i}^{\star} = &\arg\min_{\mathbf{F}_{\text{R}_i}} \left\|\mathbf{F}_{\text{opt}_i} - \mathbf{F}_{\text{R}_i}\mathbf{F}_{\text{D}_i}\right\|_F^2 \\ \text{s.t.} \quad & \left|\mathbf{F}_{\text{R}_i}^{(m,n)}\right| = 1, \quad 1 \le i \le S, \quad \forall m, n. \end{aligned} \tag{7}$$

After we determine all the optimal $\mathbf{F}_{\text{R}_i}, 1 \le i \le S$, we can construct $\mathbf{F}_{\text{R}}$ as a block-diagonal matrix. Then, we determine $\mathbf{F}_{\text{D}}$ by solving (4) with $\mathbf{F}_{\text{R}}$ known. When $\mathbf{F}_{\text{R}}$ is known, the solution of (4) for $\mathbf{F}_{\text{D}}$ is the least squares solution multiplied by the normalization factor to satisfy power constraint,

$$\mathbf{F}_{\text{D}} = \frac{\sqrt{N_s}}{\left\|\mathbf{F}_{\text{R}}\mathbf{F}_{\text{R}}^{\dagger}\mathbf{F}_{\text{opt}}\right\|_F}\mathbf{F}_{\text{R}}^{\dagger}\mathbf{F}_{\text{opt}}. \tag{8}$$

To solve (7), we follow alternating minimization approach. We start with a randomly initialized value of $\mathbf{F}_{\text{R}_i}$. At each iteration, we solve the objective function of (7) for $\mathbf{F}_{\text{D}_i}$ with known $\mathbf{F}_{\text{R}_i}$, which is $\mathbf{F}_{\text{D}_i} = \mathbf{F}_{\text{R}_i}^{\dagger}\mathbf{F}_{\text{opt}_i}$.

Algorithm 1 Proposed AltMin Hybrid Precoding Algorithm Using Alternating Minimization Method

Require: $\mathbf{F}_{\text{opt}}, N_t, C, S$.

Compute $N = \frac{N_t}{S}$, set $i = 1$.
repeat
  Set $m = (i-1)N + 1, n = iN$.
  Set $\mathbf{F}_{\text{opt}_i} = \mathbf{F}_{\text{opt}}(m:n, :)$.
  Initialize $\mathbf{F}_{\text{R}_i}^{(0)} = \exp(j\mathbf{\Theta})$, where $\mathbf{\Theta}$ is $N \times C$ matrix and $\mathbf{\Theta}^{(i,j)}$ are random phase angles, and set $k = 1$.
  repeat
    Compute $\mathbf{F}_{\text{D}_i}^{(k)} = \mathbf{F}_{\text{R}_i}^{(k-1)^{\dagger}}\mathbf{F}_{\text{opt}_i}$.
    Compute $\mathbf{F}_{\text{D}_i}^{(k)^{-}} = \mathbf{F}_{\text{D}_i}^{(k)^{H}}\left(\mathbf{F}_{\text{D}_i}^{(k)}\mathbf{F}_{\text{D}_i}^{(k)^{H}} + \epsilon\mathbf{I}\right)^{-1}$, where $\epsilon \to 0$.
    Compute $\mathbf{F}_{\text{R}}^{(k)} = \exp\left(j\angle\left(\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{D}}^{(k)^{-}}\right)\right)$.
    $k \leftarrow k + 1$.
  until $\left|\text{e}_i^k - \text{e}_i^{k-1}\right| < tol$, where $\text{e}_i^k = \left\|\mathbf{F}_{\text{opt}_i} - \mathbf{F}_{\text{R}_i}^{(k)}\mathbf{F}_{\text{D}_i}^{(k)}\right\|_F^2$ and $tol \to 0$, or $k \ge max$, the maximum number of iterations.
  Set $\mathbf{F}_{\text{R}_i} = \mathbf{F}_{\text{R}_i}^{(k)}, \mathbf{F}_{\text{D}_i} = \mathbf{F}_{\text{D}_i}^{(k)}$.
  $i \leftarrow i + 1$.
until $i \ge S$.
Set $\mathbf{F}_{\text{R}} = \mathit{blkdiag}\left(\mathbf{F}_{\text{R}_1}, \ldots, \mathbf{F}_{\text{R}_S}\right)$.
Calculate $\mathbf{F}_{\text{D}} = \frac{\sqrt{N_s}}{\left\|\mathbf{F}_{\text{R}}\tilde{\mathbf{F}}_{\text{D}}\right\|_F}\tilde{\mathbf{F}}_{\text{D}}$, where $\tilde{\mathbf{F}}_{\text{D}} = \mathbf{F}_{\text{R}}^{\dagger}\mathbf{F}_{\text{opt}}$ or $\tilde{\mathbf{F}}_{\text{D}} = \left[\mathbf{F}_{\text{D}_1}^T, \mathbf{F}_{\text{D}_2}^T, \ldots, \mathbf{F}_{\text{D}_S}^T\right]^T$.
return $\mathbf{F} = \mathbf{F}_{\text{R}}\mathbf{F}_{\text{D}}$.

We determine $\mathbf{F}_{\text{R}_i}$ with known $\mathbf{F}_{\text{D}_i}$ next. We solve (7) for unconstrained $\mathbf{F}_{\text{R}_i}$ (without modulus constraint) and approximate $\mathbf{F}_{\text{R}}$ by extracting the phase values as,

$$\mathbf{F}_{\text{R}_i} = \exp\left(j\angle\left(\mathbf{F}_{\text{opt}_i}\mathbf{F}_{\text{D}_i}^{-}\right)\right), \tag{9}$$

where $\mathbf{F}_{\text{D}_i}^{-} = \mathbf{F}_{\text{D}_i}^{H}\left(\mathbf{F}_{\text{D}_i}\mathbf{F}_{\text{D}_i}^{H} + \epsilon\mathbf{I}\right)^{-1}, \epsilon \to 0$ is the right inverse of $\mathbf{F}_{\text{D}_i}$ and $\mathbf{F}_{\text{opt}_i}\mathbf{F}_{\text{D}_i}^{-}$ is the solution of (7) for unconstrained $\mathbf{F}_{\text{R}_i}$. We repeat this procedure of minimizing (7) alternately for $\mathbf{F}_{\text{D}_i}$ and $\mathbf{F}_{\text{R}_i}$ until the convergence is achieved. After we obtain $\mathbf{F}_{\text{R}}$ by solving for each $\mathbf{F}_{\text{R}_i}$, we determine $\mathbf{F}_{\text{D}}$ using (8). Alternatively, $\mathbf{F}_{\text{D}}$ can be determined as $\mathbf{F}_{\text{D}} = \frac{\sqrt{N_s}}{\left\|\mathbf{F}_{\text{R}}\tilde{\mathbf{F}}_{\text{D}}\right\|_F}\tilde{\mathbf{F}}_{\text{D}}$, where $\tilde{\mathbf{F}}_{\text{D}} = \left[\mathbf{F}_{\text{D}_1}^T, \mathbf{F}_{\text{D}_2}^T, \ldots, \mathbf{F}_{\text{D}_S}^T\right]^T$. The proposed method is summarized in Algorithm 1. The proposed algorithm is similar, in spirit, to [9] which is used to solve phase recovery problems in which signals are retrieved from amplitude measurements. A similar method was proposed assuming unitary digital precoder for FC architecture in [10]. The complexity of the proposed method is $SN_{iter}^{i}\left(\mathcal{O}\left(NC^2\right) + C^3 + C^2N + 2C^2N_s + \mathcal{O}\left(C^3\right)\right)$, where N_{iter}^{i} is the number of iterations required for the i^{th} sub-group. The value of max is set to 50 in Algorithm 1 for simulations.

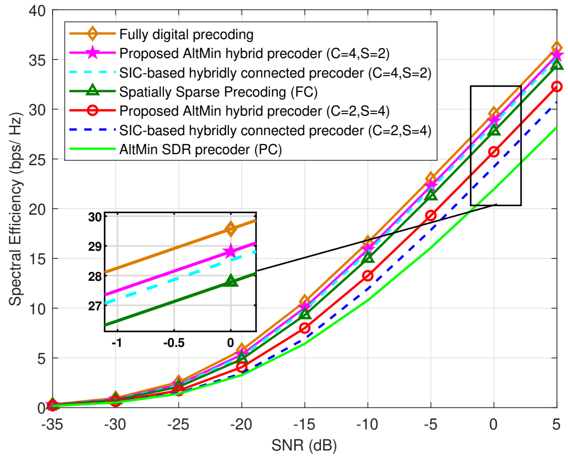

Fig. 3: Spectral Efficiency vs. SNR, $N_s = 4$, $M_t = 8$.

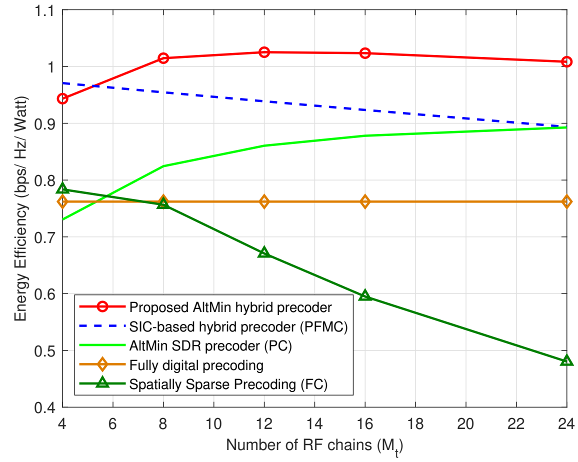

Fig. 4: Energy Efficiency vs. the number of RF chains, $N_s =$ 4, C = 2, *SNR* = 0 dB.

5 Energy Efficiency

The energy efficiency is defined as the ratio of spectral efficiency to the total power consumed [7],

$$\eta = \frac{Rate}{P_{com} + M_t P_{RF} + N_t P_{PA} + N_{PS} P_{PS}} \text{ bits/ Hz/ Joules,} \tag{10}$$

where P_{com} is the common power of transmitter, N_{PS} is the number of phase-shifters, whereas P_{RF}, P_{PA} and P_{PS} respectively represent the power of each RF chain, power amplifier and phase shifter. The number of phase-shifters in FC, PC and PFMC architectures are $N_t M_t$, N_t and $N_t C$ respectively.

6 Simulation Results

We consider both the transmitter and the receiver have ULAs with $N_t = 144$ and $N_r = 36$. The antenna elements are separated by a distance of half wavelength. The channel parameters $N_c = 5$, $N_p = 10$, $\alpha_{i\ell} \sim \mathcal{CN}(0, 1)$. The azimuth AoDs and AoAs are Laplacian distributed with mean angles uniformly distributed over $[0, 2\pi]$ and having angular spread of 7.5 degrees. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \frac{P}{\sigma^2}$. We take P_{com} = 10 W, P_{RF} = 100 mW, P_{PA} = 100 mW, P_{PS}= 10 mW [11]. We compare the performance of the proposed AltMin hybrid precoder against the fully digital precoder, spatially sparse precoder [1] which is a FC precoder, SIC-based hybridly connected precoder [8] and AltMin SDR precoder [7] which is a PC precoder. To make comparison fair, we have only considered precoding at the transmitter side for all the precoders.

The Fig. 3 shows that the spectral efficiency of the proposed precoder for $C = 2$, $S = 4$ is better than AltMin SDR precoder and lower than the spatially sparse precoder. However, when C is increased to 4 and S is decreased to 2, keeping M_t constant, the performance of the proposed hybrid precoder gets even better than the spatially sparse precoder and comes closer to the performance of fully digital precoder. In both these configurations, the proposed precoder's performance is superior to SIC-based hybridly connected precoder. Increasing C, keeping M_t constant will raise the spectral performance because the number of phase shifters also increases which adds to the beamforming gain.

The Fig. 4 that compares energy efficiency of various precoders shows that the proposed precoder exhibits the best energy efficiency among all the compared precoders. As M_t is increased, the energy efficiency of the proposed precoder increases before saturating at a point and finally decreasing. At the beginning, the increase in rate because of increasing M_t exceeds the increase in power consumption of the RF chains. But the rate can not increase beyond a point even after increasing M_t but the power consumption still increases, thereby finally causing a decline in the energy efficiency.

7 Conclusion

We establish the hybrid precoding for the PFMC architecture as a matrix factorization problem of all the sub-matrices corresponding to each subgroup of RF chains and antennas, and present an alternating minimization based hybrid precoder. The proposed precoder performs better than the existing precoder in terms of both the spectral efficiency and energy efficiency. With a properly chosen number of subgroups and the number of RF chains in each subgroup, we can reap the benefit of performance of the FC architecture, while also lowering complexity and the power consumption, an advantage of the PC structure. Thus, the PFMC architecture which provides the middle ground might be a better way forward for the mmWave MIMO architecture.

8 References

[1] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 13, pp. 1499–1513, March 2014.

[2] F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, pp. 501–513, April 2016.

[3] W. Ni and X. Dong, "Hybrid block diagonalization for massive multiuser MIMO systems," *IEEE Transactions on Communications*, vol. 64, pp. 201–211, Jan 2016.

[4] P. R. Gautam and L. Zhang, "Hybrid SLNR precoding for multi-user millimeter wave MIMO systems," in *2019 22nd International Symposium on Wireless Personal Multimedia Communications (WPMC)*, pp. 1–6, 2019.

[5] T. Lin, J. Cong, Y. Zhu, J. Zhang, and K. Ben Letaief, "Hybrid beamforming for millimeter wave systems using the MMSE criterion," *IEEE Transactions on Communications*, vol. 67, no. 5, pp. 3693–3708, 2019.

[6] L. Dai, X. Gao, J. Quan, S. Han, and C. I, "Near-optimal hybrid analog and digital precoding for downlink mmwave massive MIMO systems," in *2015 IEEE International Conference on Communications (ICC)*, pp. 1334–1339, 2015.

[7] X. Yu, J. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, pp. 485–500, April 2016.

[8] D. Zhang, Y. Wang, X. Li, and W. Xiang, "Hybridly connected structure for hybrid beamforming in mmwave massive MIMO systems," *IEEE Transactions on Communications*, vol. 66, no. 2, pp. 662–674, 2018.

[9] P. Netrapalli, P. Jain, and S. Sanghavi, "Phase retrieval using alternating minimization," *IEEE Transactions on Signal Processing*, vol. 63, no. 18, pp. 4814–4826, 2015.

[10] C. Rusu, R. Mèndez-Rial, N. González-Prelcic, and R. W. Heath, "Low complexity hybrid precoding strategies for millimeter wave communication systems," *IEEE Transactions on Wireless Communications*, vol. 15, no. 12, pp. 8380–8393, 2016.

[11] T. Rappaport, R. Heath, R. Daniels, and J. Murdock, *Millimeter wave wireless communications*. Prentice Hall, 2015.