Estimation: An inadequately operationalised national curriculum competence

Judy Sayers[1], Jöran Petersson[2], Eva Rosenqvist[3] and Paul Andrews[3]

[1]Leeds University, UK, [2]Malmö University, Sweden, [3]Stockholm University, Sweden

Research has highlighted the importance of estimation, in various forms, as both an essential life-skill and a significant underpinning of other forms of mathematical learning. It has also highlighted a lack of opportunities for learners to acquire estimational competence. In this paper, we present a review of the literature that identified four forms of estimation. These are measurement, computational, quantity (or numerosity) and number line estimation. In addition to summarising the characteristics and significance of each form of estimation, we examine critically the estimation-related expectations of the English national curriculum for primary mathematics to highlight a problematic lack of opportunity.

Keywords: Estimation; primary mathematics national curriculum.

Introduction

The ability to estimate is widely recognised not only as a core skill of everyday life but also a determinant of later arithmetical competence, particularly in respect of novel situations (Booth & Siegler, 2008). With respect to small quantities, as shown in Riess' (1943) review, estimation has long been recognised as a competence exhibited by various animals, although it is only humans that can manage large quantities. However, estimational competence of this latter form, which does not develop by chance, requires intervention (Joram, Gabriele, Bertheau, Gelman & Subrahmanyam, 2005; Peeters, Degrande, Ebersbach, Verschaffel & Luwel, 2016). In this regard, curricula internationally, while emphasising the importance of estimation, rarely engage their readers, typically teachers and researchers, in the warrant for its importance or discuss its forms and functions. Furthermore, the teaching of estimation has been a neglected skill, with many teachers having inadequate pedagogical conceptions of the topic (Alajmi, 2009; Subramaniam, 2014) and textbooks colluding in the omission (Hong, Choi, Runnalls & Hwang, 2018).

A brief summary of the literature

For many years, scholars recognised three forms of estimation, as highlighted in Sowder's (1992) well-known review chapter. These were *measurement estimation, computational estimation* and *quantity (including numerosity) estimation*. However, initial reading of recent literature identified a fourth category, *number line estimation*. Our review, therefore, focuses on these four categories. The figures of table 1 show the number of hits yielded by Google Scholar searches on the phrases, *computational estimation*, *measurement estimation*, *number line estimation* or *quantity (including numerosity) estimation* anywhere in their texts. Although there is variation across the different components, the total number of estimation-related articles has roughly trebled every decade, highlighting an exponential growth in the field. That said, fewer than a tenth of all hits were of relevance to our analysis, with the others, particularly with respect to computational, quantity and measurement estimation, drawing on estimation as a process in different STEM fields, the social sciences and health care. Moreover, the figures confirm that at the time of Sowder's review, number line estimation was effectively unknown. In the following, we summarise the literature on each of the four forms of estimation before evaluating the expectations, both statutory and non-statutory, of the English national curriculum.

	1970-79	1980-89	1990-99	2000-09	2010 -	
Computational	14	154	385	1230	2890	4773
Measurement	139	329	930	2420	7700	11518
Number line	2	3	2	109	1670	1786
Quantity	52	148	380	1578	4861	7019
	206	634	1697	5337	17121	24995

Table 1: Distribution of estimation articles over the last half-century

Measurement estimation

While measurement estimation, typically undertaken in "mathematical everyday situations in which precise calculation or measurement are contextually defined as either impossible or unnecessary" (Forrester & Pike, 1998, p.334), is an important life skill, it remains a neglected research field (Joram, Subrahmanyam & Gelman, 1998). Based on the use of reference or anchor points, it is an everyday tool of professional users of mathematics (Jones & Taylor, 2009) It is known that children who employ such reference points to their estimates are more accurate than those who do not (Joram et al., 2005) and that context familiarity improves estimates (Jones, Gardner, Taylor, Forrester & Andre, 2012). However, many teachers are uncertain how to address the teaching of measurement estimation (Joram et al., 2005), a problem exacerbated by the inadequacy of textbooks. Importantly, measurement estimation, in various forms, is linked with mathematics achievement more generally (Kramer, Bressan and Grassi (2018).

Computational estimation

The common perception, when the topic is introduced, is that estimation is an aid to computation and little else. Indeed, Sowder's (1992) well-known chapter, despite allusions to both measurement estimation and quantity estimation, focuses almost exclusively on the forms and functions of computational estimation. It has been defined as "the process of simplifying an arithmetic problem using some set of rules or procedures to produce an approximate but satisfactory answer through mental calculation" (Ainsworth, Bibby & Wood, 2002, p.28). Being able to undertake computational estimation is an essential life skill (Ganor-Stern, 2016) and is, despite teacher scepticism as to its relevance (Alajmi, 2009), an important aid to children's understanding of both place value and standard algorithms (Sowder, 1992). It is a skill dependent on both the maturity of the estimator (Ganor-Stern, 2016) and the complexity of the task (Dowker, 1997). That said, it is an under-investigated area (Lemaire & Lecacheur, 2011), not least because adults' computational estimation competence is generally poor (Anestakis & Lemonidis, 2014) and influenced by age and mathematics anxiety (Si, Li, Sun, Xu & Sun, 2016) as well as cultural context (Imbo & LeFevre, 2011).

Quantity estimation

Quantity (or numerosity) estimation, the ability to discern the number of objects in a set without recourse to counting (Crites, 1992), is reciprocally dependent on the ability to count (Barth, Starr & Sullivan, 2009) and a skill that diminishes in accuracy as the numerosity of the set of objects grows (Smets, Sasanguie, Szücs & Reynvoet, 2015). Its development is a complex interplay of set size and maturity that involves frequent shifts between logarithmic and linear conceptions of number (Sullivan & Barner, 2013). Groups typically estimate quantities better than individuals (Bonner, Sillito, & Baumann, 2007; Laughlin, Gonzalez & Sommer, 2003) and that quantity estimation is negatively associated with sensation seeking (Ginsburg, 1996). In recent years, due to their being able to facilitate the estimation of very large numbers, scholars have turned attention to the didactical use of Fermi problems across all school years (Albarracín & Gorgorió, 2019).

Number line estimation

While computational, measurement and quantity estimation can be construed as important outcomes of the learning process, number line estimation occupies a different position. As indicated in Table 1, it is a relative newcomer to the field and is principally the domain of psychologists concerned with the development of mathematical thinking more generally and the identification of learning difficulties. Broadly speaking, although there are dissenters (Ebersbach, Luwel, Frick, Onghena & Verschaffel, 2008), number line estimation competence develops with age, with young children tending to construe smaller numbers as being more widely spaced than larger numbers, creating a logarithmic pattern (Ashcraft & Moore, 2012). Successful number line estimation, which draws on a child's developing ability to exploit reference points (Sullivan & Barner, 2014), is a strong predictor of both mathematical learning difficulties (Siegler & Opfer, 2003) and mathematical achievement (Schneider, Grabner & Paetsch, 2009), particularly after focused interventions (Maertens, De Smedt, Sasanguie, Elen & Reynvoet, 2016). Indeed, instruction is important if young children's logarithmic estimations of quantity are to be replaced by linear (Siegler & Opfer, 2003).

Estimation and the English national curriculum for primary mathematics

The English national curriculum (DfE, 2013) for primary mathematics includes fifteen references to the word *estimate* and its derivatives, thirteen of which are found in the statutory requirements and two in the non-statutory. There are also ten references to *round* in the statutory requirements and five in the non-statutory, as well as three references to *approximate* and its derivatives, one in the statutory and two in the non-statutory materials. However, the clarity of these objectives varies considerably.

The situation with respect to measurement estimation is unambiguous. Year-two children should be able to "choose and use appropriate standard units to estimate and measure length… mass… temperature (and) capacity… to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels" (DfE, 2013, p14), an expectation that includes time in year three, area in year five and volume in year six. These invocations seem clear and reflective of some conception of cognitive development. However, the use of non-standard units, which research has shown to be a powerful underpinning of conceptual and procedural competence, (Chang, Males, Mosier & Gonulates, 2011), is missing from all references to estimation.

The situation concerning computational estimation is not unambiguous. In years three and four children should be able to "estimate the answer to a calculation" and, by year six, children should be able to "use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy". However, clarity with respect to such estimative processes emerges only after a reading of the requirements with respect to the process of rounding. Here, we find children in year four being expected to "round any number to the nearest 10, 100 or 1000" and "round decimals with one decimal place to the nearest whole number". This emphasis on rounding continues through the remainder of the primary years. In year five children should be able to "round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000" and "round decimals with two decimal places to the nearest whole number and to one decimal place". By year six, the single statutory statement concerning rounding is that children should be able to "round any whole number to a required degree of accuracy", with other references occurring in the non-statutory sections. That being said, the solitary reference to rounding with an explicit connection to calculation is found in year five, where children should be able to "use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy". In sum, while the process of computational estimation lacks specificity, the juxtaposition of rounding adds clarity, although the preponderance of rounding-related expectations, few of which can be directly connected to estimation, seems to imply that rounding is construed as an end in itself. Moreover, unlike authentic estimation tasks, rounding is a transformative process involving well-defined rules. Thus, since the act of rounding is not an estimative task, although computational estimation is dependent on it, the logic of including such extensive rounding-related expectations independently of the process of estimation seems difficult to discern.

Quantity estimation is effectively absent, which seems disappointing, not least because there can be few people, adult or child, who have not wondered how experts estimate the number of starlings in one of the spectacular murmurations found in various locations or asked the question, "how do they know how many stars there are in our galaxy?" Finally, expectations with respect to number line estimation lack all clarity. For example, there is a single reference, whereby year two children should be able to "…estimate numbers using different representations, including the number line". The invocation is repeated in years three and four, albeit without the reference to the number line. What does this mean? Is it an expectation of number line estimation? Sadly, the non-statutory material offers no clarification.

Summary

In this paper, albeit briefly, we have discussed the relevance to both mathematical learning and the wider world of four forms of estimation. When analysed against them, the English national curriculum for primary mathematics seems inadequate. Indeed, only measurement estimation is adequately presented, with quantity estimation and number line estimation essentially absent and computational estimation a confusing juxtaposition of vague statements about the estimation of answers and an expectation that rounding numbers, in all forms, is an end in itself.

References

Ainsworth, S., Bibby, P., & Wood, D. (2002). Examining the effects of different multiple representational systems in learning primary mathematics. *Journal of the Learning Sciences*, *11*(1), 25–61.

Alajmi, A. (2009). Addressing computational estimation in the Kuwaiti curriculum: Teachers' views. *Journal of Mathematics Teacher Education*, *12*(4), 263–283.

Albarracín, L., & Gorgorió, N. (2019). Using large number estimation problems in primary education classrooms to introduce mathematical modelling. *International Journal of Innovation in Science and Mathematics Education, 27*(2), 45–57.

Anestakis, P., & Lemonidis, C. (2014). Computational estimation in an adult secondary school: A teaching experiment. *MENON: Journal of Educational Research*, *Thematic Issue Number 1*, 28–45.

Barth, H., Starr, A., & Sullivan, J. (2009). Children's mappings of large number words to numerosities. *Cognitive Development*, *24*(3), 248–264.

Bonner, B., Sillito, S., & Baumann, M. (2007). Collective estimation: Accuracy, expertise, and extroversion as sources of intra-group influence. *Organizational Behavior and Human Decision Processes*, *103*(1), 121–133.

Booth, J., & Siegler, R. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016–1031.

Chang, K.-L., Males, L., Mosier, A., & Gonulates, F. (2011). Exploring US textbooks' treatment of the estimation of linear measurements. *ZDM*, *43*(5), 697–708.

Crites, T. (1992). Skilled and less skilled estimators' strategies for estimating discrete quantities. *The Elementary School Journal*, *92*(5), 601–619.

Department for Education (2013) The national curriculum in England: key stages 1 and 2 frameworkdocument. Available at: https://www.gov.uk/government/publications/national-curriculum-in-england-primary-curriculum (Accessed: 11 Feb 2020).

Dowker, A. (1997). Young children's addition estimates. *Mathematical Cognition*, *3*(2), 140–153.

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology*, *99*(1), 1–17.

Forrester, M., & Pike, C. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education*, *29*(3), 334–356.

Ganor-Stern, D. (2016). Solving Math Problems Approximately: A Developmental Perspective. *PLoS ONE*, *11*(5), 1–16.

Ginsburg, N. (1996). Number bias, estimation, and sensation seeking. *Perceptual and Motor Skills*, *83*(3), 856–858.

Hong, D., Choi, K., Runnalls, C., & Hwang, J. (2018). Do textbooks address known learning challenges in area measurement? A comparative analysis. *Mathematics Education Research Journal*, *30*(3), 325–354.

Imbo, I., & LeFevre, J.-A. (2011). Cultural differences in strategic behavior: A study in computational estimation. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *37*(5), 1294–1301.

Jones, M., & Taylor, A. (2009). Developing a sense of scale: Looking backward. *Journal of Research in Science Teaching*, *46*(4), 460–475.

Jones, M., Gardner, G., Taylor, A., Forrester, J., & Andre, T. (2012). Students' accuracy of measurement estimation: Context, units, and logical thinking. *School Science and Mathematics*, *112*(3), 171–178.

Joram, E., Gabriele, A., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education*, *36*(1), 4–23.

Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement estimation: Learning to map the route from number to quantity and back. *Review of Educational Research*, *68*(4), 413–449.

Kramer, P., Bressan, P., & Grassi, M. (2018). The SNARC effect is associated with worse mathematical intelligence and poorer time estimation. *Royal Society Open Science*, *5*(8), 172362–172362.

Laughlin, P., Gonzalez, C., & Sommer, D. (2003). Quantity estimations by groups and individuals: Effects of known domain boundaries. *Group Dynamics: Theory, Research, and Practice*, *7*(1), 55–63.

Lemaire, P., & Lecacheur, M. (2011). Age-related changes in children's executive functions and strategy selection: A study in computational estimation. *Cognitive Development*, *26*(3), 282–294.

Maertens, B., De Smedt, B., Sasanguie, D., Elen, J., & Reynvoet, B. (2016). Enhancing arithmetic in pre-schoolers with comparison or number line estimation training: Does it matter? *Learning and Instruction*, *46*, 1–11.

Peeters, D., Degrande, T., Ebersbach, M., Verschaffel, L., & Luwel, K. (2016). Children's use of number line estimation strategies. *European Journal of Psychology of Education*, *31*(2), 117–134.

Riess, A. (1943). Numerical quantification vs. number sense. *The Journal of Psychology*, *15*(1), 99–108.

Schneider, M., Grabner, R., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*, *101*(2), 359–372.

Si, J., Li, H., Sun, Y., Xu, Y., & Sun, Y. (2016). Age-related differences of individuals' arithmetic strategy utilization with different level of math anxiety. *Frontiers in Psychology*, *7*, 1612–1612.

Siegler, R., & Opfer, J. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, *14*(3), 237–250.

Smets, K., Sasanguie, D., Szücs, D., & Reynvoet, B. (2015). The effect of different methods to construct non-symbolic stimuli in numerosity estimation and comparison. *Journal of Cognitive Psychology*, *27*(3), 310–325.

Sowder, J. (1992). Estimation and number sense. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 371–389). NCTM.

Subramaniam, K. (2014). Prospective secondary mathematics teachers' pedagogical knowledge for teaching the estimation of length measurements. *Journal of Mathematics Teacher Education*, *17*(2), 177–198.

Sullivan, J., & Barner, D. (2014). The development of structural analogy in number-line estimation. Journal of Experimental Child Psychology, 128, 171–189.