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Liquefaction and post-liquefaction of granular material under multi-directional cyclic loading

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Abstract

Soil liquefaction can be induced by natural events that entail complicated loading directions and magnitudes. To investigate the liquefaction behaviour of granular material under complex loading conditions, a series of strain-controlled cyclic simple shear tests are conducted on the uniform-sized glass beads. These tests include uni-directional and multi-directional loading paths. An energy-based method is used to assist the understanding of the cyclic behaviour of the specimens. After the first liquefaction happens, the specimens are re-consolidated and subjected to monotonic undrained shearing to investigate their post-liquefaction behaviour. The test results indicate that the specimens subjected to multi-directional cyclic shearing are more prone to liquefy than the ones under uni-directional loading. Furthermore, the cyclic shear strain amplitude and cyclic loading path have significant influences on the soil liquefaction resistance, re-consolidation volumetric strain and post-liquefaction shear strength. Nevertheless, the total energy that is dissipated for liquefying a specimen is only dependent on its relative density.

Keywords: Multi-directional cyclic simple shear; liquefaction and post-liquefaction; granular material; settlement; energy dissipation

Introduction

It is well accepted that soil liquefaction induced by natural events is one of the major contributors to construction damages; thus, soil liquefaction and soil behaviour after liquefaction are of great interest to researchers. To investigate the liquefaction and post-liquefaction behaviour of soil, numerous studies have been carried out by using various experimental techniques and testing apparatuses *in the last decade* (Ishihara & Yasuda, 1975; Sivathayalan, 1994; McCarron *et al*., 1995; Porcino & Caridi, 2007; Wang *et al*., 2012; Bastidas *et al*., 2017; Hubler *et al*., 2017; Castelli et al., 2019; Kumar et al., 2020). On account of technical difficulties and apparatus limitations, most of these studies only focus on shear behaviour in a single direction. However, in reality, natural events, such as earthquake ~~and tsunami~~, involve complicated loading directions and magnitudes. With the assistance of advanced facilities, a handful of research with the multi-directional simple shear devices has investigated shear behaviour under such condition (Ishihara & Yamazaki, 1980; Boulanger & Seed, 1995; Kammerer *et al*., 2002; Matsuda *et al*., 2011; Mirbaha, 2017). Kammerer et al. (2002) found that, compared with the pore water pressure generation under the uni-directional cyclic loading, those under the multi-directional condition were far more rapid and complicated. Matsuda *et al*. (2011) clarified that the cyclic shear direction and shear strain amplitude have significant effects on the post-earthquake settlements.

Most of the studies mentioned above use the stress-controlled method to conduct experiments. As a result, the liquefaction resistance of tested material is interpreted based on the influence of cyclic stress ratio (cyclic shear stress amplitude/ total vertical stress) within a preset number of cycles. In this method, the criterion for liquefaction is that the single amplitude of the induced shear strain reaches 3% to 3.75% (Ishihara & Yamazaki, 1980; Boulanger & Seed, 1995; Wijewickreme *et al*., 2005). However, for loose samples, the induced shear strain can be significantly large when liquefaction initiates. In comparison to stress controlling, a few researchers believe that strain

controlling is more intuitive, since liquefaction is a displacement-dependent phenomenon. It is clarified that shear strain is the key parameter that allows control over the ground settlements and the development of pore water pressure during the cyclic loading (Dobry et al., 1982; Talaganov, 1992; Vucetic, 1992; Matsuda et al., 2011; Kang et al., 2015; Kumar et al., 2018; Dammala et al., 2019). More importantly, unlike the stress controlling method, cyclic tests controlled by shear strain do not reach uncontrolled strain condition. In other words, a full liquefaction process can be obtained, which is necessary to undertake the post-liquefaction investigation.

Although it is reported that soils are more prone to liquefaction under multi-directional condition than under uni-directional condition, most of the comparison is made based on one type of multi-directional path (circular shape). The assessment of the data obtained from different multi-directional loading paths are desired. Likewise, the interpretation of the data so obtained is expected to be made from other aspect, for example, in the view of energy dissipation. Consequently, the authors believe that our knowledge of the mechanism characterizing multi-directional cyclic liquefaction is still limited. Studies that investigate post-liquefaction and consider multi-directional loading history are even more limited. Therefore, the main objective of this work is to provide an in-depth understanding of cyclic liquefaction and post-liquefaction behaviour of granular materials under multi-directional simple shearing. To meet this end, a series of strain-controlled uni- and multi-directional cyclic simple shear tests were conducted on uniform-sized glass beads. Accordingly, the following influencing factors, namely shear strain amplitude, relative density and loading path, on liquefaction resistance, were investigated. To analyze the exact impact of these factors, an energy-based method is adopted. Lastly, the undrained monotonic shearing test was conducted on liquefied and virgin samples (without experiencing liquefaction) to study their post-liquefaction shear behaviours.

Multi-directional cyclic simple shear tests in constant volume condition

Description of the testing apparatus

The testing apparatus, known as the variable direction dynamic cyclic simple shear (VDDCSS) apparatus, is shown in Fig. 1a. Three electro-mechanical actuators control the apparatus: One vertical actuator (z-direction) provides the device with vertical load or displacement. Conversely, the other two orthogonal actuators (x- and y-direction) allow the device to exert shear stress or strain in any direction on the horizontal plane.

In this device, a cylindrical specimen (Fig. 1b & c), which is 70 mm in diameter and 22.6 mm in height, is laterally confined by several circular Teflon coated rings. Each ring is 71 mm in diameter and 1 mm in height. A latex membrane is placed between the rings and the specimen to protect the rings from being damaged and to ensure the uniform shearing of the specimen. During the test, the smooth surface of the rings allows the specimen to be sheared freely with minimum friction. The VDDCSS can perform monotonic and cyclic shear tests, and more detailed information of this apparatus can be found in Li (2016).

Testing material and testing procedure

It is reported that the factors, such as grading curve, angularity and fine composition of the tested material can affect the cyclic behaviour (references). In this work, the aim is to investigate the multi-directional loading paths on the liquefaction and post-liquefaction shear behaviour. As a

result, the uniform-distributed glass beads are selected as the tested material.Moreover, in discrete element method (DEM), simulations are often implemented on the spherical particles, for example, Zhang *et al*. (2019) employed uniform-sized glass beads to investigate static simple shear behaviour in bi-direction. The testing results obtained from glass beads in this work may provide a reference for future research either from the experimental aspect or the numerical aspect. In this research, the glass beads consists of spherical particles that primarily contain silicon dioxide with some other silicates. It is uniformly graded with a mean diameter (D_{50}) of 0.8mm (Fig. 2). Its specific gravity, maximum and minimum dry void ratios were measured according to the American Society for Testing and Materials (ASTM) standards D854, D4253 and D4254. The physical properties of this material are summarized in Table 1.

Each specimen was prepared as follows: oven-dried glass beads samples with predetermined weight were poured into the cylindrical shear box by employing the dry funnel method. This method is believed to be a proper way to duplicate the densified soil sample in earthquake regions (Li *et al*., 2018). To obtain higher relative densities, A low-energy and high-frequency shaking table was used for providing even strking (vibration), which is of 0.5mm in the amplitude and 2 Hz in the frequency.

After the sample preparation, the specimens were tested following the process shown in Fig. 3. During the stage of consolidation, the specimen was K_0 consolidated for 30 mins under the vertical confining stress σ'_{vc} = 50 kPa. The relative densities obtained at the end of consolidation are D_r = 45%, D_r = 70% and D_r = 81%, which denote medium dense, dense and very dense categories respectively. Subsequently, the undrained cyclic simple shear tests were conducted on these specimens with various shear strain amplitudes, relative densities and loading paths. During the cyclic shearing, the vertical displacement was fixed to keep the volume of the specimen constant and obtain an equivalent undrained shearing condition. In this condition, the decrease of the vertical stress that is attained in a dry sample is equal to the increase of the pore water pressure that is found in a saturated sample under the true undrained shearing test (Dyvik *et al*., 1987). The details of the performed tests are summarized in Table 2. In particular, the tests employed three different cyclic paths, namely the uni-directional Path-X, multi-directional Path-O and Path-8. The Path-X represents the conventional cyclic loading in a single direction. Fig. 4a shows the shear waveform and plan view of the strain path for this type of loading, in which the shear strain was exerted on a specimen along the x-direction. The Path-O and Path-8 represent the complex cyclic loadings with the same or different frequencies in two directions. Their typical shear waves and shear strain movement are illustrated in Fig. 4b& c, respectively. For the multi-directional tests conducted in this study, the same shear strains were simultaneously applied to the specimens with the phase difference of 90 degrees in x- and y-direction. Specifically, in the Path-O, the frequencies of x- and y-direction are the same, while in the Path-8, the frequency in x-direction is half of the one employed in y-direction. Since liquefaction is a deformation-based phenomenon, it is insensitive to loading frequency (Wong, 1971; Jong & Seed, 1988). Consequently, the loading frequency was set as 0.1 Hz in this study for ease of control. The cycle number used in this paper denotes the cycles in the x-direction. The liquefaction cycle number is recorded when the equivalent pore water pressure reaches over 95% of the vertical confining stress.

After reaching the first liquefaction, the shear displacement was returned to the original point before the specimens were re-consolidated under the same conditions as in the first consolidation. Since the cyclic shear strain amplitudes used during the tests are relatively small, the nominal relative density for each category of specimens (medium dense, dense, and very dense) after re-consolidation is considered to be the same, and they are D_r' = 50%, D_r' = 73% and D_r' = 82% respectively (The real relative density of each specimen is provided in Table 3). Finally, the monotonic undrained shearing test was conducted on the specimens in the x-direction. This stage is controlled in displacement with a speed of 0.01mm/min to ensure the adequate generation of equivalent pore water pressure. To provide a reference, the samples with no liquefaction history were also examined with the same monotonic undrained shearing test.

Test results and discussion

Initial liquefaction

In this study, the liquefaction resistance of tested specimens, which were tested by the strain-controlled method, has been estimated based on the number of liquefaction cycles and the magnitude of shear strain amplitude. Taking the tests with the shear strain amplitude of 0.1% under vertical confining stress of 50kPa and relative density of 45% as examples, Fig. 5-7 illustrate the cyclic shear behaviour of the specimens with different loading paths when tested with these conditions. Fig. 5 shows the shear stress-strain hysteresis loops in x- and y-direction. From this image, it can be seen that, as the cycle increases, the secant shear moduli (the ratio of cyclic shear stress and strain amplitude) decrease gradually to nearly zero. To compare the variation of shear stress under different loading paths more easily, Fig. 6 shows the development of shear stress against the number of loading cycles in x- and y-direction. It proves that at the first few cycles, the cyclic loading paths have little impact on the sample stiffness since their respective shear stress magnitudes are almost the same. However, as the tests continue, the decreasing rates of shear stress for Path-X, Path-O and Path-8 in the x-direction are in ascending order. Similar results are also found in the y-direction, where the specimen under the Path-8 loses its shear stress faster than under the Path-O.

From a more direct point of view, as shown in Fig. 7, the pore water pressure, which is one of the key indicators of cyclic liquefaction, is plotted against the number of cycles for different cyclic loading paths. During the cyclic shearing, the specimens tend to decrease the volume, and in a drained condition, this is called densification. However, the decreasing of this volume is prevented by the undrained condition. Similarly, the tendency of the volume to change throughout successive cycles determines the magnitude and rate of pore water pressure generation in the specimens. From Fig. 7 it could also be observed that the specimen under uni-directional cyclic path develops pore water pressure most slowly and the specimen under multi-directional cyclic Path-8 develops pore water pressure most quickly.

To closely compare the liquefaction behaviour of tested material under uni- and multi-directional loading paths, Fig. 8 displays the effects that any given cyclic loading path and relative density had on the liquefaction resistance. In this figure, the number of cycles that is necessary to reach liquefaction N is plotted against the cyclic shear strain amplitude A_γ in a logarithmic format. Fig.

8a presents the result obtained under the medium dense state and Fig. 8b & c show the corresponding results for the dense and very dense states.

Firstly, it can be seen that under the same relative density and shear strain amplitude, tests that were conducted under Path-X require far more cycles to reach liquefaction than the tests sheared multi-directionally. At the same time, the tests run under the multi-directional Path-O only require slightly more cycles than the tests performed under the Path-8. The trend of the curves is the same as the results previously presented in Fig. 5-7. The difference of liquefaction cycle numbers is resulted by the different tendencies of volume change, which may be related to the rearrangement of particles due to different loading paths. Secondly, as each graph of Fig. 8 shows, the specimens are sheared under different relative densities but by the same loading path. It is consistent with all loading paths, where increasing the relative density will increase the liquefaction resistance. It is because the specimens tend to contract less under the higher relative density, and a less contractive specimen is more difficult to liquefy.

Lastly, in this logarithm-format figure Fig. 8, it can be observed that the curves are almost parallel to each other. Moreover, any increase in the shear strain amplitude reduces the number of cycles to reach liquefaction. When the shear strain amplitude reduces significantly (close to the threshold value), the effects of cyclic loading path and relative density on the liquefaction resistance are noticeable. However, when the shear strain amplitude increases, the differences previously found between liquefaction cycle numbers for different cyclic loading paths or relative densities decline.

Energy dissipation

Nemat-Nasser and Shokooh (1979) first established an energy approach to understanding the liquefaction potential of sand under cyclic shearing. Their work indicates that the rearrangement of sand particles engendered by cyclic loading dissipate a certain amount of energy. Verified by experiments and models only considering single-directional shearing, the energy dissipated inside of the specimen is proved to be directly related to the rate of liquefaction (Figueroa et al., 1994; Liang, 1995; Baziar & Jafarian, 2007; Sonmezer, 2019). In this approach, the energy per unit volume (kJ/m^3) dissipated in one cycle is denoted by the area in the corresponding shear stress-strain loop (for example, in Fig. 5a).

Because of considering multi-directional cyclic loading, in this study, the total energy per unit volume (δW) necessary to liquefy a specimen is given by:

$$\delta W=\sum_{i=1}^{n-1}\frac{1}{2}\left(\tau_{x_i}+\tau_{x_{i+1}}\right)\left(\gamma_{x_{i+1}}-\gamma_{x_i}\right)+\frac{1}{2}\left(\tau_{y_i}+\tau_{y_{i+1}}\right)\left(\gamma_{y_{i+1}}-\gamma_{y_i}\right) \quad \text{Equation (1)}$$

Where n is the total number of recorded data before liquefaction, τ_x and τ_y are the shear stresses in x- and y-direction, γ_x and γ_y are the shear strains in x- and y-direction.

To investigate the mechanism of liquefaction under uni- and multi-directionally cyclic conditions, the total energy per unit volume calculated from Equation (1) is shown in Fig. 9. It is presented against the shear strain amplitude of different cyclic loading paths. Fig. 9a shows the curves obtained under the medium dense state. Similarly, Fig. 9b & c show the curves found under the dense and very dense state, respectively. It is found that in Fig. 9a &b, the cuves of the unit energy dissipated for liquefaction are extremely flat, while Fig. 9c shows the similar trend except for the

one obtained from the Path-X and strain amplitude of 0.1%. Refer to Fig. 8c, the number of liquefaction cycles is more than 100 for this case, and the friction error induced inside of the apparatus may be enlarged significantly to affect the energy calculation. However, in the big piture, Figure 9 illustrates that under a specific loading path and a certain relative density, the total energy per unit volume for liquefaction is generally a constant value, despite the shear strain amplitude that is exerted on the specimen. Moreover, this amount of energy grows in accordance to the increase registered in the relative density. This result is consistent with previous research findings (Figueroa *et al*., 1994; Baziar & Jafarian, 2007; Jafarian *et al*., 2012), which claimed that the total dissipated energy per unit volume up to liquefaction rises in concomitance with the increase of relative density. However, it is less dependent on the shear strain amplitude. More importantly, from this figure, it can also be observed that when the relative density is the same, the total energy dissipated per unit volume under different loading paths is, by and large, the same. A similar study conducted by Polito et al. (2013) with stress-controlled cyclic triaxial tests showed that the accumulated energy per unit volume at the onset of liquefaction is not affected by the loading shape (sinusoidal, square, triangular, irregular symmetric and irregular asymmetric).

To discuss the difference of liquefaction resistance that were found between uni- and multi-directional simple shearing, the set of tests with the shear strain amplitude of 0.1% under vertical confining stress of 50kPa and relative density of 45% is chosen as an example. Fig. 10, whose data is calculated based on Equation (1), illustrates the development of the energy dissipated per cycle as well as the accumulated energy. This figure shows that the energy dissipated drops quickly in the last few cycles, which, in turn, indicates the occurrence of liquefaction.

Since the total energy dissipated per unit volume for liquefaction is a constant value independent of the cyclic loading path, Fig. 10 evidently shows that the energy dissipated in each cycle of the specimen shearing under the Path-X is considerably lower than that under the multi-directional paths. The primary reason for this can be found in Fig. 10a, which shows that the energy dissipated per cycle in the Path-X is contributed by a single direction. While Fig. 10b &c show that the energy dissipated per cycle in the Path-O and Path-8 is composed of the x- and y-direction simultaneously. The energy dissipated per cycle in one direction is much lower than the energy dissipated in two directions. Although both in the Path-O and Path-8 energy is dissipated in two directions, the number of cycles necessary to achieve liquefaction in the Path-8 is slightly lower than in the Path-O as compared in Fig. 10b and Fig. 10c. This is the case in that the frequency of the Path-8 in the y-direction is twice that the one of the Path-O in the y-direction. In conclusion, more energy is dissipated in the Path-8 than in the Path-O in the y-direction.

Post-liquefaction

After the initial liquefaction, the specimens were re-consolidated under the same conditions as the first consolidation. Then, they were sheared in the x-direction monotonically in an undrained manner. In this study, this process is called post-liquefaction. Since the Path-O is a representative of multi-directional loading path, the set of tests, which have experienced liquefaction in the Path-O, is taken as the main example to investigate the impact that the history of the first liquefaction on the post-liquefaction stage. Fig. 11 and Fig. 12 display the volume changes that occur throughout the re-consolidation stage. Fig. 11 shows the volumetric strain development of the

specimens with a liquefaction history in the Path-O under the relative densities of 50%, 73% and 82%. It can be seen from the figure that the amount of volume reduction so observed is lower under the higher relative density. More importantly, the volumetric strain increases in accordance to the amount of the first liquefaction shear strain amplitude. Fig. 12 demonstrates the volumetric strain development of specimens with the shear strain amplitude of 0.1% under different initial liquefaction loading paths. By doing so, it could be observed that the multi-directionally liquefied specimens generate more volumetric strain than the uni-directionally ones. Same results are presented by Matsuda *et al.* (2004) by employing a multi-directional simple shear apparatus. Their tests, which were conducted with limited initial cyclic conditions, showed that the settlements obtained from the multi-directionally liquefied cases in the re-consolidation stage are larger than the ones acquired from the uni-directionally liquefied cases.

To study the static simple shear behaviour of the tested material in the post-liquefaction stage, the results of the specimens with different initial liquefaction histories are presented here. To facilitate the analysis, the data of the virgin samples are also provided as references. Fig. 13a shows the development of shear stress and stress path of specimens that liquefied in the Path-O with different strain amplitudes under the relative density of 50%. Fig. 13b &c show the specimens that liquefied under relative densities of 73% and 82% respectively. From these data, it can be observed that the specimens with liquefaction history, despite the amount of strain amplitude, have greater peak strength than the virgin samples. This phenomenon is most obvious for the medium dense specimens with D_r' = 50%. As shown in Fig. 13a, the specimens with liquefaction history shift shear behaviour from strain-softening to strain-hardening. Similar conclusions have been reported in the previous studies (Porcino *et al.*, 2009; Bastidas *et al*., 2017) that were conducted through post-liquefaction undrained cyclic tests of sand and by using uni-directional simple shear devices. They showed that, although there is no significant change in the relative density, the sand samples manifest higher post-liquefaction resistance if they have experienced small cyclic pre-shearings. This concept of small cyclic pre-shearing was first introduced by Ishihara and Okada (1978), who conducted undrained cyclic triaxial compression tests on sands. They defined the small pre-shearing as a stress history where the stress path is restricted to the domain bounded by two PT-lines (lines of phase transformation). According to this definition, the initial liquefaction loading histories used in this study in the post-liquefaction stage are all confirmed to be the small pre-shearing. The stress paths of specimens in the x- and y-direction under the vertical stress of 50kPa, relative density of 45% and strain amplitude of 0.1%, are shown in Fig. 14 as an example. Thus it is reasonable that the rest of the tests agree with the trend shown in Fig. 13.

Apart from the above observations, there is another important finding in Fig. 13. It indicates that for each category of relative density, as the initial liquefaction shear strain amplitude increases, the peak strength in the post-liquefaction stage raises as well. These results are also seen in the specimens liquefied in the Path-X and Path-8. Zhou and Chen (2005), who investigated the influence of previous strain histories on liquefaction resistance, have obtained similar results. It is found that, within the range of small pre-shearing, the liquefaction resistance is larger whenever the shear strain is larger too. This may be the case in that larger shear strains can eliminate more local instabilities in the sample structure than smaller ones.

Fig. 15 shows the undrained shear behaviour of specimens with different loading paths in the initial liquefaction stage. Data presented in this figure are extracted from the tests conducted with a shear strain amplitude of 0.1% and the relative density of 73%. The results so obtained confirm the above-discussed conclusion that the specimens with a liquefaction history (no matter which loading path caused the liquefaction) present a greater peak strength than the specimens which do not have a liquefaction history. Moreover, they prove that, the specimens that are liquefied multi-directionally, in general, exhibit a higher peak strength than those liquefied uni-directionally. For the multi-directionally samples, the result of the specimen liquefied with the Path-8 history is slightly higher than the one liquefied with the Path-O history. The same results are also observed in the tests which were run with the relative densities of 50% and 82%. This observation proves that specimens undergoing liquefaction under multi-directional loading paths are stronger in the post-liquefaction stage than those subjected to liquefaction in the uni-directional loading path.

Conclusions

To study the liquefaction behaviour of granular material under multi-directional conditions, a series of cyclic simple shear tests have been conducted on the uniform-sized glass beads. Two types of multi-directional loading paths, namely the Path-O and Path-8, are employed and compared with the uni-directional loading Path-X. After initial liquefaction, the monotonic undrained simple shearing was conducted on the re-consolidated specimens to investigate their post-liquefaction behaviour. The effects of relative density and strain amplitude have also been considered. The main conclusions are summarized below:

1) In the initial liquefaction stage, the liquefaction resistance of specimen is higher under the multi-directional path. Moreover, the number of cycles necessary to reach liquefaction are increased by an increasing relative density or a decreasing shear strain amplitude.
2) The total energy dissipated to liquefy a specimen is dependent on the relative density but independent of the shear strain amplitude and loading path. Nevertheless, during one cycle of loading, the multi-directional Path-8 dissipates energy most quickly. In contrast, the uni-directional Path-X dissipates energy most slowly. Therefore, this finding explains the difference in liquefaction resistance that a specimen exhibits under different loading paths.
3) In the re-consolidation stage, the settlements obtained with multi-directionally liquefied specimens are higher than those accrued with uni-directionally liquefied specimens. Likewise, the settlements gained with higher initial shear strain amplitudes are larger than those attained with lower initial shear strain amplitudes.
4) In the post-liquefaction stage, the initially liquefied specimens are generally stronger than the virgin specimens. The larger the initial shear strain amplitude is, the stronger the specimen becomes. Furthermore, the specimens liquefied with the Path-8 shows the highest peak strength. In contrast, the specimens liquefied with the Path-X shows the lowest one.

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References

ASTM International. (2019). ASTM D 4253-16e: Standard test method for maximum index density and unit weight of soils using a vibratory table. In *Annual Book of ASTM Standards 2019*. West Conshohocken, PA: American Society for Testing and Materials.

ASTM International. (2016). ASTM D 4254-16: Standard test method for minimum index density and unit weight of soils and calculation of relative density. In *Annual Book of ASTM Standards 2016*. West Conshohocken, PA: American Society for Testing and Materials.

ASTM International. (2014). ASTM D 854-14: Standard test method for specific gravity of soil solids by water pycnometer. In *Annual Book of ASTM Standards 2014*. West Conshohocken, PA: American Society for Testing and Materials.

Bastidas, A. P., Boulanger, R., DeJong, J., & Price, A. (2017). Effects of pre-strain history on the cyclic resistance of Ottawa F-65 sand.

Baziar, M., & Jafarian, Y. (2007). Assessment of liquefaction triggering using strain energy concept and ANN model: capacity energy. *Soil Dynamics and Earthquake Engineering, 27*(12), 1056-1072.

Boulanger, R. W., & Seed, R. B. (1995). Liquefaction of sand under bidirectional monotonic and cyclic loading. *Journal of geotechnical engineering, 121*(12), 870-878.

Castelli, F., Cavallaro, A., Grasso, S., & Lentini, V. (2019). Undrained Cyclic Laboratory Behavior of Sandy Soils. *Geosciences, 9*(12), 512.

Dammala, P. K., Kumar, S. S., Krishna, A. M., & Bhattacharya, S. (2019). Dynamic soil properties and liquefaction potential of northeast Indian soil for non-linear effective stress analysis. *Bulletin of Earthquake Engineering, 17*(6), 2899-2933.

Dobry, R., Ladd, R., Yokel, F. Y., Chung, R. M., & Powell, D. (1982). *Prediction of pore water pressure buildup and liquefaction of sands during earthquakes by the cyclic strain method* (Vol. 138): National Bureau of Standards Gaithersburg, MD.

Dyvik, R., Berre, T., Lacasse, S., & Raadim, B. (1987). Comparison of truly undrained and constant volume direct simple shear tests. *Geotechnique, 37*(1), 3-10.

Figueroa, J. L., Saada, A. S., Liang, L., & Dahisaria, N. M. (1994). Evaluation of soil liquefaction by energy principles. *Journal of geotechnical engineering, 120*(9), 1554-1569.

Hubler, J. F., Athanasopoulos-Zekkos, A., & Zekkos, D. (2017). Monotonic, cyclic, and postcyclic simple shear response of three uniform gravels in constant volume conditions. *Journal of Geotechnical and Geoenvironmental Engineering, 143*(9), 04017043.

Ishihara, K., & Okada, S. (1978). Effects of stress history on cyclic behavior of sand. *Soils and Foundations, 18*(4), 31-45.

Ishihara, K., & Yamazaki, F. (1980). Cyclic simple shear tests on saturated sand in multi-directional loading. *Soils and Foundations, 20*(1), 45-59.

Ishihara, K., & Yasuda, S. (1975). Sand liquefaction in hollow cylinder torsion under irregular excitation. *Soils and Foundations, 15*(1), 45-59.

Jafarian, Y., Towhata, I., Baziar, M., Noorzad, A., & Bahmanpour, A. (2012). Strain energy based evaluation of liquefaction and residual pore water pressure in sands using cyclic torsional shear experiments. *Soil Dynamics and Earthquake Engineering, 35*, 13-28.

Jong, H., & Seed, R. (1988). A Critical investigation of factors affecting seismic pore pressure generation and post-liquefaction flow behavior of saturated soils. *Geotechnical Engineering Research Report No. SU/GT/88-01, Stanford University*.

Kammerer, A. M., Pestana, J., & Seed, R. (2002). *Undrained response of Monterey 0/30 sand under multidirectional cyclic simple shear loading conditions.* University of California, Berkeley.

Kang, X., Ge, L., Chang, K.-T., & Kwok, A. O.-L. (2015). Strain-controlled cyclic simple shear tests on sand with radial strain measurements. *Journal of Materials in Civil Engineering, 28*(4), 04015169.

Kumar, S. S., Dey, A., & Krishna, A. M. (2020). Liquefaction Potential Assessment of Brahmaputra Sand Based on Regular and Irregular Excitations Using Stress-Controlled Cyclic Triaxial Test. *KSCE Journal of Civil Engineering*, 1-13.

Kumar, S. S., Krishna, A. M., & Dey, A. (2018). Dynamic properties and liquefaction behaviour of cohesive soil in northeast India under staged cyclic loading. *Journal of Rock Mechanics and Geotechnical Engineering, 10*(5), 958-967.

Li, Y. (2016). *Shear behaviour of sand in Bi-directional direct simple shear tests.* University of Nottingham.

Li, Y., Yang, Y., Yu, H.-S., & Roberts, G. (2018). Effect of sample reconstitution methods on the behaviors of granular materials under shearing. *Journal of Testing and Evaluation, 46*(6).

Liang, L. (1995). *Development of an energy method for evaluating the liquefaction potential of a soil deposit.* Case Western Reserve University.

Matsuda, H., Hendrawan, A. P., Ishikura, R., & Kawahara, S. (2011). Effective stress change and post-earthquake settlement properties of granular materials subjected to multi-directional cyclic simple shear. *Soils and Foundations, 51*(5), 873-884.

Matsuda, H., Shinozaki, H., Okada, N., Takamiya, K., & Shinyama, K. (2004). *Effects of multi-directional cyclic shear on the post-earthquake settlement of ground.* Paper presented at the Proc. of 13 th World Conf. on Earthquake Engineering.

McCarron, W. O., Lawrence, J., Werner, R., Germaine, J., & Cauble, D. (1995). Cyclic direct simple shear testing of a Beaufort Sea clay. *Canadian Geotechnical Journal, 32*(4), 584-600.

Mirbaha, K. (2017). Bi-directional Cyclic Behaviour and Liquefaction Analysis of a silica-carbonate sand.

Nemat-Nasser, S., & Shokooh, A. (1979). A unified approach to densification and liquefaction of cohesionless sand in cyclic shearing. *Canadian Geotechnical Journal, 16*(4), 659-678.

Polito, C., Green, R. A., Dillon, E., & Sohn, C. (2013). Effect of load shape on relationship between dissipated energy and residual excess pore pressure generation in cyclic triaxial tests. *Canadian Geotechnical Journal, 50*(11), 1118-1128.

Porcino, D., & Caridi, G. (2007). Pre-and post-liquefaction response of sand in cyclic simple shear *Dynamic response and soil properties* (pp. 1-10).

Porcino, D., Marcianò, V., & Nicola Ghionna, V. (2009). Influence of cyclic pre-shearing on undrained behaviour of carbonate sand in simple shear tests. *Geomechanics and Geoengineering: An International Journal, 4*(2), 151-161.

Sivathayalan, S. (1994). *Static, cyclic and post liquefaction simple shear response of sands.* University of British Columbia.

Sonmezer, Y. B. (2019). Energy-based evaluation of liquefaction potential of uniform sands. *GEOMECHANICS AND ENGINEERING, 17*(2), 145-156.

Talaganov, K. (1992). *Application of strain approach for investigation of post-initial liquefaction behaviour of soils.* Paper presented at the Proc. 10th World Conference on Earthquake Engineering, Madrid.

Vucetic, M. (1992). *Soil properties and seismic response.* Paper presented at the Proceedings of the Tenth World Conference on Earthquake Engineering.

Wang, S., Yang, J., & Onyejekwe, S. (2012). Effect of previous cyclic shearing on liquefaction resistance of Mississippi River Valley silt. *Journal of Materials in Civil Engineering, 25*(10), 1415-1423.

Wijewickreme, D., Sriskandakumar, S., & Byrne, P. (2005). Cyclic loading response of loose air-pluviated Fraser River sand for validation of numerical models simulating centrifuge tests. *Canadian Geotechnical Journal, 42*(2), 550-561.

Wong, R. T. (1971). *Deformation characteristics of gravels and gravelly soils under cyclic loading conditions*: University of California, Berkeley.

Zhang, M., Yang, Y., Zhang, H., & Yu, H.-S. (2019). DEM and experimental study of bi-directional simple shear. *Granular Matter, 21*(2), 24.

Zhou, Y.-g., & Chen, Y.-m. (2005). Influence of seismic cyclic loading history on small strain shear modulus of saturated sands. *Soil Dynamics and Earthquake Engineering, 25*(5), 341-353.