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# Can mode coupling chatter happen in milling?

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ABSTRACT

In milling, two different self-excited vibrations have been reported; regenerative and mode coupling chatter. The regenerative chatter mechanism has been extensively studied and validated with tests whereas mode coupling chatter mechanism was reported a long time ago for threading operations. The presented mode coupling chatter models were based only on current vibrations of the system but not on the delayed vibrations. With the increase in the research carried out on robotic milling operations, the low frequency severe self-excited vibrations at high spindle speeds were claimed to be mode coupling chatter by many researchers. However, the justification of mode coupling chatter mechanism is absent from predicted stability boundaries and a strong evidence distinguishing it from regenerative chatter mechanism is not present. Additionally, mode coupling chatter models applied to milling was based on threading operations and hence they were not capturing the characteristics of intermittent milling process. Therefore, this paper focuses on the diagnosis of mode coupling chatter in robotic milling. Mode coupling chatter principles are applied to milling process considering its intermittent characteristics. The zeroth order approximation for mode coupling chatter mechanism is adapted for milling and extended to multi frequency approximation. Mode coupling chatter stability boundaries are calculated, explored with tests and compared with regenerative chatter stability boundaries. Results show that mode coupling chatter stability boundaries are very low and vaguely dependent on the spindle speed. This contradicts the stability observed in the experimental tests. Hence, it is concluded that mode coupling chatter in milling is not possible, because the assumption of the chip thickness depending only on current vibrations does not apply to milling operations. The novelty of the presented paper is the theoretical and experimental justification that mode coupling chatter is not possible in milling operations.

## 1. Introduction

Machine tools experience two major sources of self-excited chatter vibrations which cause instability and detrimental vibrations during machining; regenerative and mode coupling chatter [1].

Regenerative chatter stability depends on the stability of the dynamic chip thickness which is the difference between vibrations left on the previously and newly cut surface. The tool propagates across the workpiece for material removal and always cuts previously machined surface. It is also the most common type of self-excited vibrations which could cause instability, poor surface finish and dimensional inaccuracy in cutting operations [2]. It has been extensively studied by many researchers [1,3,4] mainly in turning, milling and drilling operations.

Unlike regenerative chatter, mode coupling chatter is known to happen when the tool cuts a new surface at all times as in threading operations (thread boring of asymmetrical boring bar or turning with large feed), hence it happens without regeneration [5]. Fig. 1 depicts the coupled chatter system as described by Refs. [6,7].

Note the direction of feed; a new surface is always machined as in thread boring or turning operations rather than tool cutting a previously cut surface. It has also been mentioned that instability due to mode coupling can also appear in shaping and planing operations in Ref. [8]. For such a machining configuration, the mode coupling chatter depends on the instability in the dynamic chip thickness which is affected only by the current (free) vibrations at the tool-workpiece point of contact [6]. The instability in the chip thickness depends on the coupling of modes of vibrations having closely spaced natural frequencies but different modal properties and directions in at least two degrees of freedom, [9]. For threading operations, surface normal and force angle are constant. Mode coupling instability may only occur when the direction of lower natural frequency mode is between surface normal and force directions as the

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**Nomenclature**

| | |
|---|---|
| $3A_F$ | Directional dynamic milling force coefficient matrix in frequency domain |
| $A_T$ | Directional dynamic milling force coefficient matrix in time domain |
| $b$ | Depth of cut |
| $b_a$ | Axial location of the tooth |
| $b_{lim}$ | Limiting depth of cut |
| $\beta_F$ | Cutting force angle |
| $\beta_{u2}$ | Mode inclination of $u_2$ |
| $f_c$ | Chatter frequency in hertz |
| $F$ | Cutting force |
| $F_x$ | Total force along X-axis |
| $F_y$ | Total force along Y-axis |
| $FRF$ | Frequency response function |
| $\mathscr{F}$ | Fourier transform |
| $g(.)$ | Heaviside (unit) step function |
| $\gamma$ | Functional redundancy variable |
| $h(t)$ | Dynamic chip thickness in time domain |
| $h_0$ | Static chip thickness |
| $j$ | Imaginary operator |
| $K$ | Number of teeth |
| $K_{tc}$ | Dynamic tangential cutting force coefficients |
| $K_{rc}$ | Dynamic radial cutting force coefficients |
| $\lambda$ | Eigenvalues of the oriented transfer function |
| $m$ | Mass |
| $N$ | Lobe number in regenerative chatter mechanism |
| $\omega$ | Frequency in radians |
| $\omega_c$ | Chatter frequency in radians |
| $\omega_T$ | Tooth passing frequency in radians |
| $\Omega$ | Spindle speed |
| $\varphi$ | Instantaneous angular immersion |
| $\varphi_k$ | Instantaneous angular immersion of tooth, k; where k = 1,2, …,K depending on the axial location and time |
| $\Phi$ | Oriented transfer function |
| $r$ | Number of harmonics |
| $t$ | Time |
| $T_k$ | Time delay due to chip thickness regeneration |
| $u_1$ | Modal direction 1 |
| $u_2$ | Modal direction 2 |
| $v(t)$ | Radial vibrations in time domain |
| $x(t)$ | Current vibrations along X-axis |
| $y(t)$ | Current vibrations along Y-axis |

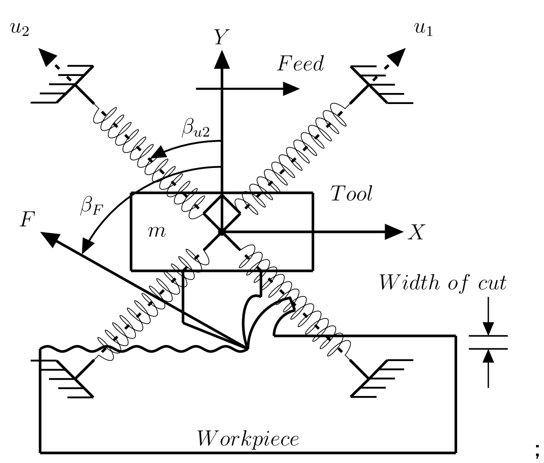


**Fig. 1.** Theoretical model of asymmetrical boring bar of a coupled system [6,7].

asymmetrical workpiece rotates [5]. At the point of instability, the system vibrates simultaneously with different amplitudes and with a difference in phase [7]. It should be emphasised that "mode coupling" should not be confused with "mode coupling chatter". Mode coupling refers to the phenomenon of coupling of modes of vibration with closely spaced natural frequencies when the structure is under free vibration. Whereas, mode coupling chatter is a specific type of self-excited vibration which causes instability as a result of the mode coupling phenomenon in operations such as threading. For this reason, it should be emphasised that mode coupling chatter is a special type of instability that could only occur in special type of operations and machine tool systems as mentioned above. The prediction of mode coupling chatter instability boundaries has been first proposed in Ref. [6] for undamped system but extend to cover damped systems in Ref. [5] for a state space system in time domain. In Ref. [10], the effect of cutting speed was shown to effect mod coupling chatter stability in face turning operation. In a very recent work, a frequency domain model of mode coupling chatter mechanism was developed by taking into account cross-FRFs of the dynamic system in Ref. [11].

Considering the underlying mechanism of mode coupling chatter, it is a special type of self-excited vibrations that can only occur in threading, shaping and planing operations. However, it has been reported that mode coupling chatter characteristics were observed when milling with flexible machine tool structures [12–15]. In Refs. [13,16], a wood milling operation carried out with a flexible machine tool was modelled as a turning operation and mode coupling stability model was applied to avoid chatter vibrations. Based on the mode coupling stability model, the wood cutting operation was stabilised by adjusting the orientation of the tool with respect to the feed direction.

When milling with a different type of flexible machine tool, i.e. robots, the appearance of both regenerative and mode coupling chatter mechanisms was also noted. Regenerative stability limits for a robotic milling operation were computed and optimised by using the functional redundancy in Refs. [17,18]. Tests proved that it is possible to improve the limits of stable cutting by utilising the configuration dependent structural dynamics of the robot. In Ref. [19], the effect of the cutting trajectory and workpiece clamping position on the regenerative stability limits and surface finish were investigated. It was shown that milling along different feed directions as well as workpiece location on the machining table have a significant influence on regenerative stability limits. In Ref. [20], regenerative stability limits in low and high spindle speeds were predicted and validated with robotic milling tests. At low spindle speeds, the stability was proven to strongly depend on the robot structural modes. At high spindle speeds, however, higher modes of vibration were shown to dominate the stability regeneratively. Similarly, the process stability was validated to be regenerative and dominated by the tool-spindle assembly mode at high spindle speeds in Ref. [21]. In Ref. [22], an eddy current damper was designed and mounted onto the spindle in an attempt to suppress chatter and increase stability limits for robotic milling. Slight improvements in the predicted regenerative stability boundaries were achieved and validated experimentally. It should be noted that.

As opposed to the predicted and validated regenerative stability mechanism in robotic milling operations, mode coupling chatter characteristics were also reported. For a robotic milling operation in Ref. [15], low frequency severe chatter at moderate spindle speeds was observed and claimed not to depend on machining parameters (such as spindle speed, feed rate, width of cut and location of the workpiece). The process was reported to be stable in a regenerative point of view hence, mode coupling chatter model was proposed as underlying chatter mechanism. In Ref. [12], for a robotic milling operation, a state space dynamics model of the robot was applied to mode coupling chatter mechanism for threading operations which includes Conservative Congruence Transformation (CCT) stiffness model. The proposed technique aims to alter the principal stiffness of the robot at tool tip and force direction by modifying the feed rate to achieve mode coupling chatter avoidance. Additionally, an online mode coupling chatter detection and avoidance algorithm was developed on the basis of controlling the feed rate to make sure the process is stable in Ref. [23]. In both [12,23], low frequency vibrations not matching to the spindle speed harmonics while milling at moderate spindle speeds were identified to be mode coupling chatter. In Ref. [24], the mode coupling chatter mechanism was utilised in robotic milling path optimisation algorithm which controls the stiffness orientation at TCP to avoid mode coupling chatter. At high spindle speeds, low frequency chatter vibrations were identified and classified as mode coupling chatter. In Ref. [14], mode coupling chatter mechanism was modelled in the state space incorporating the robot state space dynamic model based on CCT inertia and stiffness modelling. Mode coupling chatter avoidance was achieved by adjusting the cutting conditions by configuring the robot around the axis of rotation of the tool and also altering the orientation of the workpiece. The presence of low frequency chatter vibrations around the structural robot mode was claimed to be mode coupling chatter. All of the aforementioned studies applied mode coupling chatter theory for threading operations to milling whereas zeroth order approximation regenerative stability for milling operations was reduced to "mode coupling chatter in milling" by assuming negligible delayed vibrations in Ref. [25]. The highly varied helix of the end mill tool was shown as the main reason for the cancellation of the regenerative effect but proposed stability boundaries were not validated by milling tests.

Similar to flexible machine tools, the mode coupling effect in milling as a result of varying dynamic behaviour of slender milling tool was analysed with time domain regenerative digital simulation in Ref. [26]. As opposed to excitation mechanism of mode coupling chatter observed in threading operations, the structure was proposed to be excited regeneratively in milling. The structure being excited regeneratively contradicts with the excitation mechanism of mode coupling chatter (free vibration). Even though modes of vibration could couple, instability mechanism would certainly be based on regenerative excitations rather than free vibrations as in Refs. [5,6,11].

Mode coupling chatter mechanism applied to milling tend to possess inherent misrepresentation of the process coming from the nature of cutting forces being modelled as threading operation in Refs. [12–16, 24]. For such an intermittent cutting operation, dynamic forces depend on undulations left by the current and previous tooth on the chip surface. Thus, the assumption of process stability depending only on the current radial vibrations is not a valid assumption as opposed to in threading operations. Besides, the machine-tool instability was concluded to be mode coupling chatter without making any effort to differentiate it from the regenerative chatter mechanism, except in Ref. [15]. In Ref. [15], however regenerative theory for turning operations was used to predict stability of a robotic milling operation. Henceforth, stability predictions were not representative of the actual milling stability. This led to consideration of mode coupling chatter theory since low frequency modes were claimed to be not contributing to regenerative chatter vibrations at high spindle speeds. The mode coupling chatter model was applied to the robotic milling process however, it was based on thread turning operation as in Ref. [13], which does not represent the intermittent behaviour of the milling process. This approach was also applied in other works [12,14,24]. For a stronger validation of the type of chatter mechanism, theory claiming the existence of mode coupling chatter in milling [12–16,24] should have been carefully compared with the regenerative chatter mechanism theory for the same process type, within the same publication. If the publication is absent from such a detailed comparison, then (in the opinion of the current authors) it becomes impossible to justify conclusions regarding the actual mechanism of chatter that was observed in these papers. For these reasons, a detailed prediction and analysis of milling tests are required to validate which chatter mechanism is present in milling operations.

In this paper, mode coupling chatter theory and assumptions used for threading, planing, and shaping operations (self-excitation to depend on free vibrations of a two degree of freedom system) [5–8] are applied to milling scenario by utilising the dynamic cutting force model of milling as in Ref. [25]. Thus, the paper aims to identify with experimental validations whether mode coupling chatter is present in milling or not. In this respect, the theory of zeroth order approximation mode coupling chatter in milling is adapted from Ref. [25] and extended to multi frequency approximation to further investigate the spindle speed dependency of mode coupling chatter stability boundaries. In order to validate the stability predictions, stability boundaries in frequency domain are computed and validated with robotic milling tests. Finally, regenerative and mode coupling chatter stability boundaries are compared with tests to assess and identify the governing chatter mechanism within the tests. In this way, the identity of the chatter mechanism while milling with flexible machine tools (such as robots) is aimed to be addressed by this paper. Hence, this paper serves to enlighten the ambiguity of appearance of mode coupling chatter vibrations in milling operations.

## 2. Theory

In this section, the theory of mode coupling chatter mechanism in milling operations is described in zeroth order and multi frequency approximation. The zeroth order approximation developed in Ref. [25] is adapted. The mode coupling chatter mechanism is also extended to cover the multi frequency approximation for unconventional end mill tools too.

The following two assumptions are the main conditions that were previously emphasised for mode coupling chatter to happen in milling according to Refs. [12,13,16,25].

1. The system dynamics must be represented by at least two degrees of freedom, and they must have different modal directions.
2. The dynamic chip thickness must only be dependent on the current waves imprinted on the chip surface. In other words, previous undulations on the chip surface should not be present.

The first assumption is fairly valid in practice, however, the second is against the nature of the milling operation. In milling, the tool with single or multiple teeth rotates and cutting takes place as a result of the relative linear feed between the tool and workpiece. Due to the rotary motion of the tool and linear relative feed between the tool and workpiece, surface undulations left by the previous tooth are always present on the surface currently being machined. As a result, it is not possible to have a milling scenario where undulations left by the previous tooth are simply zero as stated in the second assumption. This clearly suggests that one of the main assumptions where the dynamic chip thickness is unaffected by the regeneration does not hold for milling. Nevertheless, leaving the above incongruity between the dynamic chip thickness assumption and the nature of the milling operation to aside, undulations left by the previous teeth on the surface of the chip are assumed to be negligible (zero) to simulate the mode coupling chatter phenomenon in milling. For the sake of exploring mode coupling chatter vibrations in milling, the dynamic chip thickness was assumed to only depend on the free vibrations of the system to apply mode coupling chatter principles as in thread turning, planning and shaping operations. The resulting scenario is schematically represented in Fig. 2.

The regenerative chatter formulations developed in Ref. [27] will now be adapted to consider mode coupling chatter mechanisms in milling for unconventional end mill tools. Based on the proposed

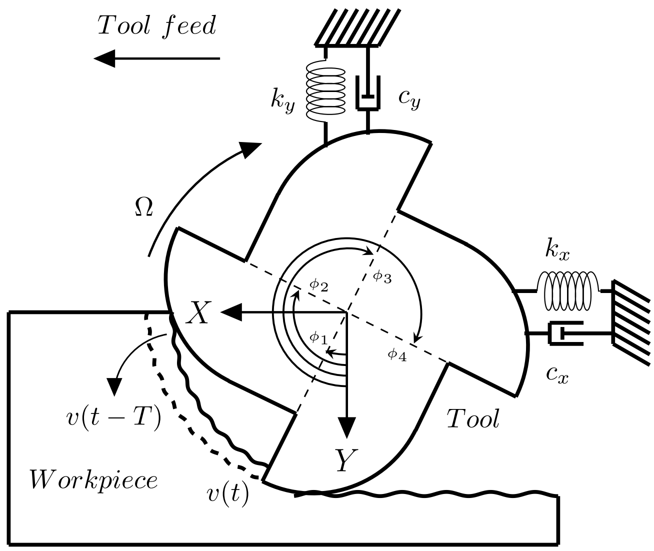


**Fig. 2.** Self-excited vibrations in milling.

approach, 2 DOF (degrees of freedom) zeroth order and multi frequency approximations for mode coupling chatter mechanism were described. The milling operation is assumed to reach steady state and the system structural dynamics to be represented by 2 DOF. Assuming the tool is fed along positive X-axis, the dynamic chip thickness, $h(\varphi_k(b_a,t))$, based on the aforementioned assumptions can be shown as;

$$h(\varphi_k(b_a,t)) = (h_0(\varphi_k(b_a,t)) - v(t))g(\varphi_k(b_a,t)) \tag{1}$$

The static part of the chip thickness is represented as $h_0(\varphi_k(b_a,t))$, where $\varphi_k(b_a,t)$ is the instantaneous angular immersion of tooth $k$ at a given time $t$ for a cutter with $K$ number of teeth with axial location, $b_a$. The $v(t)$ represents current radial vibrations at axial location, $b_a$. The chip thickness is assumed to be independent of the undulations left by the previous tooth on the surface of the chip ($v(t - T_k(b_a))$=0) as opposed to regenerative chatter mechanism [1]. The current radial vibrations, $v(t)$, at axial location, $b_a$, can be represented as,

$$v(t) = -x(t)\sin(\varphi_k(b_a,t)) - y(t)\cos(\varphi_k(b_a,t)) \tag{2}$$

where the current vibrations along X-axis and Y-axis are $x(t)$ & $y(t)$. The function $g(\varphi_k(b_a,t))$ is a Heaviside (unit) step function that determines the engagement of tooth based on the instantaneous angular immersion of tooth. Following the derivations in Ref. [27], the total cutting force in X and Y directions are represented as,

$$\begin{bmatrix} F_x(t) \\ F_y(t) \end{bmatrix} = \frac{1}{2}K_{tc}\sum_{k=1}^{K}\int_{b_a=0}^{b}[A_T(\varphi_k(b_a,t))]\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} db_a \tag{3}$$

The matrix $A_T(\varphi_k(b_a,t))$ is called the directional dynamic milling force coefficient matrix and the elements of the matrix are represented as;

$$\sum_{k=1}^{K}\int_{b_a=0}^{b}[A_T(\varphi_k(b_a,t))]db_a = \sum_{k=1}^{K}\int_{b_a=0}^{b}\begin{bmatrix} a_{xx}(\varphi_k(b_a,t)) & a_{xy}(\varphi_k(b_a,t)) \\ a_{yx}(\varphi_k(b_a,t)) & a_{yy}(\varphi_k(b_a,t)) \end{bmatrix} da \tag{4}$$

where entities with $[A_T(\varphi_k(b_a,t))]$ can be found in Ref. [1]. Taking the Fourier Transform of Equation (3) converts the cutting forces in time domain into frequency domain by application of shift theorem as in Ref. [28]. Assuming linear time-invariant combined tool-workpiece structural dynamics ($FRF(j\omega)$) and following the formulations as in Ref. [27], dynamic forces in X and Y directions become,

$$\begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix} = \frac{1}{2}K_{tc}\sum_{k=1}^{K}\int_{b_a=0}^{b} \times \sum_{r=-\infty}^{\infty} e^{jr(\varphi_{k0}+\beta_k b_a)}[A_F(r)][FRF(j\omega)]\begin{bmatrix} X(j\omega - jr\Omega) \\ Y(j\omega - jr\Omega) \end{bmatrix} db_a \tag{5}$$

where $FRF(j\omega)$ is,

$$FRF(j\omega) = \begin{bmatrix} FRF_{xx}(j\omega) & FRF_{xy}(j\omega) \\ FRF_{yx}(j\omega) & FRF_{yy}(j\omega) \end{bmatrix} \tag{6}$$

$[A_F(r)]$ is frequency domain representation of directional dynamic milling force coefficient matrix as in Ref. [1]. The stability of Equation (5) can be solved using two approaches based on the approximation of the directional dynamic milling force coefficient matrix; Zeroth Order and Multi Frequency Approximation which, are described in the following sections.

### 2.1. Zeroth order approximation

Zeroth order approximation for mode coupling chatter in milling considers the static part of the directional dynamic milling force coefficient matrix $[A_F(r)]$ when $r = 0$. Having set $r = 0$, further simplifications can be made due to the independence of the equation to the depth of cut and number of teeth. Then, Equation (5) is reduced to;

$$\begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix} = \frac{K}{2}bK_{tc}\begin{bmatrix} A_{xx}(0) & A_{xy}(0) \\ A_{yx}(0) & A_{yy}(0) \end{bmatrix}\begin{bmatrix} FRF_{xx}(j\omega) & FRF_{xy}(j\omega) \\ FRF_{yx}(j\omega) & FRF_{yy}(j\omega) \end{bmatrix}\begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix} \tag{7}$$

As it can be seen above, consideration of the static part of the directional dynamic milling force coefficient matrix ignores the effect of helix angle and pitch of the teeth giving a mean approximation for the dynamic cutting force. The static parts of the individual terms in directional dynamic milling force coefficient matrix, $[A_F(0)]$, can be found in Ref. [1]. Considering the critical process stability, the roots of the characteristic equation is obtained from the determinant of Equation (7) as;

$$\det\left(I - \frac{K}{2}bK_{tc}\begin{bmatrix} A_{xx}(0) & A_{xy}(0) \\ A_{yx}(0) & A_{yy}(0) \end{bmatrix}\begin{bmatrix} FRF_{xx}(j\omega_c) & FRF_{xy}(j\omega_c) \\ FRF_{yx}(j\omega_c) & FRF_{yy}(j\omega_c) \end{bmatrix}\right) = 0 \tag{8}$$

Equation (8) is referred in Ref. [25] as the characteristic equation for mode coupling chatter in milling. The eigenvalue of the characteristic equation is,

$$\lambda = -\frac{K}{2}bK_{tc} \tag{9}$$

Defining $[\Phi]$ to be the oriented transfer function as the product of the directional dynamic milling force coefficient and combined tool-workpiece structural dynamics matrices, the determinant in Equation (8) gives the following relationship; ,

$$(\Phi_{1,1}\Phi_{2,2} - \Phi_{1,2}\Phi_{2,1})\lambda^2 + \lambda(\Phi_{1,1} + \Phi_{2,2}) + 1 = 0 \tag{10}$$

The solution for the above quadratic equations gives two solutions, $\lambda_{1,2}$, and due to the nature of $FRF(j\omega)$, both solutions for $\lambda_{1,2}$ are complex numbers. Substituting, $\lambda$ into Equation (9) delivers;

$$b_{lim1,2} = \frac{-2(\lambda_{RE} + j\lambda_{Im})_{1,2}}{KK_{tc}} \tag{11}$$

In order for depth of cut $b_{lim}$ to be an admissible solution, it has to be a positive real number with zero imaginary part. For this reason, the depth of cut in Equation (11) is only valid for chatter frequencies which makes the imaginary part of the eigenvalues $\lambda_{Im}$ zero at the same time resulting in a negative real part of the eigenvalues $\lambda_{Re}$. Having satisfied these conditions, the admissible positive real solution for depth of cut is,

$$b_{lim1,2} = \frac{-2(\lambda_{RE})_{1,2}}{KK_{tc}} \tag{12}$$

In order to obtain a valid solution for depth of cut $b_{lim}$, a specified range of chatter frequencies ($\omega_c$ (rad/s) where $\omega_c = 2\pi f_c$, $f_c$ (Hz) being chatter frequency in Hertz) should be scanned by 1D optimisation algorithm such as golden section search method [29]. The admissible solution would give a positive $b_{lim1,2}$ with zero imaginary part for the chatter frequency, $\omega_c$. Note that, the limiting depth of cut for zeroth order approximation mode coupling in milling does not depend on spindle speed. As a result of the assumption made, the dynamic chip thickness does not depend on the undulations left on the chip surface on the previously cut side. For this reason, the dynamic chip thickness is independent from tooth passing period and spindle speed. This means that the smallest limiting depth of cut for ZOA mode coupling chatter should dominate the process stability.

### 2.2. *Multi frequency approximation*

In order to accurately acquire the stability boundary for low radial immersion milling operations with high intermittent cutting characteristics, better estimation of the cutting force profile is required. Henceforth, this necessitates the directional dynamic milling force coefficients to be computed with a non-zero harmonic number, *r*. Higher the harmonics, the more accurate the stability prediction with an expense of the computation power and time requirements. As opposed to the zeroth order approximation mode coupling chatter in milling, the multi frequency approximation depends on the spindle speed as stated in Equation (5). This is due to the directional dynamic milling force coefficients are periodic at the spindle pass frequency and its harmonics.

Equation (5) defines a relationship between the vibrations in frequency domain with itself, regulated with harmonics *r* as appears in the directional dynamic milling force coefficient. Adapting the approach from Ref. [27], it is expressed in more compact way by introducing $r = p - q$ format as;

$$\begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix}_p = \begin{bmatrix} X(j\omega + jp\Omega) \\ Y(j\omega + jp\Omega) \end{bmatrix} \tag{13}$$

$$[FRF(j\omega)]_{p,p} = [FRF(j\omega + jp\Omega)] \tag{14}$$

$$[\Psi(j\omega)]_{p,q} = \frac{1}{2} K_{tc} \sum_{k=1}^{K} \int_{b_a=0}^{b} e^{jr(\varphi_{k0}+\beta_k b_a)} [A_F(r)] db_a \tag{15}$$

Therefore,

$$\begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix}_{p,p} = [FRF(j\omega)]_{p,p} \sum_{q=-\infty}^{\infty} [\Psi(j\omega)]_{p,q} \begin{bmatrix} X(j\omega) \\ Y(j\omega) \end{bmatrix}_{q,q} \tag{16}$$

The above formulation summarises the multi frequency approximation in mode coupling chatter in milling which takes into account the tool geometrical features such as helix angle and pitch while predicting the stability boundary.

## 3. Experimental set-up

A milling trial was conducted with a serial industrial robot, ABB IRB 6640 205/2.75 equipped with a GMN HV-P 150 - 30000/26 high-speed spindle and an end mill in a Regofix tool holder with a ProMicron sensory ring and HSKA63 interface. The experimental set-up is illustrated in Fig. 3.

Machining with serial industrial robots is expected to result in low regenerative chatter stability boundaries for a given material compared to CNC machine tools [15]. This is mainly due to the low rigidity of robots compared to machine tools. For this reason, a material softer than aluminium needs to be selected to facilitate the process stability identification and minimise the effect of process parameter uncertainties on the process stability. In this respect, polyacetal (acetal polymer, Polyoxymethylene) was chosen as a workpiece material. It has been previously shown that the stability of machining acetal polymer is similar to metal cutting as in Refs. [30,31]. The predicted stability boundaries were in an excellent agreement with experimental findings. Nevertheless, possible inaccuracies coming from the application of the conventional cutting force identification model to machining acetal polymer are acknowledged.

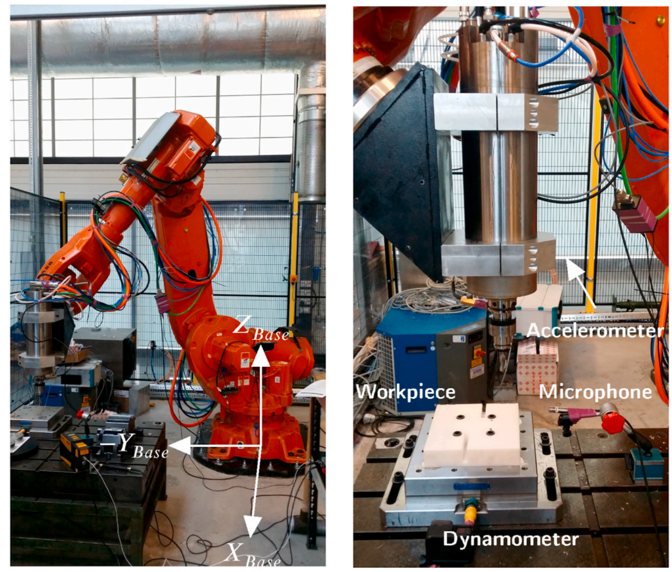


**Fig. 3.** Experimental set-up.

For chatter identification, tri-axial accelerometer was mounted onto the spindle holder, a microphone was placed close to the cutting trajectories. The details of the chatter identification procedure can be found in Appendix A. A Kistler 9255C type dynamometer was used to measure cutting forces and clamp the workpiece. It was also aligned with Base Frame ($X_{Base}Y_{Base}Z_{Base}$). Cutting tests were designed to be 50% radial immersion down milling operations with 0.15 mm per tooth feed rate. Table 1 details parameters of the regular milling tool and workpiece used in cutting tests.

For each test, cutting parameters were kept constant (radial immersion and feed per tooth) and a straight trajectory was followed. For low spindle speed tests, a square workpiece with a 50 mm cutting length was used whereas for moderate spindle speed tests a rectangular workpiece with a 250 mm cutting length was chosen. The tool axis of rotation was also set in the direction of the negative $Z_{Base}$ of the Base Frame. For all tests, the tool was fed along the positive $X_{Base}$ direction as shown in Fig. 3.

## 4. Structural dynamic identification

Manipulator structural dynamics were identified by tap testing with two different instrumented impact hammers when the manipulator was

**Table 1**
The machining parameters used throughout the machining test.

| | | |
|---|---|---|
| Tool | Type | End Mill |
| | Diameter | 16 mm |
| | Pitch | 120° |
| | Helix Angle | 35° |
| | Number of teeth | 3 |
| | Overhang Length | 51.5 mm |
| Workpiece | Material | Acetal Copolymer |
| | Tangential CFC ($K_{tc}$) | 142.2 MPa [31] |
| | Radial CFC ($K_{rc}$) | 18.9 MPa [31] |

configured to the beginning of the cutting trajectory. A Dytran 5803A impulse sledgehammer was used to excite low frequency spectrum of FRF to identify the low frequency structural modes. Due to the size of the hammer however, the structure was excited at the spindle holder rather than the tool tip in order not to damage the spindle. Higher frequencies were excited with a Kistler 9722A500 impulse hammer at the tool tip. For measuring the tool tip response, PDV-100 portable digital vibrometer was used which is able to measure low-frequency response of the system with high accuracy compared to an accelerometer.

Note that, FRFs obtained by the sledgehammer are not from the tool tip hence, this may introduce inaccuracies to stability predictions. Low frequency modes of vibration are known to come from the structural modes of vibration and the connection between spindle holder and tool interface was quite rigid. Thus, the connection between spindle holder and tool interface was assumed to have negligible effect on the low frequency response of the structure. As a result, FRFs identified by Dytran 5803A impulse sledgehammer were assumed to be tool tip FRFs and any inaccuracies are acknowledged.

6-axis serial industrial machining robots tend to have functional redundancy around the axis of rotation of the tool [18]. The orientation of the tool resulted in the functional redundancy to coincide with the negative $Z_{Base}$ and it is denoted as γ. The identification was carried out with two manipulator configurations having different functional redundancy parameters for a given 5-axis pose located around the workpiece; $\gamma = [\pi, 2\pi]$ as discussed in Ref. [32]. The manipulator configurations were named “Configuration 1” and “Configuration 2” respectively and are illustrated in Fig. 4.

During identification, joint brakes were deactivated to identify the quasi-static structural dynamics of the robot. The manipulator structural dynamics are configuration dependent however, configuration alteration as a result of tool translation is known to marginally affect the structural dynamics [20,33]. For this reason, structural dynamics variation across the cutting trajectory was assumed to be negligible and is out of the scope.

Having identified FRFs, the state space model of the structural dynamics was constructed to be applied to zeroth order and multi frequency approximations mode coupling and regenerative chatter predictions. In this respect, an experimental modal identification toolbox, Structural Dynamics Toolbox (SDT), was utilised to derive the state space model from experimental FRFs identified by both hammers. The structure was assumed to behave like linear elastic structure with modal damping, hence, the normal mode model was assumed with a symmetric pole structure. In addition, multi input multi output (MIMO) reciprocity was assumed which implies that the cross FRFs ($FRF_{xy}$ & $FRF_{yx}$) behave the same.

In low frequencies, the dominant structural modes below 40 Hz were modelled. The comparison of identified and modelled direct and cross FRFs in low frequency spectrum for Configuration 1 is represented in Fig. 5.

In higher frequencies, only the first mode coming from the spindle shaft was chosen to be modelled due to its relatively higher dynamic compliance compared to the higher frequency modes. The identified and modelled direct and cross-FRFs are also shown in Fig. 6.

It was observed that structural modes are located in low frequencies as also mentioned in various works [20,32] whereas higher modes are coming from the spindle shaft and tool. Modelled direct-FRFs were observed to have slight mismatch with identified FRFs for the considered modes. Identified cross-FRFs were observed to be mostly symmetrical in low frequencies which justifies the assumption of MIMO reciprocity. In higher frequencies, however, cross-FRFs increasingly got dissimilar relative to each other which could explain the mismatch of the modelled cross-FRFs to the measured ones. Overall, modelled and identified FRFs agree well and the mismatch could be attributed to dynamic non-linearities which do not fit to the applied assumptions. Similarly, the structural dynamics of Configuration 2 is identified, modelled and depicted in Appendix B.

## 5. Stability predictions and validations

Zeroth order and multi frequency approximation stability predictions of mode coupling chatter in milling were obtained and validated with robotic milling tests. Two regions of stability lobe diagram (SLD) were investigated based on the choice of spindle speed; low and moderate spindle speed regions. Additionally, stability predictions for regenerative and mode coupling chatter in milling were compared to conclude the chatter type of milling tests.

### 5.1. Mode coupling chatter zeroth order approximation stability predictions

Milling tests at very low spindle speeds were carried out in order to explore the presence of mode coupling chatter and effect of low frequency structural modes on the stability characteristics at this region. In this region, zeroth order approximation stability solutions for Configuration 1 are depicted in shown in Table 2. These predictions are obtained using the theory presented in Section 2.1.

Table 2 shows that the smallest limiting depth of cut for mode coupling chatter stability boundary is 0.8 mm with a chatter frequency 14.89 Hz. Having previously underlined that mode coupling chatter does not depend on spindle speed, the smallest limiting stability

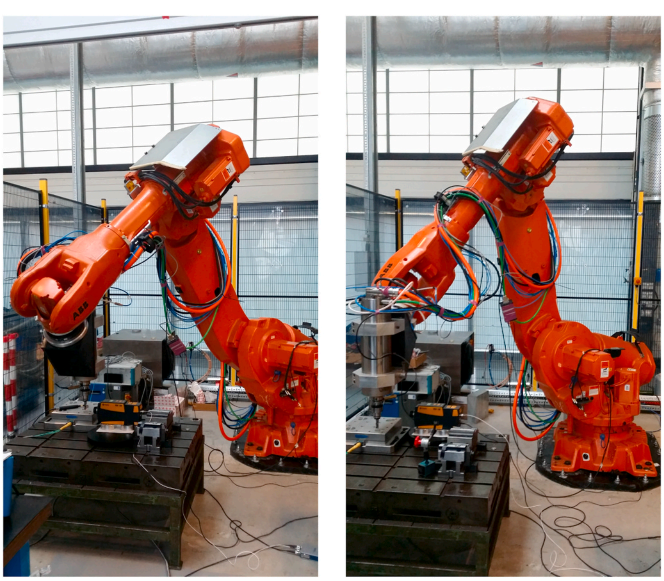

Fig. 4. Tap tested manipulator configurations with functional redundancy was set to a) γ = π (Configuration 1) b) γ = 2π (Configuration 2).

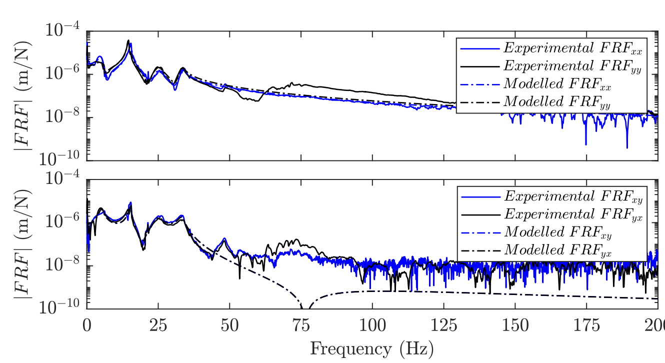


Fig. 5. Comparison of experimentally identified and modelled direct and cross-FRFs of “Configuration 1” across low frequency spectrum.

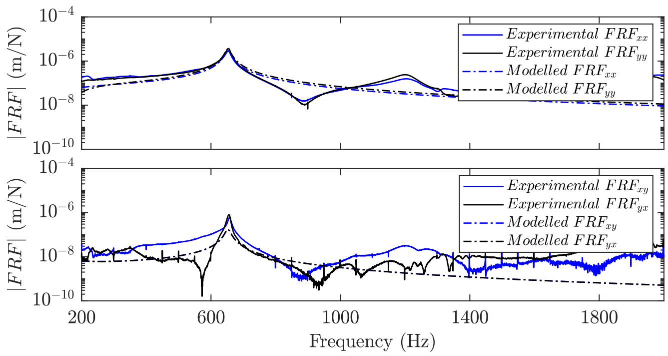


**Fig. 6.** Comparison of experimentally identified and modelled direct and cross-FRFs of "Configuration 1" across high frequency spectrum.

**Table 2**
The zeroth order approximation for mode coupling chatter stability for Configuration 1.

| | | | | | | |
|---|---|---|---|---|---|---|
| Limiting Depth of Cut, $b_{lim}$ (mm) | 0.8 | 64.0 | 21.3 | 54.6 | 5860.5 | 7.8 |
| Chatter Frequency, $f_c$ (Hz) | 14.9 | 24.1 | 29.5 | 54.5 | 81.5 | 654.5 |

boundary and stability of tests were accepted as the mode coupling chatter stability boundary. The comparison of mode coupling chatter prediction and stability of milling tests is depicted in Fig. 7.

The stability prediction was observed to be spindle speed independent and quite low. Milling tests were planned to cover axial depth of cuts of 1.5 mm and above to reduce the effect of poor pose accuracy of industrial robots on the stability of tests. For this reason, no experimental data was collected at depth of cuts below 1.5 mm. Tests showed that process could be stable even well above the predicted stability boundary. Identified process stability was also observed to be spindle speed dependent; the choice of the spindle speed has observed to affect the stability of the process. Predicted and experimentally identified mode coupling chatter frequencies, however, showed close agreement which were around 15 Hz and 14–15 Hz respectively.

Stability predictions in moderate spindle speed region were computed for both of the robot configurations. The predicted stability of Configuration 1 is validated with milling tests as illustrated in Fig. 8.

The stability prediction was observed to be spindle independent in moderate spindle speeds as well. Tests, however, showed significant mismatch with the prediction. Tests were observed to be stable at higher depth of cuts and spindle speed dependent as opposed to the prediction. Nonetheless, the process chattered around low frequency structural modes as predicted.

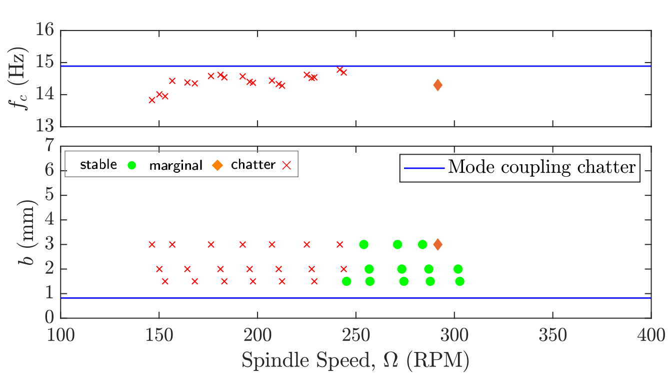


**Fig. 7.** Zeroth order approximation stability prediction & validation for lower spindle speed tests when the functional redundancy was set to $\gamma = \pi$ (Configuration 1).

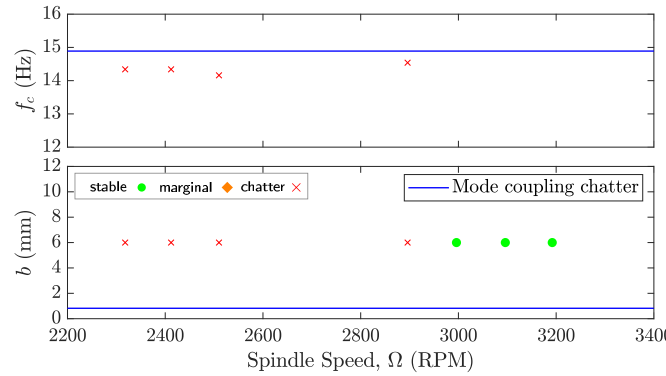


**Fig. 8.** Zeroth order approximation stability prediction validation for moderate spindle speed tests when the functional redundancy was set to $\gamma = \pi$ (Configuration 1).

For Configuration 2, possible ZOA mode coupling chatter stability solutions based on the theory presented in Section 2.1 are shown in Table 3.

The minimum limiting depth of cut for Configuration 2 was found to be 0.7 mm showing a chatter frequency of 12.6 Hz. Having identified the limiting stability boundary of mode coupling chatter, the validation of predicted stability boundary with milling tests is depicted in Fig. 9.

As in low spindle speeds, the stability boundary was observed to be spindle speed independent and low. Additionally, the stability of tests were completely disagreeing to the prediction. The stability of tests were spindle speed dependent with experimentally identified critical depth of cut being much larger than the predicted one. Identified chatter frequencies were around spindle shaft mode at higher frequencies as opposed to predicted mode coupling low frequency chatter frequency boundary which was around the most complaint structural mode.

To sum up, mode coupling chatter zeroth order approximation stability predictions were spindle speed independent as proven in Equation (12). The predicted critical depth of cut was low however, the stability of cutting tests were spindle speed dependent. This is the first proof that implies that the mode coupling chatter mechanism do not match to the experimentally identified chatter characteristics. Additionally, the predicted mode coupling chatter frequencies were always around the most compliant structural mode which is at low frequencies. The predicted and identified chatter frequencies at low spindle speed tests were in close agreement. In moderate spindle speed tests, however, the "experimentally identified critical depth of cut" and chatter frequencies were observed to depend on the choice of the spindle speed and robot configuration. Based in the choice of robot configuration, experimentally identified critical depth of cut and chatter frequencies could be higher in magnitude. Hence, this is the second proof that mode coupling chatter mechanism can not perfectly describe the experimental process stability.

Note that as the depth of cut was increased for a chosen spindle speed in Figs. 7–9, a reduction in the actual spindle speed was observed. This was due to the fact that spindle was not equipped with a control loop to make sure the chosen and actual spindle speed matches.

The fact that zeroth order approximation does not consider the effect of spindle speed on the predictions motivated consideration of the multi frequency approximation in the next section. In this way, the possibility of existence of mode coupling chatter vibrations is aimed to be further

**Table 3**
The zeroth order approximation for mode coupling chatter stability for Configuration 2.

| | | | | | | |
|---|---|---|---|---|---|---|
| Limiting Depth of Cut, $b_{lim}$ (mm) | 0.7 | 2.0 | 110.9 | 86.6 | 351.4 | 6.6 |
| Chatter Frequency, $f_c$ (Hz) | 12.6 | 13.5 | 28.3 | 31.6 | 45.1 | 655.5 |

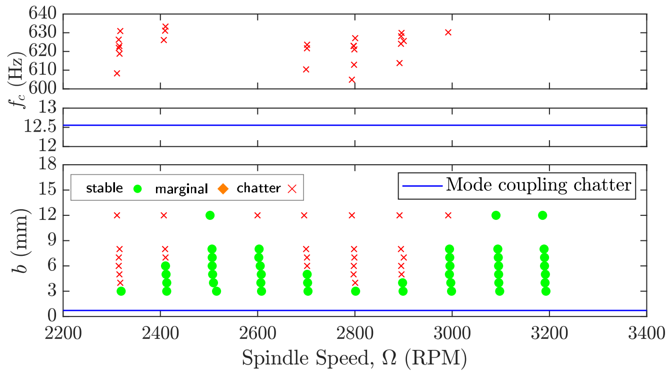


**Fig. 9.** Zeroth order approximation stability prediction validation for moderate spindle speed tests when the functional redundancy was set to $\gamma = 2\pi$ (Configuration 2).

investigated in milling.

### 5.2. Mode coupling chatter multi frequency approximation stability predictions

ZOA mode coupling chatter stability boundaries were observed to be spindle speed independent as observed in Figs. 7–9. However, consideration of multi frequency approximation (MFA) introduces spindle speed dependency to the mode coupling chatter predictions as a result of considering spindle speed harmonics as in Equation (16). For this reason, multi frequency approximation (MFA) was utilised to improve mode coupling chatter stability predictions by allowing spindle speed to be taken into account. The number of harmonics used was set to maximum and was altering based on the choice of the spindle speed and the selected maximum frequency (1000 Hz) as in Ref. [27]. In this respect, chosen machining parameters in Table 1 were used to compute stability predictions. In low spindle speed region, MFA prediction using the theory from Section 2.2 and stability of tests are illustrated for Configuration 1 in Fig. 10.

It can be seen that, for the chosen robot configuration at the beginning of the cutting trajectory, while stability boundary stayed the same, it did not show any spindle speed dependency. On the other hand, the multi frequency stability predictions for the robot configuration at the end of the cutting trajectory showed slight spindle speed dependency as shown in Appendix C. Nevertheless, the stability boundary prediction did not show strong spindle speed dependency as in milling tests. Consequently, multi frequency mode coupling chatter stability predictions were not satisfactorily describing the tests.

MFA prediction based on the theory presented in Section 2.2 and experimentally identified stability of tests are also depicted for Configuration 2 in moderate spindle speeds in Fig. 11.

MFA prediction in moderate spindle speeds did not show spindle speed dependency and an improvement in the critical depth of cut to match that of identified with tests.

All in all, MFA was not able to represent the behaviour of stability of milling tests as well. Low spindle speed stability predictions showed slight spindle speed dependency or even not spindle speed dependent behaviour depending on the robot configuration. As a result, MFA was also unable to accurately predict the process stability. Findings indicated that the governing chatter mechanism is vaguely predicted by mode coupling chatter stability predictions in milling with quantitative disagreement.

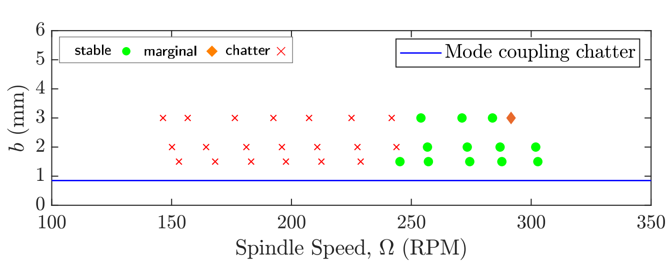


**Fig. 10.** Multi frequency stability prediction validation for low spindle speed tests when the functional redundancy was set to $\gamma = \pi$ (Configuration 1).

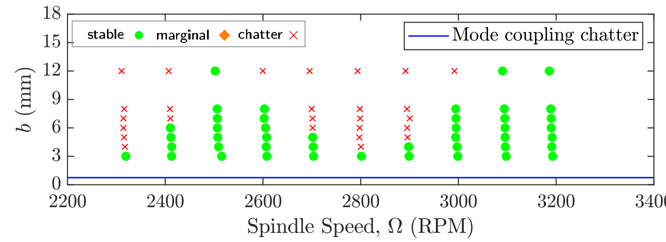


**Fig. 11.** Multi frequency stability prediction validation for moderate spindle speed tests when the functional redundancy was set to $\gamma = 2\pi$ (Configuration 2).

### 5.3. Regenerative vs mode coupling chatter

Zeroth order and multi frequency approximation stability predictions showed significant mismatch with most of the cutting tests. In order to justify the presence of mode coupling chatter mechanism and distinguish it from regenerative chatter mechanism, the zeroth order approximation stability predictions for both chatter mechanisms were compared. The stability predictions in low spindle speeds for Configuration 2 are shown in Fig. 12.

Despite the slight disagreement of regenerative stability boundaries and tests, regenerative chatter mechanism was able to predict the spindle speed dependent stability of the process as opposed to mode coupling chatter mechanism. Chatter frequencies predicted by regenerative chatter mechanism were also in close agreement with tests.

Stability predictions while milling with Configuration 2 in moderate spindles speeds are illustrated in Fig. 13.

As in low spindle speeds, regenerative chatter mechanism was able to predict the spindle speed dependent process stability in moderate spindle speeds as well unlike mode coupling chatter mechanism. The predicted regenerative chatter frequencies followed a increasing trend along the same considered lobe as the spindle speed increased in Fig. 13. However, there observed to be a shift in the location of the predicted stability boundaries compared to that of experimentally identified. Nevertheless, experimentally identified critical depth of cut was pretty accurately predicted as opposed to the very low stability boundary of mode coupling chatter. Additionally, predicted and identified chatter frequencies were found to be agreeing well for regenerative chatter

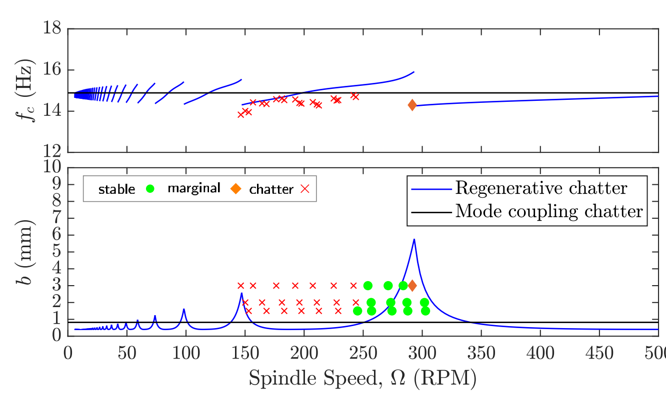


**Fig. 12.** Regenerative and mode coupling chatter zeroth order approximation stability prediction comparison at low spindle speed region when the functional redundancy was set to $\gamma = \pi$ (Configuration 1).

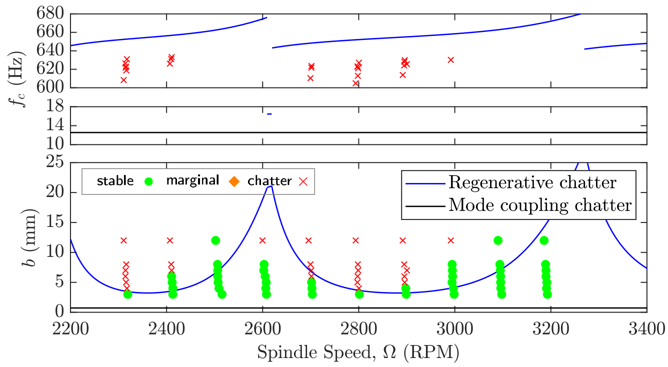


**Fig. 13.** Regenerative and mode coupling chatter zeroth order approximation stability prediction comparison at moderate spindle speed region when the functional redundancy was set to $\gamma = 2\pi$ (Configuration 2).

mechanism.

Lastly, stability predictions and tests are compared for Configuration 1 in moderate spindle speeds as in Fig. 14.

In this case, stability predictions from both regenerative and mode coupling chatter mechanisms seemed to disagree with tests. However, taking into account the shift in the location of the predicted stability boundaries compared to the experimentally identified stability as shown in Fig. 13, regenerative chatter mechanism could explain the change in the stability of the process. Despite the agreement of predicted and identified chatter frequencies for both mechanisms, regenerative chatter mechanism was again able to describe the spindle speed dependent behaviour of milling tests.

## 6. Discussion

With the aim of identifying whether mode coupling chatter happens in milling or not, ZOA mode coupling and regenerative chatter stability predictions as well as MFA mode coupling chatter predictions were compared with robotic milling tests.

Mode coupling chatter predictions and stability of robotic milling tests showed prominent mismatch. The mismatch extends across the entire spindle speed region even though chatter frequencies showed close agreement in low spindle speed region. Zeroth order approximation stability predictions did not show spindle speed dependency but the experimentally identified stability characteristics indicated that the stability depends strongly on the spindle speed. In an attempt to model the spindle speed dependency and increase the accuracy of the stability predictions, the multi frequency approximation was computed. However, MFA predictions showed either slight spindle speed dependency or non-spindle speed dependent behaviour which was not enough to explain the stability behaviour of tests.

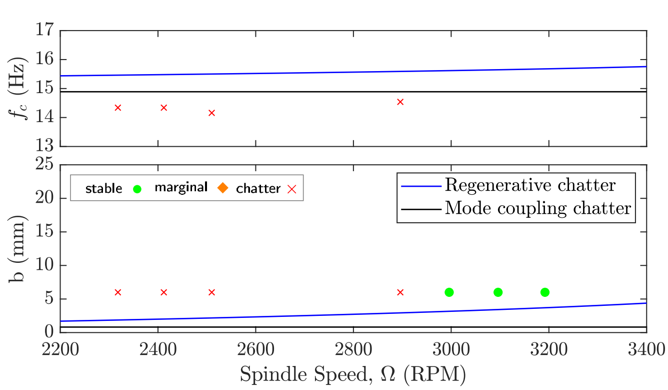


**Fig. 14.** Regenerative and mode coupling chatter zeroth order approximation stability prediction comparison at moderate spindle speed region when the functional redundancy was set to $\gamma = \pi$ (Configuration 1).

The comparison of regenerative and mode coupling stability boundaries revealed that the stability of milling tests could be better explained with regenerative chatter mechanism. The regenerative stability boundaries were able to describe the spindle speed dependent behaviour of milling tests even though predictions were not perfectly fitting to the stability of tests. The mismatch could be due to the effect of interaction of tool-workpiece on the structural dynamics of the robot shifting the natural frequencies of the modes of vibration. Identified chatter frequencies being at lower frequencies than their predicted magnitudes, especially at moderate spindle speeds, could be an indication of modes of vibration being affected by the process. Hence, this could explain why the experimentally identified process stability was shifted to lower spindle speeds with respect to the predicted stability boundaries.

The immediate implication of the significant disagreement between the theory and experiment is the hypothesis of appearance of the mode coupling chatter is incorrect, for the considered machining scenario - milling. The disagreement justifies that mode coupling chatter is not present in milling operations. This is mainly due to the false assumption which states that the dynamic chip thickness depends only on the current normal vibration on the chip surface in milling. Instead, stability predictions from both chatter mechanism indicated that the dynamic chip thickness depends not only the current undulations but also the undulations left by the previous tooth on the surface of the chip.

The stability of aluminium milling with a serial industrial robot was shown to be dominated by regenerative vibrations at low and high spindle speeds in Ref. [20]. However, no effort was made to explore the presence of mode coupling chatter. After quantitatively justifying (with stability boundary predictions and milling tests) that mode coupling chatter does not exists in milling, exploration of regenerative stability limits in milling is sufficient to determine the milling stability as concluded within this manuscript. This means that the results in Ref. [20] are in congruence with the findings of this manuscript.

Employment of acetal polymer as a workpiece material rather than a metallic part could raise concerns of whether regenerative chatter theory can be accurately applied to machining plastics or not. Regenerative chatter theory relies on the assumption that chip formation mechanism happen by shearing along a single thin plane extending from the tip of the tool to the surface of the workpiece (continuous chip formation) [1]. However, not every material exhibits continuous chip formation such as aluminium (built up edge chip formation) and titanium (discontinuous chip formation) when machined [34,35]. Nevertheless, it was shown that the process stability could be accurately predicted by regenerative theory in Refs. [36–38] for these materials. Similar to titanium, acetal polymer also exhibits discontinuous chip formation [39] however the work done in literature suggests that regenerative chatter theory was accurately applied and validated when milling acetal polymer in Refs. [30,31]. This means that chip formation mechanism does not significantly affect application of regenerative theory for stability prediction no matter the material being metallic or acetal. As per the scope of the paper, the choice of workpiece material does not violate the regenerative theory and applicability of the findings to metals.

One could argue that tests were not carried out below mode coupling chatter stability boundaries, which may not extensively explore existence of mode coupling chatter phenomenon. Choice of depth of cuts was to minimise the effect of poor pose accuracy of the robot (especially inaccuracies in actual depth of cut and radial immersion) on the stability of tests. Tool static deflection for a flexible machine tool (robot) could be large enough to cause variations in process stability characteristics as opposed to machine tools. Having carried out the tests at 1.5 mm depth of cut and above, measurements revealed that tests were subjected to ± 0.2 mm variation in depth of cut and 5% variation in radial immersion (in average). In low spindle speeds, even though test data with depth of cut lower than 1.5 mm might have existed, the closeness of stability boundaries of mode coupling and regenerative chatter would not make findings a strong evidence for chatter type differentiation considering the possible errors in practice. However, the absence of test data below mode coupling chatter stability boundaries does not invalidate the conclusions reached. Stable regions above mode coupling chatter stability boundaries as well as spindle speed dependency of tests could only be explained by regenerative and not mode coupling chatter theory. Hence, having identified stable regions at depth of cuts, where inaccuracies such as poor pose accuracy have less influence on stability of tests, proves that mode coupling chatter does not happen in milling.

It could also be argued that, for very low spindle speeds and radial immersion cuts, the regeneration effect could diminish and be dominated by rubbing between the tool and workpiece causing process damping. First of all, it should be noted that, mode coupling chatter is a special type of chatter and should not be confused with the process damping (or rubbing) effect. Process damping results in increased stability at lower spindle speeds due to the dampening of higher number of waves generated on the surface normal for the active mode of vibration as a result of regeneration. In this work, however, even though very low spindle speeds were considered, milling tests were in the region where the stability was dominated by the first lobe (the number of waves generated on the surface normal was one) for the low frequency structural mode as shown in Fig. 12. The appearance of instability at even lower spindle speeds clearly indicates that process damping was not significant at the region considered for the low frequency mode. Otherwise, it would have been expected that stability at low spindle speeds would have just been increased drastically. Additionally, detailed cutting tests with very low radial immersion proved that process damping happens where the regenerative stability is dominated by the higher lobe numbers for the considered mode of vibration at lower spindle speeds [40,41]. For this reason, the hypothesis of appearance of mode coupling chatter due to negligible chip regeneration and rubbing effect (process damping) at low radial immersions has already been eliminated by application and validation of the regenerative chatter theory with process damping in the literature.

Findings suggest that there is a misunderstanding of the mode coupling chatter mechanism in the literature. The origin of the mode coupling chatter leans on to the unstable vibrations while machining a new surface at all times such as thread cutting operations, where dynamic chip thickness is affected by the current (free) vibrations of the structure at the surface normal. Hence, mode coupling chatter principles can not be applied to milling operations in which the tool always cuts the previously cut surface resulting in regenerative dynamic chip thickness. In milling, the mechanism of dynamic chip thickness generation always depends on the difference between current and previous undulations on the chip surface due to the nature of the operation. As a result, this paper served to experimentally investigate chatter characteristics of milling with flexible machine tool systems in an attempt to identify the type of the self-excited instability of the process; mode coupling and regenerative chatter vibrations, which have not been addressed in the literature. Given the conclusions reached, mode coupling chatter is not possible in milling as the main mechanism of dynamic chip thickness generation does not involve free vibration of the system.

## 7. Conclusion

This paper aimed to explore whether mode coupling chatter vibrations exist in (robotic) milling operations or not. The theory of zeroth order approximation mode coupling chatter mechanism in milling is adapted from the literature and extended to multi frequency approximation to predict stability boundaries and validate with milling tests.

The stability of robotic milling tests was observed to depend strongly on the choice of spindle speed and depth of cut. However, mode coupling chatter stability boundaries (zeroth order and multi frequency approximations) in milling were not able to describe the spindle speed dependent process stability. Additionally, even though predicted and identified chatter frequencies were very close at low spindle speeds, there was a quantitative mismatch at moderate spindle speeds; the process was observed to chatter at much higher frequencies and experimentally identified critical depth of cut was higher than predicted. All these findings implied that mode coupling chatter predictions in milling do not comply with stability of milling tests. The comparison between stability boundaries of regenerative and mode coupling chatter mechanism revealed that regenerative chatter mechanism effectively describes the spindle speed dependent process stability. Identified and predicted chatter frequencies were also in close agreement at low and moderate spindle speeds.

Overall, regenerative chatter mechanism was shown to describe the stability of cutting tests much more effectively than mode coupling chatter in milling. As a result, mode coupling chatter is concluded to be impossible in milling with stability predictions and experimental tests, which is the main contribution of the paper. This is due to the nature of the milling operation with the rotary tool always cutting the previously cut chip surface as it removes material from the workpiece. Hence, the chip thickness variation not only depends on current but also previous undulations on the surface of the chip, which contradicts the underlying theory of mode coupling chatter in milling. This could help redirecting future research on the stability of (robotic) milling.

## Author statement

Huseyin Celikag: Methodology, Experiment, Investigation, Writing – Original Draft. Erdem Ozturk: Conceptualization, Supervision, Writing – Review & Editing, Funding acquisition. Neil D. Sims: Conceptualization, Supervision, Writing – Review & Editing, Funding acquisition.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix

### *A. Chatter identification*

In order to identify chatter, the steady state region of the milling process was manually determined from the cutting force data. Having determined the steady state region, the hall effect sensor was used to identify the exact spindle speed by taking the Fast Fourier Transform (FFT) of the voltage data which tracks the position of the key hole located on the tool holder. Similarly, the FFT of the acceleration and sound pressure from microphone was computed in order to find their frequency content. Both sensors were simultaneously used to support each other in identifying the stability of the process. Chatter frequencies were identified from the FFT of the acceleration and microphone data based on comparing the relative magnitude of a suspected chatter frequency to the third harmonic of the spindle speed (tooth passing frequency, $\omega_T = \frac{3\Omega}{60}$ Hz). An exemplary scenario where one of the operations was found stable whereas the other one was found unstable from the FFT of the acceleration data was illustrated in Figs. 15 and 16.

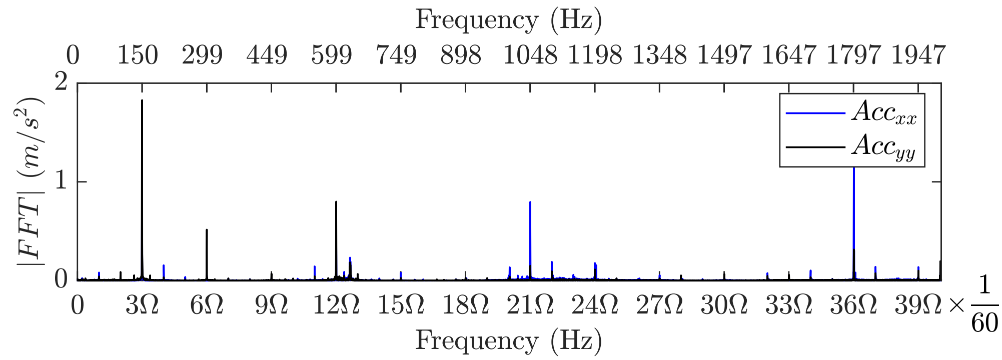


**Fig. 15.** The FFT of acceleration signals for stable cutting operation while cutting with moderate spindle speed.

The spindle speed was identified to be 2995 RPM where the third harmonic of the spindle frequency, tooth passing frequency, was observed to have the highest magnitude. A closer look into the frequency spectrum revealed that the rest of the peaks correspond to harmonics of the spindle speed due to the periodic nature of the forcing frequency (spindle frequency). This meant that the process was stable in Fig. 15.

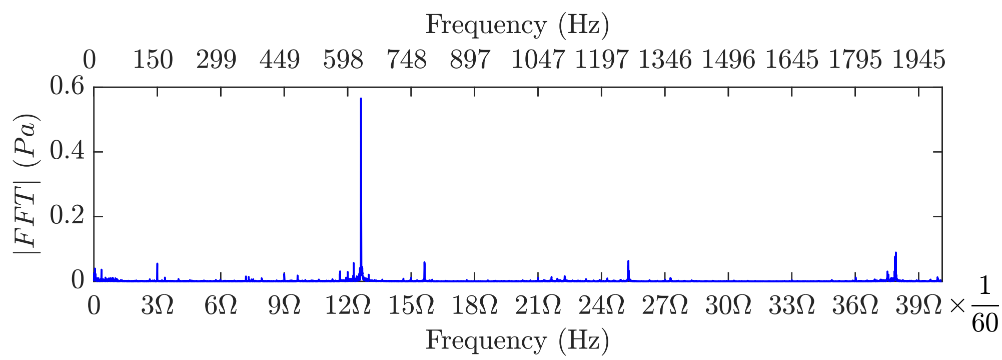


**Fig. 16.** The FFT of acceleration signals for two cutting scenarios while cutting with moderate spindle speed a) Stable b) Unstable.

In here, the magnitude of the third harmonic of the spindle frequency, tooth passing frequency, was observed to be much lower than the peak at 630 Hz. This peak was found not to comply with the forcing frequency (spindle) harmonics and was found to be located close to the tool-spindle shaft mode (around 650 Hz). All these indications strongly pointed out that the peak was a chatter frequency at tool-spindle shaft mode.

*B. Structural dynamics of "Configuration 2"*

The comparison of identified and modelled direct and cross-FRFs of Configuration 2 in low frequency spectrum for Configuration 2 is illustrated in Fig. 17.

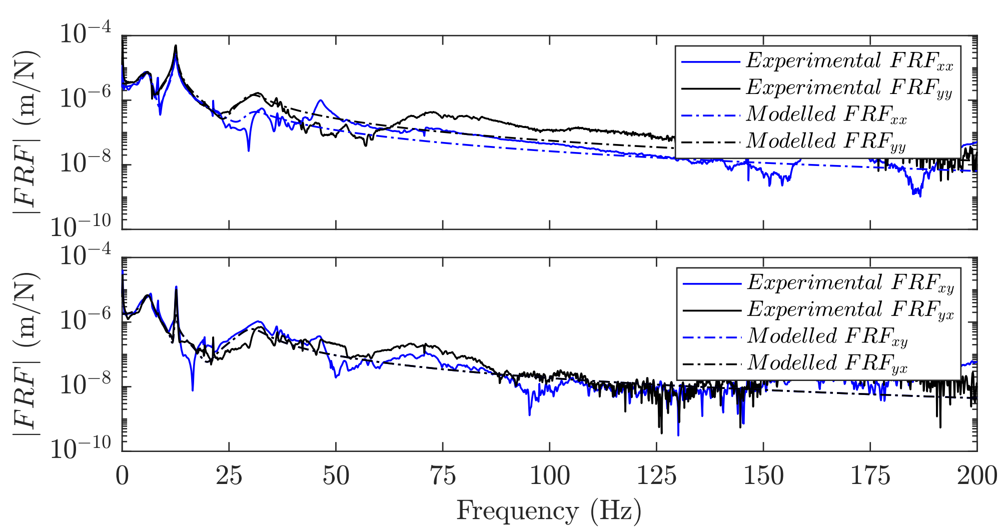


**Fig. 17.** Comparison of experimentally identified and modelled direct and cross-FRFs of "Configuration 2" across low frequency spectrum.

When compared to Fig. 5, low frequency structural modes of vibrations were observed to be configuration dependent. The configuration dependency of the structural modes also explains the configuration dependent process stability identified with the milling tests in Section 5. As before, the identified and modelled direct FRFs showed slight mismatch and cross-FRFs can be said to be mostly symmetrical considering the fact that slight magnitude deviations could be due to operator and set-up.

Similarly, the comparison of identified and modelled direct and cross-FRFs of Configuration 2 in high frequency spectrum for Configuration 2 is illustrated in Fig. 18.

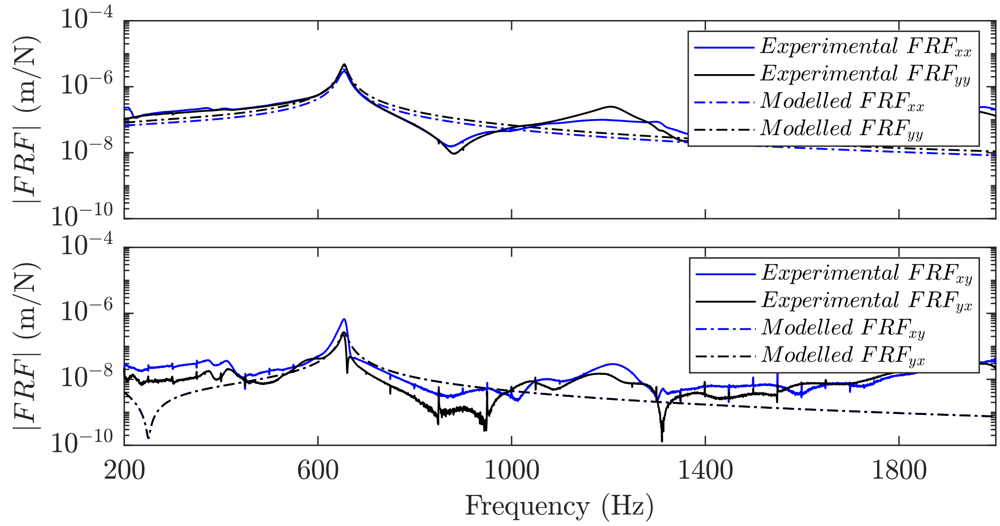


**Fig. 18.** Comparison of experimentally identified and modelled direct and cross-FRFs of "Configuration 2" across high frequency spectrum.

In high frequencies, the mode of vibrations associated with the spindle shaft and tool were largely unchanged apart from their cross-FRFs. The modelled cross-FRFs were observed to have slightly larger mismatch with their experimental counterparts compared to Fig. 6. Nevertheless, the modelled FRFs can be accepted to be accurate enough to analyse the process stability of the robotic milling tests.

*C. Multi frequency mode coupling chatter stability perdition at the end of the cutting trajectory*

MFA prediction and stability of tests are illustrated for Configuration 1 at the end of the cutting trajectory for low spindle speed tests in Fig. 19.

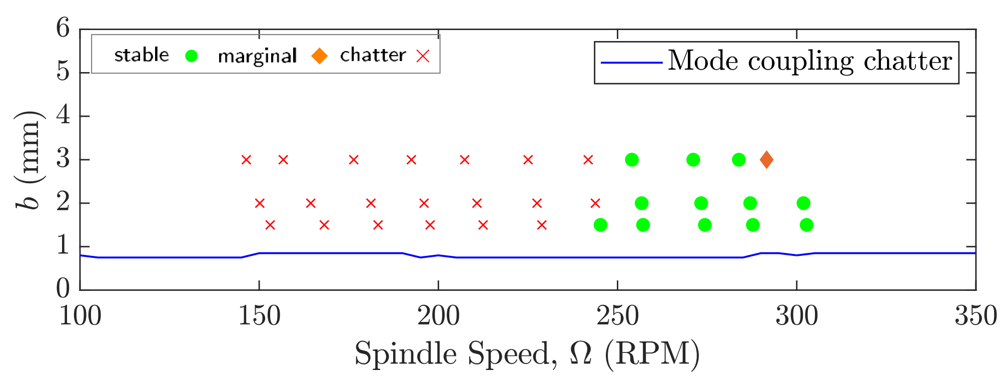


**Fig. 19.** Multi frequency stability prediction at the end of cutting trajectory for low spindle speed tests when the functional redundancy was set to $\gamma = \pi$.

The MFA mode coupling stability prediction showed slight spindle speed dependency however, it was not strong enough to match the stability of milling tests. For this reason, there is a quantitative mismatch between the MFA mode coupling chatter stability prediction with the stability of test.

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