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Incorporating e-commerce into retail location models

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The use of location models in retail businesses is well-established, particularly in the grocery sector. Many alternative methods are in use today but the spatial interaction model (SIM) has a proven record of success. To date, that success relates purely to face to face activities, modelling and predicting visits by consumers to retail outlets. However, grocery retailers are cutting back on store investments and concentrating on investment in the convenience market and e-commerce: the latter has now reached a 7.2% share of the UK grocery market, with continued growth forecast. Whilst spatial models are used extensively for helping to locate new convenience stores, so far e-commerce has not been built into existing retail location models. Yet e-commerce seems to be a spatial activity. Extensive evidence demonstrates the geography of demand and supply are as important in groceries e-commerce as they are in face to face grocery retailing. We therefore take up the challenge of incorporating e-commerce into classic location models. Methodologically, we find the standard distance deterrent term in the production-constrained SIM unsuitable for modelling e-commerce flows: we explore inverting this term and find extensive gains in prediction accuracy, an interesting finding that contributes to the ongoing applied SIM literature.

Key words: e-commerce, spatial interaction model; groceries; retail analytics

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1. Introduction

Mathematical modelling approaches which are capable of reproducing market shares and store revenues in competitive retail markets have become commonplace in the academic literature and in the commercial business practices of retailers, especially those in the grocery sector (Birkin et al., 2010, 2017). Such approaches provide powerful 'what if' market simulations and permit a variety of assessments varying from performance benchmarking to the evaluation of profitability and business impacts (see Newing et al., 2020 for contemporary examples from the grocery sector).

E-commerce is an established distribution channel in many retail product areas. For UK grocery retailing, a market share of 7.2% was recorded in 2018 (Statista, 2019). E-commerce is an important part of grocery retailers growth strategies (Davies et al., 2019) and with predicted growth of almost £6bn in the five years to 2023, online is predicted to be the fastest growing grocery channel (IGD, 2018). Notwithstanding their important contributions outlined above, spatial modelling approaches have yet to adapt to the complexities of contemporary e-commerce behaviours. The specification, testing, enhancement and evaluation of such a model in this paper offers an important and original contribution to the literature on retail location modelling and to e-commerce in the grocery market.

The literature on e-commerce highlights that geography is an important driver of e-commerce uptake amongst consumers. There are a number of reasons for this. First, there are variations in internet engagement (and specifically propensity to order groceries online) which are shaped by underlying neighbourhood level geodemographics (Alexiou, 2018). As explored in the following section, e-commerce uptake exhibits a clear relationship with consumers' gender, age and affluence. There is also evidence that demand may be driven by two competing theories of e-commerce uptake in relation to technological innovation, spreading first from major urban areas (innovation-diffusion theory) or poor access to face to face stores (efficiency theory) (Anderson et al., 2003).

The relationship between store provision and e-commerce uptake highlights the important interplay between supply and demand. In the UK the predominant model for groceries e-commerce is for customer orders to be picked, packed and delivered from a local supermarket or hypermarket. Thus there is an underlying geography to e-commerce availability driven by the geographical coverage and capacity of a retailers' store-based fulfilment network (Videira et al., *in review*). Kirby-Hawkins et al. (2019) also demonstrate that brand loyalties remain important in a grocery e-commerce setting, which are also influenced by underlying store-based provision. These studies, which are considered further in section 2, highlight the importance of a spatial perspective in understanding e-commerce in the grocery market.

E-commerce introduces a more complex set of interactions between retail demand and supply. Common location models, such as spatial interaction models (SIMs), are built on the premise that consumers travel from an origin (such as home or work) to store. With e-commerce the interaction is reversed with the retailer delivering goods to the consumer. As a result there is a more complex suite of interactions between supply and demand, with consumers able to shop interchangeably between in-store and online channels. This paper therefore aims to add e-commerce into classic retail location models as applied in the grocery sector. Although we recognise that other modelling techniques could be used (such as activity-based or regression models) the SIM is widely used by the retail industry itself for location analysis.

First we build and calibrate a classic SIM capturing face-to-face (consumers travelling to stores) interactions using the 'Yorkshire and the Humber' former region of the UK (referred to hereafter as 'Yorkshire') as our study area. Yorkshire offers a number of advantages for this study, having been extensively used as a case study area within the related literature (Clarke et al., 2015; Hood et al., 2020; Kirby-Hawkins et al., 2019) and affording a diverse range of area type and composition in terms of urban-rural classification and affluence, both key drivers of e-commerce demand (Alexiou, 2018). Second, we estimate small-area demand for groceries e-commerce, accounting for underlying small area socio-demographics and physical store access. Third we modify parameters in the SIM to capture the interactions between e-commerce demand and the major retail brands in our model. To do so we develop a novel distance deterrence term which captures the propensity for e-commerce interactions to increase with distance from store (efficiency theory), additionally taking into account brand attractiveness. We calibrate the model using previous studies in the same region which draw upon retailers' actual sales data. This model contains new model parameters which we explore in detail in order to understand how the results vary as these parameters change.

The rest of the paper is structured as follows. In section 2, we briefly explore the literature around the applied use of SIMs and the geography of e-commerce. Then, in section 3, we introduce the models and the extensions needed to add e-commerce. The calibration procedure and results are presented in section 4. We discuss our contribution to the modelling literature in section 5, considering how retailers could use these model adaptions to support their business practice and identify further enhancements required to fully operationalize these models.

2. Retail location models and the geography of e-commerce

The SIM has a long history as an applied retail location tool. With origins in gravity modelling derived from Newtonian physics (i.e. Huff, 1963; Lakshmanan & Hansen, 1965), SIMs gained greater theoretical strength when derived from first principles using entropy maximizing methods (Wilson, 1970). Wilson (1971) showed how a 'family' of spatial interaction models could be applied in a variety of sub-disciplines of geography and transport studies depending on known data and prediction requirements. During the 1980s and 1990s these models became widely applied in academia, retail consultancies, and by some retail in-house location planning teams (Birkin et al., 2010, 2017; Newing et al., 2020; Reynolds & Wood, 2010).

These models have a long track record of success in the grocery sector and were extensively used by the leading UK grocery retailers involved in the 'store wars' battle of the 1980s and 1990s (Wrigley, 1994). These models were especially adept at modelling habitual behaviours associated with the 'main weekly shop', for which the modelled flow is from consumer residence to a proximate supermarket (Newing et al., 2020; Sturley et al., 2018). A typical benchmark for success for a model used in this context is for the model to replicate existing revenues within 10% of actual, 90% of the time (Birkin et al., 2017). More complex consumer behaviours and channel mix (e.g. consumers carrying out top up shopping close to work or combining their food shop with other leisure activities) have required SIM refinement as explored below. These models have also been adapted and extended for use in non-grocery retail and leisure settings. For example, Fotheringham (1983, 1986) introduce a hierarchical modelling process to account for agglomeration, recognizing that clusters of stores may be more attractive to the consumer, especially important for modelling comparison goods. A good deal of work has also examined alternative demand functions, for example addressing linked or multi-purpose shopping trips (Arentze et al., 1993, 2005) or developing bespoke accessibility terms for leisure markets where demand might be deemed 'elastic', such as fast food or cinemas (Birkin et al., 2010; Ottensmann, 1997).

In the grocery context, models have been disaggregated to capture the complex and nuanced interplay between supply and demand which is evident in many spatial and temporal contexts. In part this has been made possible by the increasing volumes of customer level data held by retailers in this sector (see Hood et al., Forthcoming). Disaggregation has included demand side refinement to capture non-residential drivers of demand such as schools and universities (Waddington et al. 2019), workplaces (Berry et al., 2016; Waddington et al., 2019) or tourism (Newing et al., 2018, 2015). These applications of the SIM disaggregate the underlying geography of these specific demand types and their fluctuations over time (for example weekday daytime trade in many employment centres or predominantly summer trade in UK coastal tourist resorts). Incorporation of disaggregated demand side estimates have also enabled SIM calibration and parameter setting to be undertaken in an incremental fashion, allowing model-builders to fully understand the impacts of different demand types and their inferred behaviours (with regard to mobility or store choice for example) on modelled flows (Newing et al., 2015; Waddington et al., 2019).

As a result of considerable ongoing investment in model building in these contexts, the SIM has seen widespread application for performance benchmarking, new store revenue prediction and impact assessment within the grocery sector (Newing et al., 2020). Although new store build continues in some retail markets (especially in the convenience sector), a major new growth channel has been e-commerce. There is increasing interest within the retail location industry - which we take to be the academic community and location practitioners themselves - around the possibility of including e-commerce in classic retail location models (see Davies et al., 2019; Kirby-Hawkins et al., 2019; Newing et al., 2020; Sturley et al., 2018).

We choose to model e-commerce dynamics using a traditional SIM because of several reasons. First a SIM considers the retail system as a whole, that is both supply and demand. This more aggregate focus differs from efforts to model shopping behavior from an individual's decisions regarding activity participation and channel selection (Kerkman et al., 2017), and allows for the incorporation of aspects of the complex interplay between physical stores and e-commerce. Second such aggregation is also necessary to build and calibrate the model at scale for an entire region and eventually upscale to a national level. This is fundamental from an application point of view (Newing et al., 2020), but also to understand wider impacts of online shopping, for example on grocery accessibility (Videira et al., in review). Third the simple fundamentals of the SIM are extremely well fit for a first attempt to model e-commerce dynamics in the retail sector. Such attempts require a clear validation of assumptions and parameters (Murray, 2021), something which becomes less straightforward in more complex methods (Heppenstall et al., 2021). Finally, the SIM has a proven track record in retail analytics and, as highlighted above, recent academic contributions (e.g. Siła-Nowicka & Fotheringham, 2019; Waddington et al., 2019) are still developing novel innovations in this type of model. Yet the inclusion of e-commerce dynamics remain absent (Kirby-Hawkins et al., 2019). Before we attempt this in sections 3 and 4, it is useful to examine spatial dimensions under which the UK groceries e-commerce market operates.

Many consumers shop interchangeably between different channels, for example substituting in-store grocery shopping with e-commerce. Alexiou et al. (2018) highlight the importance of small area geodemographic factors in driving online engagement and e-commerce uptake. The socio-economic drivers behind the rise in grocery online shopping (both home delivery and 'click and collect') are well documented in the academic literature. Clarke et al (2015), Hood et al. (2020) and Hood et al. (Forthcoming) provide an excellent summary of the impacts of age, gender and affluence on e-groceries uptake. Higher income/education (Clemes et al., 2014; Davies et al., 2019; Hood et al., 2020; Punj, 2011; Van Droogenbroeck & Van Hove, 2017) younger age groups (predominantly mid-20s to mid-40s) (Clarke et al., 2015; Hood et al., 2020; Mortimer et al., 2016) and females (Hood et al., 2020; Mortimer et al., 2016) consistently found to be more likely to engage with e-groceries. Thus there are clear geodemographic and socio-economic drivers of e-groceries uptake which will drive underlying geographical patterns in the use of these services at a local area or store level.

The relationship between these underlying demand side characteristics and the store network is important in driving groceries e-commerce activity at the small area level. This is especially true given the small area variations in consumer access to face-to-face grocery shopping alternatives and the uneven geography in provision of home delivery and click and collect grocery services (Videira et al., *in review*). Kirby-Hawkins et al. (2019) provide evidence that access to physical stores is an important driver of e-commerce sales, utilising the Yorkshire study area. Using data from a partner retailer they demonstrated that recorded e-commerce sales were highest in those areas furthest from their physical stores. This is an important finding as it means there is a formal relationship between physical stores and e-commerce, at least in the grocery market. A number of other studies provide evidence of the so-called efficiency theory (Anderson et al., 2003), using both national and localized case studies, highlighting higher e-commerce rates in areas with lower physical retail accessibility (Beckers et al., 2018; Clarke et al., 2015; Farag et al., 2006; Hood et al., 2020; Kirby-Hawkins et al., 2019). Existing studies in a geographical context also reveal the co-existence of the innovation-diffusion theory (in parallel with the efficiency theory) evidenced by a tendency for early adoption and high uptake rates of groceries e-commerce in major urban areas (Beckers et al., 2018; Clarke et al., 2015; Kirby-Hawkins et al., 2019. Hood et al 2020). Observed innovation-diffusion in the grocery sector is likely to be driven by the geodemographic pre-disposition for consumers living in these areas to shop online (Alexiou, 2018) and the ubiquity of groceries e-commerce availability in these areas (Videira et al., *in review*).

The online sale of groceries also implies a substitution of consumer trips by last mile distribution. This new leg of the supply chain however adds a significant extra cost due the spatial fragmentation of delivery addresses (Cárdenas et al., 2017). The perishable nature of the products and consequent need for attended delivery (consumer at home to receive goods) requires retailers to offer consumers narrow/specific time windows, which further increases the complexity and cost of vehicle scheduling and routing in this last mile (Hübner et al., 2016; Morganti, 2019; Wollenburg et al., 2018). Grocery retailers typically use their existing store infrastructure to pick, pack and deliver customer orders (Aspray et al., 2013), requiring investment in in-store picking/packing capacity and in vehicles and staff for delivery to the consumers' home. As a consequence, much retailer investment has focused on major urban areas where store and population density are greatest (higher volume of customers in proximity to supply points), and consumers are more pre-disposed to use these services. If order volumes are so high in these areas that store level picking/packing capacity is reached, retailers may increase capacity by utilizing additional proximate stores for online delivery or replacing store-based infrastructure with custom warehouses or 'dark stores' (Hubner et al. 2016). Thus an uneven geography of provision may emerge, reinforcing observed urban-rural divides in groceries e-commerce engagement. This evolution has not resulted in disparities relating to costs for the consumer when it comes to the fulfilment of online orders. Delivery costs among retailers are comparable and no official price difference exists for longer distribution trips. Moreover the delivery fee is often waived when orders exceed a certain amount, which is frequently the case due the on average large basket sizes of online orders (Kirby-Hawkins et al., 2019).

Retailer investment in the provision of grocery e-commerce continues apace, yet there remains a complex geography to e-commerce provision and demand, both of which are inextricably linked. The spatial modelling frameworks (and specifically the SIM) mentioned above have gained much trust in the grocery sector due to their ability to capture the interplay between consumer demand/behaviour and the retail supply side as they vary over space and time. Based on the evidence presented above, we strongly argue that grocery e-commerce is no exception. Small area variations in demand are driven by the complex interrelationships between consumer propensity to order online and the localized provision of e-grocery services. We thus argue that there is tremendous potential to incorporate e-commerce within modelling tools traditionally used for face to face consumer interactions in this sector. In the following sections we take up that challenge, specifically considering the SIM and its application to the grocery sector in Yorkshire.

3. **The SIM and model extensions**.

The production-constrained retail SIM typically takes the following form:

$$S_{ij}^{k} = A_i^k \, O_i^k \, W_j^{\alpha^k} \, exp^{-\beta^k c_{ij}} \qquad \textit{eq. 1}$$

Where S_{ij}^{k} represents the total expenditure flow between demand zone i and store j, by consumer type k, A_i^k is a balancing factor, O_i^k is a measure of the available demand for groceries in zone i, and is disaggregated by person type (based on the UK Census Output Area Classification (OAC) supergroups); W_j represents the attractiveness of store j, measured here by store size; c_{ij} is the travel cost between demand zone and store, measured as the Euclidean distance; α^k and β^k disaggregate the model by including brand attractiveness (again disaggregated by person type – lower income groups are more attracted to the discount retailers, whilst higher income groups are especially attracted to Sainsbury's and Waitrose in the UK) and distance deterrence by consumer type respectively. This model was initially built and calibrated to allocate demand to face to face stores, as is customary in the standard production-constrained retail SIM. Calibration of the model parameters uses Acxiom consumer survey data (Clarke et al., 2015), as well as data provided by a leading UK grocery retailer. As an illustration, Fig 1 shows one of the outcome variables of the face to face traditional model, namely market share for retail brand Morrisons in our study area. Note, as expected, the strong visual association between market share and store location. The final version of the face to face SIM deducts demand allocated to e-commerce which is estimated next.

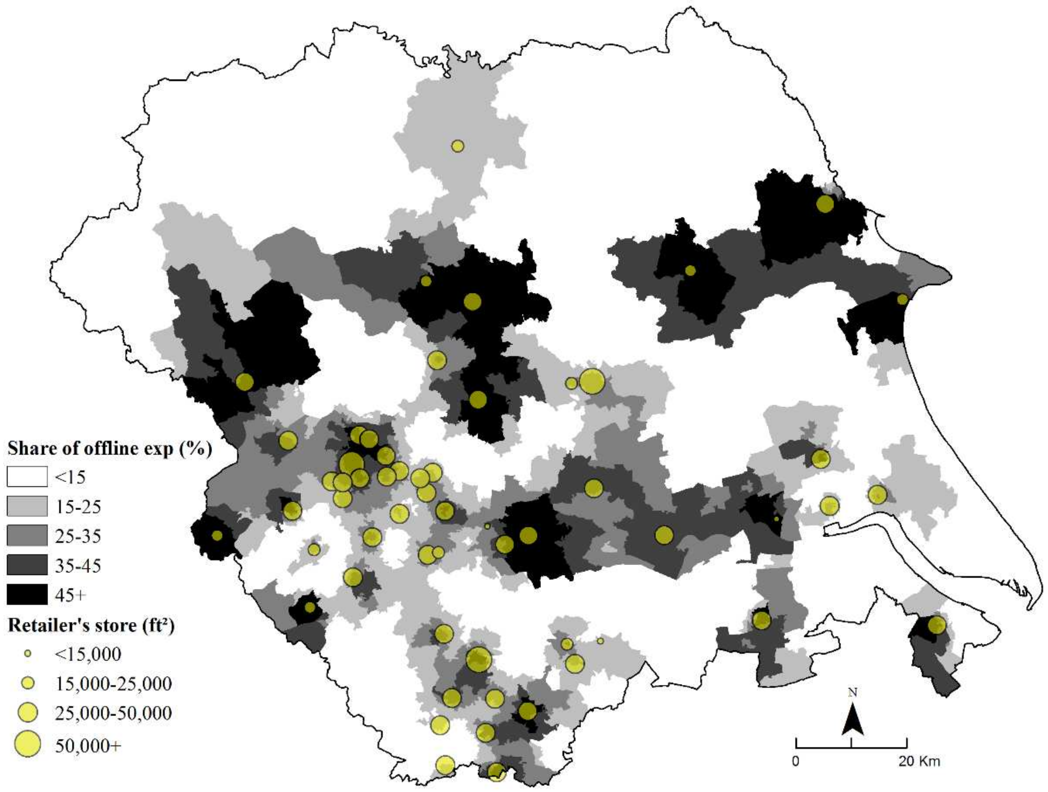


Figure 1: Estimated market share for Morrisons in Yorkshire using a traditional face to face SIM

To make progress with the e-commerce SIM we first need to modify the demand term O_i^k to account for e-commerce, which we now label $O_{i,online}^k$. The demand for e-groceries is expressed, as in the face to face SIM, as weekly expenditure in UK pounds (£) by small-area census output area (OA). This demand-term is disaggregated by customer type to reflect a variety of socio-economic and demographic factors which influence behaviour patterns (such as age and income). First, we take a fixed total expenditure on e-commerce as a percentage of the total grocery market value. Second, we need to distribute that total online expenditure among the different OAs. Given the above discussion on the potential theories on e-commerce adoption, $O_{i,online}^k$ is calculated in relation to a combination of both local geodemographics and physical store accessibility. In contrast to the face to face SIM, the demand is allocated using a constrained SIM, which allocates a share of the fixed total expenditure for e-groceries to each OA. We specifically revert the logic of the SIM here to be consistent with the idea that the pot of fixed online expenditure is spread out over each demand zone and hence include elements of both the production-constrained and attraction-constrained variant.

The online expenditure can be written as:

$$O_{i,online}^k = f\left(age, income, c_{ij^*}\right) \qquad \textit{eq. 2}$$

$$O_{i,online}^k = B_i P_i D f(c_{ij^*}) \qquad \textit{eq. 3}$$

with $$O_{i,online}^{k} \leq O_{i}^{k}$$

c_{ij^*} represents the travel cost, measured as the Euclidean distance between original zone i (each OA) and all stores j^*. j^* denotes the subset of stores with a minimum store size of 15,000ft^2 as online purchases substitute mostly larger grocery baskets (Kirby-Hawkins et al., 2019). Hence the presence of a small outlet is assumed to not greatly influence e-groceries propensity, as they are more likely to be used for top-up shopping than major weekly shops even in rural areas. P_i is now the mass term indicating the probability of residents within an OA to shop online. D is set as a fixed total for e-groceries expenditure; B_i is the balancing factor $(1/\sum_i P_i f(c_{ij^*}))$ in the constrained model. The total e-groceries expenditure, $O_{i,online}^{k}$ is further constrained in each OA i by the total possible groceries expenditure. This number is derived from the annual 'Living Costs and Food Survey' by the Office of National Statistics, which collects information about household spend across a number of categories. We do not revert to a production-attraction constrained model because the total e-groceries expenditure in each zone is not known. Instead we model this constraint as a hard cap in the model. The distance deterrence term $f(\, c_{ij^*})$ is calculated by means of an accessibility index (Hansen, 1959):

$$f(\, c_{ij^*}) = H_i^{\,-\gamma} \qquad \textit{eq. 4}$$

with $$H_i = \sum_{j^*} W_{j^*}\, exp^{(-c_{ij^*})} \qquad \textit{eq. 5}$$

For calculating P_i we use a combination of income and the age profile of each OA. Income data is only available at the MSOA level in the UK, more aggregated than the OA level. For each income class, the appropriate share of online shoppers, as presented in Clarke et al. (2015), is taken. Next, the calculation is repeated based on the age profiles at the same MSOA scale. By taking the average of both predictions we end up with a final estimated number of e-commerce shoppers. This number is then distributed among the more disaggregate OAs according to their respective age structures. The additional propensity to be an e-commerce shopper is then shaped by the Hansen accessibility term (equation 5). The parameter γ is introduced to represent the importance of the access term. The minus sign indicates the inverse relationship between store presence and online shopping behavior (i.e. efficiency theory). No parameters related to the distribution process of the e-groceries are included in the demand estimation. Little variation is found in delivery fees and hence not considered a critical factor. W_{j^*} is the attraction of store j^*, given by its size in ft^2. The results of this exercise are discussed below.

After $O_{i,online}^{k}$ has been calculated, the final step is to allocate the e-commerce expenditure to the different retailers present in the study area. We include retailers who both operate a comprehensive online channel and have a physical store presence in the study area: Asda, Morrisons, Sainsbury's, Tesco and Waitrose. (Thus Ocado is excluded because it has no physical stores as is Iceland as it's a more niche frozen food retailer). The size of the ir flow matrix is now 1600 (OAs) by 5 (the retail e-commerce brands listed above). For this allocation we make a second modification to the traditional production-constrained SIM (equation 1). As noted above, Kirby-Hawkins et al. (2019) showed a logarithmic relation between a retailer's physical store provision and its local online market share. Hence, the distance term is inverted to take this effect into account. Consequentially, the SIM for allocating the e-groceries expenditure is modelled as follows:

$$S_i^{kr} = A_i^k O_{i,online}^k W^{r\alpha^k} ln(d_{ir^*})^\beta \qquad eq.\ 6$$

With $$A_i^k = \frac{1}{\sum_r W^{r\alpha^k} ln(d_{ir^*})^\beta} \qquad eq.\ 7$$

S_i^{kr} represents the total expenditure flow between demand zone i and retailer r with $r \in \{\mathrm{Asda,\ Morrisons,\ Sainsbury,\ Tesco,\ Waitrose}\}$, disaggregated by customer type. $W^{r\alpha^k}$ is the attractiveness of retailer *r*, for e-commerce which can be measured according to their national online market share, α^k disaggregates the attractiveness for retailer *r* among different consumer types *k*. d_{ir^*} is the distance between demand zone i and the closest large store r^* of retailer *r*. Also in the allocation of the expenditure to the different brands, as noted above, no logistics parameters are considered due to little differences in delivery fees among the retailers.

To summarise, a particular brand's e-commerce revenue is based on the attractiveness of that brand to consumers in each OA (as in the face to face model) as well as how accessible their store network is to consumers in that OA – the further from a physical store of that retailer then the more attractive is their e-commerce offer.

4. Calibration and Results

It is an important next step to examine the parameters of this new model in more detail. We do this in an incremental way because we want to explore in detail how the new model parameters affect the allocation of e-groceries demand and expenditure. A sensitivity analysis where all possible combinations of parameter values are tested might result in a more 'correct' prediction of e-groceries demand and allocation, yet falls out of the scope of this analysis. In this first attempt we are concerned with identifying the parameters that are necessary to model these complex dynamics.

Because of the lack of access to retailer's data for the purpose of this study we combined different sources for the estimation and calibration of the models. In section 4a e-groceries expenditure is estimated at 5% of the groceries market, allocated at the OA level using age and income estimates from 2011. The model is operationalized based on the retail supply side and the physical store locations recorded in 2013. A common issue with retail analysis is a lack of perfect correspondence of data for the time period studied and the spatial units used. As such, we collected the best available data corresponding to the time period of our calibration data (2010 and 2014, see Table 2), whilst matching geography as closely as possible. In section 4b we use the most recent data available (2015 small area estimates, 2018 online share and 2016 physical stores) to demonstrate that the model can re-allocate expenditures based on the changing supply and demand side dynamics in this sector.

Table 1: Input data used for the different modelling steps

Input	*Original source*	*Year*	*Remarks*
Demand estimation			
OA age data	Office for National Statistics	2011	Most detailed data available

MSOA income estimates	Office for National Statistics	2011	Privacy issues prevent smaller area estimates
Retail share in total groceries expenditure	Clarke et al. (2015)	2014	Potentially volatile, especially in current context but as closely matched to time period of data as possible
Location of physical stores		2013	Detailed data subject to partnership with a retailer based on extensive data harvesting. Open source alternatives available (e.g. Geolytix), but less precise.
Allocation of expenditure			
OA age data	Office for National Statistics	2015	Nationally representative sample, but local context may vary
MSOA income estimates	Office for National Statistics	2015	Privacy issues prevent detailed estimates
Online share in total groceries expenditure	Statista (2019)	2018	Not publicly available data
Location of physical stores	GMAP Ltd (2016)	2016	

Table 2: Data sources used for calibration

	Original source	*Literature*	*Limitations*
Demand estimation			
E-commerce use among OAC	2010 British Population Survey (BPS)	Kirby-Hawkins et al. (2019)	No information on e-commerce use among updated OAC.
Postal level estimates of e-groceries share	2014 CACI estimates of e-commerce usage	Kirby-Hawkins et al. (2019)	No perfect fit between OA and Postal level boundaries

			Based on national level expenditure estimates
Allocation of expenditure			
Retailers online market shares	2014 Statista	/	Not publicly available data
Retailer share in online sales	2013 Retailer data	Kirby-Hawkins et al. (2019)	Not publicly available data, requires academic-industry partnership.

a. E-groceries expenditure

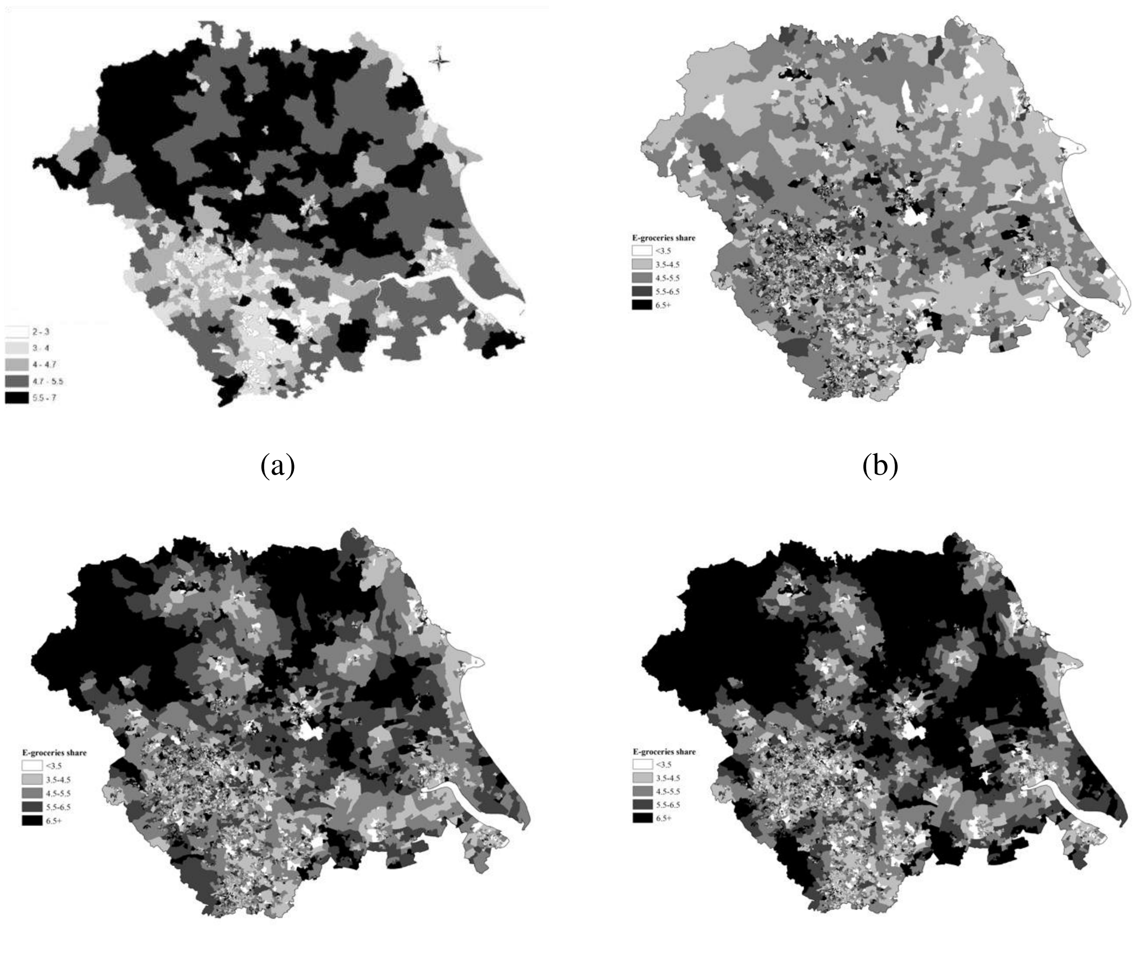
First we consider the demand estimates for e-commerce. These can be validated using a combination of e-commerce usage estimations in the British Population Survey (BPS: 2010) as well as estimates made by leading retail consultancy company CACI, discussed and presented in Kirby-Hawkins et al. (2019). The BPS assessed average e-commerce usage across different socio-economic groups using the 2001 OAC (a readily available geodemographic system provided by the Office of National Statistics in the UK – see Table 3). The results of our estimations vary in relation to the γ parameter seen in eq. 4. When γ = 0 (hence $f(c_{ij^*}) = 1$), there is a clear underestimation of the e-groceries share in the countryside and prospering suburbs locations and an overestimation of the shares of OAC groups associated with more urban areas, when compared to BPS estimates (see Table 3). Increasing γ implies a decrease of the e-groceries share in the vicinity of supermarkets (eq. 4, eq. 5). The nearest equivalence, indicated by the sum of the squared deviance per group, can be seen with a value of γ of 0.03 when compared with the BPS data.

Table 3: Calibration of eq. 3 with BPS data

	Expected share (BPS)	$\gamma = 0$	$\gamma = 0.03$	$\gamma = 0.05$	$\gamma = 0.07$
Blue collar communities	4.6	5.14	5.07	5.01	4.92
City Living	5.65	6.13	5.75	5.46	5.12
Countryside	5.35	4.66	5.46	6.15	7.01
Prospering Suburbs	5.4	4.81	4.78	4.73	4.66
Constrained by circumstances	4.2	4.44	4.29	4.15	3.99

Typical traits	5.45	5.51	5.39	5.30	5.18
Multicultural	4.5	5.41	5.04	4.76	4.45
Sum of squares		2.2	0.9	1.4	3.8

Even with the best fitting γ value, the prospering suburbs OAC group is still under-estimated against the BPS data. However against CACI data we have a better fit. Figure 2 shows the spatial extent of e-commerce demand estimation for various values of γ against the CACI estimates used by Kirby-Hawkins et al (2019). As can be seen from Figure 2a there is more e-commerce estimated in the rural areas of North Yorkshire by CACI (as shown in Kirby-Hawkins et al (2019)) when compared to our estimation when ignoring the efficiency theory (i.e. γ=0, Figure 2b).



(a) (b)

(c) (d)

Figure 2: E-groceries shares in Yorkshire. (a) Based on CACI's national on-line expenditure profiles (Kirby-Hawkins et al., 2019). (b) Model output with γ = 0. (c) Model output with γ = 0.03 (d) Model output with γ = 0.05. Note the use of postal sectors in (a) and output areas in (b), (c), (d).

Figure 2b shows a good spatial fit between the CACI estimates used in Kirby-Hawkins et al. (2019) and our estimations. We are able to replicate the large amount of e-commerce expenditure to the rural areas in the north and east of the study area with a value of γ=0.03.

b. Allocation of e-groceries expenditure

Satisfied with the outcomes of the expenditure model, we now move to the allocation of the calculated e-groceries expenditure to the main 5 retailers. For calibration of the attractiveness term $W^{r\alpha^k}$, we first set $\beta = 0$, so effectively ignoring the distance term, and $\alpha^k = 1$ for all consumer types j and retailer r. The floorspace value used for each retail brand is calibrated against observed e-commerce market shares shown in Statista (2019) but modified for the exclusion of Ocado. For example, Tesco has a value of 2500 in order to enable it to capture 51% of the market in Yorkshire (which is estimated to be the share Tesco has of the 5 retailers used here). The overall shares of the five retailers are remodelled so that they sum to 100.

Table 4: Calibration of allocation model. $W^r \in \{800, 100, 1000, 2500, 400\}$

	Observed market share	*Modelled market share (β = 0, $\alpha^k = 1$)*
Asda (W=800)	17	17
Morrisons (W=100)	1	2
Sainsbury's (W=1000)	24	21
Tesco (W=2500)	51	52
Waitrose (W=400)	7	8

Next, we vary the distance term β while keeping $\alpha^k = 1$ for all consumer types k and retailer r. For this next stage we focus on just one of the five retailers under consideration. Figure 3 depicts the modelled estimated online market share for Morrisons in the study area. In Figure 3a the distance term is ignored and the retailer maintains a constant market share of 2.2% in each output area. Figure 3(b) and (c) then demonstrate the effect of varying the β parameter given accessibility to physical stores (by brand). This results in a greater spatial variety in the retailer's market share.

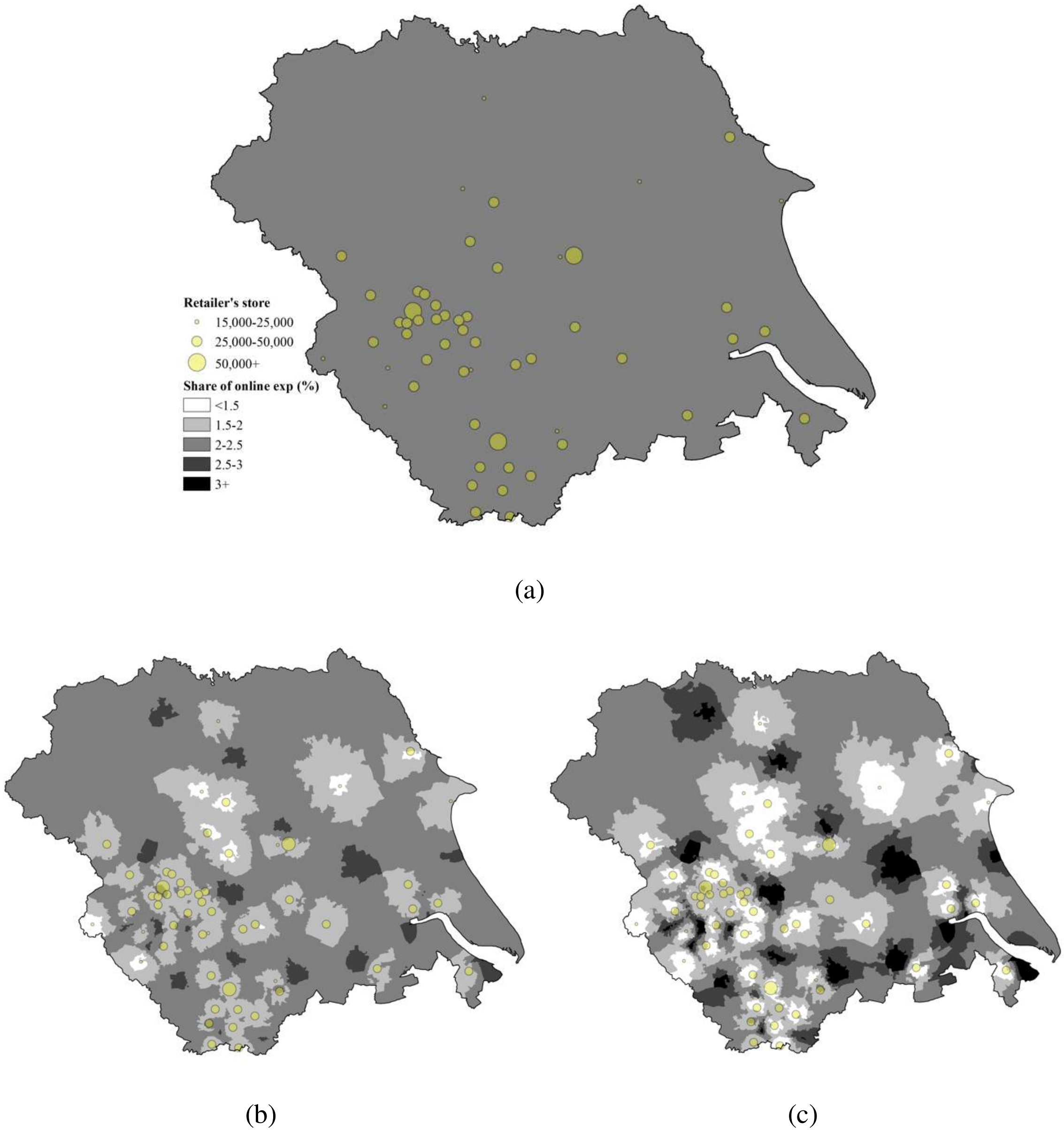


Figure 3: Impact of distance term on allocation of e-groceries expenditure to one retailer. (a): model output with β = 0. (b) model output with β = 0.5, (c) model output with β = 1.

A β value of 0.5 gives a good visual representation of the required e-commerce sales in rural areas compared to urban areas. To help validate these allocations (at least partially) we finally compare the results with the actual e-commerce sales mapped for one particular retailer in the area (presented in Kirby-Hawkins et al. (2019)). As shown in Figure 4a, where there is limited presence of their partner retailer, Kirby-Hawkins et al. (2019) found high levels of online grocery sales (e.g. south of Bradford and west/north-west Leeds (area 2), and vice-versa (area 3). Also in the city centres of both Leeds and Bradford the authors found high online shares, mostly due to the presence of young professionals (area 1 for example).

The parameter to consider next is α^k, which looks at brand attractiveness. As in the face to face model, wealthier consumers are still assumed to favour high-end retailers such as Sainsbury's or Waitrose. Figure 4b-d and Table 5 show our results for various values of α^k. It appears that the model correctly predicts low online-shares around the retailer's outlets, as in the northern suburban areas of Leeds but changing α in configurations 2 (Figure 4c) and 3 (Figure 4d) clearly impacts the outcomes. With brand attractiveness included we observe a decrease in online shares in the less affluent areas, like north of Bradford and east of Leeds where this retailer is less popular. Statistical evidence shows that larger variations in α^k improve the fit with Kirby-Hawkins (2019). The results shown in Figure 4d seem to best fit the observed patterns seen in Figure 4a although some deviations remain. Note in the northeast and southwest of our study area we overestimate the online share of the retailer. The former deviation may be a result of modelling brand attractiveness at the scale of the OAC. These are relative heterogeneous groups and it is possible that in the OAC present in that area (Prospering Suburbs) brand preferences of consumers are different which result in the purchase of online groceries from competitors. The latter deviation may result from the fact that we did not disaggregate α^k for different retailers. The preference for lower-priced retailers such as Tesco would have decreased the online share of the retailer in Figure 4.

When doing this validation exercise one of course also has to keep in mind the inconsistencies among the data sources. Whilst we attempt to align the different datasets it should be noted that part of the deviance exhibited results from using estimates at MSOA and OA levels for 2015 in combination with store locations of 2016 while the retailer's data stems from 2013.

Table 5: α^k for corresponding configurations used in Figure 4.

OAC	*Configuration 1(b)*	*Configuration 2(c)*	*Configuration 3 (d)*
Blue collar communities;	1	1	1
City Living	1	1.2	1.3
Countryside	1	1	1
Prospering Suburbs	1	1.2	1.3
Constrained by circumstances;	1	0.8	0.7
Typical traits	1	0.8	0.7
Multicultural	1	0.8	0.7

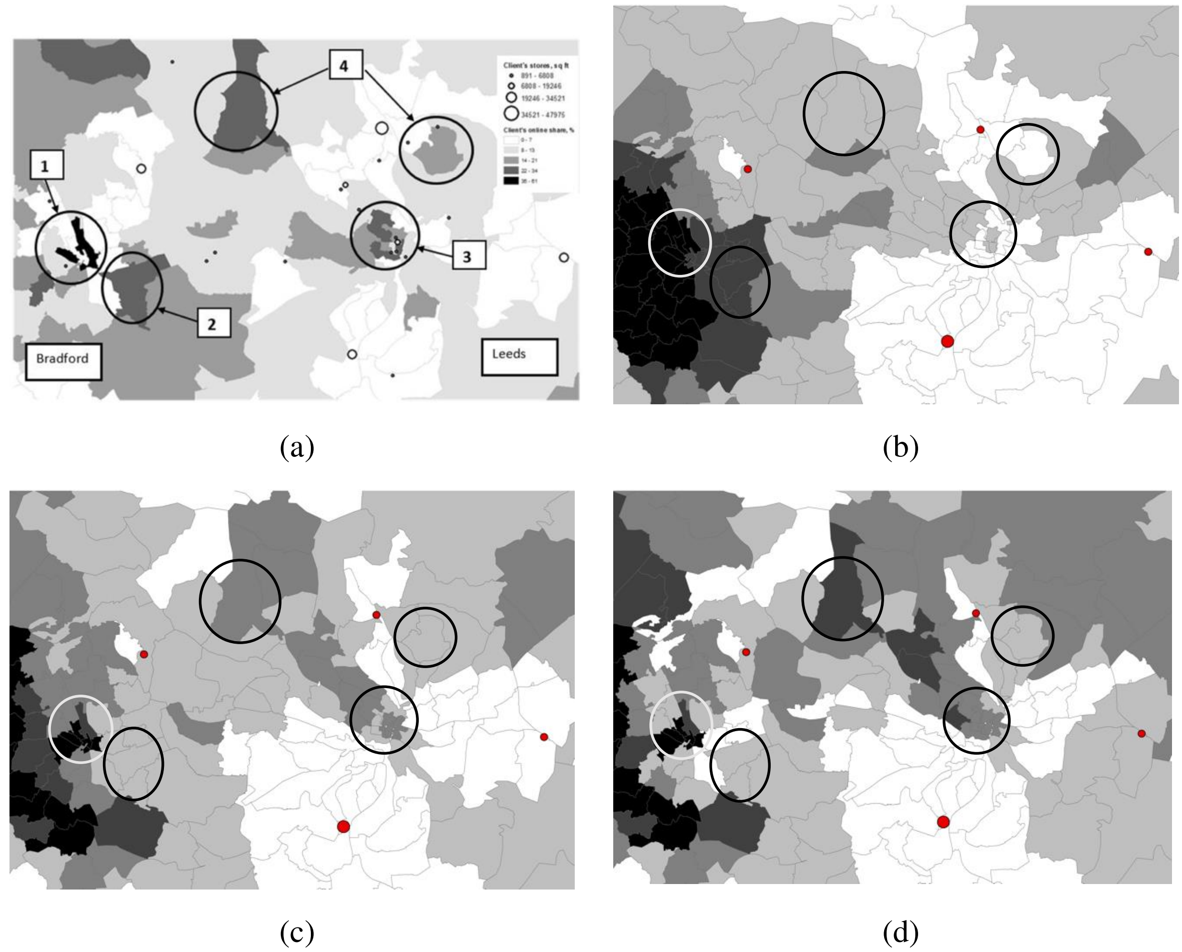


Figure 4: Effect of varying alpha by socio-demographic demand group on the retailer's online share in total groceries income. (a) Kirby-Hawkins et al. (2019). (b) Model output for configuration 1 in Table 3. (c) Model output for configuration 2 in Table 3. (d) Model output for configuration 2 in Table 3.

The inclusion of new distance-related parameters in the SIM allows us to model a double repulsion effect of physical store presence in the e-groceries context. On the level of the overall population, higher retail accessibility as modelled through the Hansen index (eq. 4) decreases e-groceries expenditure (efficiency hypothesis). On the level of a particular brand, the presence of a physical store decreases the local share in total online sales. The interaction of these relations with sociodemographic factors results in complex e-commerce dynamics as depicted in Figure 4d. The location decision for the retailer now goes beyond minimizing the distance to its consumers while accounting for competition. Instead, the retailer has to balance gaining offline consumers with losing online consumers, or vice-versa.

With online shares currently still low, the focus currently lies on the former. However, considering continuous growth rates, further boosted by the COVID-19 crisis of 2020, the balance might shift rather sooner than later and retailers might wish to improve their location models in similar ways. A simple application of our model is demonstrated in Figure 5. Here we plot the evolution of the retailers' market shares for higher levels of online grocery shopping, assuming all other parameters remain constant. If the share of e-groceries in the total groceries expenditure reaches 30%, retailers without online presence see their market shares decrease below 70% of their original (i.e. before e-commerce) values. When the current online market share is low in comparison to the F2F market share, such as in the case of Morrisons, we see a similar pattern. ASDA would be able to retain its current market position because their online market share is similar to the F2F value. The big winners of this simulation are the retailers that made an early move to the online channel, being Tesco, Waitrose and Sainsbury. The power of the model goes far beyond this simple simulation as such evolution would also result in changes in the sociodemographic profile (P_i in eq. 3) and brand attractiveness (α^k in eq. 6) of the online shoppers (innovation-diffusion theory) as well as in the influence of retail accessibility on e-groceries propensity (γ in eq. 4). Such in depth analysis would however warrants a case study on its own and is an interesting path for further research.

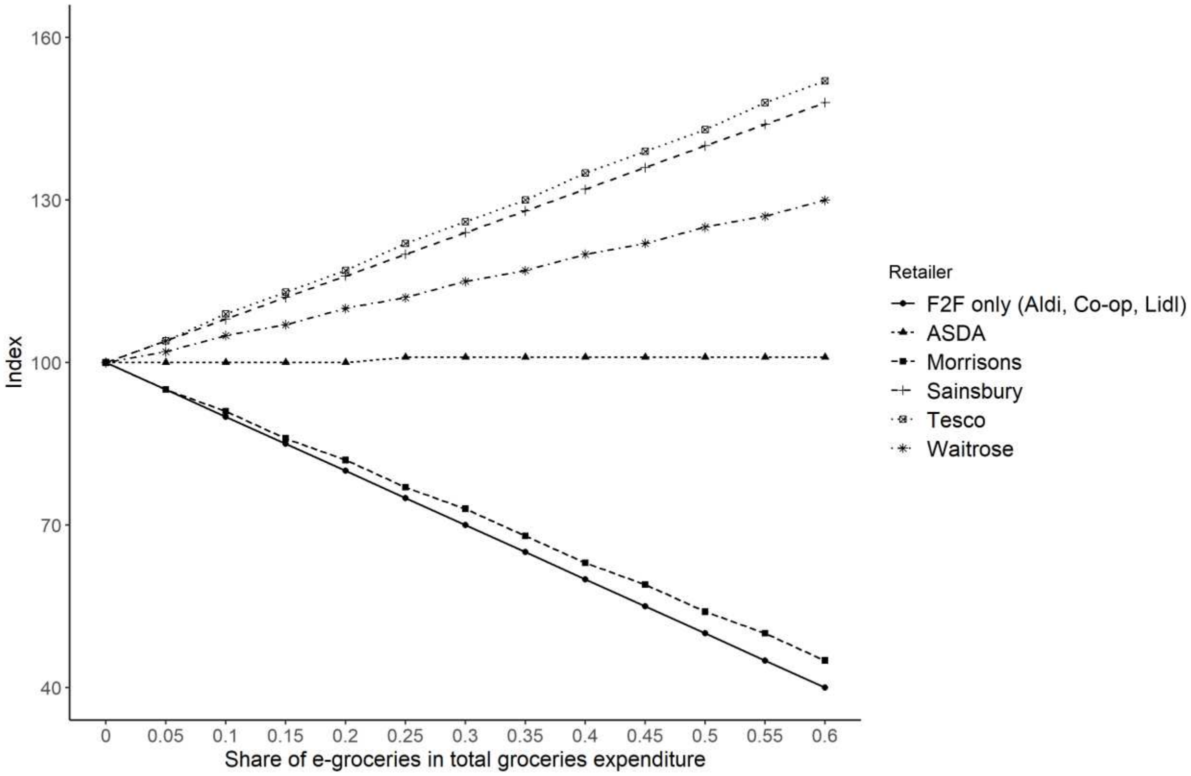


Figure 5: Evolution of retailers' market shares with increasing share of online shopping (100 = retailer's market share in only F2F setting (Kantar, 2020))

5. Conclusions

Through the progressive steps taken above, we are confident that we have arrived at a model which is capable of representing substantial empirical variations in e-commerce consumption trends for the retail grocery market. Each refinement has been justified through an analysis of observed patterns in the data relating to consumers, retail outlets and geographical neighbourhoods. The bespoke SIM that we have built offers the opportunity to include socioeconomic characteristics of local areas in the demand estimations as well as physical retail accessibility. Our SIM incorporates elements of both the production-constrained and attraction-constrained variant of the model to account for the complex nature of retail interactions present in a multi-channel environment. Using this revised SIM provides the possibility for a retailer to predict the impact of both changing consumer demand on e-commerce sales and the impacts of changes to face to face retailing on their e-commerce sales. For example, the model in its current state can estimate online and offline revenue changes resulting from the opening or closure of physical stores at the OA level. Thus, it potentially serves as a tool for retailers to better manage the organization of their different retail channels.

We cannot claim that this model has been formally optimized in relation to either its structural components or in the way it has been parameterized. Yet, the goal of this paper is to illustrate how to account for e-groceries dynamics within the SIM, given its widespread application for location decision making in this sector. We believe our quest for streamlining input and calibration data from different sources suffices to demonstrate these e-groceries dynamics. The model incorporates a rich array of trends which can be observed from retail 'big data', such as the varying e-groceries propensity among different sociodemographic classes and the inverse relation between store presence and online channel share as captured by the efficiency theory. As a result it represents an innovative attempt to simulate e-commerce activity in the grocery market.

The model is a first attempt to incorporate these dynamics and hence different paths for further improvements are identified. From a conceptual perspective, e-commerce results in a stronger intertwining between retail and logistics. Physical stores are not only selling points but are now also used for the delivery or collection of online purchases. These activities imply new constraints in a physical location model and may even broaden the focus from maximizing physical store income to minimizing the costs of the last mile delivery. Related to this an interesting follow-up would be to investigate a potential positive correlation between store presence and click&collect behaviour. This might dampen the distance inversion modelled in this paper to a certain degree.

Also other logistics dynamics could be considered. For now the accessibility and distance deterrence terms in the demand and allocation models respectively do not include any component linked to the delivery of the online purchases. This choice was deliberate due to the fact that currently no official discrimination is made based on the distance that delivery vans are required to drive. Further complexities such as price incentives and discounts for returning customers are also not considered. The model could be expanded by including such decisions in order to understand the impact of different pricing strategies. This could moderate the value of γ. We also assumed the standard delivery fee was not an extra barrier preventing potential online shoppers effectively purchasing groceries online. This assumption was made given that (i) e-groceries consist mostly of larger grocery baskets that might waive the delivery fee and (ii) e-groceries are already linked to higher incomes in our model, with consumers who are more likely to be able to pay the extra delivery fee.

From an application perspective the current model could benefit from access to more recent retailer's data. First this would help in validating the model across the sector as a whole and could improve the match of the different data sources. Second this should yield more recent insights in the profile of the online shopper, ideally in relation to small area accessible data sources. The use of the outdated OAC classification in combination with MSOA income estimates might have an impact on the value of our model for simulating current trends. More recent data at similar geographical units would greatly improve the value of our model for driving policy or business decision making.

From a more technical perspective the demand estimation could be further developed by disaggregating the γ parameter. We assume a constant balance between the efficiency and innovation-diffusion theories in our study area, yet this strongly depends on the purchasing characteristics of the consumer. For example the relative importance of the accessibility term may be different for someone frequently visiting physical stores for small purchases compared to one making one large purchase on a weekly basis. Further, the incremental calibration process chosen in this paper allows us to fully understand the effect of individual parameters. In the future it might be useful to use optimisation methods to calibrate the parameters of the model simultaneously (George et al., 1997). Open-source software packages now exist for this purpose (Dolega et al., 2016). However, we feel it is important to understand the role of each parameter fully (and their parameter space) before more black-box techniques are introduced.

Finally, from a modelling perspective we the SIM lends itself for a clear validation of assumptions and parameters. We recognize that other modelling types could be used to continue on the work done, given the findings in this work. For example activity-based models might be more suited for modelling individual shopping behaviour. However, these have to be aggregated at some stage to model the entire population. SIMs are also widely used by retail organisations as an important location tool. Specifications such as the use of travel time-based distance measures or using only those retail outlets delivering groceries ordered online could further increase the prediction power. We however encourage such elaborations in future iterations to improve the proposed model. With these remarks, we hope to have initiated a fruitful debate that will lead to a better understanding of e-grocery dynamics and its relation to retail and logistics planning decisions.

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