



This is a repository copy of *Non-numerical strategies used by bees to solve numerical cognition tasks*.

White Rose Research Online URL for this paper:  
<http://eprints.whiterose.ac.uk/171533/>

Version: Accepted Version

---

**Article:**

MaBouDi, H. [orcid.org/0000-0002-7612-6465](https://orcid.org/0000-0002-7612-6465), Barron, A.B., Li, S. et al. (8 more authors) (2021) Non-numerical strategies used by bees to solve numerical cognition tasks. *Proceedings of the Royal Society B: Biological Sciences*, 288 (1945). 20202711. ISSN 0962-8452

<https://doi.org/10.1098/rspb.2020.2711>

---

© 2021 The Author(s). Published by the Royal Society. This is an author-produced version of a paper subsequently published in *Proceedings of the Royal Society B: Biological Sciences*. Uploaded in accordance with the publisher's self-archiving policy.

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.



[eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk)  
<https://eprints.whiterose.ac.uk/>

# 1 Non-numerical strategies used by bees to solve numerical 2 cognition tasks

3  
4 HaDi MaBouDi<sup>1,#</sup>, Andrew B Barron<sup>1,2,#</sup>, Sun Li<sup>3</sup>, Maria Honkanen<sup>4</sup>, Olli Loukola<sup>4</sup>, Fei  
5 Peng<sup>3</sup>, Wenfeng Li<sup>5</sup>, James A. R. Marshall<sup>1</sup>, Alex Cope<sup>1</sup>, Eleni Vasilaki<sup>1</sup>, and Cwyn  
6 Solvi<sup>2,6,\*</sup>

7  
8 <sup>1</sup> Department of Computer Science, University of Sheffield, Sheffield S1 4DP, UK

9 <sup>2</sup> Department of Biological Sciences, Macquarie University, North Ryde, NSW 2109,  
10 AU

11 <sup>3</sup> Department of Psychology, School of Public Health, Southern Medical University,  
12 Guangzhou, China

13 <sup>4</sup> Department of Ecology and Genetics, University of Oulu, Oulu, Finland

14 <sup>5</sup> Guangdong Key Laboratory of Animal Conservation and Resource Utilization,  
15 Guangdong Public Laboratory of Wild Animal Conservation and Utilization, Institute  
16 of Zoology, Guangdong Academy of Science, Guangzhou, China

17 <sup>6</sup> School of Biological and Chemical Sciences, Queen Mary University of London,  
18 London E1 4NS, UK

19 # authors contributed equally to this work

20 \* corresponding author: c.solvi@qmul.ac.uk

## 21 22 **Abstract**

23  
24 If we are truly interested in the evolution of cognition in its many forms, it is vital to  
25 understand not simply that an animal can solve a task, but how they solve the task.  
26 We examined how bees solve a numeric-based task utilizing stimuli common  
27 amongst numerical cognition studies. Bees performed well on the task, but additional  
28 tests showed that they had learned continuous (non-numerical) cues. In simulations,  
29 a simple network model containing just nine elements was capable of learning the  
30 task using only continuous cues inherent in the training stimuli, with no numerical  
31 processing. This model was able to reproduce behaviours that have been  
32 considered in other studies indicative of numerical cognition. Our results support the  
33 idea that a sense of magnitude may be more primitive and basic than a sense of  
34 number. Our findings highlight how problematic inadvertent continuous cues can be  
35 for studies of numerical cognition. This remains a deep issue within the field that  
36 requires increased vigilance and cleverness from the experimenter. We suggest  
37 ways of better assessing numerical cognition in non-speaking animals, including  
38 assessing the use of all alternative cues in one test, using cross-modal cues,  
39 analysing behavioural responses to detect underlying strategies, and finding the  
40 neural substrate.

## 41 42 **1. Introduction**

43  
44 Mapping specific cognitive capacities to the behaviour of any animal is rarely  
45 straightforward. The difficulty is that animals may not be solving the task the way we  
46 think they are. One example of this is in our own recent work where we had bees  
47 discriminate different shapes based on relative size [1]. Bees' performance  
48 increased over training to well above chance, and in the unrewarded test they  
49 seemed to have learned to discriminate shapes based on relative size. However,  
50 analysis of first and sequential choices during training bouts and tests revealed that

51 the bees actually switched to a simpler strategy in the middle of training: win-  
52 stay/lose-switch. These results, along with other works suggesting animals are able  
53 to solve tasks in unexpected ways (e.g. [2–7]), prompted us to look deeper into the  
54 strategies of animals in numerical cognition tasks.

55  
56 Numerical cognition has been claimed in a large number of animal species (e.g. [8–  
57 39]), suggesting that a sense of number is widespread (for reviews see [40–42]). By  
58 far, the most common method for testing numerical cognition in non-verbal animals  
59 is to have subjects discriminate 2D visual displays with differing numbers of shapes  
60 (Fig 1; [8–39] all used this design). As pointed out by others (e.g. [43,44]), in these  
61 types of designs, continuous (non-numerical) cues often unavoidably covary with  
62 numerosity. These include size and shape of elements, area (total amount of colour),  
63 edge length (total boundary length of elements), convex hull (the minimum convex  
64 region covering all elements, spatial frequency (the amount of alternating dark and  
65 light regions), and illusory contour (the basic shape that outlines all elements). In  
66 Figure 1 we further describe these cues and their natural covariation with number  
67 (figure 1a-e). This covariation makes it difficult to know whether animals actually  
68 used any sense of number to solve their tasks.

69  
70 The issue of non-numerical strategies within numerical cognition studies has been  
71 highlighted by others [43–46]. It was established decades ago that cells within the  
72 visual system respond to various continuous visual features [45,47,48] and it has  
73 long been known that continuous features can be reliable discrimination cues, even  
74 for bees [49–52]. Further, several works show that animals use non-numerical cues  
75 to solve numeric-based tasks when not controlled for, e.g. size of elements [53], total  
76 area [54], and convex hull [55], and even when they are controlled (e.g. [56]; see  
77 Discussion).

78  
79 Most studies investigating numerical cognition attempt to control for at least one non-  
80 numerical cue. Several works have made valiant efforts to control for most  
81 continuous cues (e.g. [57,58]). However, we have found no studies that tested for all  
82 continuous variables. It seems clear that animals are solving these tasks, but the  
83 question we attempt to address here is how they might be solving the tasks. We set  
84 out to determine how honeybees solve a numeric-based task using stimuli common  
85 amongst numerical cognition studies.

## 86 87 **2. Materials and Methods**

### 88 89 *(a) Subjects*

90 Honeybees (*Apis mellifera*) used in the experiment were maintained at the University  
91 of Oulu (Oulu, Finland) and at Guangdong Institute of Applied Biological Resources  
92 (Guangzhou, China) in September and November 2019, respectively. Prior to  
93 training, honeybees were fed ad libitum from a gravity feeder providing 30% sucrose  
94 solution. Each focal honeybee was first lured to visit the experimental setup by  
95 allowing her to drink and walk onto a cotton bud soaked in 50% sucrose solution and  
96 then transferring the bee to the setup. Each forager that returned to the setup on her  
97 own was marked on her thorax with a coloured dot for identification.

### 98 99 *(b) Experimental setup and procedure*

100 The setup consisted of a 50 x 50 cm acrylic sheet. Stimuli were 6 x 6 cm white  
101 displays (laminated sheets of paper) with between 1 and 4 black shapes (squares,  
102 diamonds, or circles). The stimuli, identical to those used in [28], were presented  
103 vertically with a landing platform attached just below the displays. Stimuli were  
104 randomly allocated for each bee and changed when the bee returned to the hive to  
105 offload sucrose, prior to her returning to the setup. The spatial arrangement of stimuli  
106 could be randomly changed, thus excluding position orientation cues. The  
107 background acrylic sheet and landing platforms were grey coloured. The acrylic  
108 background sheet, hangers, platforms and displays were washed with water then 70  
109 % ethanol between all visits to exclude the use of olfactory cues. Two shapes were  
110 used in training, and the third shape was used for testing. During training, honeybees  
111 found either a 10 $\mu$ L droplet of 50 % sucrose solution or a 60 mM quinine  
112 hemisulphate solution, for correct and incorrect choices, respectively. Each trial, four  
113 stimuli (two identical correct; two identical incorrect) were presented simultaneously  
114 on the acrylic sheet. Stimuli positions were changed after each choice to new  
115 random positions. A choice was defined as any time a honeybee landed on a  
116 platform and touched the solution (sucrose or quinine) with their proboscis, leg, or  
117 antenna.

118  
119 One group of bees (n = 10) was trained to associate stimuli consisting of more  
120 elements with a reward, and a second group of bees (n = 10) was trained to  
121 associate stimuli consisting of fewer elements with a reward. The choices of  
122 individual bees during training were recorded until a criterion of  $\geq 80$  % for any 10  
123 consecutive choices was reached (after a minimum of 20 conditioned choices). Once  
124 an individual bee reached criterion, she was presented with a learning test followed  
125 by two additional control tests. Bees reached criterion on average in  $41 \pm 8$  choices.  
126 Each test lasted two minutes and all choices made were recorded as the dependent  
127 variable for statistical analyses. During all tests, a 10 $\mu$ l of unrewarding water was  
128 placed on each platform. Between tests, bees received two reinforced refresher trials  
129 (with the same stimuli used in training) to maintain motivation. For the learning test,  
130 bees were presented with the shape that they had not been trained on – the purpose  
131 being to test whether bees learned to solve the task. The two control tests examined  
132 whether honeybees used the number of elements or continuous visual cues. The  
133 first control test (Equal/Incongruent test) had two pairs of stimuli, each with two  
134 elements, but one pair with higher edge length, convex hull, and spatial frequency.  
135 The second control test (Incongruent/Opposite test) also had two pairs of stimuli, one  
136 pair with three elements and the other with two elements but still with higher edge  
137 length, convex hull, and spatial frequency. In all tests, total black surface area was  
138 the same across all stimuli. Experiments were performed by three different groups of  
139 individuals (MH and OL, SL, and CS) to help independently verify the results.

#### 140 141 *(c) Statistical analyses*

142 R 3.6.1 with library “lme4” was used to perform all generalised linear mixed-effect  
143 models (glmm) with binomial distribution and logit function. For the glmm evaluating  
144 the results of the tests, country and rule (more-than/less-than) were considered as  
145 fixed factors and bee ID as a random effect (Table S1). Because country and rule  
146 had no effect on performance, we display data as the mean  $\pm$  s.e.m. of all bees’  
147 data. We then removed country and rule in a second glmm (Table 2). Our second  
148 model ranked better than the first on the grounds of Akaike’s Information Criterion  
149 [59] adjusted for small sample sizes (AICc), and therefore we present data from this

150 second model in the main text. For analyses of all test videos, a blind protocol was  
 151 employed, in that each video filename was coded so that the experimenter doing the  
 152 analysis was blind to the training of each bee.

153  
 154 To calculate the spatial frequency of the training and test stimuli, a two-dimensional  
 155 Fourier transform on each image was performed, followed by a power spectrum  
 156 calculation as the square amplitude of the Fourier transform and averaged over  
 157 orientation [60]. The actual power over all frequencies was then measured by  
 158 calculating the area under the curve of the radially averaged power spectrum.  
 159 Calculations for spatial frequency, convex hull and edge length were done in  
 160 MATLAB 2018b (MathWorks, Mass., USA). Statistical analyses for the model results  
 161 were also performed in MATLAB 2018b.

162  
 163 *(d) Neural network model*

164 Our model utilizes spatial frequency encoding that is supported by bees' ability to  
 165 discriminate visual patterns based on spatial frequency [49,50] and observed  
 166 neurons in the visual lobe of insects that provide a mechanism of frequency coding  
 167 [61,62]. Analogous to the spatial frequency coding in primates [63,64], bees may  
 168 use Gabor-like filters in their visual lobe to extract spatial frequency information from  
 169 visual stimuli [65]. For our model, the stimulus,  $s$ , is encoded by the activity of a  
 170 population of neurons with different preferred spatial frequency that possess similar  
 171 response profiles. The evoked spiking activities of the seven sensory neurons were  
 172 simulated by fixed Gaussian tuning curves spanning spatial frequencies of the input  
 173 from zero to six as

174  $g_i(s, \sigma) = R_0 + R_{Max} \exp[-\frac{1}{2\sigma^2} (s - f_i)^2] + \aleph(0, \sigma_N)$ , where  $R_0 = 50$  spike/sec and  
 175  $R_{Max} = 200$  spike/sec are the spontaneous and maximum firing-rate of the sensory  
 176 neuron.  $\sigma = 2.5$  controls the degree of the selectivity of the sensory neurons to  
 177 different frequencies around the preferred frequency,  $f_i$ . Gaussian noise,  $\aleph(0, \sigma_N)$   
 178 model the randomness of neural activities.

179  
 180 Outputs of all sensory neurons drive a decision neuron through a vector of synaptic  
 181 weights,  $W$ , to create the decision neuron's activity in response to the input, as:

$$182 \quad D(s) = F \left( \sum_{k=0}^6 W_k \cdot g_k(s, \sigma) ; a, b \right)$$

183 where  $F(x; a, b) = A_0 / (1 + \exp(-a(x - b)))$  is the activation function with the  
 184 maximum activity at  $A_0 = 100$  Spike/sec. The parameters  $a = 0.05$  and  $b = 50$   
 185 control the sensitivity of the neuron to the input and spontaneous activity of the  
 186 decision-neuron, respectively.

187  
 188 Since we assume that the difference of the decision neuron's responses to the  
 189 positive ( $s_p$ ) and negative stimuli ( $s_n$ ) must be increased during the training phase,  
 190 the locally optimal synaptic weights,  $W^{opt}$ , can be obtained from maximizing the  
 191 objective function:

$$192 \quad L = \sum_{t=1}^m [D(s_p^t) - D(s_n^t)] r^t,$$

194

195 where  $t$  and  $m$  are the index over the paired stimuli and the number of presented  
 196 stimuli, respectively. Here,  $r$  presents the reinforcement signal (VUM-mx1 neuron)  
 197 that provides modulated feedback whether a stimulus is paired with the reward or  
 198 punishment ( $r = 1$ ) and  $r = 0$  for when no reinforcement signals is presented. The  
 199 (on-line) updates of the synaptic weights,  $W_i^t$  are calculated by

$$200 \quad W_i^t = W_i^{t-1} + \eta \frac{\partial}{\partial W_i} (D(s_p^t) - D(s_n^t)) r^t$$

201 where  $\eta$  is the rate of the weights change.  $W_i^{t-1}$  is the updated weight from the  
 202 iteration  $t - 1$  (with  $W_i^0$  being the initial weight), and

$$203 \quad \frac{\partial}{\partial W_i} (D(s_p^t) - D(s_n^t))$$

$$204 \quad = g_i(s_p^t, \sigma) F' \left( \sum_{k=0}^6 W_k \cdot g_k(s_p^t, \sigma); a, b \right) - g_i(s_n^t, \sigma) \cdot F' \left( \sum_{k=0}^6 W_k \cdot g_k(s_n^t, \sigma); a, b \right)$$

205 Finally, the derivatives of the activation function  $F$  is obtained as

$$206 \quad F'(x; a, b) = \frac{A_0 a \exp(-a(x - b))}{(1 + \exp(-a(x - b)))^2}$$

207 After exposing the model to conditioned stimuli in learning paradigms, the  
 208 behavioural outcomes of the model presented with a pair of the test stimuli were  
 209 evaluated as a simple subtraction of the decision neuron's responses to both test  
 210 stimuli.

### 211 3. Results

#### 212 (a) Bees use continuous cues over numerosity in a numerical cognition task

213 Using the same 2D visual stimulus set as a paradigmatic honeybee study [28], and  
 214 similar to stimulus sets used for other animals (e.g. [8–39]), we first asked whether  
 215 honeybees use numerosity to solve a numeric-based discrimination task. In this  
 216 particular stimulus set, area (total black within each stimulus) is kept constant across  
 217 all stimuli, and therefore could not be used to solve the task. But, similar to many  
 218 other numerical cognition studies, edge-length (Spearman correlation: rho=0.93,  
 219 p=1.00e-40), convex hull (Spearman correlation: rho=0.44, p=4.88e-6), and spatial  
 220 frequency (Spearman correlation: rho=0.92, p=1.00e-40) covaried with number  
 221 (figure 1f-j). We therefore aimed to train bees on this stimulus set, for which they've  
 222 already been shown to discriminate, and subsequently test bees to determine  
 223 whether they had used these particular continuous cues or rather numerosity to  
 224 solve the task.

225 We first trained honeybees ( $n = 10$ ) to find rewarding sugar solution on displays with  
 226 more shapes and an aversive quinine solution on displays with fewer shapes  
 227 (Methods; figure 2a). Another group of bees ( $n = 10$ ) was trained on the opposite  
 228 contingency. Once bees reached 80% performance (8/10 consecutive choices  
 229

238 correct), they were given an unrewarded learning test. Bees trained on a “more-than”  
239 rule preferred (landed on more often) stimuli containing more elements during the  
240 test, whereas bees trained to “less-than” preferred stimuli with fewer elements.  
241 Honeybees showed high performance in the Learning test (figure 2b left;  
242 Generalised linear mixed-effect model (glmm): 95% Confidence interval (CI) = 0.75  
243 (0.47 to 1.03),  $n = 20$ ,  $p = 1.49e-07$ ).

244  
245 To determine if bees used non-numerical cues, after the learning test and refresher  
246 trials (Methods), we tested the same honeybees on an “Equal/Incongruent test”,  
247 where two pairs of unrewarded stimuli contained the same number of elements  
248 (figure 2b middle), but differed in edge-length, convex hull, and spatial frequency  
249 (figure 2c-f). If honeybees were using numerosity, they should prefer all displays  
250 equally during this test. Conversely, honeybees more often chose stimuli with a  
251 higher level of continuous variables if they had been trained to choose stimuli with  
252 more elements, and more often chose stimuli with a lower level of continuous  
253 variables if they had been trained to choose stimuli with fewer elements (figure 2b  
254 middle; glmm: 95% CI = -0.64 (-0.89 to -0.39),  $n = 20$ ,  $p = 6.5e-07$ ). This suggests  
255 honey bees responded to continuous cues in the stimuli and not the number of  
256 elements.

257  
258 We further tested honeybees on an “Incongruent/Opposite test” where the number of  
259 elements for each pair of displays differed (2v3; figure 2b right) and the continuous  
260 cues (edge length, convex hull, and spatial frequency) were in the opposite direction  
261 to the numerical difference (i.e. higher for two elements than for three elements;  
262 figure 2c-f). In this test, honeybees behaved in the reverse manner to which we  
263 would expect if they had learned numerosity. Bees that were trained to associate  
264 more elements with reward preferred test displays with the higher level of continuous  
265 variables but fewer elements. Bees that were trained to associate fewer elements  
266 with reward preferred test displays with the lower level of continuous cues but more  
267 elements (figure 2b right; glmm: 95% CI = -0.55 (-0.79 to -0.30),  $n = 20$ ,  $p = 1.17e-$   
268 05).

269  
270 Our results indicate that honeybees use continuous properties to discriminate stimuli  
271 with varying number of shapes. This caveat may also apply to other numerical  
272 cognition studies with honeybees and other animals that used stimulus sets which  
273 controlled for one or more but not all continuous variables.

274  
275 *(b) A neural network model with no reference to numerosity can reproduce behaviors*  
276 *indicative of numerical cognition*

277  
278 Our results beg the question: what explanation is simpler and more plausible:  
279 numerical or non-numerical processing? Therefore, how simple is learning  
280 continuous variables as an explanation for the behaviour of honeybees? To explore  
281 this, we created a simple neural network model containing just nine elements  
282 arranged in three layers (figure 3a) to encode a relational rule (“more-than” or “less-  
283 than”) based only on one non-numerical cue (Materials and Methods). Seven  
284 elements acted as sensory neurons that encoded spatial frequency in the visual lobe  
285 and which projected frequency information to the eighth element, a single decision  
286 neuron in the mushroom bodies (high-level sensory integration centres involved in  
287 learning and memory). Synaptic weights between the sensory neurons and decision

288 neuron were adjusted according to the activation (by presentation of stimuli) of the  
289 ninth element, a reinforcement neuron, based on the specific learning rule (more-  
290 than or less-than). We chose spatial frequency for simplicity, and because we have  
291 yet to find any recent study that controlled/tested for it, but the model could also be  
292 applied to other continuous variables.  
293

294 We trained our model following the methods for several experiments in [28], a recent  
295 study that had honeybees discriminate 2D visual cues with different numbers of  
296 shapes. We then evaluated the model's choices when presented with test stimuli  
297 (See Methods for details and figure 3 for simplification). This simple model was able  
298 to reproduce the behaviour of honeybees in numerical cognition tasks, with a very  
299 simple computational structure using only non-numerical information. Specifically,  
300 the model could transfer a "more-than" or "less-than" rule to novel shapes, to stimuli  
301 containing a number of elements outside the range trained on, to stimuli with zero  
302 elements, and could recognise stimuli with zero elements as the lower end of a  
303 continuum (figure 3*b-e*). Thus, we are able to reproduce behavioural evidence that  
304 has been taken in honeybees (and similarly in other animals) as indicative of  
305 understanding number with a model in which there is no processing of numerosity.  
306

#### 307 **4. Discussion**

##### 308 (a) General summary

309 We are not suggesting that all numerical cognition studies are wrong or that no  
310 animal has numerical cognition. We show, however, that in a task using a 2D visual  
311 display set with differing number of shapes, non-numerical cues can be learned, they  
312 dominate over numerosity when equal to or set in opposition to number of elements,  
313 and they can be learned by simple computational systems with no reference to  
314 numerosity. Our behavioural and computational results provide a counterexample  
315 against the assumption that 2D visual stimuli with different numbers of shapes are  
316 processed by honeybees as discrete numerical elements. Our findings suggest that  
317 an alternative non-numerical explanation exists for studies using similar methods in  
318 honeybees. If other animals are sensitive to any available continuous cues, then an  
319 alternative non-numerical explanation exists for those results as well. This is vital  
320 information if we truly want to know how any animal solves the numerical problems  
321 they face in their own ecological niches.  
322  
323

##### 324 (b) *The depth of the issue*

325 It is very difficult to control for all continuous visual cues [44,66]. By controlling one  
326 parameter, another will necessarily covary with numerosity. Even varying parameters  
327 randomly during training is not enough to solve the issue. Leibovich and Henik  
328 (2014) trained adult humans on visual stimuli of differing number of dots where  
329 continuous cues were minimally correlated or uncorrelated with numerosity. Despite  
330 this, they found that in a regression analysis, half of the behavioural variance could  
331 be explained by the irrelevant continuous cues [56]. Presenting stimuli  
332 separately/sequentially may make the task more difficult (e.g. [67,68]). However,  
333 animals may store, in working memory, an accumulation of neural responses to  
334 continuous variable changes as they pass/observe stimuli, without reference to  
335 numerosity [69–72].  
336  
337



338

339 It will also not suffice to test for continuous cues separately because animals may  
340 learn multiple redundant cues and use those available when others are not [73–78].  
341 Testing all continuous variables (*that cannot be kept constant across stimuli*) and  
342 numerosity within one test can help determine if continuous variables have been  
343 learned. In one of our recent works, examining how bumblebees solved a numeric-  
344 based task, we assessed the use of continuous cues within one unrewarded test  
345 [79]. Here, bees were shown 10 stimuli simultaneously during one unrewarded test,  
346 each with different numbers of elements and levels of continuous cues. We chose  
347 the characteristics of different stimuli so that the bees' choices for some over others  
348 would reveal whether or not they had learned and used specific continuous cues to  
349 solve the task. For example, two displays both contained the same number of  
350 elements, but the elements in one of the displays had a greater edge-length. Bees  
351 chose these two displays equally in the test, suggesting they did not use edge  
352 length. However, if they had performed well on the test (i.e. more often chose stimuli  
353 based on the numerosity rule they had been trained) but had chosen one of these  
354 two stimuli significantly more than the other, this would suggest bees had learned  
355 and used edge-length instead of numerosity. We provided pairs of stimuli that varied  
356 in this way for edge-length, area, convex hull, spatial frequency and illusionary  
357 contour (Area was kept constant throughout training and tests and therefore did not  
358 need to be tested). We must keep in mind, as pointed out above, that even when this  
359 type of design suggests continuous cues were not used, as it had in our work, other  
360 strategies could still be used. Although bees' behaviour [79] indicated some form of  
361 counting, the bumblebees could have used working spatial memory to avoid recently  
362 visited shapes (cf. "inhibition of return" [80,81]). Therefore, it is possible that bees  
363 discriminated stimuli based on duration of time taken to scan all shapes within a  
364 display, or perhaps by an accumulator mechanism responding to visual changes as  
365 they scanned past each shape [69]. Either of these possible strategies do not require  
366 a true sense of number.

367

### 368 (c) *Ways forward*

369

370 How then can we address this natural, deep-seated issue? We propose that the  
371 method of assessing all continuous cues in one unrewarded test, in conjunction with  
372 varying all continuous cues during training, be set as a minimum when investigating  
373 numerical cognition in animals. But, as mentioned above we need to still keep in  
374 mind other potential non-numerical strategies.

375

376 Most numerical cognition studies utilise visual stimuli. Stimuli in other modalities  
377 come with their own set of issues regarding continuous variables. For example,  
378 number of individuals covaries with the overall complexity of their chemical/olfactory  
379 cues, and with the total volume and complexity of vocal calls. However, combining  
380 modalities does offer some promising avenues for investigation. One of the strongest  
381 pieces of evidence for numerical cognition is the ability to transfer across modalities,  
382 which seems to prevent the use of continuous cues because the only similarity  
383 across modalities should be numerosity. A nice example of this was shown in  
384 monkeys where they were able to match the sum of randomly-ordered sequentially-  
385 presented shapes and tones to a visual array with the same number of squares [82].  
386 This kind of cross-modal generalization design would certainly strengthen arguments  
387 for numerical cognition in other animals.

388

389 Video of animals solving numerical cognition tasks can help determine how animals  
390 are solving those tasks (cf. [1,2]). Automated approaches combining machine vision  
391 and learning with computational behavioural analyses have the ability to discover  
392 behavioural features that humans cannot (cf. [83]; [84]). For example, by measuring  
393 the inspection behaviour (e.g. gaze, body direction, movement) of an animal towards  
394 different numerical stimuli and comparing across different decisions (choose/reject)  
395 and different outcomes (correct/incorrect), underlying strategies may become  
396 apparent.

397

398 Ultimately, however, we must establish the underlying neural mechanisms to truly  
399 know which cues and strategies an animal utilised to solve a numeric-based task.  
400 This will provide vital information for how numerical cognition may have evolved, and  
401 how processing of numerosity compares between animals [85,86].

402

403

#### 404 **Author contributions**

405 HM, ABB and CS conceptualised the project and designed experiments. SL, MH,  
406 OJL and CS conducted experiments with the help of FP and WL. CS analysed the  
407 behavioural data with help from FP. HM analysed visual stimuli and created and  
408 analysed the model with helpful comments from ABB, JARM, AC and EV. HM, ABB  
409 and CS wrote the paper with helpful comments from the other authors.

410

#### 411 **Acknowledgements**

412 This study was supported by the EPSRC program grant Brains-on-Board  
413 (EP/P006094/1) awarded to JARM and EV. ABB and JARM were supported by a  
414 Leverhulme visiting professorship. ABB and CS were supported by the Templeton  
415 World Charity Foundation project number TWCF0266. FP was supported by the  
416 National Natural Science Foundation of China (Project no. 31700988). We thank  
417 Yonghe Zhou and Yuyi Lu for assistance with the experimental setup in China.

418

#### 419 **Conflict of interest statement**

420 All authors declare no conflict of interest.

421

#### 422 **Data availability**

423 The data supporting the findings of this study (figure 1*f, g, h* and *j*, figure 2*b*, and  
424 figure 3*b, c, d*, and *e*), the code necessary for the model, and the code for measuring  
425 the continuous visual features of the stimuli are available in the public repository  
426 figshare at <https://figshare.com/s/21c5753e31f51ece5f1c>. Please note that this is a  
427 private link for referees and will be made public upon publication.

428

#### 429 **References**

430

431 1. MaBouDi H, Solvi C, Chittka L. 2020 Bumblebees Learn a Relational Rule but Switch to a  
432 Win-Stay/Lose-Switch Heuristic After Extensive Training. *Front. Behav. Neurosci.* **14**.  
433 (doi:10.3389/fnbeh.2020.00137)

434 2. Guiraud M, Roper M, Chittka L. 2018 High-speed videography reveals how honeybees  
435 can turn a spatial concept learning task into a simple discrimination task by stereotyped

- 436 flight movements and sequential inspection of pattern elements. *Front. Psychol.* **9**,  
437 1347. (doi:10.3389/fpsyg.2018.01347)
- 438 3. Izquierdo A, Belcher AM. 2012 Rodent Models of Adaptive Decision Making. *Methods*  
439 *Mol. Biol. Clifton NJ* **829**, 85–101. (doi:10.1007/978-1-61779-458-2\_5)
- 440 4. Risko EF, Gilbert SJ. 2016 Cognitive Offloading. *Trends Cogn. Sci.* **20**, 676–688.  
441 (doi:10.1016/j.tics.2016.07.002)
- 442 5. Jolicoeur P. 1988 Mental rotation and the identification of disoriented objects. *Can. J.*  
443 *Psychol.* **42**, 461–478. (doi:10.1037/h0084200)
- 444 6. Wasserman EA, Zentall TR. 2006 *Comparative Cognition: Experimental Explorations of*  
445 *Animal Intelligence*. Oxford University Press.
- 446 7. Chittka L, Rossiter SJ, Skorupski P, Fernando C. 2012 What is comparable in comparative  
447 cognition? *Philos. Trans. R. Soc. Lond. B. Biol. Sci.* **367**, 2677–2685.  
448 (doi:10.1098/rstb.2012.0215)
- 449 8. Cammaerts M-C, Cammaerts R. 2019 Ants Correctly Locate the Zero in a Continuous  
450 Series of Numbers. *Int. J. Biol.* **11**. (doi:10.5539/ijb.v11n4p16)
- 451 9. Vonk J, Beran MJ. 2012 Bears ‘count’ too: quantity estimation and comparison in black  
452 bears, *Ursus americanus*. *Anim. Behav.* **84**, 231–238.  
453 (doi:10.1016/j.anbehav.2012.05.001)
- 454 10. Aulet LS, Chiu VC, Prichard A, Spivak M, Lourenco SF, Berns GS. 2019 Canine sense of  
455 quantity: evidence for numerical ratio-dependent activation in parietotemporal cortex.  
456 *Biol. Lett.* **15**, 20190666. (doi:10.1098/rsbl.2019.0666)
- 457 11. Rugani R, Vallortigara G, Priftis K, Regolin L. 2015 Number-space mapping in the  
458 newborn chick resembles humans’ mental number line. *Science* **347**, 534–536.  
459 (doi:10.1126/science.aaa1379)
- 460 12. Rugani R, Vallortigara G, Regolin L. 2013 Numerical abstraction in young domestic chicks  
461 (*Gallus gallus*). *PLOS ONE* **8**, e65262. (doi:10.1371/journal.pone.0065262)
- 462 13. Rugani R, McCrink K, de Hevia M-D, Vallortigara G, Regolin L. 2016 Ratio abstraction  
463 over discrete magnitudes by newly hatched domestic chicks (*Gallus gallus*). *Sci. Rep.* **6**,  
464 1–8. (doi:10.1038/srep30114)
- 465 14. Tomonaga M. 2008 Relative numerosity discrimination by chimpanzees (*Pan*  
466 *trogodytes*): evidence for approximate numerical representations. *Anim. Cogn.* **11**, 43–  
467 57. (doi:10.1007/s10071-007-0089-0)
- 468 15. Ditz HM, Nieder A. 2016 Numerosity representations in crows obey the Weber–Fechner  
469 law. *Proc. R. Soc. B Biol. Sci.* **283**, 20160083. (doi:10.1098/rspb.2016.0083)

- 470 16. Wagener L, Loconsole M, Ditz HM, Nieder A. 2018 Neurons in the Endbrain of  
471 Numerically Naive Crows Spontaneously Encode Visual Numerosity. *Curr. Biol.* **28**, 1090-  
472 1094.e4. (doi:10.1016/j.cub.2018.02.023)
- 473 17. Ditz HM, Nieder A. 2015 Neurons selective to the number of visual items in the corvid  
474 songbird endbrain. *Proc. Natl. Acad. Sci.* **12**, 7827–7832.  
475 (doi:10.1073/pnas.1504245112)
- 476 18. Yaman S, Kilian A, von Fersen L, Güntürkün O. 2012 Evidence for a Numerosity Category  
477 that is Based on Abstract Qualities of “Few” vs. “Many” in the Bottlenose Dolphin  
478 (*Tursiops truncatus*). *Front. Psychol.* **3**. (doi:10.3389/fpsyg.2012.00473)
- 479 19. Irie N, Hiraiwa-Hasegawa M, Kutsukake N. 2019 Unique numerical competence of Asian  
480 elephants on the relative numerosity judgment task. *J. Ethol.* **37**, 111–115.  
481 (doi:10.1007/s10164-018-0563-y)
- 482 20. Piffer L, Petrazzini MEM, Agrillo C. 2013 Large number discrimination in newborn fish.  
483 *PLOS ONE* **8**, e62466. (doi:10.1371/journal.pone.0062466)
- 484 21. Agrillo C, Miletto Petrazzini ME, Tagliapietra C, Bisazza A. 2012 Inter-Specific Differences  
485 in Numerical Abilities Among Teleost Fish. *Front. Psychol.* , 483.  
486 (doi:10.3389/fpsyg.2012.00483)
- 487 22. DeLong CM, Barbato S, O’Leary T, Wilcox KT. 2017 Small and large number  
488 discrimination in goldfish (*Carassius auratus*) with extensive training. *Behav. Processes*  
489 **141**, 172–183. (doi:10.1016/j.beproc.2016.11.011)
- 490 23. Vonk J, Torgerson-White L, McGuire M, Thueme M, Thomas J, Beran MJ. 2014 Quantity  
491 estimation and comparison in western lowland gorillas (*Gorilla gorilla gorilla*). *Anim.*  
492 *Cogn.* **17**, 755–765. (doi:10.1007/s10071-013-0707-y)
- 493 24. Lucon-Xiccato T, Miletto Petrazzini ME, Agrillo C, Bisazza A. 2015 Guppies discriminate  
494 between two quantities of food items but prioritize item size over total amount. *Anim.*  
495 *Behav.* **107**, 183–191. (doi:10.1016/j.anbehav.2015.06.019)
- 496 25. Bisazza A, Agrillo C, Lucon-Xiccato T. 2014 Extensive training extends numerical abilities  
497 of guppies. *Anim. Cogn.* **17**, 1413–1419. (doi:10.1007/s10071-014-0759-7)
- 498 26. Gatto E, Lucon-Xiccato T, Savaşçı BB, Dadda M, Bisazza A. 2017 Experimental setting  
499 affects the performance of guppies in a numerical discrimination task. *Anim. Cogn.* **20**,  
500 187–198. (doi:10.1007/s10071-016-1037-7)
- 501 27. Howard SR, Avarguès-Weber A, Garcia JE, Greentree AD, Dyer AG. 2019 Numerical  
502 cognition in honeybees enables addition and subtraction. *Sci. Adv.* **5**, eaav0961.  
503 (doi:10.1126/sciadv.aav0961)
- 504 28. Howard SR, Avarguès-Weber A, Garcia JE, Greentree AD, Dyer AG. 2018 Numerical  
505 ordering of zero in honey bees. *Science* **360**, 1124–1126. (doi:10.1126/science.aar4975)

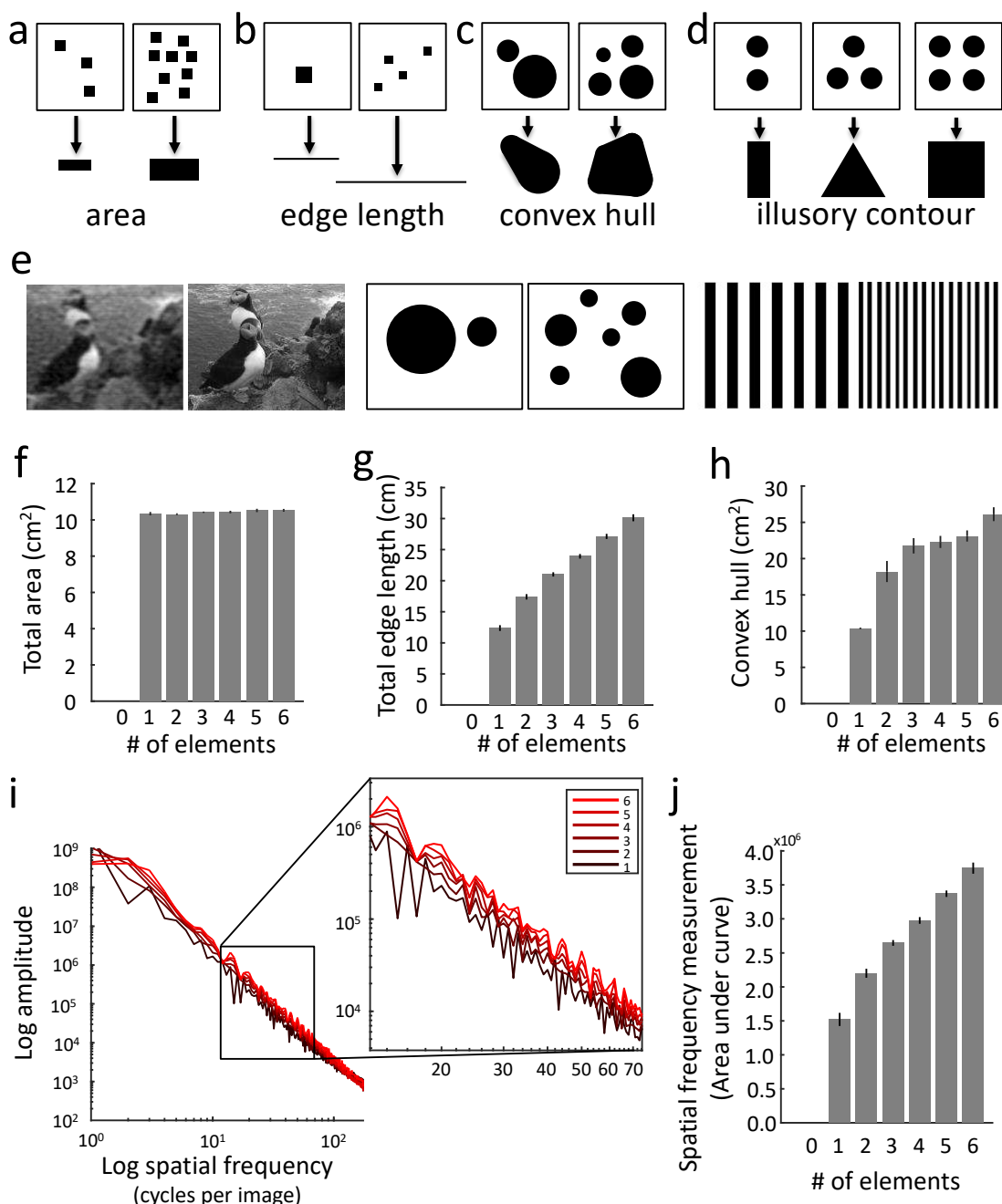
- 506 29. Howard SR, Avarguès-Weber A, Garcia JE, Greentree AD, Dyer AG. 2019 Symbolic  
507 representation of numerosity by honeybees (*Apis mellifera*): matching characters to  
508 small quantities. *Proc. R. Soc. B Biol. Sci.* **286**, 20190238. (doi:10.1098/rspb.2019.0238)
- 509 30. Howard SR, Avarguès-Weber A, Garcia JE, Greentree AD, Dyer AG. 2019 Surpassing the  
510 subitizing threshold: appetitive–aversive conditioning improves discrimination of  
511 numerosities in honeybees. *J. Exp. Biol.* **222**. (doi:10.1242/jeb.205658)
- 512 31. Bortot M, Agrillo C, Avarguès-Weber A, Bisazza A, Miletto Petrazzini ME, Giurfa M. 2019  
513 Honeybees use absolute rather than relative numerosity in number discrimination. *Biol.*  
514 *Lett.* **15**, 20190138. (doi:10.1098/rsbl.2019.0138)
- 515 32. Howard SR, Schramme J, Garcia JE, Ng L, Avarguès-Weber A, Greentree AD, Dyer AG.  
516 2020 Spontaneous quantity discrimination of artificial flowers by foraging honeybees. *J.*  
517 *Exp. Biol.* **223**. (doi:10.1242/jeb.223610)
- 518 33. de Hevia MD, Addabbo M, Nava E, Croci E, Girelli L, Macchi Cassia V. 2017 Infants'  
519 detection of increasing numerical order comes before detection of decreasing number.  
520 *Cognition* **158**, 177–188. (doi:10.1016/j.cognition.2016.10.022)
- 521 34. Jones SM, Pearson J, DeWind NK, Paulsen D, Tenekedjieva A-M, Brannon EM. 2014  
522 Lemurs and macaques show similar numerical sensitivity. *Anim. Cogn.* **17**, 503–515.  
523 (doi:10.1007/s10071-013-0682-3)
- 524 35. Okuyama S, Iwata J, Tanji J, Mushiake H. 2013 Goal-oriented, flexible use of numerical  
525 operations by monkeys. *Anim. Cogn.* **16**, 509–518. (doi:10.1007/s10071-012-0592-9)
- 526 36. Ramirez-Cardenas A, Moskaleva M, Nieder A. 2016 Neuronal Representation of  
527 Numerosity Zero in the Primate Parieto-Frontal Number Network. *Curr. Biol.* **26**, 1285–  
528 1294. (doi:10.1016/j.cub.2016.03.052)
- 529 37. Reznikova Z, Panteleeva S, Vorobyeva NS. 2019 Precise relative-quantity judgement in  
530 the striped field mouse *Apodemus agrarius* Pallas. *Anim. Cogn.* **22**, 277–289.  
531 (doi:10.1007/s10071-019-01244-7)
- 532 38. Scarf D, Hayne H, Colombo M. 2011 Pigeons on Par with Primates in Numerical  
533 Competence. *Science* **334**, 1664–1664. (doi:10.1126/science.1213357)
- 534 39. Lucon-Xiccato T, Gatto E, Bisazza A. 2018 Quantity discrimination by treefrogs. *Anim.*  
535 *Behav.* **139**, 61–69. (doi:10.1016/j.anbehav.2018.03.005)
- 536 40. Vallortigara G. 2017 An animal's sense of number. In *The nature and development of*  
537 *mathematics: cross disciplinary perspectives on cognition, learning and culture.*, pp. 43–  
538 65.
- 539 41. Bortot M, Regolin L, Vallortigara G. 2020 A sense of number in invertebrates. *Biochem.*  
540 *Biophys. Res. Commun.* (doi:10.1016/j.bbrc.2020.11.039)
- 541 42. Nieder A, Dehaene S. 2009 Representation of Number in the Brain. *Annu. Rev. Neurosci.*  
542 **32**, 185–208. (doi:10.1146/annurev.neuro.051508.135550)

- 543 43. Henik A. 2016 *Continuous issues in numerical cognition: how many or how much*.  
544 Academic Press.
- 545 44. Leibovich T, Katzin N, Harel M, Henik A. 2017 From 'sense of number' to 'sense of  
546 magnitude': The role of continuous magnitudes in numerical cognition. *Behav. Brain Sci.*  
547 **40**, e164. (doi:10.1017/S0140525X16000960)
- 548 45. Banks MS, Ginsburg AP. 1985 Infant visual preferences: A review and new theoretical  
549 treatment. In *Advances in child development and behavior* (ed HW Reese), pp. 207–246.  
550 JAI. (doi:10.1016/S0065-2407(08)60392-4)
- 551 46. Mix KS, Huttenlocher J, Levine SC. 2002 Multiple cues for quantification in infancy: is  
552 number one of them? *Psychol. Bull.* **128**, 278–294. (doi:10.1037/0033-2909.128.2.278)
- 553 47. Maffei L, Fiorentini A. 1977 Spatial frequency rows in the striate visual cortex. *Vision*  
554 *Res.* **17**, 257–264. (doi:10.1016/0042-6989(77)90089-X)
- 555 48. Pollen DA, Ronner SF. 1983 Visual cortical neurons as localized spatial frequency filters.  
556 *IEEE Trans. Syst. Man Cybern.* **SMC-13**, 907–916. (doi:10.1109/TSMC.1983.6313086)
- 557 49. Horridge GA. 1997 Pattern discrimination by the honeybee: disruption as a cue. *J. Comp.*  
558 *Physiol. A* **181**, 267–277. (doi:10.1007/s003590050113)
- 559 50. Lehrer M. 1999 Shape Perception in the Honeybee: Symmetry as a Global Framework.  
560 *Int. J. Plant Sci.* **160**, S51–S65. (doi:10.1086/314216)
- 561 51. Lehrer M. 1997 Honeybee's Use of Spatial Parameters for Flower Discrimination. *Isr. J.*  
562 *Plant Sci.* **45**, 157–167. (doi:10.1080/07929978.1997.10676681)
- 563 52. Dafni A, Lehrer M, Kevan PG. 1997 Spatial flower parameters and insect spatial vision.  
564 *Biol. Rev.* **72**, 239–282. (doi:10.1017/S0006323196005002)
- 565 53. Rivas-Blanco D, Pohl I-M, Dale R, Heberlein MTE, Range F. 2020 Wolves and Dogs May  
566 Rely on Non-numerical Cues in Quantity Discrimination Tasks When Given the Choice.  
567 *Front. Psychol.* **11**. (doi:10.3389/fpsyg.2020.573317)
- 568 54. Miletto Petrazzini ME, Wynne CDL. 2016 What counts for dogs (*Canis lupus familiaris*) in  
569 a quantity discrimination task? *Behav. Processes* **122**, 90–97.  
570 (doi:10.1016/j.beproc.2015.11.013)
- 571 55. Gatto E, Carlesso D. 2019 Spontaneous quantity discrimination in crickets. *Ethology* **125**,  
572 613–619. (doi:10.1111/eth.12912)
- 573 56. Leibovich T, Henik A. 2014 Comparing performance in discrete and continuous  
574 comparison tasks. *Q. J. Exp. Psychol.* **67**, 899–917. (doi:10.1080/17470218.2013.837940)
- 575 57. Rugani R, Regolin L, Vallortigara G. 2010 Imprinted numbers: newborn chicks' sensitivity  
576 to number vs. continuous extent of objects they have been reared with. *Dev. Sci.* **13**,  
577 790–797. (doi:10.1111/j.1467-7687.2009.00936.x)

- 578 58. Agrillo C, Piffer L, Bisazza A. 2011 Number versus continuous quantity in numerosity  
579 judgments by fish. *Cognition* **119**, 281–287. (doi:10.1016/j.cognition.2010.10.022)
- 580 59. Burnham KP, Anderson DR. 2002 *Model Selection and Multimodel Inference: A Practical*  
581 *Information-Theoretic Approach*. 2nd edn. New York: Springer-Verlag.  
582 (doi:10.1007/b97636)
- 583 60. van der Schaaf A, van Hateren JH. 1996 Modelling the Power Spectra of Natural Images:  
584 Statistics and Information. *Vision Res.* **36**, 2759–2770. (doi:10.1016/0042-  
585 6989(96)00002-8)
- 586 61. O’Carroll D. 1993 Feature-detecting neurons in dragonflies. *Nature* **362**, 541–543.  
587 (doi:10.1038/362541a0)
- 588 62. James AC, Osorio D. 1996 Characterisation of columnar neurons and visual signal  
589 processing in the medulla of the locust optic lobe by system identification techniques. *J.*  
590 *Comp. Physiol. A* **178**, 183–199. (doi:10.1007/BF00188161)
- 591 63. Sachs MB, Nachmias J, Robson JG. 1971 Spatial-Frequency Channels in Human Vision\*.  
592 *JOSA* **61**, 1176–1186. (doi:10.1364/JOSA.61.001176)
- 593 64. Silverman MS, Grosf DH, De Valois RL, Elfar SD. 1989 Spatial-frequency organization in  
594 primate striate cortex. *Proc. Natl. Acad. Sci. U. S. A.* **86**, 711–715.  
595 (doi:10.1073/pnas.86.2.711)
- 596 65. Srinivasan MV. 2010 Honey Bees as a Model for Vision, Perception, and Cognition. *Annu.*  
597 *Rev. Entomol.* **55**, 267–284. (doi:10.1146/annurev.ento.010908.164537)
- 598 66. Gevers W, Kadosh RC, Gebuis T. 2016 Sensory integration theory: an alternative to the  
599 approximate number system. In *Continuous issues in numerical cognition: how many or*  
600 *how much* (ed A Henik), pp. 405–418. San Diego: Academic Press.
- 601 67. Uller C, Lewis J. 2009 Horses (*Equus caballus*) select the greater of two quantities in  
602 small numerical contrasts. *Anim. Cogn.* **12**, 733–738. (doi:10.1007/s10071-009-0225-0)
- 603 68. Ditz HM, Nieder A. 2020 Format-dependent and format-independent representation of  
604 sequential and simultaneous numerosity in the crow endbrain. *Nat. Commun.* **11**, 1–10.  
605 (doi:10.1038/s41467-020-14519-2)
- 606 69. Meck WH, Church RM. 1983 A mode control model of counting and timing processes. *J.*  
607 *Exp. Psychol. Anim. Behav. Process.* **9**, 320–334. (doi:10.1037/0097-7403.9.3.320)
- 608 70. Gallistel CR, Gelman R. 2000 Non-verbal numerical cognition: from reals to integers.  
609 *Trends Cogn. Sci.* **4**, 59–65. (doi:10.1016/s1364-6613(99)01424-2)
- 610 71. Vasas V, Chittka L. 2019 Insect-Inspired Sequential Inspection Strategy Enables an  
611 Artificial Network of Four Neurons to Estimate Numerosity. *iScience* **11**, 85–92.  
612 (doi:10.1016/j.isci.2018.12.009)

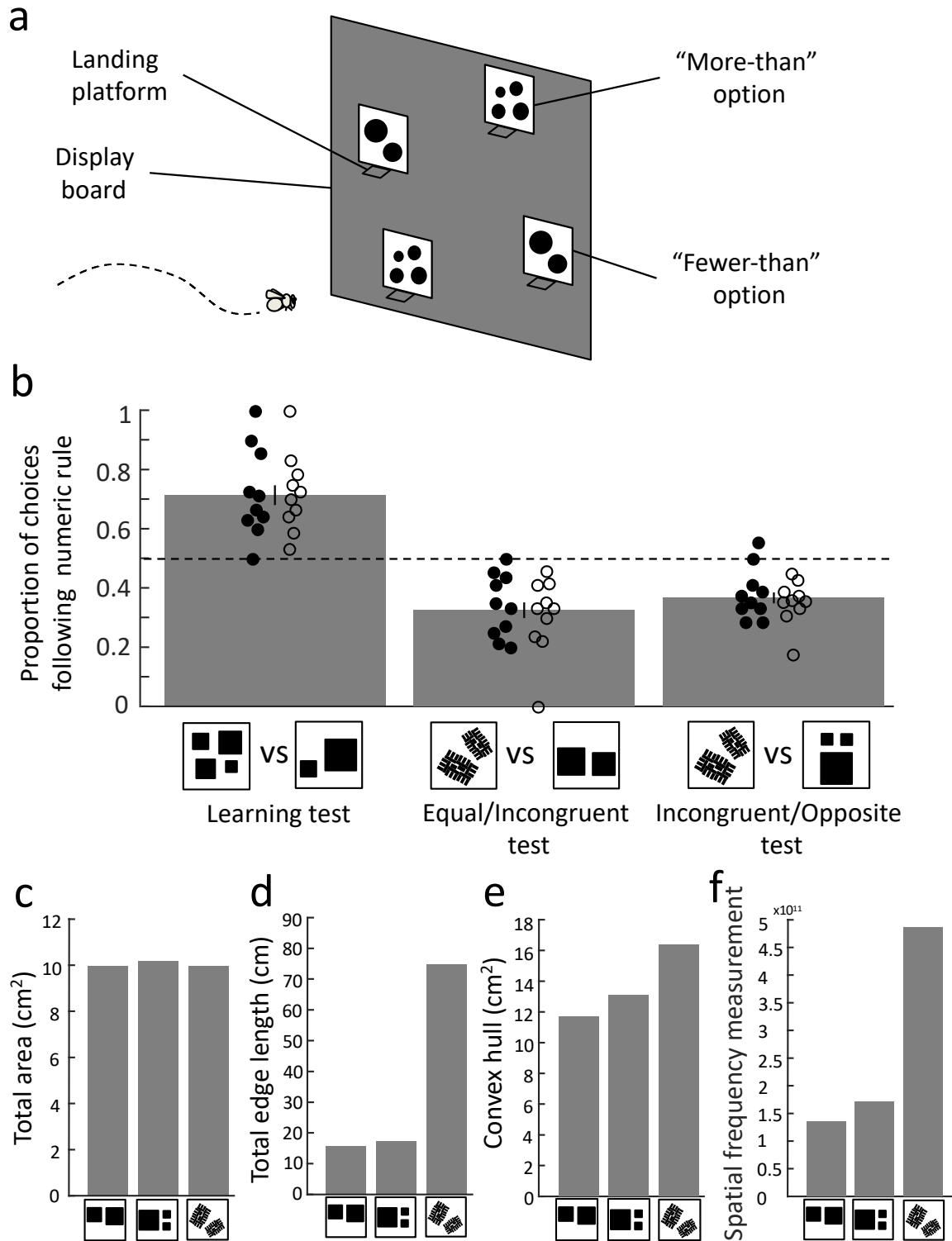
- 613 72. Rapp H, Nawrot MP, Stern M. 2020 Numerical Cognition Based on Precise Counting with  
614 a Single Spiking Neuron. *iScience* **23**, 100852. (doi:10.1016/j.isci.2020.100852)
- 615 73. Eimas PD. 1969 Multiple-Cue Discrimination Learning in Children. *Psychol. Rec.* **19**, 417–  
616 424. (doi:10.1007/BF03393868)
- 617 74. Slotnick B. 2001 Animal cognition and the rat olfactory system. *Trends Cogn. Sci.* **5**, 216–  
618 222. (doi:10.1016/S1364-6613(00)01625-9)
- 619 75. Gerber B, Smith BH. 1998 Visual modulation of olfactory learning in honeybees. *J. Exp.*  
620 *Biol.* **201**, 2213–2217.
- 621 76. Ohnishi K. 1991 Goldfish's visual information processing patterns in food-reinforced  
622 discrimination learning between compound visual stimuli. *J. Comp. Physiol. A* **168**, 581–  
623 589. (doi:10.1007/BF00215080)
- 624 77. Brembs B, Heisenberg M. 2001 Conditioning with compound stimuli in drosophila  
625 melanogaster in the flight simulator. *J. Exp. Biol.* **204**, 2849–2859.
- 626 78. Clipperton-Allen A, Cole M, Peck M, Quirt J. 2016 Pattern cue and visual cue competition  
627 in a foraging task by rats. *Learn. Behav.* **44**, 378–389. (doi:10.3758/s13420-016-0231-4)
- 628 79. MaBouDi H, Dona HSG, Gatto E, Loukola OJ, Buckley E, Onoufriou PD, Skorupski P,  
629 Chittka L. 2020 Bumblebees use sequential scanning of countable items in visual  
630 patterns to solve numerosity tasks. *Integr. Comp. Biol.* (doi:10.1093/icb/icaa025)
- 631 80. Gabay S, Leibovich T, Ben-Simon A, Henik A, Segev R. 2013 Inhibition of return in the  
632 archer fish. *Nat. Commun.* **4**, 1657. (doi:10.1038/ncomms2644)
- 633 81. Sapir A, Soroker N, Berger A, Henik A. 1999 Inhibition of return in spatial attention:  
634 direct evidence for collicular generation. *Nat. Neurosci.* **2**, 1053–1054.  
635 (doi:10.1038/15977)
- 636 82. Jordan KE, MacLean EL, Brannon EM. 2008 Monkeys match and tally quantities across  
637 senses. *Cognition* **108**, 617–625. (doi:10.1016/j.cognition.2008.05.006)
- 638 83. Egnor SER, Branson K. 2016 Computational Analysis of Behavior. *Annu. Rev. Neurosci.*  
639 **39**, 217–236. (doi:10.1146/annurev-neuro-070815-013845)
- 640 84. Skorupski P, MaBouDi H, Dona HSG, Chittka L. 2018 Counting insects. *Phil Trans R Soc B*  
641 **373**, 20160513. (doi:10.1098/rstb.2016.0513)
- 642 85. Nieder A. 2016 The neuronal code for number. *Nat. Rev. Neurosci.* **17**, 366–382.  
643 (doi:10.1038/nrn.2016.40)
- 644 86. Harvey BM. 2016 Quantity Cognition: Numbers, Numerosity, Zero and Mathematics.  
645 *Curr. Biol.* **26**, R419–R421. (doi:10.1016/j.cub.2016.03.059)





650  
 651 **Figure 1. Number of elements naturally co-varies with non-numerical cues. (a-**  
 652 **d)** Examples of 2D stimuli used in numerical cognition studies and how different  
 653 continuous cues normally covary with numerosity. Note that illusory contour does not  
 654 covary with numerosity but can still be learned and used to solve numerical cognition  
 655 tasks, especially with lower number of elements. **(e)** Spatial frequency (the amount  
 656 of alternating dark and light regions in a given area) also normally covaries with  
 657 numerosity. The more changes from black to white across an image in all directions,  
 658 the greater spatial frequency. The right images of each pair in **(e)** all have higher  
 659 spatial frequency than the left images. **(f-h)** For all stimuli in [28], from which our  
 660 stimulus set was borrowed, area (amount of total black (inset)) was kept constant **(f)**,  
 661 but edge length (total boundary length (inset); **g**) and convex hull (the minimum  
 662 convex region covering all elements (inset); **h**) covered with numerosity. **(j-i)** Spatial

663 frequency is calculated by obtaining a power spectrum (Methods) and measuring the  
664 area under the power spectrum's curve. The power spectrum plots (f and zoomed-in  
665 inset) for all stimuli in [28], from which our stimulus set was borrowed, averaged for  
666 each number of elements from one to six, shows that spatial frequency increases  
667 with numerosity (g). Note that for all covarying continuous cues, a zero-set stimulus  
668 will have zero measurement and thereby be placed naturally at the lower end of the  
669 spectrum for each of these non-numerical cues.  
670



672

673

674

675

676

677

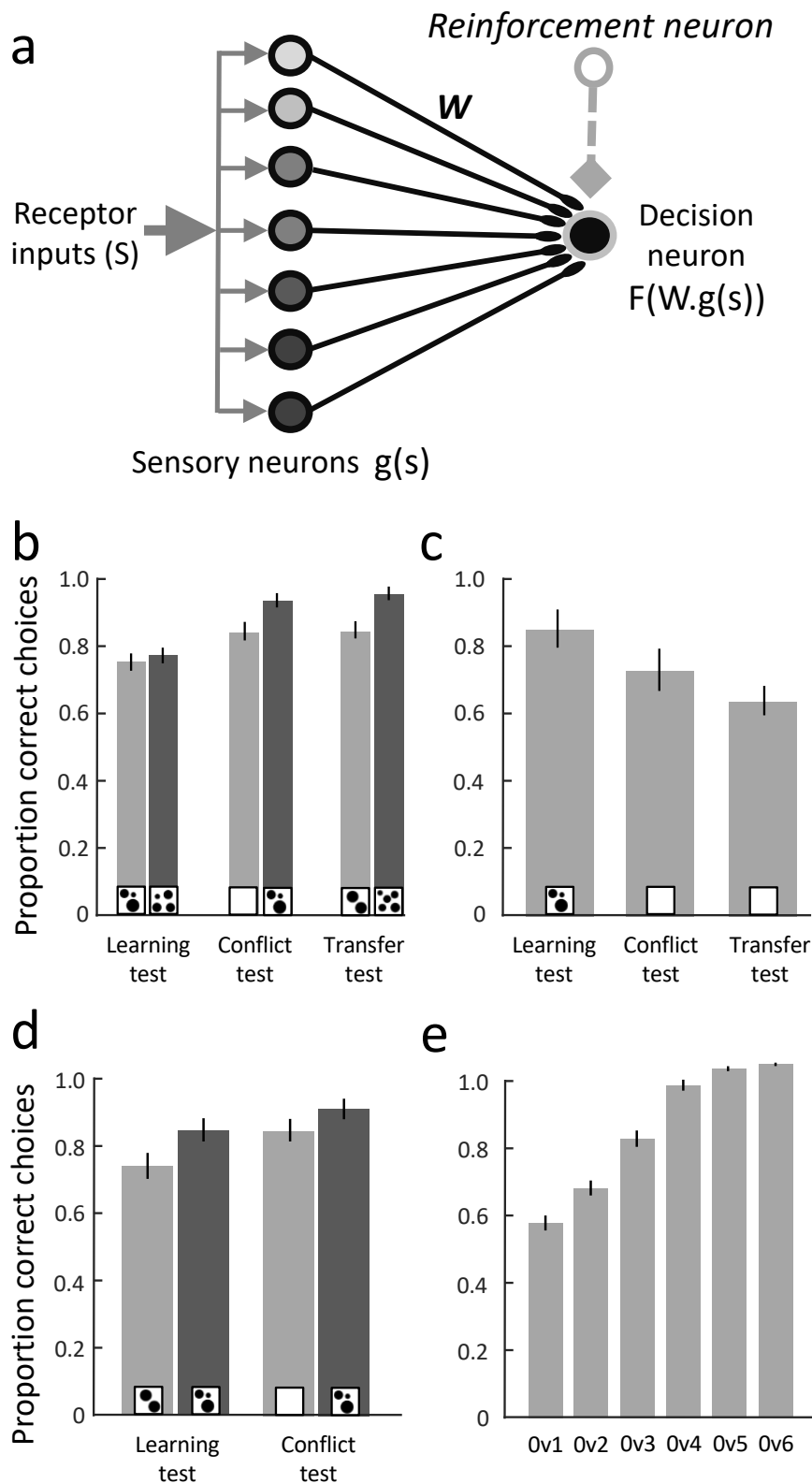
678

679

680

**Figure 2. Bees can use non-numerical strategies to discriminate numerical stimuli.** (a) Experiment setup. Honeybees were trained to find 50% sucrose solution at one of two pairs of displays showing different numbers of elements, and aversive quinine solution on the other display pair (Methods). (b) Once honeybees reached 80% performance, they were tested using displays with novel shapes. In the Learning test, honeybees more often chose stimuli following the numerical rule on which they had been trained ( $71.3 \pm 3.3\%$ ; more-than:  $70.3 \pm 4.7\%$ ; less-than:  $72.4 \pm 4.8\%$ ). However, when tested on stimuli that differed in continuous cues but not

681 number of elements (Equal/Incongruent test; middle bar;  $32.5 \pm 2.6$ ; more-than:  
682  $30.7 \pm 4.2\%$ ; less-than:  $34.2 \pm 3.4\%$ ) and separately on two pairs of stimuli where  
683 numerosity and continuous cues were set in opposition (Incongruent/Opposite test;  
684 right bar;  $36.7 \pm 1.8$ ; more-than:  $35.1 \pm 2.4\%$ ; less-than:  $38.2 \pm 2.8\%$ ), honeybees chose  
685 stimuli based on continuous cues over numerosity. Data shown are combined from  
686 the two groups trained with different numerical rules since no difference in  
687 performance was found between groups (Table 1; Methods). Dotted line = 0.5  
688 chance level. Bars = mean. Vertical lines = s.e.m. Circles = individual bees' data  
689 points (filled circles: bees trained to more-than rule; empty circles: bees trained to  
690 less-than rule). **(c-f)** Stimuli used in tests with corresponding continuous variable  
691 measurements (Methods).  
692



694  
695  
696  
697  
698  
699

**Figure 3. A simple computational model using only non-numerical cues reproduces honeybees' performance on a numerosity task. (a)** The model uses seven sensory neurons that are activated by the output of visual receptors. Each sensory neuron responds to multiple levels of a single continuous cue with different sensitivities. Firing of each sensory neuron is specific and selective to the preference

700 level modelled by a Gaussian tuning curve. Information from all sensory neurons  
701 converges at a single decision neuron. Synaptic connectivity ( $W$ ) between sensory  
702 neurons and the decision neuron are modified by an associative learning rule for  
703 encoding appetitive and aversive valences. Performance of the model is evaluated  
704 by a simple subtraction of the responses of the decision neuron to the test stimuli.  
705 Our model is able to reproduce behaviours claimed to be indicative of numerical  
706 cognition [28], without any reference to numerosity. This includes transferring a  
707 “more-than” or “less-than” rule to: **(b)** novel shapes in a “conflict test” examining  
708 preference for zero numerosity (Wilcoxon signed rank test,  $z$ -value $>6.22$  and  
709  $p<3.50e-9$ ) and a “transfer test” using displays with more shapes than in training  
710 (Wilcoxon signed rank test,  $z$ -value $>7.99$  and  $p<3.17e-14$ ). Compare to [28] Fig 1C.  
711 **(c)** stimuli containing a number of elements outside the training stimuli range, in a  
712 learning test (Wilcoxon signed rank test,  $z$ -value $=3.89$  and  $p=9.98e-05$ ), conflict test  
713 ( $z$ -value $=3.23$  and  $p=0.0012$ ) and transfer test ( $z$ -value $=2.40$  and  $p=0.016$ ). Compare  
714 to [28] Figure 1D). **(d)** novel pairs of stimuli, including stimuli with zero elements, in a  
715 learning test (Wilcoxon signed rank test,  $z$ -values $> 5.27$  and  $p<1.35e-06$ ), and  
716 conflict test (Wilcoxon signed rank test,  $z$ -values $> 5.51$  and  $p<3.49e-07$ ). Compare to  
717 [28] Figure S4. **(E)** and recognising stimuli with zero elements as the lower end of a  
718 continuum (Wilcoxon signed rank test for comparing each pair with the chance level  
719 50%,  $z$ -values $> 2.24$  and  $p<0.024$ ; Kruskal-Wallis test,  $df=299$ ;  $\chi^2=183.94$  and  
720  $p=7.71e-37$ . Compare to [28] Figure 2B. Light grey=less-than. Dark grey=more-than.  
721 Insets=test stimuli. Bars=mean. Vertical lines=s.e.m. calculated from the firing rate of  
722 the decision neuron for 50 different initial parameters that simulated 50 different  
723 model bees.

724  
725  
726

727 **Table S1. Summary of the full generalized linear mixed models examining**  
728 **factors in relation to proportion of correct choices during each test**

Dependent variable	Fixed factors	df	Estimate	SE	$z$ -value	$P$	$AICc$
Proportion correct in Learning test	Intercept	16	0.66	0.61	1.09	0.28	76.7
	Rule	16	0.19	0.29	0.65	0.52	
	Country	16	-0.11	0.30	-0.38	0.70	
Proportion correct in Equal/Incongruent test	Intercept	16	-0.68	0.57	-1.19	0.23	80.7
	Rule	16	0.91	0.26	0.35	0.72	
	Country	16	-0.06	0.26	-0.22	0.83	
Proportion correct in Incongruent/Opposite test	Intercept	16	-0.69	0.57	-1.22	0.22	76.6
	Rule	16	0.11	0.25	0.43	0.67	
	Country	16	-0.01	0.26	-0.04	0.97	

729  
730  
731  
732  
733  
734  
735  
736

For each glmm, the dependent variable was the proportion of correct choices during that test (following the numeric rule on which the bee was trained). The rule (more-than/less-than) and country where the individual bee completed training and tests (China/Finland) were included as fixed factors. Bee ID was used as a random factor.

737 **Table S2. Summary of the reduced generalized linear mixed models examining**  
 738 **factors in relation to proportion of correct choices during each test**

Dependent variable	Fixed factors	df	Estimate	SE	<i>z-value</i>	<i>P</i>	<i>AICc</i>
Proportion correct in Learning test	Intercept	18	0.66	0.14	5.25	1.49e-7	71.3
Proportion correct in Equal/Incongruent test	Intercept	18	-0.64	0.13	-4.98	6.50e-7	74.9
Proportion correct in Incongruent/Opposite test	Intercept	18	-0.55	0.12	-4.38	1.17e-5	70.8

739 For each glmm, the dependent variable was the proportion of correct choices  
 740 during that test (following the numeric rule on which the bee was trained). Bee ID  
 741 was used as a random factor.  
 742