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Endogenous Weights and Multidimensional Poverty: A Cautionary Tale *

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February 7, 2021

Abstract

Multidimensional poverty measures have become a standard feature in poverty assessments. A large and growing body of work uses endogenous (data driven) weights to compute multidimensional poverty. We demonstrate that broad classes of endogenous weights violates key properties of poverty indices such as monotonicity and subgroup consistency, without which poverty evaluation and policy targeting are seriously compromised. Using data from Ecuador and Uganda we show that these violations are widespread. Our results can be extended to other composite welfare measures such as the widely used asset indices.

Keywords: Multidimensional poverty, endogenous weights, measurement externalities, PCA weights, frequency weights.

JEL Classification: I32, C43, O18

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1 Introduction

It has become increasingly common to understand deprivation from a multidimensional perspective. Practitioners undertaking such multidimensional assessments must make several non-trivial methodological decisions, including which dimensions and indicators of deprivation to consider among the several possible and how to combine them into one single composite index of multidimensional poverty. In combining these dimensions and their indicators into a composite index, a natural question to ask is how much weight should we assign to each of them. This paper examines, both empirically and analytically, the implications of using endogenous (i.e. data driven) weights on a set of desirable properties for multidimensional poverty indices (see Bourguignon and Chakravarty, 2003; Alkire and Foster, 2011) and demonstrates their failure to satisfy these key properties under endogenous weights.

In constructing composite measures such as multidimensional poverty indices, exogenous weights which are independent of the data set and reflect the normative judgements of society, or the analyst or the policy-maker, is frequently used. In contrast, we focus on a growing body of literature which relies on the alternative of endogenous weights, which are determined by the data set, as a way to reflect the importance of the different indicators in the composite measure (OECD, 2008; Decanq and Lugo, 2013). We consider endogenous weights based on statistical methods such as Principal Component Analysis (PCA), as well as frequency-based weights, which depends on the frequency of deprivation in the different indicators.¹

The applications of these endogenous weights are widespread. For instance, targeting indices for the Mexican anti-poverty programme ‘Prospera’, popular and well-established indices such as the Social Progress Index or the Human Needs Index, and the World Food Program’s Vulnerability Monitoring Exercise, are all constructed using endogenous weights (see Dávila Lárraga, 2016; Stern et al., 2018; WFP, 2019). Studies such as Asselin and Anh (2008); Noglo (2017); Dhongde and Haveman (2017) use factorial technique such as Multiple Correspondence Analyses (MCA) to evaluate multidimensional poverty. A strand of the

¹Endogenous weights based on data-reduction factorial techniques such as PCA (see e.g. Asselin and Anh, 2008; Asselin, 2009; Alkire et al., 2015; Coromaldi and Drago, 2017; Wittenberg and Leibbrandt, 2017), use optimisation procedures applied to statistical concepts such as correlation or variance. Meanwhile, depending on the assumptions, frequency-based weights increase (or decrease) with the proportions of people deprived in a particular indicator.

literature even suggests endogenous weights as benchmarks for multidimensional poverty analyses, claiming that they can be regarded as ‘superior’ approaches in determining the weighting structures for indicators (see Pasha, 2017; Nájera Catalán and Gordon, 2019; Heshmati et al., 2008). Likewise, examples of frequency-based endogenous weights are ubiquitous in the literature on multidimensional poverty measurement (e.g. see Deutsch and Silber, 2005; Aaberge and Brandolini, 2014; Whelan et al., 2014; Alkire et al., 2015; Cavapozzi et al., 2015; Rippin, 2016; Abdu and Delamonica, 2018).

This paper demonstrates that combining a broad class of endogenous weights with a general class of multidimensional poverty indices based on the popular counting approach (Alkire and Foster, 2011), leads to the violation of two fundamental properties in poverty measurement: *monotonicity* and *subgroup consistency*.² Monotonicity states that if the poverty experience of an individual worsens in any indicator, then the overall poverty experience of the society to which this individual belongs, should not improve. Subgroup consistency requires that changes in overall poverty in a population should reflect the changes in poverty happening at the smaller population subgroup level. For instance, if poverty in a particular region of a country increases, while poverty of all other regions remains unchanged, then subgroup consistency implies that overall poverty in the country should not decrease.

Failure of a poverty index to satisfy monotonicity implies that we may observe societal poverty fall even when poverty of some individuals in that society increased, without any countervailing decrease in any other individuals’ poverty. Violation of monotonicity can lead to perverse policies whereby increasing individuals’ deprivation in some indicators can be deemed beneficial since it will lead to an overall decrease in multidimensional poverty. Meanwhile, failure of subgroup consistency can lead to a situation where increase in poverty in some regions or populations subgroups, *ceteris paribus*, may decrease societal poverty. This in turn can lead to policies that ignore increasing poverty in one region or one population subgroup because overall poverty has decreased. Without these key properties, any kind of comparative and evaluative exercise across time, regions or population groups would be seriously compromised (see Foster and Shorrocks, 1991).

²The Alkire and Foster (2011) counting approach is followed by over seventy countries and organisations including United Nations Development Programme’s (UNDP) flagship Multidimensional Poverty Index (MPI) to evaluate multidimensional poverty (see Alkire et al., 2015; OPHI, 2018).

We illustrate the violations of monotonicity and subgroup consistency using real-world data from two countries with structurally different patterns of multidimensional poverty: the 2013/14 Ecuador Living Conditions Survey and the 2015/16 Uganda National Panel Survey.³ In our context, endogenous weights generate a *measurement externality* since they depend on the distribution of deprivations across the indicators. Change in one person’s deprivation status (e.g. because she is no longer deprived in some indicator) affects the poverty scores of many other people through its impact on the weighting vector. Our appraisal of other people’s poverty is thus altered, despite the absence of any objective change in their deprivation status. By contrast, this measurement externality is non-existent if weights are set exogenously. Our results can be extended to demonstrate that societal and individual welfare measures like asset indices (see [Filmer and Pritchett \(2001\)](#)) and measures of material deprivation (see e.g. [Guio et al., 2016](#)), suffer from similar problems.

The rest of the paper is organized as follows: Section 2 introduces the notation and discusses the basic poverty measurement framework including the important properties of monotonicity and subgroup decomposability. In section 3 we provide examples of endogenous weights used in the paper. Section 4 presents an empirical illustration of violation of monotonicity and subgroup decomposability under endogenous weights, followed by the role of externalities in section 5 and the analytical results in section 6. The final section summarises the paper’s main message with some concluding remarks.

2 Preliminaries: Multidimensional Poverty Measurement

Consider a deprivation matrix $\mathbf{X}_{\mathbf{ND}}$, with each of the $N > 1$ rows representing an individual (or household) and each of $D > 1$ columns representing a deprivation indicator.⁴ We denote any individual as n , where $n = 1, 2, \dots, i, i', \dots, N$, and any indicator as d , where $d = 1, 2, \dots, j, j', \dots, D$. Let $\rho_{nd}^{X_{ND}} \in \{0, 1\}$ denote the deprivation of person n in indicator d in the deprivation matrix $\mathbf{X}_{\mathbf{ND}}$. For any individual n , poverty is determined by the deprivations faced by the individual across the different indicators, which are given by the deprivation

³For robustness, we also undertake Monte-Carlo simulations which show that the violations of monotonicity and subgroup consistency are ubiquitous. The results are presented in Supplementary Appendix D.

⁴Without loss of generality, we are not making a distinction between indicators and dimensions in the analytical part, i.e. sections 5 and 6. In the empirical part, i.e. section 4, we demonstrate our results on indicators.

vector $\mathbf{X}_{n\bullet} : \{\rho_{n1}, \dots, \rho_{nD}\}$. Further, let $\mathbf{X}_{\bullet d}$ be the column vector associated with indicator d of \mathbf{X}_{ND} , i.e. $\{\rho_{1d}, \dots, \rho_{Nd}\}$. Note that, for our purpose, we assume that individuals are either fully deprived in an indicator ($\rho_{nd} = 1$) or not at all ($\rho_{nd} = 0$).

Let each indicator of \mathbf{X}_{ND} be weighted, where weight in indicator d is represented as $w_d^{X_{\text{ND}}}$. Then we have a weighting vector of strictly positive entries: $\mathbf{w}^{\mathbf{X}_{\text{ND}}} = (w_1^{X_{\text{ND}}}, \dots, w_D^{X_{\text{ND}}})$, such that: $\sum_{d=1}^D w_d^{X_{\text{ND}}} = 1$. As alluded before, weights can be determined either endogenously or exogenously. The values of exogenous weights can, for instance, remain constant across different deprivation matrices because, by definition, they are independent from the data; whereas endogenous weights take into account the distribution of deprivation in each indicator. Thus, under endogenous weights, two different deprivation matrices \mathbf{X}_{ND} and \mathbf{X}'_{ND} will have different weights for the indicators. Specifically, $\mathbf{w}^{\mathbf{X}_{\text{ND}}} = (w_1^{X_{\text{ND}}}, w_2^{X_{\text{ND}}}, \dots, w_D^{X_{\text{ND}}})$ and $\mathbf{w}^{\mathbf{X}'_{\text{ND}}} = (w_1^{X'_{\text{ND}}}, w_2^{X'_{\text{ND}}}, \dots, w_D^{X'_{\text{ND}}})$, where for some j and j' , $w_j^{X_{\text{ND}}} \neq w_j^{X'_{\text{ND}}}$ and $w_{j'}^{X_{\text{ND}}} \neq w_{j'}^{X'_{\text{ND}}}$. We describe the specific weighting functions used in our analysis in Section 3.

2.1 Individual Poverty

Individual poverty is measured through a two-step procedure: (i) identifying whether an individual is multidimensionally deprived based on the number of indicators they are deprived in, and if so, (ii) computing a weighted aggregate of their deprivation over all the indicators (see [Alkire and Foster \(2011\)](#)). Thus, the resulting individual poverty function has two components: a poverty identification function, ψ , and a poverty severity function, s ([Silber and Yalonetzky, 2013](#)).

Similar to [Alkire and Foster \(2011\)](#), a person is identified as multidimensionally deprived if their total deprivation count is at least as high as an exogenously specified cutoff $0 < k \leq 1$. We use the identification function $\psi : [0, 1] \rightarrow \{0, 1\}$ where:

$$\psi(t_n; k) = \mathbb{I}(t_n \geq k). \tag{1}$$

and $t_n = \sum_{d=1}^D v_d \rho_{nd}$ is the total deprivation count of individual n with v_1, \dots, v_D representing

weights adding up to 1.⁵ When $0 < k \leq \min\{v_1, \dots, v_D\}$ the poverty identification function follows a *union approach whereby any person with at least one deprivation is deemed poor*. On the other extreme, when $k = 1$, poverty identification follows an *intersection approach which regards as poor only those who are deprived in all indicators*. Between both extremes, lies the intermediate approach to identification where $k \in (0, 1)$ (Alkire and Foster, 2011; Pasha, 2017). Although our results will hold for intermediate cases too, for the empirical analysis we use the union approach to identification which is adopted in practice by a swathe of the literature, especially studies using endogenous weights based on data-reduction techniques (e.g. Asselin and Anh, 2008; Asselin, 2009; Coromaldi and Drago, 2017).

The severity component, $s : [0, 1] \rightarrow [0, 1]$ measures the severity of the multiple-deprivation experience (Chakravarty and D'Ambrosio, 2006; Alkire and Foster, 2011; Silber and Yalonetzky, 2013), where for individual n the weighted deprivation score (or counting function) is:

$$C_n(\mathbf{X}_{\text{ND}}) = \sum_{d=1}^D w_d \rho_{nd} \quad (2)$$

The severity component satisfies the following properties: $s(C_i) > s(C_j)$ whenever $C_i > C_j$, $s(0) = 0$ and $s(1) = 1$. Additionally we may also include the restriction that $s''(C_n) \geq 0$ (assuming differentiability of s). Thus the severity function is monotonic in the weighted deprivation count of each individual and it increases at a non-decreasing rate. Straightforward examples of $s(C_n)$ used in the literature include $s(C_n) = C_n$ (Alkire and Foster, 2011), $s(C_n) = e^{\alpha C_n} - 1$ with $\alpha > 0$ (Chakravarty and D'Ambrosio, 2006), or $s(C_n) = (C_n)^\beta$ with $\beta \geq 1$ (Datt, 2019).

Thus, for any deprivation matrix \mathbf{X}_{ND} , the poverty function for individual n , $p_n^{\mathbf{X}_{\text{ND}}} : \{0, 1\} \times [0, 1] \rightarrow [0, 1]$, takes the form:

$$p_n^{\mathbf{X}_{\text{ND}}}(t_n, C_n; k) = \psi(t_n; k) s(C_n). \quad (3)$$

It combines the identification and the severity components to yield a measure of overall

⁵The weights \mathbf{v} are exogenous and different from $\mathbf{w}^{\mathbf{X}_{\text{ND}}}$. We suggest using exogenous weights at the identification stage in order to avoid violating the Focus axiom (see Alkire and Foster (2011, p. 480)), which captures the idea that improvements in the well-being of the non-poor should not change the level of societal poverty.

poverty at the individual level. In order to align our empirical illustration in section 4 to some of the most commonly used poverty functions in multidimensional poverty analyses (see e.g. Atkinson, 2019), we calculate individual poverty based on a linear severity function: $p_n = \psi(t_n; k)C_n$.⁶

2.2 Societal Poverty

We aggregate across all the individual poverty functions (as in 3) to derive a societal poverty function. We follow a commonly used societal poverty function $P : [0, 1]^N \rightarrow [0, 1]$, which is non-decreasing and additively separable in its constituent parts:

$$P(\mathbf{X}_{\mathbf{ND}}; k) = \frac{1}{N} \sum_{n=1}^N p_n^{\mathbf{X}_{\mathbf{ND}}}. \quad (4)$$

Societal poverty indices like P are expected to fulfill certain desirable properties. Chiefly among them is monotonicity, which requires societal poverty not to decrease if a poor individual suffers from an additional deprivation. For a formal definition, consider a deprivation matrix $\mathbf{X}_{\mathbf{ND}}$, where individual i is deemed poor, i.e. $\psi(t_i; k) = 1$. Then let $\mathbf{X}'_{\mathbf{ND}}$ be obtained from $\mathbf{X}_{\mathbf{ND}}$ by a simple increase of deprivation in indicator j of individual i in $\mathbf{X}_{\mathbf{ND}}$; meaning that: (i) $\rho_{ij}^{\mathbf{X}'_{\mathbf{ND}}} = 1$, (ii) $\rho_{ij}^{\mathbf{X}_{\mathbf{ND}}} = 0$ and (iii) $\forall (n, d) \neq (i, j), \rho_{nd}^{\mathbf{X}'_{\mathbf{ND}}} = \rho_{nd}^{\mathbf{X}_{\mathbf{ND}}}$. Then the monotonicity axiom can be written as:

Axiom 1 Monotonicity (M): Suppose $\mathbf{X}'_{\mathbf{ND}}$ is obtained from $\mathbf{X}_{\mathbf{ND}}$ by a simple increase of deprivation in indicator d of individual i , then $\Delta P = P^{\mathbf{X}'_{\mathbf{ND}}} - P^{\mathbf{X}_{\mathbf{ND}}} \geq 0$.

Another important property is *subgroup consistency* (Foster and Shorrocks, 1991; Alkire and Foster, 2011), which requires societal poverty to change (e.g. in a country across time) in the same direction of a change in the poverty levels of a subgroup (e.g. within a region across time), if the poverty levels of all other subgroups remain unchanged. For a formal definition, consider a *subgroup decomposable* deprivation matrix $\mathbf{X}_{\mathbf{ND}}$ formed by vertical concatenation of two matrices $\mathbf{X}_{\mathbf{N}_1\mathbf{D}}$ and $\mathbf{X}_{\mathbf{N}_2\mathbf{D}}$ where $N = N_1 + N_2$. We represent it as $\mathbf{X}_{\mathbf{ND}} = (\mathbf{X}_{\mathbf{N}_1\mathbf{D}} \parallel \mathbf{X}_{\mathbf{N}_2\mathbf{D}})$. Then the axiom of Subgroup Consistency can be stated as:

⁶Our empirical results also hold for the quadratic case $s(C_n) = (C_n)^2$. They are available upon request.

Axiom 2 Subgroup Consistency (SC): Suppose $\mathbf{X}_{\mathbf{ND}} = (\mathbf{X}_{\mathbf{N}_1\mathbf{D}} \parallel \mathbf{X}_{\mathbf{N}_2\mathbf{D}})$ and $\mathbf{Y}_{\mathbf{ND}} = (\mathbf{Y}_{\mathbf{N}_1\mathbf{D}} \parallel \mathbf{Y}_{\mathbf{N}_2\mathbf{D}})$ be two subgroup-decomposable deprivation matrices. P satisfies subgroup consistency if

$$[P(\mathbf{X}_{\mathbf{N}_1\mathbf{D}}; \mathbf{w}^{\mathbf{X}_{\mathbf{N}_1\mathbf{D}}}, k) > P(\mathbf{Y}_{\mathbf{N}_1\mathbf{D}}; \mathbf{w}^{\mathbf{Y}_{\mathbf{N}_1\mathbf{D}}}, k) \text{ and } P(\mathbf{X}_{\mathbf{N}_2\mathbf{D}}; \mathbf{w}^{\mathbf{X}_{\mathbf{N}_2\mathbf{D}}}, k) = P(\mathbf{Y}_{\mathbf{N}_2\mathbf{D}}; \mathbf{w}^{\mathbf{Y}_{\mathbf{N}_2\mathbf{D}}}, k)] \\ \Rightarrow P(\mathbf{X}_{\mathbf{ND}}; \mathbf{w}^{\mathbf{X}_{\mathbf{ND}}}, k) > P(\mathbf{Y}_{\mathbf{ND}}; \mathbf{w}^{\mathbf{Y}_{\mathbf{ND}}}, k).$$

3 Examples of Endogenous Weights

Consider a deprivation matrix $\mathbf{X}_{\mathbf{ND}}$. Then examples of frequency-based weights, for indicator j are given by:

$$w_j^F = \frac{f(\mathbf{X}_{\bullet j})}{\sum_{d=1}^D f(\mathbf{X}_{\bullet d})}, \quad (5)$$

where $f'(\mathbf{X}_{\bullet j}) \geq 0$ (assuming differentiability of f). One example of f with $f'(\mathbf{X}_{\bullet j}) > 0$ is $f(\mathbf{X}_{\bullet j}) = \sum_{n=1}^N \rho_{nj} \psi(t_n; k) / N$ which implies that as more people become deprived in an indicator, it becomes a more important indicator of multidimensional poverty, and hence it should carry a higher weight in the composite index. On the other hand, an example of f with $f'(\mathbf{X}_{\bullet j}) < 0$ is: $f(\mathbf{X}_{\bullet j}) = -\ln(\sum_{n=1}^N \rho_{nj} \psi(t_n; k) / N)$ (see [Deutsch and Silber, 2005](#), p. 150). Another possibility is: $f(\mathbf{X}_{\bullet j}) = 1 - (\sum_{n=1}^N \rho_{nj} \psi(t_n; k) / N)$, which essentially captures the intuition that if deprivation in one particular indicator becomes endemic, it may no longer serve as a distinguishing factor and hence should be weighted less in the composite index.

Another common way of constructing composite indices with endogenous weights consists of identifying orthogonal linear combinations of the standardized column vectors of $\mathbf{X}_{\mathbf{ND}}$, denoted as $\mathbf{X}_{\bullet d}^*$, $\forall d = 1, 2, \dots, D$, in such a way as to reproduce their variance and interlinkages as closely as possible. This logic underlies a range of factorial techniques for data reduction, including Factor Analysis, PCA and MCA ([Asselin and Anh, 2008](#)).

In our setting, let Σ denote the variance-covariance matrix of $\{\mathbf{X}_{\bullet 1}^*, \dots, \mathbf{X}_{\bullet D}^*\}$. One way to account for the binary nature of the elements in these vectors is to define the off-diagonal elements of Σ as bivariate tetrachoric correlation coefficients. Let us also denote the eigenvalues of Σ as $\lambda_1, \dots, \lambda_D$ in descending order, and as ν_1, \dots, ν_D the corresponding $D \times 1$ eigenvectors.

Then the ℓ -th principal component, a_ℓ , is given by:

$$a_\ell = \mathbf{X}_{\mathbf{ND}}^* \nu_\ell = \sum_{j=1}^D \nu_{\ell,j} \mathbf{X}_{\bullet,j}^*, \forall \ell = 1, 2, \dots, D, \quad (6)$$

and its variance is $V(a_\ell) = \lambda_\ell, \forall \ell = 1, 2, \dots, D$. In practice, the first principal component is the most commonly used ‘summary’ indicator that can be derived from PCA (Asselin and Anh, 2008). Being linear combinations of standardised variables, principal components do not have a specific unit of measure or a cardinal interpretation (see e.g. Jolliffe and Cadima, 2016). They can be re-scaled by means of a monotonic transformation that preserves the *ordering* of individuals by their scores in that component. In particular, the first principal component can be re-scaled such that $\tilde{a}_1 \equiv \frac{a_1}{\sum_{j=1}^D \nu_j}$, with $V(\tilde{a}_1) = \frac{\lambda_1}{(\sum_{j=1}^D \nu_j)^2}$. This implies that:

$$\tilde{a}_1 = \mathbf{X}_{\mathbf{ND}}^* \mathbf{w}^{PCA} = \sum_{j=1}^D w^{PCA_j} \mathbf{X}_{\bullet,j}^* \quad (7)$$

where $w_j^{PCA} \equiv \frac{\nu_j}{\sum_{j'=1}^D \nu_{j'}} \forall j = 1, \dots, D$ and $\sum_{j=1}^D w_j^{PCA} = 1$.

Intuitively, the first principal component approach gauges the extent to which each indicator contributes to reproducing the largest portion of the total variance in the dataset. This is entirely driven by the indicators’ variance-covariance matrix. After standardisation, the indicators that are, overall, highly correlated with the others will receive higher weights. The reason is that these highly correlated indicators form a ‘dominant’ indicator subset that essentially determines the first principal component. Conversely, those indicators that hold weak correlations with the elements of this ‘dominant’ indicator subset are regarded as redundant, and thus receive lower weights in the principal component.

4 Empirical Illustration

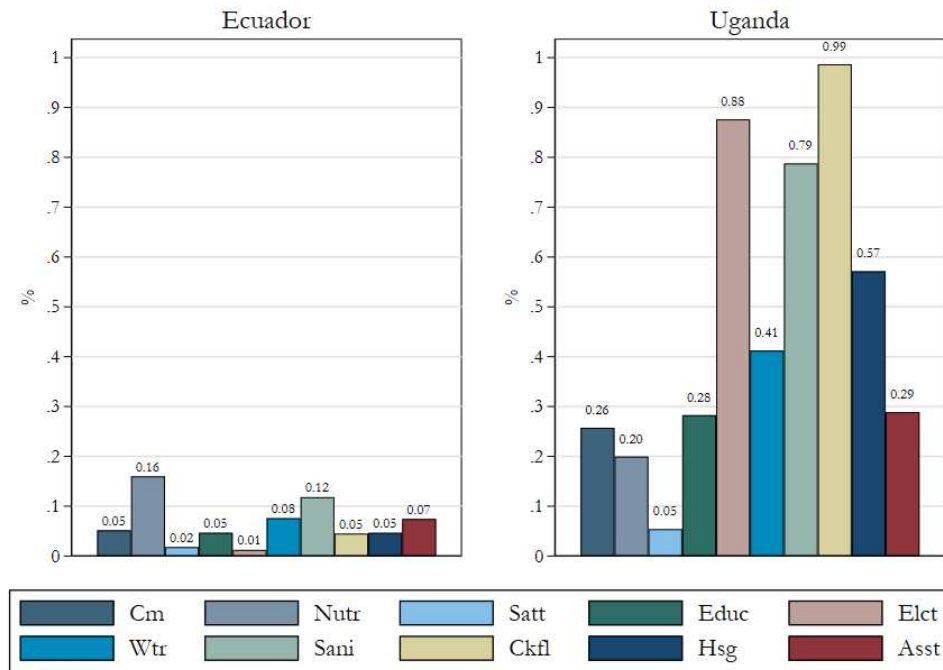
To illustrate the violations of monotonicity and subgroup consistency under endogenous weights, we consider the case of Ecuador and Uganda. As any endogenous weighting procedure, PCA and frequency weights are data-adaptive techniques, which is the reason why our empirical illustration concerns two countries with different multidimensional poverty

patterns.⁷

4.1 Data and Indicators

We use nationally representative household-level data for both countries: Encuesta de Condiciones de Vida 2013-14 for Ecuador (N=108,093) and Uganda’s National Panel Survey 2015/16 (N=17,465). Based on the Oxford Poverty & Human Development-UNDP’s global Multidimensional Poverty Index (MPI) our analysis considers ten indicators pertaining to three wellbeing dimensions namely education, health and living standards (OPHI, 2018). We argue that this index is ideal for our illustration due to its wide acceptance in academic and policy-making spheres (Atkinson, 2019; World Bank, 2018). The dimensions and their indicators along with the deprivation thresholds are presented in Table 1 (Appendix B).

Figure 1: Deprivation Headcount Ratios



Note: *Cm*: Child mortality, *Nutr*: Nutrition; *Satt*: School attendance; *Educ*: Years of schooling; *Elct*: Electricity; *Wtr*: Drinking water; *Sani*: Sanitation; *Ckfl*: Cooking fuel; *Hsg*: Housing; *Asst*: Assets

Deprivation headcount rates are lower in Ecuador compared to Uganda in every indicator

⁷See OPHI (2018) for a recent description of multidimensional poverty patterns in these countries.

(Fig. 1). Hence, for instance, if rare deprivations are considered particularly important to gauge poverty (Deutsch and Silber, 2005), then deprivations with the lowest frequencies will be assigned higher frequency-based weights (under a union approach). This is the case of electricity and school attendance in Ecuador, and school attendance and nutrition in Uganda. Conversely, deprivations with the highest frequencies will be assigned lower weights, as they are more commonly observed in the data. This is the case of nutrition and sanitation in Ecuador, and cooking fuel and electricity in Uganda.

Regarding PCA weights, note that, in Ecuador, the health indicators have relatively low tetrachoric correlations with the rest of the indicators (ranging in absolute value between 0.064 and 0.226; Table 2 in Appendix B). Meanwhile, the living standard indicators have higher correlations with the rest (coefficients ranging between 0.116 and 0.816). Thus, using first principal components, health indicators would have lower weights compared to living standards'. In Uganda, the health indicators are weakly correlated with rest of the indicators (with absolute value of coefficients between 0.07 and 0.35 and mostly below 0.20). Hence, child mortality and nutrition would have a relatively low weight.

4.2 Simulations: Weighting Vectors, Monotonicity and Subgroup Consistency

Now we compare indices based on exogenous and endogenous weights in order to show that the latter violate monotonicity and subgroup consistency. Our baseline scenario is defined by the deprivation matrices effectively observed in both datasets, \mathbf{X}_{ND} (country labels omitted for the sake of notational simplicity). We induced changes to these matrices by adding random deprivations in nutrition (*Nutr*). Based on our discussion above, we choose this indicator for our illustration because of its distinctive correlation and frequency patterns in both countries. However, we also simulated deprivations in access to safe drinking water (*Wtr*) and electricity (*Elct*) to provide a broad coverage of the heterogeneity in observed deprivation across the various indicators.⁸

As a starting point, we assign random identifiers to the population in each country that is non-deprived in nutrition. We use these identifiers to form random ventiles of non-deprived

⁸The results of changes in access to safe drinking water and electricity are available in a Supplementary Appendix C. Monte-Carlo simulations corroborating the likelihood of encountering violations of the two properties in a wide array of alternative settings are also available in a Supplementary Appendix D.

people. Then, in a *cumulative* process, we gradually assign deprivations to each random ventile; we first assign deprivations to 5% of the relevant population, then to an additional 5% (for a total of 10%) and so on until, finally, assigning the deprived status to 95% of the part of the population originally non-deprived in nutrition.

We denote the ensuing deprivation matrices as $\mathbf{X}_{\text{ND}}^{s_i}$, with $s_i = \{0, 5\%, 10\%, \dots, 95\%\}$ representing the proportion of initially non-deprived individuals who are assigned deprivations in indicator i (nutrition, in our case). Note that $\mathbf{X}_{\text{ND}}^0 = \mathbf{X}_{\text{ND}}$, and that $\forall s'_i > s_i$, $\mathbf{X}_{\text{ND}}^{s'_i}$ objectively represents a Pareto-inferior state of affairs where nobody has fewer deprivations and at least one person has more deprivations vis-a-vis $\mathbf{X}_{\text{ND}}^{s_i}$.

Throughout our analysis we adopt a union approach to identify the poor.⁹ Hence the weighting choices are only relevant for the individual poverty functions. We follow the global MPI (OPHI, 2018) to select an exogenous nested-weighting scheme, whereby each dimension is assigned an equal weight (1/3). In turn, indicators within poverty dimensions are also assigned equal weights; the education and health indicators are assigned a 1/6 (=0.1667) weight, and each living standards dimension is assigned a 1/18 (=0.0667) weight. We compute endogenous weights using (i) PCA based on the tetrachoric correlations, and (ii) frequency weighting operationalised by $f(\mathbf{X}_{\bullet, \mathbf{j}}) = -\ln(\sum_{n=1}^N \rho_{nj} \psi(t_n; k)/N)$ (see equation 5).

As expected, in the baseline PCA weighting vector for Ecuador (PCA, $s_{\text{nutr}}=0$ column in Table 3), living standard indicators tend to have higher weights, followed by the education indicators and then the health indicators. In Uganda (PCA, $s_{\text{nutr}}=0$ column in Table 3), with health indicators having low weights while living standard indicators tend to have the highest. In both countries, when nutrition deprivations increase, this indicator is assigned lower weight. That is, the added deprivations reconfigure the correlations patterns between this indicator and the rest in such a way that nutrition becomes less important in the first principal component. In compensation, all the other indicators in Ecuador are given higher weights in a relatively uniform manner. In Uganda, higher weights tend to be assigned to education, housing and assets.

⁹This ensures the fulfillment of the focus axiom. We could easily extend our analysis to different identification cut-offs based on equation 1.

Turning now to the frequency weights, as expected, in the baseline (Freq. $s_{nutr} = 0$ columns in Table 3), the highest frequency weights go to electricity and school attendance in Ecuador, and to school attendance and nutrition in Uganda. In turn the lowest weights are assigned to nutrition and sanitation in Ecuador, and to cooking fuel and electricity in Uganda.

4.2.1 Violations of Monotonicity and Subgroup Consistency

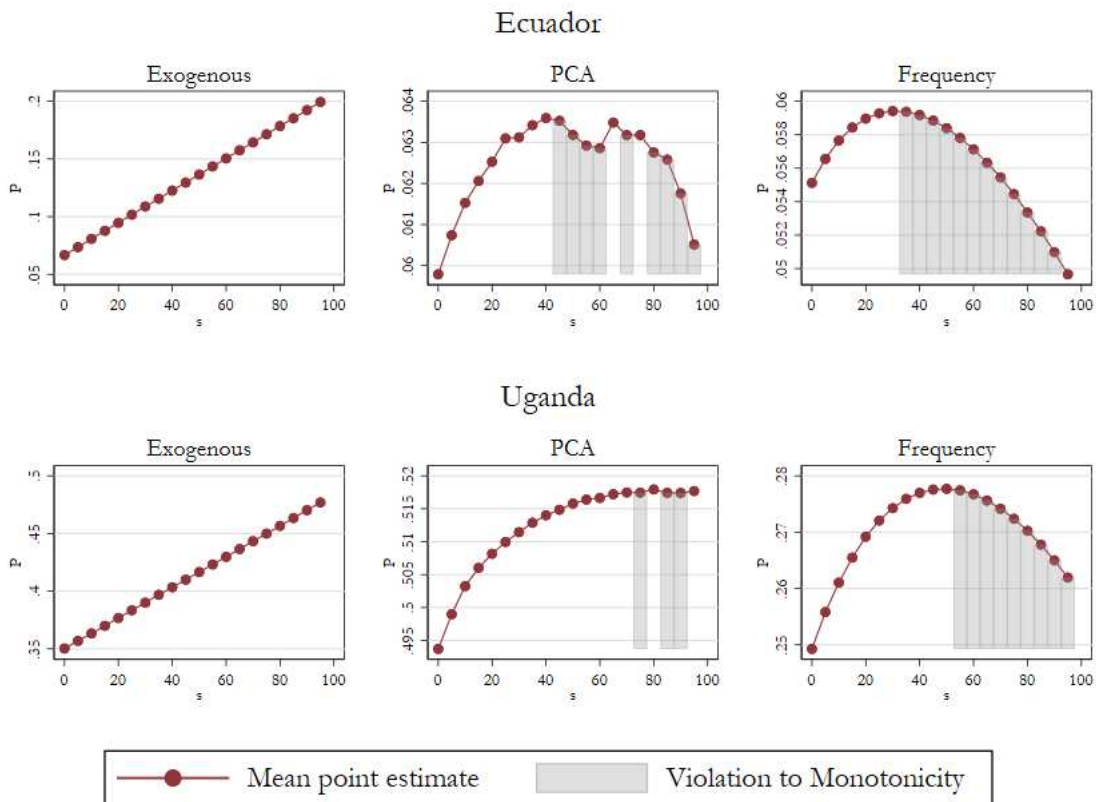
We assess the empirical response of three additively decomposable societal poverty functions with linear severity components to the gradual increase of deprivations. Each function corresponds to one weighting procedure; namely exogenous (denoted by EX), PCA, and frequency-based (denoted by F). Omitting the country index, the poverty functions are, respectively: $P_1^{s_i} = \frac{1}{N} \sum_n C_n^{EX,s_i}$, $P_2^{s_i} = \frac{1}{N} \sum_n C_n^{PCA,s_i}$, $P_3^{s_i} = \frac{1}{N} \sum_n C_n^{F,s_i}$. The poverty identification functions ψ^{s_i} are omitted purposefully, as we are adopting a union approach to poverty identification, meaning $\psi^{s_i}(t_n; 0) = 1$ for all n , in order to satisfy the focus axiom.

Our results confirm that: (i) monotonicity and subgroup consistency are never violated under exogenous weights, whereas (ii) these axioms are violated under endogenous weights.¹⁰ Regarding violations of monotonicity, in both countries (see Fig. 2), the poverty measure constructed with exogenous weights, $P_1^{s_i}$, is in theory a non-decreasing function of s_i . In our simulations it increases monotonically with a gradual increase of nutrition deprivations. That is $P_1^{s'_i} - P_1^{s_i} \geq 0 \forall s'_i \geq s_i$. This is not true for $P_2^{s_i}$ and $P_3^{s_i}$: Indeed Figure 2 shows that in Ecuador $P_2^{s_i} - P_2^{40i} < 0$ for some $45\% \leq s \leq 95\%$, and that $P_3^{s_i} - P_3^{35i} < 0$ for all $40\% \leq s \leq 95\%$; while in Uganda, $P_2^{s_i} - P_2^{70i} < 0$ for some $75\% \leq s \leq 95\%$, and that $P_3^{s_i} - P_3^{50i} < 0$ for all $55\% \leq s \leq 95\%$. Thus, there are many instances where these poverty measures based on endogenous weights *decrease* despite the constant increase in the proportion of people suffering nutrition deprivations, *ceteris paribus*. These are flagrant violations of monotonicity.

To assess possible violations of subgroup consistency, we assign another set of additional deprivations in an identical way, except that the non-deprived population eligible for

¹⁰For conciseness, we discuss the results of added nutrition deprivations ($i = nutr$). Those of added deprivations in electricity and water lead to the same qualitative conclusions and can be found in the Supplementary Appendix (C).

Figure 2: Violations of Monotonicity: Nutrition Indicator



added deprivations is *solely* concentrated in one specific sub-national region in each country. The regional poverty measures take an identical form as their national-level counterparts. Omitting the country index, the poverty functions for a generic region R are given by:

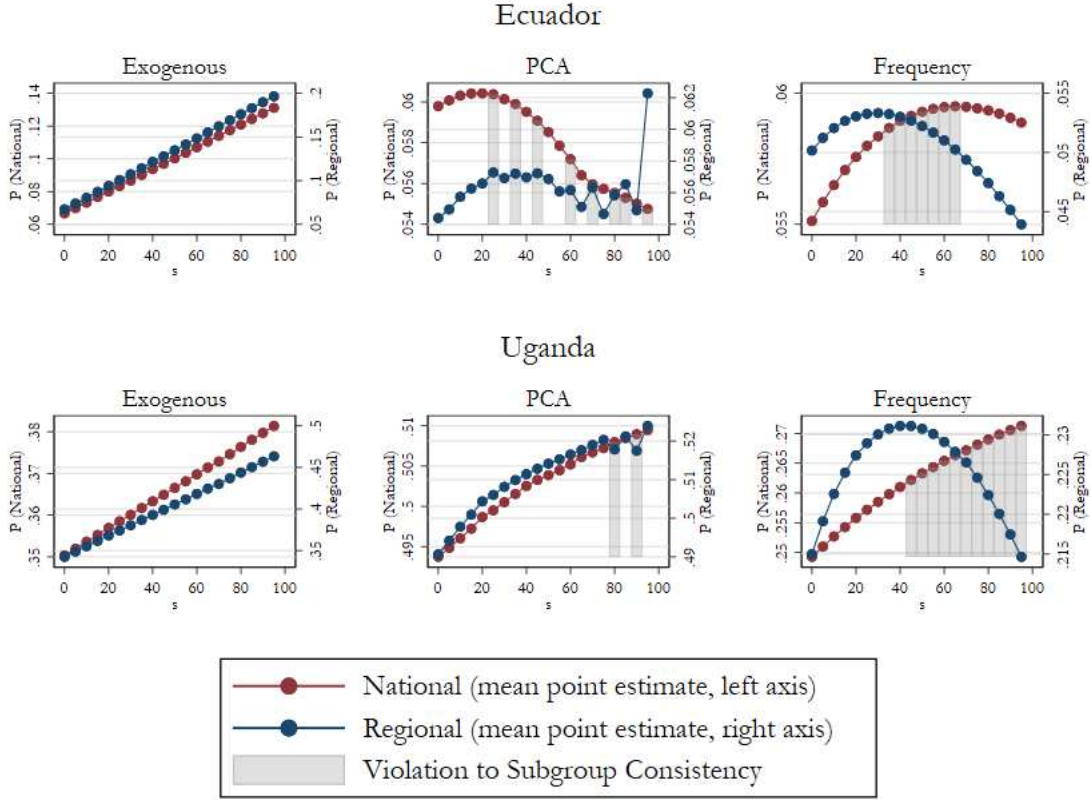
$$P_{1,R}^{s_i} = \frac{1}{N_R} \sum_{n \in R} C_{n \in R}^{EX,s_i}, P_{2,R}^{s_i} = \frac{1}{N_R} \sum_{n \in R} C_{n \in R}^{PCA,s_i}, P_{3,R}^{s_i} = \frac{1}{N_R} \sum_{n \in R} C_{n \in R}^{F,s_i}.$$

We focus on the Eastern region in Uganda (26.4% of the sample) and the Coast in Ecuador (33.1%), but we verified that choosing other regions¹¹ does not alter our main qualitative results: subgroup consistency is never violated under exogenous weights, but it can be violated under endogenous weights.

In both countries, the regional and national poverty measures under exogenous weights increase (monotonically) with additional nutrition deprivations in that region (Fig. 3).

¹¹Our data in Ecuador allows for a representative disaggregation of the additively decomposable poverty functions at the level of four geographical regions: Mountains (47.7% of the sample), Coast (33.1%), Amazon (17.6%) and Galapagos Island (1.6%). In Uganda, this is possible for four regions as well: Central (25.9%), Eastern (26.4%), Northern (23.9%) and Western (22.8%).

Figure 3: Violations of Subgroup Consistency: Nutrition Indicator



That is $(P_{1,R}^{s'_i} - P_{1,R}^{s_i} \geq 0) \implies (P_1^{s'_i} - P_1^{s_i} \geq 0), \forall s' \geq s$. By contrast, the sub-national poverty measures based on PCA or frequency-based weights often increase (decrease) while the respective national poverty measures decrease (increase), ceteris paribus. Indeed in Ecuador: $(P_{2,R}^{s'_i} - P_{2,R}^{s_i} > 0) \implies (P_2^{s'_i} - P_2^{s_i} < 0)$ for some $s' > s$ with $s, s' > 20\%$, and $(P_{3,R}^{s'_i} - P_{3,R}^{s_i} < 0) \implies (P_3^{s'_i} - P_3^{s_i} > 0), \forall 30\% \leq s, s' \leq 65\%$ such that $s' > s$ (Figure 3). Likewise, in Uganda: $(P_{2,R}^{s'_i} - P_{2,R}^{s_i} < 0) \implies (P_2^{s'_i} - P_2^{s_i} > 0)$ for some $s' > s$ with $s, s' > 75\%$, and $(P_{3,R}^{s'_i} - P_{3,R}^{s_i} < 0) \implies (P_3^{s'_i} - P_3^{s_i} > 0), \forall s, s' > 40\%$ such that $s' > s$ (Fig. 3).

5 Endogenous Weights and Measurement Externalities

Why do we observe these violations of basic properties when using endogenous weights? In this section, we investigate this issue in greater depth. We focus on the weighted deprivation score C_n (see equation 2) because the weighting choice affects the scores, which

in turn impact on the individual poverty functions under the counting approach to poverty measurement.¹²

Consider a deprivation matrix \mathbf{X}_{ND} , where individual i is identified poor. Let \mathbf{X}'_{ND} be obtained from \mathbf{X}_{ND} by a simple increase of deprivation in indicator j of individual i in \mathbf{X}_{ND} (as defined in Section 2.2). Then the change in the deprivation score of any individual $n \neq i$, who is also identified as poor, is:

$$\Delta C_n = C_n^{\mathbf{X}'_{\text{ND}}} - C_n^{\mathbf{X}_{\text{ND}}} = \rho_{nj} \Delta w_j + \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{nd} \Delta w_d, \quad (8)$$

where $\Delta C_n = C_n^{\mathbf{X}'_{\text{ND}}} - C_n^{\mathbf{X}_{\text{ND}}}$; and $\Delta w_d = w_d^{\mathbf{X}'_{\text{ND}}} - w_d^{\mathbf{X}_{\text{ND}}}$, $\forall d \in \{1, 2, \dots, D\}$. For simplicity of notation we denote $\rho_{nj}^{\mathbf{X}_{\text{ND}}} = \rho_{nj}$, $\rho_{nd}^{\mathbf{X}'_{\text{ND}}} = \rho'_{nd}$. For the i th individual who became deprived in the j th indicator, we know that $\rho'_{ij} w'_j - \rho_{ij} w_j = w'_j$. Thus, ΔC_i due to a change in the deprivation of person i with respect to indicator j , is given by:

$$\Delta C_i = w'_j + \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{id} \Delta w_d. \quad (9)$$

As long as person i is also deprived in some other indicator, the changes in the other weights produced by the change in i 's status regarding j (i.e. $\Delta w_d, \forall d \neq j$) also affect the total change in C_i . These same changes in weights led by the change in deprivation status of person i in indicator j produce, in turn, changes in the deprivation score of every other person. Proposition 1 below captures how changes in ρ_{ij} can impact weights in each indicator and, through that channel, the counting function of everybody besides person i :

Proposition 1 *Suppose \mathbf{X}'_{ND} is obtained from \mathbf{X}_{ND} by a simple increase of deprivation in*

¹²In theory the weighting choice should also affect the identification functions. But, as previously discussed, a combination of endogenous weights in the identification function with an intermediate approach to poverty identification violates the focus axiom. Meanwhile the extreme identification approaches, union and intersection, never violate the focus axiom but neither do they rely on deprivation weights.

indicator j for individual i . For all $n \neq i$:

$$(i) \text{ if } \forall d, \rho_{nd} = 0, \text{ or } \forall d, \rho_{nd} = 1, \text{ then } \Delta C_n = 0,$$

$$(ii) \text{ if } 0 < \sum_{d=1}^D \rho_{nd} < D, \text{ then } \begin{cases} \Delta C_n \leq 0 \iff \Delta w_j \leq 0 & \text{if } \rho_{nj} = 1 \\ \Delta C_n \geq 0 \iff \Delta w_j \geq 0 & \text{if } \rho_{nj} = 0 \end{cases}.$$

Proof: See Appendix A.

Note that only in indicator j we increase individual i 's deprivation. If individual n is deprived in j , then an increase (respectively decrease) in the weight of j leads to an increase (respectively decrease) in n 's deprivation score. Otherwise, if n is not deprived in j then an increase (respectively decrease) in the weight of j reduces (respectively increases) n 's deprivation score. There is no impact if n is either not deprived in any indicator or deprived in all indicators. Hence, a change in one person's deprivation status in one indicator changes the deprivation score for others too. Thus, there are clear measurement externalities among individuals which lead to violations in a poverty measure's key properties (Section 6).

6 Endogenous Weights and Societal Poverty

We show how poverty measures based on endogenous weights violate monotonicity and sub-group consistency.

6.1 Monotonicity

One of the main implications of Proposition 1 for societal poverty indices based on endogenous weights is that they violate monotonicity (Axiom 1); which implies, inter alia, that when poor individuals in a society become less deprived, societal poverty may increase and viceversa. In order to understand how this situation comes about, it is important to derive the impact produced by this change in the deprivation status of person i on the societal poverty index.

For any $\mathbf{X}_{\mathbf{ND}}$, let $\Pr[\rho_{nj} = 1 | n \neq i] \equiv \frac{1}{N-1} \sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 1 | n \neq i)$ (and similar definition for $\Pr[\rho_{nj} = 0 | n \neq i]$). Suppose $\mathbf{X}'_{\mathbf{ND}}$ is obtained from $\mathbf{X}_{\mathbf{ND}}$ by a simple increase

deprivation in indicator j of individual i . For any individual i' , let the change in poverty be denoted by $\Delta p_{i'} = p_{i'}^{\mathbf{X}'_{ND}} - p_{i'}^{\mathbf{X}_{ND}}$. Then:

$$\begin{aligned} \Delta P = & \frac{1}{N} \Delta p_i \\ & + \frac{N-1}{N} \Pr[\rho_{nj} = 1 | n \neq i] \frac{1}{(N-1) \Pr[\rho_{nj} = 1 | n \neq i]} \sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 1 | n \neq i) \Delta p_n \\ & + \frac{N-1}{N} \Pr[\rho_{nj} = 0 | n \neq i] \frac{1}{(N-1) \Pr[\rho_{nj} = 0 | n \neq i]} \sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 0 | n \neq i) \Delta p_n \end{aligned} \quad (10)$$

where

$$\Delta p_n = \psi(t_n; k) s(C_n^{X_{ND}} + \Delta C_n) - \psi(t_n; k) s(C_n^{X_{ND}}). \quad (11)$$

$(N-1) \Pr[\rho_{nj} = 1 | n \neq i]$ and $(N-1) \Pr[\rho_{nj} = 0 | n \neq i]$ represent the total number of individuals who are deprived in dimension j and not deprived in dimension j , respectively. Hence, the change in societal poverty, ΔP as given by (10), is the *population weighted* sum of (i) the change in person i 's individual poverty (Δp_i), (ii) the average change in deprivation of individuals other than i who are deprived in j , i.e. $\sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 1 | n \neq i) \Delta p_n / \{(N-1) \Pr[\rho_{nj} = 1 | n \neq i]\}$ and (iii) the average change in deprivation of individuals other than i who are not deprived in j , i.e. $\sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 0 | n \neq i) \Delta p_n / \{(N-1) \Pr[\rho_{nj} = 0 | n \neq i]\}$.

In the following discussion we show how the three components highlighted above react to an increase in one person's deprivation. First we show that for individual i whose deprivation in j th indicator increased $\Delta p_i \geq 0$.

Corollary 1 *Let \mathbf{X}'_{ND} be obtained from \mathbf{X}_{ND} by a simple increase of deprivation in indicator j of individual i . Then i 's poverty function (equation 3) does not decrease, that is $\Delta p_i \geq 0$.*

Proof: See Appendix A.

Since an increase in a person's deprivation does not decrease their individual poverty function, the main problem with counting poverty functions relying on endogenous weights lies elsewhere with the presence of *measurement externalities*. Next we assess how the poverty of other people changes as a result of the change in i 's deprivation. Two helpful corollaries stem from (8) combined with Proposition 1 and the individual poverty definition (3):

Corollary 2 *Let \mathbf{X}'_{ND} be obtained from \mathbf{X}_{ND} by a simple increase of deprivation in indicator*

j of individual i . Suppose $\Delta w_j > (<)0$. For any individual $n \neq i$:

$$\begin{aligned}\Delta p_n \geq (\leq)0 &\iff \Delta w_j > |\sum_{d=1, d \neq j}^D \rho_{nd} \Delta w_d| \quad \text{if } \rho_{nj} = 1 \\ \Delta p_n \leq (\geq)0 &\iff \sum_{d=1, d \neq j}^D \rho_{nd} \Delta w_d < (>)0 \quad \text{if } \rho_{nj} = 0\end{aligned}$$

Corollary 2 demonstrates that, with endogenous weights, $\Delta \rho_{ij} \neq 0$ is bound to produce changes in the poverty of other individuals, based on their deprivation status regarding j . In particular the poverty levels of those deprived in j and those not deprived in j will move in *opposite directions*. Hence, a priori, expression (10) may be positive, negative, or even nil. Thus we can deduce the following result:

Proposition 2 *Let \mathbf{X}'_{ND} be obtained from \mathbf{X}_{ND} by a simple increase of deprivation in indicator j of individual i . Then, for all societal poverty functions in (4) $\Delta P = P^{\mathbf{X}'_{\text{ND}}} - P^{\mathbf{X}_{\text{ND}}} \begin{matrix} \geq \\ \leq \end{matrix} 0$, thereby violating monotonicity (Axiom 1).*

This is a general result, not relying on any particular functional form of the weighting function, or any particular parameters or data. It demonstrates that the change in societal poverty, ΔP , resulting from a change in deprivation in any one indicator experienced by any one poor individual would be ambiguous, thereby violating monotonicity (Axiom 1).

For specific endogenous weights and datasets we can actually pinpoint situations in which monotonicity will be violated in the event of a simple increase of one deprivation in one individual, as stated by Proposition 2. For example, if the weight of indicator j *increases* due to a simple increase in deprivation of individual i in indicator j , then we know from corollary 2 that, on top of the increase in the individual poverty of i , the individual poverty of everyone else deprived in j will also increase, whereas the individual poverty of those not deprived in j will decrease. Then, based on equations (10) and (11), a violation of monotonicity will happen if the total sum of decreases in individual poverty is higher than the total sum of increases in individual poverty (which includes i 's). Thus, for $\Delta w_j > 0$, violation of monotonicity ($\Delta P < 0$) will happen if:

$$\sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 0 | n \neq i) |\Delta p_n| > \sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 1 | n \neq i) \Delta p_n + \Delta p_i. \quad (12)$$

The sum on the left-hand side of equation (12) depends positively on the total change in poverty of those not deprived in j ; whereas the expression on the right-hand side depends on (i) the total change in poverty of individuals initially deprived in j , and (ii) the change in poverty of i .¹³ A violation of monotonicity in this scenario will be more likely with higher proportions of people initially not deprived in j and higher reductions in their individual poverty levels (vis-a-vis the increase in individual poverty of those initially deprived in j).

If, on the other hand, the weight of indicator j *decreases* due to a simple increase in deprivation of individual i in indicator j , then we know from Corollary 2 that the individual poverty of everyone else deprived in j will decrease, whereas the individual poverty of those not deprived in j will increase, on top of the increase in the individual poverty of i . Then, following equations (10) and (11), a violation of monotonicity will occur, again, if the total sum of decreases in individual poverty is higher than the total sum of increases in individual poverty (which includes i 's), i.e. for $\Delta w_j < 0$ monotonicity will be violated ($\Delta P < 0$) will happen if:

$$\sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 0 | n \neq i) \Delta p_n + \Delta p_i < \sum_{n=1, n \neq i}^N \mathbb{I}(\rho_{nj} = 1 | n \neq i) |\Delta p_n|. \quad (13)$$

The expression on the left-hand side of equation (13) depends on (i) the total change in poverty of individuals not deprived in j , and (ii) the change in poverty of i ; whereas the sum on the right-hand side depends on the total change in poverty of individuals deprived in j . A violation of monotonicity in this scenario will be more likely with higher proportions of people initially deprived in j and higher reductions in their individual poverty levels (vis-a-vis the increase in poverty of those initially not deprived in j).

Finally, note that, by contrast, with exogenous weights the score of everybody, except i , remains unaltered: $\Delta C_n = 0, \forall n \neq i$. Consequently: $\Delta p_n = 0, \forall n \neq i$. Hence, finally, $\Delta P = \frac{1}{N} \Delta p_i$. That is, with exogenous weights, societal poverty changes coherently with the change in person i 's individual poverty, as the latter does not affect the poverty measurement of anybody else. Hence, monotonicity is fulfilled.

¹³We consider the absolute value of the change in poverty of those who are not deprived in indicator j because the change in their poverty is negative when $\Delta w_j > 0$.

6.2 Subgroup Consistency

A key implication of Proposition 1, is that societal poverty indices based on endogenous weights can also violate subgroup consistency (Axiom 2). In other words, we may find that poverty in a subgroup of the population had declined, with all the other subgroups' remaining unchanged, yet poverty of the whole society increased. Repeated application of subgroup consistency allows for comparing situations when poverty of more than one subgroup changes. In such case violation of subgroup consistency will essentially imply that societal poverty may decline even if the poverty of all the sub-populations have increased. Thus, this is a powerful axiom which ensures that changes in the poverty of the total population is consistent with the changes happening at the subpopulation level. We claim the following:

Proposition 3 *For any deprivation matrix \mathbf{X}_{ND} and any societal poverty measure given by the additively decomposable function in (4), if its value $P(\mathbf{X}_{\text{ND}}; \mathbf{w}, k)$ depends on \mathbf{w} , which represents the class of endogenous weights with weights adding up to 1, then P fails to satisfy subgroup consistency (Axiom 2).*

Proof: See Appendix A.

Proposition 3 demonstrates, in general terms, that endogenous weights lead to the violation of subgroup consistency (Axiom 2). Note that the class of endogenous weights considered is very general, which covers both the PCA and frequency based weighting used in our empirical applications.

7 Conclusions

The use of endogenous weights in multidimensional poverty measurement has enjoyed some popularity, yet the implications of letting weights depend on the dataset have not been studied in depth, above and beyond some reflections and sensible warnings (e.g. Alkire et al., 2015). In this paper we focused on a broad class of endogenous weights based on several instances of policy applications. We find that endogenous weighting leads to violations of monotonicity and subgroup consistency for a general class of multidimensional poverty

indices. Changes in the deprivation status of a household (or individual) generate measurement externalities through the endogenous weights in the form of changes in the deprivation score of other households (or people), despite the absence of any changes in their deprivation profiles.

Even though we focus on poverty indices, our analysis is equally relevant to composite welfare indices. For example, in the case of asset indices, each binary indicator could denote ownership of a specific asset (e.g. equal to one) or lack thereof (e.g. equal to zero). The asset score for each individual or household could be the weighted sum of the binary indicators. This is equivalent to the deprivation score (equation 2) in our paper. The societal poverty index in our paper can then be interpreted as the societal asset index which would essentially be the average of the individual or household asset scores. Monotonicity in that context would require that a loss of ownership of any asset (e.g. losing livestock) by an individual should not lead to an increase in overall societal asset index. Similarly, subgroup consistency would imply that if the asset score of a subgroup decreases, with asset scores of other subgroups unchanged, then the overall societal asset index should not increase. In all such cases where properties of monotonicity and subgroup consistency are required, our results hold and the use of endogenous weights would be problematic.

Of course, resorting to exogenous weights involves tricky, even potentially arbitrary choices. Best-practice suggestions for choosing exogenous weights are in their infancy, but certainly emerging. For instance, [Esposito and Chiappero-Martinetti \(2019\)](#) monitor multi-dimensional poverty in the Dominican Republic using exogenous weights generated from a separate field experiment.

One way of using endogenous weights, while satisfying monotonicity and subgroup consistency, could involve computing endogenous weights with one particular dataset and then leave them fixed toward future comparisons. This is precisely what [Asselin and Anh \(2008\)](#) do in their application to poverty comparisons in Vietnam with weights derived from MCA. However this option would not really simplify the complexity of the decision regarding weight selection, since one would still need to decide *which* dataset to use in order to compute the weights for poverty comparisons (e.g. should one use a particular dataset or pool datasets?). Moreover, as pointed out by [Alkire et al. \(2015, p. 99\)](#), if datasets are pooled to compute

weights based on data reduction techniques (e.g. MCA, principal component analysis, factor analysis, etc), there is no guarantee that a poverty comparison will be robust to *sample updating*, e.g. adding new time periods and including the new datasets in a recalculation of weights. Clearly, the latter decisions are hardly less arbitrary than choosing a vector of exogenous weights.

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A Appendix: Proofs

Proof of Proposition 1: Case (i): Suppose individual n is not deprived in any indicator. In that case, $\forall d \in \{1, 2, \dots, D\}$, $\rho_{nd} = 0$. Thus from (8), we know that $\Delta C_n = 0$. Now suppose individual n is deprived in all D indicators. Since $\sum_{d=1}^D w_d = 1$, we can deduce that:

$$\Delta w_j = - \sum_{\substack{d=1 \\ d \neq j}}^D \Delta w_d. \quad (\text{A1})$$

Hence, from (8), $\Delta C_n = 0$. Case (ii): Suppose for n , $\rho_{nj} = 1$ and $\exists d \neq j$ such that $\rho_{nd} = 0$. Then, from (A1), we can infer, $|\Delta w_j| > \left| \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{nd} \Delta w_d \right|$, since the right-hand side of the inequality aggregates over only those indicators in which individual n is deprived, except j . Thus: $\Delta C_n \gtrless 0 \iff \Delta w_j \gtrless 0$.

On the other hand if, for n , $\rho_{nj} = 0$; then from (8) we get: $\Delta C_n = \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{nd} \Delta w_d$.

This implies, $\sum_{\substack{d=1 \\ d \neq j}}^D \rho_{nd} \Delta w_d \gtrless 0$ if $\Delta w_j \gtrless 0$. Thus, $\Delta C_n \gtrless 0 \iff \Delta w_j \gtrless 0$.

Proof of Corollary 1: First we prove that $\Delta \rho_{ij} > 0$ leads to $\Delta C_i > 0$. From equation (9) we can get:

$$\Delta C_i = w'_j + \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{id} \Delta w_d, \quad (\text{A2})$$

where w'_j is the weight of indicator j in \mathbf{X}'_{ND} . Since $\sum_{d=1}^D \Delta w_d = 0$, then $\Delta w_j \gtrless 0$ implies $\sum_{d=1, d \neq j}^D \Delta w_d \gtrless 0$. Thus:

$$|\Delta w_j| = \left| \sum_{\substack{d=1 \\ d \neq j}}^D \Delta w_d \right| \geq \left| \sum_{\substack{d=1 \\ d \neq j}}^D \rho_{id} \Delta w_d \right|. \quad (\text{A3})$$

Suppose, $\Delta w_j > 0$. Thus from (A3) $|w'_j| > |\sum_{d=1, d \neq j}^D \rho_{id} \Delta w_d|$ which from (A2) implies $\Delta C_i > 0$. Likewise if $\Delta w_j < 0$, we know from (A3) $\sum_{d=1, d \neq j}^D \Delta w_d > 0$. Given $w'_j > 0$ we can deduce from (A2) that $\Delta C_i > 0$. Let t_i be the (exogenously weighted) number of indicators in which individual i is deprived and k is the cut-off for the (weighted) number of indicators one has to be deprived to be identified as poor. Then if $t_i \geq k$, given $\Delta C_i \geq 0$ and the definition of p_n , we can infer that $\Delta p_n \geq 0$. Likewise, if $t_i < k$ and $t'_i \geq k$ given $\Delta C_i > 0$, then again $\Delta p_n > 0$. Otherwise $\Delta p_n = 0$.

Proof of Proposition 3: Consider a deprivation matrix decomposed by subgroups $\mathbf{X}_{ND} = (\mathbf{X}_{N_1D} \parallel \mathbf{X}_{N_2D})$ where $N = N_1 + N_2, \forall n \in \mathbf{X}_{N_1D}, \rho_{nj} = 1$ and $\forall n \in \mathbf{X}_{N_2D}, \rho_{nj} = 0$. Suppose $\mathbf{X}'_{ND} = (\mathbf{X}_{N_1D} \parallel \mathbf{X}'_{N_2D})$, where \mathbf{X}'_{N_2D} is obtained from \mathbf{X}_{N_2D} by a simple increase of deprivation of person i in indicator j , i.e. $\Delta \rho_{ij} = 1, i \in \mathbf{X}_{N_2D}$. Suppose for \mathbf{X}'_{N_2D} : $\Delta w_j = w_j(\rho_{ij} = 1) - w_j(\rho_{ij} = 0) < 0$. To be subgroup consistent it must be the case that $\Delta P^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \underset{\geq}{\leq} 0$ if and only if $\Delta P^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} \underset{\geq}{\leq} 0$. Applying (10) we get:

$$\Delta P^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} = \frac{1}{N_2} \Delta p_i^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} + \frac{1}{N_2} \sum_{n \neq i}^{N_2} \mathbb{I}(\rho_{nm} = 0) \Delta p_n^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}}. \quad (\text{A4})$$

In (A4), $\Delta p_i^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} \geq 0$ from Corollary (1). Also $\Delta p_n^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} \geq 0 \forall n \neq i$ due to Corollary 2. Therefore, $\Delta P^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} \geq 0$. Now:

$$\Delta P^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} = \frac{1}{N} \Delta p_i^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} + \frac{1}{N} \left[\sum_{n \neq i}^N \mathbb{I}(\rho_{nm} = 0) \Delta p_n^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} + \sum_{n \neq i}^N \mathbb{I}(\rho_{nm} = 1) \Delta p_n^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \right] \quad (\text{A5})$$

Again, in (A5) $\Delta p_i^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \geq 0$. Likewise, $\sum_{n \neq i}^N \mathbb{I}(\rho_{nm} = 0) \Delta p_n^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \geq 0$. However, from Corollary 2, $\sum_{n \neq i}^N \mathbb{I}(\rho_{nm} = 1) \Delta p_n^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \leq 0$. Therefore, $\Delta P^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \underset{\geq}{\leq} 0$, unlike $\Delta P^{\mathbf{X}'_{N_2D} - \mathbf{X}_{N_2D}} \geq 0$. In fact, with $N_1 \rightarrow \infty$, we can obtain $\Delta P^{\mathbf{X}'_{ND} - \mathbf{X}_{ND}} \leq 0$.

B Appendix B: Data and Tables

Table 1: Dimensions, Observed Indicators and Deprivation Thresholds

Dimension	Indicators	A person is deprived if...
Health	Child mortality	Any child has died in the household.
	Nutrition	Any adult under 70 years of age in the household have low BMI or any child under 5 is underweight.
Education	School attendance	Any school-aged child in the household is not attending school up to the age at which he/she would complete class 8.
	Years of schooling	No household member aged 10 years or older in the household has completed five years of schooling.
Living Standards	Electricity	The household has no electricity.
	Drinking water	The household does not have access to improved drinking water or safe drinking water is at least a 30-minute walk from home, roundtrip.
	Sanitation	The household's sanitation facility is not improved, or it is improved but shared with other households.
	Cooking fuel	The household cooks with dung, wood or charcoal.
	Housing	The household has a dirt, sand, dung or other unspecified type of floor.
	Assets	The household does not own more than one radio, TV, telephone, bike, motorbike or refrigerator and does not own a car or truck.

Source: Adapted from [OPHI \(2018\)](#).

Table 2: Tetrachoric Correlation Coefficients

Ecuador										
	Cm	Nutr	Satt	Educ	Elct	Wtr	Sani	Ckfl	Hsg	Asst
Cm	1.000	0.112	0.167	-0.109	0.222	0.134	0.149	0.225	0.152	0.168
Nutr	0.112	1.000	0.226	-0.205	0.190	0.153	0.116	0.176	0.064	0.141
Satt	0.167	0.226	1.000	0.056	0.369	0.253	0.297	0.342	0.206	0.361
Educ	-0.109	-0.205	0.056	1.000	0.252	0.209	0.248	0.369	0.375	0.435
Elct	0.222	0.190	0.369	0.252	1.000	0.662	0.632	0.721	0.289	0.816
Wtr	0.134	0.153	0.253	0.209	0.662	1.000	0.500	0.542	0.165	0.513
Sani	0.149	0.116	0.297	0.248	0.632	0.500	1.000	0.537	0.406	0.587
Ckfl	0.225	0.176	0.342	0.369	0.721	0.542	0.537	1.000	0.587	0.714
Hsg	0.152	0.064	0.206	0.375	0.289	0.165	0.406	0.587	1.000	0.472
Asst	0.168	0.141	0.361	0.435	0.816	0.513	0.587	0.714	0.472	1.000
Uganda										
	Cm	Nutr	Satt	Educ	Elct	Wtr	Sani	Ckfl	Hsg	Asst
Cm	1.000	0.146	0.092	0.154	0.286	0.100	0.142	0.019	0.076	0.085
Nutr	0.146	1.000	0.171	0.191	0.346	0.109	0.226	-0.070	0.162	0.134
Satt	0.092	0.171	1.000	0.392	0.321	0.136	0.316	0.118	0.365	0.433
Educ	0.154	0.191	0.392	1.000	0.628	0.211	0.410	0.126	0.431	0.546
Elct	0.286	0.346	0.321	0.628	1.000	0.471	0.503	0.334	0.700	0.609
Wtr	0.100	0.109	0.136	0.211	0.471	1.000	0.136	0.239	0.273	0.135
Sani	0.142	0.226	0.316	0.410	0.503	0.136	1.000	0.072	0.588	0.425
Ckfl	0.019	-0.070	0.118	0.126	0.334	0.239	0.072	1.000	0.189	0.174
Hsg	0.076	0.162	0.365	0.431	0.700	0.273	0.588	0.189	1.000	0.507
Asst	0.085	0.134	0.433	0.546	0.609	0.135	0.425	0.174	0.507	1.000

Note: *Cm*: Child mortality, *Nutr*: Nutrition; *Satt*: School attendance; *Educ*: Years of schooling; *Elct*: Electricity; *Wtr*: Drinking water; *Sani*: Sanitation; *Ckfl*: Cooking fuel; *Hsg*: Housing; *Asst*: Assets

Table 3: Indicator Weights by Structure and Proportion of People Assigned Deprivations in *Nutrition*

height Ecuador							
Indicator	Exog.	PCA			Freq.		
		$s_{nutr}=0$	$s_{nutr}=50$	$s_{nutr}=95$	$s_{nutr}=0$	$s_{nutr}=50$	$s_{nutr}=95$
Cm	0.167	0.045	0.045	0.046	0.099	0.103	0.105
Nutr	0.167	0.037	0.014	0.005	0.061	0.019	0.001
Satt	0.167	0.080	0.080	0.081	0.135	0.141	0.144
Educ	0.167	0.072	0.077	0.078	0.103	0.108	0.110
Wtr	0.056	0.114	0.117	0.118	0.086	0.090	0.092
Elec	0.056	0.144	0.147	0.148	0.148	0.154	0.157
Sani	0.056	0.124	0.127	0.128	0.072	0.075	0.077
Ckfl	0.056	0.143	0.146	0.147	0.104	0.108	0.110
Hsg	0.056	0.097	0.100	0.101	0.103	0.107	0.109
Asst	0.056	0.145	0.148	0.149	0.088	0.092	0.094
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Uganda							
Indicator	Exog.	PCA			Freq.		
		$s_{nutr}=0$	$s_{nutr}=50$	$s_{nutr}=95$	$s_{nutr}=0$	$s_{nutr}=50$	$s_{nutr}=95$
Cm	0.167	0.046	0.046	0.047	0.133	0.149	0.157
Nutr	0.167	0.062	0.022	0.008	0.158	0.057	0.005
Satt	0.167	0.098	0.102	0.103	0.285	0.320	0.337
Educ	0.167	0.127	0.133	0.135	0.124	0.138	0.146
Wtr	0.056	0.075	0.078	0.079	0.087	0.097	0.102
Elec	0.056	0.155	0.160	0.162	0.013	0.014	0.015
Sani	0.056	0.119	0.123	0.124	0.023	0.026	0.028
Ckfl	0.056	0.054	0.059	0.061	0.001	0.001	0.002
Hsg	0.056	0.136	0.142	0.144	0.055	0.061	0.065
Asst	0.056	0.128	0.135	0.137	0.121	0.136	0.143
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes:

1. *Cm*: Child mortality, *Nutr*: Nutrition; *Satt*: School attendance; *Educ*: Years of schooling; *Elct*: Electricity; *Wtr*: Drinking water; *Sani*: Sanitation; *Ckfl*: Cooking fuel; *Hsg*: Housing; *Asst*: Assets. 2. s_{nutr} reflects the percentage of households with added deprivations in nutrition.

C Supplementary Appendix: Additional Results (NOT FOR PUBLICATION)

In this appendix we present the results for added deprivations in two alternative indicators: electricity and water. Ecuador has the lowest deprivation in electricity while for Uganda it represents the third highest deprivation. When it comes to water deprivation, for Uganda it is in the middle -neither high nor very low, while for Ecuador it is the third highest deprivation.

In comparing the weights of the indicators, from Table 3, one can see that in Uganda, electricity with an initial PCA weight of 0.155 and water with initial PCA weight of 0.075 are, respectively, the highest and the fourth lowest out of ten indicators. Similarly, for initial inverse-frequency weights in Ecuador, electricity with 0.148 and water with 0.086 are, respectively, the highest and third lowest out of ten indicators. Thus, between Ecuador and Uganda, the electricity and water indicators cover a wide range of PCA and frequency weights.

As before, in each country we assign random identifiers to the population that is non-deprived in each of the two indicators, *one at a time*. We use these identifiers to form random ventiles of non-deprived people in each indicator. Then, in a *cumulative* process for one indicator at a time, we gradually assign simulated deprivations to each random ventile; we first assign simulated deprivations to 5% of the relevant population, then to an additional 5% (for a total of 10%) and so on until 95%.

The figures in subsection C.1 show violations of monotonicity followed by a set of figures depicting violations of subgroup consistency in subsection C.2. In each case, we assess the consequence of changes in electricity followed by water for Ecuador and Uganda, respectively.

C.1 Violation of Monotonicity

C.1.1 Electricity

Figure 4: Violations of Monotonicity: Ecuador

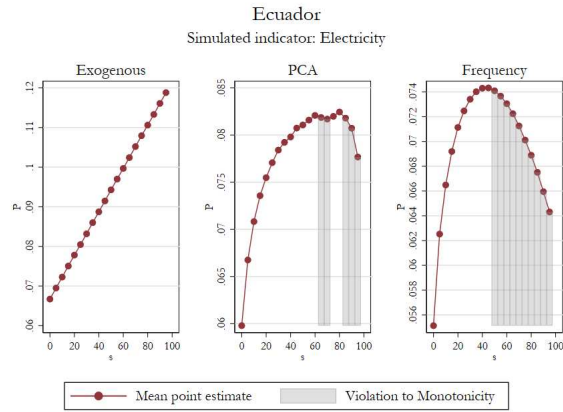
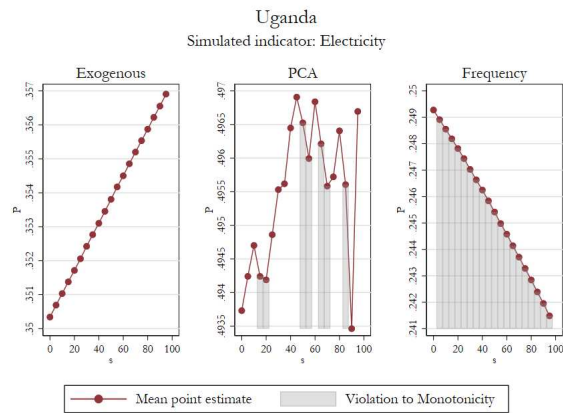


Figure 5: Violations of Monotonicity: Uganda



C.1.2 Water

Figure 6: Violations of Monotonicity: Ecuador

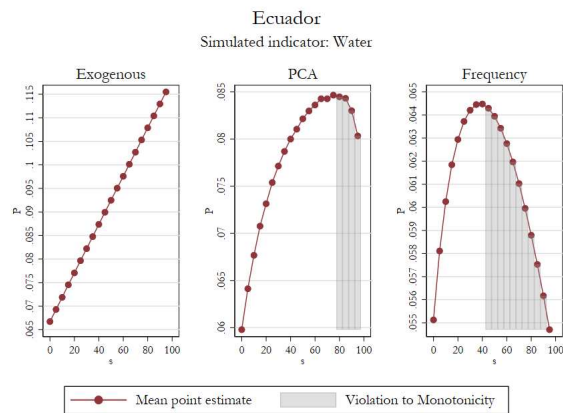
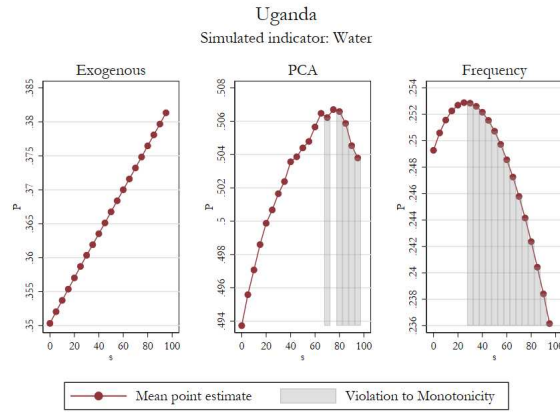


Figure 7: Violations of Monotonicity: Uganda



C.2 Violation of Subgroup Consistency

C.2.1 Electricity

Figure 8: Violations of Subgroup Consistency: Ecuador

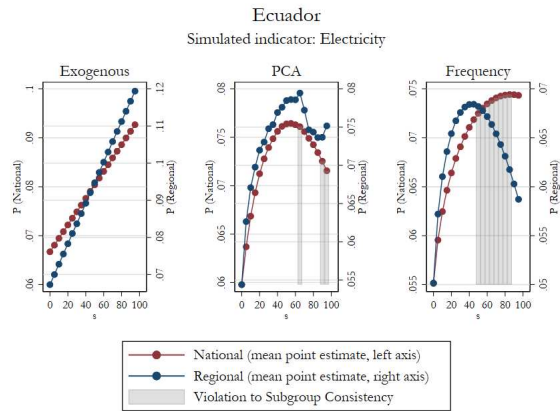
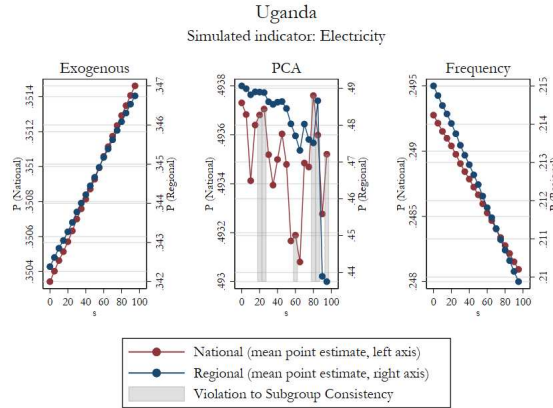
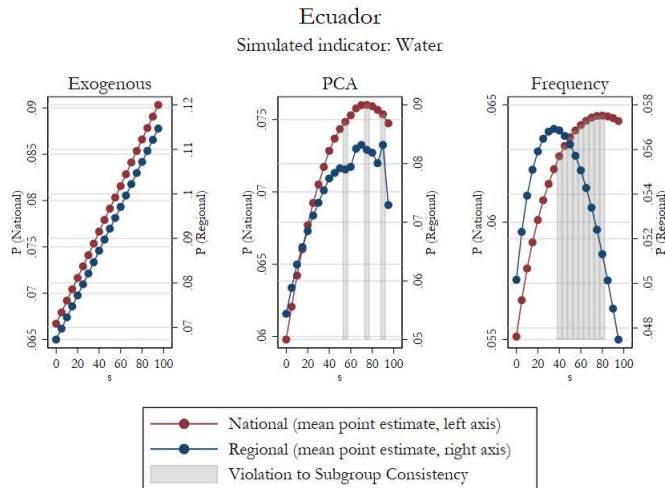


Figure 9: Violations of Subgroup Consistency: Uganda



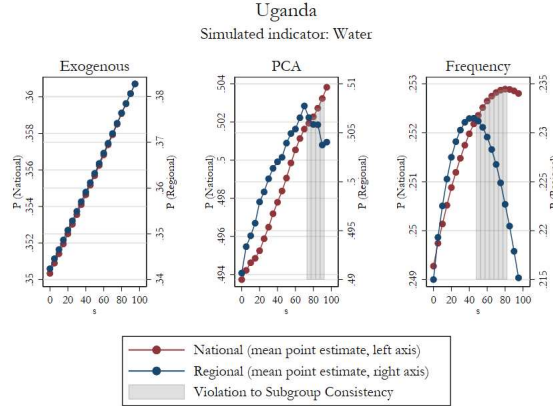
C.2.2 Water

Figure 10: Violations of Subgroup Consistency: Ecuador



In each of the cases above, one can clearly see that both monotonicity and subgroup consistency are violated by endogenous weights, but not by exogenous weights. These additional illustrations are not an exhaustive account of all possible empirical scenarios, but they cover an array of situations that enable us to safely illustrate that violations of monotonicity and subgroup consistency are widespread.

Figure 11: Violations of Subgroup Consistency: Uganda



D Illustration with Synthetic Data

We show here that violations of monotonicity and subgroup consistency are pervasive in more general contexts. To do this, we apply a Monte-Carlo procedure to create synthetic datasets consisting of four partially correlated binary indicators.

First, we generate the continuous variables underlying the binary indicators with random draws from a multivariate normal distribution such that $\mathbf{X} = (X_1, \dots, X_4) \sim \mathcal{N}_4(\mu, \Sigma)$. The mean vector $\mu \equiv (m_1, m_2, m_3, m_4)$ is set to define heterogeneous deprivation rates across indicators: $m_1 = \mathcal{U}(0.2, 0.25)$, $m_2 = \mathcal{U}(0.4, 0.5)$, $m_3 = \mathcal{U}(0.6, 0.75)$, and $m_4 = \mathcal{U}(0.8, 1)$. Thus, deprivations are least prevalent in indicator 1, and most prevalent in indicator 4. The off-diagonal elements of Σ are set to define partially correlated indicators with $\Sigma_{ij} = U(0.2, 0.8), \forall i \neq j$. Thus our parametrisation aims at defining a set of indicators that have some common variation giving sense to the notion of a ‘principal’ component in PCA analyses, while not being perfectly redundant. We then use matrix \mathbf{X} to create a dataset consisting of four vectors, one for each binary indicator, defined as $Y_j = \mathbb{I}(X_j \geq 0.5), j = 1, \dots, 4$, with Y_j sized $(1000 \times 1), \forall j$.

For any given synthetic dataset, we implement a procedure similar to the one we used to assess the real-world data and randomly assign additional deprivations among the initially non-deprived fictitious population the following way:

1. We first assign random identifiers to the observations that are non-deprived in each

one of the four indicators, one indicator at a time.

2. We then use these identifiers to form random ventiles of non-deprived observations for each indicator.
3. Finally, for each binary indicator we gradually assign deprivations to each random ventile cumulatively; we first assign deprivations to 5% of the relevant observations, then to an additional 5% (for a total of 10%) and so on, until finally assigning the deprived status to 95% of the part of the population originally non-deprived in the assessed indicator.

Note that in this synthetic data-based approach there are two sources of bias. First, there may be bias associated with the initial dataset, which comes from a random population, unlike real-world data. Second, there may be bias related to the random process of additional deprivation assignment. To account for the first source of bias, we simulate ten synthetic datasets. For each one of these, we repeat the deprivation-assignment process ten times, yielding a total of hundred realisations of our simulation procedure for each simulation setting (comprising specific choices of poverty cutoffs, weighting function, etc.). This enables us to determine the proportion of violations of the axioms, i.e. the number of times the axioms are violated out of a hundred random instances.

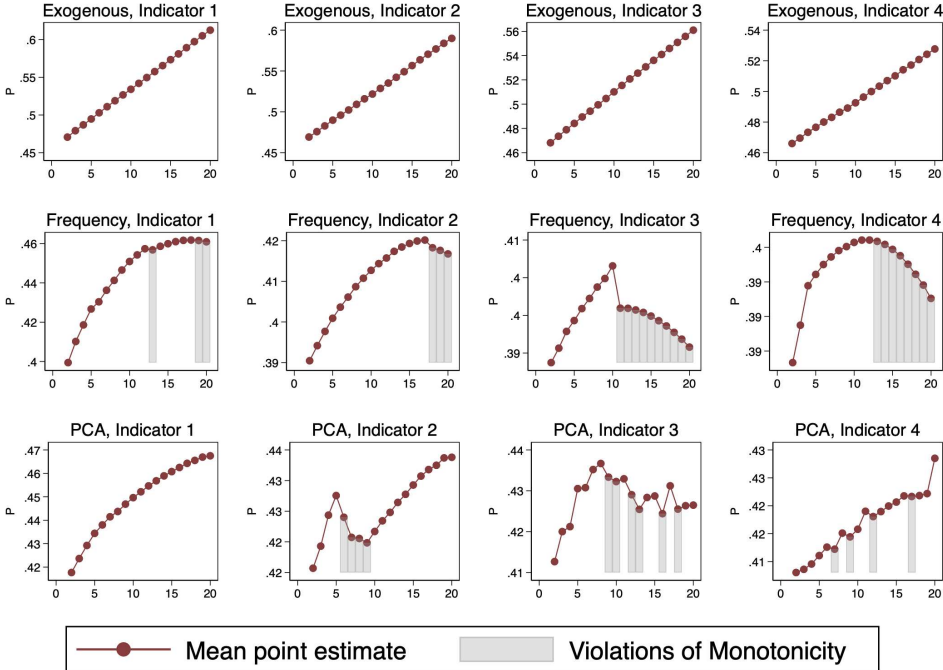
For completeness, our analysis covers a finite set of potentially meaningful poverty cutoffs (i.e. k -values), namely union ($\min\{w_1, \dots, w_D\}$), 0.25, 0.50, 0.75, and 0.99 (near intersection).

D.1 Violations of Monotonicity

We find that violations of monotonicity under endogenous weights may happen for all the considered k -values. Figure 12 shows our simulation results for $k=0.50$ (i.e. an intermediate poverty cutoff) and one out of the ten simulated datasets as an illustration. The shaded areas represent instances where societal poverty, P , decreases while deprivations among the population increase.

Violations of monotonicity are ubiquitous under frequency and PCA-based weights, while they are non-existent under exogenous weights (as expected). This is true irrespective of

Figure 12: Societal Poverty Measure $P(k = 0.5)$ for Cumulative Ventiles of Additional Deprivations: Illustration with a Synthetic Dataset

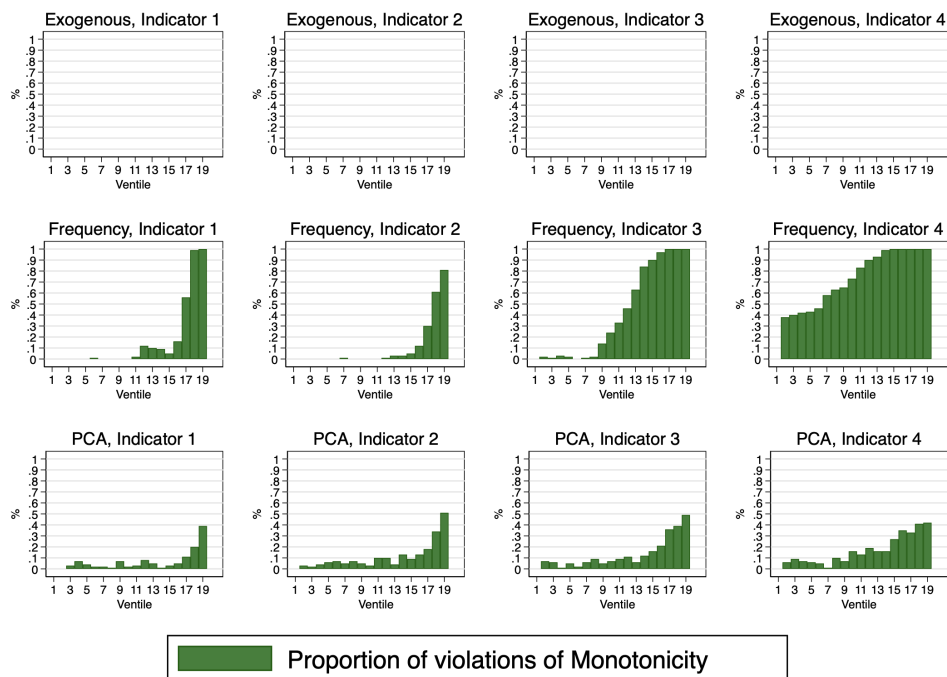


the dataset we use. Figure 13 depicts our main simulation results, namely the proportion of simulations in which violation of monotonicity occurs for each cumulative ventile. We can clearly see that, under (inverse) frequency weights, these violations are more likely to happen if additional deprivations appear in indicators with higher initial incidence (e.g. indicators 3 and 4 versus 1 and 2). Importantly, if a large proportion of the originally non-deprived population becomes deprived (say from ventile 13 onwards), violations of monotonicity occur in every single simulation under frequency weights. Violations of this axiom under PCA weights are much less predictable across indicators, yet they are hardly less problematic. If a high proportion of the originally non-deprived population becomes deprived (i.e. higher ventiles), violations of monotonicity arise in around half of the simulations.

D.2 Violations of Subgroup Consistency

To assess violations of subgroup consistency, we follow the exact same procedure and parametrisation described above to create two 500-observation datasets, each representing a subnational region. We then combine them to form a 1000-observation national dataset.

Figure 13: Proportion of Violations of Monotonicity with $P(k = 0.5)$ for Cumulative Ventiles of Additional Deprivations



Assignment of additional deprivations also follows the procedure described above, but it solely affects observations in one of the subnational regions.

For consistency, Figure 14 shows simulation results for $k=0.50$ and one dataset as an illustration, where we plot the evolution of poverty at the national (red lines) and the subnational levels (blue lines) as deprivations among the population increase. The shaded areas denote instances where poverty at the subnational level increases while it decreases at the national level.

Figure 15 shows the proportion of simulations in which we encounter violation of subgroup consistency, for each cumulative ventile. Again, violations of subgroup consistency appear widely under endogenous weights while they never occur under exogenous weights. These violations are less frequent than monotonicity violations, yet they still occur in up to 30% of the simulations. Interestingly, if deprivations appear in large part of the initially non-deprived population, violations of subgroup consistency can occur in almost every simulation under PCA weights (see indicators 1 and 2).

Figure 14: Societal Poverty Measure $P(k = 0.5)$ for Cumulative Ventiles of Additional Deprivations: Illustration with a Synthetic Dataset

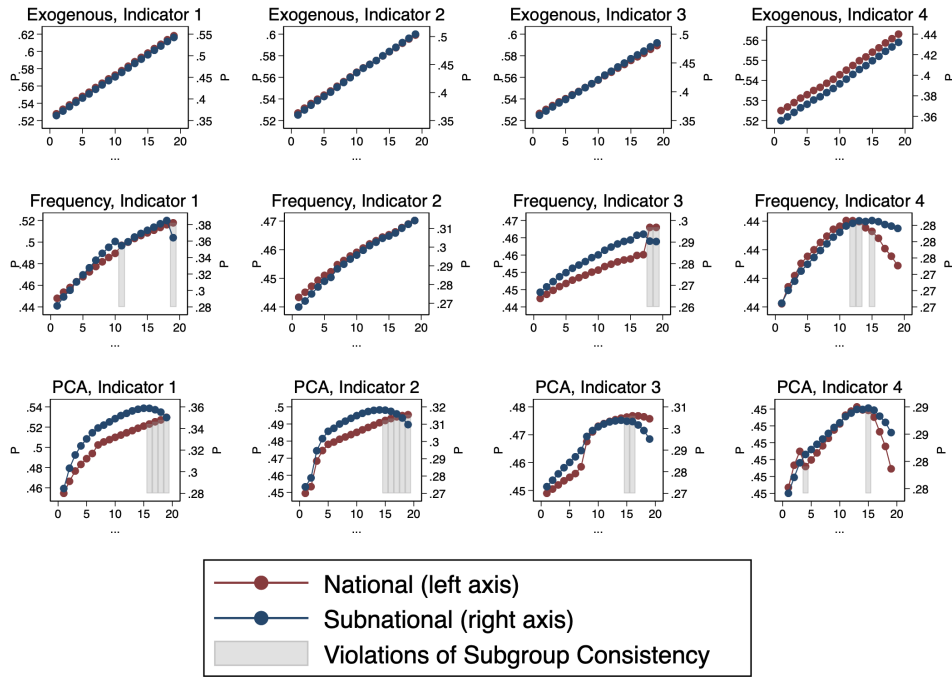


Figure 15: Proportion of Violations of Subgroup Consistency with $P(k = 0.5)$ for Cumulative Ventiles of Additional Deprivations

