Article:

Navas, A. and Nocco, A. (2021) Trade liberalization, selection, and technology adoption with vertical linkages. Review of International Economics, 29 (4). pp. 979-1012. ISSN 0965-7576

https://doi.org/10.1111/roie.12529

Trade Liberalization, Selection and Technology Adoption with Vertical Linkages.*

Antonio Navas[†] Antonella Nocco[‡]

December 2020[§]

Abstract

This paper analyses the role played by vertical linkages on the effects of trade liberalization on technology upgrading, average productivity and welfare in a model of trade with heterogeneous firms. We find that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare. Our calibration results show that vertical linkages, technology upgrading, and their interaction magnify the effects of trade liberalization on average productivity and welfare.

Keywords: trade liberalization, heterogeneity, selection, technology adoption, vertical linkages.

J.E.L. Classification: F1.

*Data sharing is not applicable to this article as no new data were created or analyzed in this study.

[†]Corresponding Author: Antonio Navas, The University of Sheffield. Department of Economics, 9 Mappin Street S102TN, Sheffield. Phone number: +44 114 222 33 25. E-mail: a.navas@she eld.ac.uk

[‡]Università del Salento, Dipartimento di Scienze dell'Economia, Ecotekne, via Monteroni, 73100 Lecce, Italia. Tel: 0039 0832 298780; E-mail: antonella.nocco@unisalento.it. Research Fellow at CESifo (Center for Economic Studies and the Ifo Institute), Munich

[§]We would like to thank the Associate Editor, Davin Chor, Paola Conconi, Sarah Brown, two anonymous referees and the participants of the Royal Economic Society Meeting (2016) in Brighton, the ETSG Meeting in Helsinki (2016) and Warsaw (2018), the Midwest Trade Meeting in Philadelphia (2018), the Annual Meeting of the Società Italiana degli Economisti in Palermo (2019) and the Annual North American Meetings of the Regional Science Association International (2020) and the participants of seminars at University of Castellon, De Montford University, University of Hull and University of Valencia for their useful comments and suggestions. Remaining errors are ours.

1 Introduction

The last two decades have witnessed the birth of a rich literature that theoretically and empirically shows that firm heterogeneity is key to understanding the impact of trade liberalization policies on an industry's average productivity. According to this literature, trade increases average productivity due to a reallocation effect that operates through a selection mechanism (trade expels the less efficient firms out of the market and reallocates production factors towards the most efficient units (Melitz, (2003)) and through a within plant effect. Both sources, selection and plant productivity growth, have been found to be empirically relevant when examining the impact of trade liberalization on an industry's average productivity (Pavnick (2002), Trefler (2004), Bloom, Draca and Van Reenen (2016)).

While the first branch of this literature has focused initially on firm heterogeneity in export activities, a very recent avenue points out that this dimension is far more complex with industries heavily relying on vertical linkages and the use of intermediate inputs. In this literature importing intermediate inputs also plays an important role in determining the ultimate effect of trade on an industry's average productivity (Kasahara and Rodrigue (2008), Halpern, Koren and Szeidl (2015)). Whilst many works document a prominent role of intermediate inputs in plant productivity growth, recent empirical evidence suggests that, following trade liberalization, firms also increase their productivity by raising their investment in R&D activities (Lileeva and Trefler (2010), Aw, Robers and Xu (2011) Bustos (2011)).[1] The current paper contributes to the literature by examining theoretically the interaction between these two sources of plant productivity growth, that is, importing intermediates (vertical linkages) and technology upgrading, a relatively unexplored mechanism in this literature. More precisely, this work investigates the role played by the presence of vertical linkages among firms that use both locally produced and imported intermediates on the impact of trade liberalization policies on technology upgrading, average productivity and, ultimately, welfare.

Vertical linkages are defined as links between firms up and down the production chain, and, theoretically, they can be modelled as the use of final output produced by other firms as intermediate inputs in the production process of a firm. The existence of these links through intermediates implies that a reduction in trade costs would generally magnify the gains from trade through the creation of an industry multiplier effect that could increase the trade volume between countries due to the back-and-forth trade in intermediates. The multiplier effect, however, will increase the demand of intermediate inputs in such a way that their prices could be eventually driven up, reducing welfare in certain cases as in Nocco (2012). The inclusion of the option for firms to technology upgrade implies that the most productive firms can also invest in innovation, which creates an extra channel of welfare gains from trade liberalization.

Although the implications of both mechanisms have been explored separately in the literature,[2] their interaction

[1] See also the seminal work by Atkeson and Burstein (2010) and a more recent contribution by Impullitti and Licandro (2018).
[2] See the literature review section below for references.

in a general equilibrium context, which constitutes one of the main contributions of this paper, could generate further implications for welfare that have been unexplored. Vertical linkages, through the multiplier effect, could enlarge firms' profits from technology adoption, encouraging firms' technology upgrading. This will have a positive impact on welfare through the direct effects of technology upgrading on the average productivity and its indirect effects on the average productivity generated by the intensification of selection, with both types of effects contributing to a reduction of the aggregate price index. However, the inclusion of technology upgrading besides vertical linkages could intensify firms' competition for scarce production factors so much that their joint action could result in an increase in the aggregate price index, reducing welfare.

Our paper is motivated not only by the fact that vertical linkages bring in these types of new channels that could modify the link between trade liberalization, productivity and welfare, but also by the recognition that they are empirically relevant. Indeed, intermediate inputs constitute a fundamental part of the production structure of an economy. To illustrate this point, Tables 1 and 2, respectively, in the online appendix, show the within country across industries average intermediate input intensity for 43 countries (including both developed and developing countries) and the within industry across countries average intermediate input intensity for 56 industries for 2014, obtained from the World Input Output Database (WIOD) built by Timmer et al. (2015). The WIOD condenses data from different national accounts to provide an international input-output matrix in which the flows of intermediate inputs are observed in country-industry pairs rather than across industries as in a standard input-output table. Although the tables display a certain degree of heterogeneity, both across industries and countries, both tables reveal that, indeed, vertical linkages are important in the production process of an economy with a country average intermediate input intensity of 54% of the total gross output. Intermediates are also responsible for a large share of the total volume of international trade with the World Trade Report 2014 (p. 43) stating that "the average import content of exports is around 25 per cent – and increasing over time – and almost 30 per cent of merchandise trade is now in intermediate goods or components". These findings not only state that vertical linkages are relevant at the sectoral level, but also suggest that heterogeneity across industries and countries in the strength of vertical linkages can result in differences in export behavior or technology upgrading decisions across firms that could have implications for welfare.

To analyze the role played by vertical linkages in determining the effects of trade liberalization on technology upgrading, productivity and welfare, we construct a trade model with heterogeneous firms that are interconnected by vertical linkages in such a way that all firms can employ, as intermediate inputs, the final goods produced by both domestic and foreign exporting firms, as in Krugman and Venables (1995) and Nocco (2012).[3] In particular, in

[3]Baldwin et al. (2005, chp 8) extensively describe this type of modelling vertical linkages (input-output linkages). Their graduate textbook on New Economic Geography suggests that: "[...] Perhaps the most important is that of the so-called "vertical linkage" models (VL models for short). [...]. The seminal paper, Venables (1996a), introduces cost linkages between an upstream sector and a downstream sector. For the sake of simplicity, Krugman and Venables (1995) and Fujita Krugman and Venables (1999 chapter 14) collapse the two sectors into one, so input-output relationships switch from 'vertical' linkages to 'horizontal' linkages (nevertheless the VL label is retained)."

our framework, firms use, to produce, a Cobb-Douglas technology that involves the use of intermediates and labor in proportions α and $1 - \alpha$, respectively. The parameter α, therefore, represents the strength of vertical linkages. These firms are also allowed to upgrade the current state of the technology they use by reducing their marginal cost to be a fixed proportion γ of its initial value, bearing a fixed cost of adoption of f_I units of the final good, as in Bustos (2011) and Navas and Sala (2015) among others. The framework is simple but rich enough to allow us to approach the research question in a tractable manner in the context of a general equilibrium model in which vertical linkages affect the variables of interest in the economy not only through a supply side effect (by affecting the costs of production) but also through a demand side effect (by increasing a firm's demand).[4] In addition, the framework is also consistent with the stylized fact that many firms within an industry do not engage in R&D activities (Klette and Kortum (2004)).

Specifically, our results highlight that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare. The model exhibits different types of equilibria depending on the parameter configuration, which are related to different hierarchies among adopters, exporters and domestic firms. The effects of trade liberalization on technology upgrading will vary with the intensity of vertical linkages and across equilibria. In particular, for the equilibrium in which firms that technology upgrade are a subset of the most productive exporters, trade liberalization will promote technology adoption when the intensity is either low (i.e. $0 < \alpha < \alpha_0$) or high ($\alpha > \alpha_2$) although in the last one, only under certain conditions.[5] For intermediate levels, however, the results are common across all equilibria: that is, trade liberalization promotes technology upgrading when the intensity of vertical linkages is intermediate-low (i.e. $\alpha_0 < \alpha < \alpha_1$) and deters technology upgrading when it is intermediate-high (i.e. $\alpha_1 < \alpha < \alpha_2$). The fact that different degrees of vertical linkages could induce a very different effect of trade liberalization on technology upgrading is the result of several opposing forces at work. On the one hand, strong vertical linkages increase the profits of technology upgrading through an expansion of the firms' market size and a reduction in the upgrading costs due to the cheaper imported intermediate inputs. On the other hand, strong vertical linkages strengthen firm's competition for scarce production factors eventually making intermediate inputs relatively more expensive.

These results on technology adoption and productivity have a clear impact on the effects of trade liberalization on welfare. Specifically, trade liberalization will have a positive impact on welfare when the intermediate input intensity is low or intermediate-low but it will have a negative impact when it is intermediate-high and high. We

[4] The specification proposed by Krugman and Venables (1995) has been widely used in New Economic Geography models, which show that vertical linkages tend to reinforce centripetal forces that promote agglomeration (Fujita et al., 1999; Puga, 1999; Nocco, 2005). Nocco (2012) introduces this type of vertical linkages among the monopolistically competitive heterogeneous firms in the variant of the Melitz (2003) model proposed by Baldwin and Forslid (2006).

[5] Here, we specifically comment on the results for this equilibrium since for our calibration exercise this is the empirically relevant one. We, however, recognize the possibility that the other types of equilibria are relevant in other contexts and for that reason a full taxonomy of all the equilibria can be found in the online appendix.

also find that both channels, vertical linkages and technology upgrading, and their interaction magnify the effects of trade liberalization on welfare. To get a flavour of the quantitative importance of these channels, we propose a calibrated version of the model for the case of the US economy. Our results suggest that these magnification effects on welfare are also quantitatively important and that small differences in the average intermediate input intensity across countries could, indeed, generate very different quantitative trade implications. For the average intermediate input intensity in the US (48%), a reduction in trade costs from 25% to 15% will increase welfare by 3.43%. This magnitude is more than double the one implied by a model without vertical linkages (1.61%). Should the US have the average intermediate input intensity of China (62%), the largest value observed in the sample, the increase in welfare would have been 49% larger. In the final part of our calibration, we compute the contribution of the interplay between technology upgrading and vertical linkages in our model by comparing the contribution of technology upgrading to welfare across different levels of vertical linkages. Our calibrated exercise suggests that ignoring the effects of trade liberalization on intermediate inputs and its consequences for technology upgrading decisions underestimates the gains from trade. In our calibrated exercise, ignoring a firm's decision to technology upgrade in a model with no vertical linkages implies that welfare gains can be underestimated by 0.060 percentage points. For the case of the US, ignoring the technology upgrading channel in a model with vertical linkages implies that welfare gains can be underestimated by 0.13 percentage points, which is more than double the underestimation without vertical linkages as reported above. In both cases the underestimation constitutes 3.72% and 3.79% of the total welfare gains respectively.

1.1 Literature review

This paper is related to different strands of the literature. The very first one is the emerging literature on macroeconomics that explores the consequences of vertical linkages for explaining cross-country income and productivity differences (Ciccone (2002), Jones (2011), Klenow and Rodriguez-Clare (2005)) or the more recent works of Fadinger, Ghiglino and Teteryatnikova (2017) and Cunat and Zymek (2017), which include vertical linkages in a multi-sector model of trade and heterogeneous firms. However, these models abstract from an important channel through which vertical linkages may affect productivity differences: technology upgrading. Vertical linkages have also been used in macroeconomics to investigate business cycle transmission across countries (Di Giovanni and Levchenko (2010)).[6]

The second one is the literature on trade which emphasizes the importance of vertical linkages for aggregate trade flows and the impact of trade liberalization on welfare (Helpman and Krugman (1987), Hummels, Ishii and Yi (2001), Nocco (2012), Costinot and Rodriguez-Clare (2014), Caliendo, Feenstra, Romalis and Taylor (2015)), Caliendo and Parro (2015), Ossa (2015). This literature outlines how vertical linkages magnify the gains from trade. In these models a reduction in trade costs generates a further reduction on final good prices through its

[6] Moreover, Lee, Padmanabhan and Whang (1997) have analyzed the so called "bullwhip effect" that underlines how small changes in final demand can cause a big change in the demand for intermediate goods along the value chain.

impact in the cost of intermediate inputs. This literature, however, leaves aside technology upgrading and, more precisely, the role played by vertical linkages on the firm's decision to technology upgrade, which is the main focus of this paper.

Another recent strand of literature explores the impact of trade on firms' technology upgrading decisions in an environment characterized by heterogeneous firms (Rubini (2014), Perla, Tonetti and Waugh (2014), Navas and Sala (2015)), where intermediate inputs are absent. More closely related to our work is the literature that analyses the role played by trade in intermediates on firms' innovation decisions. Bas and Berthou (2017), Boler et al. (2015) focus on studying the complementarity between importing and innovation decisions at the firm level.[7] These papers, however, do not consider the roundabout structure of production and, therefore, the multiplier effect, which plays an important role in our results, is absent in their frameworks. Boler et al. (2015) analyze the complementarity between R&D investments and importing intermediate inputs as a consequence of a reduction in R&D costs or a process of trade liberalization without considering selection effects or explicitly modelling the exporting activity of firms.[8] In our context, apart from the complementarities present in Boler et al. (2015), a reduction in trade costs will promote exporting which will further increase the demand of intermediate inputs and innovation profits. This will encourage more firms to technology upgrade but it will also raise the pressure on factor prices that emerges in a general equilibrium context. As specified in Boler et al. (2015) their work emphasizes supply-side complementarities while our framework considers a richer environment in which demand and supply side complementarities of multiple nature are present.

Moreover, Fieler et al. (2018) build a quantitative model of trade with firm heterogeneity, quality upgrading and input linkages in which they assume that the production of higher quality products requires higher quality inputs, to assess the impact of the 1991 Colombian's trade liberalization episode on skill intensity and the different channels behind it. Unlike Fieler et al. (2018), we focus on the impact of trade liberalization on average productivity and welfare when vertical linkages and technology upgrading are present. In addition, our paper rather than focusing on a small open economy, is solved for the equilibrium of a two symmetric open economies model and, therefore, the framework developed here is more suitable for identifying new effects of trade liberalization on technology adoption and selection when firms producing in developed countries are interconnected by vertical linkages. The different perspective in Fieler et al. (2018) is also shown by the fact that, in their case, the incentive to quality upgrade is different for exporters and importers as they state (p. 110) that "[t]rade leads exporters to upgrade because foreign has a higher demand for higher-quality goods, and it leads importers to upgrade because foreign inputs make it

[7]Bas and Berthou (2017) explore the complementarity between technology adoption and importing intermediates while Boler et al. (2015) study this complementarity in a structural model of R&D investments.

[8]In Boler et al. (2015) there is a two way complementarity between R&D investments and the imports of intermediate inputs that arises because, on the one hand, R&D increases expected future profits allowing firms to import more inputs internationally further reducing their costs and, on the other hand, the increase in imports of inputs makes R&D investments more profitable. In our work, instead, we analyze the impact of trade liberalization on both international trade and welfare with selection effects and technology upgrading decisions that are intertwined.

cheaper to produce higher-quality".

Finally, our paper is also related to a more recent literature that explores how the input network structure of firms affects the production structure of an economy and its consequences for welfare, (Antras et al. (2017), Lim (2018) and Zou (2019)). Antras et al. (2017) build a quantifiable partial equilibrium model in which firms self-select into importing by incurring a per source country fixed cost. Lim (2018) and Zou (2019) instead, consider a more sophisticated intermediate input structure in which firms need to pay an upfront fixed cost per each intermediate input variety they source either domestically or from abroad. These papers outline important novel mechanisms through which a reduction in trade costs may have an impact on output or welfare through the change in the number and the composition of inputs or source countries, but they do not explore the implications of these intermediate input structures for technology upgrading and its implications for welfare.

The results obtained in our work show that the inclusion of the possibility of firms to technology upgrade and the presence of vertical linkages affect the industry average productivity and together they shape the effects of trade liberalization on welfare. Indeed, the model unveils a new channel through which trade liberalization could affect welfare that has been unexplored in the literature: the magnified effect of trade liberalization on the firm's decision to technology upgrade provoked by the inclusion of vertical linkages. This is due to the fact that trade liberalization not only reduces the costs of serving the foreign market but also the costs of acquiring intermediate inputs from abroad, and these effects interact in a general equilibrium framework where not only the supply side effects of trade liberalization are taken into account, but also the demand side effects. As the demand for manufactured goods increases with trade liberalization and the strength of vertical linkages, its effect may eventually dominate the supply side effects resulting in an increase in the price index that, could reduce welfare and technology upgrading. In addition, the model also highlights that some of the thresholds for the strength of vertical linkages that have been obtained in Nocco (2012) not only shape the effects of trade liberalization on selection and exporting, but also those on innovation. Indeed, the model suggests that the effects of trade liberalization on technology upgrading depends qualitatively and quantitatively on the intensity of vertical linkages adding new insights to the results already obtained in the literature.

The rest of the paper is organized as follows. Section 2 presents the structure of the model and its different equilibria with explicit solutions. Sections 3 and 4, respectively, discuss the effects of trade liberalization on unit input cutoffs and on welfare. Moreover, subsection 4.2 presents a calibration of our model for the case of the US economy assessing the impact of a process of trade liberalization with an identical economy on welfare and, among other things, on the probability of firm's surviving and innovating. Section 5 concludes.

2 The Model

Let us consider a world in which there are two symmetric countries H and F, each populated by L individuals endowed with one unit of time that is dedicated entirely to work. Individuals derive utility from the consumption of two different goods, T and M, according to the following Cobb-Douglas functional form

$$U = \frac{C_T^{1-\mu} C_M^{\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}},\ 0 < \mu < 1$$

where the parameter μ represents the proportion of expenditure dedicated to the good M. The good T is a homogeneous good, produced with a linear technology (i.e., one unit of labor is required to produce one unit of output) under perfect competition and freely traded. In contrast, M is a differentiated good and individuals derive utility from a continuum of varieties (indexed by $j \in \Omega$) according to the following specification

$$C_M = \left(\int\limits_{j \epsilon \Omega} C_{\varepsilon}(j)^{\frac{\sigma-1}{\sigma}} \, dj \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the constant elasticity of substitution and $C_{\varepsilon}(j)$ denotes the individual consumption of variety j.

Each variety in the economy is produced by a single firm in a monopolistic competition environment. Unlike Melitz (2003), firms combine labor and a set of differentiated intermediate inputs to produce their variety using a Cobb-Douglas technology. More precisely, the firm producing the $j - th$ variety uses the following technology

$$q(j) = \frac{1}{a(j)} L(j)^{1-\alpha} M(j)^{\alpha}$$

where $q(j)$ is the quantity obtained combining $L(j)$ units of labor and $M(j)$ units of the differentiated intermediate inputs, and $a(j)$ represents the unit input requirements of firm j and it is an inverse measure of the productivity level of the firm. Since $\alpha \in (0,1)$ is the intermediate share and determines the importance of intermediate inputs in the production of the final good, it also represents the *strength of vertical linkages.*

Following Krugman and Venables (1995) and Nocco (2012) among others, and consistent with our empirical observations, we assume that there are vertical linkages so firms are using part of the production of the final differentiated good M as an intermediate input. The intermediate composite good used by each firm j is therefore given by

$$M(j) = \left(\int\limits_{l \epsilon \Omega} B_j(l)^{\frac{\sigma-1}{\sigma}} dl \right)^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1$$

where $B_j(l)$ denotes the amount of good l used as intermediate input by the firm producing variety j and σ controls also for the substitution between different intermediate inputs. For simplicity and common to the literature on vertical linkages (i.e., Krugman and Venables (1995), Nocco (2012)) it has been assumed that the elasticity of substitution among final goods in the consumer side is the same as the elasticity of substitution among intermediate inputs in the production function.

To enter into a market a firm needs to invest f_E units of labor to create a new variety. At the moment of entry, the firm is uncertain regarding its productivity although the firm knows that its unit input requirement $a(j)$ follows a Pareto Distribution with the cumulative density function given by $G(a) = \left(\frac{a}{a_M}\right)^{\kappa}$ with $0 \leq a \leq a_M$ and shape parameter $\kappa \geq 1$.[9] After entry, the unit input requirement is revealed to the firm and the firm has the option to leave or stay in the market. Firms that produce must incur a per period fixed cost of operation f_D in terms of the differentiated final good.

Once the firm stays, it has the possibility to export to the foreign market. Exporting, however, entails both fixed and variable trade costs. More precisely, exporters need to incur a per period fixed cost of exporting f_X in terms of the differentiated final good and a variable trade cost of the iceberg type so that $\tau \geq 1$ units of the good have to be shipped from the production country in order to sell one unit in the foreign country. At this stage, we depart from Nocco (2012) assuming that the firm can simultaneously decide to adopt a more efficient Hicks-neutral technology which reduces the marginal cost of production to be a proportion γ of the original value $a(j)$, with $0 < \gamma \leq 1$, so that the unit input requirement of the composite good of firm j that innovates is $\gamma a(j)$. Adopting the most efficient technology bears a cost of f_I units in terms of the final differentiated good.[10,11]

We assume that the homogenous good is the numeraire. This, together with the assumption of perfect competition, the linear technology (with unit input requirement equal to one) and zero trade cost assumed for this good, imply that the equilibrium wage is equal to one in both countries ($w = 1$). Finally, as common in the literature in trade and firm heterogeneity, and to keep the analysis tractable, the paper focuses on the case of symmetric economies (i.e., H and F exhibit identical parameter values, and therefore the solutions for the endogenous variables are symmetric).

[9] The Pareto distribution assumption has been widely used in the literature because of both its technical properties (it allows the derivation of a closed form solution in the Melitz (2003) model which could be important in complicated extensions like the one considered in this paper) and because of the empirical fit. Although there is a very recent literature that questions the fit of the Pareto distribution in certain contexts (Bas et al. (2017), Fernandes et al. (2018)), several papers outline the fitness of this functional form for several moments of the productivity distribution of developed countries (Axtell (2001), Eaton, Kortum and Kramarz (2011)). Specifically (Axtell, 2001) shows that for the case of the US, the case studied in the calibration exercise in our paper, the Pareto distribution fits very well the distribution of a firm's size. In addition Gabaix (1999), Luttmer (2007) show that the Pareto distribution can emerge from relatively simple stochastic models of firm's growth.

[10] Following the notation of Baldwin and Forslid (2004), the parameters f_D, f_X and f_E are, respectively, the discounted value of the fixed cost of producing for the domestic country, for export and entry. These elements consequently are the equivalent to δf_D, δf_X and δf_E in Melitz (2003). Moreover, f_I is the discounted value of the fixed cost of innovation.

[11] Unlike Navas and Sala (2015) we have assumed that the fixed costs of innovation and exporting are all in terms of the final good as in Nocco (2012). To investigate the implications of this assumption we have derived the results for a more general version of the model in which the process of entry also involves the use of intermediate inputs in a proportion λ (i.e. the fixed cost of entry is given by $f_E(P_M)^{\lambda}$). The results derived below coincide in qualitative terms with the ones derived in this more general context as long as $\lambda \neq \alpha$. As the tasks involved in exporting and innovation, are substantially different from the ones involved in the creation of a new variety, we believe that $\lambda \neq \alpha$ is the most relevant empirical case. Results relating to the more general case are available upon request.

2.1 Equilibrium

Solving the consumers' utility maximization problem, the demand function for variety j in the final good sector in each country is given by

$$C(j) = \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \mu I$$

where I represents the aggregate income of the country, $p(j)$ the price of variety j and $P_M = \left(\int_0^N p(l)^{1-\sigma} dl \right)^{\frac{1}{1-\sigma}}$ is the price index of the set of all the N differentiated varieties bought in the country, which includes both domestically produced and imported varieties.

To obtain the total demand function that each firm faces we need to specify the different components of it. Specifically, a firm can produce to satisfy the demand of both domestic and foreign consumers and firms. In general, the quantity produced in H by firm j is given by the following expression

$$\begin{aligned} q(j) &= C_D(j) + B_{HDN}(j) + B_{HDI}(j) + B_{HXN}(j) + B_{HXI}(j) + \\ &\quad + \epsilon \left(C_X(j) + B_{FDN}(j) + B_{FDI}(j) + B_{FXN}(j) + B_{FXI}(j) \right) \end{aligned}$$

where: $\epsilon = \tau$ if the firm exports and 0 otherwise; $C_D(j)$ and $C_X(j)$, respectively, denote the demand for variety j for domestic and foreign consumption; the variable $B_{vsm}(j)$ indicates the demand for variety j used as an intermediate input by firms producing in country $v = H, F$ for their domestic market (obtained when $s = D$) and to export to the other country (when $s = X$). Note that, within each case, the model distinguishes between the demand of the adopters of the most efficient technology (when $m = I$) and the non-adopters (when $m = N$) since the unit input requirements are different across the two cases. An analogous expression holds for the quantity produced by firm j producing in F.

A firm's demand for a variety used as an intermediate input is obtained by applying Shepard's lemma to its total cost function. In general terms, the total cost function of firm ι is given by

$$TC(\iota) = P_M^{\alpha} w^{1-\alpha} (f_D + \xi f_X + \zeta f_I + \gamma^{\zeta} a(\iota) q(\iota)) \tag{1}$$

where the variable ξ takes value one if the firm exports and zero otherwise, while ζ takes the value one if the firm innovates and zero otherwise. Recalling that the equilibrium wage is equal to one, the demand of firm ι for each variety j is, thus, given by $B_\iota(j)$

$$\begin{aligned} B_\iota(j) &= \frac{\partial TC(\iota)}{\partial p(j)} = \alpha \frac{p(j)^{-\sigma} P_M^{\alpha}}{P_M^{1-\sigma}} \left(f_D + \xi f_X + \zeta f_I + \gamma^{\zeta} a(\iota) q(\iota) \right) = \\ &= \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \alpha TC(\iota) \end{aligned}$$

Firms behave monopolistically and so the price that firm j charges in the domestic market is

$$p_{Dm}(j) = \left(\frac{\sigma}{\sigma - 1}\right) a(j)\gamma^{\zeta} P_M^{\alpha}$$

In the case in which firm j exports, the price that it sets abroad is $p_{Xm}(j) = \tau p_{Dm}(j)$. Thus, exporting firms set higher prices abroad and firms adopting the most efficient technology charge lower prices in both markets.

In each country the total value of the expenditure in the differentiated manufactured varieties E is given by the sum of the share of consumers' income, μI, and of the share of the total cost of production spent on intermediates, αTC, i.e., $E = \mu I + \alpha TC$. Equilibrium aggregate production for each variety j is, consequently, given by

$$q(j) = (1 + \xi\phi)\left(\frac{p_{Dm}(j)^{-\sigma}}{P_M^{1-\sigma}}\right) E$$

where $\phi \equiv \tau^{1-\sigma}$ denotes the freeness of trade. This parameter takes a value from zero to one and it becomes larger as trade is less costly. When the trade costs go to infinity, this parameter will go to zero, and this will be the autarkic case. When $\tau = 1$, there are no trade costs and international trade is free with $\phi = 1$.

A firm's operating profits with unit input requirement $a(j)$ are then given by

$$\pi(j) = (1 + \xi\phi)\Delta\gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma}$$

where $\Delta \equiv \left(\frac{E}{\sigma(P_M)^{(1-\alpha)(1-\sigma)}}\right)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$ captures aggregate variables and parameters affecting firms' demand. In the case when $\alpha > 0$, this term also includes parameters affecting its production cost levels. The term $\gamma^{\zeta(1-\sigma)}$ denotes a measure of the efficiency gains obtained by the firm when it adopts the most efficient technology (when $\zeta = 1$), with larger gains obtained for smaller values of γ.[12]

A firm with unit input requirement $a(j)$ decides to upgrade its state of technology when the benefits of adopting the new technology expressed in terms of larger operating profits overcome the costs of doing so, that is when

$$(1 + \xi\phi)\Delta\left(\gamma^{1-\sigma} - 1\right) a(j)^{1-\sigma} \geqslant f_I P_M^{\alpha}$$

with the sign of equality holding if the firm is indifferent between adopting or not adopting the new technology. Its unit input requirements, a_I, will be denoted as the adoption cutoff.

A firm decides to export if the operating profits obtained from the foreign market overcome the fixed cost of exporting, that is if

$$\phi\Delta\gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma} \geqslant f_X P_M^{\alpha}$$

[12]As smaller values of γ imply larger values of $\gamma^{(1-\sigma)} > 1$.

with the sign of equality holding if the firm is indifferent between exporting or not. The indifferent exporter is called the marginal exporter and its unit input requirement is denoted by a_X.

The firm that is indifferent between staying or leaving the market must satisfy the following condition

$$\Delta\gamma^{\zeta(1-\sigma)}a_D^{1-\sigma} = f_D P_M^{\alpha}$$

where a_D denotes its unit input requirements.

Note that if a firm finds it profitable to adopt the new technology just considering the domestic market ($\Delta(\gamma^{(1-\sigma)} - 1)a^{1-\sigma} \geqslant f_I P_M^{\alpha}$), then this firm must adopt the same technology if it decides to export. This implies that there cannot be simultaneously an equilibrium in which domestic firms adopt the most efficient technology and exporters rely on their original technology.

Depending on the parameter configurations, this model exhibits three different types of equilibria associated with different firm hierarchies regarding export and innovation activities.[13] We denote the respective variables associated with each equilibrium with the superscript $i = \{A, B, C\}$. In what follows let us focus our attention on equilibrium A, described below, as this is the equilibrium that turns out to be the relevant one according to the calibration exercise that we develop for the United States in the following Section 4.2. For the sake of completeness, we also report in the online appendix the relevant equilibrium conditions and expressions for the other two equilibria B and C, as they are, respectively, consistent with the types of rankings in Castellani and Zanfei (2007), where firms in the middle rank adopt the new technology but do not export, and in Lileeva and Trefler (2010), where firms either innovate and export or are non innovating domestic firms.

In equilibrium A firms will be sorted according to the following status: adopting exporters (the most productive ones), non adopting exporters with intermediate productivity levels and domestic firms (the least productive ones). In this equilibrium, technology adoption is a relatively expensive activity (compared to exporting) and only a subset of the most productive exporters are willing to upgrade the technology they use. This equilibrium occurs when the following condition holds

$$\frac{f_I}{(\gamma^{1-\sigma} - 1)}\left(\frac{\phi}{1+\phi}\right) > f_X > \phi f_D,$$

Specifically, in equilibrium A every firm that adopts is an exporter but only a subset of the most productive exporters adopt. Consequently, the conditions defining the three unit input cutoffs ranked as $a_I^A < a_X^A < a_D^A$ explained above, are given by

$$(1+\phi)\Delta^A(\gamma^{1-\sigma} - 1)\left(a_I^A\right)^{1-\sigma} = f_I\left(P_M^A\right)^{\alpha} \quad (2)$$

[13] See Navas and Sala (2015) for a complete characterization of all types of equilibria in a model without vertical linkages. The existence of vertical linkages does not alter the conditions determining the parameter configuration associated with each type of equilibria. This comes from the fact that external linkages affect both export and innovation activities in a similar way as they affect the production for the domestic market given that all fixed costs are using intermediates with the same intensity.

$$\phi\Delta^A \left(a_X^A\right)^{1-\sigma} = f_X \left(P_M^A\right)^{\alpha} \tag{3}$$

$$\Delta^A \left(a_D^A\right)^{1-\sigma} = f_D \left(P_M^A\right)^{\alpha} \tag{4}$$

A firm's decision to adopt the new technology, is affected by the presence of vertical linkages through two channels. The first one clearly affects a firm's demand through Δ^A. In a model without vertical linkages, total expenditure includes only domestic and foreign consumer expenditure. In this model, however, total expenditure is determined not only by consumer expenditure but also by each firm's demand for varieties used as intermediates. In principle, firms can rely on more incentives to adopt the most efficient technology since market size is larger. The existence of vertical linkages also has an impact on the firm's marginal costs of production, having an impact on global sales and operating profits through the price index P_M^A affecting Δ^A when $\alpha > 0$. This second channel, which is absent in a model without vertical linkages, also affects the cost of adoption. As the fixed costs of adoption involve the use of intermediates, any change in the cost of intermediates affects also the adoption cost.

Using (2) and (4), the proportion of surviving firms which adopt the most efficient technology is obtained as

$$\frac{N_I^A}{N_D^A} = \left(\frac{a_I^A}{a_D^A}\right)^{\kappa} = \left[\frac{(1+\phi)(\gamma^{1-\sigma}-1)f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{5}$$

where N_I^A and N_D^A are, respectively, the number of innovating and surviving firms.

Using (3) and (4) the proportion of surviving firms exporting is

$$\frac{N_X^A}{N_D^A} = \left(\frac{a_X^A}{a_D^A}\right)^{\kappa} = \left(\frac{\phi f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{6}$$

with N_X^A denoting the number of exporting firms.

Note that the presence of vertical linkages does not have an impact on the productivity distribution conditional on entry (i.e., the proportion of surviving firms adopting the most efficient technology or exporting is unchanged by the presence of vertical linkages). This is the result of both channels affecting symmetrically the variable profits of adopting, exporting and staying in the market. This can be observed by looking at conditions (5) and (6) and noting that neither Δ^A nor $\left(P_M^A\right)^{\alpha}$ are affecting these proportions.[14] However, the presence of vertical linkages has interesting implications for the survival productivity cut-off having a clear impact on the technology adoption cut-off and, ultimately, the average industry productivity and the welfare of the economy.

[14]Given that the expression for P_M^i is different across equilibria because it depends on θ^i defined in the following subsection, Δ^i changes across equilibria and it should therefore be changed accordingly.

2.2 Solution

Let us denote with E_T and L_T, respectively, the expenditure and labor devoted to the homogeneous good. An equilibrium in this economy is characterized by a vector of unit input cutoffs (a_I^i, a_X^i, a_D^i), and a vector of aggregate variables (P_M^i, E^i, N_D^i, N_X^i, N_I^i, N_E^i, E_T, L_T) that satisfy the specific equations associated with each equilibrium described above, the market clearing conditions for each good and the labor market and the Free Entry condition.

The conditions derived in each of the potential equilibria i, reveal that a_I^i, a_X^i can be expressed as a function of a_D^i. The value of a_D^i can be obtained as in the Melitz (2003) model using the Zero Profit condition (ZP) and the Free Entry condition (FE). The ZP condition is given by

$$\Delta^i \left(a_D^i\right)^{1-\sigma} = f_D \left(P_M^i\right)^{\alpha} \tag{7}$$

and the FE is given by

$$\bar{\pi}^i = \left(P_M^i\right)^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)}, \quad i = A, B, C.$$

where $\bar{\pi}^i$ represents average operating profits, $\theta^i \equiv \left[1 + \frac{G(a_I^i)}{G(a_D^i)} \frac{f_I}{f_D} + \frac{G(a_X^i)}{G(a_D^i)} \frac{f_X}{f_D}\right]$. The free entry condition states that the average expected operating profit of active producers must be equal to their expected fixed cost $\left(P_M^i\right)^{\alpha} f_D \theta^i$ - which is given by the sum of $\left(P_M^i\right)^{\alpha} f_D$, plus $\left(P_M^i\right)^{\alpha} f_I$ times the probability of being an adopter (conditional on it being a producer), plus $\left(P_M^i\right)^{\alpha} f_X$ times the probability of being an exporter (again, conditional on it being a producer) - plus the expected cost of developing a successful entrant, that is $f_E/G(a_D^i)$.[15]

Given that $\bar{\pi}^i = \bar{r}^i/\sigma$, where $\bar{r}^i$ represents the average revenues, substituting $\bar{r}^i = E^i/N_D$, we find that the free entry condition can be rewritten as

$$\frac{E^i}{\sigma N_D} = \left(P_M^i\right)^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)} \tag{8}$$

To solve the model, an expression for the aggregate price index P_M^i and one for total expenditure E^i must be obtained. Using their definitions, we obtain, respectively, the price index and the expenditure:

$$P_M^i = \left(\frac{\sigma}{\sigma - 1} a_D^i\right)^{\frac{1}{(1-\alpha)}} \left(\frac{\beta}{\beta - 1}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(N_D^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\theta^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{9}$$

$$E^i = \mu L + \alpha \left(P_M^i\right)^{\alpha} N_D^i f_D \left[1 + \frac{(\sigma - 1)\kappa}{\kappa - (\sigma - 1)}\right] \theta^i \tag{10}$$

[15] We follow Baldwin and Forslid (2006) and we consider a version of the Melitz (2003) model in which the dynamics of entry and exit stated as in Melitz (2003) is not explicitly analyzed. However it is important to note that this model of technology upgrading implies that the underlying productivity distribution (ex-ante productivity distribution) is different from the ex-post productivity distribution. Indeed, the ex-post productivity distribution will be the combination of two truncated Pareto-distributions with a hole in the middle since in steady state we should not observe any firm with an intermediate input requirement between γa_I and a_I. However, we believe that this should not have any implications for the stability of the productivity distribution as first, we have that the ex-post productivity distribution is fully determined by the ex-ante productivity distribution which is common knowledge across firms and second the ex-ante productivity distribution is not observed in the final results.

where $\beta \equiv \frac{\kappa}{\sigma-1} > 1$.[16] Conditions (7)-(10) characterize a system of equations in 4 endogenous variables (a_D^i, P_M^i, N_D^i and E^i). Solving the system for an interior solution, that is with $a_D^i < a_M$, we find that:

$$a_D^i = \left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} \tag{11}$$

$$P_M^i = \left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{-\frac{\kappa}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1}{\chi}} \tag{12}$$

$$N_D^i = \frac{\left(\frac{\beta-1}{\beta}\right)\left(\frac{\sigma}{\sigma-1}\right)^{(\sigma-1)}\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{(\sigma-1)\frac{\alpha+\kappa(1-\alpha)}{\chi}}}{\left[\frac{f_E}{f_D}(\beta-1)\,a_M^\kappa\right]^{\alpha\frac{\sigma-1}{\chi}}(\theta^i)^{\kappa\frac{\sigma-\alpha\sigma-1}{\chi}}} \tag{13}$$

where $\chi \equiv (\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma$ and $\delta_0 \equiv \alpha(\sigma-1) + \kappa\sigma(1-\alpha) > 0$.[17] In the online appendix we analyse the case of a corner solution where $a_D^i = a_M$ and the parameter range under which this solution exists.[18]

Note that χ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$, with $\alpha_1 \equiv \kappa/(\beta\sigma - 1) < 1$, while θ^i is given by:

$$\theta^i = \begin{cases} 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} & \text{if } i = A \\ 1 + \left(\phi\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta} & \text{if } i = B \\ 1 + \left[(1+\phi)\gamma^{1-\sigma}-1\right]^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta} & \text{if } i = C \end{cases} \tag{14}$$

Any change in trade costs (variable or fixed), the efficiency gains from technology adoption, γ, and its fixed cost have an impact on the survival unit input cut-off through the variable θ^i. Observing the equilibrium expression above one can conclude that θ^i can be interpreted as an indicator of trade openness and innovation opportunities generated by the levels of international integration and the conditions characterizing the innovation environment in the industry. Higher θ^i (i.e., lower variable trade costs, larger efficiency gains from technology adoption and/or lower fixed costs of exporting or technology adoption) has contrasting effects: from one side it expands the business opportunities of the most productive firms *(near the highest marginal cutoffs)*, but from the other side it intensifies competition for every firm toughening the conditions for surviving in the market. Eventually, this implies a more selective industry.

The analysis of (14) reveals that θ^i is not affected by the existence of vertical linkages. Vertical linkages affect the survival unit input cut-off and welfare through the price index P_M^i as discussed in the following section.

Finally, applying Shephard's lemma to (1), it is possible to derive the labor demand of firm ι, $L_m(\iota) = (1-\alpha)\,TC(\iota)$. Using the previous expression we obtain the total labor demand and employment in production,

[16] This condition is required to have the price index P_M^i converging to a positive value.

[17] The solution for E^i can be readily derived substituting P_M^i and N_D^i, respectively, from (12) and (13) into (10).

[18] This solution is achieved by obtaining P_M^i,N_D^i and E^i from expressions (8)-(10) making use of the labor market clearing condition.

	Welfare	Equilibrium A	
$0 < \alpha < \alpha_0$	$W \uparrow$	$a_D^A \downarrow$	$a_I^A \uparrow$
$\alpha_0 < \alpha < \alpha_1$	$W \uparrow$	$a_D^A \uparrow$	$a_I^A \uparrow$
$\alpha_1 < \alpha < \alpha_2^A$	$W \downarrow$	$a_D^A \downarrow$	$a_I^A \downarrow$
$\alpha_2^A < \alpha < 1$	$W \downarrow$	$a_D^A \downarrow$	Case 1 $a_I^A \downarrow$ Case 2 $a_I^A \uparrow$

Table 1: The effects of an increase in the freeness of trade on the productivity and innovation cutoffs for the case of an interior solution (i.e. $a_D < a_M$).

innovation and exporting activities in the manufacturing sector, that is $L_m = (1-\alpha)\left(\frac{\kappa\sigma-\sigma+1}{\kappa-\sigma+1}\right)\left(P_M^i\right)^{\alpha} N_D^i \theta^i f_D$. Hence, substituting in the expression obtained for L_m the solutions for P_M^i and N_D^i , respectively, from (12) and (13), implies that $L_m = (1-\alpha)\frac{(\kappa-1)\sigma+1}{\delta_0}\mu L > 0$.[19] Moreover, labor demand devoted to entry is given by $L_E = f_E N_E = \frac{(\sigma-1)\mu L}{\delta_0} > 0$, as $N_E = N_D\left(\frac{a_M}{a_D}\right)^{\kappa} = \frac{(\sigma-1)\mu L}{\delta_0 f_E}$, with L_E increasing in α from $L_E = \frac{\sigma-1}{\kappa\sigma}\mu L$ when $\alpha = 0$ to $L_E = \mu L$, in the limit, when $\alpha = 1$. Thus, labor devoted to the manufacturing sector including entry is given by $L_m + L_E = \mu L$. From the labor market clearing condition we have that

$$L_m + L_E + L_T = L$$

and, therefore, labour employed in the homogeneous sector, L_T, is given by $L_T = E_T = (1-\mu)L$.

3 The impact of trade liberalization on unit input cutoffs.

Table 1 summarizes and represents with arrows the effects of trade liberalization, that consists of a reduction in variable trade costs (and an increase in ϕ), on the different unit input cutoffs and on welfare (this latter effect will be discussed in next Section).[20,21] The values for α_0, α_1 and α_2^A are given respectively by $\alpha_0 \equiv \frac{\sigma-1}{\sigma}$, $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$, and $\alpha_2^A \equiv \frac{\kappa(\theta^A-(1+\phi))}{\phi(1-\beta\sigma)+(\phi+\beta\sigma)(\theta^A-1)}$.[22,23]

The baseline case, that is the one without vertical linkages ($\alpha = 0$), has the same qualitative effects as those described by the first row in table 1 with low intensity of vertical linkages ($0 < \alpha < \alpha_0$), the predominant case in

[19] A sufficient condition for $L_m < L$ to hold is that $L_m < L$ when $\alpha = 0$ and that L_m decreases with α in the range $\alpha \in (0,1)$. Indeed, it can be readily verified that $L_m = L\mu\left(\frac{\kappa\sigma}{(\sigma-1)} - 1\right)/\frac{\kappa\sigma}{(\sigma-1)}$ when $\alpha = 0$ and, in this case, $L_m < L$ as $\mu < 1 < \frac{\kappa\sigma}{(\sigma-1)}/\left(\frac{\kappa\sigma}{(\sigma-1)} - 1\right)$. Then, for $\alpha \in (0,1)$, the derivative $\frac{\partial L_m}{\partial \alpha} = L\mu(\sigma-1)^2\frac{1-\beta\sigma}{(\alpha-\alpha\sigma-\kappa\sigma+\alpha\kappa\sigma)^2} < 0$ because $\beta\sigma > 1$. This implies not only that $L_m < L$ but also that L_m is decreasing with the strength of vertical linkages α when $\alpha \in (0,1)$. Finally, notice that $L_m = 0$ when, in the limit, $\alpha = 1$, that is when only intermediate inputs are used to produce the differentiated goods.

[20] Note that Table 1 contains only the effects of trade liberalization on selection, innovation and welfare for equilibrium A. Table 3 in the online appendix includes a complete description of the effects of trade liberalization in all the equilibria.

[21] Note that the effects of trade liberalization on the number of domestic varieties N_D^i is the same as those on the domestic cutoff a_D^i because the sign of the exponents of in the solutions for a_D^i and N_D^i, respectively, in expressions(11) and (13) are the same.

[22] Note that our results do vary substantially not only with the degree of vertical linkages but also with the type of equilibrium. A full discussion of the results and the intuition behind them for equilibria B and C is provided in the online appendix.

[23] We show how the values of α_2^i can be determined in the online appendix where we derive also the signs of the derivatives $\frac{\partial a_D^i}{\partial \phi}$ and $\frac{\partial a_I^i}{\partial \phi}$ used to establish the effects of increases in ϕ on the cutoffs.

our empirical implementation in Section 4. Specifically, in the case without vertical linkages, trade liberalization increases the proportion of surviving firms that undertake technology adoption in equilibrium A.

When the economy experiences a process of trade liberalization, the benefits of technology adoption are affected by two opposing forces: Firstly, firms enjoy an expansion in their business opportunities due to better access to the foreign market. This is reflected by the multiplicative term, $(1+\phi)$, in condition (2) which is positively associated with ϕ, the freeness of trade. This is the traditional market size effect found in innovation models.[24] As the market size increases, the firm is able to apply the reduction in marginal costs associated with the new technology across more production units increasing the benefits of adoption.

However, together with the increase in market size, an increase in ϕ also reduces the potential sales of the firm in each market. This is because as trade costs are reduced, this improves the access of foreign firms into the domestic market. As each firm needs to compete with more firms within each market for consumers, their sales, ceteris paribus, are reduced in each market. This effect is captured in condition (2) by the element Δ^A that is affected by $\left(P_M^A\right)^{1-\sigma}$ which depends positively on θ^A. An increase in ϕ increases θ^A reducing the firms' market share within each market. The net effect will be the result of these two opposing forces which in this equilibrium turns out to be positive, therefore increasing the mass of firms that technology upgrade. Equation (4) helps us to conclude that the latter effect reduces the proportion of firms that survive in the market.

When $0 < \alpha < \alpha_0$, in addition to the forces discussed above, trade liberalization also reduces the cost of surviving since this depends on imported intermediate inputs which become cheaper. This latter effect, however, is not strong enough to dominate the forces discussed above and trade liberalization will have qualitatively the same impact on surviving and innovation as in the case of $\alpha = \alpha_0$. These results are, nevertheless, challenged when the intensity of vertical linkages is middle-low ($\alpha_0 < \alpha < \alpha_1$). When the intensity of vertical linkages is middle-low, trade liberalization still reduces the aggregate price index. In this case, vertical linkages are strong enough to have a positive impact on survival and technology adoption: the initial reduction in firms' domestic sales is dominated by the effect that the reduction in the aggregate price index has on the firm's production costs and the demand effect. Because production becomes cheaper, firms' operating profits increase and this will allow the least productive firms to survive: this clearly increases the survival unit input cut-off. [25] In addition, the reduction in the costs of production and adoption will increase the benefits of technology adoption. This will exert a double impact on innovation: on the one hand a larger proportion of surviving firms become innovators in equilibrium A and, on the other hand, trade liberalization increases the cutoff of firms surviving in the market. Overall the proportion of entrants adopting the most efficient technology increases after trade liberalization in all equilibria. This result is novel and thus not present in a model without vertical linkages.

[24] See Grossman and Helpman (1991), Acemoglu (2009), Peretto (2015), Impullitti and Licandro (2018), and more recently Aghion et al (2019) among others.

[25] In the particular case of $\alpha = \alpha_0$ trade liberalization has no impact on firm's survival.

Although empirically not relevant in our calibration exercise discussed in the next section, for the sake of completeness, we note that when the degree of vertical linkages is middle high $\alpha_1 < \alpha < \alpha_2$ (or high with $\alpha_1 < \alpha < 1$ if $\alpha_2 > 1$) the opposite happens. Indeed, in this case, not only does trade liberalization have a negative impact on the unit input cut-off (which makes firms' survival more difficult), but also the effect is strong enough to reduce the cutoff of firms that adopt the most efficient technology. Condition (12) reveals that trade liberalization increases the aggregate price index. Condition (9) could help us to grasp intuition behind this result. This condition reveals that, holding constant a_D^i and N_D^i, trade liberalization increases θ^i and reduces the level of the aggregate price index. This reduces production costs and it increases the demand for intermediate inputs, pushing upwards their prices. This will have an impact on the survival cutoff a_D^i and the surviving number of firms N_D^i. The overall general equilibrium effect taking into account the effects in all variables is an increase in the equilibrium price index. The increase in the equilibrium price index increases the production costs and the fixed costs of innovation, export and survival, inducing a lower proportion of firms to survive and adopt the most efficient technology. While the increase in the aggregate price index as a result of general equilibrium effects is already present in Nocco (2012), the decrease in the proportion of firms technology upgrading in all equilibria is a novel result that is not present in a model without vertical linkages.[26] Finally, note that in the cases in which α_2^i exists and α is very high so that $\alpha_2^i < \alpha < 1$, the impact of trade liberalization on technology adoption depends on the parameters of the model. [27]

4 Effects on Welfare

The previous section shows that vertical linkages shape the effect that trade liberalization has on technology adoption. In this section, we show first, how the inclusion of the possibility of firms to technology upgrade and the presence of vertical linkages shape the effects of trade liberalization on welfare and how their interaction magnifies their joint effects. Then, to illustrate the quantitative importance of these two dimensions for the impact of trade liberalization on welfare, we calibrate the model for the case of the US economy. Our main conclusion is that both channels and their interaction exacerbate the impact of trade liberalization on welfare and the ultimate effect of trade liberalization on welfare depends crucially on the strength of vertical linkages, α. The calibration results suggest that these effects are also quantitatively relevant.

[26] Navas and Sala (2015) analyse the impact of trade on technology adoption in a similar framework without vertical linkages. In that model, in equilibria A and C trade liberalization will always increase the proportion of adopting firms in the economy.

[27] We refer the interested reader to the online appendix for a detailed analysis of this case stating here that, in general, the condition to rule out the case in which $\alpha_2^A < 1$ and $\alpha_2^C < 1$ exist is that the requirement for adopting a more productive technology f_I is not too large for the given values of the other parameters. Specifically, the condition required to rule out the case in which $\alpha_2^A < 1$ and $\alpha_2^C < 1$ exist is $f_I^{\beta-1} < \frac{f_D^{\beta-1}\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}\phi}{\phi(\beta-1)-\Omega(\beta+\phi)}$ in equilibrium A, and $(f_X + f_I)^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)} f_D^{\beta-1}$ in equilibrium C.

4.1 Theoretical results

Substituting the optimal values for C_T and C_M, the aggregate indirect utility function can be expressed as

$$U = \frac{L}{(P_M^i)^{\mu}} \tag{15}$$

Thus, as it is standard in the literature, to evaluate the impact of trade liberalization on welfare it is useful to see how the aggregate price index changes with respect to changes in trade policy.[28]

The following proposition confirms that trade liberalization has a positive effect on welfare if and only if the degree of vertical linkages are not too strong, a result already present in Nocco (2012).[29]

Proposition 1 *In an interior equilibrium ($a_D < a_M$), trade liberalization has a positive (negative) impact on welfare when $0 \leq \alpha < \alpha_1$ ($\alpha > \alpha_1$). In a corner solution ($a_D = a_M$), trade liberalization has a positive impact on welfare independently of the value of α.*

Proof. *See online appendix.* ■

For the case of an interior equilibrium, our analysis reveals that the negative impact of trade liberalization on welfare, present in Nocco (2012),[30] is robust to the inclusion of technology upgrading. Specifically, when $\alpha > \alpha_1$, the price index of the composite good increases as a result of trade liberalization even in the case where firms are allowed to technology upgrade. The main reason behind this result lies in the positive effect that trade liberalization has on the demand for the composite good, because firms' willingness to become exporters increases, and this increases the costs of surviving and entry, reducing the mass of varieties produced in equilibrium. In the context of the present model these forces could be stronger for some cases since, apart from the effect on the mass of varieties produced, the increase in the cost of intermediates can have, in certain cases, a negative impact on technology upgrading.[31]

For the case of the corner solution introduced in section 2, instead, we have that trade liberalization reduces the price index by reducing the price of varieties imported, without affecting either the mass of varieties or the selection cut-off.

[28] As the utility function is Cobb Douglas, the standard expression for the aggregate indirect utility function is $U = \frac{I}{(P_M^i)^{\mu}(P_T^i)^{1-\mu}}$, where I represents the economy's aggregate income and $P_T = 1$ since good T is the numeraire. Note that in this model the free entry condition guarantees that in equilibrium the firms' aggregate profits are zero and so the total income is given by wL where $w = 1$.

[29] The subsequent analysis has been conducted under the assumption that trade liberalization episodes are small enough so that the economy does not switch from an interior to a corner solution or vice-versa. This in principle is not implausible as the notion of derivative suggests infinitesimal changes in the variable of interest. Nevertheless the calibration exercise presented in section 4.2 does not impose this restriction and we actually observe no switch due to trade liberalization.

[30] Note that the analysis in Nocco (2012) focuses on the case of an interior solution when selection is binding

[31] Even though the role played by the presence of the homogeneous good is non negligible, because it allows the nominal wage to be set to one in both countries and to have intersectoral mobility of labor, the choice of the numeraire and the presence of the homogeneous good have no impact on the emergence of welfare losses. Indeed, the choice of the numeraire simply sets the value of the wage to one. Should the wage not be equal to one because of a different choice of the numeraire, or in the case of the size of the homogeneous good sector being very small (with μ close to 1), if an increase in the demand for intermediates were to drive up the demand for labor and raise the nominal wage, the increase in the nominal wage would be smaller than the rise in aggregate price index for high values of α, still resulting in a decrease in the real wage.

While the effects of trade liberalization on welfare are qualitatively the same as in Nocco (2012), this cannot be said from a quantitative point of view. The following two propositions state that the inclusion of both technology upgrading and vertical linkages magnify the effects of trade liberalization on welfare.

Proposition 2 *The presence of vertical linkages strenghtens the impact that trade liberalization has on welfare.*

Proof. *See online appendix.* ■

Indeed, the existence of vertical linkages generates a multiplier effect on welfare. This multiplier effect comes through the direct impact of trade liberalization on the price index, which declines with low and middle-low vertical linkages ($0 < \alpha < \alpha_1$), and the specific effects of trade liberalization on selection and technology upgrading that vary according to the strength of vertical linkages and type of equilibria. In a standard model of trade with technology upgrading (i.e., Navas and Sala (2015)), trade liberalization has an impact on the price of the final composite good, through, first, the increase in the mass of varieties consumed, N, (i.e., the variety channel), second, through the reduction in the cost per imported variety (i.e., the cost-reduction channel), and third through the effects that trade has on selection and technology adoption. The presence of vertical linkages will magnify the last two channels. As the final goods are used as intermediate inputs, the reduction in trade costs will further reduce the final good prices through the reduction in the costs of intermediates.[32] Apart from the direct impact on the cost of intermediate input varieties, the reduction of trade costs will further affect the fixed cost of operation which will have implications for selection. It is important to note that, even though the results stated in Proposition 2 have already been highlighted in the literature that explores the effects of trade liberalization on welfare under the presence of intermediate inputs (i.e. Caliendo and Parro (2015)), the effects implied by our model will be different from a quantitative point of view. This is due to the fact that the technology upgrading channel, which was absent in the previous literature, is shaping those effects, as discussed below in proposition 4.

The main novelty of our paper lies on the complementary effect of technology adoption and vertical linkages in shaping the impact of trade liberalization on welfare. The first proposition below shows how technology adoption magnifies the effects of trade liberalization on welfare, while the second one explores more in-depth the complementarity observed between these two channels, vertical linkages and technology adoption.

Proposition 3 *The welfare effects of trade liberalization are magnified when technology upgrading is possible.*
Proof. *See online appendix* ■

Expression (14) shows that θ^i is larger in the technology adoption case. Note that the corresponding θ^i in the case without vertical linkages (as in Nocco (2012)) is $\tilde{\theta} = 1 + \phi^\beta \left(\frac{f_X}{f_D}\right)^{1-\beta}$. In addition, the effects of trade liberalization on welfare depend on how trade costs affect θ^i, and $\tilde{\theta}$. The main reason behind this result lies

[32] In a similar way, the increase in the costs of intermediates that comes because of the increase in the price index generated by trade liberalization when $\alpha > \alpha_1$ will lead to magnification effects on welfare.

in the fact that for the case of the equilibrium A, trade liberalization promotes technology upgrading and this, subsequently, reduces the price index, having a larger impact on welfare when $\alpha < \alpha_1$.[33] This increases welfare.

We can go further and show that the gains from trade liberalization increase with the strength of vertical linkages and the efficiency gains derived from technology upgrading (i.e. $\frac{\partial\left(\frac{\frac{\partial U}{\partial \phi}}{U}\right)}{\partial \alpha} > 0$, $\frac{\partial\left(\frac{\frac{\partial U}{\partial \phi}}{U}\right)}{\partial \gamma^{1-\sigma}} > 0$) for equilibrium A.[34] More importantly, proposition 4 establishes that each of these factors, vertical linkages and technology upgrading, reinforce each other, magnifying the initial impact of trade liberalization on welfare.

Proposition 4 *The impact of vertical linkages on the effects of trade liberalization on welfare is magnified by the possibility of firms' to technology upgrade and vice versa the impact of technology upgrading on welfare is magnified by the presence of vertical linkages* $\left(i.e. \frac{\partial^2\left(\frac{\frac{\partial U}{\partial \phi}}{U}\right)}{\partial \alpha \partial \gamma^{(1-\sigma)}} = \frac{\partial^2\left(\frac{\frac{\partial U}{\partial \phi}}{U}\right)}{\partial \gamma^{(1-\sigma)} \partial \alpha} > 0\right)$

Proof. See online appendix ■

The last proposition suggests that the effect that vertical linkages have on technology upgrading and the effect that technology upgrading has on the demand of intermediate inputs play a role in the ultimate impact of trade liberalization on welfare. The analysis of the implications of the interplay between these two channels on the effect of trade liberalization on welfare was absent in the previous literature and constitutes one of the main contributions of the present work. As the degree of vertical linkages increases, the reduction in the cost of intermediates due to trade liberalization (for the case of $\alpha < \alpha_1$) enlarges the firms' profits from technology upgrading. This encourages more firms to technology upgrade, further reducing prices and increasing welfare.

To better understand how the ultimate effects of trade liberalization on welfare operate, and to introduce the calibration exercise of the next section, one should also consider the impact that trade liberalization has on the average productivity $(\tilde{a})^{1-\sigma}$ because this effect contributes to determine the change in the price index. Following Melitz (2003) and focusing on equilibrium A, the expression for the average productivity is given by

$$\begin{aligned}\left(\tilde{a}^A\right)^{1-\sigma} = {} & \frac{N_D^A}{N_D^A + N_X^A}[\gamma^{1-\sigma}\int_0^{a_I^A} a^{1-\sigma} g_D(a)da + \int_{a_I^A}^{a_D^A} a^{1-\sigma} g_D(a)da + \\ & \left(\frac{a_X^A}{a_D^A}\right)^{\kappa} \phi \left(\gamma^{1-\sigma}\int_0^{a_I^A} a^{1-\sigma} g_X(a)da + \int_{a_I^A}^{a_X^A} a^{1-\sigma} g_X(a)da\right)]\end{aligned}$$

where $g_D(a) = \frac{ka^{k-1}}{(a_D)^k}$ and $g_X(a) = \frac{ka^{k-1}}{(a_X)^k}$. The expression above is a weighted average productivity of all firms (domestic and foreign) competing in a single country where the productivity of exporters is adjusted by the trade

[33] A complete proof of this proposition for all type of equilibria is available in the online appendix.

[34] Note that the result that $\frac{\partial\left(\frac{\frac{\partial U}{\partial \phi}}{U}\right)}{\partial \alpha} > 0$ is valid for all equilibria. See the online appendix for these derivations.

costs τ. Computing the expression above for the case of the Pareto-distribution and rearranging terms we find that

$$\left(\tilde{a}^A\right)^{(1-\sigma)} = \frac{\left(\frac{\beta}{\beta-1}\right)\theta^A}{1+\frac{N_X}{N_D}}\left(a_D^A\right)^{(1-\sigma)} \tag{16}$$

where θ^A can be rewritten as $\theta^A = 1 + \frac{N_I}{N_D}\frac{f_I}{f_D} + \frac{N_X}{N_D}\frac{f_X}{f_D}$. From (16), we establish that, ceteris paribus, the average productivity is increasing when the innovation cut-off, a_I^A, increases as the proportion of firms technology upgrading increases and consequently θ^A increases. Moreover, ceteris paribus, the average productivity is increasing when a_D^A decreases, (therefore selection toughens). Furthermore, the average productivity also changes, ceteris paribus, when $\frac{N_X}{N_D}$ changes. Since the impact of trade liberalization on both cut-offs, a_I^A and a_D^A, vary also with the level of α, the effects on the average productivity will be different for different degree of vertical linkages. Specifically, when vertical linkages are weak, $(0 < \alpha < \alpha_0)$, trade liberalization reinforces selection (decreases a_D^A) and encourages more firms to technology upgrade (increases θ^A). Both channels contribute to an increase in the average productivity and, consequently, to a decrease in the aggregate price index.[35] When vertical linkages are intermediate-low $(\alpha_0 < \alpha < \alpha_1)$ the interaction between selection and technology adoption becomes more complex as trade liberalization softens selection but boosts technology adoption. It is also important to note that changes in $\frac{N_I}{N_D}$ and $\frac{N_X}{N_D}$ capture the direct impact of trade liberalization on both innovation and exporting keeping constant a_D^A.

Taking logs and differentiating expression (16) with respect to ϕ we find that

$$\underbrace{\frac{\frac{\partial\left(\tilde{a}^A\right)^{(1-\sigma)}}{\partial\phi}}{\left(\tilde{a}^A\right)^{(1-\sigma)}}}_{\text{Average Productivity Growth}} = \underbrace{\frac{\frac{\partial\left(a_D^A\right)^{(1-\sigma)}}{\partial\phi}}{\left(a_D^A\right)^{(1-\sigma)}}}_{\text{Selection}} + \underbrace{\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A} - \frac{\frac{\partial\left(1+\frac{N_X}{N_D}\right)}{\partial\phi}}{1+\frac{N_X}{N_D}}}_{\text{Innovation and Exporting}} \tag{17}$$

Note that the average productivity growth as a consequence of trade liberalization can be decomposed in two main elements: The first one corresponds to the first element of the right hand side of the expression and it is the impact of trade liberalization on selection. The second one corresponds to the other two addends in the right hand side of the expression. These, which capture the direct impact of trade liberalization on both innovation and exporting, are intertwined and they cannot be easily decomposed, as θ^A is affected both by innovation and exporting. We label them as "Innovation and Exporting" and they are invariant to changes in α. When $\alpha_0 < \alpha < \alpha_1$, selection is softened and the selection channel is negative, as discussed in section 3. We show in the online appendix that we obtain a threshold level of vertical linkages α^{**}, such that, when $\alpha_0 < \alpha < \alpha^{**}$, the average productivity growth is positive as the change in the "Innovation and Exporting" more than compensate the negative effect of selection, and we continue to observe a decline in the aggregate price index.[36] Instead, when $\alpha^{**} < \alpha < \alpha_1$, the impact of selection

[35] See the online appendix for a computation of the average productivity of the industry $\left(\left(\tilde{a}^A\right)^{(1-\sigma)}\right)$ and how this varies depending on the degree of vertical linkages. Changes in the average productivity are at the heart of the intuition provided below for the case of $\alpha_0 < \alpha < \alpha_1$.

[36] See the online appendix for the derivation of α^{**}, where it is shown how to derive it and that $\alpha^{**} \in (\alpha_0, \alpha^*)$ with $\alpha^* \equiv \frac{(\sigma-1)^2+(\sigma-1)\kappa}{(\sigma-1)^2+\sigma\kappa} < \alpha_1$.

more than compensate the impact of innovation and exporting, and the average productivity decreases.[37] In this latter case, the effect on selection, which is softened, dominates, although the overall effect taking into account the variety change, the cost-reduction and the technology adoption channel is still able to reduce the aggregate price index and to increase welfare.

A similar decomposition can be obtained for welfare.[38] More precisely,

$$\frac{\frac{\partial U}{\partial \phi}}{U} = \underbrace{\frac{-\mu\kappa}{(1-\alpha)(1-\sigma)^2}\frac{\frac{\partial\left(a_D^i\right)^{1-\sigma}}{\partial\phi}}{\left(a_D^i\right)^{1-\sigma}}}_{\text{Number of domestic varieties}} + \underbrace{\frac{\mu}{(1-\alpha)(\sigma-1)}\frac{\frac{\partial\left(a_D^i\right)^{1-\sigma}}{\partial\phi}}{\left(a_D^i\right)^{1-\sigma}}}_{\text{Selection}} + \underbrace{\frac{\mu}{(\sigma-1)(1-\alpha)}\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}}_{\text{Theta}} \tag{18}$$

From equation (18), we observe that the impact of trade liberalization on welfare can be decomposed in three main channels. The first one is related to the impact on the number of domestic varieties, the second one is related to the impact of trade liberalization on selection, while the final element, theta, captures, similarly to the one above, the impact of trade liberalization on innovation and exporting activities keeping constant selection. The sign of the first two channels vary with the level of α and they go in the opposite direction (i.e. the number of domestic varieties produced falls (increases) when selection is toughened (softened)), while the third channel is always positive. Interestingly, as vertical linkages become stronger the size of each of these effects is magnified.[39]

This section has shown that the inclusion of both vertical linkages and the possibility of firms to upgrade technology magnifies the effects of trade liberalization on welfare. It has also shown that under certain conditions trade liberalization may have a negative impact on welfare. The next section reveals that the inclusion of these two new channels has also important quantitative implications.

4.2 Simulations

In the previous paragraphs the magnification effect of vertical linkages and technology upgrading in the impact of trade liberalization on welfare has been established theoretically. An important question to address is to what extent these effects are quantitatively relevant. In this subsection, this question is tackled by considering the effects of trade liberalization on welfare and other variables by examining the impact of a reduction in bilateral iceberg-type trade costs with a symmetric country from 25% to 15% (i.e. from $\tau = 1.25$ to $\tau = 1.15$).[40]

To undertake these exercises, a calibration of our model for the case of the US economy trading with an identical rest-of-the-world economy has been performed. The parameters regarding the elasticity of substitution, the shape parameter of the Pareto distribution, the fixed cost of operation and the fixed cost of entry follow the paper

[37] We show in the online appendix that these results for $\alpha^* < \alpha < \alpha_1$ is general (i.e. it holds for all types of equilibria).

[38] A detailed analysis of this decomposition can be found in the online appendix.

[39] This can be seen by the fact that each of the three channels is multiplied by the term $\frac{1}{1-\alpha}$ which increases with the level of α.

[40] Anderson and Van Wincoop (2004) suggest that, on average, industrialized countries face total trade costs of 21% (tax equivalent). Bernard, Jensen and Schott (2006) suggest that, for the case of the US manufacturing sector, the average trade costs is 8%. The figures provided consider the impact of trade liberalization on welfare from a moderate level of trade costs to a rough average of these two measures.

Parameter	Source
$\sigma = 4$	Melitz and Redding (2015)
$\kappa = 4.25$	Melitz and Redding (2015)
$a_M = 5$	Bernard, Redding and Schott (2007)
$f_E = 1$	Melitz and Redding (2015)
$f_D = 1$	Melitz and Redding (2015)
$L = 156$	Bureau of Labour Statistics. Average (2013-15)
$f_X = 1.5406$	Calibrated Internally
$f_I = 4.6847$	Calibrated Internally
$\gamma = 0.9131$	Calibrated Internally
$\mu = 0.66$	Impullitti and Licandro (2018)

Table 2: Parameter values

by Melitz and Redding (2015) which evaluates the gains from trade liberalization in a calibrated monopolistic competition model for the US economy with no technology upgrading or input-output structure. This implies the values for $\sigma = 4$, $\kappa = 4.25$, $f_E = 1$, $f_D = 1$.[41]

For the maximum level of unit input requirements we have followed Bernard, Redding and Schott (2007) and set the unit input requirements upper bound to $a_M = 5$.[42] For the population size we have included the average size of the US labor force during the period 2013-2015 expressed in millions obtained from the Bureau of Labour Statistics and, therefore, $L = 156$. For the manufacturing expenditure share, we follow Impullitti and Licandro (2018) and we set $\mu = 0.66$.[43] The fixed cost of exporting is calibrated internally, as explained in the following paragraphs, using the share of manufacturing firms that export in the US which is 21% as in Bernard, Eaton, Jensen and Kortum (2003).

In order to calibrate the value of the parameters related to innovation, f_I and $\gamma^{1-\sigma}$ we have obtained data for the average R&D intensity of US firms in 2014 (3.5%), and the percentage of firms reporting positive R&D expenditures between 2013-2015 obtained from the Business R&D Innovation Survey (BRDIS) provided by the National Science Foundation (3.9%). In particular, note that the average R&D intensity, defined as the total R&D expenditures over sales in the industry, in the theoretical model in equilibrium A is given by

$$R\&D_{int} = \int_0^{a_I} \left(\frac{f_I \left(P_M\right)^{\alpha}}{r(a)} \right) g_I(a) da = \frac{\gamma^{1-\sigma} - 1}{\sigma \gamma^{1-\sigma}} \frac{\kappa}{\sigma + \kappa - 1}$$

where $g_I(a) = \frac{k a^{k-1}}{(a_I)^k}$, and, therefore, with the data on R&D intensity and the values reported above we obtain

[41] Melitz and Redding (2015) use plant level data for the US reported in Bernard, Eaton, Jensen and Kortum (2003) to pin down the parameter governing the elasticity of substitution to $\sigma = 4$. Simonovska and Waugh (2010) consider a plausible range for the elasticity of substitution of $\sigma = 4.10 - 4.27$ and Costinot and Rodriguez-Clare (2010) consider a value $\sigma = 5$. A comparison of the main results provided in the online appendix reveal little variation across these alternative parameter values.

[42] Since Bernard, Redding and Schott (2007) have expressed their productivity distribution in terms of productivity rather than unit input requirements, we set $a_M = 5$ (the maximum value for the unit input requirements) which corresponds to the minimum level of productivity, φ, set in that paper (i.e. $\varphi = 0.2$). Provided that the economy is in an interior solution, the welfare response to trade liberalization is invariant to this parameter.

[43] Impullitti and Licandro (2018) set this parameter following the work by Rauch (1999) which classifies the goods by the degree of differentiation and it arrives to an expenditure share of differentiated goods over total manufactures in the US that goes from 0.64 and 0.67.

$\gamma = 0.9131$. The value of f_I is determined by using the data on γ combined with the percentage of firms reporting positive R&D, which, specifically, in the case of equilibrium A corresponds to the proportion of technology upgrading firms A, that is $\frac{N_I^A}{N_D^A} = \left(\frac{a_I^A}{a_D^A}\right)^{\kappa} = \left[\frac{(1+\phi)(\gamma^{1-\sigma}-1)f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}}$. The values of f_I and f_X have been calibrated using a value for the trade costs equal to the initial value of our exercise ($\tau = 1.25$), and they correspond respectively to $f_I = 4.6847$ and $f_X = 1.5406$.[44]

Given these parameter values, our calibration exercise confirms that the economy is characterized by an equilibrium of type A,[45] with thresholds for the middle-low level of vertical linkages and middle-high level of vertical linkages given by respectively $\alpha_0 = 0.75$ and $\alpha_1 = 0.91$. Tables 1 and 2 in the online appendix provide an idea of the empirical relevance of these thresholds. For the case of 2014, the average intermediate input share in the US is 48% (i.e. $\alpha = 0.48$) slightly lower than the average across countries which is 54% (i.e. $\alpha = 0.54$) with a standard deviation of 0.18. China exhibits the highest value of the sample (62%). Therefore, when talking about the representative sector case, all the economies would be placed below the relevant thresholds.[46] However, a quick inspection at the industry average reveals that a certain amount of manufacturing industries exhibit an average intensity close to or above α_0. Thus, from a sectoral point of view, the α_0 threshold becomes more relevant for the case of several country-industry pair experiences, although the percentage of such cases is relatively low. Therefore, we should consider these as potential theoretical possibilities with current limited empirical application given the available data.[47]

INSERT FIGURE 1 HERE

The following figures show the impact of trade liberalization on several variables for a wide range of vertical intensities, α. Specifically, in our discussion, we will comment on the impact of trade liberalization in different scenarios to illustrate the quantitative importance of our results and the various channels we identify. Our benchmark case will be the average intermediate input intensity for the US, 48%. Throughout the discussion, we will compare those results to the ones generated in an identical economy without vertical linkages, $\alpha = 0$. Moreover, to better illustrate how small differences in the strength of vertical linkages could have important quantitative implications for our relevant variables, we will compare the case of the US with that of an identical economy that has an average intermediate input intensity equal to China's, $\alpha = 0.62$. Finally in order to illustrate the quantitative properties

[44] Robustness checks for alternative values of these statistics that determine the values of $\gamma^{1-\sigma}$ and f_I, and other parameters, are provided in the online appendix.

[45] The seminal paper by Bustos (2011) found empirical relevance for this type of equilibrium in her study of technology upgrading following the MERCOSUR bilateral trade liberalization between Brazil and Argentina in the early 1990s. However, we have explored the quantitative implications of trade liberalization in the other two equilibria mentioned above. In equilibrium B the contribution of these two channels towards the impact of trade liberalization on welfare is slightly reduced from a quantitative point of view. These results are available upon request.

[46] Provided that the parameters affecting those thresholds are identical to the respective ones for the US economy.

[47] Looking at the original database which contains both manufacturing and non-manufacturing industries we find the percentile 90 to be 0.7367. At least 10% of the observations will have a value of α which is larger than 0.73. Indeed, Broda and Weinstein (2006) documents that the elasticity of substitution varies substantially across industries. Small variations in the elasticity of substitutions can, therefore, generate values of α_0 that would imply that a larger amount of industries could exhibit an intermediate input intensity above this threshold. For example, for the value of $\sigma = 3.0$ the implied value for α_0 is 0.66. Table 2 in the online appendix documents that 11 out 56 industries exhibit an average value of α that overcomes this threshold.

of the model for intense vertical linkages the results for $\alpha = 0.76$ and $\alpha = 0.85$ will be discussed. Both values of α are representative values for middle-low vertical linkages, although the effects of trade liberalization on the average productivity will be substantially different in the latter (since this value is above α^{**}).

Figure 1 shows the percentage change in the probability of surviving in the industry and the probability of innovation due to trade liberalization (unconditional).[48] As pointed out in the section above, when the vertical linkages are low (that is $\alpha < 0.75$), trade liberalization reinforces selection, worsening the probability of survival. In particular, for the case of no vertical linkages (i.e. $\alpha = 0$), the probability of surviving in the industry goes from 27.66% when $\tau = 1.25$ to 24.96% when $\tau = 1.15$. The latter constitutes a fall in that probability of 9.76%, as reported in the figure. It is interesting to point out that, provided that vertical linkages are not middle high (that is $\alpha < 0.91$), which is unlikely to occur, as vertical linkages become more important (that is, α increases) selection is softened and trade liberalization has a smaller percentage impact on selection. For the case of the US, trade liberalization worsens the probability of survival by only 7.52% which is 23% lower than the implied impact should vertical linkages had not been considered. This is because the cost-reduction effect yielded by vertical linkages allows more firms to survive, as discussed in section 3. When vertical linkages are middle-low with $0.75 < \alpha < 0.91$, trade liberalization softens selection. More precisely, trade liberalization could increase the probability of surviving by 0.83% when $\alpha = 0.76$ and 22.81% when $\alpha = 0.85$. The latter shows that the increase is indeed very sharp for this range of vertical linkages intensities.

Moreover, Figure 1 shows that trade liberalization boosts technology adoption in this equilibrium as long as vertical linkages are not middle-high and that the impact of trade liberalization on technology adoption becomes larger when the strength of vertical linkages increases. Specifically, when vertical linkages are not present ($\alpha = 0$) the increase in the probability to innovate as a consequence of trade liberalization is only 2.78%. However, when $\alpha = 0.48$, the average intermediate input share of the US, the increase is almost double and it becomes equal to 5.33%. In the case of middle-low vertical linkages the increase becomes considerably high (i.e., 14.84% when $\alpha = 0.76$ and 39.88% when $\alpha = 0.85$).

The previous figure illustrates that small differences in the average intermediate input share across countries observed in the data may actually generate important differences on selection and innovation. Should the US have had an intermediate input share that is on average the one of China, that is $\alpha = 0.62$, trade liberalization would have reduced the probability of survival by only 5.42% which is around 28% lower than the current effect for the US and it would have increased the probability of innovation by 7.72% which is almost 45% larger than the suggested effect for the case of the US. Therefore, trade liberalization is expected to have very different effects on firm's survival and innovation across countries that differ in the average intermediate input intensity. These different effects have also important consequences on the evolution of the aggregate productivity and welfare as shown in

[48] More precisely we denote the survival probability as $\left(\frac{a_D}{a_M}\right)^{\kappa}$ when $a_D \leq a_M$ and the innovation probability as $\left(\frac{a_I}{a_M}\right)^{\kappa}$ when $a_I \leq a_M$.

the next two figures.

INSERT FIGURE 2 HERE INSERT FIGURE 3 HERE

It is important to note that all figures have been carefully constructed considering the possibility of the existence of corner solutions where $a_D = a_M$ and the whole range of α (i.e. from $\alpha = 0$ to $\alpha = 0.99$).[49] The following figures report results for $\alpha < 0.86$ for clarity of exposition since, given the discussion above, this is the range of the parameter α that is empirically meaningful for the available data. Figure 2 shows the percentage change in the average productivity as a consequence of trade liberalization. Note that as the strength of vertical linkages increases, the impact on the average productivity becomes smaller. This is the result of several effects at work, discussed in subsection 4.1. Following that discussion, Figure 2 displays also the contribution to average productivity growth that can be attributed to the direct impact of trade liberalization on innovation and exporting (labelled in the figure as Innovation and Exporting) and to the impact on selection (labelled in the figure as selection). As vertical linkages increase, selection becomes less stringent and this reduces the average productivity growth. When vertical linkages are not present, the impact of trade liberalization in the average productivity is 10.96%. This is larger than the corresponding effect for the US economy whose increase is given by 9.06% which is 18% smaller. Should the US have had the average intensity of China, the effect would have been even smaller (i.e. 7.34% which is around 19% smaller than the current US one). When $\alpha_0 < \alpha < \alpha^{**}$, indeed, selection will contribute to a fall in the average productivity but the impact of the other channels (innovation and exporting) more than compensate this effect. That is the case of $\alpha = 0.76$, where the overall increase in the average productivity is much smaller (2.60%) and selection contributes to a fall in the average productivity by 0.58%. The latter is stronger when $\alpha^{**} < \alpha < \alpha_1$ and the overall effect in the average productivity growth is negative. This is the case when $\alpha = 0.85$, where the average productivity falls overall by 10.73% with the selection channel making the average productivity fall by 13.50%. This figure consequently reveals that ignoring the existence of vertical linkages leads to an overstatement of the gains in aggregate productivity due to trade liberalization.

Figure 3 shows the impact of trade liberalization on welfare. It also includes three different lines that capture the different channels through which trade liberalization affects welfare. The first channel works through the change in θ that represents the impact on innovation and exporting (labelled as "theta" in the figure). The second channel comes through changes on the threshold cutoff, a_D, labelled as "selection" and the third one comes through the impact on the number of domestic varieties accordingly labelled as "number of domestic varieties". The decomposition of the total effect in these three forces, which were discussed in the previous section, help us to convey a better intuition for the results discussed below.

Note that unlike the average productivity, the welfare gains from trade liberalization, displayed in Figure 3, are indeed increasing with the degree of vertical linkages provided that the degree of vertical linkages are not middle

[49] In the current set of results, the corner solution exists at $\alpha = 0.92$. A detailed analysis of the case in which a corner solution appears is discussed in the online appendix.

high ($\alpha < 0.91$). When vertical linkages are not present, trade liberalization generates an increase in welfare of approximately 1.61%. With a moderate increase in vertical linkages, the gains from trade liberalization increase considerably. For the case of the US, the gains from trade will more than double (3.43%). Figure 3 also reveals that the role played by the "theta" channel is relatively important in this magnification effect, as the initial effect on selection is ambiguous due to the contrasting effects between the direct impact in the unit input threshold, a_D, that contributes positively to welfare and the impact of selection on domestic varieties, that contributes negatively.[50] As in the previous figures we must remark that small differences in the average intensity of vertical linkages generate notable differences in the impact of trade liberalization on welfare. If the US had had the average intermediate intensity of China, the gains from trade would have been 5.13% which is indeed 49% larger than the case of the US. Finally, in the special case where the economy enters in the middle-low equilibrium ($\alpha > 0.75$), the increase in welfare would have been even larger (10.12% if $\alpha = 0.76$ and 27% if $\alpha = 0.85$). This is quite different from the effect of trade liberalization on the average productivity growth, where for these levels of vertical linkages, it was very small or even negative. This is the result of the fact that as vertical linkages become middle-low trade liberalization will increase rather than decrease the number of varieties produced in each country. This increase in the number of varieties, which can be also seen in Figure 4, will more than compensate the effect on selection that becomes negative for this parameter range as displayed in Figure 3.

INSERT FIGURE (4) HERE

INSERT FIGURE (5) HERE INSERT FIGURE (6) HERE

To examine the role played by technology adoption and its interaction with vertical linkages in the gains from trade, we first compute the effects of trade liberalization on the average productivity and welfare in our theoretical framework with and without innovation. Then, we compute the difference between both magnitudes. Figures 5 and 6 show respectively the contribution of technology adoption to the effects of trade liberalization on the average productivity and on welfare. The effect of trade liberalization on the average productivity in the version of our model with technology adoption is 0.72 percentage points larger than in the version without technology adoption when there are no vertical linkages. Given that the increase in the average productivity from trade liberalization in a model without technology adoption is 10.24%, we can conclude that including technology adoption increases the gains in average productivity from trade liberalization by 7.03%.

It is important to note that, as vertical linkages increase, the contribution of technology adoption to the effects of trade liberalization on average productivity shrinks in percentage points. This can be thought as a consequence of the negative impact of vertical linkages on the evolution of the average productivity due to the fact that, as

[50] Two interesting observations in this figure deserve special attention. First, the impact of the "theta" channel becomes larger as vertical linkages increase even though θ is invariant to α. Second, the effect of selection on domestic varieties is negative despite the fact that as vertical linkages increase the fall in the domestic varieties becomes smaller. In both cases, the key lies on the fact that as vertical linkages increase, the impact of trade liberalization through these two channels becomes larger due to the larger weight that intermediates have in determining the aggregate price index.

vertical linkages become more important, selection is softened. This latter effect is much more important than the effect on innovation. For the case of the average intensity of the US, the difference is 0.64 percentage points which accounts for a 7.60% increase of the original effect (i.e. without technology upgrading). If instead, the average intensity of the US had been the one of China, technology adoption would have contributed by only 0.57 percentage points, which is a 8.63% increase with respect to the original effect.[51]

Figure 6 shows the contribution in percentage points of technology adoption to the effects of trade liberalization on welfare. Note that in a situation in which there are no vertical linkages, including technology adoption increases the gains from trade liberalization by 0.06 percentage points. Although this contribution appears not to be very substantial, it is important to remark that our model predicts a gain from trade liberalization in terms of welfare of 1.61% and, therefore, technology upgrading accounts for a 3.72% of the overall gains from trade. As vertical linkages become stronger this effect magnifies in terms of percentage points and slightly in terms of percentage increase. For the case of the average intensity of the US the difference is 0.13 percentage points, which is more than double. This constitutes an increase in the overall gains from trade of 3.79% and it suggests that the interaction between technology adoption and vertical linkages has a quantitatively important impact on welfare. If the US had had the average intensity of China the effect would have been an increase in 0.20 percentage points or 3.89% of the gains from trade with such intensity of vertical linkages. If vertical linkages were stronger than those of China, the contribution of technology upgrading would be even more substantial (from 0.40 percentage points with $\alpha = 0.76$ to 1.13 percentage points with $\alpha = 0.85$. and 3.95% and 4.18% of the overall gains from trade respectively).[52] The previous results suggests that while increasing α does amplify the welfare gains from trade liberalization accounted for by technology upgrading, the size of the effect is relatively modest for the relevant range of α considered. Nevertheless, differences in the strength of vertical linkages across countries are able to generate differences in the extent to which countries benefit from trade through technology adoption.

5 Conclusion

In this paper we have analyzed the impact of trade liberalization on technology adoption, average productivity and welfare in a model with heterogeneous firms that decide whether to upgrade the state of technology they use by incurring in a fixed cost. In addition these firms are interconnected by vertical linkages. The joint effect of these two channels and the fact that they can be intertwined is a relatively unexplored dimension in the literature.

[51] When $\alpha^{**} < \alpha < \alpha_1$, the average productivity may fall because the negative selection effect is strong enough to overcome the positive impact on innovation. In figure 5 it can be seen that the inclusion of technology adoption depresses the gains from trade in terms of the average productivity because the inclusion of technology adoption reduces even more the price of production factors in such a way that contributes to an even softer selection.

[52] It should be also noted that the calibration exercise focuses on a relatively small reduction of trade costs when trade costs are already low. This could be more appropriate for the symmetric country case analysed in the paper. Melitz and Reading (2015), however, considers an initial level of trade costs $\tau = 1.83$. Considering the same trade costs reduction with that initial level of trade cost would result in an increase in the contribution of technology upgrading on the impact of trade liberalization on welfare, accounting for 12% of the total welfare gains. If, instead, we consider a trade cost reduction that goes from $\tau = 1.83$ to $\tau = 1.15$ the contribution of technology upgrading will be 0.33, 0.75 and 1.17 percentage points for $\alpha = 0, 0.48, 0.62$, respectively. In the online appendix we also show that the effect can significantly increase if we consider alternative values for the average R&D intensity.

Our analysis is motivated by the prominent role of intermediate inputs in world trade and the importance of industry-specific vertical linkages as documented in the introduction.

The paper unveils a novel set of results regarding the contribution of vertical linkages and technology upgrading on the impact of trade liberalization on average productivity and welfare. The inclusion of vertical linkages modifies the effects of trade liberalization on these dimensions and its final impact depends on the strength of vertical linkages and the type of equilibrium the economy is in, generating a very rich set of results. Focusing on the specific cases of equilibrium A and when the degree of vertical linkages is low (i.e. $0 < \alpha < \alpha_0$) or middle-low (i.e. $\alpha_0 \leq \alpha < \alpha_1$), the ones that our calibration exercise reveals as the most empirically relevant, trade liberalization encourages more firms to technology upgrade and this incentive increases with the degree of vertical linkages. Moreover, in these cases, trade liberalization has an ambiguous impact on selection. When vertical linkages are low trade liberalization reduces the survival unit-input cut-off (selection is toughened), but the opposite occurs (selection is softened) when vertical linkages are middle-low.

These have important consequences for both the average productivity and welfare. Specifically trade liberalization increases the average productivity except when $\alpha^{**} < \alpha < \alpha_1$. Vertical linkages also magnify the effects of trade liberalization on welfare as in Caliendo and Parro (2015). More importantly, unlike the previous literature, we also show that technology upgrading contributes to magnify the welfare gains from trade and that both dimensions, technology upgrading and vertical linkages, complement each other. Specifically, vertical linkages magnify the role played by technology upgrading on the welfare gains from trade and technology upgrading magnifies the role played by vertical linkages on the welfare gains from trade. This novel result suggests that to evaluate properly the contribution of each of the channels to the gains from trade, they must be jointly analysed.

Finally, to explore the quantitative importance of those effects, we have calibrated our model for the case of the US economy. The exercise reveals that ignoring both channels and their interaction leads to a substantial underestimation of the welfare gains from trade and that small differences in the average intermediate input intensity across countries, as observed in the data, could lead to large variation in the gains from trade liberalization. Finally, our exercise demonstrates that the contribution of technology upgrading to the gains from trade is quantitatively relevant and its impact increases with the intermediate input intensity confirming that the interaction between technology upgrading and vertical linkages has also quantitative implications for the gains from trade.

Thus, we conclude that our paper presents a simple but rich model that sheds some light on the analysis of the relationships that exist among trade liberalization, technology adoption, productivity and welfare, showing how its multifaceted nature can become even more complex when firms producing in open economies are interconnected by vertical linkages. The model, while containing certain simplifying assumptions, allows us to derive closed form solutions and is rich enough to provide a picture of the numerous and complex forces at play. A multi-country multi-sector quantitative model that relaxes some of these assumptions by for example, incorporating a more complex innovation structure, considering not only the extensive but also the intensive margin of innovation, and a richer intermediate input structure as in Zou (2019) and Fieler et al. (2017), would help us to provide more precise estimates about the gains from trade. This could be a fruitful avenue for future research.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*, Princeton University Press.

Aghion P., Bergeaud A, Lequien M., Melitz M.J. (2019). The Heterogeneous Impact of Market Size on Innovation: Evidence from French Firm-Level Exports. *Unpublished manuscript.*

Anderson, J.E. and Van Wincoop, E. (2004). Trade Costs, Journal of Economic Literature, *American Economic Review*, vol. 42(3), pp. 691-751.

Antràs, P., Fort, T.C. and Tintelnot, F. (2017). The Margins of Global Sourcing: Theory and Evidence from US Firms, *American Economic Review*, American Economic Association, vol. 107(9), pp. 2514-2564.

Atkeson, A. and Burstein, A. (2010). Innovation, Firm Dynamics, and International Trade, *Journal of Political Economy*, vol. 118 (3) pp.433-484.

Aw, B.Y, Roberts, M. and Xu, D., (2011). R&D Investment, Exporting, and Productivity Dynamics, American Economic Review, *American Economic Association*, vol. 101(4), pp. 1312-1344.

Axtell, R.L. (2001). Zipf Distribution of U.S. Firm Sizes, *Science*, vol. 293, pp. 1818-1820.

Baldwin, R. E. and Forslid, R., (2004). Trade Liberalization with Heterogenous Firms, *CEPR discussion paper 4635.*

Baldwin, R.E., Forslid, R., Martin, P., Ottaviano, G., Robert-Nicoud F. (2005). *Economic Geography and Public Policy*, Economics Books, Princeton University Press.

Bas, M. and Berthou, A. (2017). Does Input-Trade Liberalization Affect Firms' Foreign Technology Choice? *World Bank Economic Review*, vol. 31(2), pp. 351-384.

Bas, M., Mayer, T and Thoenig , M. (2017). From micro to macro: Demand, supply, and heterogeneity in the trade elasticity, *Journal of International Economics*, vol. 108(1), pp (1-19).

Bernard, A. B., Eaton, J. and Kortum, S.S. (2003). Plants and Productivity in International Trade, *American Economic Review*, vol. 93, pp. 1268–1290.

Bernard, A.B., Redding S.J. and Schott, P.K. (2007). Comparative Advantage and Heterogeneous Firms, *Review of Economic Studies*, vol. 74, pp. 31–66.

Bloom, N., Draca, M., Van Reenen, J. (2016). Trade Induced Technical Change? The Impact of Chinese Imports on Innovation, IT and Productivity, *Review of Economic Studies*, Oxford University Press, vol. 83(1), pp. 87-117.

Boler, E.A., Moxnes, A. and Ullveit-Moe, K.H. (2015). R&D, International sourcing and the Joint Impact on firm performance, *American Economic Review*, vol. 105(12), pp. 3704-39.

Broda, C. and Weinstein D.E. (2006). Globalization And The Gains From Variety, *Quarterly Journal of Economics*, vol. 121 pp. 541-585.

Bustos, P., (2011), Trade Liberalization, Exports and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinean Firms, *American Economic Review*, vol. 101, pp. 304–40.

Caliendo, L. Feenstra, R., Romalis, J. and Taylor, A.M. (2015). Tariff Reductions, Entry, and Welfare: Theory and Evidence for the Last Two Decades, *NBER Working Papers 21768*, National Bureau of Economic Research, Inc.

Caliendo, L. and Parro, F. (2015). Estimates of the Trade and Welfare Effects of NAFTA, *Review of Economic Studies*, vol. 82(1) pp. 1-44.

Castellani, D. and A. Zanfei, (2007). Internationalisation, Innovation and Productivity: How Do Firms Differ in Italy?, *The World Economy*, vol. 30(1), pp. 156-176.

Ciccone, A., (2002). Input Chains and Industrialization, *Review of Economic Studies*, vol. 69, pp. 565–87.

Costinot, A. and Rodriguez-Clare, A. (2014). Trade Theory with Numbers: Quantifying the Consequences of Globalization, Handbook of International Economics, Elsevier.

Cunat, A., and Zymek, R., (2017). International Value-Added Linkages in Development Accounting, *CESifo WP, 7196*.

Di Giovanni and Levchenko, A.(2010). Putting the Parts Together: Trade, Vertical Linkages, and Business Cycle Comovement, *American Economic Journal: Macroeconomics*, vol. 2(2), pp. 95-124.

Eaton, J., Kortum S., and Kramarz F. (2011). An Anatomy of International Trade: Evidence From French Firms. *Econometrica*, vol.79(5), pp. 1453-1498

Fadinger H., C. Ghiglino and M. Teteryatnikova, (2017). Income Differences, Productivity and Input-Output Networks. *Unpublished manuscript*.

Fernandes, A.M., Klenow, P. J., Meleshchuk, S., Pierola, M.D., Rodriguez-Clare, A. (2018). The Intensive Margin in Trade, *Policy Research working paper; no. WPS 8625*. Washington, D.C.: World Bank Group.

Fieler A. C., M. Eslava and Y. Xu (2018): Trade, Quality Upgrading, and Input Linkages. Theory and Evidence from Colombia, *American Economic Review*, vol. 108(1) pp. 109–146

Fujita, M., Krugman, P and Venables, A., (1999). *The Spatial Economy: Cities, Regions, and International Trade*, MIT Press Books, The MIT Press, edition 1, volume 1.

Gabaix, X. (1999). Zipf's Law for Cities: An Explanation. *The Quarterly Journal of Economics*, vol. 114, no. 3, 1999, pp. 739–767

Grossman, G. and Helpman, E. (1991). Trade, Knowledge Spillovers, and Growth, *European Economic Review*, vol. 35(2-3), pp. 517-26.

Halpern, L., Koren, M. and Szeidl, A. (2015). Imported Inputs and Productivity. *American Economic Review*, vol. 105 (12), pp. 3660-3703.

Helpman, E. and Krugman, P. (1987). Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy, MIT Press Books, The MIT Press, edition 1.

Hummels, D., Ishii J. and K.-M. Yi, (2001). The Nature and Growth of Vertical Specialization in World Trade, *Journal of International Economics*, vol. 54, pp. 75–96.

Impullitti, G. and Licandro, O. (2018). Trade, Firm Selection and Innovation: The Competition Channel, *The Economic Journal*, Royal Economic Society, vol. 128(608), pp. 189-229.

Jones, C. I., (2011). Intermediate Goods and Weak Links in the Theory of Economic Development, *American Economic Journal: Macroeconomics*, vol. 3(2), pp. 1-28.

Jones, C. I., (2013). Misallocation, Economic Growth, and Input-Output Economics In *Advances in Economics and Econometrics: Tenth World Congress, Vol. II*, edited by Daron Acemoglu, Eddie Dekel, and Manuel Arellano. Cambridge, UK: Cambridge University Press.

Kasahara, H. and Rodrigue, J. (2008). Does the use of imported intermediates increase productivity? Plant-level evidence, *Journal of Development Economics*, vol. 87, pp. 106-118.

Klenow, P. J., and Rodríguez-Clare, A. (2005). Extenalities and Growth. In *Handbook of Economic Growth*, Vol. 1A.

Klette, T.J., and S. Kortum (2004). Innovating firms and Aggregate Innovation, *Journal of Political Economy*, 2004, vol. 112 (5), pp. 986-1018.

Krugman, P. R., and A. J. Venables, (1995). Globalization and the Inequality of Nations, *Quarterly Journal of Economics*, vol. 110, pp. 857–80.

Lee, H. L., Padmanabhan, V. and S. Whang, (1997). The Bullwhip Effect in Supply Chains, *Sloan Management Review* vol. 38 (3), pp. 93-102.

Lileeva, A. and D. Trefler, (2010). Improved Access to Foreign Markets Raises Plant Level Productivity ...for some Plants, *The Quarterly Journal of Economics*, vol. 125, pp.1051-1099.

Lim, K. (2018). Endogenous Production Networks and the Business Cycle, *Unpublished manuscript*.

Luttmer, E. (2007). Selection, Growth, and the Size Distribution of Firms. *The Quarterly Journal of Economics*, vol. 122(3) pp. 1103–1144.

Melitz, M. J., (2003). The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity, *Econometrica*, vol. 71, pp. 1695–725.

Melitz, M.J., and Redding, S. (2015). New Trade Models, New Welfare Implications, *American Economic Review*, vol.105, 3, pp. 1105-46.

Navas, A., and D. Sala, (2015). Innovation and trade policy coordination: the role of firm heterogeneity. *The World Economy*, vol 38(8), pp. 1205-1224

Nocco, A., (2005). The Rise and Fall of Regional Inequalities with Technological Differences and Knowledge Spillovers, *Regional Science and Urban Economics*, vol.35, pp. 542–69.

Nocco, A., (2012). Selection, Market Size and International Integration: Do Vertical Linkages Play a Role?, *Review of International Economics*, vol. 20, pp. 960–973.

Ossa, R. (2015). Why trade matters after all, *Journal of International Economics*, Elsevier, vol. 97(2), pp. 266-277.

Pavnick , N. (2002). Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants, *Review of Economic Studies*, vol. 69, pp. 245-76.

Peretto, P. (2015). From Smith to Schumpeter: A theory of take-off and convergence to sustained growth, *European Economic Review*, Elsevier, vol. 78(C), pp. 1-26.

Perla, J., Tonetti, C. and Waugh, M. (2015). Equilibrium Technology Diffusion, Trade, and Growth, *NBER Working Papers 20881*, National Bureau of Economic Research, Inc.

Puga, D., (1999). The Rise and Fall of Regional Inequalities, *European Economic Review*, vol.43, pp. 303–34.

Rauch, F. (1999). Networks versus markets in international trade, *Journal of International Economics*, vol. 48, issue 1, 7-35.

Rubini, L. (2014). Innovation and the trade elasticity, *Journal of Monetary Economics*, Elsevier, vol. 66(C), pp. 32-46.

Timmer, M.P., Dietzenbacher, E., Los, B.,Stehrer, R. and Gaaitzen J. Vries, (2015). An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production, *Review of International Economics*, Wiley Blackwell, vol. 23(3), pp. 575-605.

Trefler, D. (2004). The Long and Short of the Canada-U. S. Free Trade Agreement, *American Economic Review*, American Economic Association, vol. 94(4), pp. 870-895.

Venables, A. (1996a). Equilibrium Locations of Vertically Linked Industries, *International Economic Review*, Department of Economics, vol. 37(2), pp. 341-359.

World Trade Organization (WTO) (2014). *World Trade Report 2014: Trade and development: recent trends and the role of the WTO*, Geneva: WTO.

Yeaple, S. R., (2005). A Simple Model of Firm Heterogeneity, International Trade, and Wages, *Journal of International Economics*, 65, pp. 1–20.

Zou, Y. (2019). Endogenous Production Networks and Gains from Trade, *2019 Meeting Papers 594*, Society for Economic Dynamics

5.1 Figures

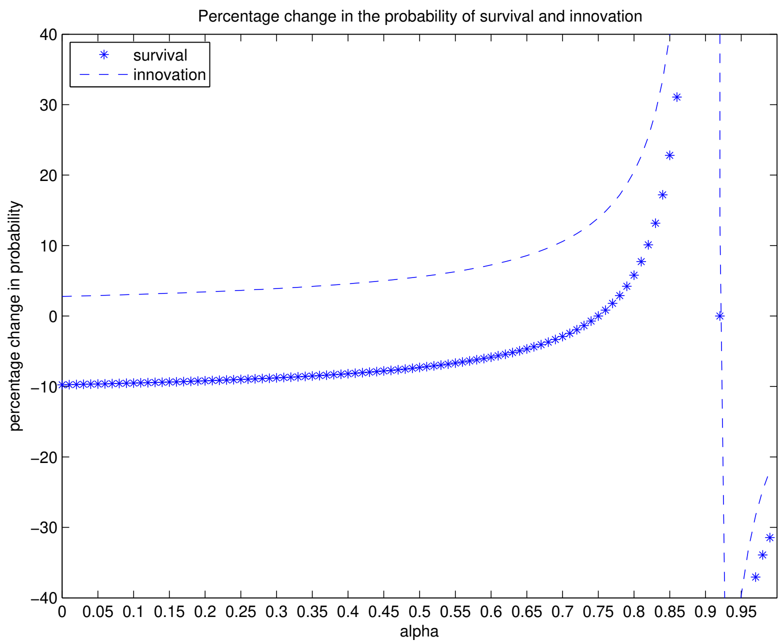

Figure 1: The impact of trade liberalization on the survival and innovation probability.

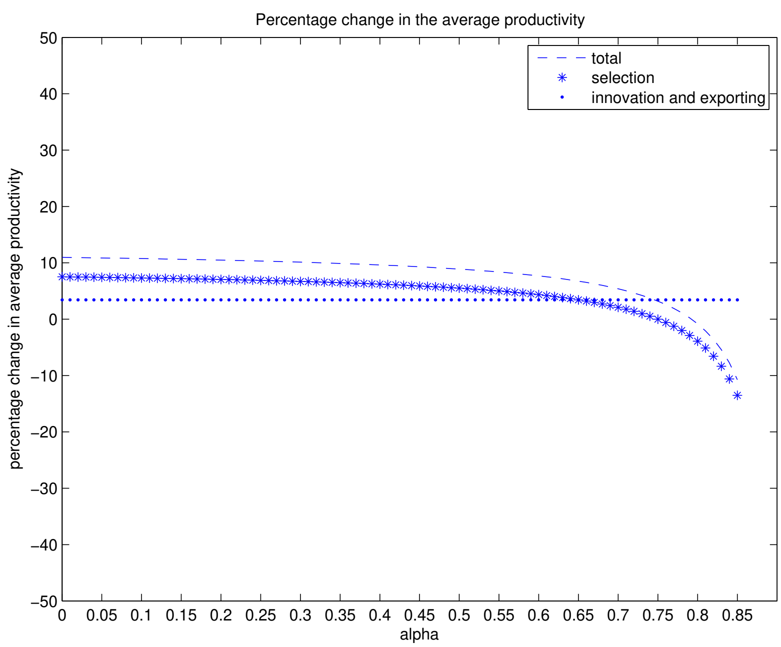

Figure 2: The impact of trade liberalization on average productivity.

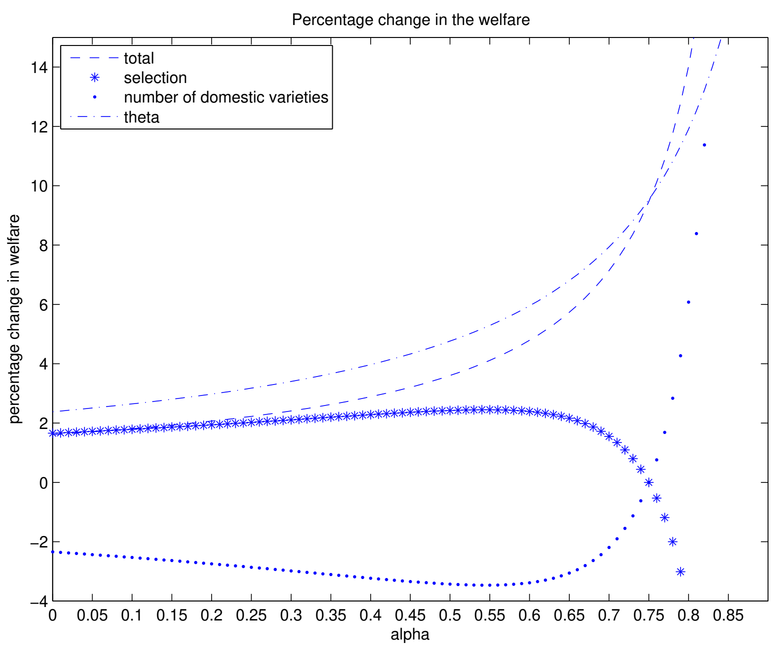

Figure 3: The impact of trade liberalization on welfare.

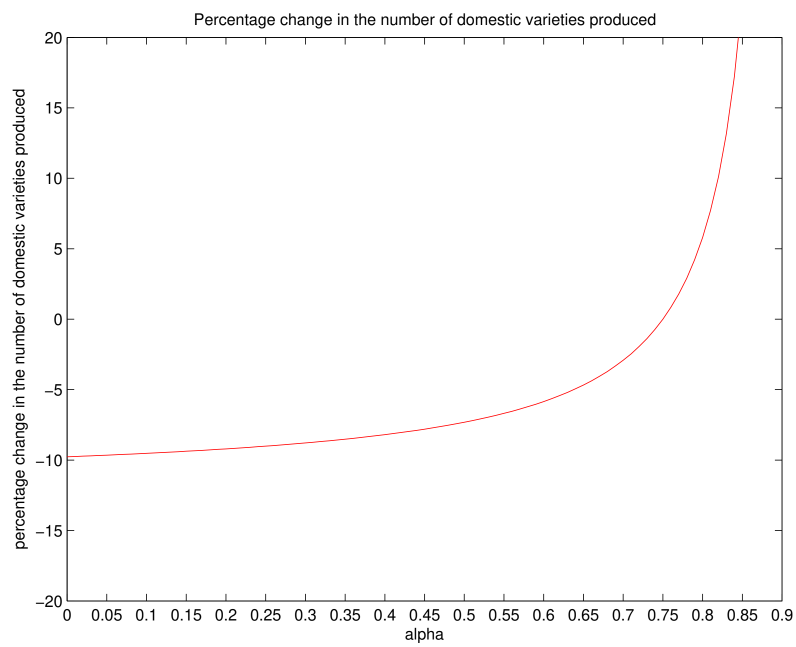

Figure 4: The impact of trade liberalization on the number of domestic varieties produced.

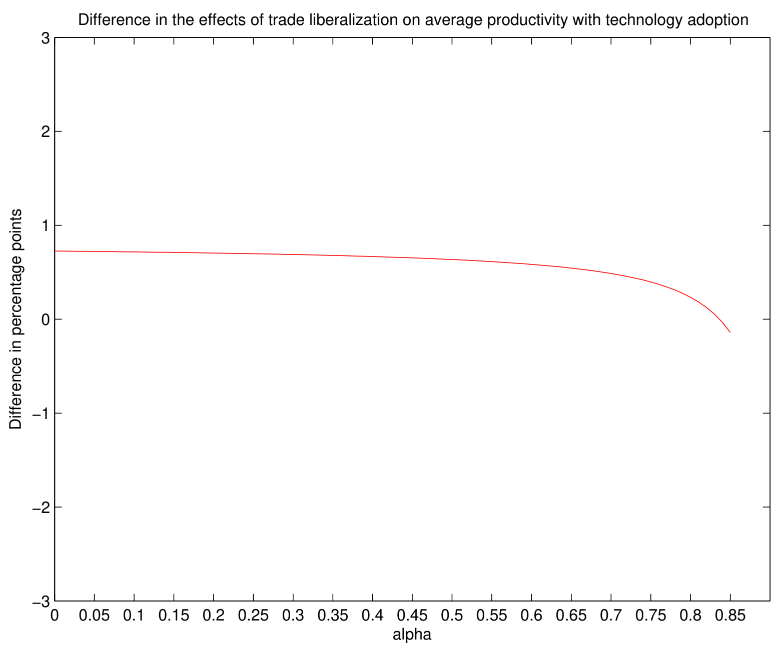

Figure 5: The contribution of technology adoption to the effects of trade liberalization on average productivity.

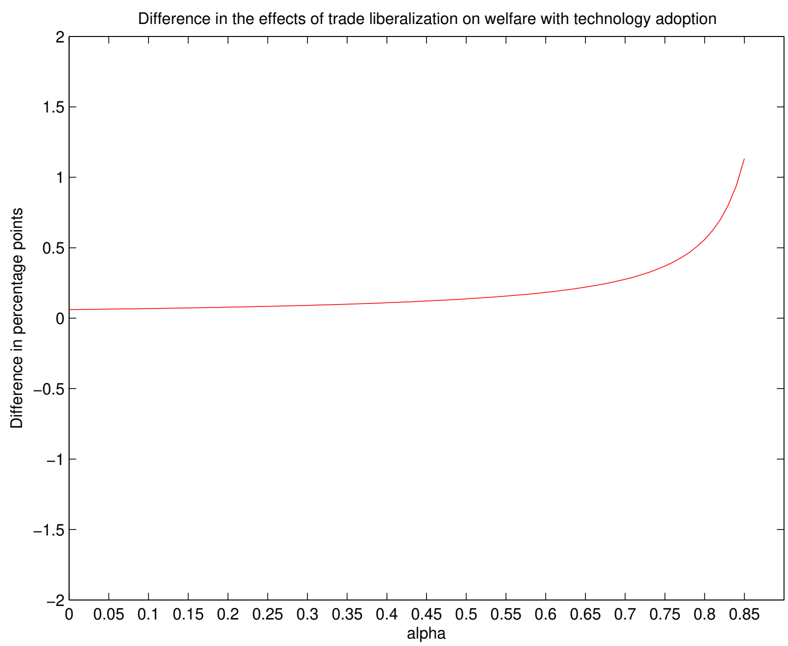

Figure 6: The contribution of the technology adoption channel to the gains from trade.

Online Appendix (Not for Publication)

December 8, 2020

1 Intermediate input intensity across industries and countries

INSERT TABLES 1 AND 2

2 Equilibria B and C and the effects of trade liberalization.

In the main manuscript we focused on an equilibrium in which innovation was the relatively most expensive activity. However, the model exhibits two other equilibria that we denote with B and C. In this section we characterize these equilibria and the conditions under which these equilibria hold.

Equilibrium B occurs when exporting is a relatively expensive activity compared to innovation. In that equilibrium firms are grouped according to the following categories: exporting adopters (the most productive ones), non exporting adopters (intermediate productivity levels) and domestic firms (the least productive ones). This equilibrium arises when the following condition holds

$$f_X > \frac{f_I}{(\gamma^{1-\sigma}-1)}\phi\gamma^{1-\sigma} > \phi\gamma^{1-\sigma} f_D,$$

Equilibrium C occurs when trade costs are intermediate. In this equilibrium firms do not find it profitable to upgrade the technology they use if they do not export and vice-versa. More precisely, this equilibrium is sustained when the following parameter configuration holds

$$\frac{f_I}{(\gamma^{1-\sigma}-1)}\frac{\phi}{(1+\phi)} \leq f_X \leq \frac{f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}\phi.$$

In this equilibrium firms are sorted according to two different types: adopters that export (the most productive ones) or just domestic firms (the least productive ones).[1]

[1]Note that $\frac{f_I}{(\gamma^{1-\sigma}-1)}\frac{\phi}{(1+\phi)} \leq \frac{f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}\phi$, since $\frac{1}{1+\phi} \leq \gamma^{1-\sigma}$.

2.1 Equilibrium B

In equilibrium B, the firm that is indifferent between adopting or relying on the original technology is a domestic firm and all exporters are in fact adopters, with the three unit input cutoffs ranked as $a_X^B < a_I^B < a_D^B$. In this case the following conditions hold

$$\phi\gamma^{1-\sigma}\Delta^B \left(a_X^B\right)^{1-\sigma} = f_X \left(P_M^B\right)^\alpha \tag{1}$$

$$(\gamma^{1-\sigma} - 1)\Delta^B \left(a_I^B\right)^{1-\sigma} = f_I \left(P_M^B\right)^\alpha \tag{2}$$

$$\Delta^B \left(a_D^B\right)^{1-\sigma} = f_D \left(P_M^B\right)^\alpha \tag{3}$$

Using (1) and (3) the proportion of surviving firms exporting is given by

$$\frac{N_X^B}{N_D^B} = \left(\frac{a_X^B}{a_D^B}\right)^\kappa = \left(\frac{\phi\gamma^{1-\sigma} f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{4}$$

and using (2) and (3) the proportion of firms adopting the most efficient technology is given by,

$$\frac{N_I^B}{N_D^B} = \left(\frac{a_I^B}{a_D^B}\right)^\kappa = \left[\frac{\left(\gamma^{1-\sigma} - 1\right) f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{5}$$

2.2 Equilibrium C

In equilibrium C, the set of adopters and exporters coincide. The firm which is indifferent between adopting or relying on the original technology knows that if it does not adopt, it won't be able to export and vice-versa. Consequently, that firm evaluates the benefits of jointly adopting and exporting instead of relying on the original technology and remaining a domestic firm. The two resulting cutoffs are ranked as follows $a_I^C < a_D^C$, where a_I^C denotes the unit input requirements associated with that firm. The conditions associated with this equilibrium are given by the following expressions

$$\Delta^C[(1+\phi)\gamma^{(1-\sigma)} - 1]\left(a_I^C\right)^{1-\sigma} = (f_I + f_X)\left(P_M^C\right)^\alpha . \tag{6}$$

$$\Delta^C a_D^{1-\sigma} = f_D \left(P_M^C\right)^\alpha \tag{7}$$

Dividing (6) and (7) and rearranging terms, the proportion of surviving firms which adopt and export is given by:

$$\frac{N_X^C}{N_D^C} = \frac{N_I^C}{N_D^C} = \left(\frac{a_I^C}{a_D^C}\right)^\kappa = \left\{\frac{\left[(1+\phi)\gamma^{1-\sigma} - 1\right] f_D}{f_I + f_X}\right\}^{\frac{\kappa}{\sigma-1}} \tag{8}$$

	Welfare	Equilibrium A		Equilibrium B	Equilibrium C	
$0<\alpha<\alpha_0$	$W\uparrow$	$a_D^A\downarrow$	$a_I^A\uparrow$	a_D^B and $a_I^B\downarrow$	$a_D^C\downarrow$	$a_I^C\uparrow$
$\alpha_0<\alpha<\alpha_1$	$W\uparrow$	$a_D^A\uparrow$	$a_I^A\uparrow$	a_D^B and $a_I^B\uparrow$	$a_D^C\uparrow$	$a_I^C\uparrow$
$\alpha_1<\alpha<\alpha_2^i$	$W\downarrow$	$a_D^A\downarrow$	$a_I^A\downarrow$	a_D^B and $a_I^B\downarrow$	$a_D^C\downarrow$	$a_I^C\downarrow$
$\alpha_2^i<\alpha<1$	$W\downarrow$	$a_D^A\downarrow$	Case 1 $a_I^A\downarrow$ Case 2 $a_I^A\uparrow$	a_D^B and $a_I^B\downarrow$	$a_D^C\downarrow$	Case 1 $a_I^C\downarrow$ Case 2 $a_I^C\uparrow$

Table 3: The effects of an increase in the freeness of trade on the productivity and innovation cutoffs for the case of an interior solution (i.e. $a_D < a_M$).

3 Results in Table 1 of the main manuscript and Table 3 in the appendix.

3.1 Derivations of the results in Tables 1 in the main manuscript and Table 3 in the appendix

This part of the Appendix contains the derivation of the signs of the derivatives of the cutoffs with respect to ϕ reported in Table 3.

In general, from equations (11) and (14) of the main manuscript it can be derived that

$$\frac{\partial a_D^i}{\partial \phi} = -\left(\frac{\sigma-1-\alpha\sigma}{\chi}\right)\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}}\left(\theta^i\right)^{-\frac{\sigma-1-\alpha\sigma}{\chi}-1}\frac{\partial\theta^i}{\partial\phi}$$

Given that $\frac{\partial\theta^i}{\partial\phi}>0$, the sign of $\frac{\partial a_D^i}{\partial\phi}$ depends on the sign of $-\frac{\sigma-1-\alpha\sigma}{\chi}=\frac{\alpha\sigma-(\sigma-1)}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ as in Nocco (2012). Hence, $\frac{\partial a_D^i}{\partial\phi}<0$ for $\alpha\in(0,\alpha_0)$ and $\alpha\in(\alpha_1,1)$, and viceversa $\frac{\partial a_D^i}{\partial\phi}>0$ for $\alpha\in(\alpha_0,\alpha_1)$ with $\alpha_0\equiv\frac{\sigma-1}{\sigma}$, $\alpha_1\equiv\frac{\kappa}{\beta\sigma-1}$ and $0<\alpha_0<\alpha_1<1$.

Turning to the sign of $\frac{\partial a_I^i}{\partial\phi}$, it has to be analyzed in detail for each case.

Equilibrium A In equilibrium A, substituting a_D^A into equation (5) of the main manuscript yields

$$a_I^A = z\left[\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\right]$$

where $z\equiv\left[\frac{\mu\kappa}{\delta_0 f_D}L\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left\{\left[\frac{f_I}{f_D}\frac{1}{(\gamma^{1-\sigma}-1)}\right]^{\frac{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}{(1-\sigma)(\sigma-1-\alpha\sigma)}}\frac{f_E}{f_D}(\beta-1)\,a_M^\kappa\right\}^{\frac{\sigma-1-\alpha\sigma}{\chi}}>0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^A}{\partial\phi} = -z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}(h-l)$$

and given that $z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}>0$, the sign of $\frac{\partial a_I^A}{\partial\phi}$ depends on the sign of $h-l$, where $h\equiv\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ and $l\equiv\frac{1}{(\sigma-1)}\frac{1}{1+\phi}/(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A})$. It can be readily verified that l does not depend on α and can be represented by a horizontal line in the cartesian plane where α is represented on the axis of abscissas (See Figures 1-2 that follow). Moreover, when the independent variable is α, h is a hyperbola, whose expression can be rewritten as follows $h = \frac{\sigma\alpha-(\sigma-1)}{[\kappa\sigma-(\sigma-1)]\alpha-(\sigma-1)\kappa}$ with center in $O\left(\frac{\kappa}{\beta\sigma-1}, \frac{1}{\kappa-\frac{(\sigma-1)}{\sigma}}\right)$, and its two asymptotes having, respectively, expressions $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$ in the case of the vertical asymptote and $h_1 = \frac{1}{\kappa-(\sigma-1)/\sigma}$ in that of the horizontal asymptote. Moreover, it can be verified that: 1) $h = \frac{1}{\kappa}$ when $\alpha = 0$; 2) $h = \frac{1}{\kappa-(\sigma-1)} > 0$ (because $\kappa > \sigma - 1$) when $\alpha = 1$; 3) and, finally, $h = 0$ when $\alpha = \frac{\sigma-1}{\sigma} \equiv \alpha_0$.

Then, substituting $\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}$ into l yields

$$l = \frac{1}{\kappa}\frac{\phi}{1+\phi}\frac{1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}}{\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\frac{\phi}{1+\phi}\left(\frac{f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l < \frac{1}{\kappa-(\sigma-1)}$ for sufficiently high levels of international integration, that is with $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$. Otherwise, for low levels of international integration, that is with $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$, we find that: $l < \frac{1}{\kappa-(\sigma-1)}$ if $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^\beta(1+\phi)^\beta\phi f_D^{\beta-1}}$ with $\Omega \equiv \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$; and $l > \frac{1}{\kappa-(\sigma-1)}$ if $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^\beta(1+\phi)^\beta\phi f_D^{\beta-1}}$.

Therefore, there are two potential cases:

Case 1 $l < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$ or when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^\beta(1+\phi)^\beta\phi f_D^{\beta-1}}$. In this case represented in Figure 1, $\frac{\partial a_I^A}{\partial\phi} > 0$ when $0 \le \alpha < \alpha_1$ as $h < l$; and $\frac{\partial a_I^A}{\partial\phi} < 0$ when $\alpha_1 < \alpha < 1$ as $h > l$.

Insert Figure 1 about here

Case 2 $l > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^\beta(1+\phi)^\beta\phi f_D^{\beta-1}}$. In this case, $\frac{\partial a_I^A}{\partial\phi} > 0$ when $0 \le \alpha < \alpha_1$ and when $\alpha_2^A < \alpha < 1$ as $h < l$; and $\frac{\partial a_I^A}{\partial\phi} < 0$ when $\alpha_1 < \alpha < \alpha_2^A$ as $h > l$.

Thus, the parameter restriction that has to hold for equilibrium A in order to have $\alpha_2^A \equiv \frac{\kappa(\theta^A-(1+\phi))}{\phi(1-\beta\sigma)+(\phi+\beta\sigma)(\theta^A-1)} < 1$, which exists only when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$, is that $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^\beta(1+\phi)^\beta\phi f_D^{\beta-1}}$ (where $\Omega \equiv \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$), and this requires that $f_I^{\beta-1} > \frac{f_D^{\beta-1}\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\phi}{\phi(\beta-1)-\Omega(\beta+\phi)}$. Otherwise, if the parameter f_I is not too large, that is if $f_I^{\beta-1} < \frac{f_D^{\beta-1}\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\phi}{\phi(\beta-1)-\Omega(\beta+\phi)}$, we can rule out case 2 and α_2^A should not be considered. Notice that when $\alpha_2^A < 1$, then it is always true that $\alpha_2^A > \alpha_1 > 0$.

Insert Figure 2 about here

Equilibrium C In equilibrium C, substituting a_D^C into (8) yields

$$a_I^C = y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}$$

where $y \equiv \left[\frac{\kappa\mu L}{f_D\delta_0}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left(\frac{f_X+f_I}{f_D}\right)^{\frac{1}{1-\sigma}}\left[\frac{a_M^\kappa f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} > 0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^C}{\partial\phi} = -y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C}\left(h-l_C\right)$$

and given that $y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C} > 0$, the sign of $\frac{\partial a_I^C}{\partial\phi}$ depends on the sign of $h-l_C$, where $l_C \equiv \frac{1}{(\sigma-1)}\frac{\gamma^{1-\sigma}}{(1+\phi)\gamma^{1-\sigma}-1}/(\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C})$ does not depend on α and can be represented by an horizontal line in the cartesian plane where α is represented on the axis of abscissas.

Then, substituting $\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C}$ into l_C yields

$$l_C = \frac{1}{\kappa}\frac{1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}}{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l_C < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)}f_D^{\beta-1}$ (Case 1), and $l_C > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)}f_D^{\beta-1}$ (Case 2). In Case 1, that is for relatively low (f_X+f_I) and relatively high levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial\phi} > 0$ when $0<\alpha<\alpha_1$ as $h<l_C$, while $\frac{\partial a_I^C}{\partial\phi} < 0$ when $\alpha_1<\alpha<1$ because $h>l_C$. In Case 2, that is for relatively high (f_X+f_I) and relatively low levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial\phi} > 0$ when $0<\alpha<\alpha_1$ and $\alpha_2^C<\alpha<1$ as $h<l_C$, while $\frac{\partial a_I^C}{\partial\phi} < 0$ when $\alpha_1<1<\alpha_2^C$, because $h>l_C$. Notice that the two cases can be represented by similar figures as those respectively used in Figure 1 and Figure 2, even if the expression of l_C should be used instead of that of l and the value of α_2^C should be used instead of that of α_2^A.

Thus we can state that the parameter restriction that has to be satisfied for equilibrium C in order to have $\alpha_2^C < 1$ is that $(f_X+f_I)^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)}f_D^{\beta-1}$. Notice that when $\alpha_2^C < 1$, then it is always true that $\alpha_2^C > \alpha_1 > 0$. On the contrary, we can rule out case 2 and α_2^C should not be considered when the fixed adopting parameter f_I is not so large that $(f_X+f_I)^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)}f_D^{\beta-1}$ holds.

Equilibrium B For the case of equilibrium B, it is clear from (5) that $sign(\frac{\partial a_I^B}{\partial\phi}) = sign(\frac{\partial a_D^B}{\partial\phi})$.

3.1.1 Discussion of cases 1 and 2 found when $\alpha_2^i < \alpha < 1$

In the first part of the Appendix (i.e. section 6.2.1), we have discussed that when $\alpha_2^i < \alpha < 1$ two different cases arise in equilibria A and C and we have provided intuition on when these two cases arise. In addition, the next proposition states parameter configurations in which the economy is in equilibrium A and it is in either case 1 or case 2 independently of the degree of variable trade costs:

Proposition 5 *Consider Equilibrium A with* $\alpha_2^A < \alpha < 1$. *if* $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{\beta+1}{\beta-1}$ *and* $\left(\gamma^{1-\sigma}-1\right) < \beta-1$ *or* $\frac{f_I}{f_D} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$ *the economy is in case 2 independently of the value of* ϕ.

Proof. In equilibrium A the economy is in case number 2 if $\left(\frac{f_X}{\phi f_D}\right)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}$ and $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > \frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))}$ with $\Omega = \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Otherwise the economy is in case number 1 (which represents the continuity from the previous equilibria). Rearranging terms in the first of the previous conditions we have that $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \phi^{\beta-1}\frac{(\beta+\phi)}{(\beta-1)}$. The right hand side is increasing in ϕ. Since ϕ is upper bounded by 1 we have that the first condition is always satisfied when $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{(\beta+1)}{(\beta-1)}$. Rearranging the second equation and replacing the value of Ω, we have that $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$. The right hand side of this expression is also increasing in ϕ. Since ϕ is upper bounded by 1 we have that the second condition is always satisfied when $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$. Notice that from equation (5) in the main manuscript we have that $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > 1$ *must be satisfied* for equilibrium A to hold. This implies that if $\frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))} < 1$ for any value of ϕ this condition will be always sustained independently of the value of ϕ. Making use of the expression for Ω and rearranging terms we have that for the previous condition to hold $\left(\gamma^{1-\sigma}-1\right) < \frac{\left((\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}\phi^{\beta-1}(\beta+\phi)\right)}{(1+\phi)}$. Notice that this expression is decreasing in ϕ. Evaluating the rhs under $\phi = 0$ we have that: $\left(\gamma^{1-\sigma}-1\right) < (\beta-1)$. If this condition holds, then $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$ for any value of ϕ. ■

The following proposition discuss parameter configurations in which the economy is in equilibrium C and it is in either case 1 or case 2.

Proposition 6 *Consider Equilibrium C with $\alpha_2^C < \alpha < 1$, then:*[2]

$$\text{if}\begin{cases}\left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the economy is in case 2}\\ \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} < \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the equilibrium of the economy}\\ & \text{depends on the value of } \phi\\ \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the economy is in case 1}\end{cases}$$

Proof. The equation governing the conditions under which the economy will be in either case 1 or 2 is the following: If $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > \frac{(1+\phi)\gamma^{1-\sigma}-1}{\beta-1}$, the economy is in case 2, otherwise the economy is in case 1 (provided that $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > 1$ which is required from (8) for being in equilibrium C). Rearranging terms we find that for the former condition to hold the following must be satisfied: $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$. Notice that the right hand side of this condition is increasing in ϕ. Because ϕ is bounded above and below, evaluating the rhs of that condition when $\phi = 1$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$, then $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ for any ϕ (we are in case 2 independently of ϕ). Consider the reverse constraint $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ (this is the condition for being in case 1). Evaluating the right hand side when $\phi = 0$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{(\gamma^{1-\sigma}-1)^{\beta}}{\beta-1}$ then $\left(\frac{(f_I+f_X)}{f_D}\right)^{\beta-1} < \frac{\left((1+\phi)\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$ for any ϕ. The economy is in case 1 independently of the value of ϕ. ■

The previous propositions conclude that for cases in which the fixed costs of exporting or technology adoption are relatively high, trade liberalization will increase the proportion of firms undertaking technology upgrading when the degree of vertical linkages are very strong in equilibria A and C and in the specific case 2. Consequently, the results suggest that while in case 1, trade liberalization induces tougher selection, and reduces the proportion of entrants undertaking technology adoption, in case 2 the latter is not so strong and the overall positive effect

[2] All of the conditions considered are consistent with the economy being in equilibrium C.

dominates. This is consistent with the fact that in case 2 the requirements for innovating and exporting are so large that very few firms decide to undertake these activities. This moderates the rise in the demand of intermediate inputs and consequently the rise in the relative cost of intermediate inputs. The latter has a moderate impact on the survival productivity threshold and the proportion of technology upgrading firms.

3.2 Discussion of the effects of trade liberalization in Table 3 in the Appendix

In the main text we have discussed the results summarized in Table 1 related to equilibrium A and here we extend the discussion to consider equilibria B and C included in the more comprehensive table 3. In equilibrium B, the marginal adopter is not an exporter. This implies that, when there is a process of trade liberalization, the marginal adopter suffers from the fall in the domestic sales due to the competition from foreign firms in the domestic market but it does not enjoy the expansion of rents from foreign markets. Consequently, in this case, trade liberalization will reduce the firms' adoption profits and therefore this will reduce the incentives to technology upgrade. Likewise, the reduction in operating profits due to the fall in domestic sales reduces the proportion of firms surviving in the market.

The presence of vertical linkages modifies some of the channels discussed in Section 3 while it brings some new forces into play. Let us consider first the case in which vertical linkages are small ($0 < \alpha < \alpha_0$) and without loss of generality the equilibrium B. In this case, substituting Δ^B into (3) gives

$$\frac{E^B \left(P_M^B\right)^{\alpha(1-\sigma)}}{\sigma \left(P_M^B\right)^{(1-\sigma)}} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(a_D^B\right)^{1-\sigma} = f_D \left(P_M^B\right)^{\alpha} \tag{9}$$

and substituting Δ^B in (5) yields

$$(\gamma^{1-\sigma}-1)\frac{E^B \left(P_M^B\right)^{\alpha(1-\sigma)}}{\sigma \left(P_M^B\right)^{(1-\sigma)}} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(a_I^B\right)^{1-\sigma} = f_I \left(P_M^B\right)^{\alpha} \tag{10}$$

Vertical linkages affect the production costs and the fixed costs of adoption, export and survival. Trade liberalization reduces the cost of imports, reducing the aggregate price index P_M^B. Since final goods are used as inputs in the production process, the reduction in the cost of imports reduces production costs, the variable ones (captured by the term $\left(P_M^B\right)^{\alpha(1-\sigma)}$ in the left hand side of (9) and (10)) and the fixed operational ones (captured by the element $\left(P_M^B\right)^{\alpha}$ in the right hand side of (9) and (10)). These two effects have a positive effect on the survival unit input cut-off a_D^B and the adoption cut-off a_I^B.[3] In addition vertical linkages also have a positive effect on each variety's demand which also increases survival (This effect is included in the element E^B in the model) and it will intensify competition. All of these effects are shaped by the parameter α which measures the strength of

[3]Note that P_M^B will be affected by trade costs, through the direct effect on export prices and through the indirect effect on the costs of intermediates.

vertical linkages. The larger is this parameter, the more the production of the firm will be dependent on the goods produced by other firms (local and foreign). In that case, a reduction of trade costs will have a larger impact on the channels mentioned above. If α is low, the negative effect on domestic sales captured by the element $\left(P_M^B\right)^{(1-\sigma)}$ in the denominator dominates. This implies that the qualitative effects of trade liberalization will be unchanged compared to a scenario in which vertical linkages are not present. When vertical linkages are stronger the effects are described in table 3.

Finally, note that for equilibrium A and equilibrium C, in the extreme cases in which α is very high $\alpha_2 < \alpha < 1$, the impact of trade liberalization on technology adoption depends on the parameters of the model. When the economy is in equilibrium A or in equilibrium C we can distinguish two cases: one in which trade liberalization deters technology adoption (Case 1) and one in which trade liberalization promotes technology adoption (Case 2). Case 1 arises when the initial level of international integration is already relatively high (either because the fixed cost of exporting is relatively small or because variable trade costs are relatively low, or both of them are small) or when the initial level of international integration is small but associated with a small fixed cost of adoption and/or a large reduction in unit requirements as a result of technology adoption. In both situations the pressures on demand for goods and intermediates are already *large* and a reduction in trade costs puts a lot of pressure on demand that increases the price index of goods and deters technology adoption. Case 2, on the contrary, arises when the initial level of international integration is relatively low (either because the fixed cost of exporting is relatively large or because trade costs are relatively high, or both of them are large) and this is associated with a high fixed cost of adoption of a new technology and/or with small gains in productivity levels associated with new technology. In this case the pressures on the demand for goods and intermediates are smaller as *initially* fewer firms innovate due to the large cost of technology adoption and/or the small potential productivity gain. Thus, a larger proportion of very productive firms would profit from trade liberalization, adopting the new technology even though the price index of intermediates is raising. Moreover, in section 6.2.2 we discuss special parameter configurations under which these two cases arise independently of the value of ϕ.[4]

4 Corner solution

Under certain parameter range the model could be in a corner solution where there is no selection (i.e. $a_D^i = a_M$). More precisely if

$$\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} \geq a_M$$

[4]See the subsection "Discussion on cases 1 and 2 found when $\alpha_2^i < \alpha < 1$" of the Appendix.

which we can rearrange to

$$\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{f_E}{f_D}\frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}} \geq (a_M)^{\frac{\alpha(\sigma-1)}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}} \tag{11}$$

all firms will survive since a_M is an upper bound for the productivity distribution. In that case, $a_D^i = a_M$. The solution for P_M^i, N_D^i and E^i could be obtained by combining expressions (8)-(10) of the main manuscript together with the labour market clearing condition. The solution is given by the following expressions

$$a_D^i = a_M$$

$$N_D^i = N_E^i = \frac{\mu L}{f_E\left(\alpha+\sigma\beta(1-\alpha)\right)}$$

$$P_M^i = \left(\frac{\sigma}{\sigma-1}a_M\right)^{\frac{1}{(1-\alpha)}}\left(\frac{\beta}{\beta-1}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}\left(N_D^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}\left(\theta^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{12}$$

$$U^i = \frac{L}{\left(P_M^i\right)^{\mu}}$$

In order to recover the values for a_X^i and a_I^i one needs to use the specific equations for each type of equilibria. In particular, for the case of equilibrium A we make use of equations (3) and (5) of the main manuscript to obtain

$$a_X^A = \left(\frac{f_x}{\phi f_D}\right)^{\frac{1}{1-\sigma}} a_M$$

$$a_I^A = \left(\frac{f_I}{(1+\phi)(\gamma^{1-\sigma}-1)f_D}\right)^{\frac{1}{1-\sigma}} a_M$$

A quick inspection of the constraint (11) suggests that the range of parameters under which a corner solution may appear depends also on the level of α. For the specific case in which $\alpha \geqslant \alpha_1$, the constraint is more likely to bind, (i.e. $a_D = a_M$) when f_E is large, as this will reduce entry and it will soften competition for the incumbent firms, as in the standard Melitz (2003) model and when L is small and a_M is large. When vertical linkages are strong, competition for scarce production factors tends to be intensified and therefore in this environment an increase in population size or a decrease in a_M tends to reinforce selection. The interior solution will not re-emerge in environments where there is less scope for selection (i.e. population size is small and/or a_M is large) as this will counter-balance the increase in competition induced by high values of alpha.

4.1 Proof of propositions.

Proof of Proposition 1 Consider first the case of an interior equilibrium (i.e. $a_D < a_M$). Deriving expression (12) of the main manuscript with respect to τ we note that:

$$sign\frac{\partial P_M^i}{\partial \tau} = sign\left(\left(\frac{1-\sigma}{\chi}\right)\left(\frac{\partial \theta^i}{\partial \tau}\right)\right) \tag{13}$$

where we know that $\frac{\partial \theta^i}{\partial \tau} < 0$ since $\frac{\partial \theta^i}{\partial \tau} = \underbrace{\frac{\partial \theta^i}{\partial \phi}}_{+}\underbrace{\frac{\partial \phi}{\partial \tau}}_{-} < 0$. The sign of the derivative of the price index clearly depends on the sign of $\frac{1-\sigma}{\chi}$, and given that expression $(1-\sigma)$ is always negative, the sign of $\frac{\partial P_M^i}{\partial \tau}$ ultimately depends on the sign of χ that depends on the value of α. More precisely, if $\alpha < \alpha_1$ then χ is positive, while χ is negative if $\alpha > \alpha_1$. Consequently, we obtain that, no matter the equilibrium in which we are, $\frac{\partial P_M^i}{\partial \tau} > 0$ if $\alpha < \alpha_1$ and $\frac{\partial P_M^i}{\partial \tau} < 0$ if $\alpha > \alpha_1$.

Consider, instead, the case of a corner solution (i.e. $a_D = a_M$). Differentiating expression (12) with respect to τ we have that

$$sign\frac{\partial P_M^i}{\partial \tau} = sign\left(\left(\frac{1}{(1-\alpha)(1-\sigma)}\right)\left(\frac{\partial \theta^i}{\partial \tau}\right)\right)$$

and then since $(1-\sigma)$ is negative and $\frac{\partial \theta^i}{\partial \tau} < 0$ then we obtain that $\frac{\partial P_M^i}{\partial \tau} > 0$. Since $\frac{\partial U}{\partial P_M^i} < 0$, then we have that $\frac{\partial U}{\partial \tau} < 0$. QED.

Proof of Proposition 2 First let us consider the case of an interior equilibrium. In order to show this proposition let us compare the two cases one with vertical linkages and one without vertical linkages. In the following lines we consider the simplest case with no technology adoption. Note that in both models from expression (15) in the main manuscript we have that $\frac{\partial lnU}{\partial \tau} = -\mu\frac{\partial lnP_M}{\partial \tau}$. Note that the expression for the price index of a model with no vertical linkages will be equivalent to expression (14) in the main manuscript with $\alpha = 0$ and $\frac{\partial lnP_M}{\partial \tau}$ will be equivalent to (13) with $\alpha = 0$. Moreover, the corresponding θ^i in the case without vertical linkages is $\tilde{\theta} = 1 + \phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Therefore, $\frac{\partial lnU}{\partial \tau} = \mu\frac{\sigma-1}{\chi}\frac{\partial ln\theta^i}{\partial \tau}$ when there are vertical linkages and $\frac{\partial lnU}{\partial \tau} = \frac{\mu}{\kappa}\frac{\partial ln\tilde{\theta}}{\partial \tau}$ when there are no vertical linkages. Note that $\theta^i = \tilde{\theta}$ as the variable θ is not affected by vertical linkages and, consequently, it will take the same value in both cases with and without vertical linkages. Thus, the effect of trade liberalization on welfare is larger in the model with vertical linkages if the following inequality holds $\frac{\sigma-1}{\chi} > \frac{1}{\kappa} \Rightarrow \kappa(\sigma-1) > \chi \Rightarrow 0 > (\sigma-1)\alpha - \alpha\kappa\sigma \Rightarrow \kappa\sigma > \sigma - 1 \Rightarrow \beta > \frac{1}{\sigma}$, which is the case since $\beta > 1$ and $\sigma > 1$. In the case of a corner solution, the price index will be given by expression (12) with $\alpha = 0$. Then the impact of trade liberalization in both models is respectively given by $\frac{\partial lnU}{\partial \tau} = \mu\frac{1}{(1-\alpha)(\sigma-1)}\frac{\partial ln\tilde{\theta}}{\partial \tau}$. Note that the previous element is increasing in α, with $\alpha = 0$ for the case of no vertical linkages. Since $\tilde{\theta}$ is not affected by vertical linkages and therefore it is the same in the model with and without technology adoption, we can conclude that the impact of trade liberalization on technology adoption is larger in a model with vertical linkages also in the case of a corner solution.

The same line of reasoning applies in a case in which we compare both models with and without vertical linkages including the possibility of firms to technology upgrade. In that case θ^i instead of $\tilde{\theta}$ will be common across both models. But the difference in the effect of trade liberalization on welfare clearly depends on the same inequality as in the previous case which clearly holds. QED

Proof of Proposition 3 To show this, first note that in a model with technology adoption the welfare effect of trade liberalization is given by: $\frac{\partial lnU}{\partial\tau} = \mu\frac{\sigma-1}{\chi}\frac{\partial ln\theta^i}{\partial\phi}\frac{\partial\phi}{\partial\tau}$ in the case of an interior solution and $\frac{\partial lnU}{\partial\tau} = \mu\frac{1}{(1-\alpha)(\sigma-1)}\frac{\partial ln\theta^i}{\partial\tau}$ in the case of a corner solution. In a model without technology upgrading we have that the effect of trade liberalization on welfare is given by: $\frac{\partial lnU}{\partial\tau} = \mu\frac{\sigma-1}{\chi}\frac{\partial ln\tilde{\theta}}{\partial\phi}\frac{\partial\phi}{\partial\tau}$ for the case of an interior solution and $\frac{\partial lnU}{\partial\tau} = \mu\frac{1}{(1-\alpha)(\sigma-1)}\frac{\partial ln\tilde{\theta}}{\partial\tau}$ for the case of a corner solution with $\tilde{\theta} = 1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Since we have that $\frac{\partial ln\theta^i}{\partial\phi} > 0$, $\frac{\partial ln\tilde{\theta}}{\partial\phi} > 0$, and the other factors $\mu\frac{\sigma-1}{\chi}$, $\frac{\mu}{(1-\alpha)(\sigma-1)}$ and $\frac{\partial\phi}{\partial\tau}$ are identical in both models, the effect will be larger in a model with technology adoption when $\frac{\partial ln\theta^i}{\partial\phi} - \frac{\partial ln\tilde{\theta}}{\partial\phi} > 0$. Note that this implies to show that $\frac{\partial\ln\left(\frac{\theta^i}{\tilde{\theta}}\right)}{\partial\phi} > 0$

Equilibrium A. In equilibrium A, $\theta^i = 1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}$. Then, we have that

$$\frac{\partial\ln\left(\frac{\theta^i}{\tilde{\theta}}\right)}{\partial\phi} = \frac{\tilde{\theta}}{\theta^i}\left\{\frac{\beta(1+\phi)^{\beta-1}\left(\gamma^{1-\sigma}-1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}\left[1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]-\beta\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\left[\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}\right]}{\tilde{\theta}^2}\right\}.$$

Note that the previous derivative is bigger than zero when the following condition holds $\left[1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\right] > \phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}(1+\phi)$. Rearranging terms, this happens when $1 < \frac{f_X}{f_D\phi}$ which is always the case as, from equation (6) in the main manuscript, we obtain that $\frac{f_X}{f_D\phi} = \left(\frac{N_D^A}{N_X^A}\right)^{\frac{1}{\beta}} > 1$.

Equilibrium B. In equilibrium B, $\theta^i = 1+\left(\phi\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta} + \left(\gamma^{1-\sigma}-1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}$. Then we need to show that,

$$\frac{\partial\ln\left(\frac{\theta^i}{\tilde{\theta}}\right)}{\partial\phi} = \frac{\tilde{\theta}}{\theta^i}\left\{\frac{\beta\phi^{\beta-1}\left(\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\left[1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]-\beta\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\left[1+\left(\phi\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}\right]}{\tilde{\theta}^2}\right\} > 0.$$

Rearranging terms we obtain that $\frac{\partial\ln\left(\frac{\theta^i}{\tilde{\theta}}\right)}{\partial\phi} = \frac{\tilde{\theta}}{\theta^i}\left[\beta\phi^{\beta-1}\left(\gamma^{1-\sigma}\right)^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\frac{\gamma^{1-\sigma}-1}{\tilde{\theta}^2}\right]\left\{\frac{\left(\gamma^{1-\sigma}\right)^\beta-1}{\gamma^{1-\sigma}-1} - \left[\frac{f_I}{\left(\gamma^{1-\sigma}-1\right)f_D}\right]^{1-\beta}\right\} > 0$. This derivative is positive when $\frac{\left(\gamma^{1-\sigma}\right)^\beta-1}{\gamma^{1-\sigma}-1} - \left[\frac{f_I}{\left(\gamma^{1-\sigma}-1\right)f_D}\right]^{1-\beta} > 0$. This is always satisfied as $\frac{\left(\gamma^{1-\sigma}\right)^\beta-1}{\gamma^{1-\sigma}-1} > 1$ because $\gamma^{1-\sigma}$, $\beta > 1$ and $\left[\frac{f_I}{\left(\gamma^{1-\sigma}-1\right)f_D}\right]^{1-\beta} < 1$, since $\frac{N_I^B}{N_D^B} = \left(\frac{a_I^B}{a_D^B}\right)^\kappa = \left[\frac{\left(\gamma^{1-\sigma}-1\right)f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} < 1$.

Equilibrium C. In equilibrium C, $\theta^i = 1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}$. Then we need to show that

$$\frac{\partial\ln\left(\frac{\theta^i}{\tilde{\theta}}\right)}{\partial\phi} = \frac{\tilde{\theta}}{\theta^i}\left\{\frac{\beta\gamma^{1-\sigma}\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta-1}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}\left[1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]-\beta\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\left\{1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}\right\}}{\tilde{\theta}^2}\right\} > 0.$$

The previous derivative is positive when

$$\gamma^{1-\sigma}\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta-1}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}\left[1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}\right] > \phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\left\{1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^\beta\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}\right\}.$$

Rearranging terms in the previous expression we find that

$$\left\{\frac{f_X+f_I}{f_D\left[(1+\phi)\gamma^{1-\sigma}-1\right]}\right\}^{1-\beta}\left[1-\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]\gamma^{1-\sigma} > \phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}\left[1-\left\{\frac{f_X+f_I}{f_D\left[(1+\phi)\gamma^{1-\sigma}-1\right]}\right\}^{1-\beta}\right].$$

Since $\gamma^{1-\sigma} > 1$, the previous inequality holds whenever $\left\{\frac{f_X+f_I}{f_D\left[(1+\phi)\gamma^{1-\sigma}-1\right]}\right\}^{1-\beta} > \phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}$, as the latter also implies that $1-$

$\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta} > 1-\left\{\frac{f_X+f_I}{f_D[(1+\phi)\gamma^{1-\sigma}-1]}\right\}^{1-\beta}$. Note that $\left\{\frac{f_X+f_I}{f_D[(1+\phi)\gamma^{1-\sigma}-1]}\right\}^{1-\beta} > \phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}$ when $\frac{f_X+f_I}{f_D[(1+\phi)\gamma^{1-\sigma}-1]} < \frac{f_X}{f_D\phi}$ since $\beta > 1$. Therefore, we need to show that $\frac{f_X}{f_D\phi} > \frac{f_X+f_I}{f_D[(1+\phi)\gamma^{1-\sigma}-1]}$, which is always true when $\frac{(1+\phi)\left(\gamma^{1-\sigma}-1\right)}{\phi} > \frac{f_I}{f_X}$. This condition always as this is one of the conditions for being in equilibrium C. QED

We can go further and show that the gains from trade liberalization increase with the strength of vertical linkages and the efficiency gains derived from technology upgrading (i.e. $\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha} > 0$, $\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{1-\sigma}} > 0$) for equilibrium A. As explained above, in an interior equilibrium, the percentage change in the indirect utility level U in equation (15) in the main manuscript due to trade liberalization is

$$\frac{\frac{\partial U}{\partial\phi}}{U} = -\mu\frac{\frac{\partial P_M^i}{\partial\phi}}{P_M^i} = \frac{\sigma-1}{\chi}\mu\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}$$

and in the case of a corner solution is given by

$$\frac{\frac{\partial U}{\partial\phi}}{U} = -\mu\frac{\frac{\partial P_M^i}{\partial\phi}}{P_M^i} = \mu\frac{1}{(1-\alpha)(\sigma-1)}\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i} \tag{14}$$

Then, for the case of an interior solution, it can be shown that

$$\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha} = \mu\left(\sigma-1\right)\frac{\kappa\sigma-(\sigma-1)}{\left(\alpha+\kappa-\alpha\sigma-\kappa\sigma+\alpha\kappa\sigma\right)^2}\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i} > 0 \tag{15}$$

with the derivative that is always positive because we know from equation (14) in the main manuscript that $\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i} > 0$ and because $\beta > 1$ and $\sigma > 1$ imply that $\kappa\sigma-(\sigma-1) = (\sigma-1)\left(\frac{\kappa\sigma}{(\sigma-1)}-1\right) > 0$. Hence, when $\alpha\in[0,\alpha_1)$, χ is positive and the larger is α, the larger is the percentage increase of utility of a larger level of economic integration ϕ. The opposite holds when $\alpha\in(\alpha_1,1)$ as, in this case, χ is negative and the larger is α, the smaller is the percentage decrease in utility of an increase in the level of integration ϕ. Note that in the case of a corner solution

$$\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha} = \mu\frac{1}{(1-\alpha)^2(\sigma-1)}\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i} > 0, \tag{16}$$

where it is evident that the expression above is larger than zero.

Note that taking the derivative of (15) with respect to $\gamma^{1-\sigma}$ we find that, for the case of an interior solution,

$$\frac{\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha}}{\partial\gamma^{1-\sigma}} = \mu\left(\sigma-1\right)\frac{\kappa\sigma-(\sigma-1)}{\left(\alpha+\kappa-\alpha\sigma-\kappa\sigma+\alpha\kappa\sigma\right)^2}\frac{\partial\left(\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}} > 0$$

In the specific case of equilibrium A it can be shown that $\frac{\partial\left(\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}} > 0$. Therefore, the latter implies that the magnification effect on the gains from trade liberalization provoked by vertical linkages is not neutral to our measure of technological progress. The larger the increase in productivity as a result of technological progress $\gamma^{1-\sigma}$, the larger will be the effect of an increase in α on the gains from trade liberalization, $\frac{\frac{\partial U}{\partial\phi}}{U}$, when $\alpha \in [0, \alpha_1)$. The opposite holds when $\alpha \in (\alpha_1, 1)$ as, in this case, χ is negative and the larger is α, the smaller is the percentage decrease in utility of an increase in the level of integration. ϕ. Moreover in the case of an interior solution,

$$\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{1-\sigma}} = \mu\frac{\sigma-1}{\chi}\frac{\partial\left(\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}} \tag{17}$$

with the sign of $\frac{\partial\left(\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}}$ clearly depending on the type of equilibrium. In the specific case of equilibrium A, the sign of the derivative $\frac{\partial\left(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}\right)}{\partial\gamma^{1-\sigma}}$ is positive and, therefore, when $\alpha \in [0, \alpha_1)$, the smaller is γ, the larger is $\gamma^{1-\sigma}$ and the larger is the impact of an increase in the level of integration ϕ on the percentage increase of utility. On the contrary, when $\alpha \in (\alpha_1, 1)$, the smaller is γ, the larger is $\gamma^{1-\sigma}$ and the smaller is the percentage decrease in utility generated by an increase in the level of integration ϕ. Taking the derivative of (14) with respect to $\gamma^{1-\sigma}$, we find that for the case of a corner solution

$$\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{1-\sigma}} = \mu\frac{1}{(1-\alpha)(\sigma-1)}\frac{\partial\left(\frac{\frac{\partial\theta^i}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}} \tag{18}$$

Note that because of θ^i is the same as in the interior equilibrium, then the third factor of the right hand side of the expression above is the same as the one of an interior equilibrium for the case of equilibrium A we have from above that $\frac{\partial\left(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}\right)}{\partial\gamma^{1-\sigma}} > 0$ and, therefore, $\frac{\partial\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{1-\sigma}} > 0$.

Proof of Proposition 4 Note that, for the specific case of equilibrium A and for the case of an interior equilibrium, taking the derivative of expression (17) with respect to α we find that

$$\frac{\partial^2\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha\partial\gamma^{(1-\sigma)}} = \frac{\partial^2\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{(1-\sigma)}\partial\alpha} = \mu\frac{(\sigma-1)\left(\kappa\sigma-(\sigma-1)\right)}{\chi^2}\frac{\partial\left(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}\right)}{\partial\gamma^{1-\sigma}} > 0$$

For the case of equilibrium A and the case of a corner solution taking the derivative of expression (18) with respect to α we find that

$$\frac{\partial^2\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\alpha\partial\gamma^{(1-\sigma)}} = \frac{\partial^2\left(\frac{\frac{\partial U}{\partial\phi}}{U}\right)}{\partial\gamma^{(1-\sigma)}\partial\alpha} = \mu\frac{1}{(1-\alpha)^2(\sigma-1)}\frac{\partial\left(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^i}\right)}{\partial\gamma^{1-\sigma}} > 0$$

Therefore vertical linkages also magnify the impact that technology upgrading has on the gains from trade liberalization and vice versa the impact of technology upgrading on welfare is magnified by the presence of vertical linkages.

5 Average productivity

Following Melitz (2003, p. 1710), let us define $\tilde{a}^i$ as the "weighted average productivity of all firms (domestic and foreign) competing in a single country (where the productivity of exporters is adjusted by the trade costs τ)". Then, the aggregate price index can be expressed as

$$P_M^i = \left(N_D^i + N_X^i\right)^{\frac{1}{1-\sigma}} \left(\frac{\sigma}{\sigma-1}\right) \left(P_M^i\right)^{\alpha} \tilde{a}^i$$

that can be rewritten as

$$P_M^i = \left(N_D^i\right)^{\frac{1}{(1-\sigma)(1-\alpha)}} \left[1 + \left(\frac{a_X^i}{a_D^i}\right)^{\kappa}\right]^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{1-\alpha}} \left(\tilde{a}^i\right)^{\frac{1}{1-\alpha}}$$

The previous expression together with equation (9) in the main manuscript yields a measure of the average productivity of firms selling in the country as follows

$$\left(\tilde{a}^i\right)^{(1-\sigma)} = \frac{\left(\frac{\beta}{\beta-1}\right)\theta^i}{1 + \left(\frac{a_X^i}{a_D^i}\right)^{\kappa}} \left(a_D^i\right)^{(1-\sigma)} = \frac{\left(\frac{\beta}{\beta-1}\right)\theta^i}{1 + \frac{N_X^i}{N_D^i}} \left(a_D^i\right)^{(1-\sigma)} \tag{19}$$

The impact of trade liberalization is assessed by analyzing how a change in variable trade barriers affect the average productivity of firms selling in the country $\left(\tilde{a}^i\right)^{(1-\sigma)}$. Note that a reduction in trade barriers will increase θ^i, $\frac{a_X^i}{a_D^i}$ and, consequently, $\frac{N_X^i}{N_D^i}$, and it has an ambiguous effect on a_D^i depending on the value of α.

Taking logs and derivating (19) with respect to ϕ and making use of $\frac{N_X^i}{N_D^i} = \left(\frac{a_X^i}{a_D^i}\right)^{\kappa}$, we find that

$$\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \phi} \frac{\phi}{\left(\tilde{a}^i\right)^{(1-\sigma)}} = \left[1 - (1-\sigma)\frac{\sigma - 1 - \alpha\sigma}{\chi}\right] \frac{\partial \theta^i}{\partial \phi}\frac{\phi}{\theta^i} - \frac{\kappa \frac{N_X^i}{N_D^i}}{\left(1 + \frac{N_X^i}{N_D^i}\right)} \frac{\partial \left(\frac{a_X^i}{a_D^i}\right)}{\partial \phi} \frac{\phi}{\frac{a_X^i}{a_D^i}} \tag{20}$$

where $\frac{\partial \theta^i}{\partial \phi}\frac{\phi}{\theta^i} > 0$ and $\frac{\partial\left(\frac{a_X^i}{a_D^i}\right)}{\partial \phi} > 0$.

Notice that the sign of $\left[1 - (1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right]$ is positive when

$$\frac{(\sigma-1)(\sigma-1-\alpha\sigma)}{(\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma} > -1 \tag{21}$$

and negative otherwise. Given that the denominator $\chi \equiv (\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma$ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$, it can be shown that if $\alpha \in [0, \alpha_1)$, then $\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right] > 0$ when $\alpha \in [0, \alpha^*)$ with $\alpha^* \equiv \frac{(\sigma-1)^2+(\sigma-1)\kappa}{(\sigma-1)^2+\sigma\kappa}$, and instead $\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right] < 0$ when $\alpha \in (\alpha^*, \alpha_1)$. If, instead, $\alpha \in (\alpha_1, 1)$, χ is negative and because both the numerator and the denominator of (21) are negative, the inequality in (21) is always true and hence the sign of $\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right]$ is always positive.

Making use of these results, we need to specify the type of equilibrium in which the economy is in order to assess the effect of trade liberalization on the average productivity $\left(\tilde{a}^i\right)^{1-\sigma}$. In the case of equilibrium A, which is the relevant one for the simulation in Section 4.2, expression (20) reads

$$\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} = \left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right]\frac{\partial\theta^A}{\partial\phi}\frac{\phi}{\theta^A} - \frac{1}{\left[1+\left(\frac{f_X}{\phi f_D}\right)^{\frac{\kappa}{\sigma-1}}\right]}\frac{\kappa}{\sigma-1}$$

where $\frac{\partial\theta^A}{\partial\phi}\frac{\phi}{\theta^A} = \frac{\beta\left(\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\phi\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^{\beta-1}\left(\frac{f_I}{f_D}\right)^{1-\beta}\right)}{1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}}$. Then, when $\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right] > 0$, $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} >$ 0 if $lhs \equiv \frac{\frac{\partial\theta^A}{\partial\phi}\frac{\phi}{\theta^A}}{\beta} > \frac{1}{\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right]\left[1+\left(\frac{f_X}{\phi f_D}\right)^{\frac{\kappa}{\sigma-1}}\right]} \equiv rhs$, and, viceversa, $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} < 0$ when $lhs < rhs$. Note that lhs does not depend on α, while rhs is increasing in α (as $\frac{\partial rhs}{\partial\alpha} = \frac{(\sigma-1)^3}{\left[1+\left(\frac{f_X}{\phi f_D}\right)^{\frac{\kappa}{\sigma-1}}\right](\alpha+\kappa+2\sigma-\sigma^2-2\alpha\sigma-\kappa\sigma+\alpha\sigma^2+\alpha\kappa\sigma-1)^2} >$ 0), with a vertical asymptote in α^*, and a positive horizontal asymptote given by $\frac{(\kappa-1)\sigma+1}{\left[1+\left(\frac{f_X}{\phi f_D}\right)^{\frac{\kappa}{\sigma-1}}\right]\left[\kappa\sigma+(\sigma-1)^2\right]}$. Note that the horizontal asymptote value is indeed smaller than $\frac{1}{\left[1+\frac{\sigma-1}{\kappa}\right]\left[1+\left(\frac{f_X}{\phi f_D}\right)^{\frac{\kappa}{\sigma-1}}\right]} > 0$ which is equivalent to the rhs evaluated at $\alpha = 0$. Since this last value is also smaller than the lhs we can conclude that for $\alpha \in (\alpha_1, 1)$, $lhs > rhs$ and therefore $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} > 0$. Note that the existence of a vertical asymptote in $\alpha = \alpha^*$ implies that there exists α^{**} such that $lhs = rhs$ and $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} > 0$ for $\alpha \in (0, \alpha^{**})$ and $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} < 0$ for $\alpha \in (\alpha^{**}, \alpha^*)$.

When $\left[1-(1-\sigma)\frac{\sigma-1-\alpha\sigma}{\chi}\right] < 0$, instead, $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} < 0$ as $\frac{\partial\theta^A}{\partial\phi}\frac{\phi}{\theta^A} > 0$. Consequently, the average productivity increases with trade liberalization (i.e., $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} > 0$), when $\alpha \in (0, \alpha^{**})$ and $\alpha \in (\alpha^*, 1)$, while it decreases (i.e., $\frac{\partial\left(\tilde{a}^A\right)^{1-\sigma}}{\partial\phi}\frac{\phi}{\left(\tilde{a}^A\right)^{(1-\sigma)}} < 0$) when $\alpha \in (\alpha^{**}, \alpha^*)$.

6 Decomposition of the effects of trade liberalization on welfare

In this subsection, we explain how we decompose the effects of trade liberalization on welfare into three main components that will convey to provide a better intuition regarding these results. Figure (3) in the main manuscript contains different lines illustrating the importance of each of these components. From the labour demand for entry given by the expression $L_E = f_E N_E = \frac{(\sigma-1)\mu L}{\delta_0}$, and from the number of entrants $N_E = \frac{(\sigma-1)\mu L}{\delta_0 f_E}$ we obtain

$$N_D^i = \frac{(\sigma-1)\,\mu L}{\delta_0 f_E}\left(\frac{a_D^i}{a_M}\right)^\kappa \tag{22}$$

Substituting expression (22) in expression (15) in the main manuscript, and rearranging terms, we have that

$$P_M^i = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{1-\alpha}} \left(\frac{\beta}{\beta-1}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\frac{(\sigma-1)\mu L}{\delta_0 f_E (a_M)^{\kappa}}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\left(a_D^i\right)^{1-\sigma}\right)^{\kappa \frac{1}{(1-\alpha)(1-\sigma)^2}} \left(\left(a_D^i\right)^{1-\sigma}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\theta^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$$

Substituting the expression above in equation (15) of the main manuscript, taking logs and differentiating with respect to ϕ we find that

$$\frac{\frac{\partial U}{\partial \phi}}{U} = \underbrace{\frac{-\mu\kappa}{(1-\alpha)(1-\sigma)^2} \frac{\frac{\partial \left(a_D^i\right)^{1-\sigma}}{\partial \phi}}{\left(a_D^i\right)^{1-\sigma}}}_{\text{Number of domestic varieties}} + \underbrace{\frac{\mu}{(1-\alpha)(\sigma-1)} \frac{\frac{\partial \left(a_D^i\right)^{1-\sigma}}{\partial \phi}}{\left(a_D^i\right)^{1-\sigma}}}_{\text{Selection}} + \underbrace{\frac{\mu}{(\sigma-1)(1-\alpha)} \frac{\frac{\partial \theta^i}{\partial \phi}}{\theta^i}}_{\text{theta}} \tag{23}$$

As it can be seen in the expression above, the impact of trade liberalization on welfare can be decomposed, into three main important channels, the impact of trade liberalization on the number of domestic varieties, the selection channel that captures the impact of trade liberalization on welfare through changes in the unit input cut-off and the "theta" channel, which captures the impact of trade liberalization on the indicator of trade openness and innovation opportunities, θ^i. Changes in this last element capture the direct impact of trade liberalization on innovation and exporting keeping constant selection.

7 Robustness checks

This section presents some additional exercises that show that our results are robust to alternative but realistic parameter configurations. The exercises also show that the contribution of technology upgrading to the gains from trade can be larger under these alternative parameter configurations.

7.1 R&D intensity

Our benchmark exercise considers the average R&D intensity for the case of the US. The BRDIS provides also data at the sectoral level for the R&D intensity. Therefore we have considered as possible alternative values for the R&D intensity, the average one for the manufacturing industries (4.4%), the average one for the non-manufacturing industries (3.2%), and three of the top-values in the sample, which are the one of the Chemical industry (6.7%), the one of the computer and electronics industry (9.8%) and the one of the Software Publishers, which is one of the highest values in the non-manufacturing industries (8.2%). In addition, we have considered alternative values of 7% which is just double the original value and 12% as the highest possible value. The exercise has been run assuming that the value for the other parameters, which can be found in Table (2) in the main manuscript, have not changed. The results reported in the table 2 considers a vertical intensity equal to $\alpha = 0.48$, the average for the US economy.

R&D int	f_I	γ	Survival	Innovation	Gains from trade	Innovation Gains	IGshare
3.2	4.17	0.9212	-7.49	5.36	3.41	0.12	3.6
4.4	6.40	0.89	-7.60	5.23	3.47	0.17	5.17
6.7	12.57	0.81	-7.85	4.95	3.59	0.29	8.8
7	13.65	0.80	-7.89	4.91	3.61	0.31	9.3
8.2	18.96	0.76	-8.04	4.73	3.68	0.39	11.65
9.8	30.13	0.69	-8.29	4.45	3.80	0.50	15.2
12	67.48	0.56	-8.71	3.97	4.01	0.71	21.5

Table 2: R&D intensity. Robustness checks

Table 2 displays the values for the fixed cost of innovation, f_I, and the innovation step, γ, calibrated using the corresponding value for R&D intensity. The table also shows the percentage change in the probability of survival, $\left(\frac{a_D}{a_M}\right)^{\kappa}$, (Column 4) and the percentage change in the probability to innovate, $\left(\frac{a_I}{a_M}\right)^{\kappa}$, (Column 5) following trade liberalization. The table also displays at Column 6 the welfare gains from trade measured in percentage points. These differences are not very large comparing with our benchmark results.

The last two columns represent, respectively, the contribution of technology upgrading to the gains from trade (column labelled Innovation Gains) and the share of the total gains from trade that this contribution represents (column labelled IGshare). This contribution is measured by the difference in percentage points of the effects of trade liberalization on welfare between a model with and without technology upgrading. It is important to mention that as the R&D intensity increases, the contribution of this channel becomes more important. This is the consequence of the fact that the increase in *R&D* intensity implies values of the parameters that induce an increase in innovation activity at the industry level. While including technology upgrading in our model implied an increase in welfare of 0.13 percentage points, should the R&D intensity being 9.8 the increase would have been 0.50 percentage points. Note that this will be more than triple the increase suggested in our benchmark exercise. Looking at the contribution in terms of the share of total gains, the increase in R&D intensity substantially rises this number. While in the benchmark case the contribution of technology upgrading represents 3.93% of the total gains from trade, the contribution rises to 15.2% for the case in which the R&D intensity is 9.8%.

7.2 Percentage of firms undertaking R&D activity

Table 3 shows the results for an alternative value for the percentage of firms reporting positive process innovation provided by the BRDIS. In particular, we have used the average for the period of 2014-2016 which is 2.8%. The results are reported for an average intensity of $\alpha = 0.48$, assuming that the rest of the parameters are the same as in table 2 (the *R&D* intensity used is the one related to the benchmark case, 3.9%). As it can be seen the results display some quantitative difference but this difference is not substantial.

Extensive	f_I	γ	Survival	Innovation	Gains from trade	Innovation Gains	IGshare
2.8	5.91	0.91	-6.53	5.35	3.42	0.12	3.62

Table 3: Mass of firms Innovating. Robustness checks.

7.3 Elasticity of Substitution

In footnote 41 of section 4.2, we report that other authors have used other values for the elasticity of substitution σ. Simonovska and Waugh (2010) consider a plausible range for the elasticity of substitution of $\sigma = 4.10 - 4.27$ and Costinot and Rodriguez-Clare (2010) consider a value of $\sigma = 5$. In what follows we report the results for these alternative values of σ.

Variable	α				
	0	0.48	0.62	0.76	0.85
$\left(\frac{a_D}{a_M}\right)^{\kappa}$ (Survival Prob)	-9.86	-7.62	-5.57	-0.30	18.54
$\left(\frac{a_I}{a_M}\right)^{\kappa}$ (Innovation Prob)	2.51	5.06	7.39	14.01	34.82
$\tilde{a}^{1-\sigma}$ (Average productivity)	11.44	9.46	7.73	3.08	-8.74
Gains from trade	1.62	3.43	5.07	9.73	23.70
Innovation gains	0.06	0.13	0.19	0.38	0.98

Table 4: Results for $\sigma = 4.1$. Robustness checks.

Variable	α				
	0	0.48	0.62	0.76	0.85
$\left(\frac{a_D}{a_M}\right)^{\kappa}$ (Survival Prob)	-10.02	-7.78	-5.80	-1.1	13.76
$\left(\frac{a_I}{a_M}\right)^{\kappa}$ (Innovation Prob)	2.07	4.61	6.86	12.96	29.06
$\tilde{a}^{1-\sigma}$ (Average productivity)	12.26	10.16	8.37	3.84	-6.27
Gains from trade	1.65	3.43	4.99	9.19	19.97
Innovation gains	0.06	0.12	0.19	0.33	0.80

Table 5: Results for $\sigma = 4.27$. Robustness checks.

Variable	α				
	0	0.48	0.62	0.76	0.85
$\left(\frac{a_D}{a_M}\right)^{\kappa}$ (Survival Prob)	-10.53	-8.31	-6.53	-2.40	5.19
$\left(\frac{a_I}{a_M}\right)^{\kappa}$ (Innovation Prob)	0.43	2.92	4.93	9.56	18.09
$\tilde{a}^{1-\sigma}$ (Average productivity)	15.59	12.96	10.93	6.50	-0.74
Gains from trade	1.74	3.42	4.77	7.85	13.40
Innovation gains	0.03	0.06	0.08	0.14	0.24

Table 6: Results for $\sigma = 5$. Robustness checks.

8 Appendix Tables and Figures

Country	Int.input share (mean)	Int.input share (Weighted Mean)
AUS	0.5178	0.4889
AUT	0.5343	0.5012
BEL	0.5839	0.5550
BGR	0.5534	0.5513
BRA	0.4688	0.4342
CAN	0.5328	0.4793
CHE	0.5343	0.5037
CHN	0.6224	0.6651
CYP	0.4804	0.4614
CZE	0.5802	0.5990
DEU	0.5163	0.4910
DNK	0.5283	0.4914
ESP	0.5207	0.4945
EST	0.5621	0.5408
FIN	0.5316	0.5184
FRA	0.5380	0.4754
GBR	0.5104	0.4696
GRC	0.4815	0.4105
HRV	0.5038	0.4776
HUN	0.5239	0.5505
IDN	0.4896	0.4924
IND	0.4317	0.4504
IRL	0.4794	0.5376
ITA	0.5369	0.5025
JPN	0.5259	0.4888
KOR	0.5612	0.5915
LTU	0.4307	0.4732
LUX	0.5651	0.7226
LVA	0.5367	0.5572
MEX	0.4581	0.4106
MLT	0.5848	0.6746
NLD	0.5322	0.5049
NOR	0.5085	0.4292
POL	0.5220	0.5242
PRT	0.5228	0.4863
ROU	0.5142	0.5289
ROW	0.5661	0.5743
RUS	0.5278	0.4966
SVK	0.5410	0.5765
SVN	0.5271	0.5221
SWE	0.5176	0.4890
TUR	0.4920	0.5003
TWN	0.5260	0.5610
USA	0.4855	0.4433

Table 1: Average intermediate input share (2014). Industry weighted means that each industry observation has been weighted by the importance of that sector's output in the country's total output. Source: Authors' computations from the World Input Output Database.

Ind name	Ind. Code	Int.Input Share
Crop and animal production, hunting and related service activities	1	0.5340
Forestry and logging	2	0.4030
Fishing and aquaculture	3	0.4470
Mining and quarrying	4	0.4496
Manufacture of food products, beverages and tobacco products	5	0.7101
Manufacture of textiles, wearing apparel and leather products	6	0.6266
Manufacture of wood and of products of wood and cork, except furniture [...]	7	0.6716
Manufacture of paper and paper products	8	0.6979
Printing and reproduction of recorded media	9	0.5930
Manufacture of coke and refined petroleum products	10	0.8021
Manufacture of chemicals and chemical products	11	0.7003
Manufacture of basic pharmaceutical products and pharmaceutical preparations	12	0.5664
Manufacture of rubber and plastic products	13	0.6955
Manufacture of other non-metallic mineral products	14	0.6314
Manufacture of basic metals	15	0.7453
Manufacture of fabricated metal products, except machinery and equipment	16	0.6253
Manufacture of computer, electronic and optical products	17	0.6101
Manufacture of electrical equipment	18	0.6655
Manufacture of machinery and equipment n.e.c.	19	0.6328
Manufacture of motor vehicles, trailers and semi-trailers	20	0.7088
Manufacture of other transport equipment	21	0.6160
Manufacture of furniture; other manufacturing	22	0.6013
Repair and installation of machinery and equipment	23	0.5322
Electricity, gas, steam and air conditioning supply	24	0.6125
Water collection, treatment and supply	25	0.4296
Sewerage; waste collection, treatment and disposal activities [...]	26	0.5496
Construction	27	0.6042
Wholesale and retail trade and repair of motor vehicles and motorcycles	28	0.4251
Wholesale trade, except of motor vehicles and motorcycles	29	0.4375
Retail trade, except of motor vehicles and motorcycles	30	0.3868
Land transport and transport via pipelines	31	0.5206
Water transport	32	0.6257
Air transport	33	0.7191
Warehousing and support activities for transportation	34	0.5270
Postal and courier activities	35	0.4403
Accommodation and food service activities	36	0.4834
Publishing activities	37	0.5383
Motion picture, video and television programme production [...]	38	0.5247
Telecommunications	39	0.4707
Computer programming, consultancy and related activities; information service activities	40	0.4361
Financial service activities, except insurance and pension funding	41	0.3670
Insurance, reinsurance and pension funding, except compulsory social security	42	0.5243
Activities auxiliary to financial services and insurance activities	43	0.4620
Real estate activities	44	0.2361
Legal and accounting activities; activities of head offices; management consultancy activities	45	0.4065
Architectural and engineering activities; technical testing and analysis	46	0.4393
Scientific research and development	47	0.3842
Advertising and market research	48	0.6035
Other professional, scientific and technical activities; veterinary activities	49	0.4716
Administrative and support service activities	50	0.4178
Public administration and defence; compulsory social security	51	0.3047
Education	52	0.2126
Human health and social work activities	53	0.3529
Other service activities	54	0.4281
Activities of households as employers; undifferentiated goods[...]	55	0.054
Activities of extraterritorial organizations and bodies	56	0.8564

Table 2: Industry average intermediate input intensity (2014). Source: Authors' calculations from the World Input Output Database.

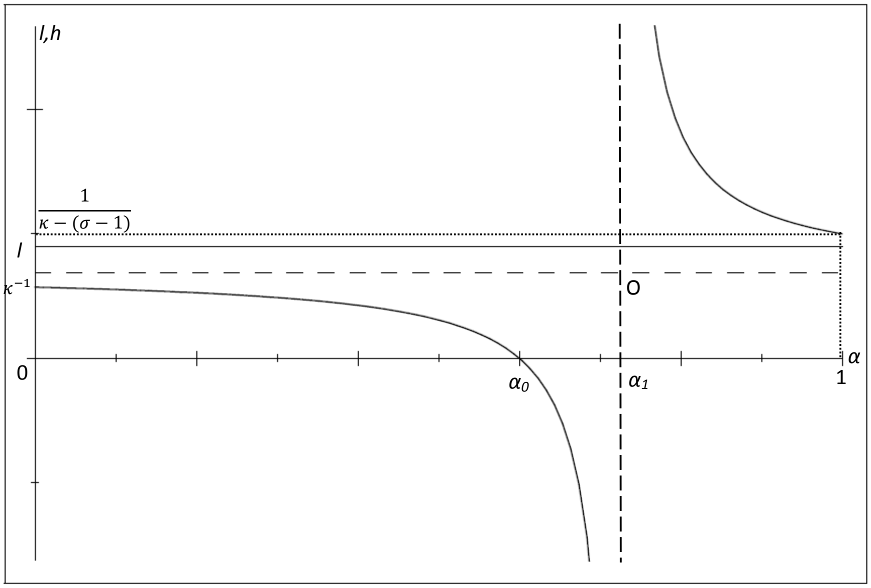

Figure 1: Parameter range for the case 1 in equilibrium A.

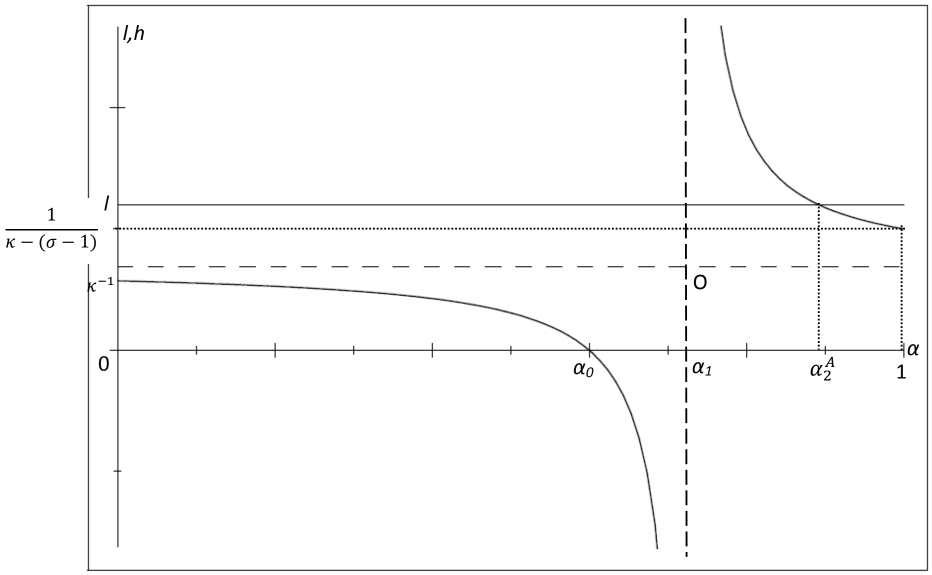

Figure 2: Parameter range for case 2 in equilibrium A.