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Validation of the Averaged Strain Energy Density Criterion for Additively Manufactured Notched Polylactide Acid Specimens

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Abstract

The applicability of the Averaged Strain Energy Density (ASED) criterion [1] to predict the failure of notched Polylactide Acid (PLA) specimens fabricated by Fused Deposition Modeling (FDM) is validated by means of experimental data reported by Ahmed and Susmel [2]. Difficulties when estimating the ASED control volume radius based on the measured fracture toughness are revealed and discussed, whereas the accuracy of the ASED criterion is found to be satisfying when a novel alternative approach is used to define the control volume size.

Keywords: Polylactide; PLA; Strain Energy Density; ASED; Critical Distances; TCD; Validation;

1. Introduction and Motivation

Although notches and cracks are omnipresent in engineering applications, they still pose problems to accurate failure prediction. For many practical applications, it is desirable to have a simple and robust way to locally predict failure of notched and cracked components of arbitrary shape using a simple linear-elastic finite element simulation and coarse meshes. Two of many methods to achieve this are the Theory of Critical Distances (TCD) [2] and the Averaged Strain Energy Density (ASED) [1] criterion. Although the latter has been used extensively for classical materials, its limits in the domain of additive manufacturing remain largely unexplored [3]. With its many potential benefits and use-cases such as rapid prototyping, complex topology optimization and massive weight reduction across many disciplines ranging from medical to aeronautical engineering, additive manufacturing clearly needs to be deeply understood in order to bridge the large gap between its capabilities and its current industrial utilization. The conceptu-

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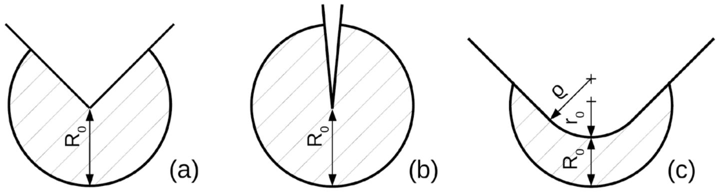

Fig. 1. Control volumes for different notch geometries, ranging from sharp v-notches (a) and cracks (b) to blunt v- and u-notches (c). Inspired from [1].

ally similar TCD was already validated for FDM-printed PLA specimens by Ahmed and Susmel [2], and the purpose of this article is to use the very same data to validate the ASED criterion.

2. A Brief Introduction to the TCD and the ASED Criterion

Instead of using the difficult-to-obtain stress in the notch root to assess failure, the TCD [4] suggests using an effective stress σ_{eff}, which can be the maximum principal stress σ_I (i) at a distance of $L/2$ on the notch bisector line (Point Method), (ii) averaged over a path of length $2L$ on the notch bisector line (Line Method), or (iii) averaged over a semicircular area of radius L in the notch root (Area Method). Especially the Line Method can be motivated easily by Neuber's structural support concept [5]. Failure occurs when $\sigma_{\text{eff}} \geq \sigma_0$, where σ_0 denotes the so-called inherent material strength. For purely brittle materials, simple relations follow from linear-elastic fracture mechanics:

$$\sigma_{\text{eff}} = \sigma_{\text{UTS}} \quad \text{and} \quad L = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_{\text{UTS}}}\right)^2. \tag{1}$$

Herein, σ_{UTS} and K_{Ic} denote the tensile strength and the fracture toughness respectively. For more ductile materials, the two material parameters L and σ_0 need to be calibrated by requiring self-consistency of the Point Method: In a plot over the notch bisector line, σ_I from the bluntest and sharpest notch within the set of considered geometries must intersect at $\sigma_I(r = L/2) = \sigma_0$. For just slightly ductile materials, the bluntest notch can be replaced by a smooth specimen, so $\sigma_0 = \sigma_{\text{UTS}}$ holds again and a single experiment suffices to calibrate L via $\sigma_I(r = L/2) = \sigma_{\text{UTS}}$.

The ASED criterion [1] is conceptually similar but differs in that the strain energy density $\psi = 1/2\ \varepsilon_{ij} C_{ijkl} \varepsilon_{kl} = 1/2\ \sigma_{ij}\varepsilon_{ij}$ is averaged instead of σ_I and the averaging domain is a crescent Ω as shown in Figure 1, where $r_0 = \frac{\pi - 2\alpha}{2\pi - 2\alpha}\rho$ and R_0 plays the role of L. Again, failure occurs when the load parameter W reaches a critical value $W = \frac{\int \psi \mathrm{d}\Omega}{\int \mathrm{d}\Omega} \geq W_c$, and the two material parameters follow from linear-elastic fracture mechanics [6]:

$$W_c = \frac{\sigma_{\text{UTS}}^2}{2E} \quad \text{and} \quad R_0 = c \cdot \left(\frac{K_{Ic}}{\sigma_{\text{UTS}}}\right)^2, \ c = \begin{cases} \dfrac{(1+\nu)(5-8\nu)}{4\pi}, & \text{plane strain} \\ \dfrac{5-3\nu}{4\pi}, & \text{plane stress} \end{cases} \tag{2}$$

Herein, E and ν denote the Young's modulus and Poisson's ratio respectively. Both the TCD and ASED criterion have been used successfully for static fracture and high cycle fatigue [4, 7, 1], have been validated for a variety of different materials [8, 9, 1] and are very simple to apply. The ASED criterion owes its success mainly to its additional capabilities to easily acccount for mixed-mode loading [8], T-stresses [10], and more. Most importantly, the ASED criterion allows for extremely coarse meshes [11].

3. Experimental Data Reported in the Literature

Ahmed and Susmel [2] recently reported an experimental campaign with failure results for notched FDM-produced PLA specimens together with the corresponding TCD predictions. The filaments were deposited in a woven orthogonal structure at a certain angle to the specimen orientation, θ_p. Different θ_p led to different tensile strengths σ_{UTS}, Young's moduli E and fracture thoughnesses K_{Ic}, which were determined from separate experiments. Note that the definition of the printing angle is slightly different as $\theta_p = 0°$ denotes diagonal fiber loading and $\theta_p = 45°$ is parallel and orthogonal. Their campaign covered many geometries, but the preliminary results presented herein are confined to the configurations shown in Table 1 together with their average failure loads for each printing angle. For the TCD predictions, the length scale L was calibrated from notched specimens and not from the measured fracture toughness. The emergent material strength σ_0 differed significantly from σ_{UTS}, indicating that the material ductility can not be neglected. Amongst other effects, the complicated mesostructure and other subtleties of the manufacturing process triggered a zigzag crack path with local mixed-mode propagation, a θ_p-dependent degree of ducility, and crack initiation at a distance from the notch root. Despite these and other complex phenomena, the Point and Area Method yielded good predictions, whereas the Line Method was not applicable due to $2L$ exceeding the specimen geometry. Further information can be taken from the reference paper [2].

Table 1. Considered configurations together with their average failure loads as reported by Ahmed and Susmel [2].

2α (°)	r (mm)	$F_{\mathrm{f,\,avg}}^{\theta_p=0°}$ (N)	$F_{\mathrm{f,\,avg}}^{\theta_p=30°}$ (N)	$F_{\mathrm{f,\,avg}}^{\theta_p=45°}$ (N)
30	0.05	1040	829	875
135	0.4	1000	754	649
135	1.0	927	693	642
135	3.0	899	722	744

4. Failure Prediction Results and Discussion

The strain energy density fields were obtained from linear elastic Finite Element simulations. ABAQUS was chosen for both meshing and solving with quadratic plane strain elements and a simple linear elastic (isotropic) material model. The simulations were made with different material parameters for each printing angle as well as using the properties averaged over θ_p in order to test the robustness of the method.

When applying the ASED criterion to the reported data, the standard approach is using Equation (2) to estimate R_0. Note that in [2], two very different sets of fracture toughnesses were reported. Obviously, the radii computed from them also differ, as can be seen in Table 2. Both sets of K_{Ic} values led to a large scatter and very conservative predictions.

However, in their analyses, Ahmed and Susmel [2] did not use K_{Ic} to obtain the material length scale L, they used the more robust approach, where L is calibrated from a part of the notched specimens. In order to create comparable conditions, an analogous procedure is applied to the ASED criterion: For the smallest notch root radius, $\psi(x)$ can be obtained from an FE computation with the boundary conditions from the failure experiment and averaged over control volumes with different radii R_0. Then, the intersection point $W(R_0) = W_c$ defines the choice of R_0 for the subsequent analyses. This idea was already presented for cyclic loading in [12], but is not commonly used for the ASED criterion. The procedure is shown in Figure 2 and the obtained R_0 are listed in Table 2.

As can be seen in Figure 3, the predictions of the ASED criterion using the more robust length scale calibration method are satisfactory. The predictions based on average material properties and one single R_0 and W_c for all data demonstrate the robustness of the method. The scatter is similar to what Ahmed and Susmel reported for the TCD, but slightly shifted towards the conservative side. This is most likely because they did not use the smallest but the second smallest notch geometry for calibration, although the stress concentration factor was around three times lower, leading to less conservative predictions.

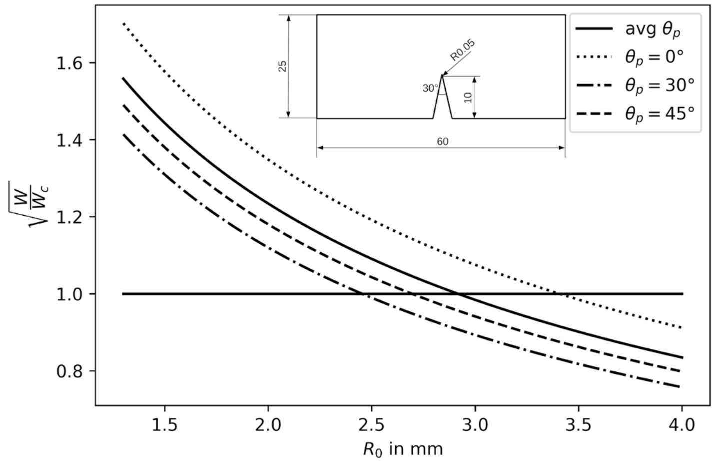

Fig. 2. Accuracy of the ASED prediction for the smallest specimen as a function of R_0. Requiring $\sqrt{W/W_c} = 1$ can serve as a way to estimate R_0 from a single experiment.

Further investigation reveals that the biggest outliers in Figure 3 come from the specimens where $\theta_p = 0°$, which means that the loading occurs at an angle of $\pm 45°$ to the fiber directions[1]. This can be explained by a change of the governing fracture mechanism from brittle to ductile when the loading changes from parallel to the layers to diagonal. As the damage initiates in the weak interface between fibers, diagonal loading allows for a significant amount of fiber reorientation and therefore energy absorption to take place. Indications for this phenomenon cannot just be found in the stress-strain curves, but also in the fracture surface photographs given by Ahmed and Susmel [2]. Since the ASED criterion as a purely brittle criterion assumes no energy dissipation, it produces conservative results when ductility plays a role.

5. Conclusions

The complicated mesostructure and other subtleties of the manufacturing process trigger a complicated zigzag crack path with local mixed-mode propagation, fibre reorientation and therefore a change from brittle to ductile frac-

[1] Note that the ASED criterion does not consider this anisotropy.

Table 2. ASED control volume sizes R_0 computed from Equation (2) for different printing angles using the K_{Ic} values reported in [2] and from the presented calibration procedure.

	K_{Ic} from SENT specimen		K_{Ic} from CT specimen		proposed calibration
θ_p in °	K_{Ic} in MPa m$^{1/2}$	R_0 in mm	K_{Ic} in MPa m$^{1/2}$	R_0 in mm	R_0 in mm
0	3.7	1.73	4.6	2.67	3.42
30	3.4	1.59	4.0	2.19	2.46
45	3.0	1.14	4.2	2.24	2.70
avg	3.4	1.50	4.3	2.40	2.92

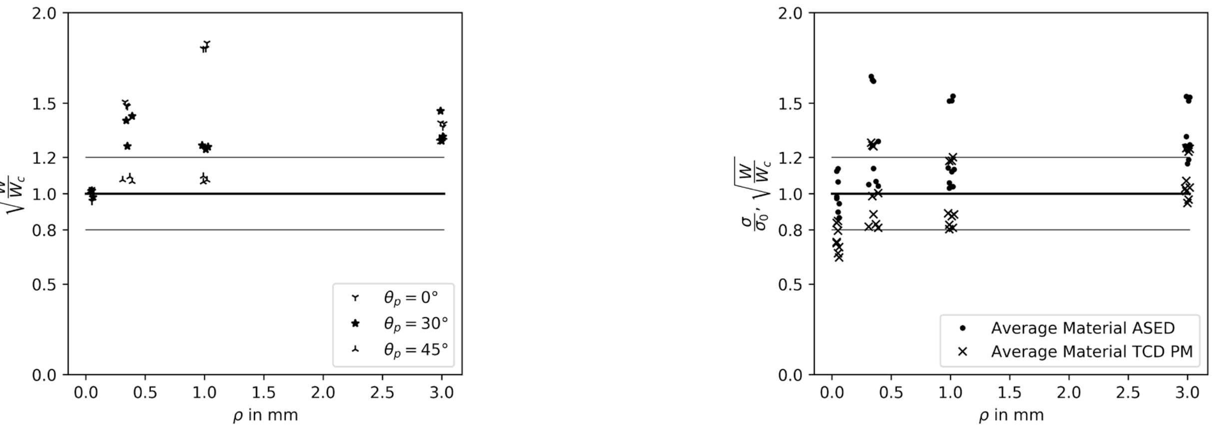

Fig. 3. Accuracy of the ASED criterion applied to the data from [2]. The thin lines denote the ±20 % scatter band. Left: Each θ_p calibrated separately. Right: Average material properties and comparison to accuracy of TCD Point Method.

ture, crack initiation somewhat away from the notch root and other complex phenomena. Considering these conditions and the simplicity of the ASED criterion, the observed scatter for the presented calibration procedure is a success. As the accuracy of the ASED criterion and the TCD is almost the same when using comparable methods for calibrating the length scale, one might question the utility of the ASED criterion. Therefore, it is worth noting that the main advantage of the ASED criterion is the high tolerance of extremely coarse meshes. Further investigations on the accuracy of the methodology on various FDM process parameters and component geometries should be performed.

References

[1] Berto, F. and Lazzarin, P., 2014. Recent developments in brittle and quasi-brittle failure assessment of engineering materials by means of local approaches. Materials Science and Engineering: R: Reports, 1–48.

[2] Ahmed, A.A. and Susmel, L., 2018. A material length scale-based methodology to assess static strength of notched additively manufactured polylactide (PLA). Fatigue & Fracture of Engineering Materials & Structures, 1–28.

[3] Razavi S.M.J., Berto F., 2019. Directed Energy Deposition versus Wrought Ti-6Al-4V: A Comparison of Microstructure, Fatigue Behavior, and Notch Sensitivity. Advanced Engineering Materials 21(8), 1900220

[4] Susmel, L. and Taylor, D., 2008. The theory of critical distances to predict static strength of notched brittle components subjected to mixed-mode loading. Engineering Fracture Mechanics 75, 534–550.

[5] Neuber, Heinz, 2013. Kerbspannungslehre: Theorie der Spannungskonzentration Genaue Berechnung der Festigkeit. Springer-Verlag.

[6] Yosibash, Zohar and Bussiba, Arie and Gilad, Ilan, 2004. Failure criteria for brittle elastic materials. International Journal of Fracture 125, 307–333.

[7] Susmel, Luca, 2008. The theory of critical distances: a review of its applications in fatigue. Engineering Fracture Mechanics 7, 1706–1724.

[8] Razavi, S.M.J. and Aliha, M.R.M. and Berto, F., 2018. Application of an average strain energy density criterion to obtain the mixed mode fracture load of granite rock tested with the cracked asymmetric four-point bend specimens. Theoretical and Applied Fracture Mechanics 97, 419–425.

[9] Torabi, A. R. and Habibi, R. and Mohammad Hosseini, B., 2015. On the Ability of the Equivalent Material Concept in Predicting Ductile Failure of U-Notches under Moderate- and Large-Scale Yielding Conditions. Physical Mesomechanics 18, 337–347.

[10] Razavi, S.M.J. and Ayatollahi, M.R. and Berto, F, 2018. A synthesis of geometry effect on brittle fracture. Engineering Fracture Mechanics 187, 94–102.

[11] Lazzarin, Paolo and Berto, Filippo and Zappalorto, Michele, 2010. Rapid calculations of notch stress intensity factors based on averaged strain energy density from coarse meshes: Theoretical bases and applications. International Journal of Fatigue 10, 1559–1567.

[12] Razavi, S.M.J. and Ferro, P. and Berto, F. and Torgersen, J., 2018. Fatigue strength of blunt V-notched specimens produced by selective laser melting of Ti-6Al-4V. Theoretical and Applied Fracture Mechanics 97, 376–384.