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Optimal dynamic volume-based price regulation*

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Abstract

We consider a model of optimal price regulation in markets where demand is sluggish and asymmetric providers compete on quality. Using a spatial model, which is suitable to investigate the health care and education sector, we analyse within a dynamic set-up the scope for price premiums or penalties on volume. Under the assumption of symmetric cost information, we show that the socially optimal time path of quality provision off the steady state can be replicated by a simple dynamic pricing rule where the dynamic part of the rule is *ex-ante non-discriminatory* in the sense that the price premium or penalty on volume is common across providers, despite their differing production costs. Whether the price schedule involves a penalty or a premium on volume relates to two concerns regarding production costs and consumer benefits, which go in opposite directions. Price adjustments over time occur only through the price penalty or premium, not time directly, which highlights the simplicity and thus applicability of this regulation scheme.

Keywords: Price regulation; Quality; Differential games.

JEL Classification: C73, I11, I14, L13.

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1 Introduction

In markets such as health care or education, prices are commonly regulated across a range of OECD countries. Providers instead compete for consumers on product quality. But quality is not easily observable to regulators, although one of the key objectives of regulation is to improve quality. Instead, regulators observe volumes (e.g., patients, pupils, students) and these can be used indirectly to incentivise quality improvements.

In this study, we investigate optimal price regulation under the assumption that demand is *sluggish*, and it is modelled such that only a fraction of consumers respond to quality changes at each point in time. This implies that it will take some time before potential demand is fully realised. Sluggish demand is a plausible assumption in health care and education, due to asymmetry of information between providers and consumers (Arrow, 1963) and related uncertainty and noise in the observed quality, or due to consumer habits, trust or confidence in particular providers.[1]

In markets where demand responds sluggishly to changes in quality, adjustments to any type of shock (e.g., exogenous cost changes or entry of new providers) are likely to take a long time, implying that such markets might seldom be characterised by allocations of qualities and demand that are close to a steady state outcome. This implies, in turn, that a price regulation scheme based on theoretical insights from static equilibrium analyses is unlikely to produce a socially optimal outcome. What is instead called for is a dynamic analysis that allows for a characterisation of the equilibrium (as well as the socially optimal) dynamic path off the steady state.

Finding simple rules for price regulation that induce a more efficient quality provision is a challenge for regulation authorities, even in a static context. Dynamic effects of price regulation on volume and quality, due to demand sluggishness, make the challenge considerably more difficult. The main contribution of the present study is to characterise the properties of a specific form of dynamic price regulation, in which the unit price paid to the provider is an affine function of demand. We suggest a price regulation scheme that allows for the price to increase with demand, a form of price *premium* on volume, or to decrease with demand, a price *penalty* on volume. This scheme is not only simple and thus applicable, since volume is easily observable while quality is not, but it is also potentially efficient. Under some assumptions, including symmetric cost information, we show that a welfare maximising regulator, by optimally choosing this pricing rule, is able to induce the socially optimal dynamic path of quality provision.

[1] See Jung et al. (2011) and Raval and Rosenbaum (2018) for empirical evidence of hospital demand sluggishness.

This kind of volume-based price regulation is in line with examples of price regulation in health care markets, which is the main motivating example for our analysis.[2] An early example is what were known as 'cost and volume' contracts when the purchaser-provider split was introduced in England in the early nineties under the internal markets. Hospitals facing higher-than-agreed volumes would face a reduction in the DRG tariff of up to 30% of the of agreed tariff below the first volume threshold, and with the tariff gradually decreasing with higher volume thresholds (Fenn et al., 1994). In recent years, concerns for cost containment have again led purchasers to make increased use of volume caps or reduced tariffs for volumes in excess of expected ones (Allen and Petsoulas, 2016).[3] There are proposals to introduce 'blended' payments that comprise a fixed amount and a variable volume-related element that reflects actual activity with a payment as low as 20% of the regular tariff (NHS Improvement, 2018). Is such a price regulation scheme, with penalties for higher volumes, likely to be socially efficient in a dynamic sense? Or would instead social welfare improve by the adoption of a pricing scheme that *rewards* higher volumes? This study identifies some general conditions that can answer these questions.

We use a Hotelling model of quality competition under regulated prices. Although this model in some sense represents a convenient simplification, such a spatial competition framework does have some features that seem particularly relevant for health care markets (and also education markets, as we discuss in Section 8). These are markets where demand decisions are in line with the unit demand assumption (e.g., each patient demands one medical treatment), and where travel distance is a key factor (in addition to quality) in determining consumers' choice of provider.[4] The assumption of fixed total demand is also a reasonable approximation to markets with non-price competition and small (or even zero) consumer copayments, implying that total demand is highly inelastic.

We consider an infinite-time horizon differential game with two providers, located at the extremes of a unit line, offering one product each. We allow providers to differ in production costs (due to differences in land, capital and labour costs), and such differences are observable to the regulator. In turn, such differences in costs can affect the optimal regulated price, which we also

[2] In Section 8 we discuss and give examples of how our analysis might also apply to other sectors, such as education and child care.

[3] There are elements of this form of price regulation in other countries. For example, in Germany and the Netherlands, the DRG-based payment systems operate within a global budget. If this budget is reached, hospitals receive a reduced tariff up to no payment. Comparable mechanisms are also in place in countries such as France, Poland and Hungary (Moreno-Serra and Wagstaff, 2010).

[4] Many empirical studies confirm that travelling distance and quality are key predictors of hospital choice (Gutacker et al., 2016; Kessler and McClellan, 2000; Tay, 2003).

allow to differ across providers. As an extension, we also consider an alternative scenario in which cost differences are not observable to the regulator, and therefore the price scheme is identical across providers.

We first characterise the optimal provider plans regarding product quality over time, under what is known in differential games as the state feedback solution, where current choice of quality depends on current demand (the state variable) which is observable by providers.[5] We then proceed by taking a social welfare perspective and deriving the optimal price regulation rule, which provides the most novel insights. We show that, under asymmetric information on costs, the socially optimal time path of quality provision off the steady state can be replicated by a dynamic pricing rule where the dynamic part of the rule is *ex-ante non-discriminatory*, so that the price premium or penalty on volume is common across providers, despite differing production costs. Instead, the fixed price component differs across providers to reflect different costs, with a higher value for the provider with lower marginal production costs. Price adjustments over time occur only through the price penalty or premium on volume, not time directly, which highlights the simplicity and applicability of this regulation scheme.

Whether the price schedule involves a penalty or a premium on volume relates to two conflicting concerns regarding production costs and consumer benefits. Under the assumption of decreasing returns to scale, concerns for cost-efficient production dictate that demand should be steered towards the provider with lower demand, which can be achieved by a quality reduction for the high-demand provider. Instead, concerns for consumer welfare dictate that the high-demand provider should invest more in quality, implying that demand is steered away from the low-demand provider. If the former concern for cost efficiency dominates, so that welfare is increased by reducing (increasing) quality of the high-demand (low-demand) provider, this can be achieved by introducing a price penalty on volume, which reduces the price-cost margin, and thus incentives for quality investments, of the high-demand provider relative to the low-demand provider. This is optimal if the convexity of production costs is sufficiently high.

Although some dimensions of costs are observable to the regulator (e.g., related to land, capital and labour costs), others dimensions of costs might not be observable. Therefore, as an extension, we assume that discriminatory (provider-specific) pricing is not possible due to asymmetric

[5] As an alternative solution concept, we also consider the open-loop solution, where providers set their quality plans at the beginning of the game, hence quality only depends on time along the equilibrium path. This solution concept might be a more likely representation of a setting where providers can (or must) commit to long-term plans when deciding on quality investments and act within a heavily regulated environment.

information on costs. We show that this might increase the scope for a price premium on volume, particularly if the cost difference between the providers is sufficiently large.

The rest of the study is organised as follows. In Section 2 we provide a brief overview of the relevant literature before presenting the model in Section 3. In Section 4 we derive the providers' optimal choice of quality under the state feedback solution concept. In Section 5 we introduce the welfare analysis by deriving the first-best solution, and in Section 6 we show how this solution can be implemented by an optimally chosen dynamic price rule. Section 7 extends the analysis to asymmetric information on costs. Concluding remarks are provided in Section 8.

2 Related literature

Our analysis relates to the large body of economic literature analysing different types of regulatory mechanisms to stimulate efficient quality provision (Sappington, 2005). A starting point of this literature is the insight developed by Spence (1975) and Sheshinski (1976), who show that an unregulated monopolist is unlikely to provide a socially optimal level of product quality and that some form of regulation might be called for.[6] A large bulk of the subsequent literature has focused on the effects of direct quality regulation (e.g., minimum quality standards) in the context of vertical differentiation, with or without competition.[7]

Our study is more closely related to the literature on quality competition in a *spatial* framework. Under the assumption of fixed provider locations, Ma and Burgess (1993) show how socially optimal quality provision can be achieved by price regulation. Brekke et al. (2006) extend this analysis to the case where firms' locations are endogenous, which implies that the socially optimal outcome cannot be achieved by simple price regulation. However, Bardey et al. (2012) show that efficient quality provision can be restored if price regulation is optimally combined with provider cost reimbursement. A further extension by Mak (2018) considers a model with multidimensional quality and a richer set of regulatory tools, including reference pricing and pay-for-performance bonuses.[8]

Our main contribution is that we identify *demand sluggishness* as an independent source of

[6] Sheshinski (1976) considers both price and quality regulation as alternative means to remedy the problem, whereas Spence (1975) suggests rate-of-return regulation as an attractive alternative due to the informational problems faced by real-world regulators.

[7] See, e.g., Besanko et al. (1987) for the case of a multi-product monopolist, and Ronnen (1991) and Crampes and Hollander (1995) for the case of competition between vertically differentiated duopolists.

[8] Another key contribution to this the literature using a spatial competition framework is Wolinsky (1997), who compares the efficiency of two different regulatory schemes, namely managed competition and regulated monopolies.

inefficiency, which introduces a dynamic dimension to the optimal price regulation problem. In this sense, our model is most closely related to Brekke et al. (2012), who investigate quality decisions under price regulation with sluggish demand. We expand their analysis in two key dimensions. First, we adopt a more general model where production costs differ across providers rather than being uniform. Second, we investigate a plausible form of price regulation, involving a price premium or penalty on volume, which has not been previously considered in the literature. In Brekke et al. (2012) price regulation mostly focuses on the steady state, where volume is fixed, and when price regulation is considered off the steady state, it depends on time, not volume.[9] However, regulators are unlikely to be able to commit to a price rule which depends directly on time. Instead, we argue that they are more likely to be able to commit to price regulation which depends on the volumes observed, without having to specify explicitly a continuum of prices over time (see Bisceglia et al., 2019b). In our model, the regulator only needs to specify two price related values for each provider.

Our analysis relates more broadly to the literature on quality competition in a dynamic context (see Brekke et al., 2018, for a review). Brekke et al. (2010) assume that demand adjusts instantaneously to quality, but quality is a stock variable which increases if its investment is higher than its depreciation rate. They show that if prices are regulated and marginal costs are increasing in volume, quality is lower under the feedback solution, which is arguably the solution concept when competition is more intense, than under the open-loop solution, when providers can commit to optimal quality plans at the beginning of the game. An analogous result is obtained by Cellini et al. (2018) when providers also compete on price in addition to quality.[10]

3 The model

We consider a market with two competing providers located at either end of a Hotelling line $S = [0, 1]$, populated by a uniform distribution of individuals, with total mass of 1.[11] At time t, each consumer demands one unit of service from one of the providers. Since prices are regulated

[9] Siciliani et al. (2013) also develop a dynamic model with sluggish demand, when providers are altruistic or intrinsically motivated, and follow a similar approach to Brekke et al. (2012) in terms of optimal price regulation.

[10] In a model with regional regulators and asymmetric providers, Bisceglia et al. (2019a) show that this result does not hold when the price which applies to the extra-regional demand, set by a national authority, is sufficiently high, since more efficient providers have strong incentives to attract consumers from another region.

[11] This is a common framework for quality competition in healthcare markets; see Calem and Rizzo (1995), Beitia (2003), Brekke et al. (2007), Karlsson (2007). A similar framework is used by Del Rey (2001) for quality competition in education markets.

and paid by a third party, consumers' choice of provider is based on provider quality and travelling costs. Let $q_i \geq \underline{q}$ denote the quality offered by Provider $i = 1, 2$, and let the marginal cost of travelling be given by $\tau > 0$.[12] The lower bound $\underline{q}$ represents the minimum quality providers are allowed to offer, which we set to zero. If a consumer located at $x \in [0, 1]$ buys the service offered by Provider i, located at $z_i \in \{0, 1\}$, the utility of this consumer is assumed to be given by

$$U(x, z_i) = v + q_i - \tau |x - z_i|, \tag{1}$$

where $v > 0$ is the utility of consuming one unit at minimum quality without having to travel. We assume that v is high enough for the market to be covered.

If consumers make utility-maximising choices, the *potential demand* of Provider i at time t is

$$D_i^* (t) = \frac{1}{2} + \frac{q_i (t) - q_j (t)}{2\tau} \tag{2}$$

with $i, j = 1, 2$ and $j \neq i$.[13] If consumers have *sluggish beliefs* about quality, the actual demand might differ from potential demand. Denoting the *actual demand* of Provider i at time $t \in [0, \infty)$ by $D_i (t)$, we assume that this demand evolves according to the following linear ordinary differential equation (ODE):

$$\dot{D}_i(t) = \gamma(D_i^*(t) - D_i(t)), \tag{3}$$

where $\gamma \in [0, 1]$ measures (inversely) the degree of demand sluggishness. We interpret γ as the fraction of consumers who, at each point in time, become aware of a previous change in the quality difference between providers, and therefore re-optimise their choice. The lower this fraction, the more sluggishly demand responds to quality changes over time. This formulation implies that (actual) demand is a state variable. Given the assumptions of unit demand and full market coverage, the demand for Provider j is $D_j (t) = 1 - D_i (t)$, which implies that the dynamic evolution of both providers' demand is described by (3).

Providers are assumed to be profit oriented, with the same (constant) preference discount rate

[12] It is straightforward to show that all the results in the paper would remain unchanged if we assume that travelling costs are quadratic instead of linear in distance. Details are available upon request.

[13] In the context of health services, suppose that only a proportion λ of individuals fall ill and demand treatment. As long as this proportion is constant along the Hotelling line, potential demand is multiplied by a fixed constant λ and all the results are qualitatively unchanged.

$\rho > 0$. Denoting by $p_i(D_i(t))$ the unit price received by Provider i, its instantaneous profit is

$$\pi_i(t) = p_i(D_i(t))D_i(t) - c_i D_i(t) - \frac{\beta}{2}[D_i(t)]^2 - \frac{\theta}{2}[q_i(t)]^2, \tag{4}$$

where $\theta > 0$, $c_i > 0$ and $\beta > 0$. We allow providers to differ in production costs c_i due to efficiency and other cost factors. We instead assume that the cost of quality captured by θ is homogeneous across providers and relates to investment in machines and capital which can be purchased from suppliers at competitive and uniform rates. We also assume that the cost function is strictly convex in output, reflecting the presence of (smooth) capacity constraints, for example in the form of congestion costs that, for a given level of demand, are also similar across providers (Brekke et al., 2012). To keep the presentation of the model simpler we assume that output and quality are cost independent. However, our main results are robust to allowing cost dependence between quality and output (see Appendix C).

The unit price received by each provider is regulated and paid by a third party. We propose a specific pricing formula where the regulated price is provider-specific and linked to the provider's demand,

$$p_i(D_i(t)) = a_i + bD_i(t), \tag{5}$$

where a_i is a *provider-specific* fixed price and b is a price component which is linear in demand and *common* across providers. If b is positive, we refer to this parameter as a price *premium* on volume. Instead, if b is negative, we refer to it as a price *penalty* on volume. Furthermore, to simplify notation we define $\sigma_i := a_i - c_i$. All else equal, a higher value of σ_i implies a higher price-cost margin for Provider i.

In principle, we could make the parameter related to a possible price premium or penalty specific to each provider. However, in Section 6 we show that, by an appropriate choice of a_i and b, the pricing rule in (5) can induce the socially optimal quality for each provider at each point in time. Therefore, a provider-specific parameter for the volume-based part of the pricing rule is not required. This result arises because the quadratic component of the cost function in quality and demand is not provider specific.[14]

One sector that closely relates to the suggested payment system is the hospital sector. In most of the OECD countries, hospitals are paid by a fixed price schedule, known as Diagnosis Related

[14] Introducing asymmetries in β and θ is analytically intractable.

Groups (DRGs) system. For a given diagnosis, the hospital receives a fixed price for every patient treated regardless of the costs sustained. Moreover, the price is adjusted across hospitals to account for differences in exogenous cost factors (related to land, capital and labour).[15] Finally, variants of the DRG system have been implemented across countries and over time, and these often involve penalties for high volumes, so that the fixed tariff decreases at higher volumes.[16]

To ensure the optimisation problems are well-behaved, we impose the following parameter restrictions:

A1. $\theta > 1/\tau$. This assumption states that the marginal cost of quality from machines and other investments is sufficiently high relative to the marginal responsiveness of demand to quality. This assumption is in line with the features of the health sector, where the cost of investing in machines (for magnetic resonance imaging, equipment for surgical operating theaters, etc.) is high, and where a large body of empirical literature suggests that demand is relatively inelastic to quality.[17]

A2. $\beta/2 > b$. This assumption is always satisfied if the price involves a penalty on volume. If the regulator instead uses a premium on volume, then we assume that the premium is small relative to the degree of convexity in production costs. In Section 6 we show that the optimal pricing rule always satisfies A2 if production costs are sufficiently convex, i.e., if $\beta > \underline{\beta}$, where $\underline{\beta} := \tau(\gamma + \rho)/\gamma$.

4 Equilibrium quality provision

Suppose that each provider chooses quality at each point in time over an infinite time horizon. In this section we derive the equilibrium under the assumption of a *state feedback* information structure, where each provider can respond to the observed evolution of the state variable, implying that the optimal investment choice at any point in time is a function of contemporaneous demand.[18] In line with the literature, we restrict attention to stationary linear Markovian strategies, in which the current value of the control variable only depends on the current value of the state variable and the rule is time invariant. We assume that each player takes the rival's strategy as given,

[15] For example, in England under Payment by Results, hospitals are paid a Health Resource Group (HRG) price (the English version of DRG prices) based on national average costs adjusted by a provider specific index, known as the market factor forces (MFF; Department of Health, 2004). The MFF adjusts the national price for local unavoidable differences in factor prices for staff, land and building costs (see Miraldo et al., 2011, for a theoretical analysis).

[16] In practice, policymakers identify volume thresholds, with the tariff decreasing further when each of the thresholds is reached. Our pricing rule can be interpreted as a continuous approximation of a step-wise function where the price decreases with volume.

[17] See Brekke et al. (2014) for a survey on the empirical evidence on hospital demand responsiveness to quality.

[18] In Appendix A.3 we show how our results are affected if we assume that the providers use *open-loop* decision rules.

implying Nash behaviour, and we look for a pair of strategies that constitute a State Feedback Nash Equilibrium (henceforth SFNE). In Appendix A.1 we show that the linear SFNE strategies are given by

$$q_i^F = \frac{\gamma}{2\tau\theta}\left(\alpha_1^i + \alpha_2 D_i\right), \tag{6}$$

where

$$\alpha_1^i = \frac{(2\varphi + 3\rho)\,\sigma_i + \varphi\sigma_j + (2\gamma + \rho)\,(\sigma_i - \sigma_j) + (\gamma + 2\rho)\,\gamma\alpha_2 - (\beta - 2b)\,(\varphi - (\gamma + \rho))}{\frac{1}{2}\,(\rho + \varphi)\,(4\gamma + 5\rho + \varphi)}, \tag{7}$$

$$\alpha_2 = -\frac{2\theta\tau^2\,(\varphi - (2\gamma + \rho))}{3\gamma^2} < 0, \tag{8}$$

for $i, j = 1, 2,\ i \neq j$, and

$$\varphi := \sqrt{(2\gamma + \rho)^2 + \frac{3(\beta - 2b)\gamma^2}{\theta\tau^2}}. \tag{9}$$

Since $\alpha_2 < 0$ and $D_i = 1 - D_j$, the SFNE is characterised by *inter-temporal strategic complementarity* (Jun and Vives, 2004), and the quality of each player responds positively to a change in demand of the other player, $\partial q_j^F/\partial D_i > 0$ and $\partial q_i^F/\partial D_j > 0$. The intuition is that a higher demand for Provider i implies lower demand and therefore lower marginal production costs for Provider j, which in turn makes it more profitable for the latter provider to increase quality. These incentives are either reinforced or dampened by the pricing scheme. A premium on volume ($b > 0$) implies that a demand reduction is accompanied by a price reduction, which weakens the inter-temporal strategic complementarity. The opposite conclusion holds for a penalty on volume ($b < 0$). The key characteristics off the steady state of the SFNE can therefore be summarised as follows:

Proposition 1 *(i) In the SFNE, quality and demand move in opposite directions on the equilibrium path to the steady state for each provider. (ii) An increase in the price parameter b reduces the quality adjustment in response to a demand change on the equilibrium dynamic path.*

Proof. Appendix B.1. ■

Part (i) of the proposition suggests that the result by Brekke et al. (2012) for symmetric providers and an exogenous fixed price holds more generally for asymmetric providers and variable prices given by (5). Part (ii) shows how the equilibrium trajectory depends on b. A higher value of b (larger price premium or a smaller penalty) reduces the absolute value of α_2 and therefore the degree of inter-temporal strategic complementarity. For symmetric providers, this translates into a smaller quality difference between providers. In turn, this reduces the amount of demand reallocation, at each point in time, towards the provider with higher quality and lower demand, and thus implies a slower convergence towards the steady state.

[Figures 1a-1c here]

The equilibrium dynamic path towards the steady state is illustrated in Figures 1a-1c. Figure 1a depicts the symmetric case ($\sigma_1 = \sigma_2$), where the quality of each provider converges to the same steady-state level, whereas Figures 1b and 1c illustrate two versions of the asymmetric case. If $\sigma_i > \sigma_j$, Figure 1b (1c) shows the equilibrium when initial demand is lower (higher) for Provider i. In all cases, an increase (decrease) in b leads to a flatter (steeper) convergence curve for each provider.

In the *steady state* of the SFNE, qualities and demand are given by

$$\overline{q}_i^F = \frac{\frac{3\gamma}{\theta\tau^2}(\beta - 2b)(\sigma_i + \sigma_j - (\beta - 2b)) + (4\gamma + \varphi + 5\rho)(2\sigma_i - (\beta - 2b))}{\frac{2\theta\tau}{3\gamma^2}(\gamma + \varphi - \rho)(\varphi + \rho)(4\gamma + \varphi + 5\rho)}, \tag{10}$$

$$\overline{D}_i^F = \frac{1}{2} + \frac{(\sigma_i - \sigma_j)\, 3\gamma^2}{2\theta\tau^2(\varphi + \rho)(\gamma + \varphi - \rho)}, \tag{11}$$

for $i, j = 1, 2$, $i \neq j$. The steady-state properties of the SFNE are summarised as follows:

Proposition 2 *In the steady state of the SFNE, (i) Provider i has higher quality and demand than Provider j if and only if $\sigma_i > \sigma_j$. (ii) An increase in the provider-specific price parameter a_i increases quality of both providers, but shifts demand towards Provider i. (iii) Compared to $b = 0$, a premium on volume ($b > 0$) amplifies quality and demand differences between providers, whereas a penalty ($b < 0$) dampens these differences.*

Proof. Appendix B.2. ■

All else equal, an increase in a_i implies that Provider i is paid a higher price p_i, which increases the marginal revenue of quality and thus increases steady state quality for this provider. An increase in a_i also leads to higher steady state quality for Provider j, even if the price of Provider j is not affected, which is caused by strategic complementarity.[19] In case of cost asymmetry, the common price parameter b serves to amplify or dampen quality (and thus demand) differences

[19] A higher quality level by Provider j implies that demand is shifted away from Provider i. The resulting reduction in marginal production costs implies stronger incentives for quality provision and Provider i will therefore respond by increasing quality.

between providers in the steady state, depending on whether b is positive or negative. In the steady state, the provider with higher quality has higher demand (Figures 1b and 1c). If $b > (<)\, 0$, the provider with higher demand is given a price *premium* (*penalty*) on volume which reinforces (weakens) the incentive for quality investments, thus amplifying (dampening) the quality difference between providers.

5 Socially optimal quality provision

We now characterise the socially optimal quality provision. Suppose that a social planner can set each provider's quality level at each point in time taking as given the demand for the two providers and its sluggishness. The socially optimal dynamic paths of quality are derived from the following problem:

$$\max_{q_i(\cdot),q_j(\cdot)} \int_0^{\infty} e^{-\rho t} W(t) dt, \tag{12}$$

subject to the dynamic constraint (3) and the initial condition $D_i(0) = D_{i0} > 0$, where $W(t)$ is the instantaneous social welfare defined as

$$W(t) = \int_0^{D_i} (v+q_i-\tau x)dx + \int_{D_i}^{1} (v+q_j-\tau(1-x))dx - \frac{\theta}{2}(q_i^2+q_j^2) - (c_i D_i + c_j(1-D_i)) - \frac{\beta}{2}(D_i^2+(1-D_i)^2). \tag{13}$$

In Appendix A.2 we show that the *feedback representation* of the first-best solution is given by

$$q_i^* = \frac{1}{2\theta}\left(\widehat{\alpha}_1^i + \widehat{\alpha}_2 D_i\right), \tag{14}$$

where

$$\widehat{\alpha}_1^i = \frac{2\theta\gamma\left(\beta + c_j - c_i\right) + \left(\theta\tau - 1\right)\left(\theta\tau\left(2\gamma + \rho\right) - \kappa\right)}{\left(\kappa + \theta\tau\rho\right)}, \tag{15}$$

$$\widehat{\alpha}_2 = \frac{\theta\tau\left(2\gamma + \rho\right) - \kappa}{\gamma}, \tag{16}$$

$$\kappa := \sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho+\gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}, \tag{17}$$

for $i, j = 1, 2,\ i \neq j$.

Proposition 3 *On the socially optimal time paths, quality and demand move in the opposite (same) direction for each provider if production cost convexity is sufficiently high (low):* $\beta > (<)\, \underline{\beta}$.

Proof. Appendix B.3. ■

The optimal dynamic relationship between quality and demand is similar for both providers, and does not depend on cost differences. If the initial demand difference is higher than the socially optimal steady state demand difference, the social planner must decide whether to attribute higher or lower quality to the provider with more demand. Because of production cost convexity in volume ($\beta > 0$), cost-efficiency is improved by reallocating demand towards the provider with *lower* demand, which suggests that higher quality should be attributed to this provider. On the other hand, the marginal benefit of quality investments is larger if these investments are made at the provider with higher demand. The latter consideration dominates if the degree of production cost convexity is sufficiently low. In this case, welfare is maximised by attributing higher quality to the provider with *higher* demand, implying that both demand and quality decreases (increases) for the provider with higher (lower) initial demand along the socially optimal dynamic path towards the steady state.

In the *steady state*, the socially optimal qualities and demand are given by

$$\overline{q}_i^* = \frac{\beta\gamma + \tau(\gamma\theta(c_j - c_i) + (\gamma + \rho)(\tau\theta - 1))}{2\theta(\beta\gamma + \tau(\gamma + \rho)(\tau\theta - 1))}, \tag{18}$$

$$\overline{D}_i^* = \frac{1}{2} + \frac{\gamma\tau(c_j - c_i)}{2\tau(\beta\gamma + \tau(\gamma + \rho)(\tau\theta - 1))}, \tag{19}$$

for $i, j = 1, 2$, $i \neq j$. In Appendix A.2 we show that this constitutes a saddle point.

In the symmetric case when production costs are the same across providers, the socially optimal quality in the steady state only depends on the marginal benefits and costs of quality given by $q^* = 1/2\theta$.[20] Instead, under cost asymmetry, the social planner faces a trade-off between production cost efficiency and travelling cost efficiency. Concerns for production cost efficiency imply that demand should be shifted towards the most efficient provider, but this would increase aggregate travelling costs, which are minimised with equal market shares. The balancing of these considerations is characterised as follows:

Proposition 4 *If production costs differ across providers, $c_i \neq c_j$, (i) socially optimal steady state quality is higher for the most cost-efficient provider; (ii) socially optimal quality difference between providers is decreasing in the degree of production cost convexity, β, and demand sluggishness, γ^{-1};*

[20] This quality level is also identical to the equilibrium quality level in a static version of the symmetrc game where providers choose both quality and price simultaneously. The social optimality of quality provision in a simultaneous price and quality game of this kind was first shown by Ma and Burgess (1993).

and (iii) a marginal increase in consumers' travelling costs, τ, increases (decreases) the socially optimal quality difference between providers if τ and θ are sufficiently low (high).

Proof. Appendix B.4. ■

The social planner can improve cost efficiency by choosing a higher quality for the most efficient provider. This shifts demand towards this provider and therefore reduces total production costs. However, because of production cost convexity, total production costs are not minimised by letting the most cost efficient provider have all demand. Thus, higher degree of cost convexity reduces the socially optimal demand and quality difference between providers. A similar relationship appears between demand sluggishness and optimal quality and demand differences across providers. A more sluggish demand (lower γ) reduces the effectiveness of quality provision as a tool to reallocate demand, implying a smaller difference in the socially optimal demand and quality between providers.

The effect of higher travelling costs τ is more complicated. For given qualities, higher travelling costs has two *allocational* welfare effects. On the one hand, it affects each consumer's trade-off between quality and distance which reduces the difference in market shares between providers. All else equal, this leads to less cost-efficient production, which can be restored by increasing the quality difference between providers. On the other hand, higher τ increases the importance of travelling cost efficiency in the welfare trade-off, leading, all else equal, to a *reduction* in the optimal market share (and quality) difference between providers. Since aggregate travelling costs are convex in τ, the second effect is stronger the higher τ is to begin with. Furthermore, a higher degree of cost convexity in quality provision (θ) makes it more costly to increase the quality difference in response to the first effect. Thus, a larger quality difference in response to higher travelling costs is socially optimal only if both τ and θ are sufficiently low.

6 Optimal dynamic price regulation

For an exogenously given fixed price $p_i = p_j = p$, quality provision in the SFNE will generally differ from the socially optimal quality provision, on and off the steady state. The key policy question is whether the socially optimal paths of quality provision can be induced by price regulation. In this section we derive the optimal dynamic price regulation rule and its properties. The next proposition shows that, by an optimal choice of a_i, a_j and b, the socially optimal solution can be implemented for a certain set of parameters.

Proposition 5 *In the state feedback Nash equilibrium, the socially optimal quality provision is replicated, at each point in time, if the regulated price is $p_i^F(t) = a_i^F + b^F D_i(t)$, with the common price parameter being set at*

$$b^F = \frac{\tau}{4\gamma^2}\left((2\gamma+\rho)\kappa - 2\gamma(2\tau\theta-3)(\gamma+\rho) - \rho^2\tau\theta\right) - \beta \tag{20}$$

and the provider-specific parameter being set at

$$a_i^F = \frac{\left[\begin{array}{c} 6\theta\gamma\left(\varepsilon\left(\beta-(c_i-c_j)\right)+\kappa c_i+\theta\tau\rho\left(\beta+c_j\right)\right) \\ +\left(\theta\tau-1\right)\left(\theta\tau\left(\varepsilon\left(6\gamma+5\rho\right)+\theta\tau\rho\left(2\gamma+\rho\right)-\kappa\left(4\gamma+5\rho\right)\right)-\kappa\varepsilon\right) \end{array}\right]}{6\theta\gamma\left(\kappa+\theta\tau\rho\right)}, \tag{21}$$

where κ is given by (17) and

$$\varepsilon := \sqrt{\theta\left(3\gamma^2\left(\beta-2b\right)+\theta\tau^2\left(2\gamma+\rho\right)^2\right)}, \tag{22}$$

for $i,j=1,2$, $i\neq j$, given that the parameter condition $\beta > \underline{\beta}$ holds.

Proof. Appendix B.5. ■

The most striking feature of this result is that optimal quality provision can be achieved by a simple pricing rule. If providers do not differ in costs, equilibrium quality is symmetric in the steady state but generally differs across providers off steady state. Nevertheless, Proposition 5 shows that the socially optimal time path of quality provision can be replicated in equilibrium by each provider through an *ex ante non-discriminatory price regulation scheme* that links each provider's price to its demand in the exact same way, through the premium or penalty parameter b given by (20).

In the optimal pricing rule, the common parameter b remains unchanged even if the providers differ in costs. In this case, prices need to be adjusted through the provider-specific parameters a_i and a_j, and set to reflect marginal production costs, with the most cost-efficient provider receiving a higher price. This leads to a larger quality difference between providers and thus cost savings through a reallocation of demand towards the most cost-efficient provider. The optimal values of a_i and a_j do not depend on time. Price adjustments over time occur only through the common parameter b, which highlights the simplicity of this regulation scheme.

The condition $\beta > \underline{\beta}$ is needed for assumption A2 to hold with the pricing rule given in Proposition 5, and requires that production costs are sufficiently convex in volume. Since this condition is identical to the one in Proposition 3, we provide the following general characterisation:

Corollary 1 *The first-best solution can be implemented by a pricing rule of the form given by (5) if and only if quality and demand move in opposite directions on the socially optimal dynamic path towards the steady state.*

Given this parameter condition, the dynamic properties of the socially optimal pricing rule from Proposition 5 can be characterised as follows:

Proposition 6 *In the state feedback Nash equilibrium, the socially optimal price involves a penalty (premium) linked to the demand of each provider, at each point in time, if the cost convexity parameter $\beta > (<) \widehat{\beta}^F$, where*

$$\widehat{\beta}^F := \frac{\tau\left(12\gamma(\gamma+\rho) - \theta\tau(2\gamma+\rho)^2 + (2\gamma+\rho)\sqrt{\theta\tau\left(\theta\tau(2\gamma+\rho)^2 + 8\gamma(\gamma+\rho)\right)}\right)}{8\gamma^2}. \tag{23}$$

Proof. Appendix B.6. ■

Whether the price schedule involves a penalty or a premium is closely related to the welfare trade-off determining the socially optimal time paths of quality provision given by Proposition 3 for the case in which $\partial q_i^* / \partial D_i < 0$. Concerns for *cost-efficient production* dictate that demand should be steered towards the provider with lower demand, which can be achieved by lower quality investments by the high-demand provider. On the other hand, concerns for *consumer welfare* dictate that the high-demand provider should invest more in quality, implying that demand is steered away from the low-demand provider.

If the *production cost* concern dominates, the optimal price scheme involves a *penalty* on volume ($b < 0$), which reduces the price-cost margin, and thus incentives for quality investments, of the high-demand provider relative to the low-demand provider. This is optimal if the convexity of production costs (β) is sufficiently high.[21] Alternatively, if the degree of cost convexity β is relatively low, the concern for *consumer welfare* dominates. In this case, the optimal price scheme has a

[21] The scope for a penalty with $b < 0$ in the optimal pricing rule is also larger if patients' transportation costs and/or the degree of demand sluggishness is lower. All else equal, a reduction in τ or an increase in γ implies that the demand response to a change in quality provision is larger, implying that cost efficiency can be improved (through demand reallocation) at a lower cost (in terms of consumer welfare).

premium on volume ($b > 0$) which makes quality investments relatively more profitable for the high-demand provider, thus amplifying the equilibrium quality difference between providers.

The structure of the optimal pricing rule suggests that the different price parameters serve distinctly different purposes in the process of replicating the first-best solution on and off the steady state. The common parameter b, which links the price to demand for each provider at each point in time, is set to induce the socially optimal *slope* of the equilibrium path towards the steady state, which determines speed of convergence of quality and demand. For a given slope, the time independent parameters a_i and a_j are then set to induce the socially optimal *level* of quality for each provider along the equilibrium path.

Additional insights about the optimal choice of b can therefore be gained by comparing the slopes of the equilibrium path and the socially optimal path. From Proposition 1 we know that quality and demand move in opposite directions over time for each provider in the SFNE, which implies that the provider with initially higher (lower) demand has quality below (above) the steady state level. By Proposition 3, the slope of the socially optimal path is also negative under the parameter condition $\beta > \underline{\beta}$, which is a requirement for the optimal pricing rule (see Corollary 1). Using (6) and (14), the *difference* in these slopes is given by

$$\frac{\partial q_i^*}{\partial D_i} - \frac{\partial q_i^F}{\partial D_i} = \frac{1}{2\theta}\left(\frac{\gamma\alpha_2}{\tau} - \widehat{\alpha}_2\right) = \frac{\theta\tau\left(2\left(\gamma+\varphi\right)+\rho\right) - 3\kappa}{6\theta\gamma}. \tag{24}$$

Both slopes, and their difference, depend on b but not on a_i or a_j. For $b = 0$, the slope difference is zero at $\beta = \widehat{\beta}^F$, as given by (23) in Proposition 6. Furthermore, it is fairly straightforward to verify that, for $b = 0$, this difference is monotonically decreasing in β, which implies that[22]

$$\left|\frac{\partial q_i^F}{\partial D_i}\right| > (<) \left|\frac{\partial q_i^*}{\partial D_i}\right| \text{ for } b = 0 \text{ if } \beta < (>)\, \widehat{\beta}^F. \tag{25}$$

Thus, if the degree of production cost convexity is sufficiently low, $\beta < \widehat{\beta}^F$, the equilibrium quality difference between the providers (for $b = 0$) is larger than what is socially optimal at any

[22] From (24) we derive

$$\frac{\partial}{\partial\beta}\left(\frac{\partial q_i^*}{\partial D_i} - \frac{\partial q_i^F}{\partial D_i}\right)\Bigg|_{b=0} = -\frac{\gamma\left(2\theta\tau\varphi - \kappa\right)\kappa}{2\theta^2\tau\varphi\left(4\gamma\left(\beta\gamma - \tau\left(\gamma+\rho\right)\right) + \theta\tau^2\left(2\gamma+\rho\right)^2\right)},$$

which is negative if $2\theta\tau\varphi > \kappa$. By squaring both sides of this inequality and collecting terms, we obtain

$$8\beta\gamma^2 + 3\theta\tau^2\rho^2 + 4\tau\gamma\left(\gamma+\rho\right)\left(3\theta\tau+1\right) > 0.$$

initial (asymmetric) state of demand. In other words, at the initial state, the high-demand provider chooses too low quality and the low-demand provider chooses too high quality. Since the marginal utility gains of quality investments are higher at the provider with more demand, social welfare would increase if more of the quality investments were channeled towards the provider with initially higher demand. This can be achieved by setting $b > 0$, which stimulates the incentives for quality investments by the high-demand provider, thereby reducing the equilibrium quality difference in the initial state, implying that the equilibrium path to the steady state is characterised by a more gradual reallocation of demand over time. On the other hand, for a sufficiently high degree of cost convexity, $\beta > \widehat{\beta}^F$, the exact opposite conclusions hold.

Figures 2a to 2c provide graphical illustrations of the above discussion. Figure 2a depicts the symmetric case, in which both providers are equally cost efficient. The equilibrium dynamic path for a volume-independent price (i.e., $b = 0$) is given by the solid line. By construction, the equilibrium for $b = 0$ coincides with the first-best solution in, but not off, the steady state. The dashed line depicts the case of $\beta < \widehat{\beta}^F$, where the first-best convergence curve is flatter, implying a slower convergence to the steady state, than the equilibrium convergence curve for $b = 0$. In this case, the equilibrium dynamic path can be changed to coincide with the first-best dynamic path by introducing a price premium on volume ($b^F > 0$). The dotted line depicts the reverse case of $\beta > \widehat{\beta}^F$, where the first-best convergence curve is steeper than the equilibrium curve for $b = 0$. Optimal price regulation implies in this case a penalty on volume ($b^F < 0$).

[Figures 2a-2c here]

Figures 2b and 2c illustrate the same two cases under provider cost asymmetry, where initial demand is either lower or higher for the most efficient provider. Notice that the effect of a particular price regulation scheme on quality differences across the providers depend crucially on whether demand for the most efficient provider is below or above its steady-state level. If initial demand is lower for the most efficient provider (Figure 2b), a price premium on volume implies that the off-steady-state difference in quality between the two providers becomes smaller, whereas a price penalty on volume reinforces this quality difference. These effects are reversed if initial demand is higher for the most efficient provider (Figure 2c).

All the above described results have been derived under the assumption of a *state feedback* information structure, where each provider can respond to the observed evolution of the state variable. An alternative solution concept is the Open Loop Nash Equilibrium (henceforth OLNE), in which each provider commits to a complete time profile of quality investments at the beginning of the game and sticks to it thereafter. Although not strongly time consistent, this solution concept might arguably be more appropriate in a setting where schools or hospitals have to commit to long-term plans when deciding on quality investments within a heavily regulated environment.[23] In Appendix A.3 we show that all of our previously derived results are qualitatively similar if we instead base our analysis on the OLNE. Thus, regardless of which of these two solution concepts we use, the optimal price scheme always involves a penalty (premium) on volume if the cost convexity parameter is sufficiently high (low). Furthermore, we show that the threshold value of β above which the optimal price involves a penalty on volume is higher in the OLNE than in the SFNE. In other words, the scope for a price penalty on volume is larger under state feedback decision rules than under open-loop decision rules.

7 Asymmetric information on costs

We have so far assumed symmetric information on costs, implying that the regulator can set different pricing rules for the high- and low-cost providers. If cost information is instead asymmetric, we assume that the regulator sets the same pricing rule for both providers: $p(D_i) = a + bD_i$. We refer to this pricing rule as *non-discriminatory (pooling) pricing.*

The technical procedure for deriving the optimal solution is laid out in Appendix A.4. The solution can only be derived numerically, so we present it in the form of two sets of simulation outputs that enable us to illustrate the main mechanisms at play. As a benchmark, Table 1 provides the case where the providers have the same costs ($c_i = c_j = 1$), which implies that the optimal pricing rule is the same under symmetric information on costs, denoted by $\left(a_i^F, a_j^F, b^F\right)$, and under asymmetric information, denoted by $\left(a^P, b^P\right)$. For the parameter values considered in Table 1, the optimal price scheme has a penalty on volume ($b^F < 0$), and the size of the penalty increases with the degree of cost convexity.

[Table 1 here]

[23] The different solution concepts also correspond to different information sets used by players in each instant of time when the game takes place. Under the open-loop rule, only the initial condition and the time are used; under the state-feedback rule, the current state of the world is considered (Basar et al., 2018; Lambertini, 2018; see also Dockner et al., 2000).

Under cost asymmetry between the providers, where $c_i < c_j$, simulation results for two different degrees of cost asymmetry are reported in Tables 2 and 3. For comparison, we show in Panel A of each of these tables the results under *symmetric* information on costs. In this case, and consistent with Proposition 5, the optimal price schedule involves a higher fixed price component for the most efficient provider (i.e., $a_i^F > a_j^F$), while the price parameter b^F is independent of the degree of cost asymmetry.

With *asymmetric information on costs*, the first-best outcome is no longer achievable. In order to replicate the first-best solution, the price schedule must ensure that the equilibrium trajectory is at the first-best *level* for each provider (and different for each provider) and that the *slope* of the these trajectories (which is equal for both providers) is also optimal. However, with only two available instruments (a and b) to achieve three targets, the first-best solution cannot be implemented. Instead, under non-discriminatory pricing, the parameter b is used to affect not only the slope but also the levels of the equilibrium trajectories for the two providers.

[Tables 2 and 3 here]

In order to explain the dual role of b, recall that, with provider-specific prices, the first-best solution is implemented by setting a higher fixed price component for the most cost-efficient provider. However, the inability to set different prices for the two providers under asymmetric information implies that, for $b = 0$, the quality difference between the providers is too low. The parameter b must therefore be used to give relatively stronger incentives for quality provision for the most cost-efficient provider, which requires that this provider receives a relatively higher price. Whether this necessitates a premium or a penalty on volume depends on whether the low-cost provider has higher or lower demand than the high-cost provider.

We know that *steady-state demand* is higher for the low-cost provider. Thus, if the low-cost provider has higher initial demand, a price premium on volume ($b > 0$) implies a relatively higher price for the low-cost provider, and thus a higher quality difference, at each point in time along the equilibrium dynamic path. However, if the low-cost provider has *lower initial demand*, there exists a turning point in time, $\widehat{t}$, such that the low-cost provider has lower (higher) demand for $t < (>) \widehat{t}$ along the equilibrium path. A premium on volume then implies that quality differences are reduced (increased) for $t < (>) \widehat{t}$, whereas a penalty on volume has the exact opposite effects. In this case, whether the optimal price schedule involves a premium or a penalty on volume depends on how quickly the turning point $\widehat{t}$ arrives, and the relative weights given to periods before and after $\widehat{t}$.

For a low degree of cost asymmetry, as in Table 2, the optimal price schedule is predominantly determined by the welfare effects in periods $t < \widehat{t}$. Compared with the symmetric information benchmark (Panel A), and for a given degree of cost convexity (β), the price parameter b decreases if the low-cost provider has lower initial demand and increases otherwise. The optimal solution has therefore a penalty on volume ($b^P < 0$) in the former case and a premium on volume in the latter case ($b^P > 0$). However, these conclusions change if the degree of cost asymmetry is higher, which makes the welfare effects in periods $t > \widehat{t}$ relatively more important. This is illustrated in Panel B of Table 3, where the optimal price schedule has a premium on volume ($b^P > 0$) regardless of initial market shares.

Notice also that b^P is always monotonically decreasing in β. Thus, a higher degree of production cost convexity either dampens the size of a price premium on volume or reinforces the size of a price penalty, because a higher cost convexity increases the efficiency gain of steering more demand towards the smaller provider. However, while this is the main factor determining whether the optimal price schedule has a premium or penalty on volume under symmetric information (cf. Proposition 6), it plays a secondary role under asymmetric information and sufficiently large cost differences between the providers.

Finally, notice that a higher (lower) value of b implies a (higher) lower price for both providers, all else equal. This implies that, if asymmetric information leads to an increase (reduction) in b, the fixed price component must be adjusted downwards (upwards) in order to avoid overprovision (underprovision) of quality. Thus, $b^P < b^F$ implies $a^P > \max\left\{a_i^F, a_j^F\right\}$ and $b^P > b^F$ implies $a^P < \min\left\{a_i^F, a_j^F\right\}$.

Although the first-best solution cannot be implemented under asymmetric information, we conjecture that a menu of incentive-compatible contracts which specify different a and b for each provider, i.e. $\left(a_i^F, b_i^F\right), \left(a_j^F, b_j^F\right)$, could further improve welfare. The main hint from the static literature on price regulation is that asymmetric information introduces rents for the providers and these can be minimised by offering a menu of non-linear contracts (see Wolinsky, 1997, and Beitia, 2003, for models of competition with asymmetric information on costs in a static setting).

8 Concluding remarks

In regulated markets where providers compete on quality, but where demand responds sluggishly to changes in quality, optimal price regulation is an inherently dynamic problem where the challenge is to ensure that the equilibrium quality provision follows a socially optimal dynamic path towards the steady state. A prime example is health care markets.

In this study we suggest an attractively simple solution to a complicated dynamic regulation problem. Under the assumption of symmetric information on costs, we show that a simple pricing rule that links each provider's regulated price to an easily observable metric, namely the provider's contemporaneous demand, can in principle ensure that the socially optimal (first-best) outcome is realised at each point in time, on and off the steady state. The key feature of this pricing rule is a price premium or a price penalty on volume. A necessary condition for such a price scheme to work is that the socially optimal dynamic path towards the steady state is characterised by demand and quality moving in opposite direction over time for each provider, which requires, in turn, that the degree of production cost convexity is sufficiently large. This conclusion is based on a differential-game version of a Hotelling duopoly framework where two exogenously located profit-maximising providers face regulated prices and compete in terms of quality to attract consumers.

Given that the first-best dynamic path can be replicated by the use of such a pricing rule, the remaining key design issue is whether the regulated price should be optimally designed with a *penalty* or a *premium* on higher volume. We show that this depends on the welfare trade-off of two opposing concerns. On the one hand, the concern for *consumer welfare* indicates that quality investments should be redirected (off the steady state) towards the provider with higher demand. This can be achieved by a regulated price that depends *positively* on demand, and therefore involves a price premium. On the other hand, the concern for *cost-efficient production* indicates that quality investments should be redirected (again, off the steady state) *away from* the provider with higher demand. This can be achieved by a regulated price that depends *negatively* on demand, and therefore involves a price penalty. As a result of this welfare trade-off, we show that a price schedule designed with a price penalty (premium) on volume is optimal if the degree of production cost convexity is sufficiently high (low).

In terms of policy implications, as discussed in the Introduction, there are several real-world examples of price regulation in hospital markets that resemble the type of pricing scheme that we suggest in this study, where the price received by each provider changes according to some volume thresholds. However, in all the examples of this kind that we are aware of, the price depends negatively on volume, and therefore involve a *penalty* for high volumes. In light of our analysis, such a pricing scheme can be socially optimal only if concerns for cost efficiency are sufficiently important. Such concerns are likely to be more important either in health systems with lower health expenditure per capita that imply tighter capacity constraints (e.g., fewer beds per capita), or within a health system that experiences budgetary restrictions following an economic downturn. For example, recent discussions in England around blended payment which involves lower payments at higher volume appear to be motivated by concerns over excess expenditure. More generally, our analysis suggests that dynamic price regulation with volume penalties is more likely to be optimal in markets where capacity constraints are important.

Although we have used health care as the prime motivating example, our analysis is also relevant for other markets characterised by price regulation and non-price competition. One example is the market for education. In most European countries, tuition fees in higher education are either absent or regulated, while funding is usually formula based and relies in part on the number of students, either through enrollment or credits awarded (Jongbloed, 2010). In England, there are price caps on the fees that universities are allowed to charge, and most universities charge the maximum effectively competing on quality to attract students. University student choice is facilitated by the comparison of scores based on the National Student Survey, which measures student satisfaction in a range of domains (e.g. teaching, assessment and feedback, learning opportunities, academic support, and overall satisfaction) and data on the proportion of students who are employed or in further education within 6 months from finishing their degree. This type of institutional arrangement across countries gives universities a monetary incentive to attract students - hopefully by improving the quality of the courses and degrees they offer. A similar incentive structure is also present in the market for compulsory education in some countries. For example, since 1992 Sweden has had a universal voucher programme in place, where free school choice is combined with a 'money-following-the-student' scheme in which each school (private or municipal) receives funding based on the number of students (Sahlgren, 2011). A similar system is in place for primary and secondary education in England, which is free of charge. Parents can choose primary and secondary schools within large catchment areas, and Offsted (the regulator) publishes reports in the public domain which allow to easily compare schools based on common metrics both on composite indicators (i.e. outstanding, good, requires improvement), education outcomes (proportion meeting expected standards in reading, writing and maths) and other specific domains (e.g. absences from school). As in regulated markets for health care, competition mainly takes place along a quality dimension, and key factors determining school choice is quality and travelling distance (Chumacero et al., 2011; Gibbons et al., 2008; Hastings et al., 2005). Furthermore, quality in education, just like quality in health care, is clearly a concern for policy makers. Another market with similar characteristics (and similar policy concerns) is the market for child care. In Norway, for example, the price paid by parents is capped at a low level and providers (public and private alike) receive government funding based on the number of children cared for (Engel et al., 2015). Again, this gives incentives for non-price competition and, since prices are fixed, parents' choice of provider is mainly determined by quality and travelling distance.[24]

As a final remark, it is worth re-iterating that our analysis is based on a theoretical framework where total demand is fixed. When lower demand for one provider is exactly offset by higher demand for the competing provider, the regulator can induce any desired demand allocation between the providers by the use of a single instrument. This is the reason why the first-best outcome can be implemented by a price regulation scheme where the dynamic part of the pricing rule is ex ante non-discriminatory (i.e., the penalty or premium on volume is common across providers). Our analysis is therefore more applicable to markets where total demand is relatively inelastic with respect to quality.

Appendix A: Supplementary calculations

In this appendix we provide supplementary calculations for the derivation of the state feedback equilibrium solution (in Section 4), the socially optimal solution (in Section 5), the open-loop solution (in Section 6) and the case of asymmetric information about costs (in Section 7).

A.1 State feedback Nash equilibrium

Since $D_j = 1 - D_i$, we can define the value function of both providers as a function of D_i. Given the linear-quadratic structure of the model, we specify the value functions as

$$V_i(D_i) = \alpha_0 + \alpha_1 D_i + \frac{\alpha_2}{2} D_i^2, \tag{A1}$$

[24] Like health care, the markets for education and child care are also examples of markets that seem particularly suited to being analysed in a spatial competiton framework. Besides the importance of travelling costs, these markets are also naturally characterised by unit demand, where each consumer demands one school admission, for example.

$$V_j(D_i) = k_0 + k_1 D_i + \frac{k_2}{2} D_i^2. \tag{A2}$$

These value functions have to satisfy the following Hamilton-Jacobi-Bellman (HJB) equations:

$$\rho V_i = \max_{q_i \geq 0} \left[(a_i + bD_i)D_i - \frac{\theta}{2} q_i^2 - c_i D_i - \frac{\beta}{2} D_i^2 + \frac{dV_i}{dD_i} \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i \right) \right], \tag{A3}$$

$$\rho V_j = \max_{q_j \geq 0} \left[(a_j + b(1 - D_i))(1 - D_i) - \frac{\theta}{2} q_j^2 - c_j (1 - D_i) - \frac{\beta}{2} (1 - D_i)^2 + \frac{dV_j}{dD_i} \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i \right) \right]. \tag{A4}$$

Using (A1)-(A2), the maximisation of the RHS of the HJB equations yields

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta} (\alpha_1 + \alpha_2 D_i), \tag{A5}$$

$$q_j = \phi_j(D) = -\frac{\gamma}{2\tau\theta} (k_1 + k_2 D_i). \tag{A6}$$

After substituting these expressions into the HJB equations, and proceeding by identification, we find that α_2 and k_2 solve the following system:

$$\rho \frac{\alpha_2}{2} = \frac{8\tau^2\theta b - 4\beta\tau^2\theta + \gamma\alpha_2(\gamma(\alpha_2 + 2k_2) - 8\tau^2\theta)}{8\tau^2\theta}, \tag{A7}$$

$$\rho \frac{k_2}{2} = -\frac{4\beta\tau^2\theta - \gamma^2 k_2 (2\alpha_2 + k_2) + 8\tau^2\theta(\gamma k_2 - b)}{8\tau^2\theta}, \tag{A8}$$

which admits four pairs of solutions:

$$\alpha_2 = 2\frac{(2\gamma + \rho)\tau^2\theta \pm \sqrt{\tau^2\theta(3(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{3\gamma^2}, \tag{A9}$$

$$k_2 = 2\frac{(2\gamma + \rho)\tau^2\theta \pm \sqrt{\tau^2\theta(3(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{3\gamma^2}. \tag{A10}$$

In order for the value functions to be concave we must have $\alpha_2 < 0$ and $k_2 < 0$, which eliminate the two positive roots. We therefore select

$$\alpha_2 = k_2 = -\frac{2\theta\tau^2}{3\gamma^2} \left(\sqrt{(2\gamma + \rho)^2 + \frac{3(\beta - 2b)\gamma^2}{\tau^2\theta}} - (2\gamma + \rho) \right), \tag{A11}$$

which is negative under assumption A2. Analogously, by collecting and equating to zero the terms containing D_i in each of the HJB equations, we obtain the following (linear) system to be solved for α_1 and k_1:

$$\rho\alpha_1 = \frac{\gamma\alpha_2(\gamma k_1 + 2(\gamma\alpha_1 + \tau^2\theta)) + 4\tau^2\theta(\sigma_i - \gamma\alpha_1)}{4\tau^2\theta}, \tag{A12}$$

$$\rho k_1 = \frac{\gamma\alpha_2(\gamma\alpha_1 + 2(\gamma k_1 + \tau^2\theta)) + 4\tau^2\theta(\beta - 2b - \sigma_j - \gamma k_1)}{4\tau^2\theta}. \tag{A13}$$

The solutions are given by

$$\alpha_1 = 2\tau^2\theta\frac{2\gamma\alpha_2((\beta - 2b - 2\sigma_i - \sigma_j)\gamma + 2\tau^2\theta(\gamma+\rho)) + 8\tau^2\theta\sigma_i(\gamma+\rho) - \gamma^3\alpha_2^2}{(4\theta\tau^2(\gamma+\rho) - 3\gamma^2\alpha_2)(4\theta\tau^2(\gamma+\rho) - \gamma^2\alpha_2)}, \tag{A14}$$

$$k_1 = 2\tau^2\theta\frac{2\gamma\alpha_2((2(\sigma_j - (\beta - 2b)) + \sigma_i)\gamma + 2\tau^2\theta(\gamma+\rho)) + 8\tau^2\theta(\gamma+\rho)(\beta - 2b - \sigma_j) - \gamma^3\alpha_2^2}{(4\theta\tau^2(\gamma+\rho) - 3\gamma^2\alpha_2)(4\theta\tau^2(\gamma+\rho) - \gamma^2\alpha_2)}. \tag{A15}$$

By substituting α_2 from (A11) into (A14) and simplifying, (A14) reduces to α_1^i as defined by (7). Using the fact that $k_2 = \alpha_2$, and keeping in mind that $D_i = 1 - D_j$, we can rewrite (A6) as

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}(\alpha_1^j + \alpha_2 D_j), \tag{A16}$$

where $\alpha_1^j = -k_1 - \alpha_2$ and is given by (7) if replacing i with j.

A.2 Socially optimal quality provision

Consider the optimal control problem of the social planner defined by (12), and let η be the current-value co-state variable associated with the dynamic constraint (3). The current-value Hamiltonian for the social planner problem is then given by

$$H = W + \eta\gamma\left[\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right], \tag{A17}$$

where W is given by (13). The solution must satisfy the following conditions: (i) $\partial H/\partial q_i = 0$, (ii) $\partial H/\partial q_j = 0$, (iii) $\dot{\eta} = \rho\eta - \partial H/\partial D_i$ and (iv) $\dot{D}_i = \partial H/\partial\eta$, along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\eta(t)D_i(t) = 0$. From conditions (i) and (ii) we derive

$$\eta = \frac{2\tau}{\gamma}(\theta q_i - D_i), \tag{A18}$$

$$\eta = -\frac{2\tau}{\gamma}(\theta q_j - (1 - D_i)), \tag{A19}$$

from which we get

$$q_j = \frac{1}{\theta} - q_i. \tag{A20}$$

The adjoint equation (iii) is given by

$$\dot{\eta} = (\rho + \gamma)\eta - (q_i - q_j) + (c_i - c_j) + (\tau + \beta)(2D_i - 1). \tag{A21}$$

From these equations we easily obtain the ordinary differential equation (ODE) system for the first-best quality and demand time paths, given by:

$$\begin{cases} \dot{q}_i = (\rho + \gamma)q_i + \frac{\gamma}{\tau\theta}\left(\beta - \frac{\tau}{\gamma}(\rho + \gamma)\right) D_i + \frac{\gamma}{2\tau\theta}(c_i - c_j - \beta) \\ \dot{q}_j = (\rho + \gamma)q_j + \frac{\gamma}{\tau\theta}\left(\beta - \frac{\tau}{\gamma}(\rho + \gamma)\right)(1 - D_i) + \frac{\gamma}{2\tau\theta}(c_j - c_i - \beta) \\ \dot{D}_i = \gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right) \end{cases}. \tag{A22}$$

The second-order conditions are satisfied if the Hamiltonian is concave in the control and state variables, which requires that the matrix

$$\begin{bmatrix} \frac{\partial^2 H_i}{\partial q_i^2} & \frac{\partial^2 H_i}{\partial q_j \partial q_i} & \frac{\partial^2 H_i}{\partial D_i \partial q_i} \\ \frac{\partial^2 H_i}{\partial q_j \partial q_i} & \frac{\partial^2 H_i}{\partial q_j^2} & \frac{\partial^2 H_i}{\partial D \partial q_j} \\ \frac{\partial^2 H_i}{\partial D_i \partial q_i} & \frac{\partial^2 H_i}{\partial D_i \partial q_j} & \frac{\partial^2 H_i}{\partial D_i^2} \end{bmatrix} = \begin{bmatrix} -\theta & 0 & 1 \\ 0 & -\theta & -1 \\ 1 & -1 & -2\beta - 2\tau \end{bmatrix} \tag{A23}$$

is negative semidefinite, which is true under assumption A1.

To obtain the first-best solution in *feedback* form, we define the value function of the social planner as

$$V(D_i) = \alpha_0' + \alpha_1' D_i + \frac{\alpha_2'}{2} D_i^2. \tag{A24}$$

This value function must solve the HJB equation given by

$$\rho V = \max_{q_i, q_j} \left[W + \frac{dV}{dD_i}\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right)\right], \tag{A25}$$

where W is given by (13). The first-order conditions with respect to q_i and q_j yield, respectively,

$$q_i = \frac{\gamma}{2\tau\theta}\alpha_1' + \left(\frac{1}{\theta} + \frac{\gamma\alpha_2'}{2\tau\theta}\right) D_i, \tag{A26}$$

$$q_j = \frac{1 - D_i}{\theta} - \frac{\gamma}{2\tau\theta}(\alpha_1' + \alpha_2' D_i). \tag{A27}$$

After substituting them into the HJB equation, and proceeding by identification, we find

$$\alpha_1' = \tau \frac{\gamma\alpha_2'(\tau\theta - 1) + 2\tau((\beta + \tau + c_j - c_i)\theta - 1)}{2\tau(\gamma(\tau\theta - 1) + \rho\tau\theta) - \gamma^2\alpha_2'} \tag{A28}$$

and

$$\alpha_2' = \tau \frac{2\gamma(\tau\theta - 1) + \rho\tau\theta \pm \sqrt{\theta\left(4\beta\gamma^2 + 4\gamma\tau(\rho + \gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}}{\gamma^2}. \tag{A29}$$

The condition that the value function must be concave leads us to select the negative root in (A29). We then define

$$\widehat{\alpha}_1^i := \frac{\gamma}{\tau}\alpha_1' \quad \text{and} \quad \widehat{\alpha}_2 := 2 + \frac{\gamma\alpha_2'}{\tau}, \tag{A30}$$

which, after substitution and re-arranging, allows us to express (A26) as (14). Similarly, (A27) is equal to (14) when i is replaced by j in the latter expression.

The equilibrium point is computed by imposing $\dot{q}_i = \dot{q}_j = \dot{D} = 0$ in the ODE system (A22), yielding the steady state qualities and demand given by (18)-(19). To determine its stability, we compute the eigenvalues of the Jacobian matrix of (A22). These are given by

$$\lambda_1 = \gamma + \rho, \tag{A31}$$

$$\lambda_2 = \frac{1}{2}\left(\rho + \frac{\sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho + \gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}}{\tau\theta}\right), \tag{A32}$$

$$\lambda_3 = \frac{1}{2}\left(\rho - \frac{\sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho + \gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}}{\tau\theta}\right). \tag{A33}$$

Clearly, $\lambda_1 > 0$ and $\lambda_2 > 0$, and it is also straightforward to verify that $\lambda_3 < 0$ under assumption A1. This proves that the steady state is a saddle point.

A.3 Open-loop Nash equilibrium

Under open-loop decision rules, Provider i takes as given its rival's strategy and solves the following optimal control problem:

$$\max_{q_i(t)} \int_0^\infty \pi_i(t)e^{-\rho t}dt, \tag{A34}$$

subject to the dynamic constraint (3) and the initial condition $D_i(0) = D_{i0} > 0$. Let $\mu_i(t)$ be the current-value co-state variable associated with the dynamic constraint (3). The current-value

Hamiltonian for Provider i is then given by[25]

$$H_i = p_i(D_i)D_i - \frac{\theta}{2}q_i^2 - c_iD_i - \frac{\beta}{2}D_i^2 + \mu_i\gamma\left[\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right]. \tag{A35}$$

The solution must satisfy the following conditions: (i) $\partial H_i/\partial q_i = 0$, (ii) $\dot{\mu}_i = \rho\mu_i - \partial H_i/\partial D_i$, (iii) $\dot{D}_i = \partial H_i/\partial \mu_i$, along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t)\,D_i(t) = 0$. Condition (i) yields

$$\mu_i = \frac{2\theta\tau}{\gamma}q_i, \tag{A36}$$

from which, by taking time derivative, we obtain

$$\dot{\mu}_i = \frac{2\theta\tau}{\gamma}\dot{q}_i. \tag{A37}$$

Condition (ii) is given by

$$\dot{\mu}_i = (\rho + \gamma)\mu_i - \sigma_i + (\beta - 2b)D_i. \tag{A38}$$

By combining (A37) and (A38) we obtain

$$\dot{q}_i = (\rho + \gamma)q_i - \frac{\gamma}{2\theta\tau}[\sigma_i - (\beta - 2b)D_i], \tag{A39}$$

which, together with the equivalent equation for Provider j and the dynamic constraint (3), constitute the ODE system which the OLNE strategies solve.

The second-order conditions are satisfied if the Hamiltonian is concave in the control and state variables, which requires that the matrix

$$\begin{bmatrix} \frac{\partial^2 H_i}{\partial q_i^2} & \frac{\partial^2 H_i}{\partial D_i \partial q_i} \\ \frac{\partial^2 H_i}{\partial D_i \partial q_i} & \frac{\partial^2 H_i}{\partial D_i^2} \end{bmatrix} = \begin{bmatrix} -\theta & 0 \\ 0 & 2b - \beta \end{bmatrix} \tag{A40}$$

is negative semidefinite, which is true under assumption A2.

To obtain the *feedback representation* of the OLNE, denoted by $q_i = \psi_i(D_i)$, we totally differentiate it with respect to time and use (A39) to obtain

$$\dot{q}_i = \frac{d\psi_i}{dD_i}\dot{D}_i = (\rho + \gamma)\psi_i - \frac{\gamma}{2\theta\tau}[\sigma_i - (\beta - 2b)D_i], \tag{A41}$$

[25] To save notation, the time indicator t is omitted in most of the subsequent expressions.

and, after substituting the state variable dynamic, we finally obtain

$$\frac{d\psi_i}{dD_i}\gamma\left(\frac{1}{2}+\frac{\psi_i-\psi_j}{2\tau}-D_i\right)=(\rho+\gamma)\psi_i-\frac{\gamma}{2\theta\tau}[\sigma_i+(2b-\beta)D_i] \tag{A42}$$

for $i,j=1,2,\ j\neq i$.

The equilibrium point is computed by imposing $\dot{q}_i=\dot{q}_j=\dot{D}_i=0$, yielding steady state qualities and demand as given by

$$\overline{q}_i^{OL}=\gamma\frac{(\beta-2b)\left((\sigma_i+\sigma_j)\gamma-2(\rho+\gamma)\tau^2\theta\right)-\gamma(\beta-2b)^2+4\tau^2\theta(\gamma+\rho)\sigma_i}{4\tau\theta(\gamma+\rho)((\beta-2b)\gamma+2\tau^2\theta(\gamma+\rho))} \tag{A43}$$

and

$$\overline{D}_i^{OL}=\frac{1}{2}+\frac{\gamma(\sigma_i-\sigma_j)}{2\left((\beta-2b)\gamma+2\tau^2\theta(\gamma+\rho)\right)}, \tag{A44}$$

for $i=1,2$ and $j\neq i$. To determine the equilibrium stability properties, we compute the eigenvalues of the Jacobian matrix of the linear ODE system which defines the equilibrium. These are given by

$$\lambda_1 = \gamma+\rho, \tag{A45}$$

$$\lambda_2 = \frac{1}{2}\left(\rho+\frac{\sqrt{\theta(2(\beta-2b)\gamma^2+(2\gamma+\rho)^2\tau^2\theta)}}{\tau\theta}\right), \tag{A46}$$

$$\lambda_3 = \frac{1}{2}\left(\rho-\frac{\sqrt{\theta(2(\beta-2b)\gamma^2+(2\gamma+\rho)^2\tau^2\theta)}}{\tau\theta}\right). \tag{A47}$$

Clearly, $\lambda_1>0$ and $\lambda_2>0$, and it is straightforward to verify that $\lambda_3<0$ under assumption A2, which implies that the steady state is a saddle point.

Let the first-best pricing rule in the OLNE be denoted by

$$p_i^{OL}(t)=a_i^{OL}+b^{OL}D_i(t). \tag{A48}$$

Following Benchekroun and Long (1998), we derive a_i^{OL} and b^{OL} by imposing that the first-best quality levels in feedback form, given by (14), solve equation (A42), which is the feedback representation of the OLNE. The price parameters a_i, a_j and b must thus solve

$$\frac{dq_i^*}{dD_i}\gamma\left(\frac{1}{2}+\frac{q_i^*-q_j^*}{2\tau}-D_i\right)=(\rho+\gamma)q_i^*-\frac{\gamma}{2\theta\tau}[\sigma_i-(\beta-2b)D_i] \tag{A49}$$

and

$$\frac{dq_j^*}{dD_i}\gamma\left(\frac{1}{2}+\frac{q_i^*-q_j^*}{2\tau}-D_i\right)=(\rho+\gamma)q_j^*-\frac{\gamma}{2\theta\tau}[\sigma_j-(\beta-2b)(1-D_i)]. \tag{A50}$$

Using (14) and the fact that $q_j^*=\frac{1}{\theta}-q_i^*$, we can write (A49) and (A50) as, respectively,

$$\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{1}{2}+\frac{\left(\widehat{\alpha}_1^i+\widehat{\alpha}_2 D_i\right)-1}{2\theta\tau}-D_i\right)=\frac{(\rho+\gamma)}{2\theta}\left(\widehat{\alpha}_1^i+\widehat{\alpha}_2 D_i\right)-\frac{\gamma\left(\sigma_i-(\beta-2b)D_i\right)}{2\theta\tau} \tag{A51}$$

and

$$-\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{1}{2}+\frac{\left(\widehat{\alpha}_1^i+\widehat{\alpha}_2 D_i\right)-1}{2\theta\tau}-D_i\right)=\frac{(\rho+\gamma)}{2\theta}\left(\frac{1}{\theta}-\widehat{\alpha}_1^i-\widehat{\alpha}_2 D_i\right)-\frac{\gamma\left(\sigma_j-(\beta-2b)\left(1-D_i\right)\right)}{2\theta\tau} \tag{A52}$$

By collecting the terms containing D_i, we obtain, from both equations, the following identity:

$$\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{\widehat{\alpha}_2}{2\theta\tau}-1\right)=(\rho+\gamma)\frac{\widehat{\alpha}_2}{2\theta}+\frac{\gamma}{2\theta\tau}(\beta-2b). \tag{A53}$$

Solving (A53) for b yields

$$b^{OL}=\frac{\tau(\gamma+\rho)}{\gamma}-\frac{\beta}{2}. \tag{A54}$$

Analogously, by collecting the other terms in (A51) and (A52), we obtain two equalities that allow us to solve for a_i and a_j, which are given by

$$a_i^{OL}=c_j+\beta, \tag{A55}$$

for $i,j=1,2,\ i\neq j$. Assumption A2 implies that the optimal solution exists if $\beta>2b^{OL}$, which is again equivalent to $\beta>\underline{\beta}$. From (A54) it follows directly that $b^{OL}>(<)\,0$ if $\beta<(>)\,\widehat{\beta}^{OL}$, where

$$\widehat{\beta}^{OL}:=\frac{2\tau(\gamma+\rho)}{\gamma}=2\underline{\beta}. \tag{A56}$$

Finally, comparing (A56) and (23), the inequality $\widehat{\beta}^{OL}>\widehat{\beta}^{F}$ can be rearranged to

$$4\gamma\left(\gamma+\rho\right)\left(\theta\tau+1\right)+\theta\tau\rho^2>\left(2\gamma+\rho\right)\sqrt{\theta\tau\left(\theta\tau\left(2\gamma+\rho\right)^2+8\gamma\left(\gamma+\rho\right)\right)}. \tag{A57}$$

Squaring both sides and collecting terms, this inequality reduces to $16\gamma^2\left(\gamma+\rho\right)^2>0$, which confirms that $\widehat{\beta}^{OL}>\widehat{\beta}^{F}$.

A.4. Non-discriminatory pricing

Suppose that the regulator sets the same pricing rule for both providers,

$$p(D_i) = a + bD_i, \tag{A58}$$

so that the fixed price component is common and not provider-specific, $a_i = a_j = a$. The SFNE strategies are

$$q_i^F(a, b, D_i) = \frac{\gamma}{2\tau\theta}\left(\alpha_1^i + \alpha_2 D_i\right), \tag{A59}$$

where, from (7),

$$\alpha_1^i = \frac{(2\varphi + 3\rho)(a - c_i) + \varphi(a - c_j) + (2\gamma + \rho)(c_j - c_i) + (\gamma + 2\rho)\gamma\alpha_2 - (\beta - 2b)(\varphi - (\gamma + \rho))}{\frac{1}{2}(\rho + \varphi)(4\gamma + 5\rho + \varphi)}, \tag{A60}$$

for $i, j = 1, 2$ and $i \neq j$, with α_2 and φ being defined respectively by (8) and (9).

The regulator's problem is to choose a and b to maximise:

$$\max_{a \in \mathbb{R}, b \leq \frac{\beta}{2}} \int_0^\infty e^{-\rho t} W(t) dt, \tag{A61}$$

subject to the dynamic constraint

$$\dot{D}_i(t) = \gamma\left(\frac{1}{2} + \frac{q_i^F(a, b, D_i) - q_j^F(a, b, D_i)}{2\tau} - D_i(t)\right) \tag{A62}$$

and the initial condition $D_i(0) = D_{i0} > 0$, where $W(t)$ is the instantaneous social welfare defined by (13) and evaluated at $q_i^F(a, b, D_i)$ and $q_j^F(a, b, D_i)$.

To solve this maximisation problem, we follow the approach suggested by Dockner et al. (2000, page 137). First, we substitute the feedback decision rules $q_i^F(a, b, D_i)$ and $q_j^F(a, b, D_i)$ into the instantaneous social welfare function, which gives

$$\begin{aligned} W(t) &= v - \frac{1}{2}(\beta + \tau) + \frac{\gamma\left(4\tau(\alpha_2 + \alpha_1^j) - \gamma\left((\alpha_1^i)^2 + (\alpha_2 + \alpha_1^j)^2\right)\right)}{8\tau^2\theta} \\ &\quad + \left(\beta + \tau - (c_i - c_j) + \frac{\gamma(\alpha_2\gamma(\alpha_1^j - \alpha_1^i + \alpha_2) + 2\tau(\alpha_1^i - \alpha_1^j - 2\alpha_2))}{4\tau^2\theta}\right) D_i(t) \\ &\quad - \left(\beta + \tau + \frac{\alpha_2\gamma(\alpha_2\gamma - 4\tau)}{4\tau^2\theta}\right) D_i(t)^2. \end{aligned} \tag{A63}$$

Second, we substitute $q_i^F(a, b, D_i)$ and $q_j^F(a, b, D_i)$ into the state dynamic equation, and solve for the equilibrium time path, which gives

$$D_i(t) = \frac{e^{-\gamma t}\left(\begin{array}{c} e^{\frac{\alpha_2\gamma^2 t}{2\tau^2\theta}}\left(\gamma(\alpha_2(2D_{i0}-1)+\alpha_1^i-\alpha_1^j)+2(1-2D_{i0})\tau^2\theta\right) \\ +e^{\gamma t}\left(\gamma(\alpha_2-\alpha_1^i+\alpha_1^j)-2\tau^2\theta\right)\end{array}\right)}{2\left(\alpha_2\gamma-2\tau^2\theta\right)}. \tag{A64}$$

Third, we substitute (A64) into (A63) giving the instantaneous social welfare $W(t)$ along the equilibrium path (omitted).

Finally, we integrate $W(t)$ over time based on (A61) to obtain $W(a, b)$, which gives a highly non-linear *static* maximisation problem with respect to a and b. The solution is an example of what is known in the differential game literature as *global Stackelberg solution* (see Dockner et al., 2000, page 141, which refers to Basar and Olsder, 1995). Given that $W(a, b)$ is heavily non-linear in a and b, the optimal solution can only be derived numerically. The results derived in Tables 2 and 3 in Section 8 are produced by first setting the parameters at specific values and then deriving the optimal values of a and b using the numerical global optimisation tool in Mathematica.

Appendix B: Proofs

This appendix contains proofs of all the propositions in the paper.

B.1 Proof of Proposition 1

(i) The result follows directly from (6) and the negative sign of α_2. (ii) The result follows directly from the fact that $|\alpha_2|$ is monotonically decreasing in b.

B.2 Proof of Proposition 2

From (10) we derive

$$\overline{q}_i^F - \overline{q}_j^F = \frac{(\sigma_i - \sigma_j)\, 3\gamma^2}{\theta\tau\,(\varphi+\rho)\,(\gamma+\varphi-\rho)}. \tag{B1}$$

The results then follow by simple inspection of (10), (11) and (B1).

B.3 Proof of Proposition 3

The result is directly confirmed by the feedback representation of the socially optimal time quality investment rule, (14), from which we derive

$$\frac{\partial q_i^*}{\partial D_i} = \frac{\widehat{\alpha}_2}{2\theta} > (<)\, 0 \text{ if } \theta\tau\,(2\gamma+\rho) > (<) \sqrt{\theta\,(4\beta\gamma^2 + 4\tau\gamma(\rho+\gamma)(\tau\theta-1) + \rho^2\tau^2\theta)}, \tag{B2}$$

which is equivalent to $\beta < (>)\, \underline{\beta}$.

B.4 Proof of Proposition 4

(i) The result follows directly from (18). (ii) The steady state quality difference is given by

$$\Delta\overline{q}^* := \left|\overline{q}_i^* - \overline{q}_j^*\right| = \frac{\gamma\tau\,|c_j - c_i|}{\beta\gamma + \tau(\gamma+\rho)(\tau\theta-1)}, \tag{B4}$$

from which it is immediately clear that $\partial\Delta\overline{q}^*/\partial\beta < 0$. Furthermore, under Assumption A1:

$$\frac{\partial\Delta\overline{q}^*}{\partial\gamma} = \frac{\tau^2\rho\,(\theta\tau-1)\,|c_j-c_i|}{(\beta\gamma+\tau(\gamma+\rho)(\tau\theta-1))^2} > 0\ . \tag{B5}$$

(iii) From (B5) we derive

$$\frac{\partial\Delta\overline{q}^*}{\partial\tau} = \frac{\gamma\left(\beta\gamma - \theta\tau^2\,(\gamma+\rho)\right)|c_j-c_i|}{(\beta\gamma+\tau(\gamma+\rho)(\tau\theta-1))^2} > (<)\, 0 \text{ if } \theta\tau^2 < (>)\, \frac{\beta\gamma}{\gamma+\rho}. \tag{B6}$$

B.5 Proof of Proposition 5

In order to obtain the first-best pricing rule in the linear SFNE, we only need to equate the providers' equilibrium strategies given by (6) with the feedback representation of the first-best solution given by (14). The optimal price parameters b^F, a_i^F and a_j^F are then found by solving the system

$$\begin{cases} \alpha_2\,(b) = \widehat{\alpha}_2 \\ \frac{\gamma}{\tau}\alpha_1^i\,(a_i, a_j, b) = \widehat{\alpha}_1^i \\ \frac{\gamma}{\tau}\alpha_1^j\,(a_i, a_j, b) = \widehat{\alpha}_1^j \end{cases} \tag{B7}$$

Under the optimal pricing rule, assumption A2 requires $\beta > 2b^F$, which holds if and only if

$$\beta > -\frac{\tau}{6\gamma^2}[2\gamma(2\tau\theta - 3)(\gamma+\rho) + \rho^2\tau\theta] \tag{B8}$$

and

$$\beta < -\frac{\tau[\gamma(8\tau\theta - 9)(\gamma+\rho) + 2\rho^2\tau\theta]}{9\gamma^2} \quad \text{or} \quad \beta > \frac{\tau(\gamma+\rho)}{\gamma} = \underline{\beta}. \tag{B9}$$

Since

$$\underline{\beta} > -\frac{\tau}{6\gamma^2}[2\gamma(2\tau\theta - 3)(\gamma+\rho) + \rho^2\tau\theta] > -\frac{\tau[\gamma(8\tau\theta - 9)(\gamma+\rho) + 2\rho^2\tau\theta]}{9\gamma^2}, \tag{B10}$$

assumption A2 is satisfied if and only if $\beta > \underline{\beta}$.

B.6 Proof of Proposition 6

The result is found by first computing

$$\frac{\partial b^F}{\partial \beta} < 0 \iff \beta > -\tau\frac{4\gamma(3\tau\theta - 4)(\gamma+\rho) + 3\rho^2\tau\theta}{16\gamma^2}. \tag{B11}$$

The second inequality in (B11) always holds since

$$\beta > \underline{\beta} > -\tau\frac{4\gamma(3\tau\theta - 4)(\gamma+\rho) + 3\rho^2\tau\theta}{16\gamma^2}. \tag{B12}$$

Thus, the supremum over the values of b^F is obtained for $\beta \to \underline{\beta}$, and it is obviously given by $\frac{1}{2}\frac{\tau(\gamma+\rho)}{\gamma} > 0$; and $\lim_{\beta\to\infty} b^F = -\infty$. This implies that there exists a unique threshold $\widehat{\beta}^F$ such that $b^F > (<) 0$ if $\beta < (>)\, \widehat{\beta}^F$, and this threshold is given by (23) in Proposition 6. Finally, it can be easily verified that $\widehat{\beta}^F > \underline{\beta}$ if and only if $(2\gamma+\rho)^2\tau\theta > \gamma(\gamma+\rho)$, which is always satisfied under assumption A1.

Appendix C: Cost dependence between quality and output

Suppose that the cost function of Provider i is given by

$$C_i(D_i, q_i) = \frac{\theta}{2}q_i^2 + c_i D_i + \frac{\beta}{2}D_i^2 + \delta q_i D_i, \tag{C1}$$

where $\delta > 0$ implies that the marginal cost of quality provision depends positively on output. This modification of our main model implies that assumption A2 must be modified accordingly, and the condition for equilibrium existence under the pricing rule considered is now given by

$$\beta > 2b + \frac{\delta^2}{\theta}. \tag{C2}$$

The optimal regulated price in the SFNE is given by $p_i^F(t) = a_i^F + b^F D_i(t)$, where the volume-dependent part is now given by

$$b^F = \frac{\tau}{4\gamma^2}\left(\left(\frac{2\delta\gamma}{\tau} + (2\gamma+\rho)\theta\right)\xi - 4\delta\gamma(2\gamma+\rho) - 2\gamma(\rho+\gamma)(2\tau\theta-3) - \rho^2\tau\theta\right) - \beta, \tag{C3}$$

where

$$\xi := \sqrt{\frac{4\beta\gamma^2 + 4\delta\gamma\tau(2\gamma+\rho) + 4\gamma\tau(\gamma+\rho)(\tau\theta-1) + \rho^2\tau^2\theta}{\theta}}. \tag{C4}$$

As before, the optimal price implies a premium (penalty) on demand if β is below (above) a threshold value $\widetilde{\beta}^F$, which is now given by[26]

$$\widetilde{\beta}^F := \frac{\left[\begin{array}{c}\theta\tau\left(12\gamma(\gamma+\rho) - \theta\tau(2\gamma+\rho)^2\right) - 4\gamma\delta\left(\theta\tau(2\gamma+\rho) - \gamma\delta\right) \\ \hline + \left(\theta\tau(2\gamma+\rho) + 2\gamma\delta\right)\sqrt{\theta\tau\left(\theta\tau(2\gamma+\rho)^2 + 8\gamma(\gamma+\rho)\right) + 4\gamma\delta\left(\theta\tau(2\gamma+\rho) + \delta\gamma\right)}\end{array}\right]}{8\theta\gamma^2}. \tag{C5}$$

Our main qualitative results are thus robust to this alternative specification of the cost function. Analogous results can be obtained using, as solution concept, OLNE. Details are available upon request.

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