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**Proceedings Paper:**

Tatchum, M, Gegov, A, Jafari, R orcid.org/0000-0001-7298-2363 et al. (1 more author) (2019) Parallel Distributed Compensation for Voltage Controlled Active Magnetic Bearing System using Integral Fuzzy Model. In: 2018 International Conference on Intelligent Systems (IS). 2018 International Conference on Intelligent Systems (IS), 25-27 Sep 2018, Madeira, Portugal. IEEE , pp. 190-198. ISBN 9781538670972

https://doi.org/10.1109/IS.2018.8710567

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# Parallel Distributed Compensation for Voltage Controlled Active Magnetic Bearing System using Integral Fuzzy Model

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**Abstract—Parallel Distributed Compensation (PDC) for current-controlled Active Magnetic Bearing System (AMBS) has been quite effective in recent years. However, this method does not take into account the dynamics associated with the electromagnet. This limits the method to smaller scale applications where the electromagnet dynamics can be neglected. Voltage-controlled AMBS is used to overcome this limitation but this comes with serious challenges such as complex mathematical modelling and higher order system control. In this work, a PDC with integral part is proposed for position and input tracking control of voltage-controlled AMBS. PDC method is based on nonlinear Takagi-Sugeno (T-S) fuzzy model. It is shown that the proposed method outperforms the conventional fuzzy PDC. It stabilizes the bearing shaft at any chosen operating point and tracks any chosen smooth trajectory within the air gap with a high external disturbance rejection capability.**

**Keywords—Takagi-Sugeno, Active Magnetic Bearings, Parallel Distributed compensation, AMB voltage-controlled**

## I. INTRODUCTION

Active Magnetic Bearing Systems (AMBS) are increasingly used in large variety of applications. Their main feature makes them attractive for solving classical rotor bearing failure. AMBS is a bearing without physical contact between the moving parts and the stationary parts, therefore losses caused by frictions can be fully eliminated. Without physical contact between the bearing and the rotor, higher operation speeds can be achieved [1].

AMB works on the principle of electromagnetic suspension of a ferromagnetic object in a free space movement (magnetic levitation). It is quite difficult to a build a magnetic bearing using permanent magnet due to the limitation described by the Earnshow's theorem stating that 'if the inverse-square law governs a group of charged particles, they can never be in a stable equilibrium' [2]. These systems are open loop unstable and are described by highly coupled differential equations, making them difficult to control. As a result, AMBS require continuous active control system to hold the load stable.

Over the years, various approaches have been used [3-4], [8-14]. In [11] a variation of the conventional Proportional Integral and Derivative (PID) control was introduced. This is achieved by cascading Proportional and Derivative controllers for AMBS position control. The advantages of this control are transparent design, simple realization as well as higher closed-loop damping and stiffness in comparison with conventional PID control. State feedback linearization was applied on a One Degree Of Freedom (DOF) AMBS and algebraically got the global linear model from its original nonlinear system [2]. PID controller was applied to the system in [9] and a nonlinear model of a voltage-controlled single DOF AMBS was obtained by combining both analytic and identified models from experimental data and a feedback linearization controller was designed for this system [8]. Furthermore, fuzzy logic control was utilized to overcome the limitation of the traditional PID control using fuzzy rules to evaluate optimal controller parameters in real-time. This has led to a significant improvement in the robustness of traditional AMBS PID controller. An hybrid controller for AMBS, was proposed in [10], the controller consists in two different controllers, a linear controller PID for operation around the linearized equilibrium point and a nonlinear controller (feedback linearization) for operation out the linearized, fuzzy controller was used to accomplish a smooth transition between the linear and nonlinear controllers.

This breakthrough has opened the way to hybrid fuzzy systems. An optimal fuzzy PID based on genetic algorithm was proposed in [10]. The controller was further developed with the introduction of a novel hybrid fuzzy control for AMBs using Radial Basic Function neural network (RBFNN) to continuously adjust controller parameters [29]. A second variance of a fuzzy logic system was introduced in [14]. The T-S controllers were widely successful in controlling various nonlinear systems [12-19]. The mean feature of this technique is to express the local dynamic of each fuzzy rule by a linear model. The completed model is achieved by using fuzzy implication to put together all the linear sub-models. The fuzzy rules are used to express fuzzy subspace and each sub-model can be easily controlled using linear control techniques such as states feedback. Hong et al, used this method on AMBS, this controller demonstrated performance that was superior to conventional PID controller [13]. New methods based on states feedback controller were further developed in [26] where T-S method is used to model AMBS and a nonlinear states feedback controller based on Parallel-Distributed Compensation (PDC) to successfully stabilize current-controlled AMBS with very low overshoot [25]. This technique was further developed in [27] using T-S fuzzy model a magnetic levitation current-controlled system and PDC to stabilize the system.

However, current-controlled AMB does not take into account the dynamics associated to the electromagnet and therefore limits their utilization to small scale applications. This is on the contrary to the voltage-controlled AMBS which take in consideration all the dynamics associated to the system. In this work, PDC controller is designed for AMBS and an improved version using an integral T-S fuzzy PDC controller is proposed for the stabilisation, input tracking and disturbance rejection of voltage-controlled AMBS.

The paper is organized as follows. Section I presents the introduction and the literature review. Section II describes the mathematical model of the thrust AMBs. Section III deals with controller design methodology of fuzzy PDC and fuzzy PDC with an integral part for voltage-controlled AMBs in detail. Section IV analyses and discusses the simulation results for the proposed control schemes.

## II. MATHEMATICAL MODELING

A schematic of a One Degree of Freedom (ODF) Thrust active magnetic bearing is illustrated in Fig. 1. The motion of the AMBs rotor-shaft can be described by the displacements $x$ of the geometric centre of the bearing. With the external forces acting on the shaft that include the magnetic $x_0$ which is the distance between the magnet and the shaft surface, when the shaft is in equilibrium, (midway between the two electromagnets).

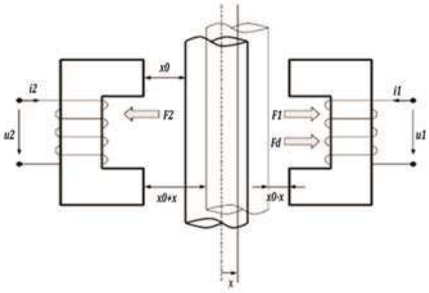


Fig. 1 One DOF Thrust AMBS

- $x_1, x_2$ are the air gaps between the magnets and the rotor.
- $x$ is the displacement of the rotor from the equilibrium position.
- $m$ is the mass of the rotor.
- $F_1, F_2$ are electromagnetically induced forces on the rotor (produced by the two magnets).
- $u_1, u_2$ are the control voltages applied to the two magnetic coils.
- $i_1, i_2$ are the currents flowing in the two magnetic coils.
- $F_d$ is an external disturbance force on the rotor shaft.

### A. AMBS Analysis

Applying the Newton equation of motion

$$m\ddot{x} = F_1 - F_2 + F_d \tag{1}$$

Where $m$ is the mass of the rotor, $F_1$ and $F_2$ are the resultant magnetic bearing forces generated by both coils. The expression for the electromagnetic force, the forces $F_1$ and $F_2$ are both produced by magnetic flux induced in the ferromagnetic cores by the current flowing in the coils.

#### 1) Magnetic force Analysis

The electromagnetic forces on the rotor $F_1$ and $F_2$ are obtained from the calculation of the flux in the air gap. The total magnetic force exerted on the rotor is noted $F$ . Resultant flux is determined via stored energy calculation in a magnetic field considering a given volume (2).

$$W_\phi = \frac{1}{2}\int_v H.dV = \int_v \frac{B^2}{2\mu_0}dV \tag{2}$$

Assuming that all the magnetic energy is stored in the two air gaps, the volume under consideration will be:

$$V = 2 \times S \quad (m^3)$$

For an infinitesimal change in the air gap of length $dx$, corresponding to a magnetic energy changed. This corresponding change can be expressed as:

$$\begin{aligned} dW_0 &= \frac{B^2}{2\mu_0} dV = \frac{B^2}{2\mu_0} \times 2Sdx \\ \frac{d}{dt}(w(\phi)) &= \frac{SB^2}{\mu_0} \\ \text{with} \quad B &= \frac{Ni\mu_0}{2x} \end{aligned} \tag{3}$$

Furthermore, the relation between the work done and the applied magnetics force on the rotor is given by:

$$dW_0 = Fdx \Rightarrow F = \frac{\phi^2}{s\mu_0} \tag{4}$$

Substituting the expression of the flux in the (4) yields:

$$\begin{aligned} F &= \frac{\varphi^2}{s\mu_0} = \frac{1}{s\mu_0}\left(\frac{i}{2x} SN\mu_0\right)^2 \\ &= \frac{S\mu_0 N^2}{4}\left(\frac{i}{x}\right)^2 \\ &= \frac{K}{4}\left(\frac{i}{x}\right)^2 \end{aligned} \tag{5}$$

K in the constant is related to the Magnetic field characteristics. Finally, the electromagnetic forces can be expressed as:

$$\begin{aligned} F_1 &= \frac{K}{4}\left(\frac{i_1}{x_0 - x}\right)^2 \\ F_2 &= \frac{K}{4}\left(\frac{i_2}{x_0 + x}\right)^2 \end{aligned} \tag{6}$$

The well-known current-controlled AMBs dynamic can be obtained by replacing the expression of the electromagnetic forces in (1)

$$m\ddot{x} = \frac{K}{4}\left(\frac{i_1}{x_0 - x}\right)^2 - \frac{K}{4}\left(\frac{i_2}{x_0 + x}\right)^2 - F_d \tag{7}$$

2) Control Voltage Analysis

From Kirchhoff law, the expression of the voltage an Inductor and Resistive circuit (LR) is expressed as follows:

$$\begin{aligned} u &= iR + L_s \frac{di}{dt} + N\frac{d\phi}{dt} \\ u &= iR + L_s \frac{di}{dt} + \frac{KN}{2N}\frac{d}{dt}\left(\frac{i}{x}\right) \end{aligned}$$

The voltage across each electromagnet is expressed as:

$$\begin{aligned} u_1 &= i_1 R + L_s \frac{di_1}{dt} + \frac{K}{2}\frac{d}{dt}\left(\frac{i_1}{x_0 - x}\right) \\ u_2 &= i_2 R + L_s \frac{di_2}{dt} + \frac{K}{2}\frac{d}{dt}\left(\frac{i_2}{x_0 + x}\right) \end{aligned} \tag{8}$$

### B. AMBS Nonlinear Model

Based on the Thrust AMBs described in Fig. 1, can define as 4 states system. Let's X the states vector:

$$X = \begin{bmatrix} x & \dot{x} & \frac{di_1}{dt} & \frac{di_2}{dt} \end{bmatrix}^T$$

The final nonlinear model of the voltage-controlled thrust AMBS [31].

$$f(X,u,F_d) = \begin{bmatrix} \dot{x} \\ \frac{K}{4m}\left(\frac{i_1}{x_0 - x}\right)^2 - \frac{K}{4m}\left(\frac{i_2}{x_0 + x}\right)^2 + \frac{F_d}{m} \\ \frac{2(x_0 - x)}{2L_s(x_0 - x) + K}\left[-Ri_1 - \frac{K}{2(x_0 - x)^2}\dot{x}i_1 + u_1\right] \\ \frac{2(x_0 + x)}{2L_s(x_0 + x) + K}\left[-Ri_2 - \frac{K}{2(x_0 + x)^2}\dot{x}i_2 + u_2\right] \end{bmatrix} \tag{9}$$

### C. AMBS Linear Model

Using Jacobean transformation, a general linear model is derived around a chosen operating point within the air gap.

$$X_i = \begin{bmatrix} x_i & \dot{x}_i & \frac{di_{1i}}{dt} & \frac{di_{2i}}{dt} \end{bmatrix}^T$$

$$\delta\dot{X} = \frac{\partial f(X,u,F_d)}{\partial X} + \frac{\partial f(X,u,F_d)}{\partial u} + \frac{\partial f(X,u,F_d)}{\partial F_d}$$

$$\dot{X} = A_1 X + B_1 u + B_2 F_d \tag{10}$$

Where:

$$A_1 = \frac{\partial f(X,u,F_d)}{\partial X}\Big|_{X=X_i}, \; B_1 = \frac{\partial f(X,u,F_d)}{\partial u}\Big|_{X=X_i}$$

$$B_2 = \frac{\partial f(X,u,F_d)}{\partial F_d}\Big|_{X=X_i}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ A_{21i} & 0 & A_{23i} & A_{24i} \\ 0 & A_{32i} & A_{33i} & 0 \\ 0 & A_{42i} & 0 & A_{44i} \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31i} & 0 \\ 0 & B_{42i} \end{bmatrix} U + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \\ 0 \end{bmatrix} F_d$$

With:

$$A_{21i} = \frac{Ki_{1i}^2}{2m(x_0 - x_i)^3} + \frac{Ki_{2i}^2}{2m(x_0 + x_i)^3};$$

$$A_{23i} = \frac{Ki_{1i}}{2m(x_0 - x_i)^2};\ A_{24i} = \frac{-Ki_{2i}}{2m(x_0 + x_i)^2}$$

$$A_{32i} = \frac{-Ki_{1i}}{(x_0 - x_i)[2L_s(x_0 - x_i) + K]}$$

$$A_{42i} = \frac{-Ki_{2i}}{(x_0 + x_i)[2L_s(x_0 + x_i) + K]}$$

$$A_{33i} = \frac{-2R(x_0 - x_i)}{[2L(x_0 - x_i) + K]};\ A_{44i} = \frac{-2R(x_0 + x_i)}{[2L(x_0 + x_i) + K]}$$

$$U = [u_1 \quad u_2]^T \quad B_{31i} = \frac{2(x_0 - x_i)}{[2L(x_0 - x_i) + K]}$$

$$B_{42i} = \frac{2(x_0 + x_i)}{[2L(x_0 + x_i) + K]}$$

## III. CONTROLLER DESIGN METHODOLOGY

### A. T-S Fuzzy Theory

The fuzzy model proposed by Takagi and Sugeno is described by fussy IF-THEN rules, which represents the local input-output of a nonlinear system; the mean feature of this technique is to express the local dynamic of each fuzzy rule by a linear model. The interred system model is achieved by using fuzzy implication to put together all the linear sub-model. The fuzzy rules are used to express fuzzy subspace [12].

Let's consider a nonlinear system

$$\begin{cases} x = f(x,u) \\ f(x,u) \in [a_1, a_2] \\ -d \le x \le d \end{cases} \tag{11}$$

Model Rules i

$$\text{if} \quad z_i(t) \text{ is } M_i^1 \text{ and } .....z(t) \text{ is } M_p^i$$
$$\text{THEN} \begin{cases} x(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{12}$$
$$i = 1,2,3..n$$

Where $A_i, B_i, C_i$ represent the state space components of each sub-system and $i_i$ is the number of rules, and the fuzzy sets $M_j^i$, $j = 1,2,3,.., p$

The fuzzy systems output is represented as:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^n \mu_i(z(t))[A_i x + B_i u]}{\sum_{i=1}^n \mu_i(z(t))} \\ &= \sum_{i=1}^n h_i(z(t))[A_i x + B_i u] \\ y(t) &= \sum_{i=1}^n h_i(t) C_i x(t) \end{aligned} \tag{13}$$

### B. PDC Control Theory

Parallel Distributed Compensation theory, each fuzzy model rule is controlled using full states feedback, the overall fuzzy controller is a fuzzy blending of each individual linear controller [26-28].

Controller rule i

$$\text{if} \quad z_i(t) \text{ is } M_i^1 \text{ and } .....z(t) \text{ is } M_p^i$$
$$\text{THEN} \begin{cases} x(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{14}$$
$$i = 1,2,3..n$$

The overall fuzzy controller rules can be represented as:

$$u(t) = -\sum_{i=1}^n h_i(t) K_i x(t) \tag{15}$$

Substituting (15) in (14) the close loop system can be represented as:

$$\dot{x}(t) = \sum_{i=1}^n \sum_{j=1}^n h_i(x) h_j(x)(A_i - B_i K_j) x(t) \tag{16}$$

For this work pole placement method is used to design the matrix $K_i$.

The equilibrium of a fuzzy system (12) is globally asymptotical stable if there exist a common positive definite matrix which verifies equation (17)[30]. I Where

$$\begin{cases} A_i^T P A_i + P < 0 \\ i = 1,2,3,\dots,n \end{cases} \tag{17}$$

B. PDC Controller Design for Thrust AMBs

In this section we used Takagi-Sugeno fuzzy model based on PDC for Thrust AMBs. The nonlinear model (9) of the Thrust AMBs can be modelled as (18), the system block diagram and the membership functions are represented respectively by Fig. 2 and Fig. 3, 5 equilibrium points are chosen within the air gap $-6\times10^{-4} < x_i < 6\times10^{-4}$ (m)

Model Rules i

$$\text{if} \quad z_i(t) \ \text{is}\ M_i^1 \ \text{and} \ldots z(t) \ \text{is} \ M_p^i$$
$$\text{THEN} \begin{cases} \dot{X}(t) = A_i X(t) + B_i u(t) \\ y(t) = C_i X(t) \end{cases} \quad (18)$$
$$i = 1,2,3,4,5$$

The equilibrium operating points are defined as follow:

$$X_1 = [-5.04\times10^{-4} \quad 0 \quad 2.1952 \quad -2.25]$$
$$X_2 = [-2.5\times10^{-4} \quad 0 \quad -1.10 \quad 1.13]$$
$$X_3 = [0 \quad 0 \quad 2 \quad 2]$$
$$X_4 = [2.5\times10^{-4} \quad 0 \quad 1.10 \quad -1.13]$$
$$X_5 = [5.05\times10^{-4} \quad 0 \quad -2.19 \quad 2.25]$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 7.17\times10^5 & 0 & 16.13 & 622.89 \\ 0 & -461.75 & -49.78 & 0 \\ 0 & -7.74\times10^3 & 0 & -21.62 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.64\times10^5 & 0 & -12.9\times10^6 & -59.61 \\ 0 & 346.08 & -46.66 & 0 \\ 0 & 1.20\times10^3 & 0 & -34.88 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6212 & 0 & 21.74 & -21.74 \\ 0 & -526.32 & -42.11 & 0 \\ 0 & 526.32 & 0 & -42.11 \end{bmatrix}$$
$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.58\times10^5 & 0 & 58.25 & 13.20 \\ 0 & -1.17\times10^3 & -34.88 & 0 \\ 0 & -354.15 & 0 & -46.66 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.85\times10^6 & 0 & -608.70 & -16.51 \\ 0 & 7.57\times10^3 & -21.62 & 0 \\ 0 & 472.52 & 0 & -49.78 \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.70 & 0 \\ 0 & 6.22 \end{bmatrix}; B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 5.26 & 0 \\ 0 & 5.26 \end{bmatrix}; B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4.36 & 0 \\ 0 & 5.83 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 5.83 & 0 \\ 0 & 4.36 \end{bmatrix}; B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 6.22 & 0 \\ 0 & 2.71 \end{bmatrix}$$

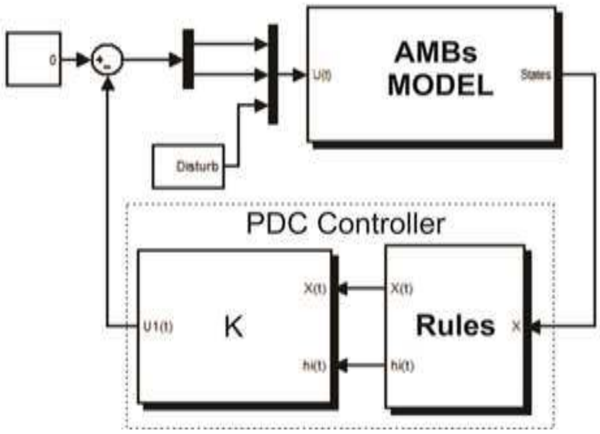


Fig. 2 PDC Control Scheme

1) Local controller design

States feedback control is a method employed in feedback control system theory to place the closed loop poles of a plant in pre-determined locations in the s-plane; the requirement system performance is chosen taking into account the complexity of AMBS:

Maximum settling time $T_s < 0.1s$

Maximum overshoot: $O_s < 1\%$

Using state space approach, the desired characteristic equation of the close look system is expressed by the Ackerman (8) [24].

$$\alpha_c(A_i) = A_i^4 + 66.9 A_i^3 - 4.1\times10^4 A_i^2 - 5.3\times10^7 A_i^1 - 1.72\times10^9 \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \quad (19)$$

$$K_i = [0 \quad 0 \quad 0 \quad 1][B_i \quad A_iB_i \quad A_i^2B_i \quad A_i^3B_i]\alpha_c(A_i)$$

The fuzzy PDC controller gains are obtained using equation (19).

$$K_1 = \begin{bmatrix}-1.87\times10^7 & -3.22\times10^3 & 55.42 & -1.55\times10^3\\ 9.10\times10^6 & 2.18\times10^3 & 13.18 & 770.99\end{bmatrix}$$

$$K_2 = \begin{bmatrix}-2.75\times10^5 & -638.21 & 129.76 & 41.33\\ -1.51\times10^6 & -3.52\times10^3 & 47.80 & 381.16\end{bmatrix}$$

$$K_3 = \begin{bmatrix}1.01\times10^6 & 3.94\times10^3 & 227.81 & -93.53\\ -1.04\times10^6 & -4.06\times10^3 & -96.46 & 231.19\end{bmatrix}$$

$$K_4 = \begin{bmatrix}1.47\times10^6 & 3.5\times10^3 & 374.66 & 50.115\\ 3.56\times10^5 & 862.41 & 56.17 & 134.62\end{bmatrix}$$

$$K_5 = \begin{bmatrix}-6.73\times10^6 & -1.78\times10^3 & 585.99 & 10.53\\ -1.77\times10^7 & -3.11\times10^3 & 1.50\times10^3 & 135.76\end{bmatrix}$$

2) Close loop stability analysis

The stability criterion described by (17) is verified, $(A_i - B_iK_i)$ represents the close loop system.

$$\begin{cases}(A_i - B_iK_i)^T P(A_i - B_iK_i) + P < 0\\ i = 1,2,3,\dots,n\end{cases}$$

Using Matlab LMI toolbox, P matrix is determined.

$$P = \begin{bmatrix}9.904 & 4.5\times10^{-4} & 0.013 & -0.0042\\ 4.58\times10^{-5} & 4.11\times10^{-7} & 3.15\times10^{-6} & -6.8\times10^{-7}\\ -0.013 & 3.15\times10^{-6} & 3.86\times10^{-5} & 1.45\times10^{-6}\\ -0.0042 & -6.83\times10^{-7} & -1.45\times10^{-6} & 5.9\times10^{-6}\end{bmatrix}$$

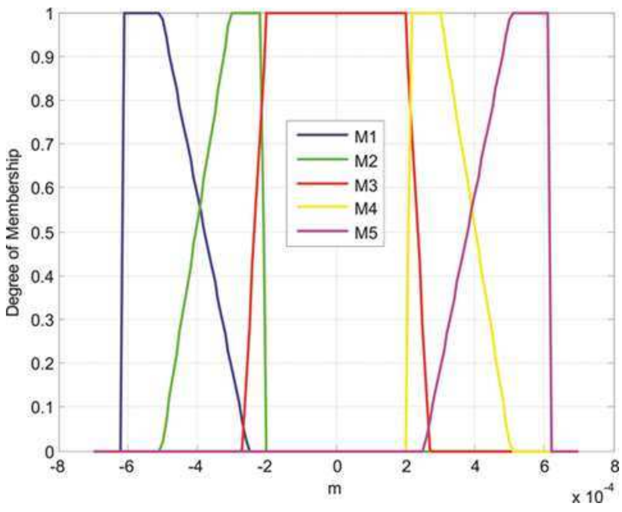


Fig. 3 Membership Functions

The designed PDC fuzzy successfully stabilizes the shaft from any initial position within the air gap, however any external disturbance results in a stationary error which deteriorate the system performance Fig. 6. In order to compensate the stationary error, an integral part is added to the PDC as per Fig. 4

Let $\bar{X}^T = [X \quad X_L]^T$ and $y_r$ be the extended state vector and the reference input respectively.

$$X_L = y_r - \sum_{i=1}^{5} h_i(t) y_i(t)$$
$$X_L = y_r - \sum_{i=1}^{5} h_i(t) C_i x(t) \quad (20)$$

The overall system T-S model is expressed as:

$$\dot{\bar{X}}(t) = \sum_{i=1}^{5} h_i(t)[\bar{A}_i\bar{X}(t) + \bar{B}_i u(t)] + B_0 y_r(t)$$
$$y(t) = \sum_{i=1}^{5} h_i(t)\bar{C}_i\bar{X}(t) \quad (21)$$

With

$$\bar{A}_i = \begin{bmatrix}A_i & 0\\ -C_i & 0\end{bmatrix}, \bar{B}_i = \begin{bmatrix}B_i\\ 0\end{bmatrix}, \bar{C}_i = [C_i \quad 0], B_0 = \begin{bmatrix}0\\ I\end{bmatrix}$$

Where *0* and *I* are matrices of proper dimension. The integral PDC control law can be written as:

$$u(t) = -\sum_{i=1}^{5} h_i(t) F_i \bar{X}(t) \quad (22)$$

Substituting the equation (22) in (21) the close loop system can be express as:

$$\dot{\bar{X}}(t) = \sum_{i=1}^{5}\sum_{j=1}^{5} h_i(X)h_j(X)[(\bar{A}_i - \bar{B}_iF_j)\bar{X}(t)] + B_0 y_r(t) \quad (23)$$

Where $F_i = [K_i \quad L_i]$

Using states feedback control law matrix L is determined.

$$L_1 = \begin{bmatrix} -14.3\times10^8 \\ 70.1\times10^7 \end{bmatrix}, L_2 = \begin{bmatrix} -21.17\times10^6 \\ -11.55\times10^7 \end{bmatrix}, L_3 = \begin{bmatrix} 7.7\times10^7 \\ -8.0\times10^7 \end{bmatrix},$$
$$L_4 = \begin{bmatrix} 11.32\times10^7 \\ 27.41\times10^6 \end{bmatrix}, L_5 = \begin{bmatrix} -51.8\times10^7 \\ -13.6\times10^8 \end{bmatrix}$$

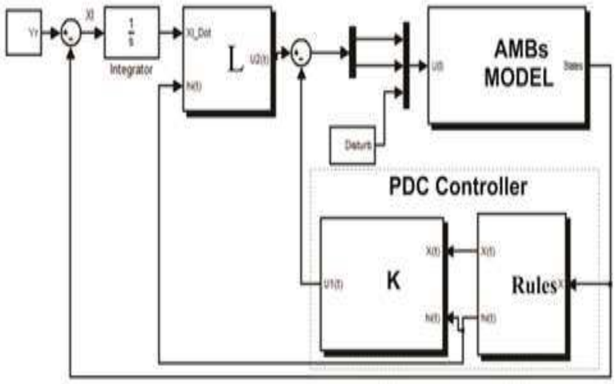


Fig. 4 PDC Control Scheme with Integral Part

## IV. SIMULATION RESULTS

The designed system is tested under different conditions, the simulation results are shown from Fig. 5 to Fig. 7, the AMBS parameters used for the simulation can be found in Table 1.

TABLE.1. AMB Simulation Parameters

| Parameter's name | Parameter's formula or symbol | values |
|---|---|---|
| Nominal air gap | xo | 0.0007m |
| Coil Resistance | R | 8 ohms |
| Coil self-inductance | Ls | 0.12 |
| K | K | $9.8\times10^{-5}$ |
| Rotor Mass | m | 4.6 |

To evaluate the performances of the designed controller based on one of the most important criteria of the AMBS the robustness and the ability to stabilize from any initial position within the air gap.

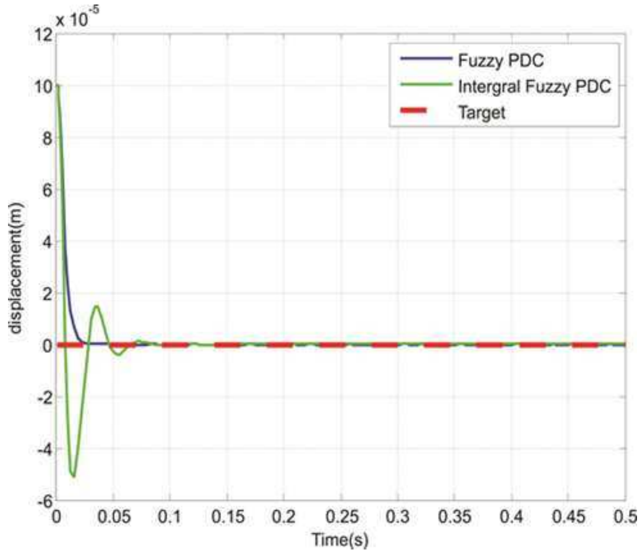


Fig. 5 . Fuzzy PDC & Integral Fuzzy PDC Stabilization Response without disturbance. Fd=0N (Initial condition $X = [0.0001 \quad 0 \quad 0 \quad 0]^T$

### A. System stabilisation without external disturbance

To evaluate the system, both Conventional PDC and the integral PDC are used to stabilize a Thrust AMBs from an initial position of (0.1mm), the result is shown on Fig. 5, both controller successfully stabilize the shaft, it should be noted that the Conventional PDC achieved it with (0%) overshoot which is slightly better compared to the integral PDC. This can be explained by the fact that during the local controller design, the damping ratio was set to 1 (system critically damped), by adding the integral part, this modify the system dynamic and the damping ration slightly dropped below 0 resulting in few oscillations.

### B. System stabilisation with external disturbance

An external disturbance of 50N is added to the system at t=0.6s and addition 20N at t=0.8s, both controllers stabilized the controller successfully stabilized the AMBS, however the integral PDC, was able to reject the disturbance and keep the stationary error to 0 as required. In contrary, the Conventional PDC stabilizes the system with a high stationary error limiting the loading of the bearing. Integral PDC Fuzzy controller has shown high robustness to external disturbance which is very important in AMBS Fig. 6.

### C. Trajectory tracking with external disturbance

The proposed PDC was further used in reference tracking. Fig. 7 shows the effectiveness on the controller.

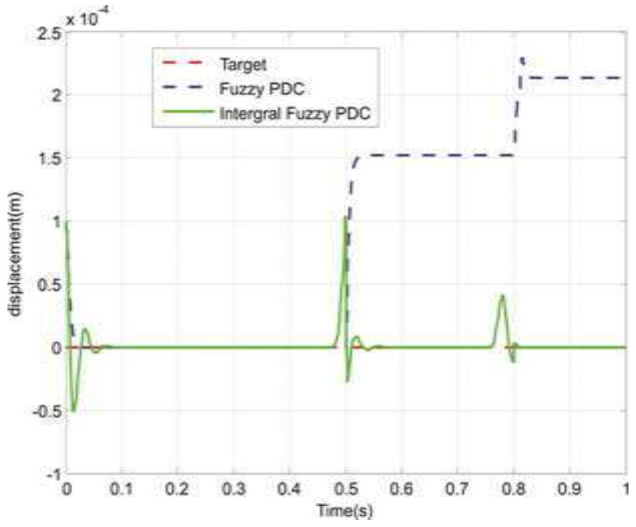


Fig. 6 Fuzzy PDC & Integral Fuzzy PDC Stabilization Response with disturbance added (t=0.5s, Fd=50N ;t=0.8s, Fd=50N+20N)

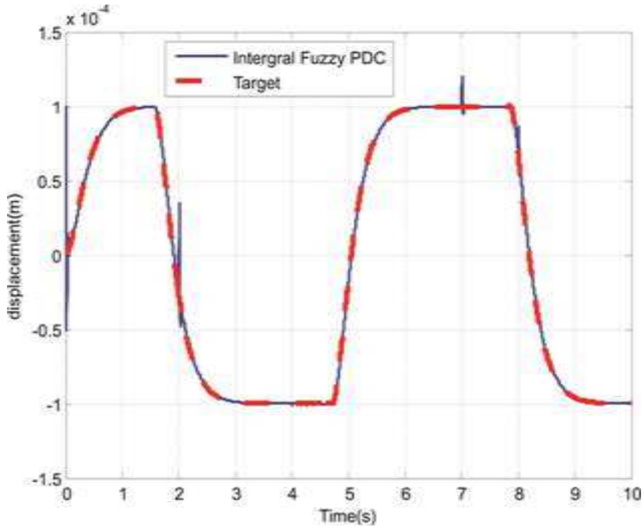


Fig. 7 Integral Fuzzy PDC Trajectory Tracking Response with external disturbance added (t=2s, Fd= 40N; t=7s, Fd=40N+20N)

### D. Analysis of Results

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - y_{refi}\right) \quad (24)$$

Based on Fig. 5 and the mean square error analysis from Table 2, it can be seen that the PDC controller with integral part under no disturbance condition produce slightly larger settling time and overshoot than the original PDC controller. However, its performances remain satisfactory with regards to the design requirement (overshoot<1%, settling time<0.1s).

The ability of the integral PDC controller to neutralise external disturbances Fig .6 makes it very attractive for AMBS application which in real industrial application is a highly disturbed system.

Variable reference tracking with external disturbance is use to further evaluate the controller robustness. Fig. 7 shows the capacity of the fuzzy PDC to track any given smooth trajectory within the air gap, the conventional PDC performance was very poor during external disturbance test, raison why its performance was not evaluated with a variable reference tracking.

TABLE 2. Mean Squared Error

| | Fuzzy PDC | Integral Fuzzy PDC |
|---|---|---|
| System without disturbance (Fig .5) | $2.28\times10^{-9}$ | $2.61\times10^{-9}$ |
| System with disturbances (Fig. 6) | $163\times10^{-10}$ | $5.58\times10^{-10}$ |
| Reference tracking with disturbance ( Fig. 7) | N/A | $5.46\times10^{-11}$ |

## V. CONCLUSION

In the present work, a novel nonlinear controller based on PDC T-S fuzzy method and enhanced by an integral action is implemented for a class of voltage-controlled AMBS to deal with stability, robustness and trajectory tracking problems. The proposed controller is applied to a thrust AMBS. Numerical simulations confirm the ability of the proposed controller to ensure stability and trajectory tracking despite the presence of external disturbances.

The major contributions of this study are the successful development of conventional fuzzy PDC for voltage-controlled thrust AMBS and fuzzy PDC with an integral part for voltage-controlled thrust AMBS. Simulations show better performance of the proposed controller compared to the conventional PDC in terms of compensating the effect of external disturbances. This is a fundamental criterion for thrust AMBS industrial applications.

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