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Modular Fluidic Propulsion Robots

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Abstract—We propose a novel concept for modular robots, termed modular fluidic propulsion (MFP), which promises to combine effective propulsion, a large reconfiguration space, and a scalable design. MFP robots are modular fluid networks. To propel, they route fluid through themselves. In this article, both hydraulic and pneumatic implementations are considered. The robots move towards a goal by way of a decentralized controller that runs independently on each module face, uses two bits of sensory information and requires neither run-time memory, nor communication. We prove that 2-D MFP robots reach the goal when of orthogonally convex shape, or reach a morphology-dependent distance from it when of arbitrary shape. We present a 2-D hydraulic MFP prototype and show, experimentally, that it succeeds in reaching the goal in at least 90% of trials, and that 71% less energy is expended when modules can communicate. Moreover, in simulations with 3-D hydraulic MFP robots, the decentralized controller performs almost as well as a state-of-the-art and centralized controller. Given the simplicity of the hardware requirements, the MFP concept could pave the way for modular robots to be used at sub-centimeter-scale, where effective modular propulsion systems have not been demonstrated.

Index Terms—Distributed control, hydraulic propulsion, modular reconfigurable robots, pneumatic propulsion.

I. INTRODUCTION

ONE OF THE principal aims of robotics is to develop machines that can carry out tasks that are difficult for

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This article has supplementary downloadable material available at <http://ieeexplore.ieee.org>. The material consists of video clips of MHP robots using the occlusion-based controller, both experimentally and in simulation. The experiment clips show 2×2 robots using both the default controller and a variant with communication enabled. The simulation clips show a 125 module cubic robot using a controller variant with obstacle avoidance, and a 125 module robot of random morphology using the default controller.

Color versions of one or more of the figures in this article are available online at <https://ieeexplore.ieee.org>.

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humans. This includes working in inhospitable or inaccessible environments, such as deep water, space, storage tanks, pipe networks, or even the human vascular network. Modular self-reconfigurable robots [1], [2] are well suited for these environments as they can adapt their morphology and functionality to cope with complex tasks and unexpected situations. Examples include shifting between configurations with longer reach or greater strength to repair equipment in space [3], using reconfiguration to deploy and retrieve the modular nodes of an underwater sensing network [4], or moving through small apertures and shoring up a damaged structure in a search and rescue scenario [5]. Additionally, a given system could be used in a wide range of scenarios, reducing production costs due to mass manufacture and lowered training time for operators.

Modular reconfigurable robotic systems are yet to realize their full potential in the real world. We hypothesize that to improve the general utility, a system should combine all of the following properties:¹

- 1) *effective propulsion*: fast and precise movement as a connected robot, independent of modular configuration;
- 2) *large reconfiguration space*: a rapid increase of the number of possible configurations with the number of modules;
- 3) *scalable design*: a performance that scales favorably as the number of modules increases or their size decreases.

Most current systems excel at one or two of the aforementioned properties, though at the expense of the remaining ones: Some systems prioritize effective propulsion. The AMOUR robot [6], distributed flight array (DFA) [7], and ModQuad [8], [9] are among the most efficacious mobile connected systems, capable of up to six degrees-of-freedom (DoF) movement. However, this comes at a cost to reconfiguration: The AMOUR robot can reconfigure only in 1-D, whereas the DFA and ModQuad robots can reconfigure only in 2-D. Moreover, current implementations do not demonstrate a high degree of scalability: The AMOUR robot relies on a centralized motion controller, the DFA relies on external sensing, and ModQuad relies on a centralized path planner. For some other systems, the modules can move effectively on their own, yet connected robots have only limited propulsion effectiveness [10]–[13].

Some systems prioritize a large reconfiguration space, but lack effective movement capabilities as a connected robot [14]–[16]. The M-Blocks [15], for example, can self-assemble into 3-D structures that produce net movement primarily via progressive

¹In addition, the utility of modular systems improves if they are capable of self-reconfiguration.

repositioning of the modules. As a result, the robot's propulsion speed is limited by its reconfiguration speed. M-TRAN [17] can move effectively in some, but not all configurations.

Some systems prioritize scalability [18]–[20]. Scaling systems up in number of modules and down in size allows modular robots to take a given form with a higher resolution. Such systems should be able to adapt with greater precision to the task at hand. In addition, decreasing the size of the modules allows the robots to fit into smaller environments. Current systems that prioritize scalability exhibit limited propulsion efficacy. For example, the Pebbles robot [20], while having an impressive module side-length of only 12 mm, features no explicit propulsion mechanism. Similarly, claytronics robots can only ambulate slowly via progressive repositioning of individual modules [18], [21].

In this article, we introduce the *modular fluidic propulsion* (MFP) concept. The concept combines a large reconfiguration space and scalable design with the ability to move effectively in any configuration. An MFP robot is a modular fluid network. By rearranging the constituent modules, robots (i.e., fluid networks) of different shapes can be built. MFP robots self-propel by routing fluid through themselves. We consider a directed motion task, where an MFP robot has to move toward a goal. We present a fully decentralized motion controller, which is inspired by an occlusion-based object transportation controller developed for swarms of mobile robots [22]. The controller runs independently on each module face, uses 2 bits of sensory information and requires neither run-time memory, nor communication. It could be realized on hardware that does not offer arithmetic computation, thereby engendering the future miniaturization of the system.

This article extends preliminary work on a related, hydraulic propulsion concept [23]. In particular,

- 1) It proposes the MFP concept, and discusses a pneumatic implementation in addition to the hydraulic implementation proposed in [23].
- 2) It derives the number of unique forces and torques that 2-D orthogonally convex MFP robots can produce.
- 3) It proposes two novel variants of the decentralized motion controller presented in [23]: one using communication to reduce energy consumption; the other using range sensors to avoid obstacles. By simulation, it evaluates the performance of the new variants, and compares them to both the original controller and/or the state-of-the-art (centralized) controller in [24].
- 4) It proves for the first time that 2-D MFP robots reach the goal or a morphology-dependent distance from it, if of orthogonally convex or arbitrary shape, respectively.
- 5) It presents a new, smaller 2-D hydraulic prototype, which additionally features communication and range sensing.
- 6) By systematic experiment, it shows that the new prototype successfully completes the directed motion task when using the original controller in [23], and that 71% less energy is expended when using the communication-enabled variant of this controller.

The rest of this article is organized as follows. Section II provides further related work. Section III outlines the MFP concept, along with hydraulic and pneumatic examples of

TABLE I
OVERVIEW OF RELATED RECONFIGURABLE MODULAR FLUIDIC SYSTEMS

System	Self-propulsion	Propulsion	Reconfigurability	Decentralized control	Number of modules
Neubots [25]	Yes	3-D	3-D	No	(10+)
Hydron [26]	Yes	3-D	–	No	2
AMOUR [6]	Yes	3-D	1-D	No	1
ANGELS [27]	Yes	3-D	1-D	No	9
DFA [7]	Yes	3-D	2-D	No	12
REMORA [28]	Yes	3-D	1-D	No	3
TEMP [29]	Yes	2-D	2-D	No	33
Naldi et al. [30]	Yes	3-D	3-D	No	2
ModQuad [8], [9]	Yes	3-D	2-D	Yes	7
Roboat [31]	Yes	2-D	2-D	No	3
MFP	Yes	2-D	2-D	Yes	4
Inou et al. [32]	No	–	2-D	No	2
White et al. [33]	No	3-D	3-D	Yes	4
Tribolon [35]	No	2-D	2-D	Yes	6
Tolley et al. [34]	No	3-D	3-D	Yes	14
Lily [36]	No	2-D	2-D	Yes	10
Ganesan & Chitre [37]	No	3-D	3-D	Yes	(120)

Number of modules are those within a same experiment (in parenthesis if only simulated). Data for our system (MFP) are shown in boldface.

implementations. Section IV analyzes the static forces and torques that an orthogonally convex 2-D MFP robot can generate. Section V describes the directed motion task and controllers. Section VI provides a formal analysis of the controllers. Section VII presents simulations of a 3-D hydraulic MFP system. Sections VIII and IX describe a 2-D hydraulic MFP hardware prototype and the experiments, respectively. Finally, Section X concludes this article.

II. FURTHER RELATED WORK

We discuss further related work on reconfigurable modular fluidic systems. In particular, the propulsion capabilities of these systems are analyzed to serve as points of comparison to the MFP system (see Table I). We classify the work into two groups, systems with self-propelling modules, and those which rely on an external source of propulsion.

A. Self-Propelled Modular Systems

In early work [25], controllers and morphologies for the Neobot underwater modular robots are evolved. Each module has six connectors, giving the system a 3-D lattice structure. Rotational joints between connected modules allow robots to self-propel in 3-D through coordinated movement.

Hydron modules [26] move underwater in 3-D. An impeller and collar mechanism controls horizontal movement, whereas a syringe controls buoyancy, allowing the modules to move vertically. Simulated modules can self-assemble into 3-D structures, although a connection mechanism is not proposed.

The AMOUR robot can perform 6-DoF motion underwater. It is conceptualized to self-reconfigure in 1-D, and has manually reconfigurable thruster pose [6]. The controller can handle arbitrary thruster configurations. An inertial measurement unit

provides input to the controller. In [38], the AMOUR robot can learn its own thruster configuration.

ANGELS [27] is an underwater modular system that can form chain configurations. Individually, each module can translate and rotate using propellers. Connected modules can collectively produce an undulatory swimming motion. Buoyancy can be controlled using a swim bladder.

The REMORA project [39] examines the use of aquatic modular robots for the inspection and maintenance of off-shore structures, such as oil platforms. The robots can move underwater in 3-D. Furno *et al.* [28] present a hardware prototype of the system and perform experiments in which three robots reconfigure from an “I” shape to an “L” shape.

The tactically expendable marine platform (TEMP) [29] is envisaged for use as a rapidly deployable structure for humanitarian missions. The modules can self-assemble into 2-D structures on the surface of water. Both individual modules and connected structures can self-propel in 2-D.

Naldi *et al.* [30] study the dynamics and control of a modular aerial robot. Each module consists of a ducted fan and a set of vanes that allow the robot to move in 3-D. The design allows modules to be attached both horizontally and vertically, and still fly. However, only configurations with horizontal connections are tested.

Wang *et al.* [40] introduce the Roboat, a modular robotic boat with a force-efficient thruster configuration. The modules can move and reconfigure in 2-D, thereby dynamically forming floating structures. Docking experiments, in the presence of turbulent currents, are reported in [41]. The Roboats are further used for shapeshifting experiments [31].

B. Externally Propelled Modular Systems

Inou *et al.* [32] develop a set of mechanical modules which reconfigure by pumping air through a set of bellows but are not capable of translational motion. The air is provided by an external pump to each module individually, although the authors propose a future system for routing air internally from one module to another. The robots can form a 2-D lattice, although experiments are restricted to two modules.

White *et al.* [33] present two systems in which individual modules move in 3-D inside an oil-filled tank. The oil is agitated externally to provide stochastic motion. The modules can connect to a base plate to construct immobile 3-D structures. The modules of one of the systems have a similar internal structure to the MFP system, allowing fluid to pass through them. However, they are passive, with no propulsion method or internal energy source of their own. Tolley *et al.* [42] and Tolley and Lipson [43] investigate different self-assembly strategies for similar systems.

The Tribolon system [35] uses vibrating pager motors as a source of propulsion. This achieves a similar effect to externally induced stochastic motion. Power is provided externally via a pantograph. The modules can reconfigure in 2-D.

The Lily robots float on the surface of water, which is agitated by external submerged pumps [44]. The robots both move and reconfigure in 2-D. The system is used for the study of stochastic self-assembly [45], for which the authors develop a simple

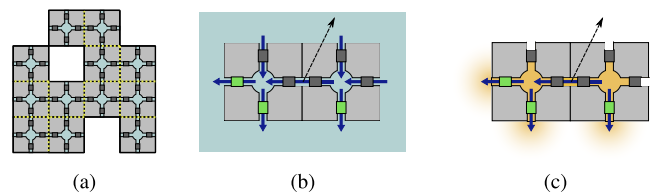


Fig. 1. MFP concept and two implementation options: MHP and MPP. (a) Generic 2-D MFP robot consisting of 12 modules. Each module has one actuator per face, which it can use to route fluid. By routing fluid, the robot propels. Yellow lines denote internal faces, all other faces are external. (b) MHP robot operating in a liquid environment (shown in blue). Green and dark gray boxes indicate active and inactive actuators, respectively. (c) MPP robot containing a gas propellant (shown in yellow). In (b) and (c), blue arrows indicate the direction of fluid flow, and the dashed black arrow indicates the resultant direction of motion.

physics model that avoids the use of complex computational fluid dynamics [46].

Ganesan and Chitre [37] investigate simulated modules that move and self-assemble stochastically in 3-D, in a fluidic environment. A rule-based structure is used to self-assemble different structures, and an analysis of the dynamics of the system is provided.

III. MODULAR FLUIDIC PROPULSION (MFP)

The MFP concept is illustrated in Fig. 1. An MFP *robot* is a modular fluid network. By rearranging the constituent *modules*, robots (i.e., fluid networks) of different shapes can be built. MFP robots self-propel by routing fluid through themselves. The modules are homogeneous squares in 2-D or cubes in 3-D. With a slight abuse of terminology, we use *face* to refer to both 3-D module faces and 2-D module sides. A module can be physically connected to other modules via each of its faces. A module face that is in contact with another module is *internal*. All other faces are *external*. Each module incorporates an internal fluid reservoir. Routing pipes run from the reservoir to each face. The routing pipes of adjoining internal faces are connected to form a direct link between the corresponding reservoirs.

Each routing pipe includes an actuator to control the fluid flow through the pipe. The actuator’s state is binary—active or inactive, meaning that not only the actuators can themselves be mechanically simple, but also the need for complex control circuitry is reduced. This simplicity could enable future miniaturization of the system, and allow a large number of modules to be produced. At the same time, the system would retain effective movement capabilities; even a robot with a small number of modules has a wide range of motion available to it. A 3-D robot with eight modules arranged in a $2 \times 2 \times 2$ cube would have 24 actuators on its external module faces. Without considering internal actuators, a total of $2^{24} \approx 10^7$ unique actuator firing configurations exist. A 3-D robot with 1000 modules arranged in a $10 \times 10 \times 10$ cube would have $2^{600} \approx 10^{180}$ unique actuator firing configurations.

A. Modular Hydraulic Propulsion (MHP)

MHP is a specific implementation of MFP that uses a liquid as the routing fluid. For this implementation, the robot requires

a liquid environment, such as a lake or water pipe. The robot can either float on the surface of the liquid and move in 2-D, or move in 3-D beneath the surface. The interior of the robot (i.e., the reservoirs and routing pipes) is permeated by the liquid [see Fig. 1(b)]. The actuators take the form of pumps. Active pumps can displace liquid between modules, or between module and environment. Inactive pumps allow liquid to pass through in any direction. As liquid is pumped from the robot into the environment, liquid from the environment is drawn into the robot via the inactive pumps of other faces.

B. Modular Pneumatic Propulsion (MPP)

An alternative implementation, MPP uses gas as the routing fluid. The gas is stored within the robot, at a higher pressure than the environmental medium. The actuators take the form of valves. When active, a valve on an external module face can release gas from within the robot into the environment, providing propulsion [see Fig. 1(c)]. Valves on internal module faces control the diffusion of gas throughout the interior of the robot. One advantage of this implementation over MHP is the ability of the system to work in a vacuum, such as in space. However, this comes at the cost of a limited supply of fuel. As the gas used for propulsion is stored inside the robot, and cannot be reclaimed, once it is exhausted, the robot will not be able to control its own movement.

C. Sensing and Communication

In addition to the aforementioned core characteristics of the MFP concept, we consider sensing and communication abilities that may be specific to the tasks we attempt to solve with the MFP system. Each module has the following.

- 1) Binary *connection* sensors, one per face. These sensors detect whether any neighboring modules are present, that is, physically attached.
- 2) Binary *goal* sensors, one per face, mounted in the face center. These sensors detect whether a goal object is visible, that is, the ray between the face center and goal is not blocked by any part of the robot. They have a hemispherical field of view with unlimited range.
- 3) Binary *obstacle* sensors, one per face, mounted in the face center. These sensors detect the presence of nearby obstacles. They have an infinitesimal field of view (a single ray along the face normal) with limited range.
- 4) The capability to communicate with neighboring modules directly connected to each of its faces.

D. Formal Model

Consider an N -dimensional robot ($N = 2$ or 3) contained in an unbounded fluid environment. The robot consists of m identical square or cubic modules that are connected at their faces. The problem is scaled so that each module has unit size and each face can exert a unit force in the direction of its normal vector.

Let $\mathbf{q}_i \in \mathbb{R}^N$ denote the position of the i th module. Each module has the same uniform density. Then, the robot's center

of mass is

$$\mathbf{q}_c = \frac{1}{m} \sum_{i=1}^m \mathbf{q}_i.$$

The robot propels itself by exerting forces at its external faces. We make the following assumptions about the actuator force model.

- 1) Actuators on external faces can be active (expelling fluid) or inactive (allowing fluid to pass through freely).
- 2) Actuators on internal faces are inactive.
- 3) Forces due to fluid being drawn into or routed through the robot are not considered.

The resultant actuator force on the robot is the sum total of the force due to active actuators. These forces result in a net force and torque of

$$\begin{aligned} \mathbf{f} &= \sum_{\phi \in \Phi} \mathbf{f}(\phi) \\ \boldsymbol{\tau} &= \sum_{\phi \in \Phi} (\mathbf{q}(\phi) - \mathbf{q}_c) \times \mathbf{f}(\phi) \end{aligned}$$

where Φ is the set of external faces, $\mathbf{f}(\phi)$ is the force exerted by face ϕ , $\mathbf{q}(\phi)$ is the position of the center of that face, and \times denotes the cross product.

We assume that the total force and torque is small and the environment is highly viscous. We, therefore, neglect inertia and treat the robot motion as quasi-static. Under this assumption, the drag forces acting on the robot—which depend on its velocity—exactly counteract the propulsive forces. For highly viscous fluids, the robot's velocity is proportional to the net actuator force, and so the robot's dynamics can be described by

$$\dot{\mathbf{q}}_c = \mathbf{K} \mathbf{f} \quad (1)$$

where \mathbf{K} is a positive-definite diagonal matrix. In two dimensions, we write the components of (1) as

$$\dot{x}_c = k_x f_x \quad (2)$$

$$\dot{y}_c = k_y f_y \quad (3)$$

where $\mathbf{q}_c = (x_c, y_c)$ are the coordinates of the center of mass and $\mathbf{f} = (f_x, f_y)$ is the net actuator force on the robot. \mathbf{K} is the inverse of the matrix of drag coefficients, and each element on the main diagonal is inversely proportional to the projected size of the robot along the corresponding axis. We can also apply

$$\dot{\theta} = k_\theta \tau \quad (4)$$

where θ is the angle of the robot with respect to some initial reference frame and $\tau = |\boldsymbol{\tau}|$ is the net torque due to the individual torques developed by each actuator. In (2)–(4), we have used constants $k_x, k_y, k_\theta > 0$ whose exact values depend on the robot geometry and fluid properties.

Definition 1 (Orthogonal convexity): A shape $S \subset \mathbb{R}^2$ is orthogonally convex if for any pair of points $(x_0, y_0), (x_1, y_1) \in S$ with either $x_0 = x_1$ or $y_0 = y_1$, the line segment connecting the two points is fully contained within S .

Orthogonal convexity is a geometric property that plays an analogous role to ordinary convexity, when analyzing rectilinear



Fig. 2. Orthogonally convex shape (left) intersects with any horizontal or vertical line at most twice (on the boundary). A shape that intersects any horizontal or vertical line more than twice (right) is not orthogonally convex.

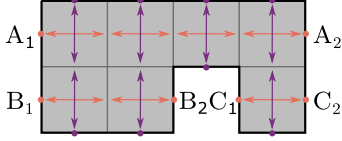


Fig. 3. Each external face of an MFP robot can be paired with another that points in the opposite direction. Paired faces lie on each end of a connected chain of modules (e.g., see A, B, and C).

objects. For an orthogonally convex shape, each vertical or horizontal line intersects with the boundary of the shape at most twice (see Fig. 2). This implies that for each module of an orthogonally convex MFP robot, there exists exactly one pair of external faces with the same x coordinate and exactly one pair of external faces with the same y coordinate.

IV. STATIC FORCE AND TORQUE ANALYSIS

In this section, we analyze the range of motions available to a 2-D MFP robot. We do not consider a specific controller, and assume that the activation state of each actuator can be chosen individually. We define the module positions such that $\mathbf{q}_c = \mathbf{0}$. We use $w, h \in \mathbb{N}$ to denote the width and height of the robot. The number of external faces in a 2-D robot with m modules is bounded by

$$|\Phi| \leq 2 + 2m$$

and is even due to the simple fact that each external face can be paired with another external face, which points in the opposite direction (see Fig. 3). We denote the set of pairs of faces aligned with each axis by \mathcal{P}_x and \mathcal{P}_y . The sizes of these sets are related to the number of faces by

$$|\Phi| = 2(|\mathcal{P}_x| + |\mathcal{P}_y|).$$

By summing over all the pairs of faces aligned with a given axis, we can compute the total force along that axis. As all faces exert the same magnitude of force when active, and paired faces point in opposite directions, each pair of faces exerts a force of either -1 , 0 , or 1 . The scenario where both faces are active is equivalent to the one where both are inactive.

A. Forces Generated by a 2-D Robot

A pair of horizontal faces, $p \in \mathcal{P}_x$, generates a horizontal force

$$f_x(p) = \begin{cases} 1, & \text{only leftward face active} \\ -1, & \text{only rightward face active} \\ 0, & \text{otherwise.} \end{cases}$$

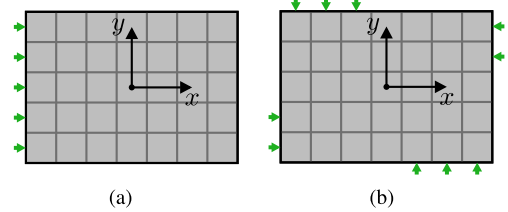


Fig. 4. 2-D MFP robot maximizing (a) force along the x -axis, and (b) torque (in the counterclockwise direction). Green arrows indicate active actuators.

Similarly, a pair of vertical faces can generate three different forces. The force along one axis is maximized when all face pairs aligned with that axis each have one active face of the same direction [see Fig. 4(a)]. As all component forces are integers, the total force is an integer, and

$$f_x \in \{-|\mathcal{P}_x|, -|\mathcal{P}_x| + 1, \dots, |\mathcal{P}_x| - 1, |\mathcal{P}_x|\} \quad (5)$$

$$f_y \in \{-|\mathcal{P}_y|, -|\mathcal{P}_y| + 1, \dots, |\mathcal{P}_y| - 1, |\mathcal{P}_y|\}. \quad (6)$$

As the active faces can be chosen independently, all configurations of values shown in (5)–(6) are possible. Therefore, the number of unique forces is $(2|\mathcal{P}_x| + 1)(2|\mathcal{P}_y| + 1)$.

B. Torques Generated by a 2-D Robot

The torque generated by a force $\mathbf{f} = (f_x, f_y)$ acting at $\mathbf{q} = (x, y)$ is

$$\tau = |\mathbf{q} \times \mathbf{f}| = f_x y - f_y x.$$

As the force generated by a horizontal pair, $p \in \mathcal{P}_x$ has no vertical component and the two faces have the same y coordinate, the pair's torque

$$\tau(p) = \begin{cases} y(p), & \text{only rightward face active} \\ -y(p), & \text{only leftward face active} \\ 0, & \text{otherwise} \end{cases}$$

where $y(p)$ is the shared y coordinate of the two faces. Similarly, the torque generated by a pair of vertical faces can take three values, which depend on the pair's shared x coordinate, $x(p)$. Torque is maximized when one face from each pair is active depending on that pair's location [see Fig. 4(b)]. In this configuration, the torque is

$$\tau_{\max} = \sum_{p_x \in \mathcal{P}_x} |y(p_x)| + \sum_{p_y \in \mathcal{P}_y} |x(p_y)| \quad (7)$$

where $y(p_x)$ is the y coordinate of a pair of horizontal faces and $x(p_y)$ is the x coordinate of a pair of vertical faces.

For an orthogonally convex robot, we can simplify the sums in (7). Let w_ℓ, w_r, h_a , and h_b be the maximum dimensions of the robot with respect to the center of mass (see Fig. 5). If we additionally assume that the center of mass is located at the corner of some module, then all of these variables are integers and all $y(p_x)$ and $x(p_y)$ are half more than an integer. In this

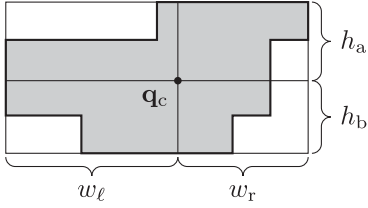


Fig. 5. Parameters w_l , w_r , h_a , and h_b define the bounding box of a 2-D MFP robot with respect to its center of mass.

case, (7) is

$$\begin{aligned} \tau_{\max} &= \sum_{i=1}^{w_l} \left(i - \frac{1}{2}\right) + \sum_{i=1}^{w_r} \left(i - \frac{1}{2}\right) \\ &+ \sum_{i=1}^{h_a} \left(i - \frac{1}{2}\right) + \sum_{i=1}^{h_b} \left(i - \frac{1}{2}\right) \\ &= \frac{w_l^2 + w_r^2 + h_a^2 + h_b^2}{2}. \end{aligned}$$

Additionally, as all individual torques are integers, it is possible to obtain any smaller integer value of torques, so the set of possible torques is

$$\tau \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, \tau_{\max} - 1, \tau_{\max}\}$$

and the number of possible torques is $\mathcal{O}(|\Phi|^2)$ as $|\Phi| = 2(w_l + w_r + h_a + h_b)$. If the center of mass is not located at the corner of a module, the maximum torque and number of torques are different but are still $\mathcal{O}(|\Phi|^2)$.

V. DIRECTED MOTION TASK AND CONTROLLERS

To validate the MFP concept, we consider a task that requires an MFP robot to move to a goal, and present three versions of an occlusion-based motion controller to solve it.

A. Task Definition

We consider an environment with an MFP robot and a static goal point $\mathbf{g} \in \mathbb{R}^N$. Unless otherwise stated, the environment is free of obstacles. The robot has the following sensors (one per module face): connection sensors, goal sensors, and, where relevant, obstacle sensors.

We assume that the robot starts far away from the goal. The robot should approach the goal with the ultimate objective of reaching it. The goal is considered to be reached if within the robot's bounding box.

B. Occlusion-Based Motion Controller ($\bar{\text{dec}}$)

The occlusion-based motion controller [see Fig. 6(a)], referred to as $\bar{\text{dec}}$, is adapted from an occlusion-based cooperative transport controller, which was used for controlling swarms of mobile robots [22]. Although the operating principle is the same, previously passive objects were moved by externally induced forces whereas MFP robots can produce these forces by themselves. The adapted controller is fully decentralized and is executed independently on each module face of the robot.

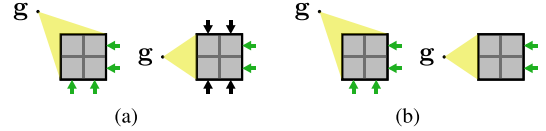


Fig. 6. 2^2 MFP robot moves toward a goal using an occlusion-based motion controller: $\bar{\text{dec}}$, $\bar{\text{dec-com}}$, or $\bar{\text{dec-obs}}$. (a) Robots running the $\bar{\text{dec}}$ controller. (b) Robots running the $\bar{\text{dec-com}}$ controller do not activate actuators on opposite sites. The third controller, $\bar{\text{dec-obs}}$ (not shown), also enables obstacle detection. Arrows indicate the forces caused by active actuators. Forces that make a net contribution to the total force are indicated in green.

The controller requires only 2 bits of sensory information, c and v : 1) c is true if and only if a connection is detected; 2) v is true if and only if the goal is visible. The controller maps the sensor state directly to the actuator state

$$a = \bar{c} \wedge \bar{v}.$$

This requires neither run-time memory, nor communication. Actuators of internal faces are never activated. All external module faces that are occluded from the goal have active actuators ($a = 1$), and those that are not occluded have inactive actuators ($a = 0$). The net result is that the robot moves toward the goal. This is shown to be true for convex robots in [23], and proved to be correct for non-convex robots in Section VI.

Note that the structure of an MFP robot can fully enclose a section of the environment (see Fig. 1(a), for a 2-D example). For a 2-D robotic platform, such as our surface-water MHP robot (see Section VIII), this presents no problem. For a 3-D robotic platform, however, the fluid would have nowhere to move other than into the robot. For 3-D pneumatic implementations, all valves surrounding the enclosed section would open, which is acceptable. For 3-D hydraulic implementations, all pumps surrounding the enclosed section would fire while the fluid would not move, causing a large load.

C. Communication-Enabled Motion Controller ($\bar{\text{dec-com}}$)

The $\bar{\text{dec-com}}$ controller [see Fig. 6(b)] allows for communication between paired faces (see Fig. 3). It requires 3 bits of sensory information: c , v , and \hat{v} , where \hat{v} is the binary state of the goal sensor of the paired face, if no connection is detected (c is false), and false otherwise. The actuator state is given by

$$a = \bar{c} \wedge \bar{v} \wedge \hat{v}.$$

Assuming instantaneous, faultless communication, this controller is equivalent to $\bar{\text{dec}}$. However, it prevents actuators on opposite sides of the robot from being active at the same time and expending additional energy.

D. Obstacle Avoidance Motion Controller ($\bar{\text{dec-obs}}$)

The $\bar{\text{dec-obs}}$ controller is designed for environments with obstacles. It requires 3 bits of sensory information: c , v , and o , where o is true if and only if an obstacle is detected. The actuator state is given by

$$a = \bar{c} \wedge (\bar{v} \vee o).$$

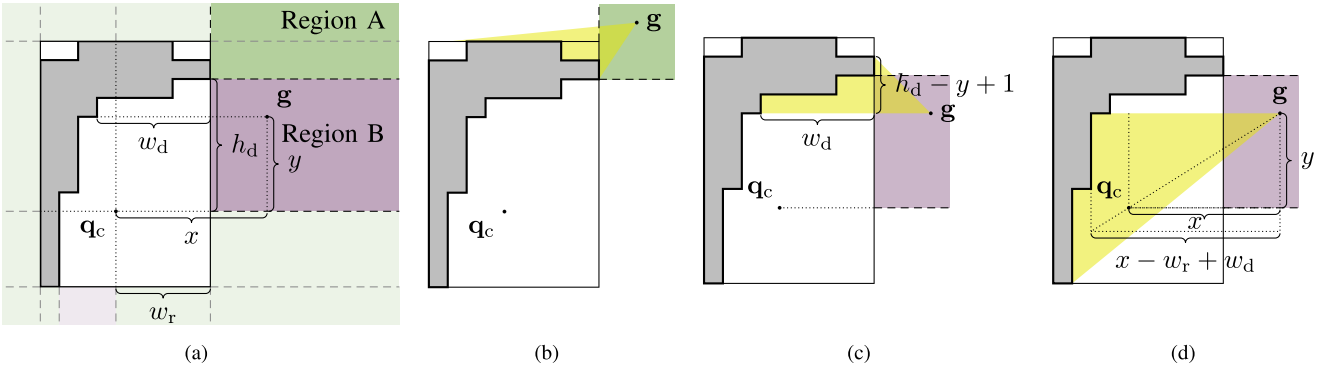


Fig. 7. Illustrations for Theorem 1 (not to scale). (a) Goal \mathbf{g} can be in Region A or B (excluding symmetric cases). (b) In Region A, at least one rightward face at the edge of the bounding box detects the goal, and no leftward or downward faces can detect the goal. Thus, $f_x > 0$ and $f_y \geq 0$. (c) and (d) In Region B, exactly w_d downward faces detect the goal and no upward face can detect the goal. Thus, $f_y = -w_d$. Moreover, no leftward face can detect the goal. Below the goal (d), f_x can be bounded, using similar triangles, by $(x - w_r + w_d)\frac{y}{x}$.

The controller is equivalent to dec , except the actuator is also active if an obstacle is detected. As a consequence, the robot tends to move away from obstacles, resulting in fewer collisions while still moving toward the goal.

VI. CONTROLLER ANALYSIS

In this section, we analyze the use of the dec controller for 2-D MFP robots in an obstacle free environment with a goal.² First, we prove that orthogonally convex robots are guaranteed to reach the goal. Second, we prove that robots of arbitrary shape are guaranteed to move toward the goal up to a morphology-dependent distance.

Throughout this section, we describe the dynamics of the robot in its local coordinate system. In this coordinate system, the module positions are defined such that $\mathbf{q}_c = \mathbf{0}$, and the goal location moves while the robot is stationary. Transforming (2)–(4) into this coordinate system results in the goal having dynamics

$$\dot{x} = -k_x f_x + k_\theta y \tau \quad (8)$$

$$\dot{y} = -k_y f_y - k_\theta x \tau \quad (9)$$

where $\mathbf{g} = (x, y)$ is the location of the goal, f_x, f_y, τ are the net forces and torques exerted by the robot, and $k_x, k_y, k_\theta > 0$ are constants. As the goal is a single point, it does not have rotational dynamics.

By symmetry, we can consider only the region for which the goal is above the center of mass and to the right of the bounding box [see Fig. 7(a)]. We further divide this region into Regions A and B, based on which modules touch the right edge of the bounding box. Let h_d be the y coordinate of the bottom of the lowest module that touches the right edge of the bounding box. If $h_d > 0$, we define Region A as the region with $y \geq h_d$ and Region B as the region with $y < h_d$. Else, we define Region A as the region with $y \geq 0$ (above the center of mass); in this case, Region B does not exist.

Lemma 1: In Region A, $f_x > 0$ and $f_y \geq 0$.

²An equivalent analysis can be conducted for the dec-com and dec-obs controllers, but it is omitted here.

Proof: In Region A, $f_x > 0$ as 1) no leftward face can detect the goal, and 2) there must be at least one rightward face, at the edge of the bounding box, which detects the goal [see Fig. 7(b)]. Moreover, as the robot is orthogonally convex, no downward face can detect the goal. Therefore, $f_y \geq 0$. ■

Lemma 2: In Region B, $f_y = -w_d$.

Proof: Let $w_d > 0$ be the horizontal distance at height y between the right side of the robot and the right side of the bounding box [see Fig. 7(c)]. Exactly w_d downward faces detect the goal, and no upward face can detect the goal, thus

$$f_y = -w_d. \quad (10)$$

Lemma 3: In Region B, $f_x \geq h_d + 1 + \frac{(w_d - w_r)y}{x}$.

Proof: The goal cannot be detected by any leftward face. It is detected by all rightward faces above the goal but below (w_r, h_d) , as well as by at least one rightward face at the edge of the bounding box [see Fig. 7(c)]. Thus

$$f_x \geq h_d - y + 1.$$

Below the goal, all rightward faces of the robot have an x coordinate less than or equal to $w_r - w_d$. The line-of-sight ray that passes through the center of mass of the robot must hit the robot (the latter being a single entity); the horizontal ray hits the robot too. As the robot is a single entity, any ray in between these two are guaranteed to hit the robot [see Fig. 7(d)]. The number of rightward faces is minimized when the faces are as close to the goal as possible. The minimum rightward force is generated if all the faces have an x coordinate equal to $w_r - w_d$. Using similar triangles, the force from rightward faces below the goal is thus bounded by $(x - w_r + w_d)\frac{y}{x}$, and the total rightward force is thus bounded by

$$\begin{aligned} f_x &\geq h_d - y + 1 + (x - w_r + w_d)\frac{y}{x} \\ &= h_d + 1 + \frac{(w_d - w_r)y}{x}. \end{aligned} \quad (11)$$

Theorem 1: If $k_x = k_y$ and the robot is orthogonally convex, then any initial robot configuration will eventually result in the goal being contained in the bounding box.

Proof: We use the Lyapunov function,

$$V = \frac{1}{2} (\mathbf{g} - \mathbf{q}_c)^\top (\mathbf{g} - \mathbf{q}_c). \quad (12)$$

This function is positive semidefinite with $V = 0$ if and only if $\mathbf{g} = \mathbf{q}_c$. To prove the theorem, it is sufficient to show that $\dot{V} < 0$ whenever the goal is not contained in the bounding box of the robot, which means the goal is getting closer to the robot's center of mass.

By differentiating (12) using the dynamics of the goal location, (8)–(9), the Lyapunov derivative is

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} \\ &= -xk_x f_x - yk_y f_y. \end{aligned} \quad (13)$$

Without loss of generality, we assume that the problem is scaled such that $k_x = k_y = 1$. Then the goal has dynamics

$$\dot{V} = -x f_x - y f_y. \quad (14)$$

In Region A, from Lemma 1, $f_x > 0$ and $f_y \geq 0$. As in this region $x \geq w_r > 0$ and $y \geq 0$, \dot{V} is negative. Thus, the goal approaches the robot's center of mass.

In Region B, using the bounds (10)–(11) in the Lyapunov derivative (14), we obtain

$$\begin{aligned} \dot{V} &\leq -x \left(h_d + 1 + \frac{(w_d - w_r)y}{x} \right) + y w_d \\ &= -x (h_d + 1) - y (w_d - w_r) + y w_d \\ &= -x (h_d + 1) + y w_r. \end{aligned}$$

Region B was defined by $x > w_r$ and $y < h_d$, and so

$$\begin{aligned} \dot{V} &< -w_r (h_d + 1) + h_d w_r \\ &= -w_r < 0. \end{aligned}$$

Therefore, whenever the goal is in Region B, it approaches the robot's center of mass. We can conclude that whenever the goal is not in the bounding box, it approaches the robot's center of mass, and will eventually enter the bounding box. ■

In Theorem 1, we exploited the fact that for orthogonally convex robots, no more than two directions of faces can detect the goal at any given time. For a general MFP robot, it is possible for three different directions of faces to detect the goal at the same time. As a consequence, the goal could recede from the center of mass of the robot at a given instant (see Fig. 8, for an example). However, in the following, we show that whenever the goal is sufficiently far away, it approaches the robot. By symmetry, we can consider only the region for which the goal is above the center of mass and to the right of the bounding box (see Fig. 9). We further divide this region into Regions C and D. We define Region C as the region with $y \geq h_a$, and Region D as the region with $y < h_a$.

Lemma 4: In Region C, $f_x > 0$ and $f_y > 0$.

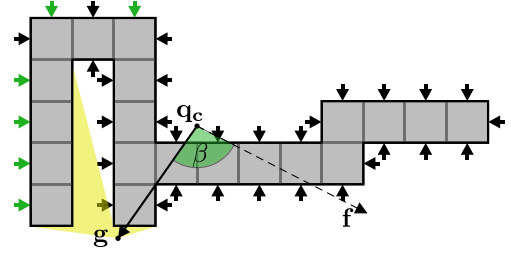


Fig. 8. MFP robot composed of 20 modules. The robot has center of mass \mathbf{q}_c . The goal point \mathbf{g} lies outside of the robot's bounding box. The small arrows indicate the forces generated by active actuators. Those forces that make a net contribution to the total force acting on the robot are indicated in green. The angle β between the total force vector \mathbf{f} and the vector $\mathbf{g} - \mathbf{q}_c$ is greater than $\pi/2$. Note that while this means the robot does not move toward the goal point at this instant, it does not imply that the robot never reaches the goal. In this example, once the goal has changed position, a different set of actuators is activated, which do provide $\beta \leq \pi/2$.

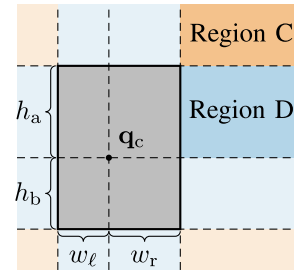


Fig. 9. In the analysis of Theorem 2, we consider the area outside the bounding box of an MFP robot of an arbitrary shape, with its center of mass at \mathbf{q}_c . The goal can be in Region C or D; all other cases are symmetrically equivalent.

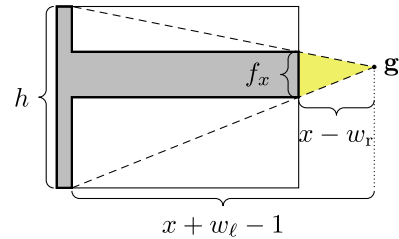


Fig. 10. In Region D, for a given bounding box, the number of rightward faces, which detect the goal, is minimized when all the rays hit modules at the right side of the bounding box, which occlude any faces that are further to the left.

Proof: In Region C, $f_x > 0$ as 1) no leftward face can detect the goal, and 2) there must be at least one rightward face, at the edge of the bounding box, which detects the goal (see Fig. 9). By symmetry, we can also conclude that $f_y > 0$. ■

Lemma 5: In Region D, $f_x \geq \frac{h(x-w_r)}{x+w_l-1} > 0$.

Proof: In Region D, $x > w_r$ and so no leftward face can detect the goal. There must be modules touching the top and bottom edges of the bounding box, so the rays passing through $(-w_l + 1, h_a)$ and $(-w_l + 1, -h_b)$ as well as all rays in between must hit a module (see Fig. 10). If these rays all hit faces at the right edge of the bounding box, the total force—computed using similar triangles—is

$$f_x = \frac{h(x - w_r)}{x + w_l - 1}.$$

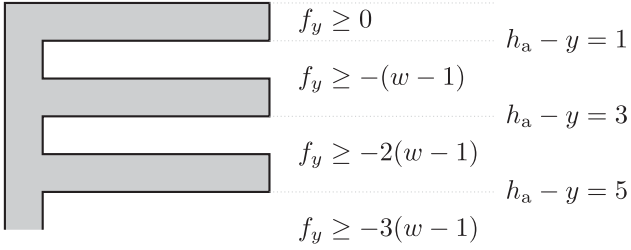


Fig. 11. In Region D, downward forces are maximized by robots with alternating rows of modules and empty space.

Any other robot with the same bounding box must have at least the same amount of rightward faces that detect the goal. As $x > w_r$, this force is always positive. ■

Lemma 6: In Region D, $f_y \geq -\frac{1}{2}(w-1)(h_a+1-y)$.

Proof: In Region D, if $w = 1$ or $h = 1$, then $f_y = 0$ and the lemma holds. If $w > 1$ and $h > 1$ it is possible for both upward and downward faces to detect the goal so f_y can have either sign. The bound is determined by counting the occluded downward faces. Downward faces can only detect the goal if they are located above g and are directly above an empty cell. Therefore the downward force is maximized by a robot with alternating rows of modules and empty space at all coordinates above the goal (see Fig. 11). When the goal is far enough away, the downward faces of every row of modules above the goal detect it. This row's length is at most $w-1$ as at least one cell in the row of empty space must contain a module so that the robot is connected. Therefore, if $h_a - y$ is an odd integer, then

$$f_y \geq -(w-1) \left(\frac{h_a - y}{2} + \frac{1}{2} \right). \quad (15)$$

If $h_a - y$ is not an odd integer, then the downward force are bounded by the same bound as the nearest smaller odd integer. However, as the right-hand side of (15) increases monotonically with $h_a - y$, it bounds this stepwise bound and is, therefore, a valid bound for any $y \in [0, h_a]$. ■

Theorem 2: If $k_x = k/h$ and $k_y = k/w$ for some constant k , an MFP robot of arbitrary shape is guaranteed to reach a distance of at most $\frac{1}{2}(d+1)^2$ from the goal where $d = \max\{w_\ell, w_r, h_a, h_b\}$.

Proof: We use the Lyapunov function (12). In Regions C and D, $x > w_r > 0$ and $y > 0$. By Lemma 4, in Region C, $f_x, f_y > 0$ and as $k_x, k_y > 0$, (13) becomes $\dot{V} < 0$. Therefore, when the goal is in one of the four Region Cs, it always approaches the center of mass of the robot. In Region D, Lemma 5 guarantees that $f_x > 0$ but it is possible that $f_y < 0$, and so \dot{V} is not necessarily negative everywhere. The drag coefficients are related to the robot's dimensions by $k_x = k/h$ and $k_y = k/w$ for some constant k . Without loss of generality, we assume that the problem is scaled so that $k = 1$. Combining the bounds from Lemmas 5 and 6, the Lyapunov derivative (13) is then bound by

$$\dot{V} \leq -\frac{1}{h}x \frac{h(x-w_r)}{x+w_\ell-1} + \frac{1}{2} \frac{1}{w}y(w-1)(h_a+1-y). \quad (16)$$

We cancel the term $\frac{w-1}{w} < 1$ to obtain a slightly larger bound. This bound includes a quadratic function of y , which is restricted to $y \in [0, h_a]$. By differentiating this term, we can find its maximum over this interval

$$\frac{d}{dy}(y(h_a+1-y)) = h_a+1-2y.$$

This derivative is 0 when $y = \frac{1}{2}(h_a+1)$, and the second derivative is negative, and so we can further bound (16) by

$$\dot{V} < -x \frac{x-w_r}{x+w_\ell-1} + \frac{1}{8}(h_a+1)^2. \quad (17)$$

Next, we provide an upper bound for the term $\frac{x-w_r}{x+w_\ell-1}$. Letting $a = x - w_r$, this term's inverse is

$$\left(\frac{x-w_r}{x+w_\ell-1} \right)^{-1} = \frac{a+w-1}{a} = 1 + \frac{w-1}{a}.$$

Suppose that $x \geq \frac{1}{3}(w-1) + w_r$. Then, $a \geq \frac{1}{3}(w-1)$ and so the inverse is bounded by

$$\left(\frac{x-w_r}{x+w_\ell-1} \right)^{-1} \leq 4.$$

Inverting this bound and using it in (17) results in

$$\dot{V} < -\frac{1}{4}x + \frac{1}{8}(h_a+1)^2.$$

Then, if $x \geq \frac{1}{2}(h_a+1)^2$, the derivative is negative. We have used two bounds for x , so a sufficient condition is that x satisfies both

$$x \geq \max \left\{ \frac{1}{3}(w-1) + w_r, \frac{1}{2}(h_a+1)^2 \right\}.$$

So far we have found a bound for x in one of the eight Region Ds. By symmetry, we get similar results on the other seven Region Ds, but with different permutations of w_ℓ, w_r, h_a , and h_b . As d bounds all of these dimensions, a sufficient condition for all eight regions is

$$|x|, |y| > \max \left\{ \frac{1}{3}(2d-1) + d, \frac{1}{2}(d+1)^2 \right\}.$$

However, it holds that $\frac{1}{2}(d+1)^2 > \frac{1}{3}(2d-1) + d$ for all real numbers d , and therefore, the condition is equivalent to

$$|x|, |y| > \frac{1}{2}(d+1)^2.$$

While this condition is fulfilled, the Lyapunov derivative is negative and so the goal approaches the robot's center of mass. ■

VII. SIMULATIONS

In this section, we study MFP robots using physics simulations. Compared with Section VI, where quasi-static motion was assumed, the simulations enable us to validate the concept under more practical conditions, with a Reynolds number on the order of 10^4 . The simulated robots are a 3-D implementation of the hydraulic MFP concept, hence, in the following, we refer to them as MHP robots. They operate in an underwater

environment. We analyze their speed and energy consumption as they move toward a goal, as well as their ability to avoid obstacles. We compare the decentralized controller variants from Section V against the state-of-the-art (centralized) controller from the literature.

A. General Setup

The simulator is based on the Open Dynamics Engine (ODE) [47], an open source 3-D physics library. Modules are modeled as solid cubes of neutral buoyancy with an edge length of 8 cm. A simplified model of fluid drag is used to avoid a full, computationally expensive, fluid dynamics treatment. The drag force on each external module face is calculated individually without reference to the overall shape of the robot. It is assumed to follow the quadratic drag equation for turbulent flow. This means the drag force on each external module face is

$$\mathbf{f}_d = \begin{cases} -\frac{1}{2}\rho C_D s^2 \dot{\mathbf{q}} \cdot \mathbf{u} \dot{\mathbf{q}}, & \dot{\mathbf{q}} \cdot \mathbf{u} > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\rho = 1 \text{ g}\cdot\text{cm}^{-3}$ is the density of the surrounding fluid (water), $C_D = 0.8$ is the drag coefficient, s is the module side length, \mathbf{u} is the face normal vector, and $\dot{\mathbf{q}}$ is the velocity of the face center.

Three types of robot configuration are used, *cubic*, *orthogonally convex*, and *unconstrained*. Robots of cubic shape comprise 125 modules, arranged in a 5^3 cubic (and thus convex) configuration. Robots of unconstrained shape comprise 125 modules, arranged as follows: The configuration is initialized with a single module. Additional modules are added one at a time. A new module is added to a face that is chosen randomly from all available external module faces on the robot. This repeats until the robot consists of the desired number of modules. Robots of orthogonally convex shape comprise 125 modules, and are arranged in a similar manner to unconstrained robots. However, additional modules may only be added in such a manner that the robot remains orthogonally convex.

Each module has six pumps, one per face. Firing (active) pumps apply a force of 640 dyn to the center of the module face, along the inward face normal. The net resultant force and torque acting on a robot are integrated by ODE. As none of the controllers fire pumps of internal module faces, only the pumps of external module faces are considered. The routing of fluid throughout the robot is not simulated.

Each module has one connection sensor, one goal sensor, and one obstacle sensor, on each of its six faces. Goal sensors use the ray casting functionality of ODE to check line of sight between the module face center and the goal. Line of sight can be occluded by the body of the robot itself. Obstacle sensors use a ray of length 40 cm, cast from the center of the face, parallel to the face normal. The sensor returns true if and only if it intersects with an obstacle, which could be another module of the same robot. The specific ray length was chosen following preliminary trials.

When using controller *dec-com*, a module's face may communicate with a paired face (see Fig. 3). Each face simply knows



Fig. 12. Image taken from a simulated trial with an unconstrained 3-D MHP robot moving toward the goal on the right (green sphere). Green and white module faces represent active and inactive pumps, respectively.

the required information from its paired face, communication delay, and information loss are not simulated.

In addition to the decentralized controllers, we use the state-of-the-art centralized controller, *cen*, taken from the literature [24]. This controller uses multiple PID controllers, one per actuator, to generate the desired output. The reference input for these PID controllers is calculated by solving equation systems, taking the positions and orientations of every actuator into account. The controller can handle both translation and rotation. To maximize performance the controller rotates the robot into an orientation that minimizes drag while translating toward the goal. Although the original controller uses multiple sets of parameter values, each one specifically calibrated for a particular actuator, we use the same set of parameter values for all actuators. This set was obtained by performing a grid search using a 5^3 cubic robot. The same set was also used for robots of random morphology. Using a single set of values could result in a lower than optimal performance.

Throughout this section, we use the two-tailed Mann-Whitney test. Each test uses a base alpha level of 0.01. Given that each set of data has two tests performed upon it, we use a Bonferroni adjusted alpha level of 0.005. We report the *a posteriori* p -values obtained by the tests.

B. Time to Completion

This section evaluates the success rate of, and the time taken by, simulated 3-D MHP robots to reach the goal. We consider the goal to be reached once the robot has been in physical contact with it.

1) *Setup*: The simulation environment is shown in Fig. 12. It is continuous and unbounded, and contains a single robot and a goal.

The robot starts at a distance of 500 cm from the goal, with a uniformly random orientation. The trial is deemed successful if the robot reaches the goal within 250 s.

We ran nine sets of trials, one per combination of robot configuration (cubic, orthogonally convex, and unconstrained) and controller (*dec*, *dec-com*, and *cen*). Each set consisted of 100 trials, using the same 100 random starting orientations.

2) *Results*: Every trial was successfully completed. Fig. 13 shows the time taken to reach the goal. The horizontal line across each graph indicates the theoretical smallest time taken for a 5^3 cubic robot to reach the goal. Controllers *dec* and *dec-com* produce identical distributions. This is because the effect of allowing communication is only to deactivate pairs of pumps that would otherwise both be firing and producing no net force and torque. Therefore, controllers *dec* and *dec-com* give rise

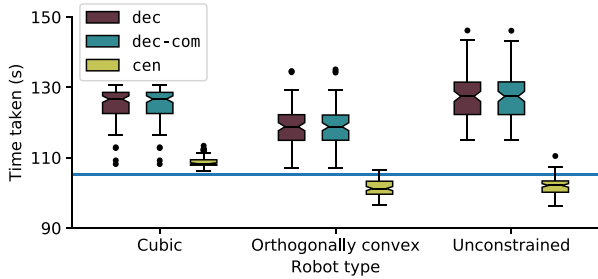


Fig. 13. Time taken for simulated 3-D MHP robots of cubic, orthogonally convex, or unconstrained shape to reach the goal using the `dec`, `dec-com`, and `cen` controllers. Each robot is composed of 125 modules. Each box represents 100 trials. The horizontal blue line represents the theoretical minimum time for a cubic (5^3) robot to reach the goal. Note that the y -axis is truncated for readability.

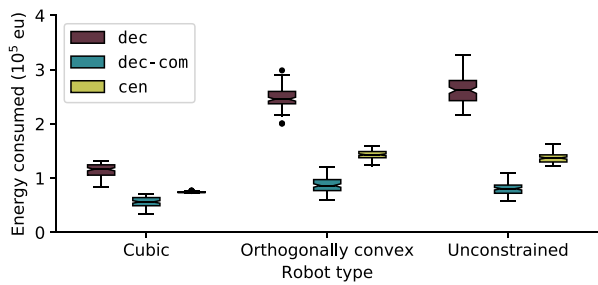


Fig. 14. Energy consumed by simulated 3-D MHP robots of cubic, orthogonally convex, or unconstrained shape to reach the goal using the `dec`, `dec-com`, and `cen` controllers. Each box represents 100 trials.

to the same motion of the robot (when discarding the effects of noise).

Controller `cen` outperforms `dec` for cubic robots ($p < 0.001$), orthogonally convex robots ($p < 0.001$), and unconstrained robots ($p < 0.001$), and unconstrained robots ($p < 0.001$). The `dec` controller does not perform significantly worse with unconstrained robots than with cubic robots ($p = 0.025$). Somewhat surprisingly, cubic robots are outperformed by both unconstrained robots ($p < 0.001$) and orthogonally convex robots ($p < 0.001$) when using `cen`. This suggests that only calibrating the PID values for a single robot shape does not unduly hamper the controller in our scenarios.

C. Energy Expenditure

This section evaluates the performance of simulated MHP robots with regard to energy expenditure. Each active pump expends 1 energy unit (eu) per second. Inactive pumps expend no energy. The energy demands of processing and sensing are considered negligible.

1) *Setup*: We analyze the same set of simulation runs reported in Section VII-B. The energy consumption is measured throughout each trial.

2) *Results*: Fig. 14 shows a box plot of the cumulative energy consumption during the trials. For each controller, cubic robots perform better than unconstrained robots ($p < 0.001$). This is explained by the potential concavity of unconstrained robots, which may result in more pumps firing for a net zero effect. The

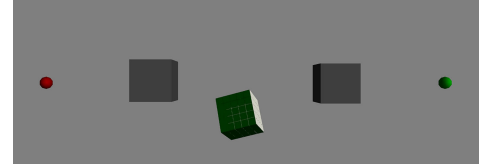


Fig. 15. Image taken from a simulation run showing a 5^3 cubic MHP robot (green cube) in an environment with obstacles (gray cubes). The robot uses the `dec-obs` controller and moves from its starting position on the left (red sphere) to the goal on the right (green sphere).

`dec-com` controller outperforms the `dec` controller for cubic robots ($p < 0.001$), orthogonally convex robots ($p < 0.001$), and unconstrained robots ($p < 0.001$). The largest improvement is shown for unconstrained robots, as communication helps to counter the effect of the concavity. For all types of robots, the `cen` controller outperforms the `dec` controller ($p < 0.001$ in all cases) and is outperformed by the `dec-com` controller ($p < 0.001$ in all cases). The `dec-com` controller reduces energy required by 51% for cubic robots, 65% for orthogonally convex robots, and 69% for unconstrained robots compared to the `dec` controller. Note that the `cen` controller was optimized to minimize the time taken, rather than to minimize the energy expenditure.

D. Negotiating Obstacles

We compare the `dec`, `dec-obs`, and `cen` controllers in an environment with obstacles. We evaluate their performance with regards to success rate, time taken, and collision rate.

1) *Setup*: An example environment is shown in Fig. 15. The setup is as reported in Section VII-B. The environment additionally contains a pair of obstacles situated between the goal and the robot. Each obstacle is cubic with a side length of 50 cm. The obstacles cannot be moved by the robots. Their positions are (125, 0, 0) cm and (375, 0, 0) cm, plus a uniformly random shift of up to 25 cm along each axis.

We ran three sets of trials, one for each of the `dec`, `dec-obs`, and `cen` controllers. Each set consisted of 100 trials, using the same 100 random starting orientations. In each trial, we recorded the number of time steps in which the robot was in contact with an obstacle.

2) *Results*: For the `dec` and `dec-obs` controllers, all trials were successful. For the `cen` controller, 95 trials were successful. Fig. 16 shows the times taken for the trials. The `dec-obs` controller is not significantly faster or slower than the `dec` controller ($p = 0.549$). The `cen` controller is not significantly slower than the `dec` ($p = 0.014$) controller, but is significantly slower than the `dec-obs` ($p = 0.002$) controller.

Fig. 17 shows the percentage of time that the robot was in physical contact with an obstacle. The `dec-obs` controller shows significantly less collisions than the `dec` controller ($p < 0.001$), with an average reduction in the collision rate of 77%. The `cen` controller shows significantly more collisions than both the `dec` ($p < 0.001$) and `dec-obs` ($p < 0.001$) controllers. The relatively poor performance of the `cen` controller in this scenario can be explained by the fact that the controller has no knowledge of the obstacles, and causes the robot to move directly

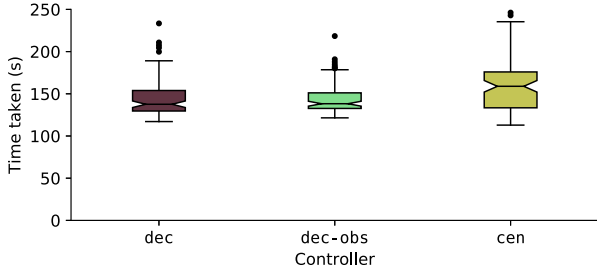


Fig. 16. Ability of simulated 5^3 MHP robots to negotiate obstacles while moving toward the goal. Time taken for the robot to reach the goal using the *dec*, *dec-obs*, and *cen* controllers. The boxes represent 100, 100, and 95 successful trials, respectively.

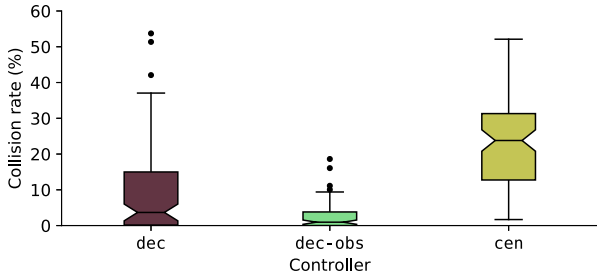


Fig. 17. Ability of simulated 5^3 MHP robots to negotiate obstacles while moving toward the goal. The percentage of time the robot was in physical contact with an obstacle when using the *dec*, *dec-obs*, and *cen* controllers. The boxes represent 100, 100, and 95 successful trials, respectively.

toward the goal. This increases the number of encounters the robot has with the obstacles, and reduces speed. Performance of the controller could be improved by setting waypoints between the starting position and the goal.

E. Effect of Sensor Angle

Thus far, the goal sensors have been assumed to have a hemispherical field of view, which may not be the case in a real implementation. This section relaxes this constraint, by studying the impact of different fields of view.

1) *Setup*: We consider cubic, orthogonally convex, and unconstrained MHP robots. The field of view of each goal sensor is a cone. We tested cone half-angles between 90° (hemisphere) and 45° , in steps of 5° . The setup is otherwise as reported in Section VII-B. We ran 30 sets of trials, one per combination of robot configuration (cubic, orthogonally convex and unconstrained) and sensor angle (45° , ..., 90°). Each set consisted of 100 trials, using the same 100 random starting orientations.

2) *Results*: Fig. 18 shows the times taken to reach the goal. Interestingly, for all robot types, a sensor angle of 85° degrees produces the lowest median time; however, for cubic robots, this comes at the expense of a much higher variance compared to a sensor angle of 90° .

F. Low Reynolds Number Environments

The simulations so far have dealt with high Reynolds number environments, that is, environments in which inertial forces are large compared to viscous forces. However, the theoretical model in Section VI presupposes scenarios involving highly

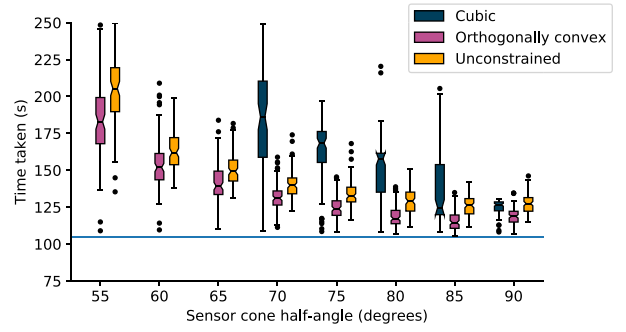


Fig. 18. Time taken for simulated 3-D MFP robots to reach the goal for various fields of view of the goal sensor. Only sets for which 20 or more trials were successful are shown. The boxes represent $-96/94$, $-100/100$, $-100/100$, $85/100/100$, $100/100/100$, $100/100/100$, $100/100/100$, and $100/100/100$ successful trials with robots of cubic/orthogonally convex/unconstrained shape, respectively. The horizontal blue line represents the theoretical minimum time for a cubic (5^3) robot to reach the goal.

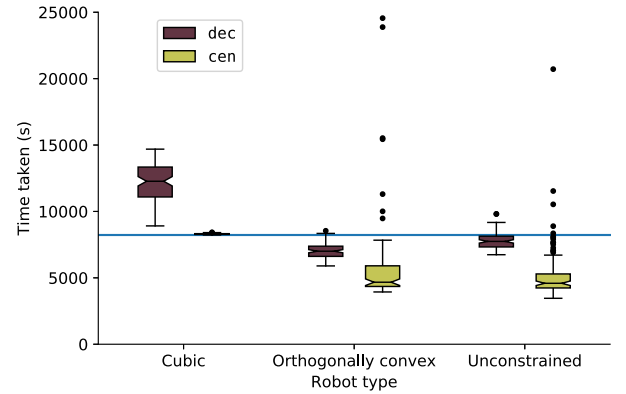


Fig. 19. Time taken for simulated 3-D MFP robots to reach the goal in a low Reynolds number environment. The boxes represent only successful trials (100, 100, 100, 98, 100, and 97 trials). The horizontal blue line represents the theoretical minimum time for a cubic (5^3) robot to reach the goal.

viscous fluids or low-speed actuation. In this section, we compare the *dec* and *cen* controllers in a low Reynolds number environment. The fluid density remains $1 \text{ g}\cdot\text{cm}^{-3}$ as previously, and a fluid dynamic viscosity of $10 \text{ dyn}\cdot\text{s}\cdot\text{cm}^{-2}$ is used. These values could represent an oil at low temperature. The thruster force is reduced by a factor of 100 to 6.4 dyn , resulting in a Reynolds number of approximately 0.1. The drag model is changed to a linear model, using Stoke's law with the robots approximated as spheres. The trial is deemed successful if the robot reaches the goal within 25 000 s. The PID parameters for the *cen* controller were recalibrated for a 5^3 robot under the new conditions.

1) *Setup*: We ran 100 trials per combination of robot configuration (cubic, orthogonally convex, and unconstrained) and controller (*dec* and *cen*). The setup is otherwise as reported in Section VII-B.

2) *Results*: Fig. 19 shows the times taken to reach the goal. For the *dec* controller, all trials were successfully completed. Cubic robots are outperformed ($p < 0.001$) not only by orthogonally convex robots, but also by unconstrained robots (see also Fig. 13). For the *cen* controller, 100%, 98%, and 97% of trials were successfully completed by cubic, orthogonally convex, and unconstrained robots, respectively. This controller outperforms

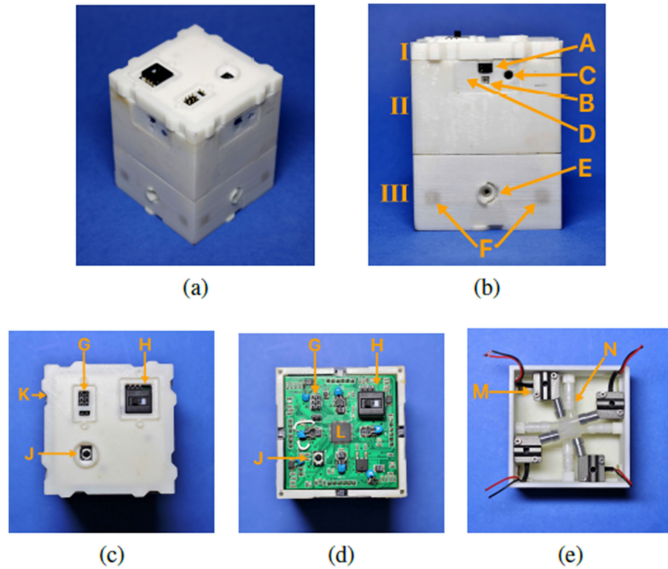


Fig. 20. Physical implementation of the hydraulic MFP (MHP) concept. (a) Assembled module. (b) Side view. (c) Top view. (d) Internal circuitry. (e) Internal reservoir and pumps. I: lid. II: upper hull. III: lower hull. A: IR transceiver (communication and range sensing). B: Phototransistor (goal detection). C: Magnet for connection detection. D: Hall effect sensor. E: Pump input/output port. F: Attachment magnets. G: Programming pins. H: Module switch and charging pins. J: IR receiver (remote control). K: Alignment features. L: Microcontroller. M: Micropump. N: Pump-port connecting pipe.

the *dec* controller for each robot type ($p < 0.001$). However, not all the trials were successfully completed, and a small number of successful trials displayed long times taken, suggesting that the *cen* controller did not cope well with certain random shapes in this scenario.

VIII. ROBOTIC PLATFORM

To validate the MFP concept in a real-world environment, we developed a physical implementation of a hydraulic MFP system, that is, an MHP robot. This section describes the module design and implementation. The modules represent a 2-D version of the MHP concept. They float on the surface of the water in a tank, and are capable of actuating in the horizontal plane. In this section, we use the term *faces* to refer to only the four vertical faces of the module.

A. Hull Design

The module is cuboid, with a footprint of 6.75×6.75 cm, a height of 8.15 cm, and a weight of 199 g. Its body consists of an *upper hull*, a *lower hull*, and a lid (see Fig. 20). All three parts of the body were 3-D printed using ABS plastic.

The lower hull contains the fluid routing systems of the module, as shown in Fig. 20(e). During operation, this section is fully submerged and acts as a water reservoir. Four ports, one per face, allow water to move into and out of the module. Each port is centered on its respective face of the module. Eight magnets (two per face) are placed close to the corners of the lower hull and are used as the attachment mechanisms between modules.

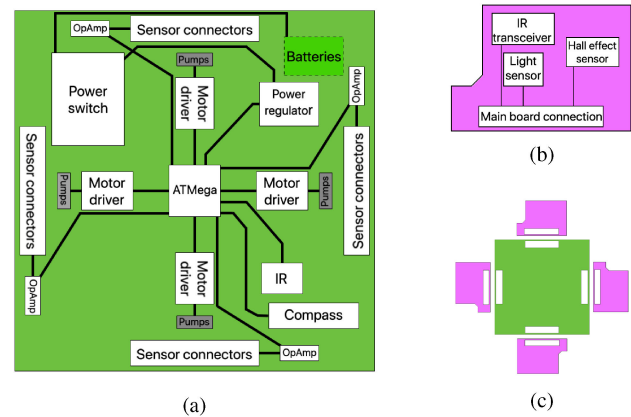


Fig. 21. Simplified schematic of the PCBs used in the physical MHP modules. (a) Main board containing all of the components with the exception of face sensors. (b) Daughter board containing the sensors for each face. (c) Relative positions of all the boards for a single MHP module.

The upper hull houses the module electronics [see Fig. 20(d)]. Each face center contains openings for the sensors. These openings are covered with a layer of transparent polyethylene. This allows the sensors to operate while preventing water from entering. The upper section is protected by a lid mounted on top of the module. The lid has openings that are used to power and program the modules.

The upper hull is 4.25 cm in height, whereas the lower hull is 3.5 cm in height. When in operation, the waterline is approximately 5.7 cm from the base of the module. This means that the module lid and the sensor openings are above the waterline, reducing the chance that water enters into the upper hull through the lid openings. To ensure full waterproofing of the upper hull of the robot, it is coated with resin.

Each module has a set of complementary tabs and notches on each face. These help to align connected modules.

B. Electronics

The electronics, with the exception of the sensors for each face, are integrated into a main printed circuit board (PCB) located in the upper hull. A simplified schematic of the circuits is shown in Fig. 21. The main board features an ATmega324P processor, and four H-bridge motor drivers for the pumps. It also accommodates a set of light emitting diodes (LEDs) that are used to indicate pump and connection states, as well as for debugging. The LEDs are visible through the closed module lid, allowing the state of the pumps to be ascertained during operation. Operational amplifiers are used to amplify the signals from the sensors. The sensors for each face are placed in a daughter board that connects perpendicularly with the main board. Each module contains four daughter boards, one per face. The module is powered by two rechargeable 3.7 V lithium polymer batteries. These batteries can be charged via a set of battery charging pins, which can be accessed through the module lid. A power switch, also accessible through the lid, is used to toggle the module between operating and charging modes.

C. Actuation

The module is actuated by four submersible centrifugal micropumps (M200S-SUB from TCS micropumps). Each pump has dimensions of $2.9 \times 1.6 \times 1.6$ cm, and is capable of a maximum flow rate of $11 \text{ mL} \cdot \text{s}^{-1}$. Each pump outlet is connected via a pipe to one of four ports on the lower hull. When active, a pump extracts water from the reservoir and discharges it into the environment (or a neighboring module). Correspondingly, fluid is drawn into the reservoir from the environment (or a neighboring module) via inactive pumps. This routing process provides the motive force for the module.

D. Sensing

Each module face has a magnet–hall sensor pair, a phototransistor, and an infrared (IR) transceiver. The magnet–hall sensor pair provides the ability to detect connections between modules. The phototransistor allows the module to sense visible light. It is used to determine whether the goal is visible to the face. If the face is connected, the IR transceiver can be used for communication. If not, the IR transceiver can be used as an obstacle sensor.

An IR receiver mounted on the main board allows the module to receive information from an overhead controller or a conventional television remote control. The combination of the IR receiver and the power switch allows for three operating modes: the *off/charging* mode, the *standby* mode, and the *active* mode. In the *off/charging* mode, the batteries are disconnected from the rest of the circuit. In the *standby* mode, the batteries are connected to the circuit allowing the sensors and LEDs to be active, but the pumps do not respond. In the *active* mode, the pumps respond to the sensor readings in accordance with the controller implemented. An electronic compass in each module can provide orientation sensing. It returns the angular offset of the module, relative to an initial orientation.

IX. EXPERIMENTS

This section details the physical experiments undertaken with the prototype MHP system. As we could not realize reliable real-time communication of the robot’s pose with respect to the goal via the IR receivers, the centralized controller was not tested. In the following, we compare the performance of the *dec* and *dec-com* controller variants from Section V.

A. General Setup

Fig. 22 shows an illustration of the experimental environment, a water tank of 115 cm length and 55 cm width. The environment is unobstructed, containing only a single robot. The water level is deep enough such that the robot does not touch the bottom of the tank. Elastic bands are attached to three sides of the tank to prevent the robot from sticking to the glass. The controllers use binary sensor readings. This is realized by using predefined threshold values, which were calibrated experimentally to suit the environmental setup.

At the beginning of each trial, a 2^2 MHP robot (a square of four modules) starts at one end of the tank, approximately equidistant

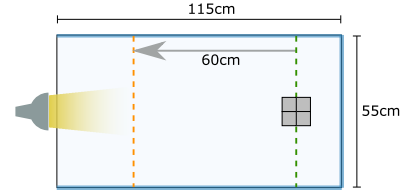


Fig. 22. Illustration of the experimental setup. A water tank of dimensions 115 cm and 55 cm with a 2^2 MHP robot shown on the right. The goal is represented as a lamp on the left, outside of the tank. The dashed green and orange lines represent the start and finish lines, respectively. Thick blue lines indicate the presence of an elastic-band boundary.

between the tank’s long sides, as illustrated in Fig. 22. The robot’s starting orientation is chosen randomly. The robot is tasked to move toward the goal, which is represented by a white LED lamp (806 lumens) positioned just outside the other end of the tank. All trials are recorded using an overhead camera. Each trial lasts 120 s. A trial is deemed successful if the robot’s centroid reaches the finish line, which is 60 cm nearer to the goal than the start line; the start and finish lines are determined by postanalysis of the video recordings.

Throughout this section, we use the two-tailed Mann–Whitney test. Each test uses a base alpha level of 0.01. Given that each set of data has three tests performed upon it, we use a Bonferroni adjusted alpha level of 0.003. We report the *a posteriori* *p*-values obtained by the tests.

B. Time to Completion

We evaluate the success rate of the physical MHP robots and the time taken for them to reach the goal.

1) *Setup*: The MHP robot comprises four modules arranged in a 2^2 configuration. The orientation of each module within the robot is chosen randomly for each trial. We performed 30 trials with the *dec* and *dec-com* controllers (i.e., 60 trials in total). We measured the time taken for successful trials.

2) *Results*: The *dec* and *dec-com* controllers had a success rate of 90% (27 out of 30 trials) and 100% (all 30 trials), respectively. Fig. 23 shows a series of snapshots from a typical trial using the *dec* controller. A video recording of this trial and of a trial using the *dec-com* controller is included in the online supplementary material. Video recordings of all 60 trials are available in [48].

Fig. 24 shows the time to completion for both controllers. The *dec* and *dec-com* controllers, although having a similar minimum completion time, require an average time of 19.5 s and 38.0 s, respectively, across the successful trials. The difference is significant ($p < 0.001$), and corresponds to an increase in time taken of 94.9% when using the *dec-com* controller. These findings are not in agreement with the theory (see Section V-C) and simulation results (see Section VII-B), which predict that completion times for both controllers should be identical. This is further analyzed in Section IX-D.

C. Energy Expenditure

We evaluate the performance of physical MHP robots with regard to energy expenditure.

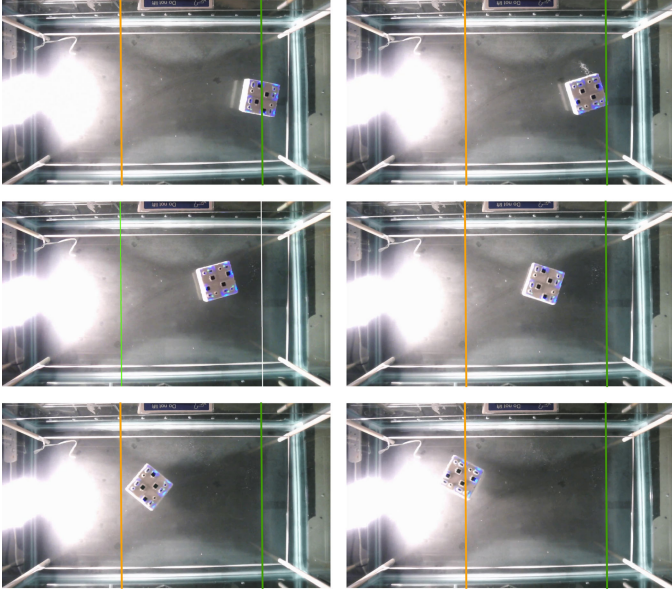


Fig. 23. Snapshots of a trial with a physical 2^2 MHP robot using the `dec` controller, taken at (from top left to bottom right) 0 s, 4 s, 9 s, 12 s, 18 s, and 22 s. The green and orange lines represent the start and finish lines, respectively.

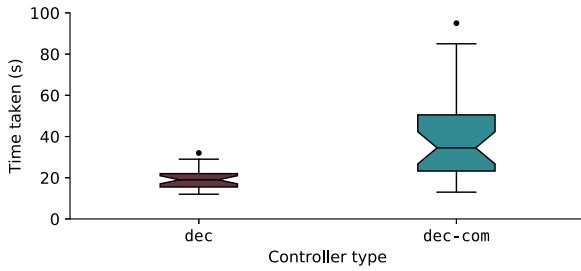


Fig. 24. Time taken for a physical 2^2 MHP robot to move by 60 cm toward the goal using the `dec` and `dec-com` controllers. Successful trials only; the boxes on the left and right represent 27 and 30 trials, respectively.

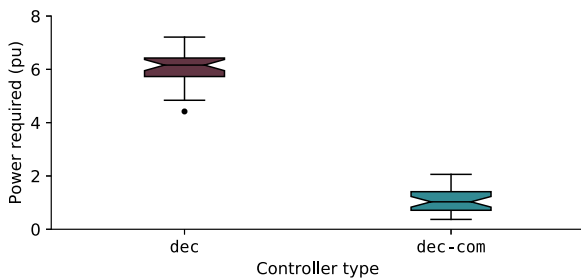


Fig. 25. Power required to move a physical 2^2 MHP robot by 60 cm toward the goal using the `dec` and `dec-com` controllers. Successful trials only; the boxes on the left and right represent 27 and 30 trials, respectively.

1) *Setup*: To obtain an estimate of the energy expenditure, we define 1 energy unit (eu) as the amount of energy expended by a single pump in 1 s. Moreover, we define 1 power unit (pu) as the number of energy units expended by the MHP robot per second. We calculate these performance metrics from the video footage, for both sets of trials reported in Section IX-B. This is facilitated by the module's LEDs; activated LEDs correspond to activate pumps.

2) *Results*: Figs. 25 and 26 compare the power usage and energy consumption, respectively, during the trials. The `dec-com`

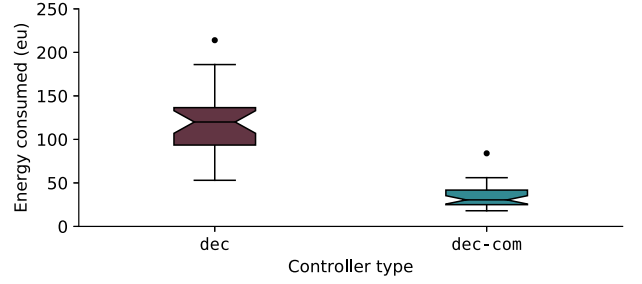


Fig. 26. Energy consumed by a physical 2^2 MHP robot to move by 60 cm toward the goal using the `dec` and `dec-com` controllers. Successful trials only; the boxes on the left and right represent 27 and 30 trials, respectively.

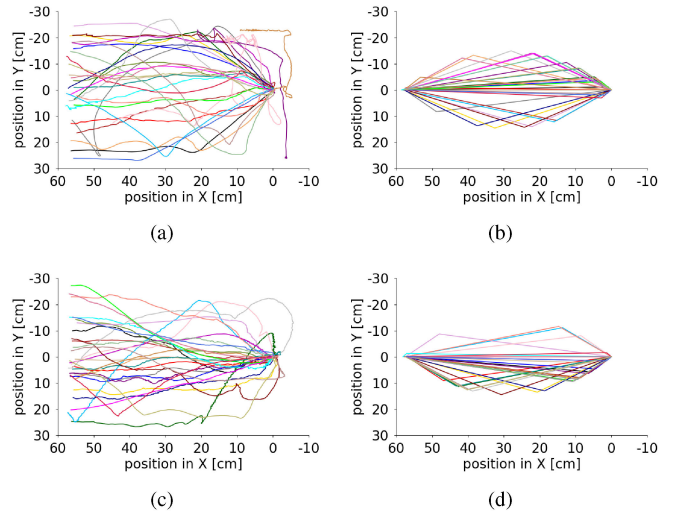


Fig. 27. Experimentally observed versus theoretically predicted trajectories for 2^2 MHP robots moving toward the goal (to the left, and outside the shown range). Thirty trials using the `dec` controller. (a) Real trajectories. (b) Predicted trajectories. Thirty trials using the `dec-com` controller. (c) Real trajectories. (d) Predicted trajectories.

controller uses significantly less energy and has a significantly lower power rate than the `dec` controller ($p < 0.001$ for both). The successful trials of the `dec` controller use, on average, 118.9 eu per trial at a rate of 6.1 pu. The successful trials of the `dec-com` controller use, on average, 34.8 eu per trial at a rate of 0.9 pu. This corresponds, on average, to an energy reduction of 70.7% and a power reduction of 85% when using the `dec-com` controller, rather than the `dec` controller.

D. Reality Gap

When testing a robotic system in simulation and then in reality some discrepancies are to be expected. This phenomenon is referred to as the reality gap [49]. To help visualize the reality gap, Fig. 27 presents the trajectories of MHP robots as observed in the 60 experimental trials with the `dec` and `dec-com` controllers, as well as the corresponding trajectories that our theoretical model predicts, assuming the same initial pose of the robot in each trial. Although for about half of the trials, the real and predicted trajectories are in broad agreement, overall substantial discrepancies can be observed.

In the following, we analyze two successful trials of similar length, one for each controller. Fig. 28 compares the number of pumps active throughout each trial. For the `dec` controller,

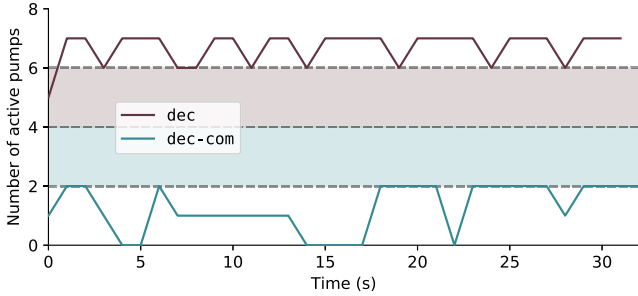


Fig. 28. Number of active pumps of a physical 2^2 MHP robot during two successful trials using the `dec` controller and `dec-com` controller, respectively. Shaded areas correspond to the respective theoretically possible range (see Fig. 6).

between five and seven pumps were active at all times. For the `dec-com` controller, between zero and two pumps were active. In both cases, this represents a departure from theory: Fig. 6 assumes a point goal and a fault-free system, and shows that either four or six pumps should be active for the `dec` controller, whereas either two or four pumps should be active for the `dec-com` controller. One explanation for this difference is that, according to the model, goal sensors have a hemispherical field of view, whereas, in reality, they have a smaller field of view. For this reason, false-negative readings may occur. From the sample trials, it is apparent that false-negative readings affect the two controllers in different ways. When using the `dec` controller, false-negative readings cause an activation of pumps, resulting in a higher than expected number of pumps firing. Conversely, when using the `dec-com` controller, false-negative readings cause a deactivation of pumps, resulting in a lower than expected number of pumps firing. Indeed, there are periods of time when no pumps are firing; the robot then moves only due to inertia and random drift. This is particularly problematic if it happens at the start of a trial, where inertial forces are negligible, and hence, the robot remains almost stationary.

Another discrepancy is that the real MHP robots tend to undergo rotational movements, whereas the theory for a convex-shaped robot predicts purely translational movement.³ This can be attributed not only to the aforementioned false-negative readings, but also to noise and inaccuracies in the pumps. The theory assumes that each active pump produces the same constant force along the face normal. However, real pumps produce forces that are of similar magnitudes and slightly offset from the face normal. In addition, the theory does not consider the internal routing of fluid through the robot, which could introduce additional torques or forces on the real robot.

The `dec-com` controller outperforms the `dec` controller in terms of power usage, and it does so to a greater extent in reality than the theory predicts. This can be attributed to the aforementioned widened gap in the number of active pumps between the two controllers (see Fig. 28). Although the `dec-com` controller requires about twice the time to complete the task compared to the `dec` controller, the total energy consumed is still substantially lower (see Fig. 26).

³For a convex robot, all actuators on a given robot face will either be active or inactive. Net force will, therefore, be symmetric with respect to the robot center of mass, and no rotation will occur.

X. CONCLUSION

In this article, we introduced a novel concept—MFP—whereby modular robots move by routing fluid through themselves. We posited that the concept would enable systems to combine effective propulsion, a large reconfiguration space, and a scalable design. We proposed two MFP implementations: 1) MHP robots, which route liquid from the environment through themselves, and 2) MPP robots, which store pressurized gas within themselves and selectively release it into the environment. MFP robots of sufficient resolution have the potential to produce torques and forces with high precision, despite using simple, binary actuators. We derived expressions for the number of unique torques and forces that orthogonally convex robots can produce.

To test the MFP concept, we considered a task where a robot had to move toward a goal. To solve this task, we proposed an occlusion-based controller (`dec`), which was inspired by studies of swarms of ground robots that cooperatively transport a tall and heavy object toward a goal [22]. We also considered two controller variants, one that uses communication between modules (`dec-com`) and one that is capable of obstacle avoidance (`dec-obs`). We then analyzed the controllers for an environment free of obstacles. Under certain assumptions about the drag model, we proved that orthogonally convex MFP robots are guaranteed to reach the goal, and that MFP robots of arbitrary shape are guaranteed to approach the goal up to a morphology-dependent distance.

The occlusion-based controller is fully decentralized, and runs on every module face of the robot. The controller does not require run-time memory or communication. It uses only 2 bit of input—whether the face is connected to another module, and whether the goal is visible. The controller maps this 2-bit input onto a 1-bit output—the state of the actuator on the face. To the best of our knowledge, it is the simplest solution to a nontrivial control problem in modular robotics to date. The simplicity of the solution is on par with the so-called *computation-free* solutions reported for groups of loosely coupled mobile robots [50]. We hypothesize that exceedingly simple solutions, such as MFP, can make modular robots more valuable at the subcentimeter scale, where effective modular propulsion systems have not been demonstrated.

Using computer simulations, we tested robots consisting of 125 modules moving in 3-D. All three decentralized controllers were tested, along with the state-of-the-art centralized controller (`cen`). The `dec-com` controller required the same time to complete a trial as the `dec` controller, but saved on average 51% and 69% energy for cubic and unconstrained robots, respectively. In an environment with obstacles, the `dec-obs` controller was slightly slower than the `dec` controller, but reduced collisions by 77% on average.

To further validate the concept, we developed a new set of MHP prototype modules, which move in 2-D on the surface of water. Each module has four faces, which can detect light, obstacles, and neighboring modules, if any. Modules can communicate when connected. We reported a series of experiments where an MHP robot of four modules had to move toward a light source. The `dec` and `dec-com` controllers completed the

task in 90% and 100% of the experimental trials, respectively. The `dec-com` controller saved 71% energy on average, at the expense of increasing the time taken to reach the goal by 95% on average.

Future work could include a physical implementation with modules of subcentimeter size, targeted at operations in highly confined spaces, such as pipe networks, and possibly in 3-D. In addition to effective watertightness, a 3-D MHP system would have to consider the effects of submersion on sensors and communication (e.g., attenuation). Buoyancy control could be provided via existing actuators, or the use of swim bladders. Pneumatic prototypes of MFP could be considered, for applications in space. Incorporating self-reconfiguration into the design, potentially by repurposing the propulsion actuators, would further increase the utility of the system.

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