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Kadomtsev-Petviashvili equation for magnetosonic waves in Hall plasmas and soliton stability

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Abstract

In this article we study nonlinear waves in Hall plasmas. We consider magnetosonic waves propagating at the angles with respect to the equilibrium magnetic field that are not close to either 0 or $\pi/2$. Using the reductive perturbation method we derive the three-dimensional Kadomtsev-Petviashvili (KP) equation. We use the KP equation to show that both fast and slow magnetosonic solitons are unstable with respect to transverse perturbations. We confront our results with the investigation of soliton stability in anisotropic media using the study of soliton self-refraction.

Keywords: plasma, nonlinear waves, stability

1. Introduction

When the characteristic time of a problem related to plasma motion is much larger than the inverse ion cyclotron frequency it can be adequately described by the classical magnetohydrodynamics (MHD). However, when it becomes comparable to inverse ion cyclotron frequency we have to take the Hall current in Ohm's law into account. As a result, we have an additional term in the induction equation determining the magnetic field behaviour, and we arrive at Hall MHD. Hall MHD is widely used in description of various astrophysical processes, like flux expulsion in neutron star crusts [1], angular momentum transport in weakly ionised protoplanetary discs [2, 3], and the formation of intensive flux tubes in the solar atmosphere [4]. It is also used in the application to fusion plasmas [5–7].

One of the first studies of wave propagation in Hall plasmas was carried out in [8]. After that waves in Hall plasmas have been studied by many authors. In particular, the linear waves in Hall plasmas were studied in [9, 10]. In [11, 12] the propagation of sausage and kink waves in a magnetic slab was investigated. The parametric instabilities of circularly polarised small-amplitude Alfvén waves were considered in [13]. In [14] dispersive shocks in resistive Hall plasmas with the application to waves in the solar wind were studied.

In this article we deal with solitons. The first observation of a soliton on the water surface was reported by John Scott Russell [15]. It took 50 years before the Korteweg–de Vries (KdV) equation that provided the theoretical explanation of the phenomenon observed by John Scott Russell was derived in [16]. After that it was practically forgotten for very long time, and then almost suddenly became very popular when it turned out that it describes various types of waves in plasmas. It received an additional impetus when it was shown in [17] that exact solutions of the KdV equation can be obtained using the inverse scattering transform method. This article initiated the whole new branch of applied mathematics called solitonics. At present, the KdV equation is one of the most popular nonlinear equations both in physics and applied mathematics.

A naturally arising question is if a soliton described by the KdV equation is stable. It is definitely stable with respect to one-dimensional perturbations when both the soliton and its perturbations are described by the KdV equation. Moreover, the collision of two solitons is elastic meaning that they preserve their form and amplitudes after the collision. However, when a one-dimensional soliton propagates on the water surface infinite in two directions the soliton can be unstable with respect to two-dimensional perturbations. This problem was addressed in [18] where the two-dimensional generalisation of the KdV equation called the Kadomtsev-Petviashvili (KP) equation was derived. It was shown in that article that the stability of KdV solitons with respect to the two-dimensional transverse perturbations depends on the sign of wave dispersion. When the wave dispersion is negative, that is the wave frequency is a monotonically decreasing function of the wave number, then the solitons are stable, while they are unstable when the dispersion is positive. In particular, solitons on the water surface are stable unless the surface tension dominates the gravitational force. Usually the KP equation is called the KPI equation in the case of positive dispersion, and the KPII equation in the case of negative dispersion.

In fact, the KP equation is also valid for any waves described by the KdV equation that propagate in an isotropic medium. In [19] the method describing the nonlinear evolution of a two-dimensional soliton with a nonplanar front was developed. In particular, the authors used their method to study the soliton stability with respect to transverse perturbations. In the case when solitons propagate in an isotropic medium they obtain the same result as in [18]. This method was also applied to studying stability of solitons propagating in an anisotropic medium. The conclusion was that a soliton is stable no matter what is the sign of dispersion unless it propagates at a small angle with respect to one of the extremal directions defined by the condition that the derivative of the phase speed with respect to the propagation angle is zero at these directions.

One example of an anisotropic medium is a Hall plasma. The anisotropy is related to the presence of magnetic field. As we have already pointed out, the motion of a Hall plasma is described by Hall MHD that is used in application to a verity of astrophysical problems. In particular, it can be used to describe magnetosonic waves in the solar atmosphere, solar wind, and in the magnetosphere of Earth and other planets. In [20, 21] observations of long-periodic compressional waves in coronal holes and the inter-plume regions were reported. These waves were interpreted as slow magnetosonic waves. It was suggested in [22] that the high-speed solar wind that originates in coronal holes is accelerated by magnetosonic solitons. Later this model was further developed in [23]. In [24] the recent observations of slow MHD waves in the Earth's magnetosheath obtained by Magnetospheric Multiscale (MMS) NASA mission were reported, while it was suggested in [25] that the observed periodic variations of Saturn's magnetosphere is controlled by compressional waves.

To develop adequate theories involving MHD waves as well as to provide correct interpretation of observations of MHD waves in space plasmas it is necessary to study the properties of these waves. In particular, it is important to study the stability of magnetosonic solitons. In the case when solitons are stable with respect to transverse perturbations magnetosonic waves can exist in the form of one-dimensional solitons. However, when they are unstable their temporal evolution results in their decomposition in an array of two-dimensional solitons [26]. These two-dimensional solitons are solutions to the KPI equation [27].

The aim of this article is twofold. The first aim is to derive the KP equation for magnetosonic waves in a Hall plasma and use it for studying the soliton stability with respect to transverse perturbations. The second aim is to use the results of the stability study to verify the statement made in [19] on the soliton stability propagating in an anisotropic medium. The paper is organized as follows. In the next section we formulate the problem and present the governing equations. In section 3, we briefly describe the linear theory of wave propagation in Hall plasmas. In section 4, we derive the KP equation for magnetosonic waves propagating in a Hall plasma. In section 5, we study the soliton stability with respect to transverse perturbations. In section 6, we confront our results of the stability study with those obtained in [19]. Finally, we present the summary of obtained results and our conclusions in section 7.

2. Problem formulation and governing equations

To describe the plasma motion we use the Hall magnetohydrodynamic (MHD) equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \nabla \cdot \mathbf{B} = 0, \tag{1a}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{1b}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{m_i}{e\mu_0 \rho}\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}], \tag{1c}$$

$$p = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}. \tag{1d}$$

Here ρ is the density, p the pressure, $\mathbf{v}$ the velocity, and $\mathbf{B}$ the magnetic field; ρ_0 and p_0 are the equilibrium density and pressure, respectively; m_i is the ion mass, e the elementary charge, μ_0 the magnetic permeability of free space, and γ the adiabatic exponent. Below we use Cartesian coordinates x, y, z. In the equilibrium $\mathbf{v} = 0$ and $\mathbf{B} = \mathbf{B}_0$, where

$$\mathbf{B}_0 = B_0(\cos\alpha, \sin\alpha, 0). \tag{2}$$

When writing down equation (1c) we assumed that the electrons and ions have the same temperature implying that the electron pressure p_e is determined by equation (1d) with

p_0 substituted by $p_0/2$. This asumption enables to eliminate ∇p_e from equation (1c).

3. Linear theory

We write

$$\rho = \rho_0 + \rho', \quad p = p_0 + p', \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \tag{3}$$

substitute these expressions in equations (1), linearise the obtained equations with respect to ρ', p', $\mathbf{v}$, and $\mathbf{b}$, and take all variables proportional to $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, where $\mathbf{k} = (k_x, k_y, k_z)$ and $\mathbf{r} = (x, y, z)$. As a result, we obtain

$$\omega\rho' - \rho_0(\mathbf{k}\cdot\mathbf{v}) = 0, \quad \mathbf{k}\cdot\mathbf{b} = 0, \tag{4a}$$

$$\rho_0\omega\mathbf{v} = \mathbf{k}p' - \frac{1}{\mu_0}[\mathbf{b}(\mathbf{k}\cdot\mathbf{B}_0) - \mathbf{k}(\mathbf{b}\cdot\mathbf{B}_0)], \tag{4b}$$

$$\omega\mathbf{b} = \mathbf{B}_0(\mathbf{k}\cdot\mathbf{v}) - \mathbf{v}(\mathbf{k}\cdot\mathbf{B}_0) + \frac{im_i}{e\mu_0\rho_0}(\mathbf{k}\cdot\mathbf{B}_0)(\mathbf{k}\times\mathbf{b}), \tag{4c}$$

$$p' = c_s^2\rho', \tag{4d}$$

where $c_s^2 = \gamma p_0/\rho_0$ is the square of the sound speed. Eliminating all variables in favour of $\mathbf{b}$ yields

$$\omega^2\mathbf{b} = \frac{k^2(\mathbf{b}\cdot\mathbf{B}_0)[\omega^2\mathbf{B}_0 - c_s^2\mathbf{k}(\mathbf{k}\cdot\mathbf{B}_0)]}{\mu_0\rho_0(\omega^2 - c_s^2k^2)} + \frac{\mathbf{k}\cdot\mathbf{B}_0}{\mu_0\rho_0}[\mathbf{b}(\mathbf{k}\cdot\mathbf{B}_0) - \mathbf{k}(\mathbf{b}\cdot\mathbf{B}_0)] + \frac{im_i\omega}{e\mu_0\rho_0}(\mathbf{k}\cdot\mathbf{B}_0)(\mathbf{k}\times\mathbf{b}). \tag{5}$$

Now we choose the x-axis in the direction of $\mathbf{k}$, so that $\mathbf{k} = (k, 0, 0)$. Then $\mathbf{k}\cdot\mathbf{B}_0 = kB_0\cos\alpha$, and taking the scalar product of this equation with $\mathbf{B}_0$ and $\mathbf{k}\times\mathbf{B}_0$ we obtain

$$\begin{aligned}\omega^2(\omega^2 - c_s^2k^2)(\mathbf{b}\cdot\mathbf{B}_0) &= V_A^2k^2(\omega^2 - c_s^2k^2\cos^2\alpha)(\mathbf{b}\cdot\mathbf{B}_0)\\ &\quad - i\ell k^2V_A^2\cos\alpha(\omega^2 - c_s^2k^2)\mathbf{b}\cdot(\mathbf{k}\times\mathbf{B}_0),\end{aligned} \tag{6a}$$

$$\omega^2\mathbf{b}\cdot(\mathbf{k}\times\mathbf{B}_0) = k^2V_A^2\mathbf{b}\cdot(\mathbf{k}\times\mathbf{B}_0)\cos^2\alpha + i\ell\omega k^3V_A\cos\alpha(\mathbf{b}\cdot\mathbf{B}_0), \tag{6b}$$

where the Alfvén speed V_A and the characteristic dispersion length ℓ are defined by

$$V_A^2 = \frac{B_0^2}{\mu_0\rho_0}, \quad \ell = \frac{m_iB_0}{e\mu_0\rho_0V_A}. \tag{7}$$

Equations (6) constitute the system of two linear homogeneous algebraic equations for $\mathbf{b}\cdot\mathbf{B}_0$ and $\mathbf{b}\cdot(\mathbf{k}\times\mathbf{B}_0)$. The condition that it has non-trivial solutions is that its determinant is zero. This gives the dispersion equation

$$(\omega^2 - V_A^2k^2\cos^2\alpha)[\omega^4 - (c_s^2 + V_A^2)k^2\omega^2 + c_s^2V_A^2k^4\cos^2\alpha] = \ell^2\omega k^5V_A^3(\omega^2 - c_s^2k^2)\cos^2\alpha. \tag{8}$$

Now we use the long wavelength approximation and assume that $k\ell \ll 1$. Then we derive the approximate dispersion relations

$$\omega = kV_A\cos\alpha + \frac{\ell^2k^3(c_s^2 - V_A^2\cos^2\alpha)}{2V_A\sin^2\alpha}, \tag{9a}$$

$$\omega = a_\pm k + \beta_\pm k^3, \tag{9b}$$

where $a_- < a_+$, and $a_\pm$ and $\beta_\pm$ are defined by

$$a_\pm^4 - (c_s^2 + V_A^2)a_\pm^2 + c_s^2V_A^2\cos^2\alpha = 0, \tag{10}$$

$$\beta_\pm = \frac{\ell^2V_A^3(a_\pm^2 - c_s^2)\cos^2\alpha}{2(a_\pm^2 - V_A^2\cos^2\alpha)[2a_\pm^2 - (c_s^2 + V_A^2)]}. \tag{11}$$

Equation (9a) corresponds to Alfvén waves, while equation (9b) to magnetosonic waves, where the plus and minus signs are for the fast and slow magnetosonic waves, respectively. We note that β+ > 0 and β- < 0. If we neglect the dispersion described by the second term on the right-hand side of equation (9b) then the group velocity of magnetosonic waves is given by

$$\mathbf{V}_g = \frac{\partial(a_\pm k)}{\partial\mathbf{k}} = \frac{a_\pm\mathbf{k}}{k} - \frac{c_s^2V_A^2\mathbf{e}_y\sin2\alpha}{2a_\pm[2a_\pm^2 - (c_s^2 + V_A^2)]}, \tag{12}$$

where $\mathbf{e}_y$ is the unit vector in the y-direction.

4. Derivation of Kadomtsev–Petviashvili equation

To derive the KP equation for magnetosonic waves we use the reductive perturbation method [28, 29]. We consider nonlinear magnetosonic waves with the characteristic dimensionless amplitude $\epsilon \ll 1$ propagating along the x-axis. We assume that waves are weakly dispersive, so that the second term on the right-hand side of equation (9b) is much smaller than the first one. We also assume that α is not close to either 0 or $\pi/2$. When α is close to 0 one-dimensional nonlinear waves are described by the modified Kortewed–de Vries (mKdV) equation, and the two- and three-dimensional waves by the equation derived in [30, 31]. When α is equal to $\pi/2$ the term on the right-hand side of equation (8) that describes the dispersion is zero, and the wave dispersion is related to the account of the electron inertia. Hence, when α is close to $\pi/2$ the electron inertia must be taken into account.

We are going to derive the equation that describes the competition between the nonlinearity and dispersion. The ratio of the second and first term on right-hand side of equation (9b) is of the order of ℓ^2 divided by the wavelength squared. In order to have the competition between the nonlinearity and dispersion it should be of the order of ϵ. Hence, we consider perturbations with the characteristic spatial scale equal to $\epsilon^{-1/2}\ell$. On the time scale of the order of the characteristic spatial scale divided by the wave phase speed the wave propagation is described by the linear theory, so that all the perturbations are the functions of $x - at$, where either $a = a_-$ or $a = a_+$. The effect of nonlinearity and dispersion only occurs on a much longer time scale, of the order of ϵ^{-1} times the previous time scale. We also assume that the perturbations can vary in the y and z direction with the characteristic spatial scale equal to $\epsilon^{-1/2}$ times the spatial scale in the x-direction. Following this discussion we introduce the scaled variables

$$\xi = \epsilon^{1/2}(x - at), \quad \eta = \epsilon y, \quad \zeta = \epsilon z, \quad t_1 = \epsilon t, \quad \tau = \epsilon^{3/2} t. \tag{13}$$

The necessity of introducing the intermediate time t_1 will be clear later. Using these scaled variables we transform equations (1) to

$$\epsilon\frac{\partial\rho}{\partial\tau} + \epsilon^{1/2}\frac{\partial\rho}{\partial t_1} - a\frac{\partial\rho}{\partial\xi} + \frac{\partial(\rho u)}{\partial\xi} + \epsilon^{1/2}\nabla_\perp\cdot(\rho\mathbf{v}_\perp) = 0, \tag{14a}$$

$$\epsilon\frac{\partial u}{\partial\tau} + \epsilon^{1/2}\frac{\partial u}{\partial t_1} - a\frac{\partial u}{\partial\xi} + u\frac{\partial u}{\partial\xi} + \epsilon^{1/2}(\mathbf{v}_\perp\cdot\nabla_\perp)u = -\frac{1}{\rho}\frac{\partial P}{\partial\xi} + \frac{B_x}{\mu_0\rho}\frac{\partial B_x}{\partial\xi} + \frac{\epsilon^{1/2}}{\mu_0\rho}(\mathbf{B}_\perp\cdot\nabla_\perp)B_x, \tag{14b}$$

$$\epsilon\frac{\partial\mathbf{v}_\perp}{\partial\tau} + \epsilon^{1/2}\frac{\partial\mathbf{v}_\perp}{\partial t_1} - a\frac{\partial\mathbf{v}_\perp}{\partial\xi} + u\frac{\partial\mathbf{v}_\perp}{\partial\xi} + \epsilon^{1/2}(\mathbf{v}_\perp\cdot\nabla_\perp)\mathbf{v}_\perp = -\frac{\epsilon^{1/2}}{\rho}\nabla_\perp P + \frac{B_x}{\mu_0\rho}\frac{\partial\mathbf{B}_\perp}{\partial\xi} + \frac{\epsilon^{1/2}}{\mu_0\rho}(\mathbf{B}_\perp\cdot\nabla_\perp)\mathbf{B}_\perp, \tag{14c}$$

$$\epsilon\frac{\partial B_x}{\partial\tau} + \epsilon^{1/2}\frac{\partial B_x}{\partial t_1} - a\frac{\partial B_x}{\partial\xi} = \epsilon^{1/2}(\mathbf{B}_\perp\cdot\nabla_\perp)u - \epsilon^{1/2}(\mathbf{v}_\perp\cdot\nabla_\perp)B_x - \epsilon^{1/2}B_x\nabla_\perp\cdot\mathbf{v}_\perp - u\frac{\partial B_x}{\partial\xi}, \tag{14d}$$

$$\begin{aligned}\epsilon\frac{\partial\mathbf{B}_\perp}{\partial\tau} + \epsilon^{1/2}\frac{\partial\mathbf{B}_\perp}{\partial t_1} - a\frac{\partial\mathbf{B}_\perp}{\partial\xi} &= B_x\frac{\partial\mathbf{v}_\perp}{\partial\xi} - u\frac{\partial\mathbf{B}_\perp}{\partial\xi} \\ &- \mathbf{B}_\perp\frac{\partial u}{\partial\xi} - \epsilon^{1/2}\frac{\ell\cos\alpha}{V_A}\mathbf{e}_x\times\frac{\partial^2\mathbf{B}_\perp}{\partial\xi^2}[1 + \mathcal{O}(\epsilon^{1/2})] \\ &- \epsilon^{1/2}\mathbf{B}_\perp\nabla_\perp\cdot\mathbf{v}_\perp + \epsilon^{1/2}(\mathbf{B}_\perp\cdot\nabla_\perp)\mathbf{v}_\perp \\ &- \epsilon^{1/2}(\mathbf{v}_\perp\cdot\nabla_\perp)\mathbf{B}_\perp,\end{aligned} \tag{14e}$$

where $\mathbf{e}_x$ is the unit vector in the x-direction, u the x-component of the velocity, $\mathbf{v}_\perp = (0, v, w)$, $\mathbf{B}_\perp = (0, B_y, B_z)$, and

$$\nabla_\perp = \left(0, \frac{\partial}{\partial\eta}, \frac{\partial}{\partial\zeta}\right), \quad P = p + \frac{B^2}{2\mu_0}. \tag{15}$$

Equation (1*d*) remains unchanged. Next we look for the solution to equations (14a) in the form of asymptotic expansions

$$\begin{aligned}\rho &= \rho_0 + \epsilon\rho_1 + \epsilon^{3/2}\rho_2 + \epsilon^2\rho_3 + ..., \\ p &= p_0 + \epsilon p_1 + \epsilon^{3/2}p_2 + \epsilon^2 p_3 + ..., \\ u &= \epsilon u_1 + \epsilon^{3/2}u_2 + \epsilon^2 u_3 + ..., \\ \mathbf{v}_\perp &= \epsilon\mathbf{v}_{\perp1} + \epsilon^{3/2}\mathbf{v}_{\perp2} + \epsilon^2\mathbf{v}_{\perp3} + ..., \\ B_x &= B_0\cos\alpha + \epsilon B_{x1} + \epsilon^{3/2}B_{x2} + \epsilon^2 B_{x3} + ..., \\ \mathbf{B}_\perp &= \mathbf{e}_y B_0\sin\alpha + \epsilon\mathbf{B}_{\perp1} + \epsilon^{3/2}\mathbf{B}_{\perp2} + \epsilon^2\mathbf{B}_{\perp3} + ...\end{aligned} \tag{16}$$

4.1. The first order approximation

Substituting the expansions given by equation (16) in equation (1*d*) and (14) and collecting terms of the order of ϵ we obtain

$$a\frac{\partial\rho_1}{\partial\xi} - \rho_0\frac{\partial u_1}{\partial\xi} = 0, \quad p_1 = c_s^2\rho_1, \quad \frac{\partial B_{x1}}{\partial\xi} = 0, \tag{17a}$$

$$a\frac{\partial u_1}{\partial\xi} = \frac{1}{\rho_0}\frac{\partial p_1}{\partial\xi} + \frac{B_0\sin\alpha}{\mu_0\rho_0}\frac{\partial B_{y1}}{\partial\xi}, \tag{17b}$$

$$a\frac{\partial\mathbf{v}_{\perp1}}{\partial\xi} + \frac{B_0\cos\alpha}{\mu_0\rho_0}\frac{\partial\mathbf{B}_{\perp1}}{\partial\xi} = 0, \tag{17c}$$

$$a\frac{\partial\mathbf{B}_{\perp1}}{\partial\xi} = B_0\mathbf{e}_y\sin\alpha\frac{\partial u_1}{\partial\xi} - B_0\cos\alpha\frac{\partial\mathbf{v}_{\perp1}}{\partial\xi}. \tag{17d}$$

The condition that this linear homogeneous system of equations has non-trivial solutions is that $a = a_\pm$ defined by equation (10). Imposing the condition that all perturbations vanish as $\xi \to -\infty$ we obtain from this system of equations

$$w_1 = 0, \quad B_{x1} = B_{z1} = 0, \tag{18a}$$

$$\rho_1 = \frac{\rho_0 V_A^2\sin\alpha}{B_0(a^2 - c_s^2)}B_{y1}, \quad p_1 = \frac{\rho_0 c_s^2 V_A^2\sin\alpha}{B_0(a^2 - c_s^2)}B_{y1}, \tag{18b}$$

$$u_1 = \frac{aV_A^2\sin\alpha}{B_0(a^2 - c_s^2)}B_{y1}, \quad v_1 = -\frac{V_A^2\cos\alpha}{aB_0}B_{y1}. \tag{18c}$$

4.2. The second order approximation

Collecting terms of the order of $\epsilon^{3/2}$ in equations (1*d*) and (14), and using equation (18) yields

$$a\frac{\partial\rho_2}{\partial\xi} - \rho_0\frac{\partial u_2}{\partial\xi} = \frac{\partial\rho_1}{\partial t_1} + \rho_0\nabla_\perp\cdot\mathbf{v}_{\perp1}, \quad p_2 = c_s^2\rho_2, \tag{19a}$$

$$a\frac{\partial u_2}{\partial\xi} - \frac{1}{\rho_0}\frac{\partial p_2}{\partial\xi} - \frac{B_0\sin\alpha}{\mu_0\rho_0}\frac{\partial B_{y2}}{\partial\xi} = \frac{\partial u_1}{\partial t_1}, \tag{19b}$$

$$a\frac{\partial\mathbf{v}_{\perp2}}{\partial\xi} + \frac{B_0\cos\alpha}{\mu_0\rho_0}\frac{\partial\mathbf{B}_{\perp2}}{\partial\xi} = \frac{\partial\mathbf{v}_{\perp1}}{\partial t_1} + \frac{\nabla_\perp p_1}{\rho_0} + \frac{B_0\sin\alpha}{\mu_0\rho_0}\left(\nabla_\perp B_{y1} - \frac{\partial\mathbf{B}_{\perp1}}{\partial\eta}\right), \tag{19c}$$

$$a\frac{\partial B_{x2}}{\partial\xi} = B_0\cos\alpha\nabla_\perp\cdot\mathbf{v}_{\perp1} - B_0\sin\alpha\frac{\partial u_1}{\partial\eta}, \tag{19d}$$

$$\begin{aligned}a\frac{\partial\mathbf{B}_{\perp2}}{\partial\xi} - B_0\mathbf{e}_y\sin\alpha\frac{\partial u_2}{\partial\xi} + B_0\cos\alpha\frac{\partial\mathbf{v}_{\perp2}}{\partial\xi} &= \frac{\partial\mathbf{B}_{\perp1}}{\partial t_1} + B_0\sin\alpha\left(\mathbf{e}_y\nabla_\perp\cdot\mathbf{v}_{\perp1}\right. \\ &\left. - \frac{\partial\mathbf{v}_{\perp1}}{\partial\eta}\right) + \ell V_A\cos\alpha\,\mathbf{e}_x\times\frac{\partial^2\mathbf{B}_{\perp1}}{\partial\xi^2}.\end{aligned} \tag{19e}$$

This is a linear inhomogeneous system of equations with respect to the variables of the second order approximation. Its homogeneous counterpart coincides with the system of the first order approximation. Hence the system of equations (19) is compatible only if the right-hand side satisfies the compatibility condition. To obtain this condition we eliminate all the variables of the second order approximation from equation (19). Then, using equation (10) we obtain

$$\frac{\partial B_{y1}}{\partial t_1} - \Upsilon\frac{\partial B_{y1}}{\partial \eta} = 0, \quad \Upsilon = \frac{c_s^2 V_A^2 \sin 2\alpha}{2a[2a^2 - (c_s^2 + V_A^2)]}. \tag{20}$$

It follows from this equation that B_{y1} must depend not on t_1 and η separately but on their linear combination $\theta = \eta + \Upsilon t_1$. We note that, in accordance with equation (12) $\Upsilon = -V_{gy}$. The result that B_{y1} depends on $\theta = \eta + \Upsilon\, t_1$ is related to the fact that the wave energy propagates in the direction of the grope velocity $\mathbf{V}_g$ rather than in the direction of phase velocity. In an isotropic medium the two directions coincide. In particular, when the wave propagates in the x-direction then $V_{gy} = 0$ and we obtain that there is no dependence on t_1. Hence, there is no need to introduce the intermediate time t_1 when deriving the KP equation in an isotropic medium. We only need to introduce t_1 when the medium is anisotropic.

Using equations (18) and (20) we obtain from equation (19)

$$\frac{\partial \rho_2}{\partial \xi} = \frac{\rho_0 V_A^2 \sin\alpha}{B_0(a^2 - c_s^2)}\frac{\partial B_{y2}}{\partial \xi} + \frac{\rho_0 V_A^2 (c_s^2 - V_A^2)\cos\alpha}{B_0(a^2 - c_s^2)[2a^2 - (c_s^2 + V_A^2)]}\frac{\partial B_{y1}}{\partial \theta}, \tag{21a}$$

$$\frac{\partial u_2}{\partial \xi} = \frac{a V_A^2 \sin\alpha}{B_0(a^2 - c_s^2)}\frac{\partial B_{y2}}{\partial \xi} + \frac{c_s^2 V_A^2 (c_s^2 - V_A^2\cos^2\alpha)\cos\alpha}{aB_0(a^2 - c_s^2)[2a^2 - (c_s^2 + V_A^2)]}\frac{\partial B_{y1}}{\partial \theta}, \tag{21b}$$

$$\frac{\partial v_2}{\partial \xi} = -\frac{V_A^2 \cos\alpha}{aB_0}\frac{\partial B_{y2}}{\partial \xi} + \frac{c_s^2 V_A^2 (c_s^2 - V_A^2\cos^2\alpha)\sin\alpha}{aB_0(a^2 - c_s^2)[2a^2 - (c_s^2 + V_A^2)]}\frac{\partial B_{y1}}{\partial \theta}, \tag{21c}$$

$$\frac{\partial w_2}{\partial \xi} = \frac{V_A^2}{B_0(a^2 - V_A^2\cos^2\alpha)} \times \left(\frac{a^3 \sin\alpha}{a^2 - c_s^2}\frac{\partial B_{y1}}{\partial \zeta} - \ell V_A \cos^2\alpha \frac{\partial^2 B_{y1}}{\partial \xi^2}\right), \tag{21d}$$

$$\frac{\partial B_{x2}}{\partial \xi} = -\frac{\partial B_{y1}}{\partial \theta}, \tag{21e}$$

$$\frac{\partial B_{z2}}{\partial \xi} = -\frac{a V_A \cos\alpha}{a^2 - V_A^2\cos^2\alpha} \times \left(\frac{a V_A \sin\alpha}{a^2 - c_s^2}\frac{\partial B_{y1}}{\partial \zeta} - \ell \frac{\partial^2 B_{y1}}{\partial \xi^2}\right). \tag{21f}$$

4.3. The third order approximation

Collecting terms of the order of ϵ^2 in equations (1*d*), (14*a*), and (14*b*), and in y-components of equations (14*c*) and (14*e*) yields

$$a\frac{\partial \rho_3}{\partial \xi} - \rho_0\frac{\partial u_3}{\partial \xi} = \frac{\partial \rho_1}{\partial \tau} + \Upsilon\frac{\partial \rho_2}{\partial \theta} + \frac{\partial(\rho_1 u_1)}{\partial \xi} + \rho_0\frac{\partial v_2}{\partial \theta} + \rho_0\frac{\partial w_2}{\partial \zeta}, \tag{22a}$$

$$a\frac{\partial u_3}{\partial \xi} - \frac{1}{\rho_0}\frac{\partial p_3}{\partial \xi} - \frac{B_0\sin\alpha}{\mu_0\rho_0}\frac{\partial B_{y3}}{\partial \xi} = \frac{\partial u_1}{\partial \tau} + \Upsilon\frac{\partial u_2}{\partial \theta} + \frac{B_{y1}}{\mu_0\rho_0}\frac{\partial B_{y1}}{\partial \xi} + u_1\frac{\partial u_1}{\partial \xi} - \frac{\rho_1}{\rho_0^2}\left(\frac{\partial p_1}{\partial \xi} + \frac{B_0\sin\alpha}{\mu_0}\frac{\partial B_{y1}}{\partial \xi}\right) + \frac{B_0\sin\alpha}{\mu_0\rho_0}\frac{\partial B_{x2}}{\partial \theta}, \tag{22b}$$

$$a\frac{\partial v_3}{\partial \xi} + \frac{B_0\cos\alpha}{\mu_0\rho_0}\frac{\partial B_{y3}}{\partial \xi} = \frac{\partial v_1}{\partial \tau} + \Upsilon\frac{\partial v_2}{\partial \theta} + u_1\frac{\partial v_1}{\partial \xi} + \frac{B_0\cos\alpha}{\mu_0\rho_0^2}\rho_1\frac{\partial B_{y1}}{\partial \xi} + \frac{1}{\rho_0}\frac{\partial p_2}{\partial \theta} + \frac{B_0\cos\alpha}{\mu_0\rho_0}\frac{\partial B_{x2}}{\partial \theta}, \tag{22c}$$

$$a\frac{\partial B_{y3}}{\partial \xi} + B_0\cos\alpha\frac{\partial v_3}{\partial \xi} - B_0\sin\alpha\frac{\partial u_3}{\partial \xi} = \frac{\partial B_{y1}}{\partial \tau} + \Upsilon\frac{\partial B_{y2}}{\partial \theta} + \frac{\partial(u_1 B_{y1})}{\partial \xi} + B_0\sin\alpha\frac{\partial w_2}{\partial \zeta} - \ell V_A\cos\alpha\frac{\partial^2 B_{z2}}{\partial \xi^2}, \tag{22d}$$

$$p_3 = c_s^2\rho_3 + \frac{c_s^2(\gamma - 1)}{2\rho_0}\rho_1^2. \tag{22e}$$

The homogeneous counterpart of this system obtained by taking the right-hand sides of all equations equal to zero has a non-trivial solution $\rho_3 = \rho_1$, $p_3 = p_1$, $u_3 = u_1$, $v_3 = v_1$, and $B_{y3} = B_{y1}$. This implies that the system of equations (22) is compatible only if the right-had sides of the equations in this system satisfy the compatibility condition. To obtain this condition we eliminate all the terms of the third order approximation. As a result, using equations (18), (20) and (21) we obtain

$$\frac{\partial}{\partial \xi}\left(\frac{\partial B_{y1}}{\partial \tau} - \Upsilon\frac{\partial B_{y1}}{\partial \theta} + N B_{y1}\frac{\partial B_{y1}}{\partial \xi} - \beta\frac{\partial^3 B_{y1}}{\partial \xi^3}\right) + D_y\frac{\partial^2 B_{y1}}{\partial \theta^2} + D_z\frac{\partial^2 B_{y1}}{\partial \zeta^2} = 0, \tag{23}$$

where

$$N = \frac{a V_A^2 \sin\alpha}{2B_0[2a^2 - (c_s^2 + V_A^2)]}\left(3 + (\gamma + 1)\frac{a^2 - V_A^2}{a^2 - c_s^2}\right), \tag{24a}$$

$$D_y = \frac{a^2(c_s^2 + V_A^2) - c_s^2 V_A^2 + \Upsilon^2(c_s^2 + V_A^2 - 6a^2)}{2a[2a^2 - (c_s^2 + V_A^2)]}, \tag{24b}$$

$$D_z = \frac{a^3}{2[2a^2 - (c_s^2 + V_A^2)]}, \tag{24c}$$

and the quantities β and Υ are defined by equations (11) and (20). Introducing $b = \epsilon\, B_{y1}$ and returning to the original

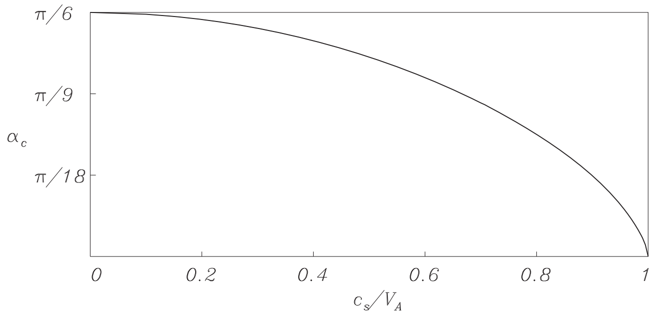

Figure 1. Dependence of α_c on c_s/V_A for $c_s \leqslant V_A$. The same curve shows the dependence of α_c on V_A/c_s when $c_s \geqslant V_A$.

independent variables we transform equation (23) to

$$\frac{\partial}{\partial x}\left(\frac{\partial b}{\partial t} + a\frac{\partial b}{\partial x} - \Upsilon\frac{\partial b}{\partial y} + Nb\frac{\partial b}{\partial x} - \beta\frac{\partial^3 b}{\partial x^3}\right) + D_y\frac{\partial^2 b}{\partial y^2} + D_z\frac{\partial^2 b}{\partial z^2} = 0. \quad (25)$$

Using the inequalities valid for $\alpha \neq 0$,

$$a_+ > \max(c_s, V_A), \quad a_- < \min(c_s, V_A), \quad (26)$$

we obtain

$$\Upsilon_+ > 0, \quad N_+ > 0, \quad \beta_+ > 0, \quad D_{z+} > 0, \quad (27a)$$

$$\Upsilon_- < 0, \quad N_- < 0, \quad \beta_- < 0, \quad D_{z-} < 0. \quad (27b)$$

It is proved in appendix that $D_{y+} > 0$. It is also proved that $D_y < 0$ for $\alpha < \alpha_c$ and $D_y > 0$ for $\alpha > \alpha_c$, where α_c is defined by the equation $D_y(\alpha_c) = 0$. The angle α_c is a function of the dimensionless parameter c_s/V_A. When c_s/V_A varies from 0 to 1, α_c monotonically decreases from $\pi/6$ to 0. The dependence of α_c on c_s/V_A is shown in figure 1 for $c_s \leqslant V_A$. Using equations (10), (20), and (24*b*) we can see that D_y is invariant with respect to the transposition of c_s and V_A. Hence, for $c_s \geqslant V_A$ the dependence of α_c on c_s/V_A is defined by the relation $\alpha_c(c_s/V_A) = \alpha_c(V_A/c_s)$. When V_A/c_s varies from 1 to 0, α_c monotonically increases from 0 to $\pi/6$.

When b is independent of y and z equation (25) reduces to the KdV equation derived for nonlinear waves in Hall plasmas in [32].

5. Soliton stability

In this section we study the stability of magnetosonic solitons decribed by the KdV equation derived in [32] with respect to the transverse perturbations. To do this we introduce the dimensionless variables

$$T = \frac{t}{t_0}, \quad U = \frac{\chi t_0 Nb}{6}, \quad X = \chi(x - at),$$
$$Y = (y + \Upsilon t)\sqrt{\frac{3\chi\sigma^2}{t_0 D_y}}, \quad Z = z\sqrt{\frac{3\chi\nu^2}{t_0 D_z}}, \quad (28)$$

where t_0 is an arbitrary constant with the dimension of time, and

$$\chi = -\frac{1}{\sqrt[3]{t_0\beta}}, \quad \sigma^2 = \mathrm{sgn}(\chi D_y), \quad \nu^2 = \mathrm{sgn}(\chi D_z). \quad (29)$$

Using these dimensionless variables we reduce equation (25) to

$$\frac{\partial}{\partial X}\left(\frac{\partial U}{\partial T} + 6U\frac{\partial U}{\partial X} + \frac{\partial^3 U}{\partial X^3}\right) + 3\sigma^2\frac{\partial^2 U}{\partial Y^2} + 3\nu^2\frac{\partial^2 U}{\partial Z^2} = 0. \quad (30)$$

First we consider the stability of solitons with respect to perturbations that are independent of Z, so that the last term on the left-hand side of equation (30) disappears and this equation reduces to the standard KP equation. Since $\beta_+ > 0$ and $D_{y+} > 0$, it follows that $\chi_+ < 0$ and $\sigma_+^2 = -1$. Then equation (30) becomes the KPI equation and, in accordance with the results obtained by [18], it follows that the fast magnetosonic solitons are unstable with respect to the transverse perturbations. When $\alpha < \alpha_c$ we have $D_{y-} < 0$. Since $\beta_- < 0$ it follows that $\chi_- < 0$ and $\sigma_-^2 = -1$, equation (30) reduces to the KPI equation and the slow magnetosonic solitons are also unstable with respect to the transverse perturbations. Finally, when $\alpha > \alpha_c$ we have $D_{y-} > 0$, $\chi_- > 0$ and $\sigma_-^2 = 1$, equation (30) reduces to the KPII equation and, in this case, the slow magnetosonic solitons are stable with respect to perturbations propagating in the plane defined by the directions of magnetic field and soliton propagation.

The magnetosonic solitons that we studied in this article propagate at sufficiently large angles with respect to the extremal directions that are defined by $\alpha = 0$ and $\alpha = \pi/2$. Still both fast and slow solitons are unstable with respect to the transverse perturbations propagating in the plane defined by the directions of magnetic field and soliton propagation. Hence, the results obtained in this article do not agree with the statement made in [19] about the stability of solitons propagating in anisotropic media.

Now we consider the stability of slow magnetosonic solitons with respect to perturbations independent of Y. As a result, the last but one term on the left-hand side of equation (30) disappears and this equation again reduces to the standard KP equation. Since $D_{z-} < 0$ and $\beta_- < 0$, it follows that $\chi_- > 0$, $\nu^2 = -1$, and equation (30) reduces to the KPI equation. It is straightforward to see that the fast magnetosonic waves are also unstable with respect to perturbations independent of Y. Hence, we conclude that slow magnetosonic solitons are unstable with respect to perturbations independent of Y for any value of α. Summarising, we see that both fast and slow magnetosonic solitons in Hall plasmas are unstable with respect to transverse perturbations.

6. Discussion

We now confront our results on the soliton stability with those obtained in [19]. As we have already mentioned, these authors showed that solitons propagating in an anisotropic medium are stable with respect to transverse perturbations unless they propagate at a small angle with respect to one of the extremal directions defined by the condition that the derivative of the phase speed with respect to the propagation angle is zero at these directions.

In [19] a two-dimensional problem was considered. Hence, we can compare their results with our study of the soliton stability with respect to perturbations that either independent of y or independent of z. In the first case we consider the stability of solitons propagating in the x-direction with respect to perturbations that depend on x and z. Since the equilibrium magnetic field is in the xy-plane, the x-direction is an extremal direction because the angle between the equilibrium magnetic field and the propagation direction takes its minimum when a soliton propagates in the x-direction. This implies that the phase speed of a fast magnetosonic wave takes its minimum in this direction, and that of a slow magnetosonic wave takes its maximum. Hence, the result that both fast as well as slow magnetosonic solitons are unstable with respect to transverse perturbations independent of y does not contradict to the result obtained in [19].

Now we proceed to the case where the perturbations are independent of z. The solitons again propagate in the x-direction. The extremal directions are defined by $\alpha = 0$ and $\alpha = \pi/2$, where α is the angle between the x-axis and the equilibrium magnetic field. In our study we assumed that α is not close to either 0 or $\pi/2$ meaning that the propagation direction of solitons is not close to the extremal directions. However, we found that fast magnetosonic solitons are always unstables with respect to perturbations independent of z, and slow magnetosonic solitons are unstable if they propagate at the angle with respect to the equilibrium magnetic field larger than α_c. These results contradict to those obtained in [19].

One possibility to reconcile the results obtained in this article and those obtained in Paper I is the following. In this article we studied the soliton stability with respect to normal modes that are harmonic perturbations with the infinite extension in the y-direction. It seems that the equations describing the soliton dynamics were derived in [19] using an implicit assumption that perturbations are bounded in the y-direction. In this case we must distinct between the absolute and convective instability [33–35]. When the instability is absolute perturbations grow exponentially at any fixed spatial position. On the other hand, when the instability is convective the perturbation amplitude grows exponentially, but at the same time perturbations spread out of any finite spatial region so fast that they decay at any fixed spatial position. If the instability is absolute or convective depends on the reference frame. It can be absolute in one reference frame and convective in another. The reference frame used in Paper I is well defined by the condition that far from the soliton the medium is at rest. Now we make the following conjecture. We assume that it is not proved in Paper I that the solitons propagating in an anisotropic medium are stable with respect to normal modes. Rather it is proved that if the solitons are unstable with respect to normal modes, then the instability is always convective. The physical explanation given in [19] 'Anisotropy is thus seen to enhance the stability of solitons because the difference between the phase and group velocity results in the deformation of perturbations which spread out over the front and do not succeed in accumulating at a particular point' seems to support the conjecture that we made. If our conjecture is correct then there is no contradiction between the results obtained in this article and those obtained in [19].

Equation (23) differs from the standard KP equation by the presence of terms proportional to Υ. As we have seen, this term can be easily removed by the variable transformation corresponding to choosing a new reference frame moving in the y-direction with the velocity equal to the y-component of the group velocity. This change of reference frame does not affect the stability investigation with respect to normal modes. But it is very important for studying the absolute and convective instability. As we have already pointed out, the distinction between the absolute and convective instability is frame-dependent. Hence, it can be convective in the original reference frame but absolute in the reference frame moving in the y-direction.

Usually studying the absolute and convective instabilities is split in two steps. The first step is studying the stability with respect to normal modes and derivation of the dispersion equation. The second step is finding if the instability is absolute or convective in a particular reference frame. This paper can be considered as a half of the first step. We only studied the stability with respect to normal modes in the long wavelength approximation. It is obtained in [18] that the instability increment is proportional to the wave number. This implies that the initial values problem is ill-posed because the increment is not bounded. It tends to infinity when the wave number increases. To obtain the increment bounded for all wave numbers one needs to carry out the analysis of stability with respect to normal modes without using the long wavelength approximation. This will be the second half of the first step. After that we can study the absolute and convective instability using Brigg's method [33]. This is the plan for future study.

7. Summary and conclusions

In this article we studied the stability of magnetosonic solitons in a Hall plasma with respect to transverse perturbations. Using the reductive perturbation method we derived the KP equation describing three-dimensional nonlinear magnetosonic waves. The magnetosonic waves can be considered as typical examples of waves propagating in an anisotropic medium. Hence, we believe that the main results obtained for these waves are quite general and remain the same for other waves propagating in anisotropic media. These results are the following:

(i) When solitons propagate in an isotropic medium they are stable with respect to two-dimensional transverse perturbations when the wave dispersion is negative and unstable when it is positive (e.g. [36]). However, this statement does not hold when solitons propagate in an anisotropic medium.

This is clearly seen in the case of slow magnetosonic solitons. The dispersion of slow magnetosonic waves is negative, however the slow magnetosonic solitons are still unstable with respect to transverse perturbations.

(ii) The equation that describes multi-dimensional propagation of nonlinear waves differs from the standard KP equation by a term proportional to the component of the group velocity that is transverse to the soliton propagation direction. This term can be removed by the variable transformation corresponding to changing to the reference frame moving in the direction transverse to the soliton propagation direction. This change of the reference frame does not affect the stability investigation with respect to normal modes. However, it is important for studying the absolute and convective instability because the distinction between these two kinds of instabilities is frame-dependent.

(iii) Using the derived KP equation we showed that both the fast and slow magnetosonic solitons are unstable with respect to normal modes propagating in the plane defined by the soliton propagation direction and the equilibrium magnetic field. This result is in a seeming contradiction with the statement made in [19] that solitons propagating in an anisotropic medium are stable with respect to the transverse perturbations unless they propagate at small angles with respect to the extremal directions. To reconcile our results with those obtained in [19] we make a conjecture that in [19] the soliton stability was studied with respect to perturbations having the finite extension in the transverse direction. If this is the case then we must distinct between the absolute and convective instability. We speculate that the exact formulation of the result obtained in [19] on the stability of a soliton propagating in an anisotropic medium is the following: If a soliton propagating in an anisotropic medium is unstable with respect to transverse perturbations then the instability is always convective. To confirm or disprove this conjecture an additional study much more involved than that carried out in this article is needed.

Appendix A. Appendix. Evaluation of D_y

In this section we investigate the sign of D_y. Since the expression for D_y as well as quation (10) determining a are symmetric with respect to c_s and V_A it is enough to consider the case where $c_s \leqslant V_A$. Using equation (10) and the identify following from this equation,

$$c_s^2 V_A^2 \sin^2\alpha = (a^2 - c_s^2)(a^2 - V_A^2), \tag{A.1}$$

after long but straightforward calculation we transform equation (24*b*) to

$$D_y = \frac{a^2 F(a)}{2[2a^2 - (c_s^2 + V_A^2)]^3}, \tag{A.2}$$

where $F(a)$ is given by

$$F(a) = V_A^2(V_A^2 - c_s^2)^2 + (a^2 - V_A^2)[4(V_A^2 - c_s^2)^2 + 3(a^2 - V_A^2)(2a^2 + V_A^2 - 3c_s^2)]. \tag{A.3}$$

It follows from the first inequality in equation (26) that $F(a_+) > 0$. Since the denominator on the right-hand side of equation (24*b*) is positive for $a = a_+$, we conclude that $D_{y+} > 0$.

Using equation (A.3) we obtain

$$\frac{dF(a_-)}{da_-} = 4a_-[2(c_s^2 - V_A^2)^2 + 9(a_-^2 - c_s^2)(a_-^2 - V_A^2)] > 0. \tag{A.4}$$

Since a_- is a decreasing function of α, it follows that

$$\frac{dF(a_-)}{d\alpha} = \frac{dF(a_-)}{da_-}\frac{da_-}{d\alpha} < 0. \tag{A.5}$$

We have $a_- = c_s$ when $\alpha = 0$ and $a_- = 0$ when $\alpha = \pi/2$. We also have

$$F(c_s) = c_s^2(V_A^2 - c_s^2)^2 > 0,$$
$$F(0) = -c_s^2 V_A^2(c_s^2 - V_A^2) < 0. \tag{A.6}$$

It follows from equations (A.4) and (A.6) that there is $\alpha_c \in (0, \pi/2)$ such that $F(a_-) > 0$ for $\alpha < \alpha_c$ and $F(a_-) < 0$ for $\alpha > \alpha_c$. Since, in accordance with equation (26), $2a_-^2 - (c_s^2 + V_A^2) < 0$, we conclude that $D_{y-} < 0$ for $\alpha < \alpha_c$ and $D_{y-} > 0$ for $\alpha > \alpha_c$.

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