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Closure

Closure to "Wavelet spectral analysis of the free surface of turbulent flows", by Giulio Dolcetti and Héctor García Nava

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The Discusser proposes a new equation to calculate the wavelength of the stationary waves oriented parallel to the flow, in alternative to Eq. (16) in Dolcetti and García Nava (2018) (hereafter DGN18). The alternative equation (Eq. (4) in the Discussion) is obtained by assuming a time-averaged streamwise velocity profile with the form (the notation and system of reference are the same of DGN18)

$$U(z) = (1 - n)\, U_0 + \frac{nU_0}{d} z, \tag{1}$$

where the shear rate, nU_0/d, equals the gradient of the power function profile (Eq. (13) of DGN18) at the surface. The height, z, is defined between $z = 0$ (bottom) and $z = d$ (mean surface level). Eq. (1) is a simplified approximation of the velocity distribution in a real rough open channel flow. The power function profile used by DGN18 can approximate this distribution more accurately, and its exponent n carries a physical link with fundamental hydraulic parameters such as the Reynolds number and friction factor (e.g., Cheng (2007)). The two equations still yield almost identical values of the wavelength λ_0 of the stationary waves across the range of conditions discussed by DGN18, so that the results of the analysis presented there are not affected. The small shear approximation of the dispersion relation presented in Eq. (15) of DGN18 still applies with the equation proposed by the Discusser, although in that case G_0 should be calculated as $G_0 = (1 + n\mathsf{F}^2)^{-1/2}$.

Although the formulation used by DGN18 is believed to be more rigorous, the relation suggested by the Discusser is simpler, and has an apparently comparable accuracy. In its non-dimensional form, it shows very clearly the effect of the velocity profile (shear rate) on the wavelength of the stationary waves. This clear link suggests a possibility to infer or estimate the characteristics of the flow below the water surface from the observation of the surface shape, paving the way for the future development of new flow rate monitoring technologies. With this greater goal in mind, it seems worth clarifying the range of validity of the two approximations, and discussing their relative accuracy over a broader range of flow conditions than that investigated originally by the Authors.

The derivation of the linearised dispersion relations for both cases has been reported widely in the literature, e.g., Biesel (1950) for constant shear flow, and Burns (1953) for the power function velocity profile. Here we only report the details that are most important for the discussion. The surface tension is neglected, as it would introduce an additional parameter and it is only important for very short wavelengths. The two-dimensional stationary wave problem is defined in terms of the Rayleigh equation (e.g., Burns (1953)),

$$U(z)\left[\phi''(z) - k_0^2\phi(z)\right] - U''(z)\phi(z) = 0, \tag{2}$$

with boundary conditions

$$\phi(0) = 0, \tag{3a}$$

$$U^2(d)\phi'(d) = \left[U(d)U'(d) + g\right]\phi(d) \tag{3b}$$

where the apex indicates derivation with respect to z, and ϕ is a stream function, which is proportional to the vertical velocity induced by the wave. The shear rate appears explicitly only in the surface boundary condition. The constant-shear solution can be obtained as an approximation of the power function velocity profile by neglecting terms of order $\sim n(n-1)(k_0 d)^{-2}$, so that

$U''(z) = 0$. With this assumption, the wavelength of the stationary waves is found to be

$$\frac{\lambda_0^{(\Gamma)}}{d} = \frac{2\pi\mathsf{F}^2}{(1+n\mathsf{F}^2)\tanh\left(\frac{2\pi d}{\lambda_0}\right)}, \tag{4}$$

which is the same as Eq. (4) derived by the Discusser, with an additional finite-depth correction factor.

For the power function velocity profile, there are two solution of Eq. (2), which behave near $z = 0$ like z^{1-n} and like z^n, respectively. Both Burns (1953) and Fenton (1973) chose the one that behaves like z^{1-n}, since this decays faster than $U(z)$ when $n < 1/2$. This solution yields the formula reported by DGN18 as Eq. (16). One would expect to be able to retrieve Eq. (4) as the limit of that formula for $n \to 1$, but this is not possible. When $n = 1$, in fact, the solution chosen by Burns (1953) and Fenton (1973) does not satisfy the boundary condition Eq. (3a). When $n > 1/2$, it also decays more slowly than $U(z)$ and than the second solution. These aspects where not noticed by either Burns (1953) or Fenton (1973), since they were focused on a specific velocity profile with $n = 1/7$. In order to extend the validity of the theory to $n > 1/2$ and compare with the constant-shear solution, we propose a more general solution with form $\phi(z) = \sqrt{z}\,\mathrm{I}_{(0.5-n)s}(k_0 z)$, where $s = \mathrm{sgn}(0.5 - n)$. This solution behaves like z^{1-n} when $n < 1/2$, and like z^n when $n > 1/2$. For $0 \leq n \leq 1$, the wavenumber and wavelength of the stationary waves can be found from the following non-dimensional but implicit equations:

$$k_0 d = \frac{1}{\mathsf{F}^2}\frac{\mathrm{I}_{(0.5-n)s}(k_0 d)}{\mathrm{I}_{-(0.5+n)s}(k_0 d)} \implies \frac{\lambda_0}{d} = 2\pi\mathsf{F}^2\frac{\mathrm{I}_{-(0.5+n)s}\left(\frac{2\pi d}{\lambda_0}\right)}{\mathrm{I}_{(0.5-n)s}\left(\frac{2\pi d}{\lambda_0}\right)}. \tag{5}$$

Equation (5) coincides with Eq. (16) of DGN18 when $n \leq 1/2$, but also remains valid for $n > 1/2$. It coincides with Eq. (4) when $n = 0$ and $n = 1$.

The difference between the two approximations is evaluated in terms of the wavelength ratio $\lambda_0/\lambda_0^{(\Gamma)}$, where λ_0 was calculated numerically for the power-function profile based on Eq. (5), and $\lambda_0^{(\Gamma)}$ was calculated based on the constant-shear model using Eq. (4) of the discussion (deep-water case, Fig. 1a), and using Eq. (4) (finite-depth case, Fig. 1b). The difference increases with $n(n-1)(k_0 d)^{-1/2}$, as expected. When the waves are short (F is small), the difference is always smaller than 1 %, and $n = 1/2$ has the largest deviation. At $\mathsf{F} > 0.5$, the difference increases rapidly. In Fig. 1a, the largest error is observed for $n = 0$, where the two equations should give identical results. This is an effect of neglecting the shallow water effects. When finite-depth effects are accounted for, the constant-shear solution tends to overestimate the wavelength of the stationary waves by up to 20 % at $\mathsf{F} \approx 1$ when $0 < n < 1/2$. Smaller errors are observed for $n > 1/2$. When the waves are long, the solution with the largest error has $n < 1/2$. This fact can be explained by noticing that the maximum of the curvature occurs for lower n near the bed.

In conclusion, the alternative formulation proposed by the Discusser is a useful contribution, which allows a simpler way to represent the dependence of the characteristic surface scales on the fundamental parameters of the flow. Such a formulation can be used as an effective, accurate alternative to Eq. (16) of DGN18 when the Froude number is smaller than 0.5. For larger Froude numbers, the waves are mainly affected by the flow near the bottom, which is slower and more strongly sheared. In this case, the equation presented originally, and its extension for $n > 1/2$ presented here, are believed to be more accurate.

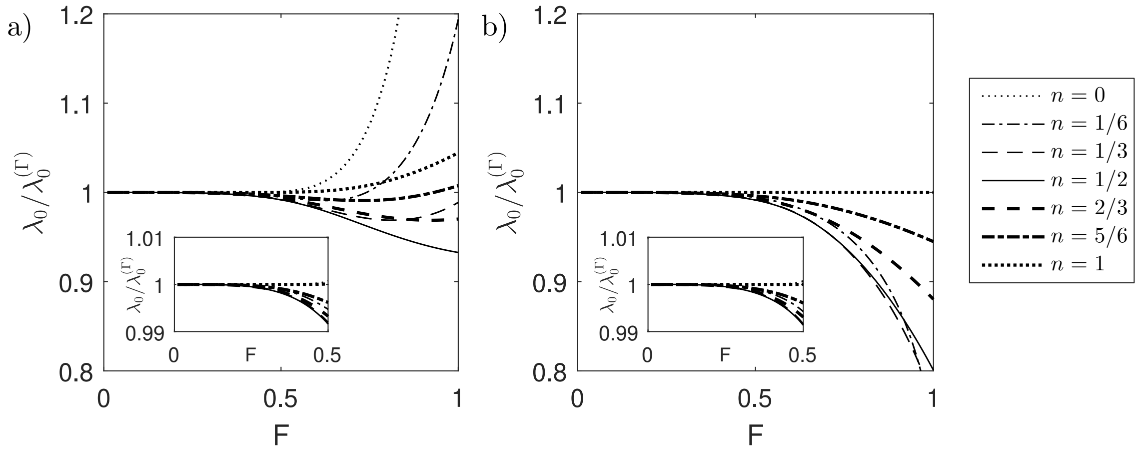

Figure 1 Ratio of the wavelength of stationary waves calculated for different velocity profiles. λ_0 was calculated for a power function velocity profile using Eq. (5), which extends the theory used by DGN18 to $n > 1/2$. $\lambda_0^{(\Gamma)}$ was calculated for a linear velocity profile (a) with Eq. (4) proposed by the Discusser, and (b) with Eq. (4) which includes finite-depth effects.

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