

A survey on underactuated robotic systems: bio-inspiration, trajectory planning and control

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Abstract: Underactuated robotic systems have become an important research topic aiming at significant improvement of the behavioural performance and energy efficiency. Adopting some bio-inspired ideas and properties, the self-organisation and main tasks of the robotic systems can be achieved by coordination of the subsystems and dynamic interaction with the environment. Conversely, biological systems achieve energy efficient and adaptive behaviours through extensive autologous and exogenous compliant interactions. The "trick" that give rise to the lifelike movements is appropriate application of the bio-inspired ideas and properties, and construction of control systems in a generally underactuated system. In this paper, we aim to strengthen the links between two research communities of robotics and control by presenting a systematic survey work in underactuated robotic systems, in which both key challenges and notable successes in bio-inspiration, trajectory planning and control are highlighted and discussed. One particular emphasis of this article lies on the illustration of roles of bio-inspired properties, control algorithms and prior knowledge in achieving these successes and specifically, how they contribute to the taming of the complexity of the linked domains. We demonstrate how bio-inspiration and control methods may be profitably applied, and we also note throughout open questions and the tremendous potential for future research.

1. Introduction

During the past decade, there has been a surge of studies in fields of underactuated robotic systems (URSS) and bio-inspiration, aiming at the significantly improving the behavioural performances and energy efficiency of the robotic systems. Bio-inspiration implies the understanding of principles underlying the behaviours of animals and humans and transfers these principles into the development of robots. Biological systems naturally perform dynamic behaviours in complex environment with fantastic energy efficacy, adaptability and robustness. Active and dynamic compliances are created and enhanced from musculoskeletal system (joint-space) to external environment (task-space) amongst the underactuated motions. Human body has incredible number of muscles as actuators and has multiple muscles to actuate one point, nevertheless, the control system becomes underactuated when jumping through the air that, no combination of muscle inputs exists to change the ballistic trajectory of the centre of mass. However, control of URSS is still intractable, in that their self-organisation and overall tasks must be achieved by coordinating the subsystems and dynamically interacting with the environment. Towards the discrepancy of behaviour/motor control in biological and robotic systems, URSS have attracted significant attentions for manoeuvrable, efficient, and adaptive behaviours in the real world. One important question to raise is: How can we design control systems to achieve efficient locomotion, while adapt to dynamic conditions as the living systems do?

URSS are characterized with fewer independent control inputs than configuration variables. The terminology

underactuation is referred to as the system which has a difference between the number of degrees of freedom (DOF) and the number of control actions [1]. Studying underactuation in the context of locomotion, as reported in the seminal work [2], is likely to lead to an improved understanding of locomotion in biological systems. Basically, underactuation describes the property of a system to have an input vector with smaller dimension than the configuration space of the system. The dimension of the configuration space is the number of DOF. These systems are extensively utilized in the real-world, such as mobile robots, helicopters, underwater vehicles, legged robots [3], self-propelled robots [4,5], aircrafts, spacecrafts and underactuated manipulators. Underactuation originates from: (1) natural dynamics of the system, such as spacecraft, aircraft, helicopters, underwater vehicles; (2) designing for reduction of the cost or some practical purposes, such as flexible-link robots and satellite systems with two thrusters; (3) being imposed artificially to create complex low-order nonlinear systems to gain insight into the control of high-order underactuated systems, e.g., the Pendubot [6], the Acrobot [7], the TORA [8]; and (4) the actuator failure.

To achieve a thorough understanding of URSS, it is necessary to scrutinize from the dynamic characterizations in terms of modelling, bio-inspiration, trajectory planning and nonlinear control over the past decade. On the other hand, the complexity is increased by the restricted control authority, resulting into less applicability of classical control approaches, such as feedback linearizability and passivity-based methods. Practical requirements are raised from the advanced applications, for instance, underactuated soft robots

rely on compliance to mitigate uncertainties and adapt to dynamic environment and tasks, novel algorithms to the control of soft robots that account for their material properties need to be explored [9]. These difficulties motivate the studies on modelling, nonlinear control, as well as motion trajectory generation, etc. However, despite these studies, there are a few particularly significant challenges that are related to the control of nonlinear dynamics derived from autologous compliant interaction between the subsystems and exogenous physical interaction with the environment.

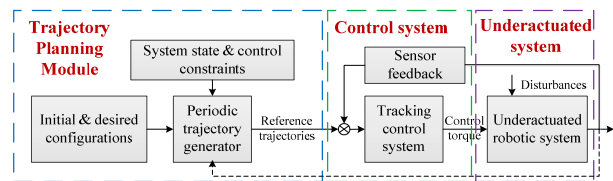


Fig. 1. Block diagram of trajectory planning and control of URSSs.

Although these problems and challenges are nontrivial, there are several potentially promising research directions which, we believe, significantly contribute to the progress in this exciting research domain of URSSs. Recently, there are some survey papers in literature concerning the main subject of underactuated robotics. The topics of classification of URSSs, common mechanisms and open-close loop control methods were discussed in [10], which positioned itself based on the viewpoints of classical mechanisms and control systems. The authors considered nonholonomic constraints to classify URSSs, summarized their common mechanisms, and discussed the control flow of URSSs mainly from a perspective of fuzzy systems. Whilst in this paper, we aim to explore the recent advances in adopting bio-inspired properties, modelling, trajectory planning and control algorithms to strengthen the links between two research communities of underactuated robotics and control. We have put a survey on underactuated mechanical systems, seven years ago as in [11], investigating the topics of modelling, classification and classical control methods. With the advances in computational and powers technology, things are changing rapidly, particularly the introduction and application of bio-inspiration and intelligent control systems. Therefore, in this article, we provide a background for describing URSSs that builds on discussions from the perspective of biological inspiration and intelligent systems, which is the first time in literature form a perspective of survey. The objective of this article is to present a systematic survey work in bio-inspiration and control in underactuated robotics, in which both key challenges and notable successes are highlighted and discussed. A particular focus of this article lies on the roles of bio-inspired properties, control algorithms and prior knowledge in achieving these successes and specifically how they contribute to the taming of the complexity of the linked domains. This article discusses four prevailing directions of research and technological challenges that will potentially lead to significant breakthroughs in dealing with bio-inspired URSSs. The references discussed in this review are selected with rationale for representing the critical information that delineate the state-of-art perspectives and addressing particular research issues and problems in underactuated systems. Fig. 1 shows the relationship of four directions from

the system level. The block region in purple presents the underactuated systems to be controlled, where the issues of modelling (Section 2) and bio-inspired design (subsection 3.1) are discussed. The block region in blue shows the trajectory planning module where the desired trajectory is generated, this module is discussed in Section 4. The block region in green demonstrates the control system for URSSs, the studies on bio-inspired control and nonlinear control system design are investigated in Subsection 3.2 and Section 5, respectively.

The review investigates in detail at recent efforts and successes towards bio-inspiration and control in underactuated robotics. The paper has seven main Sections. In Section 2, we present a focused investigation into the modelling issue. Section 3 presents studies related to bio-inspired properties and bio-inspired control, whereas Section 4, it summaries efforts towards trajectory planning and optimization. Optimized trajectory planning is an important topic for underactuated robotics. Hence, Section 5 reports on the main results of nonlinear control systems by providing an overview of the state of the art. In Section 6, challenges, difficulties and future research directions in bio-inspiration and control in underactuated robotics are discussed from both theoretical and practical perspectives. Finally, in Section 7, we summarize conclusions, and place forward few remarks.

2. Modelling of URSSs

Different analytical solutions for robotic application have been developed through the understanding of the fundamental first principles which precisely portray the robot dynamics. Generally speaking, a set of differential equations are formulated from the basis of mathematical models whose solutions predict the evolution of the configuration variables in time in the presence of a given sequence of external generalized forces which referred to as control input torques. For an object system with n -DOF ($n > 1$), the governing equation [12,13] can be given by

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{ij} \Gamma_{ij}^k(q)\dot{q}_i\dot{q}_j + g_k(q) = p_k^T B(q)u \quad (1)$$

where $q = [q_1, \dots, q_n]^T$ represent the generalized coordinates vectors that belong to an n -dimensional configuration manifold, $u = [u_1, \dots, u_p]^T$ denote the vector of p external forces applied on the systems. $B(q)$ is the input force matrix and assumed to be of full column rank, together with $B(q)u$ describing the generalized forces resulting from the control inputs u . $k = 1, 2, \dots, n$, p_k is the k^{th} standard basis in \mathcal{R}^n , d_{kj} is the inertia matrix element, $g_k(q) = \frac{\partial V(q)}{\partial q_k}$, and $\Gamma_{ij}^k(q)$ are Christoffel symbols [14] and is defined as

$$\Gamma_{ij}^k(q) = \frac{1}{2} \left(\frac{\partial d_{kj}(q)}{\partial q_i} + \frac{\partial d_{ki}(q)}{\partial q_j} - \frac{\partial d_{ij}(q)}{\partial q_k} \right) \quad (2)$$

The vector form of (2) can be obtained as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)u \quad (3)$$

where $D(q)$ is a symmetric and positive-definite matrix of inertias, $c_{ij} = \sum_{k=1}^n \Gamma_{ij}^k(q)\dot{q}_k$ is the element of $C(q, \dot{q})$. Two types of terms are involved in $C(q, \dot{q})\dot{q} \in \mathcal{R}^n$ which are called Centrifugal terms (when $i = j$) and Coriolis terms (when $i \neq j$), $G(q)$ represents the gravitational terms.

A robotic system described by Eq. (3) is referred to as an underactuated system if $m = \text{rank}(B(q)) < n$, which means it has fewer independent control inputs m than the degree of freedom n , and as such $k = n - m$ DOF cannot be directly actuated. Assuming that $B(q) = [0, I_m]^T$, without loss of generality, (3) can be rewritten in a generic form and

further partitioned as $q = [q_p, q_a]^T \in \mathcal{R}^{n-m} \times \mathcal{R}^m$, where q_p and q_a respectively represent the unactuated (passive) and actuated configuration vectors, we have

$$\begin{bmatrix} D_{pp}(q) & D_{pa}(q) \\ D_{ap}(q) & D_{aa}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_p \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} C_p(q, \dot{q}) \\ C_a(q, \dot{q}) \end{bmatrix} \begin{bmatrix} \dot{q}_p \\ \dot{q}_a \end{bmatrix} + \begin{bmatrix} G_p(q) \\ G_a(q) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (4)$$

where the inertia matrix $D(q) = \begin{bmatrix} D_{pp}(q) & D_{pa}(q) \\ D_{ap}(q) & D_{aa}(q) \end{bmatrix}$ is symmetric positive-definite, the matrix $C(q, \dot{q}) = \begin{bmatrix} C_p(q, \dot{q}) \\ C_a(q, \dot{q}) \end{bmatrix} \in \mathcal{R}^{n-m}$ contains the Centrifugal and Coriolis forces, $G(q) = \begin{bmatrix} G_p(q) \\ G_a(q) \end{bmatrix}$ represents the gravitational forces applied on the passive and actuated configurations.

Definition 1. The set of DOF of URSSs can be partitioned into two subsets [15], which referred to as collocated subset with its cardinality contains the actuated DOF and equals the number of control inputs; and non-collocated subset accounts for the remaining non-actuated DOF.

Modelling of URSSs has been extensively investigated in various domains over the past decade, from prevailing benchmarks such as the cart-pole system [16,17] to novel underactuated systems [18–27]. It's also worth mentioning the walking and running of biped robots with point feet [28,29] are one of the important URSSs. The modes of these robotic systems have different degrees of underactuation in each phase of motion. Towards the modelling of URSSs, most of the studies have been conducted based on fundamental Lagrangian mechanical system. However, many practical considerations have been simplified or omitted, for instance, modelling of the interactions with actuators, sensors, dynamic frictions, and (structured or unstructured) uncertainties and external disturbances. Towards this end, researchers have been trying to design efficient control systems that are able to guarantee the adaptability and robustness to the inaccuracies. Nevertheless, any achievement in adaptive and robust control becomes intractable due to the underactuated dynamics. For many URSSs (e.g., self-propelled robots [30], soft robotic hands [31], UAVs [32], underactuated ships [33]), their system performance mainly or partially relies on the non-collocated subsystem. Therefore, it is meaningful to model the URSSs in a systematic way, particularly if the frictions, uncertainties and external disturbances are existing in the non-collocated subsystem of URSSs which are not directly controllable. In this regard, internal dynamics/coupling and interconnection between collocated and non-collocated subsystems play a vital role to account for the mis-matching.

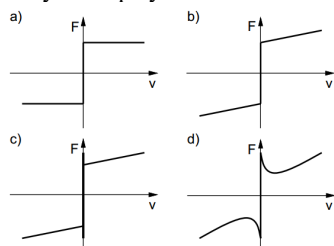


Fig. 2. Friction models [34]: (a) the Coulomb model; (b) the Coulomb viscous damping model; (c) Stiction plus Coulomb and viscous friction; (d) seven-parameter model

Towards engineering systems with high fidelity, accurate modelling and prediction of nonlinear frictional

dynamics has always been a nontrivial and intractable aspect of scientific research. Frictional instabilities are typically required to be eliminated or compensated through efficiently designed controllers. Simplified static friction model using the Rayleigh dissipation function (see Fig. 2(b)) has been employed in very few literatures, in which the friction force was considered proportional to the velocity of the object. Subsequently, accompanied by the requirements of underactuated systems in the industrial applications, substantial efforts have been devoted to the modelling of more realistic frictions for practical control purposes [35–38]. The dynamic friction model proposed in these works normally refers to as the LuGre friction model, which is capable of reproducing some of the experimentally observed friction distinctive behaviour, such as hysteresis, Stribeck effect and Coulomb friction. A discontinuous friction model was applied on the unactuated joint in [39] for a class of 2 DOF underactuated system, which was based on the Coulomb friction model (Fig. 2(a)). Recently, a modified nonlinear friction model based on the LuGre model utilized for the passivity-based control (PBC) of an underactuated system was proposed in [40]. And the PBC law together with the interconnection and damping assignment was successfully demonstrated by an underactuated double pendulum with friction effect. For more realistic application, the considerations of modelling the frictions need to be more practical. As novel underactuated microrobotic models, the capsule robotic systems have attracted significant interest in various applications such as medical assistance [18,41–44], pipeline inspection [5,45–47], maritime search [48], etc. For self-propelled capsule systems, friction plays pivotal roles in capsule propulsion and locomotion, particularly for the vibro-driven underactuated systems, the dynamic coupling between the driving mechanism and the system body are utilized to generate efficient stick-slip motions. Hence, accurate prediction of dynamic interactions in the sticking, presliding as well as pure sliding regimes becomes crucial. For frictions exist in the unactuated subsystems of URSSs, the passive dynamics can be explored to indirectly control the friction-induced stick-slip motions to improve the system performance [26].

The uncertainties and disturbances are other important issues need to be considered in modelling, which contain parameter uncertainty, environmental noises and uncertain perturbations. The inclusion of disturbances and uncertainties in the system dynamics has always been one of the pivotal issues particularly in the control system design. During the past years, the development of control algorithms is accompanied with the deepening understanding and improving of robustness in the presence of various type of uncertainties and disturbances. Among them, most of the researches modelled the system dynamics considering relatively simple parameters with uncertain boundaries [49,50] and utilized robust control approach. More recently, the issues of robust tracking control for an underactuated surface vessel with parameter uncertainties was addressed in [51]. An adaptive neural network tracking control was proposed for underactuated systems with matched and mismatched disturbances [52]. For nonholonomic mobile manipulators with an underactuated joint, adaptive motion/force control by dynamic coupling and output feedback is considered by [53], in the presence of parametric and functional uncertainties. An integral sliding-mode

controller was proposed in [54] on a two-wheeled mobile robot with the friction modelled as the combination of viscous friction and Coulomb friction. Most of the studies were conducted from the viewpoint of control, i.e. developing robust controllers for underactuated systems with uncertainties, however, relatively a few considerable works took this issue to the modelling stage. Therefore, the issue of modelling of underactuated robots is still challenging in accurate representation of the interactions with actuators, sensors, dynamic frictions, and (structured or unstructured) uncertainties and external disturbances.

3. Bio-inspired properties and bio-inspired control

Previous studies on URSs or underactuated systems in general has demonstrated their fascinating characteristics in energy efficiency, manoeuvrability and robustness through explicitly exploring the passive dynamics. However, there are challenging issues of control and coordination of the nonlinear dynamics derived from the internally (between the subsystems) and externally (with the environment) physical interactions. As such, precise control of the URSs becomes difficult because of these nonlinear dynamics-induced interactions, which leads to very limited variations of behaviour patterns of the system [55]. In this section, we exploit how bio-inspiration could help tackling these challenges by discussing and summarizing some plausible principles of bio-inspiration from the perspectives of design and motor control.

3.1. URSs with Viscoelastic and Soft Property

Nature has always been a source of inspirations and ideas for researchers and practitioners from robotics and control communities. The terminology of bio-inspiration implies the understanding of fundamental principles that underlie the motions/behaviours of animals and humans and transfers these principles into development of robotic systems. For example, the muscles, during walking, constantly change their stiffness and damping when the leg is swinging forward and the foot is put on the ground [56]. This idea enables design of robotic systems with compliant elements—viscoelasticity to mimic the compliant motion of biological muscles.

During the past few decades, the effective utilizations of compliant elements into the robotic locomotion have attracted significant interests. The motivations are diverse, for instance, to build up safer interactions with humans [57–59], to improve the model accuracy of the robotic systems [60,61], to achieve higher level of manoeuvrability [62], high bandwidth mechanical compliance, flexibility, agility, controllability [63], adaptability, and efficacy in fulfilling large scope of tasks in unstructured and hazardous environment. Multi-fingered grasping robotic hands are underactuated systems that are also typically of crucial needs in robotics, especially for industrial applications. Adopting underactuation as their transmission architectures of the robotic hands becomes a dominating principle for designing during the last decade [64]. Underactuated transmission design allows passive/adaptive movements between the DOFs, which are often used to allow the adaptation of the hand shape to the grasped object. The literature has witnessed a distinct growth after the year 2000 in the use of compliant/soft actuation systems and simplified architectures

for multi-fingered robotic hands which are essentially underactuated [65–67]. These systems use fewer motors which save space, weight, and cost. The DLR hand arm system as shown in Fig. 3 has Series Elastic Actuators (SEAs) that employ compliant and complaint elements (e.g., springs) at the joints. Variable stiffness enhances the robustness of the robotic hand and provides a low-pass filtering of impacts and allows stiffness adjustments depending on the task. More importantly, introducing of compliant/soft elements serve as one of the essential factors to improve the energy efficiency of the overall robotic system.

Extensive endeavours have been devoted to these research domains. The online estimation problem of transmission stiffness in robots driven by variable stiffness actuators in antagonistic or serial configuration was studied in [68] without the need for joint torque sensing. A viscoelastic models were proposed in [69] for a soft robotic mechanism horizontally actuated by two dielectric elastomer actuators. To maximize the energy dissipated in transparent laminates under low velocity impact, a genetic algorithm was employed in [70] to optimize a model built as thermo-elasto-visco-plastic materials. In the presence of hysteresis and friction, the impact on stiffness and damping characteristics of elastic robot joints were discussed in [71]. To design an optimal motion trajectory of flexible mobile manipulators, Pontryagin’s minimum principle was adopted in [72] and the optimal control issue was converted into a two point boundary value problem. However, for mobile systems, the challenge is how to utilize the system dynamics in the forms of optimally synthesized trajectory and effectively designed controller, particularly in the presence of viscoelasticity. Structural simple systems may perform rich system dynamics, and even a tiny variation in parameters may lead to dramatic qualitative changes in the system outputs.

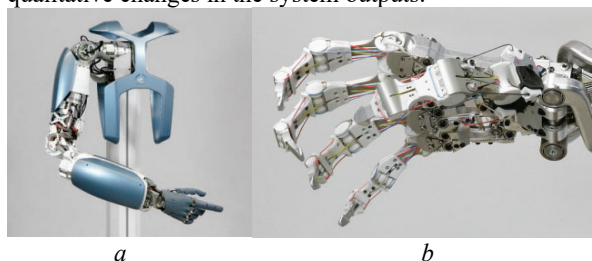


Fig. 3. The DLR hand arm system [73]

Recently, along with the engineering application requirements and the rising research interest in nonlinear dynamics, the vibro-impact characteristic of active mechanisms have been widely applied to a large range of practical mechanical systems. During these applications, correlative relationships between the model parameters and dynamic performance can be achieved. Driven by external harmonic excitations, these implementations are capable of motions such as rectilinear [74,75], unidirectional [76] and bidirectional [77] by utilizing a periodically driven mass/inertia interacting with the main body. A newly developed three masses model was analysed and compared with a low dimensional model in [75]. More interestingly, the authors considered three main control parameters which were referred to as the applied static force, the amplitude and the frequency of the applied dynamics force, which were optimally chosen through the higher dimensional model

simulations. As a practical application in robotics domain, the trajectory planning of a capsule robot was studied in [21], which consists of a capsule main body interacting with an internal pendulum driven by a harmonic excitation. Notably, the dynamic models developed by these works have been proved to be useful for uncovering the interactive dynamic performance of such systems in real-world applications. Moreover, the related studies have contributed abundant information of the fundamental characteristics to the non-smooth motions of practical mechanical systems especially with impacts. It is noted that most of these researches are, in nature, based on linear motions with the consideration of viscoelastic characteristic. However, for the systems that are intrinsically nonlinear, limited studies have been considered modelling, analysis and optimal parameter selection for active rotational motions with viscoelastic properties.

3.2. Bio-Inspired Behaviour/Motor Control

Biological systems naturally exhibit energy efficient, robust and adaptive behaviours in complex environment, whilst the existing robotic systems are still suffering from insufficient capabilities of sensory-motor and learning. To bridge the gap between biological and robotic systems in behaviour control, there has been a surge of research interests in URSs that operate in the real world.

Due to the nature of underactuation, the behaviours of URSs are constrained by their passive dynamics, which characterize the motion control in biological systems [55]. The passive dynamics bring three advantages: (1) most of the behaviours of underactuated robots are regulated by passive dynamics due to less number of motors, e.g. Passive Dynamic Walkers [78,79]; (2) the locomotion velocity is plausible to be improved through exploiting the passive dynamics, and the limitation on maximum speed of each actuator can be sufficiently relaxed; (3) underactuated systems have simpler mechanical structures and therefore control architectures on account of less number of motors and sensors. Therefore, passive dynamics play a vital role for URSs in achieving controlled behaviours and self-adaptability.

3.2.1 Active Impedance Modulation/Control for Compliance Interactions: It is well-established that appropriate utilization of impedance modulation/control is able to improve the interaction ability of robots through modulation of high mechanical impedance. Over the years, it has attracted significant research interests in the domains where the robots are required to work in close vicinity or interact with the unknown and dynamic environments or humans.

Active impedance modulation/control means control the actuator through software to mimic the impedance behaviour. The software controller calculates the correction based on the measured output state, the correction is then set through the (stiff) actuator. As a merit, this approach controls impedance by adapting online both the stiffness and damping in a theoretical infinite range with infinite speed [80]. It is plausible to adopt active impedance idea for control the compliance interactions of URSs, particularly URSs with flexible elements at joints (e.g., SEAs). However, an important problem, that related to controllability and stability, is how to integrate active impedance control with passive design-based actuators in the URSs. A carefully designed control architecture is needed to exploit the joint flexibility when using impedance modulation/control for underactuated

systems with flexible joints. Some bioinspiration-based control schemes such as a feedforward action would work well than using the standard feedback control schemes which make the system stiffer [81]. Under this circumstance, a novel human-like learning controller to interact with unknown environments was proposed in [82], which can deal with unstable situations that are typical of tool use and gradually acquire a desired stability margin. An adaptive impedance control scheme was presented in [83] that adapts the robotic assistance according to the disability level and voluntary participation of human subjects. Interestingly as shown in Fig. 4, an impedance model with virtual force was considered in [84] to design the model reference control of robot dynamics, which provides a kind of cushion effect (compliance) for better user experience. It is noted that the determination of the architecture of active impedance control is dramatically related to specific application and required performance of impedance regulation, including stability bandwidth, desired impedances, passivity, working frequency, and other mechanical and electrical features of the robotic systems.

3.2.2 Appropriate Mechanical Feedback for Self-Stabilization: Mechanical feedback is an important and useful notion that proposed and studied by many researchers from various fields of biological research. Its main idea is that, in biology, many mechanical processes effectively act to assist in the self-stabilization of tasks, and therefore, serve functionally as a first level of feedback control [85]. Using neural feedback has been proved insufficient to control many tasks of biological systems, and therefore more appropriate perspectives in feedback control in neuro-mechanical systems are needed when designing bio-inspired robot and control system architectures.

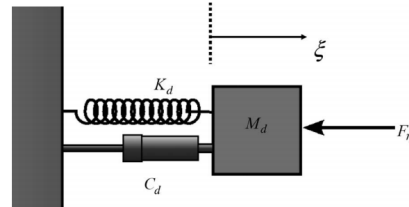


Fig. 4. Virtual mass-spring-damper impedance model [84]

It is also plausible that the motions of underactuated robot are able to be mechanically regulated through appropriate design inspired from the biological systems. Mechanical feedback for self-stabilization in periodic motions has been proved applicable to different kinds of underactuated robot models. The study in [86] based on the Passive Dynamic Walker is a good example as shown in Fig. 5, which can walk on level ground and induce behaviour patterns with small active power sources substituted for gravity. More interestingly, the undesired motion deviations due to the robot-environment interactions can be mechanically regulated. Mechanical feedback is an important and useful notion that proposed and studied by many researchers from various fields of biological research.

3.2.3 Optimized Morphological Design for Behavioural Variability: Morphological computation can be loosely defined as the exploitation of the shape, material properties, and physical dynamics of a physical system to improve the efficiency of a computation [87]. Morphological control is the application of morphological computing to a control task. The nonlinear dynamics of underactuated robots that derived from

their morphological constraints have attracted many research interests in to the modest control system design. Morphology plays a vital role in underactuated systems with respect to the behavioural variability, since many of them merely capable of limited periodic behavioural patterns [55].

The study in [56] demonstrates reduction of the energy cost of human walking through designing and utilization of an unpowered exoskeleton. A lightweight elastic device was designed as shown in Fig. 6, it acts in parallel with the user's calf muscles, off-loading muscle force and thereby reducing the metabolic energy consumed in contractions. Interestingly, there is no mechanical work is done by the actuators, and the springs store and return energy through the fact that the kinetic and potential energy of the body remain constant on average. A powered prosthetic ankle joint was designed in [88] for walking and running as shown in Fig. 7. The active spring design improve the motion/behavioural variability in certain range and relax the limitations in positive work output of passive walking and running feet. These studies demonstrate how various kinds of motion/behaviour can be created through the nonlinear dynamics that are significant in motion adaptability as well. It is noted that not only behavioural variability is achieved through appropriate computational procedure of the motor control, but also it is dramatically determined by the interaction dynamics with simple motor action and the reaction force from the environment.

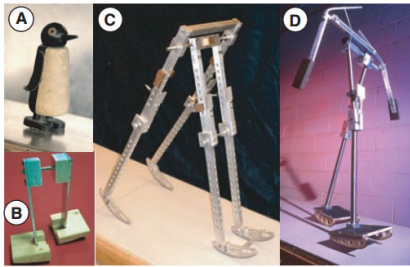


Fig. 5. Bipedal robots based on passive-dynamic walkers [86]

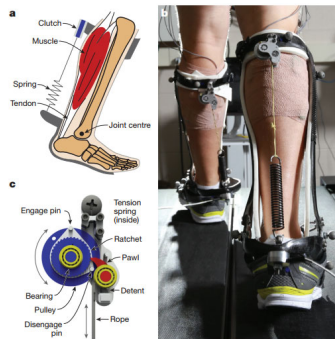


Fig. 6. Unpowered exoskeleton [56]

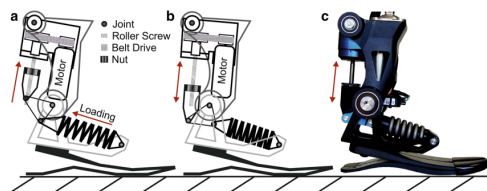


Fig. 7. Springactive Walk-Run ankle [88]

One of the interesting challenges is how to generate desired and substantially different motor/behaviour patterns

through appropriate design and control of the morphological parameters, e.g., coefficient of elasticity and viscosity. As such, new optimal motion control schemes are to be constructed with energy efficiency and adaptability.

3.2.4 Optimal Dynamics Control for Motor Control Learning: URSs have less number of motors, simpler mechanical structures and control architectures, as such, a large part of their behaviours are regulated by passive dynamics. The appropriate design of mechanical feedback for self-stabilization has been proved to be of great significance in the research of underactuated robotics, whilst the challenge in kinematic trajectory control is still an intractable issue due to the unactuated/passive dynamics. As the recent advances in computational intelligence, it is plausible to adapt computational learning/optimization techniques into the motor control of URSs to account for the discrepancy of behaviour control in animals and robots [55]. There has been a rising interest in utilization of computational optimization, which is able to tackle with the automatic reasoning of nonlinear dynamics through evaluation of single scalar value. A reinforcement learning algorithm was presented to acquire in-hand manipulation skills of an underactuated robotic hand [89]. A novel approach to reinforcement learning is proposed in [90] for parameterized control policies based on the framework of stochastic optimal control with path integrals. A method that learns to generalize parametrized motor plans by adapting a small set of global parameters is studied in [91], called meta-parameters. The arm reaching dynamics was thoroughly explored in [92] to achieve reductions of metabolic cost during motor learning. The studies in [93] presented a method to learn discrete robot motions from a set of demonstrations, global asymptotic stability at the target was guaranteed through defining of sufficient conditions.

The cutting-edge researches on motor control learning including control and trajectory planning have demonstrated significant preliminary steps in bio-inspired control of URSs, whilst there are several challenging issues need to be uncovered. The reduction of the number of trial-and error iterations is the nontrivial and intractable one. Towards this end, it is plausible to explore the design of more generalized state representations, and improvement in autonomy of mechanical model generation of the robot itself [94].

3.3. Undulatory Locomotion and Bio-Inspired Self-Propulsion

3.3.1 Undulatory Locomotion and Serpentine Robotic Systems: Movement is one of the vital existential requirements of microbial and animal life on the earth. Many terrestrial animals adopt limbs to support their weight and to cope with the gravitational forces. Some smaller animals have employed a great number of forms that keep them close to the ground or even underground to minimise the effects of gravity. Whilst flying, subterranean and marine animals have to deal with various kinds of physical environments.

Undulatory locomotion is a primitive and relatively simple mode of locomotion that relies on the generation and propagation of waves along the animal body. It is remarkably widespread across a wide range of biological systems from motile bacteria and worms to snakes. It is evident that the body's interplay with the physical environment is the key to undulatory locomotion. Various forms of undulations are adopted by animals, which can be categorized into direct

(same as the motion direction) or retrograde (opposite to the motion direction), horizontal or vertical, and longitudinal or transverse. Generally speaking, retrograde waves are used to propagate opposite to the motion direction such that the body move in a given direction. Specifically, the environment applies forward forces to the body if the body wave travels backward. For example, some worms and protozoa, when their body is moving forwards or backwards, have their body lined with so-called ‘bristles’ that jut out at right angles to the long axis and act as paddles to generate sufficient drag forces.

It is evident that undulatory locomotion is typically constrained by frictional or drag forces of the physical environment rather than the gravitational forces. Significant endeavours have been made in the development of robotic systems with undulatory locomotion that is inspired from worms or snakes [95–98], e.g., the worm-inspired robot as shown in Fig. 8. These systems typically consist of a chain of rigid segments linked by articulated joints actuated by motors and normally restricted to planar bending motions. They propel themselves by changing their body configurations. The snake robot Anna Konda [99,100] is a typical example that is able to push against external obstacles apart from a flat ground and capable of obstacle-aided locomotion. There are also some robotic systems using alternative actuation systems such as pneumatics [101] and shape-memory alloys [102,103].

The forward propulsion by means of undulatory locomotion requires the actuators are controlled in a manner that the propulsive wave propagates along the robot body, this feature is significantly different from the traditional wheeled, legged or tracked robotic systems whose forward motion is obtained simply by driving the motors on the wheel or leg. Therefore, undulatory locomotion has the potential capabilities of robustness and versatility with suitably designed control systems. Undulatory rectilinear motion can be generally partitioned in two different forms: rectilinear motion using vertical waves as shown in Table 1 and rectilinear motion using expanding/contracting segments as shown in Table 2. See Appendix 1 for table of results.

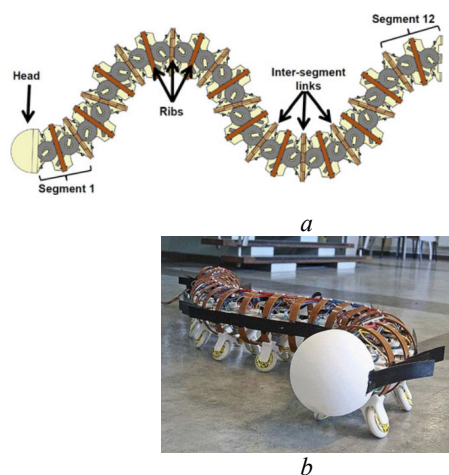


Fig. 8. Worm-inspired robot [95]: (a) 2-D schematic of the robot, (b) Physical robot

Viscoelastic property helps understanding the efficient, compliant and adaptive behaviours of biological systems through bio-inspired design of the URSs, the

problem is how to realize optimal morphological design such that the behavioural variation can be increased while maintaining fascinating characteristics of URSs? Besides, compliant interactions can be obtained through active impedance modulation/control, self-stabilization can be realized by appropriate mechanical feedback using passive dynamics and motor learning is important preliminary steps in bio-inspired control of URSs.

3.4. Bio-inspired/Soft Robotics: Insights into Non-Minimum Phase Systems and Feedforward Control

A system is said to be non-minimum phase if it has zeros in the right-hand side of the complex plane, meaning that trajectories of its zero dynamics are not divergent [104]. These unstable zeros bring difficulties in guaranteeing the robustness of the system. Many real-world URSs fall into this category that their input force matrix/vector $B(q)$ in Eq. (3) is represented by a nonlinear function rather than a simplified dynamic equation. Some example URSs are hypersonic vehicles [105], surface ships [106], VTOL [107], and Inertia Wheel Inverted Pendulum [108], etc. Many bio-inspired/soft robotic systems have demonstrated the non-minimum phase characteristic in that they have strong input couplings and the input force matrix/vector is highly nonlinear, e.g., snake-like robots [95], continuum robots [109], robots with flexible links [110]. It appears that if a robot is designed to be biologically inspired or soft, the strong simplification of the input force matrix/vector does not hold, which makes it not fully feedback linearizable. Being minimum phase is an essential property for a system to have, as it enables formulating the regulation of the output as a control goal. The non-minimum phase nature restricts direct application of recently developed nonlinear control methodologies. Therefore, a better understanding of the minimum phase w.r.t a meaningful output should be regarded as a major challenge in the control of bio-inspired/soft robots.

The method of approximate input–output linearization has been applied to deal with a class of slightly non-minimum phase nonlinear systems. However, its limitation is that only the weak non-minimum phase system can be processed, which is not the case of bio-inspired/soft robotic systems. Besides, from the feedback/feedforward perspective, the control challenge arises because typical feedback-based methods have fundamental performance limits for non-minimum-phase systems. In some applications with strong input couplings, feedback might not be easily implemented. Feedforward scheme can alleviate such control challenges—for non-minimum phase systems. This is apparently true if considering the biological systems (e.g., animals and human beings), they can walk/run efficiently on uneven terrains with the aid of feedforward control, which contributes to the muscle activities to responsible for the adaptations to the ground contacting [111,112]. Therefore, proper combination of feedforward control with passive (mechanical) feedback or active feedback into the control design for URSs is a meaningful research direction to be explored.

There are several plausible approaches to confront this problem by demonstrating ideas (e.g., output selection, using of feedforward schemes) to produce a minimum phase system. For flexible-link robots, three auxiliary signals are used to redefine the outputs to achieve fast regulation of the non-

minimum phase endpoint force [113]. The control performance, such as bandwidth, robustness, and error performance are significantly improved. Towards a class of soft robots, the constant approximation of the curvature is used as control output to produce a minimum phase system for advanced control system design for the soft robots [114]. It is plausible that if a priori information of the disturbance is available, feedforward approaches can overcome the limits of feedback methods for non-minimum phase systems. A feedforward-based control approach was developed in [115] for non-minimum-phase systems when the disturbance is not known a priori, and the performance was demonstrated through a simple flexible robot. A real-time approach of walking pattern generation was proposed in [116], which combined a feedback and a feedforward controller. The feedback controller is used to improve the system stability by employing a pole placement method which shifts the poles of the robotic system. The feedforward controller is designed to account for the non-minimum phase property by adopting advanced pole-zero cancelation by series approximation method.

4. Trajectory planning

Trajectory planning is a terminology that extensively used in robotics and control communities, which generally includes motion planning and trajectory optimization for the process of finding a feasible trajectory to fulfil certain tasks that minimizes or maximizes some measure of performance within prescribed constraint boundaries.

The concentrations on periodic trajectory planning are twofold depending on the dimensions of the input space. For the mechanical systems whose DOF is equivalent to the dimensions of the input space (referred to as fully-actuated systems), the procedure of trajectory planning falls into the task of generating trajectories that integrally reveals the system dynamics and satisfies specific constraints, for instance, bounded input torques, constraints in various motion stages, obstacles avoidance in the work space. In terms of the motion execution, the feedback linearization technique which shed light on the tracking issue of a pre-designed reference trajectory is convenient to be extended to more general cases. On the other hand, when the reduced dimensions of the input space appear, the underactuation is an essential factor needs to be considered, which makes finding a feasible trajectory for a specific task highly nontrivial. Moreover, it becomes more complicated in the presence of nonholonomic dynamic constraints [117].

Towards the issue of trajectory planning for URSSs, extensive efforts have been made in diverse ways. A feedback motion-planning algorithm was proposed by [118] to efficiently evaluate regions of attraction for smooth non-linear systems, which utilized rigorously computed stability regions to build a sparse tree of LQR-stabilized trajectories. Optimized adaptive control and neural network-based trajectory generation was studied in [84] for a class of wheeled inverted pendulum (WIP) models of vehicle systems for dynamic balance and motion tracking of desired trajectories. The proposed control method considers the presence of various uncertainties, including both parametric and functional uncertainties. An optimal offline minimum-time trajectory planning (MTTP) approach for underactuated overhead cranes was proposed in [119], which

simultaneously considers various constraints, including the bounded swing angle for the payload, bounded velocity, acceleration, and even jerk for the trolley. A point-to-point motion planning algorithm was presented in [120] that is based on the natural frequency of the pendulum-like free motion with unconstrained degree of freedom. The virtual holonomic constraint approach was utilized in [121] to generate the feasible periodic motion along a path founded through the computation of the reduced-order dynamics. Towards the nonholonomic constraints and nonlinear dynamic coupling, [122] used a special inertia distribution on the manipulator arm to achieve the differential flatness property of mobile manipulators, such that the issues of trajectory planning and control were addressed. However, dynamic constraints and the evaluation of objective function may result in computational complexity and subsequent slow convergence, particularly in the presence of higher DOF and higher degrees of underactuation.

For biped robots with point feet, the issues of planning and stable control of their gait have obtained different solutions, but they are still open research problems in this area. ATRIAS [29] is an underactuated bio-inspired biped robots that attracts much attentions during the past decade. SLIP (spring loaded inverted pendulum) [123,124] and PMB (point mass biped) [125] are basic underactuated models to generate and control biped running gaits with natural properties, where PMB has been shown to be capable of generating more general gaits than SLIP.

Kinematic coupling was elaborately considered in [126] to plan the motion trajectory of overhead crane systems with the objectives of smooth trolley transportation and small payload swing. An anti-swing mechanism was developed into an S-shape reference trajectory based on analytical studies on the coupling behaviour between the payload and the trolley. The combined trajectory was tuned through a designed iterative learning scheme to ensure precise trolley positioning. The trajectory planning scheme proposed in this study was proved to be robust against payload variations, and it guarantees accurate trolley positioning and efficient swing elimination. However, globally describing and characterizing the coupling behaviour including kinematic and dynamic couplings, which are of vital importance particularly for efficient trajectory planning, are still difficult and challenging tasks for URSSs.

Motion behaviours are important aspect to the trajectory of underactuated systems. A behaviour-based control approach was proposed in [127] for the trajectory tracking control of an underactuated planar capsule robot. The basis behaviours and required behaviour-sets to track the trajectory were elaborately defined in this study. Four motion behaviours, four switching behaviours and one stationary behaviour were proposed for the motion trajectory generation. A selection algorithm was designed to determine the appropriate behaviour-set to track each piece of the trajectory. Nevertheless, the issue of robustness to uncertainties and external disturbances were not investigated.

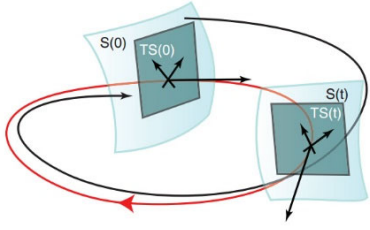


Fig. 9. A visualization of Poincaré surfaces and transverse linearization of a periodic orbit (red) and a trajectory converging to it (black) [128]

There have been a rising research interests in employing limit cycle reshaping/control for trajectory planning of underactuated robots [129–134]. This approach is motivated by various practical engineering applications whose motion behaviours are repetitively, for instance, walking [135,136], running [137,138], etc. Limit cycles are periodic trajectories defined on the phase space, accordingly the utilization of limit cycles can be regarded as curve tracking in the phase space. The common difficulty exists in the determination of the existence of limit cycles for a given set of differential equations. It is also challenging to plan these periodic orbits as feasible trajectory candidates which can be served as the dynamic behaviour of the closed-loop system. The utilization of limit cycle control falls into the existence of limit cycles and the orbital stability analysis. Confronting both tasks, Poincaré map analysis is a popular and promising approach. The method of Poincaré sections and return maps has been widely used to determine the existence and stability of periodic orbits in a broad range of system models. Poincaré maps are able to sample the solution of a system according to an event-based or time-based rule, and then evaluate the stability properties of equilibrium points (or fixed points) of the sampled system. Periodic solutions correspond to fixed points in Poincaré map. The stability of the periodic solutions can be guaranteed through the stability of the fixed points in Poincaré map which is determined by the eigenvalues of the Poincaré map linearized about these points as shown in Fig. 9.

Comparison among trajectory planning algorithms based on key features is demonstrated in Table 3. See Appendix 2 for table of results. To sum up, underactuated systems have reduced dimensions of the input space, thus underactuation is an essential factor needs to be considered, which makes finding a feasible trajectory for a specific task highly nontrivial. Moreover, it becomes more complicated in the presence of nonholonomic dynamic constraints [117]. Describing and characterizing the coupling behaviour including kinematic and dynamic couplings, which are of vital importance particularly for efficient trajectory planning, are still difficult and challenging tasks for URSs.

5. Control systems of URSs

The issue of control of URSs is an active domain of research in robotics and control engineering, which generates interesting topics and requires systematic nonlinear approaches. The difficulties of designing controller for URSs are originated from the nature of underactuation, which results in the partially linearizable feedback. Some well-established approaches and properties of nonlinear systems such as feedback linearizability and passivity are not directly

applicable in the presence of URSs. The traditional approaches to nonlinear control laws design are, for instance, backstepping [139–142], forwarding [143,144], predictive control [145–147], and SMC [51,54,148,149]. This is resulted from the fact that these approaches are unable to transform URSs into cascade nonlinear systems.

Nonholonomy is one of the important characteristics that related to underactuation [150]. It can be described that a nonholonomic constraint in form of $\phi(q, \dot{q}, t) = 0$ cannot be integrated into a holonomic constraint which is in the form of $\phi(q, t) = 0$. The former typically restrain the way in which the possible configurations of the system can be reached, instead of doing so directly on those configurations. Whilst the later reduces the number of a system’s DOF by one but it doesn’t apply to a nonholonomic constraint. It is crucial to consider nonholonomy for the control of a subclass of URSs (e.g., using underactuated system for dexterous manipulation or mobile manipulators). Exploration into the non-integrity of nonholonomic constraints for development of controllable URSs is an important research direction and it has been attracting much attentions [151–155].

The linearized dynamics of some underactuated systems (e.g., Acrobot and Cart-Pole systems) about an unstable fixed point (e.g. upright position) are proofed to be controllable [156]. This reveals an interesting property of the controllability of URSs that, if their nonlinear equations are linearized and if started way from the zero state, they can be returned to the zero state in finite time. Thus, the controllability of these systems presents an important point that URSs are not necessarily uncontrollable that although arbitrary trajectories cannot be followed by URSs, they do have the capability of arriving at arbitrary points in state space. However, there are some underactuated systems whose linearized dynamics are uncontrollable at any fixed point, as such, their controllability is one of the meaningful topics to be explored.

During the past decade, considerable nonlinear control algorithms have been developed for the underactuated characteristics based on passivity, feedback linearization, Lyapunov theory, etc. However, nonlinear control systems design for URSs is still regarded as a major open challenge [157–160].

5.1. Classification

Based on the introduction in Section 1, this subsection concentrates on the underactuation due to the origination that imposed artificially to create complex low-order nonlinear systems for gaining insight into the control of higher order URSs. These systems are classified into two types in [13] according to the object to be controlled, which are named as Type-I systems and Type-II systems.

Type-I systems is defined as the URSs that contain a pendulum or a system of pendulums, such as the Acrobot, the Pendubot, the IWP (inertia-wheel pendulum) system, the rotating pendulum system, the cart-pole system, etc. Based on the system properties, the main control objective is to regulate the configuration variables asymptotically convergence to the set-point references. Two essential issues have been facing towards these URSs. The first one is devoted to swing the pendulum from the hanging position to the upright position [161–163]. The second issue is dealing with the problem of upward pendulum stabilization [164–166], including stabilizing the system around its unstable equilibrium point,

on condition that the pendulum is initially above the horizontal plane, or lies inside an open vicinity of zero, i.e. the attraction region of the closed-loop system. Numerous control schemes have been developed, e.g. Bang-Bang Control [119,167,168], Fuzzy Logic [149,169–171], energy based [163], state feedback based [172], Sliding Mode [173], Backstepping, PID adaptive [53], Time Optimal [174], Switching [175], Neural Network [176], Prediction [177], etc. The issues of trajectory planning and optimized adaptive control was investigated in [84] for a class of WIP vehicle models. Under the control objective of shaping the controlled vehicle dynamics with minimized motion tracking errors and angular accelerations, the linear quadratic regulation optimization approach was employed to achieve an optimal reference model. Variable structure technique was used for adaptive control to guarantee the reference model to be accurately matched in a finite-time horizon, even in the presence of internal and external uncertainties. Interestingly, a neural network-based adaptive generator of implicit control trajectory of the tilt angle was proposed to indirectly manipulate the forward velocity.

Type-II systems is defined as the URSs that contain car-like subsystems such as the mobile robot [178], VTOL aircraft [107], UAV [33], underwater vehicles [179], etc. The control objectives of these kind of URSs are to regulate the configuration variables asymptotically convergence to the pre-designed trajectories. This trajectory tracking problem has twofold cases: kinematic tracking or dynamic tracking which is depended on whether the systems is represented by a kinematic or dynamic model. Some studies have been made on the kinematic tracking issue, for instance, [33,78,180,181]. However, considering the tracking problem in a dynamics point of view is more realistic and practical than its kinematic counterpart, which needs to be uncovered elaborately.

5.2. Control Systems Construction

5.2.1 Partial Feedback Linearization: Partial feedback linearization (PFL) is an interesting property which can be applied for the control of URSs. For URSs with symmetry, the authors proposed natural global changes of coordinates according to the Lagrangian of the system that transform nonlinear models into strict feedback ones. PFL approach is presented in detail as follows.

Lemma 1 [182]: Consider the actuated configuration vector q_2 in Eq. (4), there exists a global invertible change of control in the form below

$$u = \alpha_1(q)\tau + \beta_1(q, \dot{q}) \quad (5)$$

that partially linearizes the dynamics of Eq. (4) in the following form

$$\begin{aligned} \dot{q}_1 &= p_1 \\ \dot{p}_1 &= f_0(q, p) + g_0(q)\tau \\ \dot{q}_2 &= p_2 \\ \dot{p}_2 &= \tau \end{aligned}$$

where $\alpha_1(q)$ is a $m \times m$ positive-definite symmetric matrix and

$$\begin{aligned} f_0(q, p) &= -D_{11}^{-1}(q)h_1(q, \dot{q}) \\ g_0(q) &= -D_{11}^{-1}(q)D_{12}(q) \end{aligned}$$

The procedure of PFL using Lemma 2.1 is named as the collocated partial linearization, which copes with the dynamics of the actuated configuration vector. The advantages of the PFL are both a conceptual and a structural simplification of the control problem. It is always used as an initial simplifying step for reduction and control of

underactuated systems, regardless of the method used for decoupling of the actuated and unactuated subsystems. There are a few control approaches, such as energy-based control (EBC), adaptive control, and SMC have been developed based on the PFL technique.

5.2.2 Energy-based Control: EBC is one of the most popular control approaches for URSs particularly for the set-point regulation problem. This idea is originated from the energy existing in the system dynamics. Obtaining the derivative of total energy [11] gives

$$\dot{H}(q, \dot{q}) = \dot{q}^T [B(q)u - \frac{\partial p(\dot{q})}{\partial \dot{q}}] \leq \dot{q}^T B(q)u \quad (6)$$

where $\dot{H}(q, \dot{q})$ denotes the total energy of the systems, $p(\dot{q})$ is the dissipation term of URSs, $B(q)$ is the input force matrix. (2.4) implies that the system is passive with respect to the input u and output \dot{q} . As an essential characteristic of URSs, the passivity enables the stable origin and existence of feedback control law for $\dot{H}(q, \dot{q}) \leq 0$. Therefore, passivity has always been a main property considered in energy-based control. The main idea of passivity-based control is to regulate the total energy of the system to the equivalent value of a desired equilibrium.

Most EBC algorithms integrate with the PFL technique to deal with the swing-up control of the pendulum-like (Type-I) URSs. Energy-based swing-up control was studied in [163] for a remotely driven Acrobot which is a 2-link planar robot with the first link being underactuated and the second link being remotely driven by an actuator mounted at a fixed base through a belt. The global motion analysis was conducted based on the behaviour of the closed-loop solution and the stability of the closed-loop equilibrium points. An energy coupling-based output feedback control scheme was proposed in [183] for 4 DOF overhead cranes with saturated input constraints. The concept of virtual payloads was introduced with a designed energy storage function to efficiently explore the crane dynamics. A new energy shaping control design was presented in [184] for a class of underactuated systems including flexible joint robots, Series Elastic Actuators, and Variable Impedance Actuated Robots. Passivity property was utilized to conduct Lyapunov-based analysis for arbitrarily low feedback gains. Interestingly, non-collocated feedback was considered for the control scheme to shape the kinetic energy of the system.

5.2.3 Sliding Mode Control/Variable Structure Control:

In the control system construction, uncertainty is a common but intractable problem to be considered, particularly for URSs. One of the notable forms is the discrepancies between the practical system and the theoretical model built up through some well-established principles. These discrepancies are mainly due to the unmodeled dynamics, parameter uncertainty and external disturbances. Therefore, adaptability and robustness have attracted significant interests from the control engineering community in the past decade. Among them, two of the main approaches are adaptive control and robust control.

Robust control aims to make the system insensitive to all uncertainties using a fixed structure, but is only suitable for coping with small uncertainties. On the other hand, adaptive control uses on-line identification in which either the system parameters are identified using the predictive errors, or the controller parameters are adjusted using tracking errors. It is applicable to a wide range of parameter variations, but is sensitive to the unstructured uncertainties.

The difficulty of control law designs for URSs results from the reduced dimension of the input space, and it becomes folded when taking uncertainty into consideration. Thus, the control of URSs with uncertainty has been received extensive attentions. One interesting approach is Sliding Mode Control (SMC), which is a specific type of Variable Structure Control (VSC). This method has been successfully applied to various URSs. For example, an adaptive neural network sliding-mode controller design approach with decoupled method was proposed by [185], which presented a simple way to achieve asymptotic stability for a class of fourth-order nonlinear systems. SMC was employed to stabilize a class of underactuated systems which are in cascaded form in [186]. A novel SMC method was introduced by [187] based on the coupling sliding surface, the semi-globally asymptotically stable zero dynamics over the upper half-plane was generated. A cascade adaptive fuzzy sliding-mode control (AFSMC) scheme including inner and outer control loops is investigated in [188] for the stabilizing and tracking control of a nonlinear two-axis inverted-pendulum servomechanism. Hybrid controller design is developed by [189] for a class of 2-DOF underactuated mechanical systems with dry friction in the joints. It is noted that both of the unactuated and actuated joints were regulated, and the convergence of error dynamics and robustness to small variations of Coulomb friction coefficients were guaranteed. A robust-velocity-tracking scheme was proposed in [148] using two SMC methods to deal with the parametric uncertainties and external disturbances. To suppress the pendulum sway motion of an offshore container crane in load/unload operations, [190] designed a new mechanism for anti-sway control through a sliding surface design. Taking into consideration of frictions and uncertainties, A hierarchical sliding-mode under-actuated control scheme was developed in [191] for trajectory tracking of a differential mobile robot. Direct and indirect reference inputs were elaborately planned with separately defined sliding surfaces for the collocated and non-collocated subsystems.

5.2.4 Artificial Intelligence-based Learning and Approximation: Despite the sustained active research on control of underactuated robotics over the past decades, the key technical problems such as adaptive learning of varying nonlinear dynamics, the improvement of robustness, and the removal of effects of unmodeled dynamics, external disturbances and uncertainties remain to be the main research issues that have attracted consecutive attention. Extensive researches have been carried out towards these issues. One of the prevailing objectives is to make the existing controller more intelligent. Artificial intelligence is regarded as one of the key future intelligent systems technologies and has been studied and applied in addressing different kinds of practical problems. It contains various advanced techniques such as Neural Networks (NNs), Fuzzy Logic (FL), Evolutionary Computation (EC), which are paradigms for mimicking human intelligence and smart optimization mechanisms observed in the nature to solve problems that are too large or too complex to be solved with traditional techniques [192].

The structure of NNs is inspired by observed processes in natural networks of brain neurons. The learning process is conducted by adjusting the weights which represent the interconnection strength of neurons based on specific learning algorithms. NNs have an inherent learning

ability and are able to approximate a nonlinear continuous function to arbitrary accuracy. As such, a surge of researches has been devoted using NNs-based approach for underactuated robot control. An active adaptive NNs-based controller for WIP models was proposed in [193], wherein NN scheme was utilized for motion control of the actuated subsystem, and the passive subsystem was indirectly controlled through the dynamic coupling with the planar forward motion of its actuated counterpart. The energy-based controller integrated with radial basis function (RBF) NN compensation was developed in [6] to swing up the Pendubot. In this study, NNs was employed to compensate the effect of dynamic friction of the system. Multiple underactuated underwater vehicles were considered in [179], where the leader-follower formation control system was proposed using NNs to approximate model parametric uncertainties and unknown disturbances for the follower.

FL is a form of multivalued logic derived from fuzzy set theory to address vague instead of precise reasoning, wherein the degree of truth of a statement is ranging from zero to one. Fuzzy systems provide an alternative representation framework to present problems which are difficult to be expressed using deterministic and probabilistic mathematical models. As such, FL is chosen as one of the prevailing approximator for the control problems of URSs. Nonholonomic mobile manipulator was considered in [169] in the presence of parametric and functional uncertainties, and designed an adaptive control for the actuated subsystem using FL approximation. The reference trajectory was developed through FL-based motion generator, and the unactuated subsystem is indirectly controlled through dynamic coupling. A Takagi-Sugeno-type FL controller was presented in [194] for a two-wheeled mobile robot to facilitate position control of the wheels while keeping the pendulum around the upright position. The proposed FL controller synthesizes the heuristic knowledge and the model information of the considered system. The output parameters of the controller are chosen through comparison of the output with a linear controller at certain operating points, which avoids the tedious manual tuning work. To sum up, nonlinear control systems design for underactuated systems is still regarded as a major open challenge [11,157–160]. The existence of underactuation and other undesirable properties like possessing an undetermined relative degree or being in a non-minimum phase, give rise to complex theoretical problems and less generality in which conventional techniques are not directly applicable.

See Appendix 3 for table of results.

6. Challenges and future directions

Based on the investigations in modelling, bio-inspired design and bio-inspired control, trajectory planning and nonlinear control of URSs, we may observe that the evolutions of relevant techniques are relatively slower than the speed of development of sophisticated robotic prototypes. This drives us wonder that why this discrepancy exists when the above technical issues are supposed to be significant aspects of integrally functioning of URSs.

6.1. Theoretical Challenges and Common Difficulties

Analysis of Frictional Interaction Dynamics: As discussed in Section 2, for high fidelity engineering systems,

accurate modelling or prediction of nonlinear friction force is a nontrivial while intractable aspect of scientific research. Conventionally, the frictional instabilities are required to be eliminated or compensated through efficiently designed controllers. For instance, the practical engineering problems historically reside in the circumstances where robust friction models with instabilities are essentially required. Therefore, accurate predictions of friction-induced dynamic responses in sticking, presliding as well as pure sliding regimes become crucial. Several friction models with an arbitrary degree-of-complexity (i.e. numbers of parameters to be identified and controlled) have been proposed in literature which incorporates varying physical phenomena corresponding to friction. However, an accurate representation of friction for given practical applications of URSs is required to capture several experimentally observed dynamic phenomena reported in literature. The static friction models are merely determined by the relative velocity between surfaces in frictional contact, and the dropping friction characteristics in the low relative velocity regime and the hysteretic loops are not captured.

Optimal Morphological Computation and Motor Control Learning: As discussed in Subsection 3.1, URSs introduce several beneficial properties including mechanical self-stability, passivity/adaptivity, energy efficiency and manoeuvrability, however, there remains some challenges that are related to optimal morphological design process of nonlinear mechanical dynamics and their robust and accurate control. To realize efficient trajectory planning and tracking control, bio-inspired morphology constraints need to be elaborately considered, such that the behavioural variation can be increased while maintaining fascinating characteristics of URSs. Another challenge is to reduce the number of trial-and-error iterations in motor control learning in URSs. Designing more generalized state representations and enforcing the system to generate autonomously an appropriate mechanical model of its own body are necessary methods to approach this challenge [55].

Producing Minimum Phase Systems with Strongly Coupled Inputs: Being minimum phase is an essential property for a robotic system to have, as it enables formulating the regulation of the output as a control goal. The non-minimum phase nature restricts direct application of many nonlinear control methodologies. Therefore, a better understanding of the minimum phase w.r.t a meaningful output should be regarded as a major challenge in the control of bio-inspired/soft robots. Input-output linearization can be applied to deal with a class of slightly non-minimum phase URSs, but it is not applicable to systems with strong input couplings such as bio-inspired/soft robotic systems.

Efficient Operation/Locomotion: It implies efficient operation/locomotion during each motion cycle in terms of travelling distance and energy consumption, either for the Type-I URSs [195,196] that are fastened to the environment or, type-II URSs that are designed to move and interact with the environment [197,198]. The operation/locomotion index is typically set as distance-optimal or energy-optimal, as such, the challenges become how to generate optimal trajectory and how to design effective control system to satisfy the designed index.

Dynamic Coupling Characterization with System Performance: Describing and characterizing the coupling behaviour, which are difficult and challenging, are of vital

importance particularly for efficient trajectory planning. Unfortunately, a majority of reported results in the literature are mainly devoted to the couplings characterization in part of the motion stage, the underactuated (passive) motion stage is usually neglected. This is mostly owing to the underactuated kinematic and dynamic coupling behaviours and the relevant analysis is a difficult and challenging task. Towards trajectory construction, it is worth mentioning that there are several significant studies for overhead cranes systems based on phase plane analysis of crane kinematics [126,199], whilst as locomotion systems, the locomotion-performance indexes (e.g., average locomotion velocity, energy efficiency) were not examined. Indeed, it is a tough task to achieve steady-state periodic motion of the driving mechanism and efficient system performance simultaneously.

Planning of Optimized Motion Trajectories:

Generating periodic motions that can be seen in various natural locomotion of biological systems has always been a challenging issue. URSs have reduced dimensions of the input space, thus underactuation is an essential factor needs to be considered, which makes finding a feasible trajectory for a specific task highly nontrivial. Moreover, it becomes more complicated in the presence of nonholonomic dynamic constraints [117] and viscoelastic property [72].

Dealing with Uncertainties and Disturbances:

Uncertainties in system dynamics are critical and challenging issues either for control design or for trajectory planning of the URSs, including structured and unstructured uncertainties and time-varying matched and unmatched external disturbances. As such, the construction of adaptive control schemes or approximator-based (e.g., NNs, FL) approaches tends to be promising solutions. However, the uncertainty lies in different loops requires different treatments, especially in the non-collocated subset that is unmatched with the control action, which is nontrivial and intractable for adaptive control system design.

6.2. Trends and Future Directions

Through the investigations into the characteristics and state-of-arts of URSs and bio-inspired approaches, it is apparent that studying on URSs is meaningful and significant and has always been a popular and active domain of research in robotics and control communities. Based on the investigations, several essential research issues, trends and promising future research directions of URSs are summarized and presented as follows.

Novel Bio-Inspired Design and Development: With increasing requirements in real life, current machines and equipment become unable to satisfy new applications and new explorations. What can be further developed based on the current framework of URSs to deal with the presence of new issues in real-life control systems? For example, the tasks of monitoring, sensing and intervention in narrow and restricted space such as pipeline that are inaccessible to human beings require the robot to undertake minimally invasive operation/locomotion. The robot therefore needs to adopt some principles inspired from animals that excel in moving in such environments. Therefore, novel bio-inspired design and development of URSs are required for a natural understanding of motion/behaviour principles of biological systems, the achievement in diversified motion/behaviour patterns of URSs. It is believed that this is a promising research direction of URSs in applications in military, healthcare, medical assistance, industry, etc.

Exploiting Feedforward Control Schemes: In some applications with strong input couplings, feedback might not be easily implemented, and it has fundamental performance limits for non-minimum-phase systems. Feedforward scheme can alleviate such control challenges—for non-minimum phase systems. With the aid of feedforward control, the biological systems (e.g., animals and human beings) can walk/run efficiently on uneven terrains, which contributes to the muscle activities to responsible for the adaptations to the ground contacting [111,112]. Therefore, proper combination of feedforward control with passive (mechanical) feedback or active feedback into the control design for URSs is a meaningful research direction to be explored.

Accurate Modelling and Prediction of Dynamic Frictional Interactions: Friction plays an important part in the motion of URSs, however, it is easily ignored or simplified in the works during the past decades. Moreover, the investigation of nonlinearities of the friction effects is still open. Therefore, attentions are to be paid to the characterization of frictional dynamics. Besides, investigations from the viewpoint of chemical and material science are also promising directions to characterize the dynamic interactions with the environment.

Analysis of Underactuated Dynamics with Bio-Inspired Viscoelastic Property: For locomotive URSs, there has always been a lack of thorough understanding of system dynamics and their efficient utilization. Therefore, efforts are to be made in how to achieve a systematic way of utilizing system dynamics in the forms of optimally synthesized trajectories and effectively designed controllers, particularly when bio-inspired viscoelastic elements are employed. Moreover, to the best of our knowledge, for the systems consisting of a pendulum or a system of pendulums that are essentially nonlinear, unfortunately, there is little analytical research.

Optimal Planning of Periodic Motion Trajectories: Dynamical underactuated locomotion of robotic systems corresponds to the existence of limit cycles in the state space of the URSs. The generation of periodic motion trajectory and the design of controllers that induce limit cycles, while a challenge in its own right, are made significantly even more difficult by the aforementioned difficulties. The objectives of optimal planning are typically containing time-optimal, distance-optimal and energy-optimal. Therefore, attentions are to be paid to how to construct the periodic motion trajectories and how to design efficient control laws that induces limit cycle locomotion and holds stability.

Adaptive and Robust Control in the Presence of Matched and Unmatched Uncertainties: It is well-established that tracking control has always been a vital control issue of URSs due to unknown unactuated trajectory, less control actuator, and nonlinear behaviour, etc. Compared with their fully-actuated counterparts, challenges still remain in trajectory tracking control of URSs, particularly in the presence of matched and unmatched uncertainties. When the dynamic parameters are uncertain or unknown in practice, and kinematics relationship is not accurate, what adaptive control scheme is feasible for this nonlinear system where linear parameterization does not hold and linear structured adaptive control scheme is not valid.

7. Conclusions

Given the importance that URSs have been gaining in the past decades, particularly with recent advances in introduction and application of bio-inspiration and intelligent control systems, in this article we have presented a systematic review of the state of the art of URSs and its current limitations. In particular, we have covered four rapidly developing ideas and technologies in modelling, bio-inspiration, trajectory planning and control systems, which will potentially lead to significant breakthroughs in handling URSs. Non-minimum phase system is a key to bridge the gap between bio-inspiration and URSs. We have reported that proper combination of feedforward control with passive (mechanical) feedback or active feedback into the control design, for bio-inspired/soft URSs with strong input coupling, is a meaningful research direction to be explored. Finally, we have discussed theoretical challenges and common difficulties, and how bio-inspiration and control approaches may be profitably applied. We have also pointed out the tremendous potential and trends for future research in URSs throughout these challenges and open questions.

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9. References

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Fuzzy Sets and Systems 2016;290:158–77.
<https://doi.org/10.1016/j.fss.2015.08.013>.

10. Appendices

Appendix 1

Table 1 Undulation-based bio-inspired robots using vertical waves





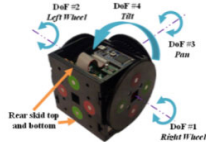
Robots /Features	Bio-inspiration /Biomimetic	Locomotion	Perception /Sensors	Power	Examples
Inchworm robots	Inchworm	Extension & flexion; Autonomous	Tactile; Infrared	Tethered	[200–203]
Snake robots	Snake	Obstacle-aided Autonomous	Visual camera	Tethered	[96,98–100]
Reconfigurable robots 	Snake	Sinusoid serpentine-like; Rolling track; Caterpillar-like; Autonomous	Visual camera	Electrical	[204,205]
Modular robots 	Snake	Serpentine -like; Climbing, Swimming & Crossing gaps; Autonomous	Video camera	Tethered	[206]

Table 2 Undulation-based bio-inspired robots using linear expansion

Robots /Features	Bio-inspiration /Biomimetics	Locomotion	Perception /Sensors	Power	Examples
Slim Slime robot 	Snake	Snake-like creep; Snail-like pedal wave; Lateral rolling & pivot turning; Autonomous	Visual camera	Tethered	[207]
Planar inchworm robots 	Inchworm; Snake	Snake-like creep; Autonomous	Visual camera	Tethered	[208] Planar Walker
Self-Reconfigurable robots 	Snake	Contracting & expanding; Conecting & disconnecting from neighbour modules; Autonomous	Not reported	Electrical module	[209] SMORES

Appendix 2

Table 3 Comparison among trajectory planning algorithms for URSs based on key features

Algorithms /Features	Controlled system	Control objectives	Novelties /Merits	Uncertainties ^a	Holonomic /Nonholonomic constraints ^a	Dynamic couplings ^a	Kinematic couplings ^a	Examples	Comments
Feedback motion planning	Constrained nonlinear system	To build a sparse tree of LQR-stabilized trajectories	LQR-trees; Sums-of-squares approach	No ^b	No	No	No	[118]	Randomized motion planning
NN-based trajectory generation	WIP	Dynamic balance & motion tracking	Optimized trajectory model	Yes	Yes	No	No	[84]	Implicitly controlled passive dynamics
MTPP	Overhead cranes	Minimum-time trajectory planning	State & control constraints	Yes	Yes	No	No	[119]	Off-line ^c trajectory planning
Point-to-point planning	Cable -suspended robot	Regulation of prescribed poses	Natural frequency of unconstrained DOF	No	No	No	No	[120]	Unconstrained motion dynamics
Periodic motion planning	Underactuated helicopter	To track prescribed motion trajectory approach	Virtual constraints-based	Yes	Yes	No	No	[121]	Control problem of linearized system
Kinematic coupling-based planning	Overhead cranes	Accurate trolley positioning	S-shape reference trajectory with coupling	No	No	No	Yes	[126]	Off-line trajectory planning
Behaviour-based planning	Planar capsule robot	To track predefined behaviour-sets	Design of basis behaviours	Yes	No	No	No	[127]	Off-line trajectory planning
Controlled invariants	Underactuated systems	To Create invariants via feedback	Reduction and representation of dynamics	No	Yes	No	No	[210]	Virtual holonomic constraints

^a Uncertainties, holonomic/nonholonomic constraints, dynamic couplings and kinematic couplings mean the corresponding considerations in the construction of trajectory planning algorithms.

^b A ‘yes’ or ‘No’ means the corresponding property is or is not considered in the trajectory planning algorithm, respectively.

^c ‘Off-line’ is compared to on-line or real-time, it means the trajectory planning and/or optimization are/is undertaken during a pre- or post- motion/operation stage.

Appendix 3

Table 4 Comparison among nonlinear control algorithms for URSs based on key features

Algorithms	Properties	Adaptability	Robustness	Advantages	Limitations	Examples
/Features	to uncertainties ^a	to disturbances ^b	/Merits	/Demerits		
PFL	Linearization for dynamics of the actuated/unactuated configuration vector	Poor ^c	Poor	A conceptual and a structural simplification of the control problem	Low-level control ^d	[18,211–213]
EBC/ Passivity -based control	Regulation of the total energy to the equivalent value of a desired equilibrium state	Weak	Weak	Investigation through passive dynamics	Conditions of passivity need to be satisfied	[40,163,214–216]
SMC/VSC	Alteration of the dynamics by applying discontinuous control signal	Good	Good	Robust to input disturbances	Control input chattering; Assumption on known or fixed uncertainty bounds	[51,190,191,217]
NNs-based control	Approximation of nonlinear continuous function	Good	Good	Learning ability; Arbitrary approximation	Design of the NN structure; Optimal determination of the NN parameters	[6,6,84,218,219]
FL-based control	Representation of nonlinear continuous function by quantification	Good	Good	Learning ability; Arbitrary approximation	Design of fuzzy rules	[142,149,176,220,221]

^a ‘Adaptability to uncertainties’ means, by adopting the corresponding control approach, the ability of the robotic system to adapt itself or its behaviour efficiently or fast according to changes/uncertainties in its circumstances or parts of the system itself, including structured and unstructured uncertainties.

^b ‘Robustness to disturbances’ means, by adopting the corresponding control approach, the ability of the robotic system to be strong and effective in tolerating external disturbances/perturbations.

^c A ‘poor’, ‘weak’ or ‘good’ means the performance of the corresponding control approach in achieving the corresponding property is not desirable, partly achieved or completely achieved, respectively.

^d ‘Low-level control’ means the corresponding approach makes use of the dynamics of a subset to achieve a local rather than a global solution in controlling the robotic system. In terms of the URSs, it refers to partially linearization of the robotic dynamics (collocated subset with its cardinality contains the actuated DOF and equals the number of control inputs; or non-collocated subset that accounts for the remaining non-actuated DOF) in order to reach a control authority of the system.