Proceedings Paper:

Andrievsky, B. and Selivanov, A. orcid.org/0000-0001-5075-7229 (2020) Historical overview of the passification method and its applications to nonlinear and adaptive control problems. In: 2020 European Control Conference (ECC). 2020 European Control Conference (ECC), 12-15 May 2020, Online conference. IEEE , pp. 791-794. ISBN 9781728188133

Historical Overview of the Passification Method and its Applications to Nonlinear and Adaptive Control Problems

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***Abstract*— The present survey paper provides a historical overview of the method of passification and its applications to nonlinear and adaptive control problems from 1980 to present days.**

***Index Terms*— passification, nonlinear control, adaptive control**

The concept of *passivity* came to control theory from the theory of linear electric circuits [1], where it was used for denoting circuits that do not contain internal energy sources. The detailed exposition of the notion passivity and its generalizations, such as the *incremental passivity*, *input feedforward passivity*, etc. may be found in [2]–[12].

A solution to the *passification problem* for linear time-invariant plants was given in the framework of the *Passification method*, founded by A.L. Fradkov in 1974 as a part of the consideration of the adaptive stabilization problem for the output of a linear dynamic plant with uncertain parameters. The result, as an auxiliary statement, is formulated in the form of a lemma [13, Lemma 1], which establishes a connection between the existence of solutions of the Lyapunov matrix inequality with linear relations and the existence of a stabilizing static output controller, from one side, and the minimum-phase property of the transfer function of the controlled plant, from another one.

Since the mentioned Lemma establishes the conditions for the existence of feedback that makes the plant transfer function strictly positive real, rendering the closed-loop system passive, this Lemma became known as the *Passification theorem* (sometimes called “The Fradkov theorem”, cf. [14], [15]). Based on this Theorem, the Passification method was developed in [16]–[20]. Its more detailed exposition may be found in [21]–[26], including the surveys [8], [12], [27] and the monographs [17]–[19].

In the present paper some application results of the Passification theorem to the problems of nonlinear and adaptive control are outlined in the chronological order.

The passification method was employed in [13] to find the adaptive feedback that uses the plant output measurements only. In this framework the *Implicit Reference Model* (IRM) concept was originated. This approach has been extended to spatially distributed systems in [26], [28] and to systems with time delay in [29]. In [17, Ch. 7] it is shown that the proposed in [13] adaptive control law may also be used for a class of nonlinear systems with sector nonlinearities acting additively to the control signal. In [18], [30], [31] it is shown that if there are stable multipliers with small time constants in the denominator of the plant transfer function, then one may try to drop them and carry out design by the reduced model. The passification-based IRM method is extended to the tracking problem in [17], [18], [32], [33]. In [18], [24], [27], [34] the passification-based signal-parametric controllers are introduced. It is demonstrated, particularly, that the sliding-mode motion for passifiable systems may be ensured without measurements of the full state vector, but by the output measurements only. The IRM design method for nonlinear systems is described in [35], allowing one to overcome some structural obstacles following from matching conditions and leads to a simplified adaptive controller design procedure. In addition, the restriction of the relative degree can be mitigated using the so-called “shunt” (parallel feedforward compensator) [8], [22], [36]–[39]. Papers [40], [41] are devoted to the problem of adaptive synchronization. They provide a general statement of this problem and, based on the passification method, present a synchronization scheme for two subsystems. This approach is applied in [42]–[44], to the problem of widespread information transmission based on the chaotic generator modulation. Paper [45] proposes two passivation-based adaptive control schemes for the so-called *G-passifiable systems*, cf. [20]. In this case, the passivity of the closed-loop system can be ensured with respect to the linear combination of the plant outputs, defined by matrix G, both square and a rectangular one. Based on the passification design method and the IRM approach, the adaptive control laws are designed and experimentally studied for Quanser/LAAS *Helicopter Benchmark* in [46]. Robustness of the adaptive control systems based on the passification method is studied in [47]. It is shown that a closed-loop adaptive system provides better (or at least not worse) value of $\mathscr{L}_2$ performance index than the static feedback found by the solution of the LMI for known plant parameters. Robustness of passivation-based adaptive control with respect to time-dependent uncertainties is studied in [48]. Since convergence of the state vector to the

*This work was partially supported by the RFBR (grants No 17-08-01728, 18-38-20037, 19-18-50428).

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equilibrium point cannot be proved, it is replaced by convergence to a small neighborhood of the origin. The attractor can be made as small as required by choosing the parameters of the adaptation algorithm. The results of [47] were employed in [49] to obtain a procedure for developing simple adaptive controllers for "*almost stable*" systems. Paper [50] is devoted to the problem of robust passification of linear time invariant MIMO systems, closed by a static (non-adaptive) feedback. The variant of the passification theorem for state observers is derived in [51]. In the series of papers [52]–[54], passification-based methods for the harmonic disturbances compensation with a time-delayed output measurements for a parametrically and functionally uncertain nonlinear plant is elaborated. The method of [14] is used in [55]–[57] for adaptive control of the satellite libration angle. Fault tolerance of adaptive controllers with the IRM is demonstrated in [58]. The passification method has been applied for developing the algorithms of nonlinear systems observation and synchronization over the limited capacity communication channel, see [59]–[65]. Paper [66] is aimed to parametrize the stabilizing controllers, which are the static output feedback for continuous time linear systems with Markov switching. The control problem for plants affected by stochastic disturbances is considered in [67], [68], where the influence of both coordinate and parametric disturbances such of a white noise kind is considered. In [69], the passification method is used to synthesize a robust angular motion control system for a non-rigid aircraft. To stabilize the characteristics of the closed loop in a wide range of aircraft parameters, sliding modes are used. The first order shunt is introduced for making the extended plant passifiable. Synchronization of nonlinear dynamical systems in the master-slave configuration with time delay in the communication channel, subjected to external disturbances is studied in [70]. In [71], the problem of synchronization by states in the network of identical linear agents is considered by applying consensus output feedback. In the series of papers [72]–[74] the IRM adaptive controller is used for control of quadrotors. Decentralized adaptive synchronization of nonlinear dynamic networks with delay is described in [75]. Application of the Passification theorem to adaptive control with variable time delay in control and measurements is performed in [76].

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