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Review of Alain Bensoussan's book
'Estimation and Control of Dynamical Systems'

This is no ordinary book. Neither a textbook nor a monograph. It is a guide to the world of deterministic and stochastic control. Leading the reader from classical results to the most recent methods through the eyes of one of the founders of the stochastic control theory. This view and the subjective choice of material make this book different. I have become accustomed to watch my PhD students struggling to recreate a broader picture of stochastic control from research papers and highly technical monographs. This book can change it.

Although the title does not reveal it, this book is mostly concerned with deterministic and stochastic control. Deterministic control theory is often used to introduce methods which are then applied to study stochastic problems. The presentation starts from explicitly solvable linear-quadratic problems (with full and partial observation) and then moves to general methods for non-linear systems. Principal agent problems and differential games are covered in detail in final chapters. 'Estimation' in the title refers mostly to classical statistical techniques (both static and dynamic) but presented with a control twist (this part of the book can be read without any prior knowledge and interest in control methods). Stochastic control with partial observation is limited to linear-quadratic systems.

1 Estimation

The title could suggest that the presentation of estimation techniques is geared towards control problems. This is only partly true. Stochastic control problems with partial information are only studied in the linear-quadratic framework, i.e., the state is modelled as a linear system with a Gaussian noise and the functional is quadratic in the state and control. Broader treatment of partially observed controlled systems would be very welcome, but the reader can always consult another book by the same author, [1], where they will find the theory presented here with complete proofs as well as the general theory of control with partial observation.

The innovations approach in the framework of discrete-time Kalman filter is beautifully presented in Section 4.7 explaining some of the magic that many feel when faced with the classical introduction of the approach in continuous time (Section 7.1). The most part of Chapter 4 is a crash course in frequentist and Bayesian statistics which, in my opinion, is too quick to learn the material: a reader unfamiliar with the basics of mathematical statistics may need to consult a statistics textbook. Chapter 5 covers mostly generalised linear models (GLMs) and can be skipped by readers interested in optimal control. Sections 5.1-5.4 present classical material commonly found in statistical textbooks. However, in Section 5.5 one finds a dynamic version of GLM where the parameter of interest evolves over time. Here, one needs to use approximations (or rather postulate a certain form of conditional distribution) - a very pragmatic approach. I was excited to see such practical thoughts (one can find more of them in Chapter 4) which are rarely seen in the world of stochastic control theory but often crucial for real applications of this theory.

2 Control of linear systems

Chapters 2 and 3 present classical problems in control of deterministic linear systems, i.e., systems of the form $x'(t) = F(t)x(t) + G(t)v(t)$, where F, G are known matrix-valued functions and v is the control variable. It is interesting to read about controllability and observability problems before turning to optimal control problems which are more familiar to the stochastic control audience. In those problems, the functional to be minimised is of linear-quadratic form where the running cost is a quadratic function of the state and control, and the terminal cost is quadratic in the terminal state. This theory is notationally cumbersome but a lot can be derived explicitly without resorting to general results thus providing useful insights. An infinite horizon problem akin to controllability is discussed in depth.

In the stochastic version the state is perturbed by a Brownian motion, i.e., in the simplest case, it satisfies the stochastic differential equation $dx(t) = \big(F(t)x(t) + G(t)v(t)\big)dt + dW(t)$, where $(v(t))$ is a stochastic process adapted to the filtration generated by the Brownian motion $(W(t))$. The objective is the expectation of the above mentioned quadratic functional. In Chapter 8, the author builds on intuitions developed for the deterministic problem to show tricks that apply in several types of stochastic problems, including a risk-sensitive functional in Section 8.3 (called in the book 'Exponential-of-integral payoff').

Chapter 9 in the only place where the reader will see the interplay between control and estimation. The state $x(t)$ is not directly observable by the controller. Instead, all the control actions must be based on observations of another linear process whose dynamics depends on $x(t)$. The book introduces a fundamental concept of separation principle which states that the solution of the problem can be split into two phases: filtering and then control of a fully observable system. Here, again, the linear form of the state dynamics allows for explicit methods of solution and relevant 'tricks' are nicely presented.

It could seem that the framework of linear-quadratic control is nothing but dead from the research perspective. Just the opposite. This is a model of choice for new control problems such as mean-field games, control with non-Gaussian noise (e.g., fractional Brownian motion or Rosenblatt process) or principal-agent problems.

3 Control of non-linear systems

There are two broad methods for solution of optimal control problems: Pontryagin's Maximum Principle and Dynamic Programming. The first one formulates necessary conditions for optimality which, only under additional assumptions (such as convexity) become sufficient. Dynamic programming, on the other hand, is aimed at providing sufficient conditions. The book follows a well trodden path of explaining those two concepts for deterministic systems in Chapter 10 before moving to stochastic systems in Chapter 11. Apart from discussing the maximum principle and dynamic programming equation (Hamilton-Jacobi-Bellman equation or HJB equation), it also directs reader's attention to more delicate issues. The link between the two concepts is intuitively explained (more formal presentation can be found, e.g., in Chapter 5 of [4]). I like very much that the maximum principle is related to the Gâteaux differential of the functional at the optimum providing a first-order condition interpretation of the procedure.

The treatment of stochastic systems involves a lot of technicalities. Instead of delving into details, the author replaced many proofs by sketches or intuitive derivations. These flag up main ideas which are often less clear in formal treatments and allow the reader to absorb a large number of sophisticated techniques in a relatively short time. Topics covered include weak approach to stochastic control (via Girsanov transformation), the relationship to backward stochastic differential equations (BSDEs) and viscosity solutions to deal with cases when the value function is not smooth enough to apply (generalised) Itô formula. The latter theory is vast and covered in an excellent monograph [3], but the present book provides sufficient details to allow the reader to appreciate this methodology and understand its usefulness. It should be noted that the presentation of the HJB approach is limited to problems in which control affects the drift only. Interested reader can find more general results, e.g., in [4].

BSDEs appear in two areas described above: giving dynamics of the adjoint process in stochastic maximum principle and in weak formulation of the dynamic programming approach. There is a further link with non-linear parabolic PDEs (with non-linearity in the first derivative). The existence and uniqueness of solutions to such PDEs is proved in the first part of Chapter 12. It is worth mentioning that the PDEs appearing in stochastic analysis are usually defined on the whole space while the classical PDE theory is concerned with boundary value problems. Solutions to BSDEs with a driver of quadratic growth, which are linked to the above PDEs, are explored in Section 12.3. A reader unfamiliar with this theory may need further reading to appreciate the results as some technical details are omitted, e.g., in which sense to interpret the solution of the PDE, or under what conditions the link between BSDEs and PDEs is valid.

In Chapter 13 the reader will find there presentation of techniques for explicit solution of optimal control problems motivated by financial applications. These problems involve control of the diffusion term (not covered in the theoretical part of the book) or problems of control and stopping. An interesting study of non-Markovian stochastic control problems is presented in Chapter 14. There, the reader will appreciate in more details the difference between strong and weak formulation of the problem as well as the use of backward partial differential equations.

4 Principal agent problems, differential games and target problems

The last part of the book, Chapters 15-18, is concerned with problems which are related to control or which use control techniques. Some of the theory is relatively new (developed in the last 10 years). The coverage is very wide. The presentation is centred on ideas and general mathematical techniques with assumptions largely left out. The reader should not expect to find theorems ready to cite in their work but will benefit from a coherent and unclogged view of various difficult problems in applications of control theory.

In the framework of differential games, there are multiple concepts of equilibria depending on the information available to each player. Open-loop equilibria are related to the Pontryagin's maximum principle, while closed-loop equilibria to Bellman principle and HJB equations. Examples of non-zero sum and zero-sum games are presented, including the linear-quadratic case. The reader will also find a deep discussion of the case when the Issac's condition for a zero-sum game fails: this gives rise to a duality gap between an upper and lower value to the game. Interpretation of those values as values of the game with different information structure available to players is provided. Solution techniques for stochastic games are presented as generalisations of those for deterministic games. Chapter 17 discusses Stackelberg differential games in which players are no longer equal. In Stackelberg games there are two players: a leader and a follower. The follower chooses his strategy in response to the leader. There are multiple variations of this response mechanism which are cleanly presented in the book. I found it very helpful to see examples which showed how each of these different mechanisms affects players' strategies.

Principal-agent problems also feature two players but the principal does not control the underlying dynamics directly. Instead, he offers a contract (remuneration) to the agent to steer him into applying controls favoured by the principal. The contract itself does not have to be of feedback form, so direct application of stochastic control methods is not possible. The approach of [2] is deeply rooted in BSDE methods while the presentation in this book tries to offer a more direct analytical approach. To appreciate it fully the reader should be familiar with the aforementioned monograph.

The last chapter of the book shows a small selection of hedging problems on the Black-Scholes market: superreplication with convex constraints and quantile hedging. To fully understand the material, the reader must already be familiar with continuous-time contingent claim pricing beyond what is discussed in Chapter 13. The solution of a quantile hedging problem is particularly worth mentioning because the author uses elegant probabilistic arguments.

5 Conclusions

This book is a great resource for graduate students and those who want to learn and understand stochastic control theory. It is also a great read for experts who want to gain a broader overview of the subject and see connections between different techniques. The book is not an ultimate reference with the strongest statements of all results and complete proofs. I concur with the author that it is better to present ideas clearly rather than have proofs mired in technicalities. What could be further expanded is an introduction to each chapter on its content and how it relates to the rest of the material in the book to help readers to see the broad picture and connections between different topics and techniques. Nevertheless, this is an excellent book and a great complement to the current offering in stochastic control.

References

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