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Robust Hybrid Linear State Estimator Utilizing SCADA and PMU Measurements

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***Abstract*—This paper intends to improve the accuracy of power system State Estimation (SE) by introducing a hybrid linear robust state estimator. To this end, automatic bad data rejection is accomplished through an M-estimator, i.e. a Schweppe-type estimator with Huber loss function. The method of Iteratively Reweighted Least Squares (IRLS) is used to maximize the likelihood function in the M-estimator. Leverage measurements are also treated by a simple yet effective formulation. To run the algorithm for real-world large-scale grids, cumbersome construction of the Jacobian matrix at each iteration is avoided. In addition, convergence to the local minima faced in the large-scale Gauss-Newton algorithm is not a concern as the proposed formulation is linear with no approximation. As observability and redundancy considerations mandate SE to take advantage of traditional SCADA measurements along with available PMU measurements, the linearity of the proposed SE formulation is guaranteed regardless of whether PMU-only, SCADA-only or hybrid SCADA/PMU measurements are utilized. In this regard, covariance matrix for measurements weights is derived for both types of measurements. Thanks to the linear formulation and therefore swiftness of the proposed algorithm, SE could be run for different power systems with a few up to thousands of buses.**

***Index Terms*—Huber loss function, PMU, Power system operation, RTU, SCADA, Schweppe-type estimator, State estimation.**

I. Nomenclature

V_a	True voltage amplitude at bus a.
$\boldsymbol{V}_a^{meas}$	PMU measurement of voltage at bus a.
V_a^{meas}	Remote Terminal Unit (RTU) measurement of voltage amplitude at bus a.
$\boldsymbol{V}_a$	Unknown true complex voltage at bus a with respect to the phase angle of reference bus.
I_{ab}	True current amplitude through line a-b.
φ_{ab}	True phase-angle of the current through line a-b (with respect to $\boldsymbol{V}_a$).
I_{ab}^{cal}	RTU calculated/measured current amplitude through line a-b.
φ_{ab}^{cal}	RTU calculated/mesured phase-angle of the current through line a-b (with respect to $\boldsymbol{V}_a$).
P_{ab}^{meas},Q_{ab}^{meas}	RTU active and reactive power measurements through line a-b.
$\boldsymbol{I}_{ab}^{loc}$	True complex current through line a-b (with respect to $\boldsymbol{V}_a$).
$\boldsymbol{I}_{ab}$	Unknown complex current through line a-b with respect to the reference bus.
$\boldsymbol{I}_{ab}^{meas}$	PMU measurement of complex current through line a-b with respect to the reference bus.
$\boldsymbol{I}_{a,inj}^{loc}$	True complex current injection at bus a (with respect to $\boldsymbol{V}_a$).
$\boldsymbol{I}_{a,inj}$	True complex current injection at bus a with respect to the reference bus.
ε_{V_a}	Measurement error of V_a^{meas}.
$\boldsymbol{\varepsilon}_{\boldsymbol{V}_a}$	Complex measurement error of $\boldsymbol{V}_a^{meas}$.
$\varepsilon_{I_{ab}}$	Measurement error of I_{ab}^{meas}.
$\boldsymbol{\varepsilon}_{\boldsymbol{I}_{ab}}$	Complex measurement error of $\boldsymbol{I}_{ab}^{meas}$.
ε_{δ_a}	Measurement error of δ_a^{meas}.
δ_a	Unknown phase-angle of complex voltage at bus a.
L_a	Set of branches connected to bus a.
$\mathbf{z_{ab}}$	Series impedance of transmission line a-b.
$\mathbf{y_{ab}}$	Shunt admittance of transmission line a-b.
$[I]$	Identity matrix.
$[Y_{se}]$	Diagonal matrix of series admittances of branches.
$[A]$	Bus-branch incident matrix of the network.
$[Y_{sh}]$	Diagonal matrix of shunt admittances of branches.
$\underline{V}^{meas}$	Vector of ${V_a}^{meas}$ values ($a \neq 1$).
$\underline{\boldsymbol{V}}^{meas}$	Vector of ${\boldsymbol{V}_a}^{meas}$ values.
$\underline{I}^{cal}$	Vector of ${I_{ab}}^{cal}$ values ($a \neq 1$)
$\underline{\varphi}^{cal}$	Vector of ${\varphi_{ab}}^{cal}$ values ($a \neq 1$).
$\underline{\boldsymbol{I}}^{meas}$	Vector of $\boldsymbol{I}^{meas}$ values.
$\underline{I}_1^{cal}$	Vector of ${I_{ab}}^{cal}$ values ($a = 1$).
$\underline{\varphi}_1^{cal}$	Vector of ${\varphi_{ab}}^{cal}$ values ($a = 1$) .
$[Y_{bus}]$	Bus-admittance matrix of the network.
$\underline{I}_{inj}^{cal}$	Vector of measured/calculated injected-current amplitudes by RTUs excluding the reference bus.
$\underline{\varphi}_{inj}^{cal}$	Vector of measured/calculated phase angles of injected current by RTUs excluding the reference bus.
$\underline{\boldsymbol{V}}$	Vector of true complex voltage at buses excluding the reference bus.
$\underline{\delta}$	Vector of voltage phase-angles excluding the reference bus.
$\underline{\boldsymbol{\varepsilon}}$	Vector of complex measurement errors.
m	Number of measurements.
n	Number of buses.

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II. INTRODUCTION

POWER system State Estimation (SE) is a prerequisite for a number of energy management system (EMS) applications running in real time in power system control rooms. It provides input data for economic dispatch, optimal power flow, contingency analysis, ancillary services and wide area protection and control applications [1], [2]. Existing EMSs around the world still rely on measurements provided by supervisory control and data acquisition (SCADA) system. At a hierarchically lower level of the SCADA system, remote terminal units (RTUs) interface various meters to the SCADA system by transmitting telemetry data, such as voltage, current, active and reactive powers, circuit breaker status and other measurements to the master station. The aim of SE is to estimate the system state, i.e. voltage amplitude and phase-angle at all network buses, using the aforementioned measurements [3].

SCADA measurements, however, are nonlinear functions of the system state variables, and therefore, the SE problem is traditionally solved by using iterative algorithms such as Newton's iterative method [4]. Currently, the weighted-least-squares (WLS) estimator is the most widely used approach for solving SE using SCADA measurements [5]. There are, however, several technical challenges related to the application of the WLS estimator to the non-convex SE problem, such as lack of guaranteed convergence. Besides, cumbersome calculation of Jacobian matrix at each iteration as well as the need for post-processing of the WLS estimation for bad data detection and identification (BDDI) impose time-consuming extra efforts to ensure the estimator will function desirably.

In comparison with the WLS estimator, the concept of robust estimator [6] is more effective for solving the estimation problems, in the sense that it inherently counteracts the inclusion of bad data in the measurement set. However, in practice, the application of robust estimators to SE has been quite limited due to the heavy computational burden of the nonlinear state estimator, which involves time-consuming iterations [7]–[9].

In order to overcome the nonlinear nature of the SE process, semidefinite (SDP) and conic programming have been used in [10]–[13] for convexification of power flow equations. These techniques enable solving robust SE problems for large-scale systems in a reasonable time. However, dropping the rank-one constraint might not work in all conditions of the network topology and measurements as observed in the case of optimal power flow problem [14].

Another avenue of research utilizes synchrophasor measurements [15] making the SE problem linear. This requires placing a minimal number of PMUs at some certain locations to ensure system observability [2], [16]–[18]. However, scarcity of PMU measurements makes PMU-only SE unattainable in many today's power systems. Therefore, SE by hybrid SCADA/PMU measurements has been investigated extensively to bridge the gap between the past and future [19]–[22]. The problem formulation though mostly resembles that of SCADA-based SE [23] with the same practice for BDDI.

This paper presents a novel linear robust state estimator, which is suitable for large-scale power systems due to its linear formulation as well as automatic bad data rejection qualities. In particular we build on the work of Schweppe [24] and apply a Schweppe-type estimator with Huber loss function [6] to deal with multiple bad data, automatically. As discussed in [9], leverage measurements are not accounted for properly in the Schweppe-type estimator [24]. According to [6], leverage points, i.e. measurements with diagonal hat matrix values greater than 0.5, are better to be avoided in the estimation. Therefore, we modify the Schweppe-type estimator for these leverage points by using the same weights as Schweppe [24] for non-leverage points but modified weights for leverage points based on the hat matrix elements. In contrast with [9], no additional computations regarding the projection statistics is required as the hat matrix is already available.

Robust state estimation based on M-estimator or IRLS has been studied in the literature. Reference [25] uses nonlinear constrained optimization considering zero-injection buses. A Huber M-estimator is used to reject bad data and an iterative Newton-based primal-dual interior-point approach is used to solve the problem. In [26] an exponential function for the M-estimator is used for solving SE. An iterative Newton-based algorithm is utilized that needs updating both Jacobin matrix and measurement functions at each iteration. Our proposed method, in contrast to [25] is not modeled as a non-linear optimization problem. Moreover, compared to [26], the proposed method does not need calculating Jacobian and measurement functions at each iteration, thanks to the constant coefficient matrix in the proposed formulation.

Using IRLS, [27] basically solves the problem in [9] by orthogonal decomposition and Given Rotations in order to reduce the computational burden of the algorithm. References [28], [29] solve least-absolute-value (LAV) estimator by IRLS instead of linear programming. They use the conventional nonlinear formulation of the SE problem and rely on Newton's algorithm to iteratively solve the problem by updating the measurement function and Jacobian. This is in contrast to the proposed algorithm where both measurement functions and Jacobian matrices remain constant, thereby reducing the computational effort. Reference [30] considers measurement dependencies, yet still needs Jacobian and measurement functions to be updated at each iteration.

This paper extends the linear formulation of [31] in order take advantage of PMU measurements to solve the hybrid SE problem. The generalized M-estimator used will highly speed up the hybrid SE process, while rejecting bad data during the process. It is worth mentioning that IRLS is a tool to solve the proposed linear M-estimator with the least computational effort. This tool is widely utilized in statistics and has been applied to the proposed formulation as well. Moreover, the derivations of covariance matrix of PMU/RTU measurements corresponding to the developed system of equations are rigorously obtained. The linearity of the coefficient matrix in our formulation makes it suitable for robust Schweppe type estimator, which involves much more iterations than the conventional WLS-based SE.

The proposed SE algorithm is generalized in the sense that its input data do not have to be limited to PMU measurements.

TABLE I
CONTRIBUTION OF THE PROPOSED METHOD OVER PREVIOUS SE ALGORITHMS

Reference	[1], [3]	[24]	[7], [8]	[10]–[13]	[16], [17]
Algorithm	WLS	M-E	LAV	SDP	WLS
Measurements	SCADA	SCADA	SCADA	SCADA	PMU
Linear	No	No	No	No	Yes
Robust	No	Yes	Yes	Yes	No
Need GPS	No	No	No	No	Yes
Convexified	No	No	No	Yes	No
Leverage	No	No	No	No	No
Need init. guess	Yes	Yes	No	No	No
Reference	[31]	[9]	[19]–[21]	[22]	Proposed
Algorithm	WLS	M-E	WLS	LAV	M-E
Measurements	SCADA	SCADA	Hybrid	PMU	SCADA PMU or Hybrid
Linear	Yes	No	No	Yes	Yes
Robust	No	Yes	Yes	Yes	Yes
Need GPS	No	No	Yes	Yes	No
Convexified	No	No	No	No	No
Leverage Meas.	No	Yes	No	No	Yes
Need init. guess	No	Yes	Yes	No	No

This paves the way for integrating PMU measurements into existing SCADA measurements while maintaining the linearity of the hybrid state estimator, ensuring a fast solution process.

In the case of purely PMU measurements, the proposed method replicates the state estimator presented in [22]. The contribution in this specific case will be the utilization of an M-estimator instead of LAV estimator and the treatment of leverage measurements if PMU measurements include injection measurements. In other words, in contrast with [22] that assumes no injection measurement exists, the proposed algorithm does not pose such restriction and can readily deal with leverage measurements resulting from injection measurements. Table I compares the proposed and previous methods in terms of different technical characteristics. In brief, with identical measurement inputs, the proposed method overcomes difficulties of traditional state estimator such as the need for initial guess, challenges with convergence and sensitivity to bad data.

III. PRELIMINARIES AND NOTATIONS

The variables referring to voltages and currents in the rest of this paper can be categorized into true, measured, calculated and estimated values. To distinguish between the variables of these categories, $(.)^{meas}$, $(.)^{cal}$ and $\hat{(.)}$ denote measured, calculated and estimated values, respectively. True values are denoted with no superscript. In particular, calculated values are confined to branch-current amplitudes and phase angles of complex current with respect to the phase angle of the sending-end complex voltage (see (5) below). All complex-valued variables (matrices and vectors) are printed in bold, while regular font is used for real-valued matrices and vectors. Matrices and vectors are denoted by $[.]$ and $\underline{.}$ respectively. In particular, $[I]$ represents the identity matrix and $\underline{0}$ denotes all-zero vector of appropriate size. $[(\underline{.})]$ denotes a diagonal matrix whose elements are vector $\underline{.}$. Complex voltages and currents with respect to a certain reference angle (i.e. phase angle of voltage at the reference bus) cannot be measured except by using PMUs (synchrophasors). Nonetheless, branch currents can be expressed as complex values with respect to the voltage phase-angle at the same bus. These complex variables measured by corresponding local RTUs are denoted by $(.)^{loc}$.

IV. EXACT LINEAR FORMULATIONS FOR MEASUREMENTS

In what follows, firstly SCADA measurements used for SE are formulated based on the results from [31], [32]. Next, the derivations corresponding to PMU measurements including the phase-angle measurements and voltage and current phasor measurements are developed. It should be noted that sampling rates of SCADA and PMU measurements are not the same. This problem also known as the time skew problem can be dealt with by buffering PMU measurements [33], [34].

A. SCADA Voltage Amplitude Measurements

Voltage amplitude at any bus is related to the associated complex bus voltage as

$$\boldsymbol{V}_a = V_a e^{j\delta_a} \tag{1}$$

Consequently, the non-ideally measured voltage amplitude at bus a is related to the corresponding complex voltage as

$$\boldsymbol{V}_a = V_a^{meas} e^{j\delta_a} + \varepsilon_{V_a} e^{j\delta_a} \tag{2}$$

where ε_{V_a} is the measurement error for bus voltage a. The phase-angle operator $e^{j\delta_a}$ does not appear in (1) for the slack bus, whose phase angle is set to zero.

B. SCADA Branch Flow Measurements

Let us consider the complex current through line a-b, which can be expressed as

$$\boldsymbol{I}_{ab}^{loc} \triangleq I_{ab} \measuredangle \varphi_{ab} \tag{3}$$

where I_{ab} and φ_{ab} are the current amplitude and current phase-angle (with respect to the complex voltage $\boldsymbol{V}_a$), respectively. These quantities can be obtained by RTUs, and hence (3) can be rewritten as

$$\boldsymbol{I}_{ab}^{loc} = (I_{ab}^{meas} + \varepsilon_{I_{ab}}) \measuredangle (\varphi_{ab}^{cal} + \varepsilon_{\varphi_{ab}}) \tag{4}$$

where φ_{ab}^{cal} is calculated as follows

$$\varphi_{ab}^{cal} = tg^{-1}\left(-\frac{Q_{ab}^{meas}}{P_{ab}^{meas}}\right) \tag{5}$$

I_{ab}^{meas}, P_{ab}^{meas} and Q_{ab}^{meas} are the measured current and active and reactive power through line a-b (from bus a to bus b), respectively. If I_{ab} measurement is not communicated by RTUs it can be calculated by voltage and active and reactive power measurements as addressed in [31].

With reference to the slack bus, the complex current through line a-b may be expressed as

$$\boldsymbol{I}_{ab} = \boldsymbol{I}_{ab}^{loc} e^{j\delta_a} \tag{6}$$

where δ_a is the unknown phase-angle of complex voltage at bus a, with reference to the slack bus. Utilizing the transmission line pi model shown in Fig. 1, we have

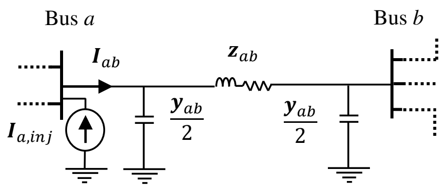

Fig. 1. Illustration of branch current measurement.

$$\boldsymbol{I}_{ab}^{loc} e^{j\delta_a} = (\frac{\mathbf{y_{ab}}}{2} + \frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_a + (-\frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_b \tag{7}$$

In terms of available measurements, (7) can be rewritten as

$$(I_{ab}^{meas} \measuredangle \varphi_{ab}^{cal}) e^{j\delta_a} + \boldsymbol{e}_{\boldsymbol{I}_{ab}} = (\frac{\mathbf{y_{ab}}}{2} + \frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_a - (\frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_b \tag{8}$$

where $\boldsymbol{e}_{\boldsymbol{I}_{ab}}$ is the complex measurement error written as

$$\boldsymbol{e}_{\boldsymbol{I}_{ab}} = \varepsilon_{I_{ab}} \measuredangle (\varphi_{ab}^{cal} + \delta_a + \varepsilon_{\varphi_{ab}}) + \varepsilon_{\varphi_{ab}} I_{ab}^{meas} \measuredangle (\delta_a + \varepsilon_{\varphi_{ab}} + \frac{\pi}{2}) \tag{9}$$

C. SCADA Injection Measurements

Fig. 1 shows current injection at bus a. Similar to (8), and based on the first Kirchhoff's law, one obtains:

$$(I_{a,inj}^{meas} \measuredangle \varphi_{a,inj}^{cal}) + \boldsymbol{e}_{\boldsymbol{I}_a} = \sum_{b \in L_a} (\frac{\mathbf{y_{ab}}}{2} + \frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_a - (\frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_b \tag{10}$$

where $I_{a,inj}^{meas}$ and $\varphi_{a,inj}^{cal}$ are the calculated/measured injected current amplitude and phase angle, respectively, obtained by RTU measurements. They may be either measured directly or calculated indirectly from injected active and reactive power flow measurements similarly to (5). L_a is the set of branches connected directly to bus a.

D. PMU Phase-Angle Measurements

PMUs are capable of measuring time-synchronized bus-voltage phase angles, bus-voltage amplitudes and branch current phasors, all with respect to the phase angle of voltage at the reference bus. Phase-angle measurement of voltage at bus a can be expressed by first-order Taylor series approximation of $e^{j\delta_a}$ as

$$e^{j\delta_a} \approx e^{j\delta_a^{meas}} + je^{j\delta_a^{meas}} \varepsilon_{\delta_a} \tag{11}$$

where δ_a and δ_a^{meas} are true and measured phase angle of voltage phasor at bus a, respectively, and ε_{δ_a} is the measurement error.

E. PMU Voltage Phasor Measurements

Phasor measurement of voltage at bus a can be written as

$$\boldsymbol{V}_a^{meas} + \boldsymbol{\varepsilon}_{\boldsymbol{V}_a} = \boldsymbol{V}_a \tag{12}$$

where $\boldsymbol{\varepsilon}_{\boldsymbol{V}_a}$ is the complex voltage phasor measurement error expressed in terms of magnitude and phase-angle measurement errors as follows.

$$\boldsymbol{\varepsilon}_{\boldsymbol{V}_a} = e^{j\delta_a}(\varepsilon_{V_a} + j\varepsilon_{\delta_a} V_a) \tag{13}$$

F. PMU Current Phasor Measurements

In contrast to RTUs, PMUs can directly measure $\boldsymbol{I}_{ab}$ [35] in (8) as

$$\boldsymbol{I}_{ab}^{meas} + \boldsymbol{\varepsilon}_{\boldsymbol{I}_{ab}} = (\frac{\mathbf{y_{ab}}}{2} + \frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_a + (-\frac{1}{\mathbf{z_{ab}}})\boldsymbol{V}_b \tag{14}$$

where $\boldsymbol{\varepsilon}_{\boldsymbol{I}_{ab}}$ is the complex current phasor measurement error expressed as

$$\boldsymbol{\varepsilon}_{\boldsymbol{I}_{ab}} = e^{j\theta_{ab}}(\varepsilon_{I_{ab}} + j\varepsilon_{\theta_{ab}} I_{ab}) \tag{15}$$

where I_{ab} and θ_{ab} are current phasor magnitude and phase angle, respectively.

V. Covariance Matrix of Measurements

A. SCADA Measurements

SCADA measurements formulated in this paper consist of voltage magnitude as well as current local phasor measurements. The variance of voltage measurement introduced in (2) can be obtained as

$$\sigma_{V_a^{Re}}^2 = \mathbb{E}\{\{\varepsilon_{V_a} cos(\delta_a)\}^2\} - \{\mathbb{E}\{\varepsilon_{V_a} cos(\delta_a)\}\}^2 = \sigma_{V_a}^2 cos(\delta_a)^2 \tag{16}$$

And similarly

$$\sigma_{V_a^{Im}}^2 = \sigma_{V_a}^2 sin(\delta_a)^2 \tag{17}$$

The real and imaginary parts of current phasors given in (8) and (10) have the following variances.

$$\sigma_{I^{Re}}^2 = (\sigma_I cos(\varphi))^2 + \sigma_\varphi^2 (I sin(\varphi))^2 \tag{18}$$

$$\sigma_{I^{Im}}^2 = (\sigma_I sin(\varphi))^2 + \sigma_\varphi^2 (I cos(\varphi))^2 \tag{19}$$

It is assumed that the current amplitude measurement as well as its associated variance, i.e. σ_I^2, is available. The phase angle of current phasors in (8) and (10) are given by (5). Accordingly, σ_φ in (18) and (19) is obtained in terms of the associated active and reactive power measurements and their variances, i.e. σ_P^2 and σ_Q^2, as follows.

$$\sigma_\varphi^2 = \frac{Q^2\sigma_P^2 + P^2\sigma_Q^2}{(Q^2 + P^2)^2} \tag{20}$$

B. PMU Measurements

With reference to (11) real and imaginary parts of phase angle measurements are given by

$$\sigma_{\delta^{Re}}^2 = \sigma_\delta^2 sin(\delta)^2 \tag{21}$$

$$\sigma_{\delta^{Im}}^2 = \sigma_\delta^2 cos(\delta)^2 \tag{22}$$

where σ_δ^2 is the known variance of phase angle measurements. According to (12) the real and imaginary parts of voltage synchrophasors have the following variances.

$$\sigma_{V^{Re}}^2 = (\sigma_V^{PMU} cos(\delta))^2 + \sigma_\delta^{PMU^2} (V^{PMU} sin(\delta))^2 \tag{23}$$

$$\sigma_{V^{Im}}^2 = (\sigma_V^{PMU} sin(\delta))^2 + \sigma_\delta^{PMU^2} (V^{PMU} cos(\delta))^2 \tag{24}$$

where V^{PMU} and δ are respectively the magnitude and phase angle of the measured voltage synchrophasor. Standard deviations of these measurements, i.e. σ_V^{PMU} and σ_δ, are known. Quite similarly for current synchrophasors given in

(14) the following variances are developed for the real and imaginary parts.

$$\sigma^2_{I^{Re}} = (\sigma_I^{PMU} cos(\theta))^2 + \sigma_\theta^{PMU^2} (I^{PMU} sin(\theta))^2 \quad (25)$$

$$\sigma^2_{I^{Im}} = (\sigma_I^{PMU} sin(\theta))^2 + \sigma_\theta^{PMU^2} (I^{PMU} cos(\theta))^2 \quad (26)$$

where I^{PMU} and θ are the magnitude and phase angle of measured current synchrophasor, respectively. The covariance matrix of measurement errors for the system of equations (30) below can therefore be written as

$$[R] = \begin{array}{c} diag[\underline{\sigma}^2_{V^{Re}_{RTU}}, \underline{\sigma}^2_{I^{Re}_{RTU}}, \underline{\sigma}^2_{\delta^{Re}}, \underline{\sigma}^2_{V^{Re}_{PMU}}, \underline{\sigma}^2_{I^{Re}_{PMU}} \\ \underline{\sigma}^2_{V^{Im}_{RTU}}, \underline{\sigma}^2_{I^{Im}_{RTU}}, \underline{\sigma}^2_{\delta^{Im}}, \underline{\sigma}^2_{V^{Im}_{PMU}}, \underline{\sigma}^2_{I^{Im}_{PMU}}] \end{array} \quad (27)$$

VI. Generalized Linear Formulation Utilizing Hybrid SCADA and PMU Measurements

While PMU measurements can be sufficient for power system SE if observability is ensured [36], using hybrid SCADA and PMU measurements are favorable for SE in existing power systems [37]. The proposed hybrid SE formulation is developed first by complex variables and next by real variables.

A. Complex Formulation

Equations (2), (8) and (10) show nonlinear relationships between measurements and system states, i.e. bus voltage amplitudes and phase angles. The key idea here is to properly rearrange these equations in order to come up with a new linear formulation of the problem. To this end, the exponential phase-angle operators appearing in (1), (7) and (10) are included in the vector of state variables, together with the complex voltages.

By combining SCADA and PMU measurements, one obtains the following system of linear equations:

$$\begin{bmatrix} [I] & \left[\begin{pmatrix} 0 \\ -\underline{V}^{meas} \end{pmatrix}\right] \\ [Y_{se}][A]+[Y_{sh}] & \left[\begin{pmatrix} \underline{0} \\ -\underline{I}^{meas}\measuredangle\underline{\varphi}^{cal} \end{pmatrix}\right] \\ [Y_{bus}] & \left[\begin{pmatrix} 0 \\ -\underline{I}^{meas}_{inj}\measuredangle\underline{\varphi}^{cal}_{inj} \end{pmatrix}\right] \\ [0] & [I] \\ [I] & [0] \\ [Y_{se}]\,[A]+[Y_{sh}] & [0] \end{bmatrix} \begin{bmatrix} V_1 \\ \underline{\boldsymbol{V}} \\ e^{j\underline{\delta}} \end{bmatrix} + \boldsymbol{\underline{\varepsilon}} = \begin{bmatrix} \begin{bmatrix} V_1^{meas} \\ \underline{0} \end{bmatrix} \\ \begin{bmatrix} \underline{I}_1^{meas}\measuredangle\underline{\varphi}_1^{cal} \\ \underline{0} \end{bmatrix} \\ \begin{bmatrix} \underline{I}^{meas}_{1,inj}\measuredangle\underline{\varphi}^{meas}_{1,inj} \\ \underline{0} \end{bmatrix} \\ [e^{j\underline{\delta}^{meas}}] \\ [\underline{\boldsymbol{V}}^{meas}] \\ [\underline{\boldsymbol{I}}^{meas}] \end{bmatrix} \quad (28)$$

where $[(.)]$ denotes a diagonal matrix consisting of elements in vector $(.)$. In (28) the last three rows are related to PMU measurements whereas the other rows are related to SCADA measurements. In a more compact form, (28) becomes

$$[\boldsymbol{H}]\,\underline{\boldsymbol{x}} + \underline{\boldsymbol{\varepsilon}} = \underline{\boldsymbol{z}} \quad (29)$$

where $[\boldsymbol{H}]$ is the $m \times (2n-1)$ state matrix, $\underline{\boldsymbol{x}}$ is the $(2n-1) \times 1$ vector of unknown state variables, $\underline{\boldsymbol{z}}$ is the $m \times 1$ vector of complex measurements and $\underline{\boldsymbol{\varepsilon}}$ is the $m \times 1$ vector of complex measurement errors. Both $[\boldsymbol{H}]$ and $\underline{\boldsymbol{z}}$ are composed of either measurements in (2), (8), (10)-(14) or network parameters, and hence known.

B. Real Formulation

Separating (29) into real and imaginary parts yields

$$\begin{bmatrix} [H^R] & [-H^I] \\ [H^I] & [H^R] \end{bmatrix} \begin{bmatrix} \underline{x}^R \\ \underline{x}^I \end{bmatrix} + \begin{bmatrix} \underline{\varepsilon}^R \\ \underline{\varepsilon}^I \end{bmatrix} = \begin{bmatrix} \underline{z}^R \\ \underline{z}^I \end{bmatrix} \quad (30)$$

where $(.)^R$ and $(.)^I$ denote the real and imaginary parts of the complex argument, respectively. In a compact form, this real-valued system of equations is now written as

$$[M]\underline{y} + \underline{e} = \underline{b} \quad (31)$$

The unknown state variable vector is comprised of

$$y = \begin{array}{c} [V_1 \ V_2 cos\delta_2 \ ... \ V_n cos\delta_n \ cos\delta_2 \ ... \ cos\delta_n \\ V_2 sin\delta_2 \ ... \ V_n sin\delta_n \ sin\delta_2 \ ... \ sin\delta_n]^T \end{array} \quad (32)$$

which is obtained by weighted linear least-squares estimation as follows.

$$y = (M^T R^{-1} M)^{-1} M^T R^{-1} b \quad (33)$$

where $R = \mathbb{E}\{\underline{e}\underline{e}^T\}$ has been obtained in the previous section. Since $[\boldsymbol{H}]$ in (28) and therefore $[M]$ is not constant, δ_2,...,δ_n in (32), which are estimated in (31), are used to transfer all measurements to the right hand side of (31). If $[\boldsymbol{H}]$ in (28) is partitioned as $[\boldsymbol{H}] = [[\boldsymbol{H}_1]\,[\boldsymbol{H}_2]]$, where $[\boldsymbol{H}_1]$ is a constant matrix comprising of first n columns of $[\boldsymbol{H}]$, then (29) can be rewritten as

$$[\boldsymbol{H}_1] \begin{bmatrix} V_1 \\ \underline{\boldsymbol{V}} \end{bmatrix} + \underline{\boldsymbol{\varepsilon}} = \underline{\boldsymbol{z}} - [\boldsymbol{H}_2]\, e^{j\hat{\underline{\delta}}} \quad (34)$$

After separating (34) into real and imaginary parts, a real-valued system of equations is obtained as

$$[N]\underline{u} + \underline{\epsilon} = \underline{w} \quad (35)$$

where $[N]$ is constant and unknown vector $\underline{u}$ is as follows.

$$\underline{u} = [V_1 \ V_2 cos\delta_2 \ ... \ V_n cos\delta_n \ V_2 sin\delta_2 \ ... \ V_n sin\delta_n]^T \quad (36)$$

Once (35) is solved by the proposed linear robust M-estimator, the unknown state variables are attainable from the elements of $\underline{u}$ as

$$\hat{V}_i = \sqrt{\hat{u}_i^2 + \hat{u}_{n+i}^2} \quad (37)$$

$$\hat{\delta}_i = tg^{-1} \frac{\hat{u}_{n+i}}{\hat{u}_n} \quad (38)$$

VII. Robust Linear M-Estimator for Hybrid SE

An M-estimator is a generalization of the weighted least-squares estimator, maximizing a likelihood function. In contrast to the WLS estimator, an M-estimator is able to deal with outliers more effectively [6].

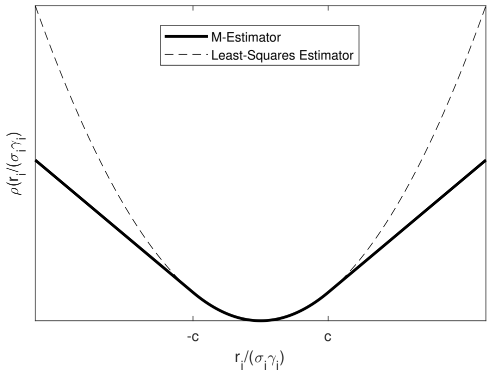

Fig. 2. Schweppe-type M-Estimator.

A. Background

The objective function of the Schweppe-type M-estimator is shown in Fig. 2. Mathematically, this objective function is expressed as

$$\rho(\frac{r_i}{\gamma_i \sigma_i}) = \begin{cases} \frac{1}{2}(\frac{r_i}{\gamma_i \sigma_i})^2 & \text{for } |(\frac{r_i}{\gamma_i \sigma_i})| \leq c, \\ c(|(\frac{r_i}{\gamma_i \sigma_i})| - \frac{1}{2}c), & \text{otherwise.} \end{cases} \tag{39}$$

In [24] it is shown that $r_i \sim \mathcal{N}(0, \sigma_i^2(1-h_i))$, where r_i is the residual of the $i$the measurement and h_i is the $i$the diagonal element of hat matrix, i.e. $N(N^T R^{-1} N)^{-1} N^T R^{-1} w$ in (35). Accordingly, Schweppe $et\ \ al.$ in [24] pick $\gamma_i = \sqrt{1-h_i}$ so that an outlier whose residual is c times larger than its standard deviation lies in the linear part of the objective function. The application of the M-estimator in SE resembles the WLS algorithm, except that in an iterative process the weights of measurements are updated based on their associated residuals in the previous iteration. This approach is known as iterative reweighted least squares (IRLS) and is traditionally applied to SCADA-based SE as follows [9], [24].

$$\Delta \underline{x}^{(k)} = (H^{T^{(k)}} R^{-1} Q^{(k)} H^{(k)})^{-1} H^{T^{(k)}} R^{-1} Q^{(k)} (\underline{z} - \underline{h}(x^(k))) \tag{40}$$

where $H^{T^{(k)}} = \left[H_1^{T^{(k)}} H_2^{T^{(k)}} ... H_m^{T^{(k)}} \right]$, $H_i{}^{(k)} = \frac{\partial h_i}{\partial \underline{x}^{(k)}}$, k is the iteration number, Q is the diagonal weight matrix and $\underline{h}(\underline{x})$ is the vector of measurement function.

B. Comments on Previous Research

1) Leverage Measurements: The problem of leverage measurements arises in the foregoing formulation with $\gamma_i = \sqrt{1-h_i}$. Leverage measurements, by definition are those with the property of $h_i \to 1$. Therefore ith leverage measurement with even a gross error e_i may not lie in the linear part of the ρ function. The reason is as follows.

$$r_i = z_i - \hat{z}_i = \sum_{k=1}^{m} S_{ik} e_k \approx S_{ii} e_i = (1-h_i) e_i \tag{41}$$

where S is the residual covariance matrix. It can be seen that the argument of ρ function in (39) will be

$$\frac{r_i}{\gamma_i \sigma_i} = \frac{\sqrt{1-h_i}}{\sigma_i} e_i \tag{42}$$

Therefore, a bad leverage measurement still lies in the quadratic part of the objective function as $h_i \to 1$. This problem has been discussed in [38]. However, alternative weights, i.e, γ_i in (39), used in the literature are too time-consuming to compute, in particular for large-scale networks with lots of measurements.

2) Updating $H^{(k)}$ and $\underline{h}(\underline{x}^{(k)})$ in (40): Although the problem of leverage measurements in [24] is addressed in [9], computational burden is still an issue in both of them. The reason is the need for updating the Jacobian matrix $H^{(k)}$ and measurement function $\underline{h}(\underline{x}^{(k)})$ at each iteration k. For example a 10,000-bus network with measurement redundancy of 2 will have a Jacobian matrix of the size 40,000*20,000 and 40,000 measurement functions. Based on various conditions, including the amount of bad data, the number of iterations is variable, but is reported to have an average number of 10 [6, p.187].

C. Proposed Method

Thus far, we have presented a generalized linear formulation for SE consisting of either SCADA or PMU or hybrid SCADA/PMU measurements and determined the measurement weights.

The linear formulation in this paper aims to address the two technical difficulties mentioned. To effectively deal with leverage measurements, one needs to be cautious about measurements with $h_i > 0.5$ as recommended in [6]. Accordingly, we modify the weights of residuals in (39) as follows.

$$\gamma_i = \begin{cases} \sqrt{1-h_i} & \text{for } h_i < 0.5, \\ 1-h_i & \text{otherwise.} \end{cases} \tag{43}$$

It should be noted that given $0 \leq h_i \leq 1$ [3], we choose smaller values of γ_i for leverage measurements. Also, according to (42), we expect that bad leverage measurements lie in the linear part of the ρ function. It should be noted that there is a trade-off between re-weighting and not re-weighting the leverage measurements. The downside is that by decreasing their weight, their ability to identify other bad data is compromised. However, if the measurement set is large enough, as assumed here and is the case in practice, this will not be a problem. Moreover, with high redundancy level, re-weighting bad leverage measurements help identify and suppress them in the SE algorithm, which is our goal. Regarding the second issue above, the linear formulation in this paper leads to the following system of equations in contrast to (40).

$$\underline{u}^{(k)} = (N^T R^{-1} Q^{(k)} N)^{-1} N^T R^{-1} Q^{(k)} \underline{w} \tag{44}$$

where Q is a diagonal matrix whose $i$the element is 1 if $|r_i/(\sigma_i \gamma_i)| \leq c$ and $c(\sigma_i \gamma_i / r_i).sign(r_i/(\sigma_i \gamma_i))$ otherwise. In comparison with (40), at each iteration k there is no need to update Jacobian matrix $H^{(k)}$ and measurement function $\underline{h}(\underline{x}^{(k)})$. This expedites the algorithm for large-scale networks. The IRLS algorithm is adopted to determine the updated elements of $Q^{(k)}$ based on the value of $r_i^{(k-1)}$. The iteration begins with conventional WLS, where Q is the identity matrix.

It should be noted that observability is a requirement for the proposed method. When facing inadequate measurements

TABLE II
AVERAGE RMSE FOR DIFFERENT SYSTEMS (NOISY AND ERRONEOUS MEASUREMENTS)

System	WLS	IRLS
14-bus	0.0057	0.0048
30-bus	0.0074	0.0066
57-bus	0.0089	0.0081
118-bus	0.0046	0.0036
300-bus	0.0109	0.0099
1354-bus	0.0046	0.0043
9241-bus	0.0041	0.0036

we may resort to pseudo-measurements with lower accuracy (larger variance) levels [1]. However, measurement redundancy is large enough in today's systems. For example, in [39] it is reported that the redundancy level (number of measurements divided by number of states) for the Spanish system is 3.6.

VIII. CASE STUDIES

In this section, the proposed linear M-estimator is compared with the linear WLS estimator introduced in [31] as well as the conventional Gauss-Newton-based algorithm [3] . The performance index used for comparison was the root-mean-square error (RMSE) defined as

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} |\hat{\boldsymbol{V}}_i - \boldsymbol{V}_i|^2}{n}}. \tag{45}$$

where $\hat{\boldsymbol{V}}_{\boldsymbol{i}} = \hat{V}_i e^{j\hat{\delta}_i}$ is the estimated complex voltage at bus i obtained by (37) and (38). In practice, the true complex voltages $\boldsymbol{V}_i$ are never known. This is not the case in a simulation environment, where true $\boldsymbol{V}_i$ values are simulated as the output of the load flow function in MATPOWER [40].

A. SCADA-Only State Estimation

The proposed M-estimator is tested on the IEEE 118-bus test system [41]. Voltage and current measurement errors are assumed to be non-correlated Gaussian zero-mean noise with standard deviations (STD) of 0.001 and 0.002 pu, respectively. Standard deviations of active and reactive power measurements are 0.002 pu. To generate bad data, 20% of randomly chosen branch currents have been polluted by non-correlated Gaussian zero-mean noise with a standard deviation of 0.1 pu. The detection threshold is set to c=3 [3]. Fig. 3 reflects the results of 100 Monte Carlo simulations. In this figure, it is assumed that branch currents from both ends of each line are available.

Table II summarizes the estimation results for other IEEE test systems as well as two large-scale systems. It can be observed that similarly to Fig. 3, the proposed M-estimator outperforms the WLS estimator in presence of bad data.

Figs. 4 and 5 present the RMSE statistics associated with conventional Gauss-Newton-based nonlinear WLS and the proposed linear WLS, respectively. The conventional SE algorithm is available in $doSE.m$ in MATPOWER toolbox [40]. For large-scale networks the algorithm has been modified by defining all large matrices as sparse matrices to avoid memory problem faced in the original code. The maximum tolerance

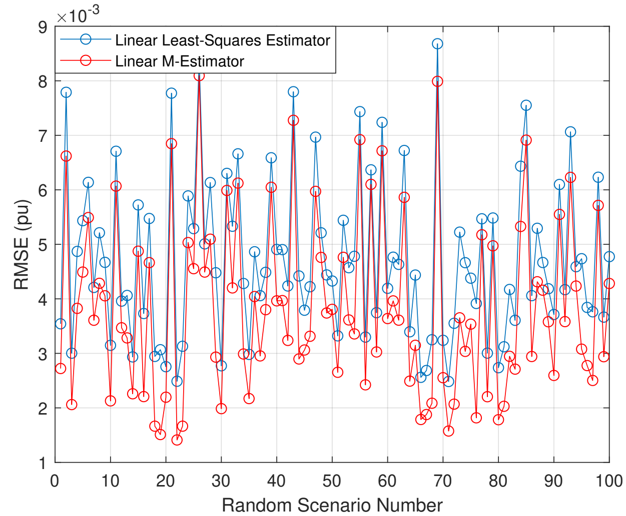

Fig. 3. IRLS versus WLS estimator for the IEEE 118-bus system.

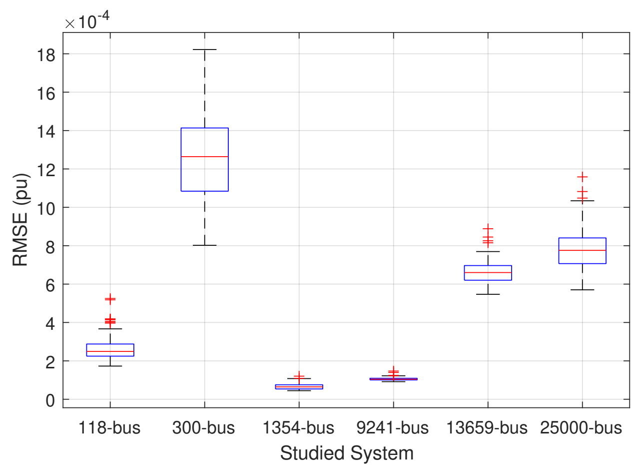

Fig. 4. RMSE statistics for conventional nonlinear WLS (No gross error).

for convergence was set to $1e$-6 and the algorithm converges in 5 iterations.

The measurement set includes voltage measurements from all buses and flow measurements from both ends of every line. The measurement set does not include any gross error so that the two algorithms can be compared fairly based on the RMSE index. It can be observed that the accuracy of the proposed algorithm is comparable to the conventional algorithm for different case studies in Figs. 4 and 5. It should be noted that WLS is the best linear unbiased estimator. However, the conventional SE algorithm based on nonlinear WLS is not guaranteed to be the best estimator as can be seen from Figs. 4 and 5.

Fig. 6 shows the impact of a gross error on WLS- and proposed M-Estimator. The system under study is the IEEE 14-bus network and the measurement configuration is taken from [9]. The studied leverage measurement is the injection at bus 6 whose corresponding hat value is $h = 0.75$ as shown in Fig. 6. This injection measurement is multiplied by 0.8, 0.9, 1.1 and 1.2 to model the gross error in the measurement set.

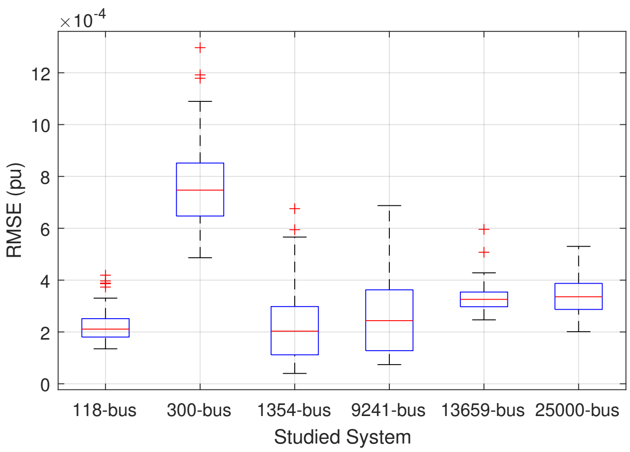

Fig. 5. RMSE statistics for proposed linear WLS (No gross error).

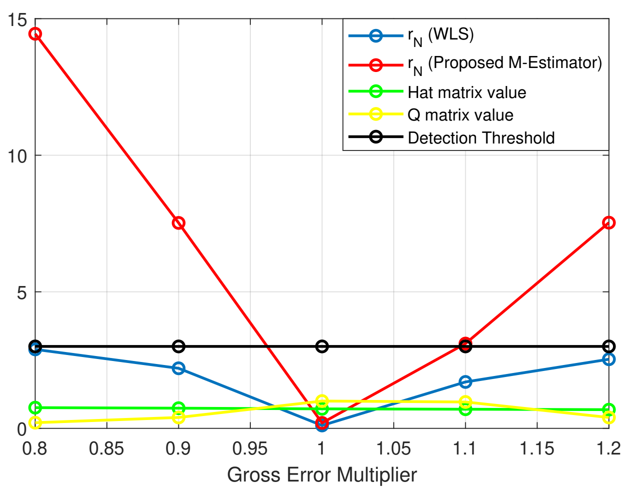

Fig. 6. Impact of gross error on WLS- and proposed M-Estimator (Gross error is injection measurement at bus 6 in IEEE 14 bus network)

It can be seen that in all cases, the normalized residual related to this measurement defined by (42) lies below threshold 3, causing the largest normalized residual test [3] in WLS-estimator to fail. However, $\frac{r_i}{\gamma_i \sigma_i}$ where γ_i is defined by (43) in the proposed algorithms lies above the threshold, making this measurement down-weighted by the factor calculated in matrix Q. In order to compare the proposed algorithm with conventional IRLS, the injection at bus 6 is multiplied by 1.25 to simulate a gross error. As shown in Table III, this is a leverage measurement. The weights for measurements for the conventional and proposed algorithms are reflected in Tables IV and V, respectively. It is evident from (43) that the proposed method suppresses this bad data more severely than the conventional IRLS.

Note that not all leverage measurements are down-weighted, while non-leverage measurements may be down-weighted due to interaction with bad data at the first iteration. However, as the iterations proceed and the actual bad data is severely suppressed, other measurements will not be down-weighted eventually.

TABLE III
LEVERAGE MEASUREMENTS IN 14-BUS NETWORK

Leverage Measurement	Hat matrix value (h_i)
I_{5-4}	0.6212
Inj_2	0.7357
Inj_4	0.7784
Inj_6	0.7552
Inj_{10}	0.5982
Inj_{11}	0.6635
Inj_{12}	0.6053
Inj_{13}	0.7586
Inj_{14}	0.6016

TABLE IV
MEASUREMENT WEIGHTS IN THE CONVENTIONAL IRLS [24]

iter.	I_{5-6}	Inj_1	Inj_6	Inj_{11}	Inj_{12}	Inj_{13}
1	0.85	0.976	0.49	0.69	0.65	0.60
2	0.83	0.982	0.58	0.75	0.72	0.68
3	1.00	0.969	0.39	0.73	0.67	0.58
4	0.94	0.975	0.50	0.71	0.68	0.62
5	1.00	0.968	0.34	0.80	0.71	0.60
6	1.00	0.972	0.48	0.69	0.66	0.58
RMSE	9.8161e-04					

Table VI compares the proposed and conventional state estimation in terms of computation time. For each studied system, an average time of 100 Monte-Carlo simulations is reported. Moreover, as both approaches need diagonal elements of the hat matrix, the associated computation times that are far larger than those of the estimators, are reported separately. It can be seen that the computation time of both estimators are comparable. The proposed linear estimator is shown to be faster than the conventional nonlinear one for all cases. It is well-known that the computation burden for calculating diagonal elements of the hat matrix is the most challenging in the SE. It should be noted that the figures reported in Table VI in this regard aim to make a comparison between the proposed and conventional algorithms. They can, however, be optimized for example by sparse inverse method [3].

B. Hybrid SCADA-PMU State Estimation

Fig. 7 shows the effect of including additional PMUs into the SCADA-based state estimation. For any number of PMUs considered, an optimization problem is run to determine the optimal locations of PMUs in terms of maximizing the observability. Two cases for measurements are studied, where the first case contains no erroneous measurements while in the second case 20 % of measurements are polluted with gaussian noise with standard deviation of 0.1 pu. Fig. 7 demonstrates the advantage the M-estimator over WLS-estimator. For the case of no erroneous measurement, both WLS- and M-estimator give the same result. However, with gross errors in measurements, the M-estimator results in more accurate estimates compared to the WLS-estimator. As expected, the more the number of PMUs in the system, the more accurate the state estimation. It should be noted that The saturation effect of additional PMUs on the accuracy of the estimation [18] is also evident in this figure.

TABLE V
MEASUREMENT WEIGHTS IN THE PROPOSED IRLS

iter.	I_{5-6}	Inj_6	Inj_{11}	Inj_{12}	Inj_{13}	Inj_{14}
1	0.79	0.29	0.62	0.54	0.37	0.92
2	1.00	0.18	0.77	0.64	0.32	1.00
3	1.00	0.13	1.00	0.87	0.35	1.00
4	1.00	0.11	1.00	1.00	0.44	1.00
5	1.00	0.10	1.00	1.00	0.57	1.00
6	1.00	0.09	1.00	1.00	0.74	1.00
7	1.00	0.09	1.00	1.00	0.94	1.00
8	1.00	0.09	1.00	1.00	1.00	1.00
RMSE	4.6067e-04					

TABLE VI
COMPARISON BETWEEN COMPUTATION TIME OF THE CONVENTIONAL NONLINEAR AND PROPOSED LINEAR STATE ESTIMATION

Network	Estimator Computation (s)		Hat Matrix Computation (s)	
	Conventional	Proposed	Conventional	Proposed
118-Bus	0.0079	0.0022	0.4538	0.1924
300-Bus	0.0139	0.0042	2.751	0.9383
1354-Bus	0.0560	0.0180	69.201	40.3445
9241-Bus	1.0174	0.1680	4992.9	3158.3
13659-Bus	1.5365	0.2171	8919.3	5578.8
25000-Bus	2.9926	0.5891	27161.4	17074.3

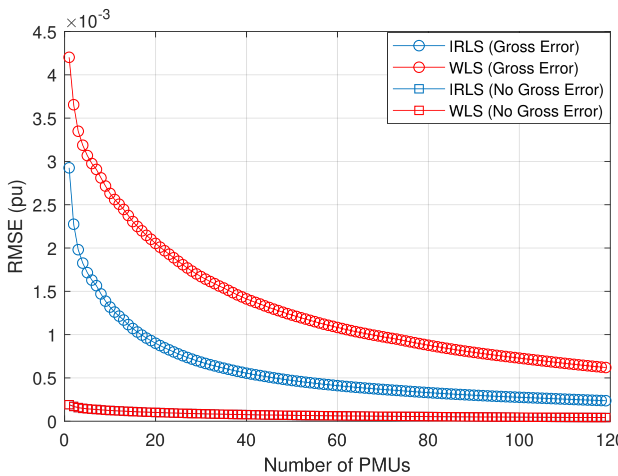

Fig. 7. Influence of additional PMUs on RMSE of the 118-bus system.

IX. CONCLUSION

In this paper a robust hybrid linear state estimator, dealing with bad data in power system state estimation, has been presented. In contrast to previous robust state estimators, the proposed algorithm is generalized in the sense that it can make use of SCADA data, PMU data and a combination of these two as input data. The proposed estimator is linear with no approximations, resolving the problem of convergence to local minima in state estimators formulated in a non-linear form. Moreover, the proposed algorithm deals with a constant matrix in the iterative reweighted least squares method while the conventional method requires building a new Jacobian matrix in each iteration. These two features of the proposed method distinguish it from the conventional robust state estimation. An extension of the proposed method can include zero injections as equality constraints by the Lagrangian method. This will still keep the formulation linear and would be an area for further research.

Simulation results show that the proposed robust estimator and the conventional WLS estimator are comparable in terms of accuracy and computational burden. In presence of gross measurement errors, the proposed method outperforms the WLS algorithm by reweighting the outliers and therefore decreasing their impact.

Hybrid PMU/SCADA formulation has also been tested with different number of PMUs in the system. The improvement in the accuracy of the results is significant after adding a few PMUs while the improvement slows down as the number of PMUs increases. As today's power are flooded with various SCADA and PMU measurements, the proposed robust estimator can be an effective tool to enhance the accuracy of SE without sacrificing the speed of the process.

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