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Since the 1970s, scholars have begun to pay attention to the presentation of women in Bede's *Ecclesiastical History of the English People*, the main source for the early history of Britain (from the first to eighth centuries AD). Vastly different conclusions have been drawn, ranging from positivist approaches which saw the period as a golden age for women to rather more negative assessments, which argue that Bede suppressed the role of women. By analysing the concept of communicability and relevance of certain nodes in complex networks we show how Bede's *Ecclesiastical History* affords women complex and nuanced social roles. In particular we can show the independent importance of certain abbesses, which is a significant result and challenges much of the existing scholarship on Bede's attitude to female power.

Keywords: Gendered Networks and Communicability and Node Relevance and Medieval History

1. Introduction

Over the last few years, there has been an explosion in the use of network science as a quantitative approach to analysing interpersonal relationships in human societies [1, 2]. Notwithstanding the fact that applications of networks in the Social Sciences date back more than 5 decades [3], this renewed interest in the subject is mainly due to the increase in computational capability at our disposal. As a result, historians have also begun looking to these digital methodologies with the intention of gaining new insights into their subject matter [4–6]. Much of the work of historians in this area tries to reconstruct 'real' social networks, based on data from a variety of sources, to understand the power of communities in the past [7]. This article, by contrast, builds on recent studies of character networks in narratives, fictional or otherwise, which seek to reconstruct how individual texts or sets of texts present the connections between actors they describe, and with what results [8–11]. A benefit of the study of character networks is that it not only allows us to unveil complex hidden structures in past societies, as network science generally does, but to reveal unspoken principles underlying the representation of human relationships through investigation of who the authors of these texts connected with whom. For this reason, we are seeking to show that it lends itself particularly well to the study of the role of women in past societies, as it allows us to understand whether gendered attitudes - by authors of specific texts and potentially the societies they lived in - affected how these roles were represented. The employment of network analysis to investigate gender stereotypes and the social roles of women is a surprisingly underused approach in the Humanities at least, but can perhaps be traced back to a general tendency, also noticeable in in data science, to not systematically collect data on gender from the historical record [12].

In order to do this, we apply this approach to the representation of women in Bede's *Historia Ecclesiastica Gentis Anglorum* (henceforth called *HE*) a text completed in the north-east of Britain in AD 731 [13]. Bede's *HE* offers an unusually comprehensive and coherent picture of events in Britain in this period. However, although his account is without parallel for our knowledge of British history in the seventh and early-eighth centuries, we should remember that Bede was a monk, who had entered his monastery at the age of seven and spent his entire life in a monastic milieu. Since the 1970s, much scholarly attention has been paid to Bede's presentation of women and vastly different conclusions have been drawn. These vary from positivist approaches which saw the period as a golden age for women [14]; negative assessments which argue that Bede actively suppressed the role of women, both queens and abbesses, in his book [15, 16]; and more recent studies which argue that, when Bede's *HE* is read alongside his far more numerous works on theology, his subtle presentation of the agency of queens in the *HE* becomes more apparent [17]. This article presents the preliminary results of our network analysis of Bede's *HE*, and our findings challenge the argument that Bede suppressed the role of women in the book.

We specifically analyse the communicability of nodes, *i.e.* their ability to send or receive information [18–20]. We compare temporal measures with aggregate ones and discuss the importance of characters according to the definitions of [21–23]. By 'importance' we mean the characters that our analysis reveals were important to Bede in his construction of the narrative. Our results show that some women were indeed very important within the network structure of the narrative and this importance is unrelated to the powerful men to which they were connected. The constitutive role of certain women in this historical character network would not emerge without the techniques applied here.

This paper is organized as follows: in the next section we briefly discuss Bede's work in its historical context and the reason for choosing it as a case study. This is followed by a section on communicability for aggregate and temporal networks and the definition of node relevance. We then present our results, with particular emphasis on women and discuss those from a historical viewpoint. We conclude with some perspectives for future work and discussion on the use of networks in history.

2. Bede's Ecclesiastical History of the English People and its Network

The Venerable Bede (*c.* 673 – 735) was a monk in Northumbria and one of the foremost scholars of Western Europe of his time. He wrote extensively on theology, history, mathematics, natural phenomena and time reckoning [24–26].

Notwithstanding the extensive range of his literary production, he has gone down in history as the 'Father of English History' because his work is our main source for the early history of Britain. Bede was part of extensive intellectual and ecclesiastical networks, including the archbishopric of Canterbury and other monasteries and churches throughout Britain, many of whom assisted in his efforts to learn about the history of his people. Thanks to the efforts of the founding abbot of his monastery, Benedict Biscop (d. 690), who visited Rome five times and always returned with books and other ecclesiastical resources, Bede had access to one of the best stocked libraries anywhere in Europe at this time. The depth of his learning is apparent in his historical, theological and scientific writings. In the *HE*, Bede endeavoured to reveal his own people's role in the unfolding of salvation history by recounting the story of their conversion to Christianity and burgeoning membership of the universal church. His work encompasses approximately 800 years, from the Roman conquest of Britain by Julius Caesar in 55 *BC* to AD 731.

Bede's *HE* is divided into five books which are to a certain extent chronologically ordered. These books are further divided into chapters, ranging from 20 to 34 depending on the book. Book 1 opens with a description of Britain's geographical position and covers the longest period of time running from Julius Caesar's first invasion in the first century BC up to the late sixth century and the arrival in Kent of Christian missionaries sent from Rome. Books 2 to 5 recount the gradual movement of Christianity from Kent to the northern kingdom of Northumbria, including the significant setbacks experienced by the missionaries, and by the conclusion of the book Bede indicates that the whole island has accepted Christianity. Books 3 and 4 chart the movement from early Christianity to the mature establishment of the Church including the proliferation of monasteries in the second half of the seventh century. Historians have long noticed the importance of books 3 and 4 for the *HE*'s overarching narrative and our examination has confirmed that also, in terms of the number of characters and connections, books 3 and 4 dominate the narrative.

2.1. *Network and data gathering*

In this project we have developed a data model that classifies the types of relationship we encounter in our sources into 21 different categories of relationship; here we will focus on 15 undirected categories as explained below. This data model offers depth of analysis as we can distinguish the relationship between spouses, from those of family members, members of a monastic community, or military combatants to name but some. In some cases however these categories generate networks composed of very small clusters which are not connected to each other, making their analysis problematic from a mathematical point of view. Thus bearing in mind that the goal of this article is to understand the representation of women in Bede's *HE* and given that some of the categories in his work are indeed very sparse, we opted for a coarse grained approach and did not use the classification of links. This allowed us to have more dense and interconnected networks, making the analysis meaningful.

Whenever two or more persons are described by Bede in some sort of interaction, an edge between them is recorded. That is, we connect characters if we are explicitly told that they are connected, such as when two characters are presented as brothers, or implicitly when two characters are presented as part of the same enterprise at the same time, for example, members of the same missionary team. This is done manually as this is the only way that we can record networks with such depth.Moreover, many of the characters are unnamed or they are a collective group and any automated reading method would not be able to single them out (see Fig. 2 for an example network; note that some of the blue circles are unnamed women). We also work from the original Latin. Edges are weighted, that is we associate with them a value $\geqslant 1$ which accounts for the number of times the corresponding nodes interacted throughout the narrative. When categories are combined, edge weights are added.

As we shall argue in more detail below, when studying the ability of nodes to communicate we must differentiate between their ability to send or receive information. For directed networks this is a given, but for undirected ones this difference arises as an effect of temporal ordering of links [18]. We explain this apparently contradictory idea in the Appendix, taking as an example a simple synthetic network. Since we were interested in this time-generated asymmetry, we studied only the network which excludes directed links, such as letter recipients, textual transmission and patronage. We effectively worked with a network composed of 15 undirected categories. In this case the network has 473 characters, of which 62 are women. The number of links is 2340. Below we present a table with the number of people P and links L for each book:

	P	L	% of links
Book 1	87	276	12
Book 2	89	456	19
Book 3	143	712	31
Book 4	144	572	24
Book 5	117	324	14
total	473	2340	100

Table 1: Number of people/vertices P and undirected links/edges L of Bede's *HE*. The last column is the percentage of links of each book relative to the total number of links.

Even though there is one more character in Book 4 as compared to Book 3, the latter has a considerably larger number of links and consequently more weight than all other books. The effect of this dominance of book 3 will be discussed in the section where we present our results.

3. Methods

In this paper we use two measures to assess the relevance of characters in the narrative of Bede: communicability and importance[a]. We discuss both measures in details in what follows.

3.1. *Communicability*

The communicability between a pair of nodes is usually defined as the shortest path connecting them. It owes its name to the idea that even when any two nodes are not directly connected, a signal starting from one can reach the other through intervening nodes. In this context, communicability plays a role in epidemic spreading, signal transmission and the submission/adjustment to social normal, to name a few examples [20]. Intuitively it is also clear that in certain situations the communication between any two given nodes might not happen through the shortest possible path between them but by means of longer ones. Thus one may generalize the idea of 'path' between any two nodes i and j to that of a 'walk' [18]. A path of length k is a sequence of k *different* edges $e_1, e_2, \cdots e_{k-1}, e_k$ connecting i to j. On the other hand, a *walk* of length k between i and j is a sequence of not necessarily different edges connecting both nodes, *i.e.* edges can be traversed more than once on the way from

[a]Henceforth we use the terms relevance and importance interchangeably.

i to j. A path implies an edge being crossed only once, whereas in a walk we are allowed to pass the same edge again and again as long as it is undirected. Directed edges can be traversed only once in a specific direction. This idea can be made more precise as follows.

A network is a set of vertices $\{V\}$ and edges $\{E\}$ that can be depicted as a graph $G = (V, E)$. Let $|V| = P$ be the number of vertices and $|E| = L$ the number of edges. For this graph one may define the adjacency matrix $A(G) = \mathbf{A}$ whose elements are given by:

$$A_{ij} = \begin{cases} 1 & \text{if } i \, \text{and} \, j \text{ are connected} \\ 0 & \text{if } 0 \text{ otherwise.} \end{cases} \tag{1}$$

It is a known fact in the field of algebraic graph theory that the $(i, j)-$entry of the k-th power of the adjacency matrix $(\mathbf{A}^k)_{ij}$ is equal to the number of walks of length k between nodes i and j [18, 27]. Therefore one may define the communicability C_{ij} between nodes i and j as the sum of all walks between them:

$$C_{ij} = \mathbf{A}_{ij} + (\mathbf{A}^2)_{ij} + (\mathbf{A}^3)_{ij} + \cdots \tag{2}$$

Undirected links can make this sum diverge. In order to circumvent this problem we can modify it by including a 'damping' factor $\alpha < 1$ so that the sum converges for a properly chosen value of α [29]:

$$\mathbf{C} = \alpha\mathbf{A} + \alpha^2\mathbf{A}^2 + \alpha^3\mathbf{A}^3 + \cdots = (\mathbf{I} - \alpha\mathbf{A})^{-1} = Res\,(\mathbf{A}) \tag{3}$$

In the passage from the 2nd to the 3rd equation we used the fact that

$$1 + ar + (ar)^2 + (ar)^3 + \cdots = \frac{1}{1 - ar} \tag{4}$$

is a geometric series of ratio ar but properly defined for a matrix. This quantity is also called the resolvent of matrix A, $Res(\mathbf{A})$ for short and $\mathbf{I}$ the identity matrix. It can be shown that if $\alpha = 1/|\lambda_{max}|$ then convergence of (3) is guaranteed. λ_{max} is the largest eigenvalue of $\mathbf{A}$ and $|\lambda_{max}|$ is the so-called spectral radius of $\mathbf{A}$.

Equation (3) can be applied to adjacency matrices representing directed or undirected networks. For the context of the present work its most advantageous property is that it can be easily extended to treat temporal networks [18, 29, 34]. For the sake of completeness we briefly discuss this extension before we present our results.

When one has a series of discrete-time representations of a dynamic network, the resulting network may be represented by a *time-ordered sequence of adjacency matrices*, one for each time layer. This system can be represented as one single higher-dimensional block matrix which respects the arrow of time. In order to achieve this goal the authors of [29] introduced the notion of a dynamic walk and the temporal version of Eq. (3) as follows:

$$\mathbf{C} = (\mathbf{I} - \alpha\mathbf{A}^{(\mathbf{1})})^{-1}\,(\mathbf{I} - \alpha\mathbf{A}^{(\mathbf{2})})^{-1} \cdots (\mathbf{I} - \alpha\mathbf{A}^{(\mathbf{M})})^{-1} \tag{5}$$

$\mathbf{A^{(t)}}$ represents the adjacency matrix at a given time t ($t = 1, 2, \cdots M$). It is important to note that in this case, to guarantee the convergence of the product, the parameter α is defined as before but using for its definition the largest eigenvalue among the M highest eigenvalues of each adjacency matrix $\mathbf{A^{(t)}}$, that is $\alpha = max\{1/\lambda^{(1)}_{max}, 1/\lambda^{(2)}_{max}, \cdots, 1/\lambda^{(M)}_{max}\}$. Defined this way, the communicability can be viewed as the number of walks across space and time using a time-ordered sequence of adjacency matrices $\{\mathbf{A^{(t)}}\}_{t=1}^{M}$.

From this one may define two different instances of communicability, a so-called *in*–communicability (receive-communicability) and an out-communicability (broadcast-communicability). These terms are borrowed from the literature and in the present context should not be taken to represent the same as in communication networks. It is given by the sum of the entries of a row or column of $\mathbf{C}$ respectively [29]:

$$C^{out} = \mathbf{C}^T \mathbf{1} \qquad C^{in} = \mathbf{C}\ \mathbf{1} \tag{6}$$

In this equation $\mathbf{1}$ is an all-ones vector and $\mathbf{C}^T$ is the transpose of $\mathbf{C}$.

3.2. *Importance*

The entropy of a network characterizes the amount of information encoded in the network structure. So, by removing one node and recalculating the entropy of the new network, we can rank the changes in entropy, or network structure, from its highest to its lowest value and thus see how the removal of a given node affects it [21]. Nodes which cause the largest entropy change are supposedly the most influential. We compute the contribution of a given node j to the entropy as [22, 23, **?**]:

$$s_j = -\rho_j \log \rho_j \tag{7}$$

where ρ_j is a probability density defined via the communicability C_j of node j:

$$\rho_j = \frac{C_j}{\sum_{j=1}^{P} C_j} \tag{8}$$

The entropy of a system reads:

$$S = \sum_{j=1}^{N} s_j = -\sum_{j=1}^{N} \rho_j \log \rho_j \tag{9}$$

To see how a given node j affects the structure of the network, one redefines a new network where j and all its links are removed. The remaining $N-1$ nodes form a new network for which the entropy S_{-j} is now defined as

$$S_{-j} = \sum_{k \neq j}^{N-1} s_k \tag{10}$$

Please note that s_k is defined as in (8) but with new values of C_k calculated over a new network of $N-1$ nodes. The importance I_j of node j can be defined as the difference between the overall entropy, when all nodes are considered, and the same entropy when node j is left out

$$I_j = \frac{S - S_{-j}}{S} \tag{11}$$

In the following section these concepts are employed on the data described in Section 2.

4. Data Analysis

We are interested in determining which women, if any, play an important role in Bede's *HE* network. In order to find this, we first calculated the communicability in the aggregate and temporal approaches. In the aggregate one, for undirected links, there is no distinction between receiver/broadcaster (in/out) communicability. However, in the temporal case, the same undirected links may give rise to two different values of communicability due to links being time-ordered, as explained in the Appendix. The results are depicted in Fig. (1). For the sake of clarity we show only the 30 highest-ranking characters (highest communicability value normalized to 1.0).

First of all one can see some remarkable differences between in- and out-communicabilities (lower panel), the largest difference being that of Saint Wilfrid: as a founder and ruler of many monasteries, educator and an artistic patron his influence was widely felt in Northern Britain. As a Northumbrian noble, he entered religious life at a young age and spent part of his formative years studying in Gaul and Rome. He was regarded as the spokesman for Roman rules of the church, as opposed to the Irish monks at Iona and played an important role in the Synod of Whitby, held in 664, where these questions were discussed. Second to Wilfrid is Theodore of Canterbury, an archbishop and church reformer who appointed many bishops to different sees in Britain. Other notable differences are that of Ecgfrith and Aldfrith, both sons of King Oswiu, and later became kings of Northumbria.

Another feature to notice is that the relative height of the bars and the order of most of the characters, from lowest to highest communicability, remain practically unchanged when one compares the temporal and the aggregate versions of the network. One would expect these measures to differ because as explained in the Appendix, in the case of time-ordered links the undirected ones lead to asymmetries. So in temporal networks the distinction between the two types of communicability comes naturally and is a consequence of time-ordering [29]. It goes without saying that networks representing historical characters can be modelled as temporal ones. However, looking at Eq. (5) it can also be seen that the parameter α is selected from a given realization $\mathbf{A}^{(t)}$ which has the largest eigenvalue among all other adjacency matrix. This leads, in some cases, to one particular time frame dominating over

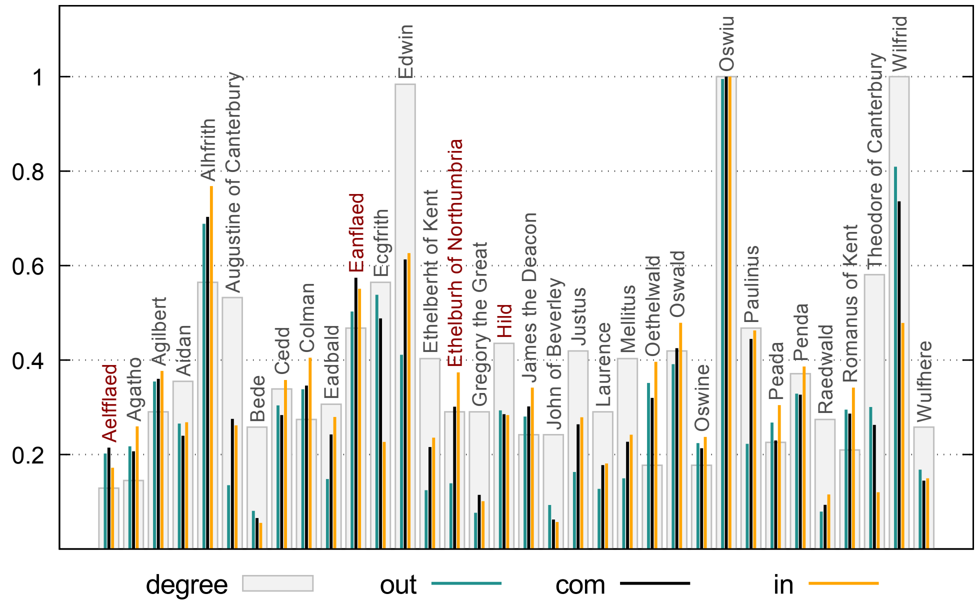

Fig. 1. Top tier characters in communicability values. The grey bars indicate the degree of a character. The black lines are the communicabilities when one considers the aggregate network, that is when one combines all instances of the network into one large network. The green and yellow lines are the in and out communicabilities when using a temporal approach.

others, in which case one will not see much difference in the aggregate or temporal version of the network.

This is exactly the case for Bede and can be understood in terms of the way his narrative was written: each book and chapter of Bede's *History* encompass a different number of years, as some events he reports took place hundreds of years before his time while he was contemporary to some others. So the question is how to break them into more or less self-contained historical periods. In the case of Bede our choice was to use the books he divided his work into as a natural division of time, as they are temporally ordered in spite of not representing the same length of time. As Book 3 is larger than 4, it will dominate the narrative and this explains why there will be little difference between the temporal and aggregate versions of the network. In the temporal case, the parameter α comes from the adjacency matrix of book 3. In the aggregate case, the portion of the entire network which comes from book 3 is denser.

Notwithstanding these important differences, given that we are interested in gendered networks and that the same women (names in red in Fig. 1) appear in both versions with the same relative rank order and their values differ slightly in the aggregate and temporal cases, we will concentrate in the forthcoming discussion on the aggregate version only. Although this is not the appropriate choice in general, since for some networks temporal order may yield rather profoundly different results, it is computationally simpler and justified in our particular case.

A surprising result for many will be the appearance of four women among the main characters, one of them – Eanflaed – having the fifth highest communicability value. Based on the historical context of the narrative, the appearance of Eanflaed raises a red flag: she was Queen of Northumbria and her husband, King Oswiu of Northumbria, is the historical persona with the highest communicability. This could mean that her result might be a reflection of his connections. This becomes clear when one examines the network depicted in Fig. (2). In this sense, Bede's narrative appears to reflect reality, a finding that is significant considering many historians have argued that Bede suppressed the real role of women, especially queens and abbesses in his book [15, 16, 30–32].

In order to check if a woman's high communicability is a reflection of her being attached to high-ranking men, we verified the relevance of characters by assessing how the topology of the network is changed by their removal. This can be done in different ways but we opted to follow the definition of entropy [21] as this measure is very sensitive to changes in the overall structure of the network [35, 36]. This is discussed in what follows (see section 3.2 for the appropriate definitions).

The results for I_j are depicted in the upper plot of Fig. (3) for all 473 characters, where for the sake of clarity only the names of important characters are shown. The removal of most characters basically leaves the entropy unchanged but for Edwin, Oswiu and Paulinus the changes are significant. Among the women, Eanflaed and Ethelburh of Northumbria are the ones who change the topology of the network more prominently. In this sense these two women are the most relevant ones. Notice that by removing characters one may get a positive or negative variation in entropy value. A positive change means that by removing that particular node the new entropy is smaller, that is the links in the network tend to be less regularly distributed. A negative change means exactly the opposite: the removal of a node makes the entropy larger, links are more evenly distributed: the character removed concentrated around him a larger number of links when compared to the average distribution.

With a list of important characters we re-evaluated the communicability of the four women whose communicability values were more prominent in Fig. (1): Aelfflaed, Eanflaed, Ethelburh of Northumbria and Hild. This was done by seeing how their communicability values changed when characters were removed one by one. The results are presented in the lower plot of Fig. (3). The base line of each one fluctuates around their nominal communicability values, that is their values for the full networks. The spikes correspond to their new C values when the characters represented by straight vertical lines were removed. The two women whose values are most affected by the removal of men are Eanflaed (upper blue curve) and Ethelburh of Northumbria (black curve). These changes can be explained due to their kinship to important men: Ethelburh of Northumbria was the wife of King Edwin and mother

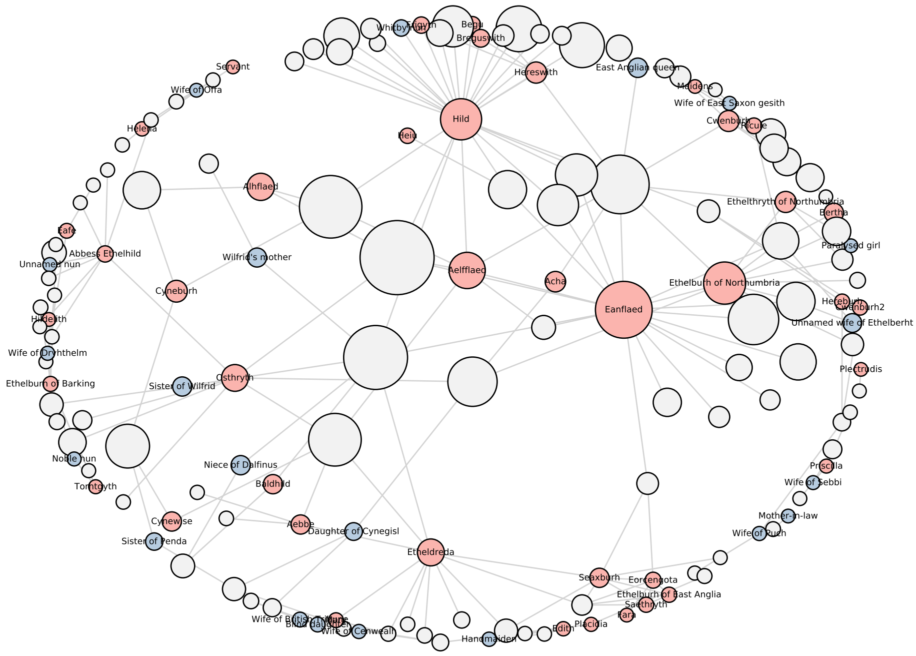

Fig. 2. The network of characters according to their communicability. The size of each circle is proportional to the communicability of a given node. Women are depicted as salmon-coloured. Blue circles represent women who are unnamed in the text. Men are depicted as grey circles. Note how Queen Eanflaed is directly connected to the largest circle which represents the character with largest communicability, King Oswiu, her husband. For the sake of clarity, we did not encode the weights of the edges in width of the lines that represent them.

of Eanflaed, who later married Oswiu. Note also that Ecgfrith's removal – son of Eanflaed and grandson of Ethelburh – also affects their communicability, as does the removal of Alhfrith, son of Oswiu through his first wife (Eanflaed was Oswiu's second wife).

Less affected by removals of other characters are Aelfflaed, daughter of Eanflaed and Oswiu, and Hild. This means that their communicability should be attributed largely to themselves and not to their connections to relevant men. Historically this might be explained by the fact that Aelfflaed was handed over to a religious order at the age of one, and was thus not under the direct influence of her parents for most of her life. Her communicability is still mostly affected by the removal of her mother and a bit less by that of her father as one can see in Fig. (3). Hild, however, who brought Aelfflaed up, was the foundress and abbess of Whitby and appears to have been an influential figure. This is reflected in her 'independent' communicability, and these results allow us to get a more nuanced view of Hild's and Aelfflaed's relevance in the network even though their nominal values of communicability are smaller than that of Eanflaed and Ethelburh of Northumbria.

The independent relevance of abbesses in these social networks is a more significant finding than may at first appear. While the power and authority of queens rested on their husbands and was usually lost on the death of the king, abbesses should have been better insulated from such vagaries. However, while we might expect abbesses to be important in society, their prominence in a work written by a monk is less expected. This is especially so as many readers of Bede's work using models of literary criticism and other approaches have come to a diametrically opposed position: they argue that Bede set out to marginalise women in his book, and Hild and Aelfflaed have been presented as particular victims of his supposed 'misogyny' [15, 16, 33]. Our approach has shown the opposite, and revealed the independent influence of these women, at least as represented by Bede. We should also not assume that Bede unknowingly recorded the complex connections of these women. Hild and Aelfflaed were abbesses of a prominent monastery that contained communities of both monks and nuns, known as double-monasteries and these were dis-continued after the eighth century. Aebbe was a contemporary of theirs, a sister and aunt of successive kings of Northumbria, and abbess of the prominent double-monastery of Coldingham. She does not feature so prominently in our metrics, indicating that the results for Hild and Aelfflaed are unlikely to be accidental. In this, Bede may have been accurately representing their actual influence on their society, or subtly revealing his support for their endeavours while downplaying those of others. Whatever his reasons, our results challenge a significant strand of the existing scholarship on Bede and his attitude to women.

In order to capture this feature we also studied the communicability of women in Bede's *HE* for a 'women-only-network' to see how women compare under themselves. We constructed a women-only-network by keeping men only if they were directly connected to a woman, that is all edges between male characters are not taken into account. One can clearly see from our results, depicted in Fig. (4), how Hild stands out. In this network, men's communicability is reduced by construction, albeit it is now equal to zero as some still represent nodes in the paths of women.

The contrast between the whole network and the women-only-network and how it affects Hild and Eanflaed's communicability can be also seen in Fig. (5). In this figure we plot only Hild and Eanflaed's direct connections. The upper two figures represent their communicability in the complete network, where one clearly sees that Eanflaed's communicability is higher (larger central circle). However, when we represent the women-only-network in the same way, Hild's communicability value surpasses that of Eanflaed, showing clearly how in the former case Eanflaed's score is

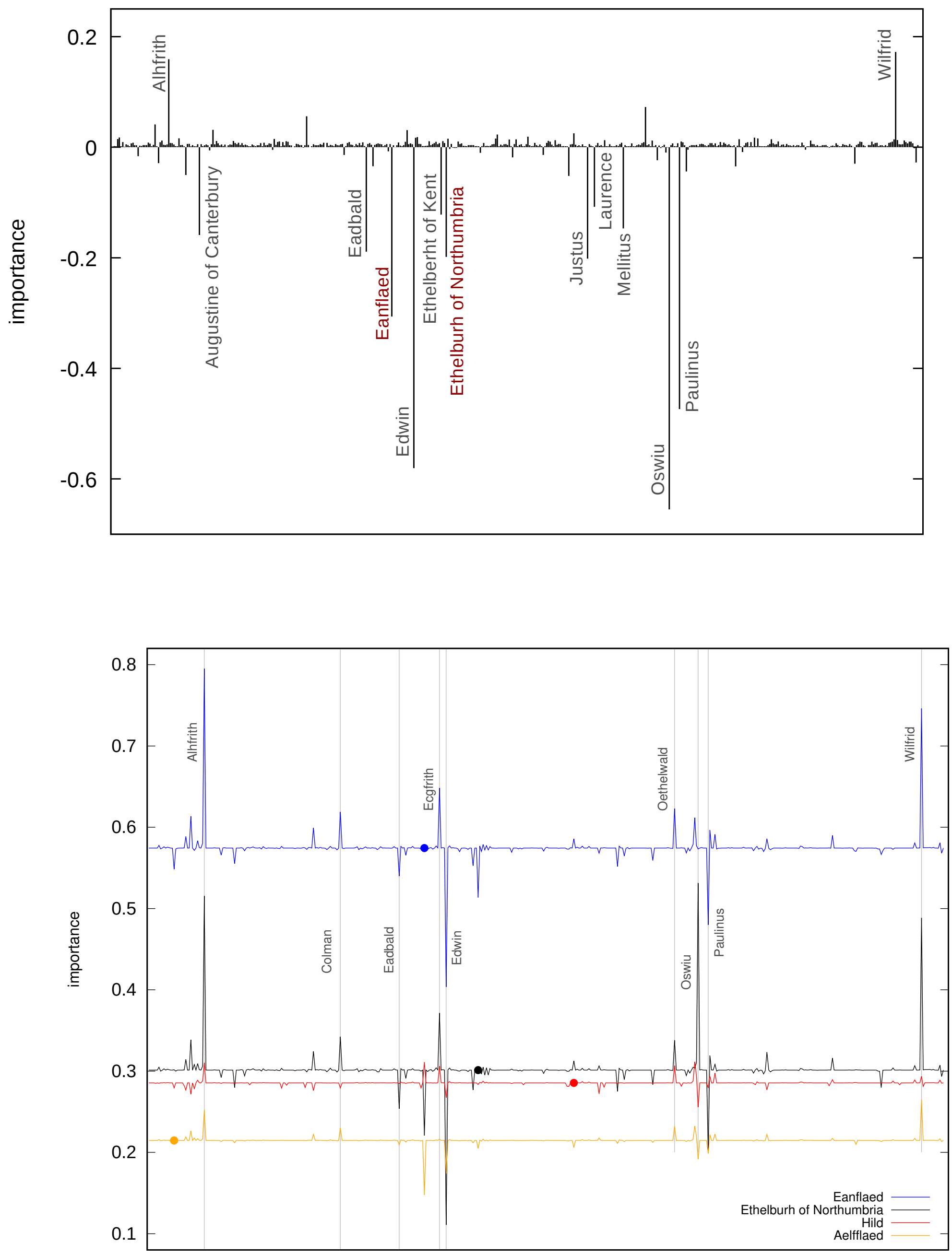

Fig. 3. Upper panel: the importance of all characters given by the relative deviation of the network's entropy as defined by Eq. (11). Lower panel: the variation of the communicability of the four most relevant women when characters are removed from the network. The removed characters with highest variation are represented by vertical lines with their names attached to them. The dots on the curves represent their nominal communicability value and the effect of their own removal can be seen as spikes in the other curves.

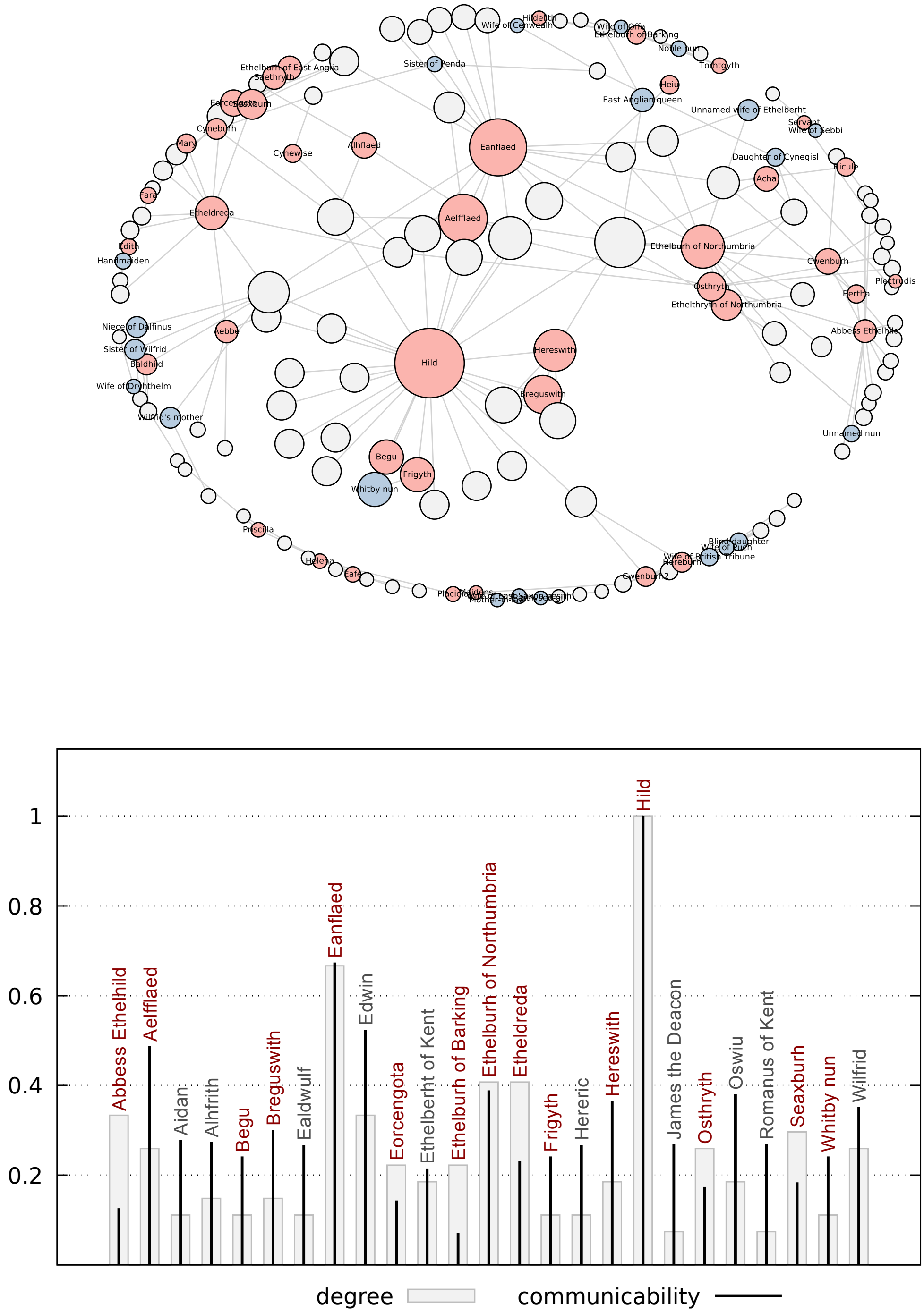

Fig. 4. The communicability of the women-only- network. The upper figure depicts the network where the size of the circle is proportional to the communicability of the characters. The lower figure depicts the values of communicability for the 28 highest-ranking characters. Women's names are magenta, men's black.

due mainly through her connections with important men. Another relevant feature of these plots is that women are sparsely connected to each other, although Hild is the one with the largest number of connections: 5.

Any quantitative method of finding key players or the most influential character or set of characters in a representation of a historical network is highly contingent upon the way we look at them or the way we weigh and classify their connections. This fact is not new and is widely stated in the literature of social networks [35, 36].

In table 2 we show three usual centrality measures for the six women with highest degrees: normalized degree, eigenvector centrality, and betweenness.

women	degree	eigenvector centrality	betweenness
Eanflaed	0.127	0.251	0.263
Hild	0.119	0.119	0.183
Ethelburh of Northumbria	0.081	0.120	0.024
Etheldreda	0.051	0.038	0.150
Seaxburh	0.041	0.077	0.037
Aelfflaed	0.031	0.098	0.089

Table 2: Degree, eigenvector centrality and betweenness (all normalized) for the six highest-ranking women in the aggregate network.

Eanflaed is the woman with the highest degree and eigenvector centrality and the one with the highest communicability (Fig. 1). On the other hand, her communicability is more susceptible to the presence of prominent men than Hild (see Fig. (3)). Hild's centrality values place her as the second highest ranking woman, even though her eigenvector centrality is half of that of Eanflaed. Also, Aelfflaed's communicability is larger than the East Anglian princesses, Etheldreda and Seaxburh (both not shown in Fig. (3)), since she has higher eigenvector centrality than both of them and smaller betweenness when compared to Seaxburh. The possible connection between these results calls for a more detailed study of how measures correlated over different time scales. Work in this direction is currently under way [37].

5. Conclusions

A conventional historical analysis of Bede's *HE* prioritises the actions and interactions of male characters, whereas in this article we have foregrounded female characters. Some historians have previously speculated that the women described by Bede had power, although this power is usually seen as derived from their male connections. Others have argued that Bede suppressed women's activities.

In this paper we have used communicability which quantifies the propensity of communication between two characters and the notion of relevance taken from the topological variation caused by the removal of a character (entropy variation) as a way to quantify what we could call their numerical importance. Our approach has

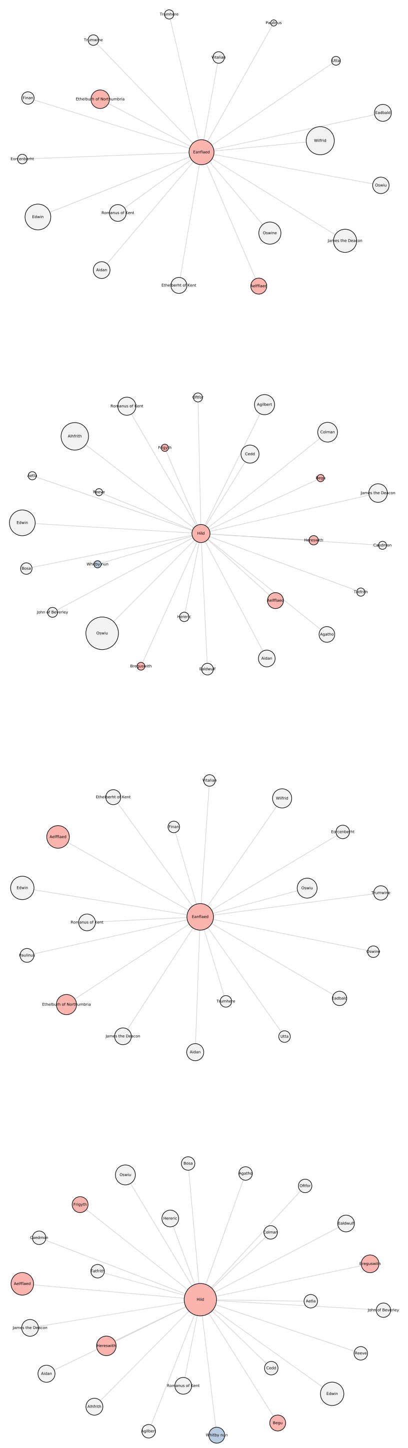

Fig. 5. Eanflaed and Hild's communicability values – represented as networks – for the complete network (upper graphs) and the women-only-network (lower graphs).

shown that it is true that some women's power, for example that of queens, was dependent upon their male connections, such as husbands, sons and even step-sons. However, we have identified at least one woman, Hild, abbess of Whitby, whose position in the network as derived from Bede was unaffected by her male connections. For now, this indicates the complexity in Bede's presentation of women, and we will apply this approach to other texts to determine whether there is more to say about the power of women elsewhere in the early Middle Ages.

Our computation by no means replaces other methods of assessing the historical relevance of these women. However, the idea of applying tools of social networks to historical accounts is an attempt to bring to the fore different interpretations or less intuitive views on the role of these women.

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Appendix. Communicability in temporal networks

One of the most unusual features of a temporal network is the asymmetry that appears in the in- and out-communicability even when links are undirected. To illustrate the idea we present a simple network of 4 nodes interacting at 2 different times.

we can represent the interactions by the table below where 1 stands 'connected' and 0 for 'unconnected'. For the first time step the table reads:

	node 1	node 2	node 3	node 4
node 1	0	1	0	0
node 2	1	0	1	0
node 3	0	1	0	0
node 4	0	0	0	0

while for the second we have:

	node 1	node 2	node 3	node 4
node 1	0	1	1	0
node 2	1	0	0	0
node 3	1	1	0	1
node 4	0	0	1	0

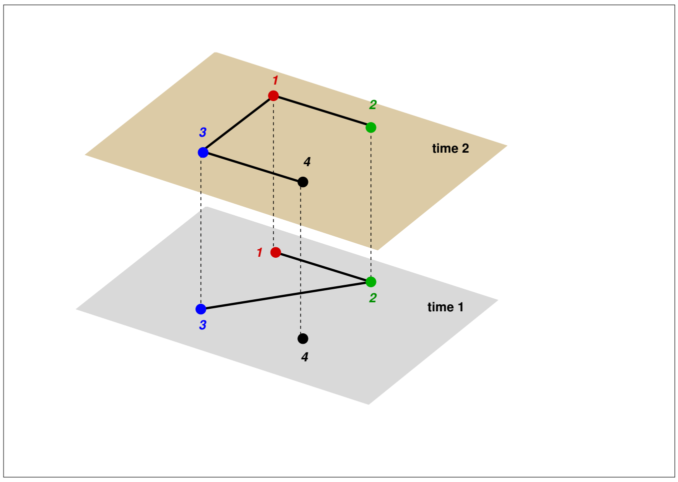

Fig. 6. A simple network is made up of 4 people interacting at two different times. At time $t = 1$ node 1 connects with node 2 and node 2 with node 3. Node 4 is not connected at this point. At $t = 2$ 1 is connected to 2 and 3 and 3 to 4.

These realizations of the network at different times can be represented by two adjacency matrices as follows:

$$\mathbf{A_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{A_2} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{12}$$

One important feature of an adjacency matrix is that the element A^k_{ij} of its k-th power is equal to the number of walks of length k between nodes i and j. As an example, if one multiplies $\mathbf{A_1}$ by itself once, one would get:

$$\mathbf{A_1} \times \mathbf{A_1} = \mathbf{A_1}^2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

Each entry represents the number of walks of length 2 (2-step walks) from a given node to another node or back to itself. We notice for instance that the entry equal to 1 for nodes 1 and 3 means that the latter can be reached form the former in two steps and vice-versa, even though they are not connected directly connected in the original adjacency matrix. The fact that there is a 2-step walk between a node and itself represents the fact that one can step towards the next neighbor and retrace the step back to the original position. If we now sum all powers of a given matrix

$$\mathbf{C} = \mathbf{A_1} + \mathbf{A_1}^2 + \mathbf{A_1}^3 + \mathbf{A_1}^4 + \cdots \tag{14}$$

the entry C_{ij} of $\mathbf{C}$ is simply the number of all walks of all possible lengths between any two nodes i and j. This number of course is infinite, so in order to make it converge one may introduce an attenuation (damping) factor α:

$$\mathbf{C} = \alpha\mathbf{A_1} + \alpha^2\mathbf{A_1}^2 + \alpha^3\mathbf{A_1}^3 + \alpha^4\mathbf{A_1}^4 + \cdots \tag{15}$$

The meaning of α is that small values of this parameter prioritizes short lengths over long ones. Moreover the sum above can be written as

$$Res(\mathbf{A}) = (\mathbf{I} - \alpha\mathbf{A})^{-1} \tag{16}$$

where we use use the geometric series applied to a matrix (see main text). $Res(\mathbf{A})$ is the so-called *resolvent* of matrix $\mathbf{A}$. The sum over all entries of a given row i is defined as the communicability of node i [*cf.* eq. (6)]

$$C_i = \sum_j [Res(\mathbf{A})]_{ij} \tag{17}$$

For the temporal matrix in our example one can extends the definition of communicability to [19]

$$Res(\mathbf{A_1}) \times Res(\mathbf{A_2}) = (I - \alpha\mathbf{A_1})^{-1} \times (I - \alpha\mathbf{A_2})^{-1} \tag{18}$$

To consider now how the temporal features may give rise to asymmetric communicabilities even when links are undirected we go back to our simple model. As an example we will count walks starting/ending in node 1.

For time layer 1, there are two possible ways out of 1: from 1 to 2 and from 1 to 3. These walks are depicted in the upper right and lower left corner of the figure below. However you can also have something originating at 1, moving to 3 and then being carried by 3 to the next time layer, reaching 4. This is one walk which moves across time layers. So a message passed by 1 to 3 in the past can reach 4 in the future. When it comes to time layer 2, you have three possible walks from 1, depicted below: directly to node 2, directly to node 3 or to node 4 using node 3 as intermediary. Notice that in this last case node 1 would be reinforcing his message to node 4 via node 3, so this walk is an additional and is shorter (2 walks) as compared to the path from 1 to 4 via node 3 in the past (3 steps). Note that from time layer 2 there is no way down to time layer 1, for this would mean going back in time.

There is a total of 6 walks that one can take starting from 1 to reach the other 3 nodes.

We can now count walks towards node 1. We depict them in the figures below. You can reach node 1 from node 2 or node 3. However, the only way for node 4 to reach node 1 is in the second time layer. At this time you can also reach node 1 starting at node 2 or node 3. Moreover, you could have started at node 2 within time layer 1, reach out to node 3 and have the message carried to the next time layer to be

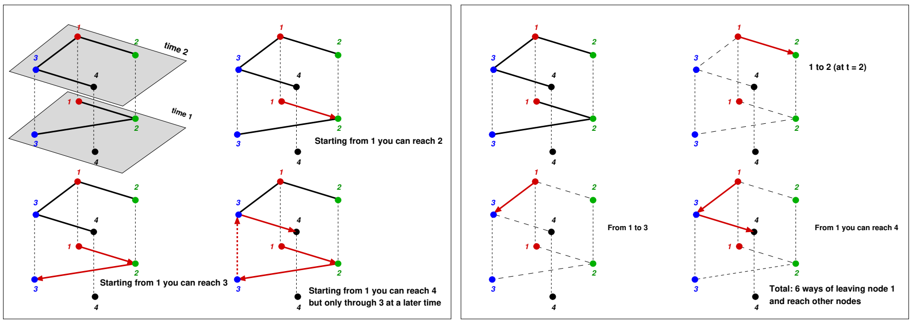

Fig. 7. Walks having node 1 as a starting point. From node 1 you can reach node 2 and node 3, but also node 4 at a later time through node 3.

delivered to 1. Or, start out from 3, pass it along to 2 and have 2 carry it to the next time layer, delivering it to 1 then.

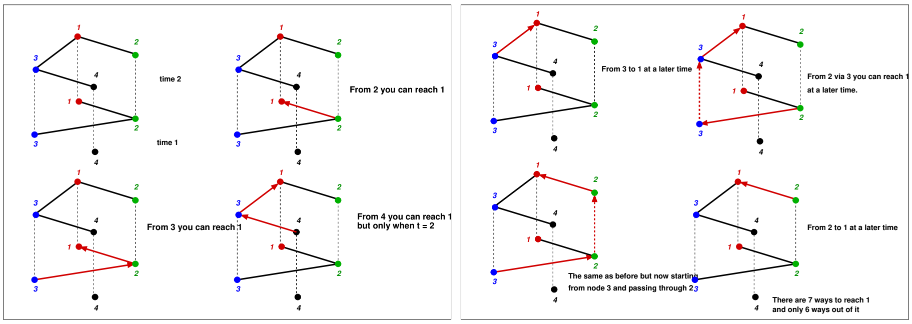

Fig. 8. Walks having node 1 as ending point.

Summing up these results there are 7 different ways of node 1 being reached. So, even though node 1 is involved in undirected (symmetric) relations, when we allow messages or relationships to be carried along time, he ends up receiving more than he was able to give.

If we extend this counting to other nodes we get the following table:

	in	out
node 1	7	6
node 2	5	9
node 3	7	7
node 4	5	3

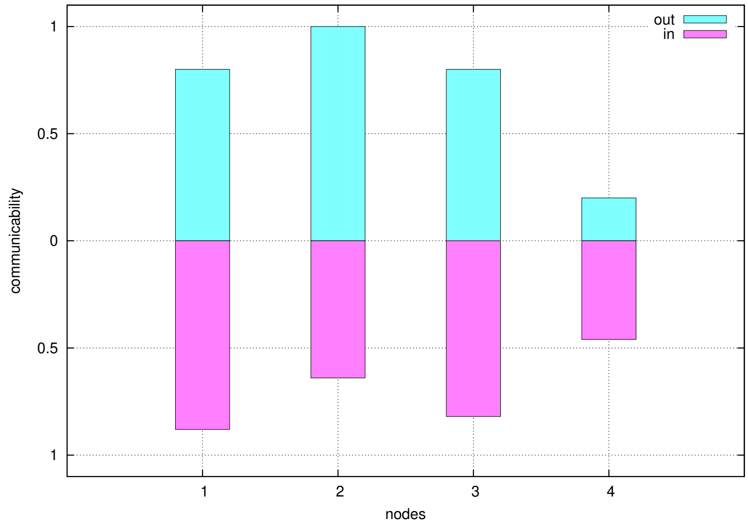

Fig. 9. The communicability of the 4 nodes normalized by the highest communicability value of node 2.

This simple model clearly shows how starting from a undirected network one may get asymmetric communicability values if this network evolves over time.

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