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Aygün, Orhan and Lanari Bo, Inacio (Accepted: 2020) College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case. American Economic Journal: Microeconomics. ISSN 1945-7669 (In Press)

https://doi.org/10.1257/mic.20170364

College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case

By Orhan Aygün and Inácio Bó*

In 2012 Brazilian public universities were mandated to use affirmative action policies for candidates from racial and income minorities. We show that the policy makes the students' affirmative action status a strategic choice, and may reject high-achieving minority students while admitting low-achieving majority students. Empirical data shows evidence consistent with this type of unfairness in more than 49% of the programs. We propose a selection criterion and an incentive-compatible mechanism that, for a wider range of similar problems and the one in Brazil in particular, removes any gain from strategizing over the privileges claimed and is fair.
JEL: C78, D63, D78, D82
Keywords: Mechanism design, matching with contracts, college admissions, affirmative action, diversity.

I. Introduction

Following an increasing need for affirmative action for students of African descent and of low-income families in terms of access to public universities, in August 2012 the Brazilian congress enacted a law[1] establishing the implementation of a series of affirmative action policies throughout the federal higher education system.[2] Since then, these policies have had a significant impact on the lives of hundreds of thousands of students who join its undergraduate programs every year.

* **Aygün**: Boğazici University Department of Economics Natuk Birkan Building, Room 205D Bebek, Istanbul 34342, website: http://www.econ.boun.edu.tr/aygun; e-mail: orhan.aygun@boun.edu.tr. **Bó**: University of York, Department of Economics and Related Studies, York, United Kingdom; website: http://www.inaciobo.com; e-mail: mailto:inacio.lanaribo@york.ac.ukinacio.lanaribo@york.ac.uk. The authors would like to thank Samson Alva, Rustamdjan Hakimov, Aysun Hiziroglu, Scott Kominers, Utku Ünver, Tayfun Sönmez, Bertan Turhan, Alexander Westkamp, and two anonymous referees for helpful comments. We would also like to thank the Brazilian Ministry of Education for providing the data used in this paper.

[1]Brazilian federal law 12.711 of August 29, 2012.

[2]For detailed information on the history of affirmative action in Brazil, see Moehlecke (2002).

The law established that certain proportions of the students accepted into each program in those institutions[3] should have studied in public high-schools, come from a low-income family, and/or belong to a racial minority. This objective was implemented by partitioning the seats in each program, reserving them for different combinations of these characteristics. Some seats, for example, were reserved for those who claim coming from a public high-school and from a low-income minority, while other seats were reserved for those who claim coming from a public high-school and belonging to a racial minority, etc. The students with the highest grades in a national exam, for each group of seats, would then be accepted into the program.

In this paper we show that, while the method proposed by the government makes the cohorts of students satisfy ratios specified in the law, it has some important deficiencies. First, it is **unfair** in the sense that it may reject *high-achieving* students who are the target of the affirmative action policies while accepting *low-achieving* students who do not have privilege priority status. For example, a low-income minority student with a high exam grade may be rejected while a high-income white student with a low exam grade is accepted. This is not just a theoretical observation: our analysis of the cutoff grades in the 2013 admissions shows evidence consistent with unfairness in the assignments in about 49% of the more than 3,000 programs available. Second, it gives an advantage to students who **strategize over the privileges that they claim**. A student who makes that choice based on good information about other students' choices and their exam grades can improve their chances of being accepted at their preferred programs.

We show how the problems that we observe in the data and in the incentive properties of the procedure currently being used come from a combination of two factors. First, it treats differently students who are eligible to claim a set of privileges from students who *could credibly act as if they were eligible for them.* For example, if a seat gives priority to students who claim low income but not minority privileges over those who claim both, then a student who is low income and a minority could benefit by not claiming her minority privilege. Second, it seems to have in its design the implicit assumption that students who are eligible

[3]In Brazil, as in many other countries including Turkey (Balinski and Sönmez, 1999), students apply directly to the university for a specific program whereas in other countries, like the US, students simply apply to the university and, once accepted, must then choose the majors or programs to pursue.

to claim privileges have lower grades than those who are not. More specifically, if low-income students always have lower grades than those who are not low-income, and likewise for minorities and students from public HS, the procedure being used in Brazil would not suffer from any of the shortcomings we identify.

Based on these and other insights that we obtain from our application, we provide a new choice procedure to be used by the programs that eliminates the problems above. It does so by guaranteeing that no student could be worse off by claiming additional privileges. Moreover, we show that the choice function that is defined by that procedure can be combined with the cumulative offer mechanism to provide a strategy-proof mechanism for matching students to programs under the proposed policies.

This paper is related to the literature on affirmative action in college admissions and school choice mechanisms. The incorporation of affirmative action policies into those mechanisms, in the form of majority quotas,[4] has already been studied in early papers on the use of centralized mechanisms for school choice and college admissions (Abdulkadiroğlu and Sönmez, 2003; Abdulkadiroğlu, 2005). Hafalir, Yenmez and Yildirim (2013) present an alternative method for implementing affirmative action, denoted minority reserves. With minority reserves, schools or colleges give higher priority to minority students up to the point where the minorities fill the reserves. They show that the use of these reserves, as opposed to maximum quotas, leads to a Pareto improvement for the students.[5]

Kojima (2012) showed, however, that the use of affirmative action in the form of majority quotas may paradoxically hurt every minority student. Doğan (2016), moreover, showed that this problem is also present when using minority reserves: the introduction of those policies can make all minority students weakly worse off. Moreover, the author shows that these situations are not rare cases, but are indeed pervasive.

The kind of policy that we evaluate in this paper is closely related to the concept of minority reserves, in that seats in university programs give a higher priority to certain types of students, leaving those seats available to other students when those are not occupied. Similarly to Kojima (2012) and Doğan (2016), we show that the method used in Brazil also leads to perverse effects that may also hurt the

[4]That is, an upper bound on the number of "majority" students who can be matched to a school or college.

[5]Generalizations of the use of minority reserves for multiple types of affirmative action objectives are also present in the soft-bound quotas for multiple types in Ehlers et al. (2014) and Bo (2016).

intended beneficiaries of these policies. Differently from their cases, however, the problem here is a consequence of a miscoordination that is caused by the fact that the procedure we evaluate makes the privileges a strategic variable. Moreover, to the best of our knowledge this is the first paper to provide empirical evidence consistent with those negative effects.

The remainder of this paper is structured as follows. In section II we present the mechanism suggested by the ministry of education and currently used by the universities surveyed. In section III we introduce the matching with contracts model that we apply to the college admissions problem with multidimensional privileges, and define the desirable properties that a procedure for selecting students into programs should satisfy. In section IV we show that the currently used Brazil Reserves choice function induces a game in which strategically sophisticated students may obtain better outcomes by strategizing over which privileges to claim. We show that there is a trade-off between fairness and a legalistic interpretation of the affirmative action objectives, which is embedded in the Brazil Reserves procedure. Moreover, while the current procedure satisfies the latter, it comes at the cost of the former and also of bad incentives. Section V provides empirical evidence on how the situations that lead to those problems were pervasive in the year 2013. In section VI, we introduce the multidimensional privileges choice function, which provides a general solution for problems with multidimensional reserves, and we apply it to the Brazilian case. Moreover, we build upon the choice function defined to describe a mechanism – the student-proposing stable mechanism – that matches students to colleges using a centralized system, satisfies stability, is strategy-proof, and fair. All the proofs are given in the Appendix.

II. The Ministry of Education's Guidelines

For the most part, until 2010 college admissions in Brazil essentially worked in a decentralized way. Students first applied to a specific program at each university of their choice (e.g., history at the University of Brasilia or biology at the Federal University of Minas Gerais). Then, by using a combination of grades in a national exam and sometimes exams particular to those programs, the universities ranked them and accepted the top applicants to each program up to the programs' capacities, putting the remaining ones on waiting lists. Among those accepted, typically some would not enroll because they had also been accepted for admission at another university. The universities would then proceed to a second round, accepting students from the wait list following their ranking. Depending on the university, this might be followed by a third, fourth, or more rounds.

The law introducing the use of affirmative action in the access to the federal universities did not change the decentralized nature of the entire system itself, but it has changed the rules the universities use to choose among students who apply to them, in an attempt to satisfy the affirmative action objectives. Although since 2010 an increasing number of universities and students have been using a centralized mechanism to determine the students' matches, our analysis and proposals can be applied to improve both decentralized and centralized systems. [6]

The law established that 50% of the seats in each program offered by federal universities should have its access determined by affirmative action policies. In order to have higher priority access to those seats, a student must complete three years of high school at a public institution. When assigning students to at least 50% of those seats, the university should also give higher priority to students who claim belonging to a low-income family (and provide documented proof). Additionally, when assigning a number of seats in the same proportion as the aggregate number of African and native Brazilian descendents (referred to here as "minorities") in the state in which the institution is, the university should give higher priority to students who claim being a minority. Since the status associated with these claims constitute a special right that they have, we say that those students claim specific **privileges**, and we denote these as "**public HS privilege**," "**low-income privilege**," and "**minority privilege**."

In a state where minorities constitute 25% of the population, a program with 80 seats will have 40 seats, for example, giving higher priority to students claiming public HS privilege. At least 20 of those should give a higher priority to those claiming low-income privilege, and 10 for those claiming minority privilege.

One key distinctive issue presented by the privileges proposed in the law is the fact that they are multidimensional. That is, students may well belong to one or more of the groups specified. For instance, a low-income white student from a public high school would qualify for the low-income and public HS privileges, but

[6] More specifically, our solution will say how universities should decide which students to choose when facing a pool of applicants, which is the typical case when universities select students in a decentralized way. That same solution, however, can be the criterion for selecting from students in an algorithm that is used to produce allocations in a centralized system.

not for the minority privilege.

In October of the same year, Brazil's Ministry of Education published an ordinance[7] specifying some details on the implementation of the affirmative action law as well as a suggested mechanism for choosing students while satisfying those policies. Starting in the student selection processes of 2013, those recommendations were widely adopted as the new selection criteria. We denote these rules proposed by the government to determine the set of students to be chosen from any set of applicants as the class of *Brazil Reserves Choice Functions* (or simply *Brazil Reserves*). It suggests that the seats for each program should be split into five subsets. Let r be the proportion of minorities in the population of the state in which the program is. For any program with capacity q, the five distinct subsets are:

- A set with $\lceil \frac{q}{4} r \rceil$ seats which gives priority to students who claim public HS, minority, and low-income privileges.
- A set with $\lceil \frac{q}{4} (1-r) \rceil$ seats which gives priority to students who claim public HS and low-income privileges only.
- A set with $\lceil \frac{q}{4} r \rceil$ seats which gives priority to students who claim public HS and minority privileges only.
- A set with $\lceil \frac{q}{4} (1-r) \rceil$ seats which gives priority to students who claim public HS privilege only.
- A set with the remaining seats.

Given the students who apply for each of these subsets, those better ranked in the entrance exam are accepted up to the capacity of the set. It is easy to see that if there are enough applicants for each of those sets, the affirmative action objectives, as described by the law, are satisfied. If the number of students who apply for some of those sets is smaller than their capacity, those seats will be filled following different priority structures, which are detailed in the Appendix. We denote any procedure for selecting students that follows the criteria above as an *implementation of Brazil Reserves.*

[7] Normative ordinance 18 of October 11, 2012.

III. Model

There are finite sets $S = \{s_1, \dots, s_n\}$ and $P = \{p_1, \dots, p_\ell\}$ of students and programs. Each program p has its own capacity q_p. Each student s has a vector of exam grades $\theta(s) = (\theta_{p_1}(s), \dots, \theta_{p_\ell}(s))$ such that $\theta_p(s)$ indicates the grade of student s in program p. There are no ties in the exam grades of each program, that is, for any two students $s, s' \in S$ and program $p \in P$, $\theta_p(s) = \theta_p(s') \iff s = s'$. Each student s has a vector of available privileges she can claim, $t_s = \left(t_s^h, t_s^m, t_s^i\right)$ where t_s^h, t_s^m, t_s^i represents public HS, minority, and low-income privileges, respectively, and complete strict preferences $(\succ_s^*)_{s\in S}$ over programs in P and remaining unmatched, represented by $\emptyset$. We assume that students are indifferent between the vectors of privileges claimed and only care about which program (if any) they are matched to. This is justified mainly by the fact that no benefit or assistance given by the university is associated with the privileges claimed, and that the privileges claimed are not made public.

Each element of t_s is binary, where 1 means that the student is eligible for the privilege. This notation allows us to make ordering comparisons between vectors of privileges: if t^* is such that $t^* \geq (1,0,0)$, for example, then $t^* \in \{(1,0,0), (1,1,0), (1,0,1), (1,1,1)\}$. To reduce confusion, we will typically represent this vector by using lowercase and uppercase letters, so $(1,0,1)$ will be represented by (H, m, I), for example. For each combination of privileges, a program may reserve some seats for those claiming them. For example, $Q_p^{(H,m,i)}$ is the set of seats reserved for students who claim public HS privileges. Also, when referring to those seats, we denote by q_p^t the number of seats in Q_p^t. Following the ministry of education guidelines, therefore, $q_p^{(H,m,i)} = \left\lceil \frac{q}{4}(1 - r_p) \right\rceil$, where r_p is the proportion of minorities in the state where program p is. In the Brazilian system, if a student claims public HS, minority or low-income privileges she is required to provide documental proof related to those classifications.[8] Therefore, some students may opt to not claim a privilege associated with a group she belongs to, but students who do not belong to a group (and therefore do not have any proof of belonging to it) are unable to claim that privilege.

For simplicity, we will make use of the *matching with contracts* (Hatfield and

[8]Unlike the public HS and low-income privileges, in order to claim minority privileges a student only has to identify herself as a minority. Therefore, in principle, it is possible for a white student to declare herself a minority (Ritter, 2018). This possibility, however, is ignored in this paper.

Milgrom, 2005) notation.[9] In this context, a **contract** x is a tuple (s, p, t), where $s \in S$, $p \in P$ and t represents the set of privileges the student is claiming. A contract (s, p, t) is **valid** if $t \leq t_s$. For a contract x; x_S, x_P and x_T represent the student, the program, and the vector of privileges that student s is claiming in contract x, respectively. Let X be the set of all valid contracts. For ease of notation, for a set of contracts Y, Y_i is the subset of Y that contains only the contracts that include $i \in S \cup P$. Similarly, Y_t is the subset of Y that only contains the contract with the privilege vector t. We denote by $X_{i,t}$ the set of contracts that include i and the privilege vector t. Let $s(Y)$, moreover, be the set of students with contracts in Y, that is, $s(Y) = \{s \in S : \exists(s, p, t) \in Y\}$. A **feasible allocation** is a set of contracts $X' \subset X$, such that for every $s \in S$ and every $p \in P$, $|X'_s| \leq 1$ and $|X'_p| \leq q_p$. Let χ be the set of all possible feasible allocations.

The null contract, meaning that the student has no contract, is also denoted by $\emptyset$. A contract is acceptable if it is preferred to the null contract. While students have preferences over programs, we need to translate them into preferences over contracts. We denote these preferences, for each student s, by $\succeq_s$. These preferences are complete over X_s and are derived in such a way as to make them consistent with the relation $\succ^*_s$, and the assumption that students only care about the program they are matched to:

$$\forall s \in S, \forall p, p' \in P \text{ and } t, t' \leq t_s : (s, p, t) \succ_s (s, p', t') \iff p \succ^*_s p'$$

$$\forall s \in S, \forall p \in P \text{ and } t, t' \leq t_s : (s, p, t) \sim_s (s, p, t')$$

Next, the choice function of program p, $C_p : 2^X \to 2^X$ is such that for $Y \subset X$, $C_p(Y) \subset Y_p$. The set $C_p(Y)$ has a cardinality of at most q_p and has at most one contract per student.

The choice functions that we will present in this paper are all instances of choice functions using *slot-specific priorities*, described in Kominers and Sönmez (2016). Under slot-specific priorities, each seat in a program has its own priority ordering over contracts. Given a set of contracts, each seat "accepts" the top

[9]While it would be possible for us to formulate our problem (and solution) in terms of the matching with complex constraints presented in Westkamp (2013), due to the simplicity of the representation of the entire argument in terms of slots and contracts, we opted to follow that model.

contract with respect to that seat's priority ordering, among those which have not yet been accepted. As shown in that paper, the set of contracts accepted may depend on the order in which those seats are filled, and therefore that order is also a parameter of the problem.

More specifically, under slot-specific priorities, each seat i in a program p has its own priority order $\blacktriangleright_p^i$ over elements of X, and each program p has an order of precedence over its seats $\rhd^p$. The interpretation of $i \rhd^p i'$ is that, whenever possible, the program p fills seat i before filling i'. When filling seat i in program p, the contract with the highest priority with respect to $\blacktriangleright_p^i$, among those available, is chosen. As we will show, this model is rich enough for us to represent both the current procedures being used and our proposed solution.

Let Φ be the set of all strict preferences over P and $\Theta_s = \left\{t \in \{0,1\}^3 : t \leq t_s\right\}$ be the set of privilege vectors that student s can claim. A mechanism is the strategy space $\Delta_s = \Phi \times \Theta_s$ for each student s along with an outcome function $\psi : \prod_{s \in S} \Delta_s \to \chi$ that selects an allocation for each strategy vector $\prod_{s \in S} \delta_s \in \prod_{s \in S} \Delta_s$. Given a student s and a strategy $\delta_s \in \Delta_s$, let δ_{-s} denote the strategy of all students except student s. Moreover, we use the notation ψ_s for the contract involving student s selected by ψ, that is, $\psi_s(\cdot) = \psi(\cdot) \cap X_s$.

A. *Desired properties of choice functions and mechanisms*

Below, we define the properties that we consider desirable for both the allocations and choice functions used by programs and for a centralized mechanism that assigns students to programs. The first one is privilege monotonicity.

Definition 1. *A choice function $C_p : 2^X \to 2^X$ is* ***privilege monotonic*** *if for any given set of contracts $Y \subset X$, and any student s with no contract in Y,*

$$(s,p,t_s) \notin C_p(Y \cup \{(s,p,t_s)\}) \implies (s,p,t') \notin C_p(Y \cup \{(s,p,t')\}), \forall t' \leq t_s.$$

Privilege monotonicity suggests that when a student applies to a program, claiming an additional privilege should not decrease her chance of being accepted. As a result, when a student applies to a program that uses a choice function with that property, it is always safe for her to claim all the privileges that she can. This creates a strategic simplicity for those students when it comes to the decision of which privileges to claim. When the choice function is not privilege monotonic there are circumstances in which, in order to be accepted, the student should not claim some privilege. This, in general, gives an advantage to students who strategically manipulate the set of privileges claimed. By removing any gain from those manipulations, the use of privilege monotonic choice function "levels the playing field" for those students (Pathak and Sönmez, 2008), eliminating those strategic aspects from the outcomes obtained by them.

Moreover, as an additional benefit, the use of privilege monotonic choice functions makes the privileges claimed a more reliable information. That is, similarly to the fact that strategy-proofness turns the preferences submitted by students into more reliable information for welfare estimations, here privilege monotonicity does the same for the composition of candidates in terms of their affirmative action characteristics.

Fairness of the choice function, as we use it here, indicates that if a student is not chosen, those contracts that are chosen include students who either have higher test grades or are there due to the fact that those accepted claimed more affirmative action characteristics.

Definition 2. *For any given set of contracts Y, the chosen set $C_p(Y)$ is **fair in** p if for any $x \in Y_p$:*

$$x \notin C_p(Y) \implies \forall y \in C_p(Y), \text{ either } \theta_p(y_S) > \theta_p(x_S) \text{ or } x_T \ngeq y_T$$

*We say that a **choice function** $C_p : 2^X \to 2^X$ is **fair** if for any given subset $Y \subset X$, $C_p(Y)$ is fair in p. An allocation Y is **fair** if for any given pair of contracts $x, y \in Y$*

$$y_P \succ^*_{x_S} x_P \implies \text{ either } \theta_{y_P}(y_S) > \theta_{y_P}(x_S) \text{ or } x_T \ngeq y_T$$

*We say that a **mechanism** ψ is **fair** if the allocations it produces are fair.*

That is, an allocation is fair if the reason why a student is not matched to a program is because every student matched to that program either has a higher exam grade or is claiming strictly more privileges. Note that since we are concerned about the fairness of a program both in isolation and in conjunction with other programs, we have two definitions of fairness of allocations, as well as for choice functions and mechanisms.

Definition 2 only focuses on pairwise comparisons of chosen and not chosen students, and in particular, does not place any restrictions on total numbers of chosen students of various types. For example, a choice function that simply ranks students based on their grades is fair. Similarly, a choice function that is based on privilege vectors (e.g., simply ranking students by the number of privileged categories that they belong to), and only uses exam grades to break ties on the margin, is also fair.

The Brazilian legislation, and many other affirmative action programs, uses specific numbers of reserved seats as a method to quantify the proportion of them which can be used to assist the students who are target of these policies. As we have described in section II, there are four combinations of privileges that have seats reserved for those claiming them, and their numbers are derived from formulas in the Ministry of Education's guidelines. Our next definition is designed to satisfy these types of conditions. It codifies the extent to which students with certain combinations of privileges have their access to universities assisted in the number of seats reserved for them.

Definition 3. *For any given set of contracts Y, the chosen set $C_p(Y)$* ***legalistically satisfies the affirmative action objectives at program*** *p if for all vectors of privileges t, $|C_p(Y) \cap Y_t| \geq \min\{q^t, |Y_t|\}$. Moreover, a choice function $C_p : 2^X \to 2^X$ legalistically satisfies the affirmative action objectives at program p if for every Y, $C_p(Y)$ legalistically satisfies the affirmative action objectives in p.*

As said in the name of the property itself, the definition above takes the notion of "satisfying the affirmative action objectives" very literally and legalistically, and most importantly, it ignores an inherent relationship between combinations of privileges. For example, both sets of seats $Q_p^{(H,M,I)}$ and $Q_p^{(H,m,I)}$ give priority to those claiming public HS (H) and low-income (I) privileges, ignoring therefore the fact that those who are also minority students are eligible to apply to both of them. As we will show in section IV, indeed, ignoring this relationship is at the heart of the flaws of the Brazil Reserves procedure. It is important to note, therefore, that as opposed to the other properties presented in this section, legalistically satisfying the affirmative action objectives is not among the things that we necessarily "want", especially if it comes at the cost of violating fairness.

If we consider the historical context in which most affirmative action policies are introduced, it seems clear that its goal is to help disadvantaged groups of applicants, and not to mandate an exact makeup of various combinations of advantages and disadvantages. There is, however, a more flexible way to interpret them, which we describe below.

Definition 4. *A choice function $C_p : 2^X \to 2^X$* ***satisfies the spirit of the affirmative action objectives at program*** *p if for every $Y \subseteq X$ and vector of privileges t:*

$$\sum_{t' \geq t} |C_p(Y) \cap Y_{t'}| \geq \sum_{t' \geq t} \min \left\{ q_p^{t'}, |Y_{t'}| \right\}$$

That is, we say that a choice function satisfies the spirit of the affirmative action objectives if it takes an inclusive interpretation of the number of seats reserved: seats reserved to students claiming some privileges might also be used to accept those claiming even more privileges. Definition 4 better embodies the intention behind the use of affirmative action policies when compared to its "legalistic" counterpart: reserving seats for those claiming a privilege might help anyone who is eligible, not only those who do not claim other ones as well. Notice that this definition constitutes a weakening of definition 3, in that a choice function that legalistically satisfies the affirmative action objectives also does so in spirit. As we will see, however, having the objective of satisfying this weaker interpretation of the affirmative action objectives will not come at the cost of the other desirable properties, as opposed to the stricter one.

Next, a commonly desired property for an allocation in a matching market is stability.

Definition 5. *An allocation Y is* ***stable*** *under preferences $(\succeq_s)_{s \in S}$ and choice functions $(C_p)_{p \in P}$ if*

(i) For all $s \in S$ and for all $p \in P$, $Y_s \succ_s \emptyset$, $C_p(Y) = Y_p$; and

(ii) $\nexists (p, s) \in P \times S$, and contract $x \in X \backslash Y$, such that $x \in C_p((Y \setminus Y_s) \cup \{x\}), x \succ_s Y_s$.

In college admissions processes like the one in Brazil, the choice function used by the programs embodies a legal requirement, establishing who among the applicants has the right to be admitted into a program. Stability, therefore, is a natural desirable characteristic for an allocation. If each student applies to only one program, stability requires that the rules encoded in the choice function determine which students should be selected. While the law currently does not say anything about the allocation of students to colleges when students apply to multiple colleges, stability presents a natural way to solve this ambiguity: a student will only be matched to a less desirable program if, by following the rules of acceptance there, she would not be accepted given the students who have been matched to that program. Unstable allocations, therefore, have the potential to lead to lawsuits from dissatisfied students.

Next, we define incentive-compatibility in our setup, where students have not only preferences but also privilege vectors.

Definition 6. *A mechanism ψ is **incentive-compatible** if*

$$\nexists s \in S, \delta_{-s} \in \prod_{j \in S \setminus \{s\}} \Delta_j, \delta' \in \Delta_s, \text{ such that } \psi_s \left(\delta', \delta_{-s}\right)_P \succ_s^* \psi_s \left(\left(t_s, \succ_s^*\right), \delta_{-s}\right)_P .$$

In other words, it would be in the best interest of any student that we consider, no matter what her true preferences are or which privileges are available to her, to reveal her true preferences and claim all the privileges that she is eligible for.

IV. Current Mechanism Revisited

When we look at the motivation behind the implementation of most affirmative action policies, we usually find that it comes from the fact that without them the individuals targeted by these policies would be underrepresented in the population that is accepted by these institutions. In fact, when access was determined solely by exam grades, low-income and racial minorities were significantly underrepresented among students accepted to Brazilian public universities (McCowan, 2007). This is therefore associated with some negative correlation between performance in the exam and a student belonging to one of the targeted populations. Since this fact and the way that it relates to the problems we identify in the current mechanism are crucial to our analysis, we will use the notion below.

Definition 7. *A set of contracts Y satisfies **weakly disadvantaged minorities** if, for any program p and privilege vector t for which $|Y_{p,t}| > q_p^t$, the student s with the $\left(q_p^t + 1\right)^{th}$ highest grade in p among students in $s\left(Y_{p,t}\right)$, is such that for every $s' \in s\left(Y_{p,t'}\right)$, where $t' < t$, $\theta_p(s) < \theta_p(s')$.*

A population of students with one contract per student in Y that satisfied weakly disadvantaged minorities, therefore, have the students who are at the *margin* of the quotas for their characteristics having a grade lower, at that program, than students claiming fewer privileges. The definition also contains, as a special case, a stronger (but simpler) condition:

Remark. *If, for any pair of contracts x, y in a given set of contracts Y $x_T < y_T \implies \theta_p(x_S) > \theta_p(y_S)$, then Y satisfies weakly disadvantaged minorities.*

That is, if students claiming more privileges is associated with having lower grades then the set of contracts satisfies weakly disadvantaged minorities. Our definition still allows for the existence of multiple minority students with grades that are higher than those who do not belong to a minority group, however. If the population of students satisfy that condition, in fact, Brazil Reserves satisfies most of the desirable characteristics that we listed in section III.A:

Proposition 1. *Let C_p^{BR} be any implementation of the Brazil Reserves. Then, C_p^{BR} legalistically satisfies affirmative action objectives. Moreover, given a set of contracts Y that satisfies weakly disadvantaged minorities, $C_p^{\text{BR}}(Y)$ is fair in p.*

The proposition below shows, however, that when some subset of contracts associated with a program does not satisfy weakly disadvantaged minorities, an incompatibility between fairness and legalistically satisfying affirmative action objectives emerges.

Proposition 2. *For any college admission problem with multidimensional privileges where for every p and t $|X_{p,t}| \geq q_p^t$, if there exists a set of contracts $Y \subseteq X$ violating weakly disadvantaged minorities, then no choice function exists which legalistically satisfies the affirmative action objectives and is fair.*

Proposition 2 shows that, when the policymaker wants to target individuals with different combinations of privileges in the composition of the accepted cohort, this generally comes at the cost of rejecting students with high grades and claiming more privileges.

When the affirmative action policy involves only one type of privilege, such as minority reserves (Hafalir, Yenmez and Yildirim, 2013), this incompatibility does not happen because there is a clear hierarchy in the access to seats: majorities have no quota associated with them. As a result, minority students are only rejected when their grades are lower than all accepted minority and non-minority students.

Next, we show that when contracts do not satisfy weakly disadvantaged minorities, implementations of the Brazil Reserves may fail to satisfy privilege monotonicity, also implying the violation of incentive-compatibility in stable mechanisms in which programs use that choice function.

Example 1. *Consider a program p with eight seats and where $r_p = \frac{1}{2}$, and the following set of students, in descending order of grade, and their vectors of available privileges:*

s_1	(h, m, i)
s_2	(h, m, i)
s_3	(h, m, i)
s_4	(h, m, i)
s_5	(H, M, I)
s_6	(H, M, I)
s_7	(H, m, I)
s_8	(H, M, i)
s_9	(H, m, i)

Under the Brazil Reserves choice function, student s_6 is rejected. Suppose, however, that student s_6 does not claim minority privileges. Then, the students applying to each set of seats is the following:

$Q^{(H,M,I)}$	$Q^{(H,m,I)}$	$Q^{(H,M,i)}$	$Q^{(H,m,i)}$	$Q^{(h,m,i)}$
s_5	s_6, s_7	s_8	s_9	s_1,s_2,s_3,s_4

By claiming fewer privileges student s_6 is chosen by any implementation of the Brazil Reserves, while if she claims all that she is eligible to claim she is not. Therefore, implementations of Brazil Reserves may ***not be privilege monotonic****. As a result, stable mechanisms using that family of choice functions will therefore* ***not be incentive-compatible****, since a student who prefers to be matched to a program may be accepted by claiming fewer privileges than she is eligible for.*

The example above shows that since the Brazil Reserves choice functions give priority for some seats to students who claim subsets of the privileges that a student may claim, some students may have an incentive to not claim all of her privileges. More specifically, if, for example, a minority student from public HS knows that a large number of high-scoring candidates may be applying to the seats in $Q^{(H,M,i)}$, she might increase her chances of being accepted by applying to $Q^{(H,m,i)}$ instead. This could give an unfair advantage to students who, for example, obtain information about other students' choices or who exchange information with their peers about their choices.

The proposition below shows that these incentive problems induced by the Brazil Reserves choice functions are intrinsically related to violations of the property of weakly disadvantaged minorities.

Proposition 3. *Let C_p^{BR} be any implementation of the Brazil Reserves. Let also Y be a set of contracts that satisfies weakly disadvantaged minorities and in which (i) no student has two contracts with different privilege vectors, and (ii) for every t we have that $|Y_{p,t}| \geq q_p^t$. Then, for every student s with a contract $(s,p,t) \in Y$ where $(s,p,t) \notin C_p^{\mathrm{BR}}(Y)$, for every $t' < t$ it is the case that $(s,p,t') \notin C_p^{\mathrm{BR}}(Y \backslash \{(s,p,t)\} \cup \{(s,p,t')\})$.*

That is, as long as there are enough students with each combination of privileges applying to a college, contracts satisfying weakly disadvantaged minorities guarantees that no student can manipulate the privileges they claim and become accepted by a college.

The shortcomings of the Brazil Reserves identified in this section rely on configurations in which the set of students' contracts violate weakly disadvantaged minorities. In the next section, we use empirical data to see whether these concerns are relevant or just a theoretical possibility.

V. Empirical Evidence

Although in the last section we showed that under the government's guidelines outcomes may not be fair and students may not be accepted to a program unless they strategize over the privileges that they claim, one may wonder how empirically relevant those situations are. After all, when there are more candidates than seats (which is the case in the vast majority of federal programs in Brazil,) in order for a student to successfully manipulate her claims she must have an exam grade that is higher than that of a student who is accepted but did not claim some privileges. For example, a low-income minority student who is not accepted when claiming all of her eligible privileges (and therefore has an exam grade that is lower than all those who are claiming those privileges) can only successfully manipulate her claims if her exam grade is higher than some student who has been accepted despite claiming fewer privileges. Since the affirmative action program is implemented to increase the access of those students when compared to a system that selects based simply on exam grades, it seems reasonable to expect these opportunities of manipulation to be rare. Note, moreover, that these allocations, which would allow for a successful manipulation of privileges claimed, are not fair: a student who is able to profitably manipulate is not accepted to a program and has a higher exam grade than a student who is accepted and is claiming fewer privileges.

We obtained the cutoff exam grades (that is, the lowest exam grade among those accepted in the program) for each of the five sets of seats described in section II for all the 3,187 federal higher education programs that participated in the SiSU in 2013 and implemented the guidelines described in section II.[10] Following the timeline specified in the law, during this first year of implementation of the new policies, the universities could opt to allocate only a quarter of the seats that would ultimately be allocated for the affirmative action policy. That is, instead of 50% of the seats in each program, the universities could opt to offer 12.5% or more. The ratios of those seats reserved for students claiming low income and minority privileges, however, remain at 0.5 and r_p, respectively. [11]

Let $\theta_p^*(H, m, i)$ be the cutoff grade at program p for the set of seats designated for students who claim the vector of privileges (H, m, i). For example, $\theta_p^*(H, m, I)$ is the cutoff grade at program p for the seats designated for students who claim public HS and low-income privileges. A necessary condition for a student to be able to successfully manipulate her claimed privileges is that the cutoff grade for seats designated with a certain set of privileges is higher than the cutoff grade for seats designated with a subset of those privileges. This

[10] Although the SiSU centrally matches students to programs, using a deferred acceptance procedure in which students' reported preferences are restricted to only two acceptable programs, for the sake of the analysis presented in this section those details are unimportant, since the conditions for the manipulation of the privileges claimed that we argue here, assuming a decentralized system, immediately translate to manipulations in the SiSU.

[11] As mentioned in section II, the value of r_p is the proportion of minorities in the overall population in the state where program p is located.

means that there may be a student who was not accepted but has an exam grade that is high enough to be accepted when applying to a set of seats designated for a lower number of privileges. For example, suppose there is a program $p \in P$ such that $\theta_p^* (H, m, I) > \theta_p^* (H, m, i)$. Let there be a student s with a vector of available privileges $t_s = (H, m, I)$ and an exam grade $\theta_p (s)$ such that $\theta_p^* (H, m, I) > \theta_p (s) > \theta_p^* (H, m, i)$. If she claims all of her available privileges she will not be accepted, since $\theta_p^* (H, m, I) > \theta_p (s)$. However, if she does not claim a low-income privilege she will be accepted, since $\theta_p (s) > \theta_p^* (H, m, i)$. Notice that when this is the case, the set of contracts associated with the set of students applying to that program violates weakly disadvantaged minorities.

Since we do not have data on the grades of the students who were not accepted, however, we are not able to determine whether there are, in fact, students who could have been accepted if they had manipulated the privileges they were claiming. However, given the high competition for seats in those programs — in total there were 1,757,399 candidates and 129,319 seats, an average of 13.59 candidates per seat[12] — it is reasonable to use the occurrence of those disparities in cutoff grades as an indication of the existence of opportunities for manipulation. We therefore looked for instances in which the cutoff grades for a set of seats reserved for students claiming a certain set of privileges were higher than the cutoff grades for seats, in the same program, reserved for students who were claiming a subset of those privileges. The results are presented in Table 1.

Regarding the values in Table 1, the first fact to note is how pervasive the issue is. In more than 54% of the programs there is at least one instance in which those conditions for manipulability are observed. That is, there is a reasonable chance that in those programs, the allocation is not fair: students that are the target of the affirmative action policies are not being accepted even though they have higher exam grades than those who are accepted. One might wonder how significant the differences are between the cutoff grades presented above, since when these are too small it becomes less likely that some student would have the opportunity to successfully manipulate her outcome. The grades obtained in the exam, across all students who took it, range from 261.33 to 971.5. Since the competition for seats is very high, however, the more relevant information is how those differences compare to the range of grades that allow for acceptance in some

[12]Source: http://g1.globo.com/educacao/noticia/2013/01/sisu-registra-quase-2-milhoes-de-inscricoes-diz-ministerio-da-educacao.html (Accessed on October 16, 2017).

	Number of occurrences (out of 3,187)	Average difference (standard deviation)
$\theta_p^*(H,M,I) > \theta_p^*(H,M,i)$	935	11.56 (13.24)
$\theta_p^*(H,M,I) > \theta_p^*(H,m,I)$	398	12.60 (14.70)
$\theta_p^*(H,M,I) > \theta_p^*(H,m,i)$	161	13.67 (15.19)
$\theta_p^*(H,M,I) > \theta_p^*(h,m,i)$	51	8.88 (8.53)
$\theta_p^*(H,M,i) > \theta_p^*(H,m,i)$	217	14.85 (17.29)
$\theta_p^*(H,M,i) > \theta_p^*(h,m,i)$	79	13.20 (12.25)
$\theta_p^*(H,m,I) > \theta_p^*(H,m,i)$	452	15.19 (16.29)
$\theta_p^*(H,m,I) > \theta_p^*(h,m,i)$	181	12.15 (13.25)
$\theta_p^*(H,m,i) > \theta_p^*(h,m,i)$	384	13.06 13.79
Number of programs with at least one of the cases above	1,730 (out of 3,187)	

Table 1—Instances in which the observable conditions for the manipulability of the current guidelines are met and the average difference in the cutoff grades. Source: Brazilian Ministry of Education.

programs. By observing the distribution of cutoff grades across all programs we therefore have a better idea of the range of exam grades obtained by those who are closer to the borderline between being accepted or not. Table 2 shows the difference between the 5% quantile and the 95% quantile, for each, 90% of them are in the 500–750 range and 61.14% are in the 600–700 range. Moreover, 64.67% of the differences summarized in Table 1 are greater than or equal to 5 points.

Although we could not find any data on the number of candidates and seats for each of the programs above, we did discover some information for one of the universities, UNIFESP, which published the number of candidates per seat for its 56 programs, 38 of them among those with the issue above. Table 3 shows the values of the cutoff grades and the number of candidates per seat for each type of seat for four programs in that university. These give an indication of the likely

Seats	$Q^{(H,M,I)}$	$Q^{(H,M,i)}$	$Q^{(H,m,I)}$	$Q^{(H,m,i)}$	$Q^{(h,m,i)}$
Average	628.18	634.59	639.28	652.02	665.76
5% Quantile	563.02	564.67	572.46	578.45	593.82
95% Quantile	703.14	718.92	722.29	738.06	752.36
Difference	140.12	154.25	149.84	159.61	158.54

Table 2—Quantiles of cut-off grades

reason why the numbers in Table 1 are so dramatic: the competition for seats reserved under the affirmative action policy is very high, and therefore there are enough students who claim those privileges and have high exam grades to push up the value level of the cutoff grades. Since the number of seats allocated for affirmative action will increase to its target 50% in the coming years, the proportion of programs with this issue will likely be reduced. But given how extreme the differences are in the competitiveness of the seats, it is reasonable to expect it to remain significant.

Seats	$Q^{(H,M,I)}$		$Q^{(H,m,I)}$		$Q^{(H,M,i)}$		$Q^{(H,m,i)}$		$Q^{(h,m,i)}$	
	Cutoff	C/S	Cutoff	C/S	Cutoff	C/S	Cutoff	C/S	Cutoff	C/S
Philosophy	652.76	28.50	671.53	18.00	657.70	25.50	688.18	30.50	675.93	10.69
History	684.29	71.00	667.92	36.67	669.67	42.50	678.29	50.00	685.78	19.00
Economics	682.82	83.50	732.68	111.00	696.24	60.00	719.46	117.00	719.26	41.65
Pharmacy	681.58	88.67	679.82	81.40	673.94	70.67	703.66	105.00	704.88	30.01

Table 3—Cutoff grades and candidates per seat (C/S) for programs at UNIFESP in 2013.
Source: UNIFESP

VI. Student-proposing stable mechanism

As we have shown in section IV, the Brazil Reserves choice function suffers from serious shortcomings when the contracts available to students violate weakly disadvantaged minorities. High-achieving low-income and/or minority students may not be accepted into a program while students without those characteristics are. Moreover, students who strategically manipulate the privileges they claim may obtain an unfair advantage. The empirical evidence in the last section shows evidence compatible with a pervasiveness of these problems.

In this section we take a closer look at the flaws in the design of the Brazil

Reserves which lead to these problems, and develop a solution that solves those issues for the Brazilian college admissions and for a wide range of similar problems. For that, we propose a choice function, that could be used by the universities even in the absence of a centralized mechanism to produce assignments. That is, if a university simply faces a pool of applicants, the choice function could be used to determine which ones should be accepted by that university. We also aim to design a mechanism that carries out our choice function's properties and produces stable allocations.

We are proposing a new choice function, which consists of a choice procedure with slot-specific priorities, where the priorities are designed in such a way that any possible gain from strategizing over the privileges claimed is removed.

The intuition behind the way in which the slot-specific priorities are designed is that whenever a set of contracts Y_t are in a slot's priority ordering, contracts $Y_{t'}$ claiming more privileges (that is, $t' > t$) must either have a higher priority than those in Y_t or must be ordered by grade together with Y_t. For example, suppose a program p has a single seat and can accept contracts claiming the vectors of privileges (H, M, I), (H, m, I) and (H, m, i), with priorities between these contracts as follows:

$$Y_{(h,m,i)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(H,M,I)}$$

Priorities among contracts within the same indifference class are determined by the students' exam grades. Under those priorities, a student claiming the vector of privileges (H, M, I) would only be accepted if there were no students claiming (h, m, i) or (H, m, I), regardless of their exam grades. If that student instead claims (h, m, i) and her exam grade is high enough then she could be accepted to that seat. That is, a manipulation of her privilege vector would be profitable. In the priority orders used in the Brazil Reserves, we see this problem: in the seats in $Q^{(H,m,i)}$, for example, contracts claiming (H, M, i) have lower priority than any contract claiming (H, m, i).

Consider instead the following two alternative priorities:[13]

[13] In the second alternative we give, the priority between the contracts in the first indifference class is lexicographic, first based on students' exam grades, and then based on any arbitrary order of privileges claimed. In practice, as we will show later, this order of privileges within the indifference classes is inconsequential, since students will only use contracts with a single vector of privileges. As a result, the priority order between contracts involving two different students within an indifference class is determined only by their grades.

$$Y_{(H,M,I)} \blacktriangleright' Y_{(H,m,I)} \blacktriangleright' Y_{(h,m,i)}$$

$$Y_{(H,M,I)} \cup Y_{(H,m,I)} \blacktriangleright'' Y_{(h,m,i)}$$

In both cases, no manipulation of the vector of privileges claimed can be profitable: a student who is not chosen while claiming a vector of privileges would also not be chosen by claiming fewer privileges. Notice, however, that under $\blacktriangleright'$ whenever there is at least one student claiming the vector (H, M, I) the chosen student will be the one claiming that vector, whereas under $\blacktriangleright''$ a student claiming the vector (H, M, I) will only be chosen if her exam grade are greater than all the students claiming (H, M, I) or (H, m, I) in Y.

That is, under $\blacktriangleright''$ students who claim (H, m, I) are still eligible for preferential treatment in the presence of students claiming (H, M, I). In order to eliminate gains that the latter could have by misrepresenting their privileges, however, this preferential treatment is conditional on having grades that are higher than some students claiming (H, M, I).

Next, we generalize this insight and apply the solution to the Brazilian case.

A. *The multidimensional privileges choice function*

We will now extend the notation we have been using for an arbitrary set of privileges, while keeping it consistent with what we have done so far. There is a list of privileges $\Gamma = \left(\gamma^1, \gamma^2, \ldots, \gamma^k\right)$ that students can claim, and each student s can claim some subset of those privileges. We denote the vector of available privileges of student s by $t_s = \left(t_s^1, t_s^2, \ldots, t_s^k\right)$, where $t_s^i \in \{0, 1\}$. Student s can claim privilege γ^i if and only if $t_s^i = 1$.

The set of combinations of privileges that can be claimed therefore consist of a list of vectors $T = \left(t^1, t^2, \ldots, t^{2^k-1}\right)$, ordered by the number in the binary base that they represent. For example, let $k = 3$. Then t^0 refers to claiming no privilege — $(0, 0, 0)$ — t^1 for claiming only the privilege γ^1, that is, the vector $(0, 0, 1)$, t^6 for claiming both γ^2 and γ^3, that is, the vector $(1, 1, 0)$, and so on. Each program p has a list of affirmative action objectives, which are non-negative values associated with each vector of privileges in T: $\left(q_p^{t^0}, q_p^{t^1}, \ldots, q_p^{t^{2^k-1}}\right)$. The total number of reserved seats equals the capacity of a program: $\sum_{i=0}^{2^k-1} q_p^{t^i} = q_p$. Seats that are not reserved for any privilege vector are "reserved" for those claiming t^0.

In line with our observations, in each seat that is reserved for some privilege vector t, we will combine, together, contracts claiming a superset of those privileges. Therefore, for every set of contracts Y, vector of privileges t, and seat reserved for it Q_p^t, the slot-specific priority is:

$$Q_p^t : \left\{ \bigcup_{t' \in \{0,1\}^k : t' \geq t} Y_{t'} \right\} \blacktriangleright Y_{t^{2^k-1}} \blacktriangleright Y_{t^{2^k-2}} \blacktriangleright \cdots \blacktriangleright Y_{t^0}$$

No contract is ordered more than once, so if a contract is in the first class, it is not repeated down the ordering.[14] Contracts within a class are ordered lexicographically, as follows:

$$\left(s, p, t'\right) \blacktriangleright \left(s', p, t''\right) \implies \theta_p\left(s\right) > \theta_p\left(s'\right) \text{ or } s = s' \text{ and } t' > t''$$

We denote any choice function that is based on the slot-specific priorities above by a **multidimensional privileges choice function**, or C_p^{MCF}. In these choice functions, in any seat reserved for contracts claiming a vector of privileges t, the top class combines those contracts with t with those contracts claiming all the privileges in t and some extra privileges. Since students may choose not to claim some privilege that is available to them, what this does is combine contracts from students claiming t with contracts from *students who could choose to claim t*. The way that the contracts within indifference classes are ordered makes sure that high-achieving students claiming more privileges can compete for seats reserved for those claiming fewer privileges, ensuring fairness, and that the contract claiming the largest set of privileges is the one chosen among those available to a student. The rest of the priority order, in decreasing order of privileges claimed, removes the possibility of a strategic manipulation of privileges when there are not enough contracts in the first indifference class.

In addition to these characteristics related to fairness and privilege monotonicity, the way the top indifference classes combine contracts makes sure that whenever there are enough high-scoring students claiming each combination of privileges, they will be matched to the seats reserved for them. All of these facts combine into the theorem below.

[14]We do not make that explicit in the notation for simplicity of exposition.

Theorem 1. *Every choice function C_p^{MCF} is privilege monotonic, fair, and satisfies the spirit of the affirmative action objectives. If $Y \subseteq X$ satisfies weakly disadvantaged minorities then $C_p^{\mathrm{MCF}}(Y)$ legalistically satisfies the affirmative action objectives for any program p.*

The properties of fairness and satisfaction of the spirit of affirmative action objectives can also be modeled as a *matching with distributional constraints* (Kamada and Kojima, 2017) (KK). In the KK model, the matchings of students to programs are deemed feasible if they satisfy some capacity constraints, which can include not only capacities for a single program, but also for groups of programs. Our notions of fairness and affirmative action objectives are closely related to their model in the cases where, whenever a program p chooses from a set of contracts Y, it contains at least q_p^t contracts with type t, for each t such that $q_p^t > 0$. In this case, for each matching of students that is fair and satisfies the spirit of affirmative action objectives, we are able to construct an instance of the KK model, in which a corresponding matching is *strongly stable* with respect to their model. Notice, however, that when the number of students claiming each vector of privileges is not large enough, this relation does not involve affirmative action objectives.

We can now apply the definition of the multidimensional privileges choice function to the Brazilian affirmative action policies. Given a set of contracts Y, the slot-specific priorities for the seats in a program are as follows:

Set	**Number of seats**	**Slots priorities**
$Q^{(H,M,I)}$	$\lceil \frac{q_p}{4} r_p \rceil$	$Y_{(H,M,I)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(H,M,i)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$
$Q^{(H,M,i)}$	$\lceil \frac{q_p}{4} r_p \rceil$	$Y_{(H,M,I)} \cup Y_{(H,M,i)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$
$Q^{(H,m,I)}$	$\lceil \frac{q_p}{4} (1 - r_p) \rceil$	$Y_{(H,M,I)} \cup Y_{(H,m,I)} \blacktriangleright Y_{(H,M,i)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$
$Q^{(H,m,i)}$	$\lceil \frac{q_p}{4} (1 - r_p) \rceil$	$Y_{(H,M,I)} \cup Y_{(H,m,I)} \cup Y_{(H,M,i)} \cup Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$
$Q^{(h,m,i)}$	$Q - 2\left(q^{(H,M,I)} + q^{(H,m,i)}\right)$	$Y_{(H,M,I)} \cup Y_{(H,m,I)} \cup Y_{(H,M,i)} \cup Y_{(H,m,i)} \cup Y_{(h,m,i)}$

The precedence order in which those slots are filled is left as a choice for the policymaker. Although the order that is chosen does not impact any of the results presented in this paper, different orders of precedence may lead to accepting different sets of students (Kominers and Sönmez, 2016; Dur et al., 2018).[15] We will denote this implementation of the multidimensional privileges choice function for the Brazilian case by $C^{\mathrm{BR-MCF}}$.

As shown in Theorem 1, when using $C^{\mathrm{BR-MCF}}$ allocations will be fair in any program and incentives for strategically manipulating the privileges claimed are eliminated. This represents a clear improvement over the problems identified in the Brazil Reserves. The objective of legalistically satisfying affirmative action objectives, however, goes in the opposite direction. While the Brazil Reserves always produces allocations that satisfy them, $C^{\mathrm{BR-MCF}}$ may not. This is not a surprise: we saw in proposition 2 that fairness is, in general, incompatible with legalistically satisfying affirmative action objectives.

The choice between Brazil Reserves and $C^{\mathrm{BR-MCF}}$, therefore, may be seen as a choice between (i) legalistically satisfying affirmative action objectives or (ii) fairness and privilege monotonicity. While this points to a seemingly straightforward trade-off, the lack of privilege monotonicity in Brazil Reserves may lead to manipulations on the part of students that lead to the set of accepted students legalistically violating affirmative action objectives, as shown in example 1.

B. The student-proposing stable mechanism

We now present a mechanism that, given students' preferences over programs and their vectors of claimed privileges, produces an allocation of students to programs. For each student s we collect her preference ranking over programs $\succ_s^*$ and the vector of privileges claimed t_s. We then use the cumulative offer mechanism (Hatfield and Milgrom, 2005), which is a generalization of the Gale-Shapley deferred acceptance mechanism for the problem of matching with contracts. While the preference relation over contracts $\succeq_s$ contains indifferences, we make a small modification to the mechanism by making students propose only contracts with the vector of privileges they submitted. As shown in Kominers and Sönmez

[15]To illustrate why this is the case, consider a simple example of a program with two sets of seats, Q^1 and Q^2, both with unit capacity, where Q^1 orders contracts claiming a privilege γ above those not claiming any privilege. Let the students, in descending order of grades, be $S = \{s_1, s_2, s_3\}$, where only s_1 and s_3 claim the privilege γ. If the precedence order is $Q^1 \rhd Q^2$ student s_1 will be allocated to seat Q^1 and s_2 to Q^2, so the set of students accepted is $\{s_1, s_2\}$. If the precedence order is $Q^2 \rhd Q^1$, instead, then s_1 will be allocated to seat Q^2, and s_3 will be allocated to seat Q^1, since that seat will prioritize s_3 over s_2, due to the fact that the former is claiming γ. As a result, the set of students accepted becomes $\{s_1, s_3\}$.

(2016), when the programs' choice functions are based on slot-specific priorities, such as C^{MCF}, the outcome of that mechanism is stable. While in principle, due to the elimination of indifferences, the outcome would not necessarily also be stable with respect to $\succeq_s$, we show below that this is also the case here. The overall procedure, therefore, is denoted the student-proposing stable mechanism, or SPSM. A detailed description of the mechanism is given in the Appendix.

Although we have shown that the choice function that we proposed satisfies the desired fairness and incentives properties, we are also interested in knowing whether the corresponding properties are satisfied by the overall allocation when the SPSM mechanism is used to match students to programs. The first properties that we analyze are stability and fairness.

Proposition 4. *The student-proposing stable mechanism, ψ^{SPSM}, is stable under $(\succeq_s)_{s\in S}$ and C^{MCF}, and is fair.*

The next property that we present here is the incentive-compatibility of the mechanism, which is a desired characteristic in mechanism design. Incentive-compatibility in this context can be described as a property that guarantees that students cannot be better off by strategizing over the preferences being submitted or privileges being claimed. In our problem, the students' strategy spaces consist not only of the preferences over schools but also the privileges claimed. Although it may be tempting to conclude that the incentive-compatibility of the SPSM immediately follows as a corollary of the well-known incentive properties of the SPSM mechanism, due to the wider strategy space for students the result must be obtained explicitly.

Proposition 5. *The student-proposing stable mechanism, ψ^{SPSM}, is incentive-compatible.*

C. Beyond the Brazilian case

We showed that multidimensional privileges choice function and the student-proposing stable mechanism in association with these functions, constitute proposals with desirable characteristics for the Brazilian affirmative action policies. The problems that we identified, and the solution, however, potentially have a much wider applicability. The main characteristics that a matching or assignment problem should have to create the incentives to misrepresent their privileges, but be solved with our proposal, are listed below.

Preferential treatment based on multiple characteristics. — One crucial aspect of the affirmative action policies in Brazil that we describe is that they target multiple characteristics that individuals may have: income, ethnicity, and the type of institution they studied. Most importantly, they may have different combinations of these characteristics. When that is not the case, existing minority reserves mechanisms, such as minority reserves (Hafalir, Yenmez and Yildirim, 2013) and the "soft bounds" mechanism in Ehlers et al. (2014) provide most of the benefits we have shown.

Optionality of not claiming privileges. — In the Brazilian affirmative action policy, the opportunity to claim any privilege is entirely optional. A student who belongs to a low-income family, for example, may choose not to claim that fact when applying to universities. The same for black students or those from public HS. In order for our solution to remove the incentives for manipulation, however, that optionality must go only in one direction. A low-income student may choose not to claim that privilege, but a high-income one cannot choose to do so. Clearly, when that is not the case and agents are able to costlessly claim privileges or not, whenever that characteristic is used in a beneficial or detrimental way, strategical considerations will be unavoidable. Whenever there is the need for some documental proof associated with the privilege, such as, for example, a medical documentation certifying a disability, however, this directed optionality is satisfied.

Indifference among different types of seats. — We have assumed, throughout the paper, that students have preferences only over the programs to which they are matched. That is, that they are indifferent to which specific seat was used in their acceptance. When that is not the case then our solution would have different strategic implications and would not be incentive-compatible. In the Indian engineering schools, whether a student is admitted to a reserved seat determines whether college housing is provided, for example. In that case, a solution that considers that preference may restore incentive-compatibility (Aygün and Turhan, 2020).

VII. Conclusion

In this paper we analyzed, under the perspective of a market designer, the affirmative action policy implemented in the selection of students for federal universities in Brazil. We showed that the method chosen by the policymakers, in which students claiming the same set of privileges compete on the basis of exam grades, may lead to unfair matchings and create incentives for students to strategize over the privileges that they claim.

The empirical evidence that we provided indicates that these problems are likely not just a purely theoretical possibility but may be affecting the outcomes of a large number of students. The solution that we provide is based on a simple principle: when students are applying for seats reserved for those claiming some privileges, those claiming more privileges must be able to compete for them as well. Otherwise, by claiming fewer privileges students may have access to seats that would otherwise be out of their reach.

In a broader sense, some of the lessons that we learned from the policy analyzed also apply to other market design problems. The initial motivation that led to the implementation of the affirmative action policies was the observation that, under the old criterion for acceptance based solely on exam grades, the affected populations were underrepresented. This was, in large part, due to the fact that students from public HS, minorities, and from low-income families obtained, on average, lower exam grades. Giving those students a higher priority in sets of seats proportional to its population seems at first sight to be a solution that would work well. One problem is that not all of these students have low grades and, moreover, they may have different preferences. If low-income students are more likely to prefer a particular program than those with a high income, for example, the competition for the low-income seats would be substantially higher than that for the high-income, therefore leading to fairness and incentive problems. Policymakers must, whenever possible, design mechanisms that are robust to different configurations and assumptions about the characteristics of the participants.

REFERENCES

Abdulkadiroğlu, Atila. 2005. "College admissions with affirmative action." *International Journal of Game Theory*, 33(4): 535–549.

Abdulkadiroğlu, Atila, and Tayfun Sönmez. 2003. "School Choice: A Mechanism Design Approach." *The American Economic Review*, 93(3): 729–747.

Aygün, Orhan, and Bertan Turhan. 2020. "Dynamic reserves in matching markets." *Journal of Economic Theory*, 188: 105069.

Balinski, Michel, and Tayfun Sönmez. 1999. "A tale of two mechanisms: Student placement." *Journal of Economic Theory*, 84(1): 73–94.

Bo, Inacio. 2016. "Fair implementation of diversity in school choice." *Games and Economic Behavior*, 97: 54–63.

Doğan, Battal. 2016. "Responsive affirmative action in school choice." *Journal of Economic Theory*, 165: 69 – 105.

Dur, Umut, Scott Duke Kominers, Parag A. Pathak, and Tayfun Sönmez. 2018. "Reserve Design: Unintended Consequences and the Demise of Boston's Walk Zones." *Journal of Political Economy*, 126(6): 2457–2479.

Ehlers, Lars, Isa E Hafalir, M Bumin Yenmez, and Muhammed A Yildirim. 2014. "School choice with controlled choice constraints: Hard bounds versus soft bounds." *Journal of Economic Theory*, 153: 648–683.

Hafalir, Isa E, M Bumin Yenmez, and Muhammed A Yildirim. 2013. "Effective affirmative action in school choice." *Theoretical Economics*, 8(2): 325–363.

Hatfield, John William, and Paul R. Milgrom. 2005. "Matching with Contracts." *The American Economic Review*, 95(4): 913–935.

Kamada, Yuichiro, and Fuhito Kojima. 2017. "Stability concepts in matching under distributional constraints." *Journal of Economic Theory*, 168: 107 – 142.

Kojima, Fuhito. 2012. "School choice: Impossibilities for affirmative action." *Games and Economic Behavior*, 75(2): 685–693.

Kominers, Scott Duke, and Tayfun Sönmez. 2016. "Matching with slot-specific priorities: Theory." *Theoretical Economics*, 11(2): 683–710.

McCowan, Tristan. 2007. "Expansion without equity: An analysis of current policy on access to higher education in Brazil." *Higher education*, 53(5): 579–598.

Moehlecke, Sabrina. 2002. "Ação afirmativa: história e debates no Brasil." *Cadernos de pesquisa*, 117(11): 197–217.

Pathak, Parag A, and Tayfun Sönmez. 2008. "Leveling the playing field: Sincere and sophisticated players in the Boston mechanism." *The American Economic Review*, 98(4): 1636–1652.

Ritter, Carolina. 2018. "A política de cotas na educaçãoo superior: as (a)simetrias entre o acesso nas universidades federais e o desenvolvimento social brasileiro." PhD diss. Programa de Pós-Graduação em Serviço Social, Escola de Humanidades.

Westkamp, Alexander. 2013. "An analysis of the German university admissions system." *Economic Theory*, 53(3): 561–589.

Appendix

Proofs

PROOF OF PROPOSITION 1: Let a program's choice function C_p^{BR} be an implementation of the Brazil Reserves. Then, for any privilege vector $t > (0,0,0)$ that students can submit, C_p^{BR} requires q^t seats to give contracts with the privilege vector t highest priority. Now, consider any given set of contracts Y and any given privilege vector $t > (0,0,0)$. During the choice procedure, some contracts with privilege vector t may be chosen before C_p^{BR} handles the group of seats Q_p^t. Therefore, when the choice function handles the group of seats Q_p^t, the number of remaining contracts with the privilege vector t will be either less than or equal to q^t, or greater than q^t. In the former case, the choice function chooses all remaining contracts, in the latter case, q^t contracts with the privilege vector t. So, from any given contract set Y, the choice function C_p^{BR} chooses either all contracts with the privilege vector t, or q^t contracts with the privilege vector t. Hence, C_p^{BR} legalistically satisfies the affirmative action objectives.

As the second part of the proof, one should note that given that C_p^{BR} legalistically satisfies the affirmative action objectives, for any privilege vector $t > (0,0,0)$ if a contract $x \in Y$ with privilege vector t is rejected, then x_S is not among the top q^t students who claimed the privilege vector t. Thus, x_S has a lower grade than the owners of any contract with privilege vector $t' < t$, by weakly disadvantaged minorities. By definition, fairness is violated only if there exists a chosen contract whose owner has a lower grade with a privilege vector $t' < t$. Since their owners have higher grades, C_p^{BR} is fair.

Proof of Proposition 2: By assumption, for every p and t $|X_{p,t}| \geq q_p^t$. Assume there exists a set of contracts $Y \subseteq X$ violating weakly disadvantaged minorities. So, there exists a program p and a pair of contracts x, x' such that $p(x) = p(x') = p$, $t(x) > t(x')$, $s(x)$ has the $(q_p^{t(x)} + 1)^{th}$ highest grade in p among students in $s\left(Y_{p,t(x)}\right)$ and $\theta_p(s(x)) > \theta_p(s(x'))$.

Now consider a set of contracts Y' which includes the contracts of the top $(q_p^{t(x)} + 1)$ students in $s(Y_{p,t(x)})$ with the privilege vector $t(x)$, $q_p^{t(x')}$ contracts with privilege type $t(x')$ including x' and q_p^t contracts with the privilege vector t, for all $t \notin \{t(x), t(x')\}$.

Let C be an arbitrary choice function legalistically satisfying affirmative action objectives. It is easy to see that C chooses $Y' \setminus \{x\}$ from the set Y', i.e., $C(Y') = Y' \setminus \{x\}$. Since contract x is rejected, contract x' is accepted, $\theta_p(s(x)) > \theta_p(s(x'))$ and $t(x) > t(x')$ we say C is not fair. Hence, there is no choice function which legalistically satisfies affirmative action objectives and is fair.

Proof of Proposition 3: Let a program's choice function C_p^{BR} be an implementation of the Brazil Reserves. Then, by proposition 1, C_p^{BR} legalistically satisfies affirmative action objectives, which requires that for any privilege vector t, if a contract x is rejected, then $\theta_p(x_S)$ is not among the top q_p^t grades of students who claimed the privilege vector t. Moreover, for any $t' < t$, after replacing x with (x_S, p, t'), $\theta_p(x_S)$ will not be among the top $q_p^{t'}$ grades of students who claimed the privilege vector t', by weakly disadvantaged minorities. Therefore, (x_S, p, t') will be rejected due to the property of legalistically satisfying affirmative action objectives.

Proof of Theorem 1: First, consider privilege monotonicity. Suppose, for the sake of contradiction, that there is a set of contracts $Y \subset X$, and a student s with no contract in Y, where $(s,p,t_s) \notin C_p^{\text{MCF}}(Y \cup \{(s,p,t_s)\})$ and $(s,p,t') \in C_p^{\text{MCF}}(Y \cup \{(s,p,t')\})$, for some $t' \leq t_s$. Since the only difference between the two sets are the contracts (s,p,t_s) and (s,p,t'), the contract (s,p,t') has a higher priority than (s,p,t_s) at some slot. However, since by construction of C_p^{MCF} there is no slot giving higher priority to (s,p,t') than (s,p,t_s), we have a contradiction. Hence, C_p^{MCF} is privilege monotonic.

Second, we prove the fairness property. For any set of contracts Y, if a contract $x = (s,p,t)$ is rejected, $x \notin C_p^{\text{MCF}}(Y)$ then it is not chosen by any slot of program p. Since by construction of $C_p^{\text{MCF}}(Y)$, there is no slot giving higher priority to a contract with privilege vector $t' < t$, chosen contracts have either higher privilege vector or have owners with higher grades than s. Therefore, $C_p^{\text{MCF}}(Y)$ is fair.

Third, consider satisfying the spirit of the affirmative action objectives property. Let $\{Q_p^{t^{2^k-\alpha}}\}_{\alpha=1}^{2^k-1}$ be the sequence of groups of seats processed by C_p^{MCF}. We use mathematical induction to prove the statement.

Base Case: For $\alpha = 1$, since agents claiming all privileges have highest priority, we have $|C_p(Y) \cap Y_{t^{2^k-1}}| \geq \min\left\{q_p^{t^{2^k-1}}, \left|Y_{t^{2^k-1}}\right|\right\}$.

Inductive Step: We now show that for any α our condition is satisfied if it is satisfied for any $\alpha' < \alpha$.

Let $C_p^{\alpha'}(Y)$ be the cumulative set of contracts chosen from the set Y up to step α'. If the condition is satisfied by any $\alpha' < \alpha$, then at the beginning of step α either we have

$$\sum_{t' \geq t^{2^k-\alpha}} \left|C_p^{\alpha-1}(Y) \cap Y_{t'}\right| \geq \sum_{t' \geq t^{2^k-\alpha}} \min\left\{q_p^{t'}, |Y_{t'}|\right\}, or$$

$$\sum_{t' \geq t^{2^k-\alpha}} \min\left\{q_p^{t'}, |Y_{t'}|\right\} > \sum_{t' \geq t^{2^k-\alpha}} \left|C_p^{\alpha-1}(Y) \cap Y_{t'}\right| \geq \sum_{t' > t^{2^k-\alpha}} \min\left\{q_p^{t'}, |Y_{t'}|\right\}$$

In the former case, condition is satisfied by α. In the latter case, since contracts with privilege vectors $t' \geq t^{2^k-\alpha}$ have highest priority, in step α, either $q_p^{t^{2^k-\alpha}}$ more contracts are accepted with privilege vectors $t' \geq t^{2^k-\alpha}$, or all remaining contracts with privilege vectors $t' \geq t^{2^k-\alpha}$ are accepted. In any case, we have $\sum_{t' \geq t^{2^k-\alpha}} \left|C_p^{\alpha}(Y) \cap Y_{t'}\right| \geq \sum_{t' \geq t^{2^k-\alpha}} \min\left\{q_p^{t'}, |Y_{t'}|\right\}$. Therefore, for any α our condition is satisfied. Hence, every choice function C_p^{MCF} satisfies the spirit of the affirmative action objectives.

Finally, assume that a set of contracts Y satisfies weakly disadvantaged minorities. Let $\{Q_p^{t^{2^k-\alpha}}\}_{\alpha=1}^{2^k-1}$ be the sequence of groups of seats processed by C_p^{MCF}.

First, consider the privilege vector $t^{2^k-1} = (1, 1, \ldots, 1)$. Since $q^{t^{2^k-1}}$ seats are reserved for t^{2^k-1}, it is guaranteed that $C_p^{\text{MCF}}(Y)$ has at least $\min\{q^{t^{2^k-1}}, |Y_{t^{2^k-1}}\}$ contracts with privilege vector t^{2^k-1}. Next, as an induction strategy, we will show that for any α, the number of accepted contracts after the group of seats $Q_p^{t^{2^k-\alpha}}$ processed by C_p^{MCF} is no less than $\min\{q^{t^{2^k-\alpha}}, |Y_{t^{2^k-\alpha}}\}$. We already show induction assumption for $\alpha = 1$. Now, we will show that the assumption is true for a given α if it is satisfied for all $\alpha' < \alpha$.

Since for all $\alpha' < \alpha$ the assumption is satisfied, in the first class of contracts for the group of seats $Q_p^{t^{2^k-\alpha}}$ either there is no contract with a privilege vector higher than $t^{2^k-\alpha}$ or, due to weakly disadvantaged minorities, the owners of any remaining contract with a higher privilege vector have lower grades than the owners of the contracts with $t^{2^k-\alpha}$. Therefore, when the group of seats $Q_p^{t^{2^k-\alpha}}$ is processed, the number of chosen contracts with the privilege vector $t^{2^k-\alpha}$ will be no less than $\min\{q^{t^{2^k-\alpha}}, |Y_{t^{2^k-\alpha}}\}$. Hence, for any set of contracts Y that satisfies weakly disadvantaged minorities, C_p^{MCF} legalistically satisfies the affirmative action objectives.

Proof of Proposition 4: Assume that the student-proposing stable mechanism, ψ^{SPSM}, is not stable under $(\succeq_s)_{s\in S}$ and C^{MCF}. Therefore, either the individual rationality condition is violated or there exists a blocking pair. By construction of the cumulative offer algorithm, in each step students offer one of their acceptable contracts. Therefore, no student has a contract worse than being unmatched. If there is a blocking pair then there exists a pair (p, s) and a contract x' such that $x' \succ_s \psi^{\text{SPSM}}(s)$ and $x' \in C_p^{\text{MCF}}(Y_p \cup \{x'\})$. Let $\psi^{\text{SPSM}}(s)$ be x. Since $x' \succ_s x$ there exists another contract x'' of student s which is offered before x and has the same privilege vector. Therefore, by IRC, $x'' \notin C_p^{\text{MCF}}(Y_p \cup \{x''\})$ and by privilege monotonicity, $x' \notin C_p^{\text{MCF}}(Y_p \cup \{x'\})$. Hence, ψ^{SPSM} is stable under $(\succeq_s)_{s\in S}$ and C^{MCF}.

Next, assume that the mechanism is not fair. That is, we can find $x, y \in X'$ such that $y_P \succ^*_{x_S} x_P$, $\theta_{y_P}(y_S) < \theta_{y_P}(x_S)$ and $x_T > y_T$. Since we have $y_P \succ^*_{x_S} x_P$, there exists a contract x' such that $x' = (x_S, y_P, x_T)$ and $x' \succ_{x_S} x$. By the design of the cumulative offer mechanism, x' must be offered by x_S and be rejected before the final step K. Therefore, at step K, we have $y, x' \in A_{y_P}(K)$ and $X'_{y_P} = C^{\text{MCF}}_{y_P}(A_{y_P}(K))$. Since, by theorem 1, $C^{\text{MCF}}_{y_p}$ is fair, x' is rejected from $A_{y_P}(K)$ and y is accepted, then either $\theta_{y_P}(y_S) > \theta_{y_P}(x_S)$ or $x_T \ngeq y_T$ must be true. A contradiction. Hence, ψ^{SPSM}, is fair.

Proof of Proposition 5: For an arbitrary student s, assume that $\delta' = (t', \succ'_s) \neq (t_s, \succ_s)$. Let her assigned program from $\psi^{\text{SPSM}}(\delta', \delta_{-s})$ be p^*. Since for any fixed submitted privilege vector profile choice functions and SPSM is an example of a cumulative offer mechanism induced by slot-specific priorities. According to Theorem 3 of Kominers and Sönmez (2016), the SPSM cannot be manipulated via preferences over programs. Therefore, for a strategy δ'' in which we have privilege vector t' and preference where only contract (s, p^*, t') is acceptable, we must have $\psi^{\text{SPSM}}(\delta'', \delta_{-s}) = p^*$.

Since, by construction of C^{MCF}, no slot of any program gives less priority if student s applies with t_s instead of any $t' < t_s$. So, under Theorem 4 of Kominers and Sönmez (2016), we have that for a strategy δ''' in which we have privilege vector t_s and preference where only contract (s, p^*, t_s) is acceptable, we must have $\psi^{\text{SPSM}}(\delta''', \delta_{-s}) = p^*$.

Finally, again under Theorem 3 of Kominers and Sönmez (2016) SPSM cannot be manipulated via preferences over programs. Therefore, for a strategy $\delta_s = (t_s, \succ_s)$, we must have $\psi^{\text{SPSM}}(\delta_s, \delta_{-s}) \succeq_s p^*$.

Therefore, we have $\psi^{\text{SPSM}}(\delta_s, \delta_{-s}) \succeq_s \psi^{\text{SPSM}}(\delta', \delta_{-s})$. Hence, ψ^{SPSM}, is incentive-compatible.

The cumulative offer mechanism

Below, we provide a brief description of the cumulative offer process, which is used to produce the student-proposing stable matching. As mentioned in section VI, we make a small modification to the description of the original procedure, described in Hatfield and Milgrom (2005).

Step 1: One randomly selected student s_1 offers her most preferred contract x^1, according to her preferences $\succ_{s_1}$, that contains the privilege vector she submitted. The program that receives the offer, $p_1 = x^1_P$, holds the contract. Let $A_{p_1}(1) = x^1$, and $A_p(1) = \emptyset$ for all $p \neq p_1$.

In general,

Step $k \geq 2$: One of the students for whom no contract is currently held by a program, say s_k, offers the most preferred contract, according to her preferences $\succ_{s_k}$, that has not been rejected in previous steps and contains the privilege vector she submitted. Let us call the new offered contract, x^k. Let $p_k = x^k_P$ hold $C_{p_k}(A_{p_k}(k-1) \cup \{x^k\})$ and reject all other contracts in $A_{p_k}(k-1) \cup \{x^k\}$. Let $A_{p_k}(k) = A_{p_k}(k-1) \cup \{x^k\}$, and $A_p(k) = A_p(k-1)$ for all $p \neq p_k$.

The mechanism terminates when either every student is matched to a program or every unmatched student has no contract left to offer. The mechanism terminates in a finite number K of steps due to there being a finite number of contracts. At that point, the mechanism produces an allocation $X' = \bigcup_{p \in P} C_p(A_p(K))$, i.e., the set of contracts that are held by some program at the terminal step K.

Priority ordering for vacant seats under Brazil Reserves

Below, we list the full priority order used for each set of seats in the Brazil Reserves choice functions, for any set of contracts $Y \subseteq X$. These are used when there are vacant seats after considering all the candidates applying with the relevant privileges for that set.

$$Q^{(H,M,I)} \; : \; Y_{(H,M,I)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(H,M,i)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$$

$$Q^{(H,m,I)} \; : \; Y_{(H,m,I)} \blacktriangleright Y_{(H,M,I)} \blacktriangleright Y_{(H,M,i)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(h,m,i)}$$

$$Q^{(H,M,i)} \; : \; Y_{(H,M,i)} \blacktriangleright Y_{(H,m,i)} \blacktriangleright Y_{(H,M,I)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(h,m,i)}$$

$$Q^{(H,m,i)} \; : \; Y_{(H,m,i)} \blacktriangleright Y_{(H,M,i)} \blacktriangleright Y_{(H,M,I)} \blacktriangleright Y_{(H,m,I)} \blacktriangleright Y_{(h,m,i)}$$

It is not specified, however, in which order those seats are filled following those priorities.[16]

[16] Although not explicitly stated in the government document, we assume that universities do not give higher priority to students claiming some privileges for the open access seats ($Q^{(h,m,i)}$).