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# Game Theory For Self-Driving Cars

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**Abstract**—Pedestrian behaviour understanding is of utmost importance for autonomous vehicles (AVs). Pedestrian behaviour is complex and harder to model and predict than other road users such as drivers and cyclists. In this paper, we present an overview of our ongoing work on modelling AV-human interactions using game theory for autonomous vehicles control.

## I. INTRODUCTION

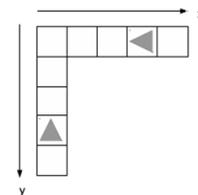
Autonomous Vehicles (AVs) also called “self-driving cars” are appearing on the roads. The technology is claimed by many automotive companies and their arrival on the market was announced for 2020 [1]. But their future interaction with other road users raise some concerns. Autonomous vehicles currently lack of the ability of human drivers to read the personality of other road users, predict their future behaviour and then interact with them. A comprehensive review of pedestrian modelling techniques for AVs was recently proposed, ranging from low level sensing, detection and tracking models introduced in [3] to high level interaction and game theoretic models of pedestrian behaviour presented in [4].

A European project called CityMobil2<sup>1</sup> used transport data science [12] to reveal a drawback of highly safe and perfect autonomous vehicles. This project launched a trial with an autonomous minibus in two European cities, in La Rochelle (France) and in Trikala (Greece). After a few days of driving, people became used to the minibus and they learnt its driving behaviour, the AV’s behaviour was easily predictable, as it would avoid any obstacle by a stop. Thus, pedestrians started stepping intentionally in front of the minibus [14]. In most of these cases, the minibus was slowed down or stopped for fun.

This inability of current AVs to accurately predict pedestrian crossing intent is known as “the big problem with self-driving cars” [2]. Pedestrians do not exhibit the same behaviour with human drivers, hence the European project interACT<sup>2</sup>, to which this work is part of, is investigating current human drivers-road users interactions. From these observations, we are trying to understand how these interactions occur in order to develop new behavioural models for road users e.g. [13] [15] [16] and new eHMI (external Human-Machine Interface) solutions that could facilitate the communication for autonomous vehicles in mixed traffic environments, i.e. with human-driven cars, cyclists and pedestrians.



(a) Game of Chicken



(b) Sequential Chicken Model

Fig. 1: Game of Chicken: two agents try to cross over an intersection as quickly as possible while avoiding a collision. The first agent to pass wins the game (reward), the second loses (small penalty) and they are both bigger losers if there is a collision (large penalty).

## II. GAME THEORY MODEL

As a solution to the minibus problem, we started using a game theory model called the game of chicken, as shown in Fig. 1a. Game theory is a well-known framework used for modelling decision-making between rational agents. We proposed a mathematical model for the game of chicken [13], a discrete sequential game theory model called the Sequential Chicken Game, for negotiations between an autonomous vehicle and a pedestrian at an unsignalized intersection, as shown in Fig. 1b. This model shows that not only the first agent to yield is more likely to lose the game but also if the AV only uses its position to signal its intent, there must exist a small probability for a collision to occur. This collision probability can be used as a threat for the pedestrian, preventing them from stepping intentionally in front of the AV.

## III. EMPIRICAL EXPERIMENTS

### A. Board Game Experiment

A first empirical study [11] expanded the sequential chicken model using empirical data to measure behaviour of humans in a controlled plus-maze experiment with participants playing the game of chicken as a board game. This study provided an empirical understanding of the human factors required by future autonomous vehicles. In the first three games, i.e. natural games, players were simply told to cross over the intersection as quickly as possible. After playing the natural games, each group played a further three games in which specific chocolate rewards were specified in advance, i.e. chocolate games. With these two game types, we found that more collisions occurred during the chocolate game than in the natural game. The results showed that participants had a preference for saving time  $U_{time}$  rather than avoiding a

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<sup>1</sup>[https://www.youtube.com/watch?v=ls6xsj\\_fCWU](https://www.youtube.com/watch?v=ls6xsj_fCWU)

<sup>2</sup><https://www.interact-roadautomation.eu/>

collision  $U_{crash}$ . Such parameters ( $U_{time}$ ,  $U_{crash}$ ) of the model could be inferred via a Gaussian Process regression.

### B. Physical Experiment

We later developed a novel empirical method [5] based on tracking real humans in a semi-structured environment, in order to model and predict their behaviour with game theory. We made use of dynamic programming to compute the optimal game theoretic solution form, then found the behavioural parameters via empirical observation and a Gaussian Process regression analysis. This model formed a step towards game-theoretic controllers for autonomous vehicles in similar real-world situations such as negotiations over priority at un-signalised road-crossings. This second study showed that participants were globally playing rationally, 11% of them deviated from their optimal behaviour. It also confirmed participants preference for time saving rather than collision avoidance, this unusual result was due to the high safety conditions of the experiment.

## IV. SEQUENCE ANALYSIS OF PEDESTRIAN-VEHICLE INTERACTIONS

### A. Sequence Patterns Recognition

We collected a large scale data from real-world human road crossings at the intersection near the University of Leeds, UK. Pedestrian-vehicle interactions were decomposed into sequences of independent discrete events [9]. We looked for common patterns of behaviour that can predict the winner of an interaction, which can thus be integrated into game-theoretic AV controllers to inform real-time interactions. We used probabilistic methods – logistic regression and decision tree regression – and motif analysis to analyse sub-sequences of actions used by both pedestrian and human drivers while crossing. We found predictive features that could inform the AV about the eventual winner of an interaction.

### B. Filtration Analysis

We then used the same dataset of pedestrian-vehicle interaction sequences to study the temporal orderings (filtration) in which features (including signals from the pedestrian) can be revealed to an autonomous vehicle and their informativeness over time during pedestrian-vehicle interactions [8]. This framework suggests how optimal stopping controllers may then use such data to enable an AV to decide when to act (by speeding up, slowing down, or otherwise signalling intent to the pedestrian) or alternatively, to continue at its current speed in order to gather additional information from new features, including signals from that pedestrian, before acting itself. In this study, we found that the AV should wait and observe about 7 to 10 features before acting/making its decision.

## V. PEDESTRIAN INTENTION ESTIMATION

To optimally interact with pedestrians, autonomous vehicles must be able to predict their crossing intent. Thus, we developed a model inspired by the Sequential Chicken model. It appeared that a heuristic method, simply based on

tracking data, was found to be very efficient in estimating crossing intent for most of the interactions [10]. However, this heuristic model would fail in more complex and maybe critical interaction scenarios.

## VI. VR EXPERIMENTS

As virtual reality (VR) offers the opportunity to experiment on human behaviour in simulated real world environments that can be dangerous or difficult to study, we used it to develop three simple experiments about pedestrian-AV interaction at non signalized crossings. VR allows us to better understand pedestrian crossing behaviour in more realistic conditions than in our previous artificial laboratory experiments and also to improve the AV game theoretic behaviour model.

In a first experiment [6], we asked participants to cross the road as they would do in every day life. We recorded their trajectories in order to learn their behaviour preferences, i.e. time delay vs collision avoidance. The virtual AV's decision-making was based on the Sequential Chicken model [13], which is a discrete model, thus the car had a slow and a fast speed. Our analysis of the data showed that participants were more cautious in crossing and often yielded for the AV.

In a second and third experiments [7], we wanted to learn from the participants which combination of space and time parameters (from the game theory model) would make the car behave more “naturally” and also to discover if there is any behavioural change in crossing in different environments and with different car models. Experiment 2's environment was a wide tarmac road with a narrower pathway and the AV was a normal sized-car whereas in Experiment 3 the environment looked more like a park/garden and the car looked like a small podcar. Participants were presented each time with an AV that had different parameters, they were asked whether they found the interaction with the virtual AV “natural” or “un-natural”, they had to rate it on a scale from 1 (un-natural) to 5 (natural). Two methods were used to change the parameters of the car:

- Brute Force: we used predefined orderly sets of parameters one after the other
- Gradient Descent: we started with a hypothetical optimal parameter and then changed the parameters following the preferences expressed by each participant.

The results show that pedestrians prefer an AV that makes its decisions quickly and that pedestrians behave similarly in different environments.

## VII. CONCLUSIONS AND FUTURE WORK

This is a work in progress on self-driving car technology. We present game theory as a tool to model future human interactions with autonomous vehicles. Semi-structured empirical and VR experiments with human participants and interaction sequence analysis provide a better understanding of human behaviour by inferring their behaviour parameters using Gaussian Process regression. Future work will look into developing the game theory model on a real self-driving car and to test its validity by performing some experiments with human participants.

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