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Population aging, social security and fiscal limits

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ABSTRACT

We use an overlapping generations (OLG) life-cycle model with distortionary taxation on labor and capital to derive a *threshold dependency ratio,* i.e. a point in the cross-section distribution of the population beyond which tax revenues can no longer sustain the planned level of transfers to retirees. We quantify the level of the threshold; the distance of the economy from the threshold; and the probability of reaching the threshold at some point in the future. The model is calibrated on the United States and fourteen European countries which have dependency ratios among the highest in the world. We examine the effects on the threshold and welfare of a number of policies often advocated to improve the sustainability of pension systems. New tax data on dynamic Laffer effects are provided.

1. Introduction

Background. This paper examines the sustainability of pension systems using an OLG life-cycle model with distortionary taxation that takes into account the possibility of an upper limit on the real value of tax revenues raised through direct taxation. The limit exists because tax revenues are subject to dynamic Laffer effects (DLEs) due to the distortionary taxation of the factors of production. Our main result is that, in an OLG life-cycle model, DLEs imply the existence of an upper bound, or threshold, on the dependency ratio (the number of retirees as a proportion of the labor force) of the economy. This threshold identifies a critical point in the cross-section of the age-distribution of the population beyond which tax revenue from direct taxation can no longer sustain the planned level of transfers to retirees. We refer to this as the *threshold dependency ratio*. This is determined by the structure of the economy, the design of fiscal policy and evolves over time due to demographic changes. The threshold dependency ratio can be used as an indicator of the sustainability of a pension system and to evaluate the impact of pension policy reforms.

We show that the threshold dependency ratio is derived from a subset of the competitive equilibria achievable in an OLG life-cycle economy. This subset includes all competitive equilibria in which the government chooses tax policy to maximize tax revenue. The threshold is then derived from the government budget constraint. We are interested in characterizing the

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level of the threshold in a given period and its projection over the medium and long term, with a view of comparing this against existing demographic projections.

Demographic projections possess a significant degree of uncertainty. We exploit this to derive a statistical measure of the *distance* between the projected dependency ratio and the threshold. We use this distance in conjunction with the distribution of stochastic demographic forecasts to measure the probability of an economy reaching the threshold at some point in the future. The distance from the threshold indicates to what extent the government can exploit its ability to raise revenue through direct taxation in order to maintain current levels of publicly funded support for older people. The probability of reaching the threshold indicates how likely a government is to be able to sustain the pension system through direct taxation in the medium and long run. The probability of reaching the threshold also provides a direct comparison of the effects of policy on the sustainability of a pension system, including changes in the retirement age.

The existence of the threshold affects both the beneficiaries of and the contributors to the social security net. Once the dependency ratio reaches the threshold - the distance is then zero - the government can no longer sustain the social security net for older people through an increase in direct taxation. It then faces a choice of either partially reneging on its social security commitments, for example, by reforming the pension system and making people retire later, or of increasing indirect taxation, or possibly reducing other types of public spending.

Quantitative studies on dynamic fiscal policy based on large-scale (life-cycle) simulation models typically focus on the United States. Our analysis breaks new ground by covering, in addition to the United States, fourteen European (EU14) countries. This extension is particularly interesting as the results for these EU14 countries are much more dramatic than those for the United States; their dependency ratios have reached some of the highest values in the world by 2010, and are projected to increase very rapidly by 2100. For each country, we start by quantifying the current size of the fiscal space as measured by the potential increase in tax revenue that could be achieved if tax rates on income from capital and labor were set to maximize tax revenues. This gives an indication of a country's ability to sustain the pension system through increase in direct taxation alone. We then measure the threshold over the period 2010–2100 and use stochastic population forecasts to quantify the distance from the threshold and the probability of reaching the threshold in the medium and long run.

The threshold dependency ratio is a useful statistics to evaluate the effects of different types of policy intervention. For this reason, we consider four alternative policy scenarios. The first covers the case of no-policy change (S1-NPC). The remaining three policy scenarios reflect reforms typically advocated for improving the sustainability of existing pension systems (Council, 2012): increasing the consumption tax rate by 5 percentage points (S2-ICT), reducing the replacement ratio of pensions by 10 percentage points (S3-RRR) and increasing the retirement age from 65 to 70 (S4-IRA). We examine the contribution that these policy changes may make in increasing the distance from the threshold and/or reducing the probability of reaching the threshold in the medium and long term. We also rank these reforms based on their welfare effects on the cohorts of individuals alive during 2010–2100.

Quantitative results We find that the size of the fiscal space in the United States ranges between 32 and 47% in 2010 (depending on whether the public sector is committed to maintain either the level or the replacement ratio of pensions, respectively) and is expected to grow over the period 2010–2100, though not fast. The threshold dependency ratio in the United States in 2010 is about three times larger than the actual dependency ratio (61 vs 22%). If no policy change is implemented, the probability of reaching the threshold is zero in 2050 but about 4% in 2100. Under the policy scenario S2-ICT the probability of reaching the threshold by 2100 declines to about 2%. Under the policy scenarios S3-RRR and S4-IRA the probability of reaching the threshold falls to zero by 2100.

The outlook is very different for the EU14 countries. Compared to the United States, they have, on average, narrower fiscal spaces, more generous pension systems, are older (higher dependency ratios) and are expected to age much faster. On average across the EU14 countries the threshold dependency ratio is only 0.2 times larger than the actual dependency ratio in 2010. If no policy change is implemented, dependency ratios in all EU14 countries are expected to overtake the threshold well before 2100. Under the policy reform scenarios S2-ICT, S3-RRR and S4-IRA, respectively, the number of countries that are expected to exceed the threshold dependency ratio before 2100 reduces to thirteen, eleven and nine. The outlook is worst for Austria, Belgium, France, Greece, Italy, Spain and the three Scandinavian countries. If no change in policy is under-taken, on average, these countries are expected to exceed the threshold dependency ratio by 2030. This date is postponed by 5, 15 and 40 years under the policy reform scenarios S2-ICT, S3-RRR and S4-IRA, respectively. These results highlight how imminent is the need of significant pension system reforms for the public finances of the EU14 countries. We also consider the projected impact of pension policy reforms carried out by the EU14 countries on their thresholds using data from the 2018 Ageing Report of the European Commission.[1] The results vary across countries since the thresholds tend to follow trajectories that mimic the demographic evolution of the projected dependecy ratio. This is consistent with the observation that most of these reforms include mechanisms that automatically index key pension parameters to demographic changes. Nevertheless the increase in the threshold resulting from these reforms appears to be limited to few countries and modest is size.

The welfare analysis compares the effects of three alternative changes to policy that would give the same degree of protection, and hence sustainability, to existing pension provision through yielding the same distance from the threshold by 2050. The three policies are a change to the consumption tax rate, to pension contributions and to the retirement age. For

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[1] See EPC (2018).

the United States we find that of the three policy reforms, the greatest welfare gains are obtained through an increase of the taxation of consumption, as this leads to the largest reduction of the distortionary taxation on income from capital and labor. A similar result for the United States is found by De Nardi et al. (1999), Kotlikoff et al. (2007) and Conesa et al. (2016). In contrast, we find that this is not necessarily the best policy option for most of the EU14 countries, as increasing the retirement age and/or reducing pension contributions achieve greater welfare gains than increasing the taxation of consumption. These contrasting welfare results reflect differences in tax burdens, demographic structures and discount factors among the EU14 countries.

A by-product of our numerical analysis is the quantification of revenue maximizing tax rates in an OLG life-cycle model. This contributes to the existing quantitative literature on DLEs which is based on infinitely-lived agent models. We provide a new data set of revenue-maximizing tax rates on capital and labor for the United States and the EU14 countries based on a life-cycle model. When keeping constant the replacement ratio of pensions, these tax rates are generally in line with those obtained by Trabandt and Uhlig (2011) using infinitely-lived agent models. The OLG life-cycle model, however, gives significantly lower revenue-maximizing tax rates on capital and labor when the level of pension per-capita is kept constant. We also use the OLG life-cycle model calibrated on the United States to highlight how population aging impacts on the position and shape of the Laffer curves, and how uncertainty about demographic projections impacts on DLEs.

Related literature. The paper is related to the extensive literature on the implications of aging for the sustainability of social security systems based on multi-period OLG models. See, for example, Auerbach et al. (1983), Auerbach and Kotlikoff (1987), De Nardi et al. (1999), Fuster et al. (2007), Kotlikoff et al. (2007), Heer and Irmen (2014), Conesa et al. (2016) and Imrohoroğlu et al. (2016).[2] These studies evaluate how aging is likely to increase the tax burden required to fund the social security system over a given period of time and how the resulting welfare cost could be mitigated through various reforms of the social security system, including partial financing with a consumption tax, reduction of social security transfers or increase in the eligibility age. This paper contributes to this literature by providing a measure of the limits faced by tax policy in maintaining the sustainability of pension systems through the threshold dependency ratio and by assessing the probability that an economy will reach a point at which reforms will be inevitable. A further contribution is the quantitative analysis of the pension systems of 14 European countries as well as that of the United States. This is particularly important as the sustainability of the pension systems in Europe is an even more pressing issue than it is for the United States.

Our paper is also related to the growing literature on the implications for public finances and macroeconomic policy of DLEs, typified by the works Davig et al. (2010), Trabandt and Uhlig (2011), Bi (2012); D'Erasmo et al. (2016); Polito and Wickens (2014, 2015).[3] The common denominator among these studies is their use of infinitely-lived agent models. We contribute to this literature by studying DLEs in a life-cycle model and by considering their implications for the sustainability of pension systems. As noted above, the paper contributes to the Laffer curve literature by providing new data on the peak of the Laffer curves for the same country coverage of Trabandt and Uhlig (2011), using instead an OLG model. The present paper also highlights how Laffer curves across countries are affected by the way that tax revenue is shared among retirees, aging and uncertainty about demographic projections.

Park (2012) studies how aging affects the size of the fiscal space in G7 countries using a neoclassical growth model with infinitely-lived agents. Ma and Tran (2016) consider the same issue for Japan and the United States, but using a model with OLG households. Holter et al. (2018) and Guner et al. (2016) consider DLEs in a large-scale model with overlapping generations. Their aim is to quantify how much extra tax revenue can be generated in the United States by increasing the progressivity of the tax system. In these two studies, DLEs impose an upper bound on a government's ability to redistribute resources in the economy. The present paper differs in several respects from these works. Our main focus is on how DLEs contribute to determining the threshold dependency ratio. The threshold imposes an upper bound on a government's ability to sustain the pension system in the medium and long terms. We study the probability of reaching this threshold at some point in the future and, in a wider quantitative analysis, the effects on the threshold of demographic shifts throughout 2010–2100. We also study the welfare implications for major pension-system reforms.

A number of issues concerning particular features of existing pension systems are beyond the scope of this paper. These include normative questions such as why we have the pension systems that we do and whether there is a socially optimal level of redistribution from workers to older people.[4] Our analysis is positive, being confined to the financial sustainability of a pension system in the presence of fiscal limits, the policy changes that can be implemented to maintain the social security net for older people and the welfare costs that societies may incur in implementing these changes.

Paper structure. The paper is organized as follows. Section 2 describes the OLG life-cycle model used for the analysis of dynamic fiscal policy and derives the threshold dependency ratio, the distance from the threshold and the probability of reaching the threshold. Section 3 describes the benchmark calibration used for the quantitative analysis. Sections 4 and

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[2] A related branch of this literature focuses on the macroeconomic effects of reforms of the United States' tax system, see for example Altig et al. (2001), Conesa et al. (2009), Guner et al. (2012, 2016).

[3] In these works, DLEs are quantified by calculating competitive equilibria over given grids for the tax rates. Babel and Huggett (2017) illustrate how to predict the top of the Laffer curve directly using the sufficient statistic approach.

[4] Diamond (2004) and Diamond and Orszag (2005) present various economic arguments underpinning the existence of social security contributions. Shiller (2005) and RMWJ et al. (2011) survey advantages and disadvantages of individual savings accounts for social insurance. Volume 19, issue 2, of the Journal of Economic Perspectives collects a series of different views on social security and reforms of social security systems.

5% the results for the United States and the EU14 countries, respectively. In Section 6 we consider a number of extensions of the benchmark model: income heterogeneity, non-linear taxation, labor decisons at the extensive margin, variation in college attainments. Section 7 concludes.

2. The model

The economy is described by a life-cycle model comprising a large number of OLG of households with a finite life, a representative firm and government. Each household includes one individual who makes consumption, saving and labor supply decisions to maximize lifetime utility. The firm uses aggregate capital and labor to maximize profits, while operating a neoclassical production technology. Consumption, income from labor and income from capital are subject to proportional taxes. The government uses tax revenue and issues debt to finance the provision of public consumption goods and the social security system, which includes transfers to all individuals and pension payments.

2.1. Demographics

In each period $t \geq 0$ a new cohort of individuals is born and denoted by its date of birth. Individuals in each cohort live for $J + 1$ periods, with $J \geq 1$. In $t = 0$, J cohorts of individuals are already alive, each indexed by their date of birth $(-1, -2, \ldots, -J)$. We denote by j_t^0 the age of an individual in $t = 0$, so that for any cohort born in $t \geq -J$, $j_t^0 = \max\{-t, 0\}$. The probability of surviving until age j in period $t + j$, conditional on being alive at age $j - 1$ in period $t + j - 1$ is $\phi_{t,j}$.[5] This is non-zero in each age j, other than in the last period, i.e. $\phi_{t,j} \neq 0$ for $j \in (j_t^0, J - 1)$ and $\phi_{t,J+1} = 0$. The population grows at the rate $n > -1$. The share of individuals of age j in the population, μ_j, is given by $\mu_j = \mu_0/(1 + n)^j$ for $j \in (1, J)$, with $\sum_{j=0}^{J} \mu_j = 1$.

Individuals work in the first $j_R - 1$ periods of their life and retire from age j_R onwards, with $j_R \in (2, J)$. For the numerical analysis, each period, t, corresponds to five years. Newborns have a real-life age of 20–24 ($j = 0$), retire at age 65($j_R = 9$) and live up to age 94 ($J = 14$). The dependency ratio d is defined as:

$$d = d\left(\left(\mu_j\right)_{j=0}^{J}, n, j_R\right) = \frac{\mu^R}{\mu^W}, \tag{1}$$

where $\mu^R = \sum_{j=j_R}^{J} \mu_j$ and $\mu^W = \sum_{j=0}^{j_R-1} \mu_j$ denote the shares in the population of retirees and workers, respectively. The dependency ratio is determined by four factors: the maximum life duration J, the distribution of age-j individuals in the population, the growth rate of the population and the retirement age. The first three are affected by population aging, through reductions in birth rates and increases in life expectancy. Given life expectancy, a decline in the birth rate results in a reduction of n that leads to an increase in the number of retirees relative to workers in the population. Given the birth rate, an increase in life expectancy, for example through a reduction in the mortality rate, leads to increase in the dependency ratio, as would a change in μ_j and/or J for any given n. Without loss of generality, we abstract from exogenous changes in the cross-section distribution of the population due, for example, to migration.[6] We treat J, μ_j's and n as exogenous although, in practice, they could be related to the economic environment and policy, and hence be endogenous. Making these three variables endogenous would not affect our qualitative results. The retirement age j_R indicates the age from which individuals start receiving old-age social security contributions. This could be either an endogenous variable chosen by the individual conditional on the minimum retirement age set by the government or a policy parameter, depending on how social security eligibility is regulated in the economy.[7] The OLG life-cycle model of the economy described here is compatible with both these interpretations, since the existence of the threshold and its related statistics (distance and probability) do not depend on the mechanism underlying the choice of j_R. We appraise the effect of variation of j_R in the quantitative analysis.

2.2. Environment

Households. Individuals within each cohort are the same. Newborns start their life with no assets and do not leave bequests, thus $a_{t,0} = a_{t,J+1} = 0$. They are also endowed with one unit of time at each age of their life. This is shared between labor and leisure during the working age. No labor is supplied during retirement. Each unit of time devoted to labor provides $z_j \geq 0$ units of productivity.

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[5] Throughout the paper, unless otherwise indicated, the first subscript denotes the date in which an individual is born, whereas the second denotes the age of the individual. Thus the sum of the two subscripts is the current period. Variables with only one time subscript are not age dependent and the subscript denotes the period in which they are observed.

[6] In the quantitative analysis we account for the impact on any factor influencing the demographic structure of the population, including migration. This is because our measure of the distance from the threshold and the probability of reaching the threshold depend on forecasts of dependency ratios in the medium and long-term that account for these factors.

[7] All the studies based on life-cycle models cited in the Introduction assume that the retirement age is exogenous. Fehr et al. (2013) and Kitao (2014), among others, study a large-scale lfe-cycle model with endogenous retirement.

Individual preferences depend on consumption and leisure. For any $t \geq -J$, these are ordered by the expected lifetime utility:

$$U^t = \sum_{j=j_t^0}^{J} \beta^{j-j_t^0} \left(\prod_{s=j_t^0}^{j} \phi_{t,s} \right) u(c_{t,j}, l_{t,j}), \tag{2}$$

where $\beta = (1+\rho)^{-1}$ is the common discount factor, with ρ denoting the discount rate; where $\phi_{t,s}$ denotes the probability of surviving until age s in period $t+s$, conditional on being alive at age $s-1$ in period $t+s-1$; $c_{t,j}$ and $l_{t,j}$ are the consumption and the labor supply of an individual of age j born in period t, respectively. Utility u is strictly increasing in consumption and leisure, twice continuously differentiable, strictly concave and satisfies the Inada conditions. Following Trabandt and Uhlig (2011), the instantaneous utility is specified as:

$$u(c_{t,j}, l_{t,j}) = \frac{1}{1-\eta} \left(c_{t,j}^{1-\eta} \left[1 - \kappa (1-\eta) l_{t,j}^{1+1/\varphi} \right]^{\eta} - 1 \right), \tag{3}$$

where $\kappa > 0$ is the weight attached to labor disutility, φ is the Frisch elasticity of labor supply, and η is the inverse of the intertemporal elasticity of substitution. Individuals have perfect foresight. The budget constraints faced by individuals for $j \in (j_t^0, J)$ are:

$$q_{t,j} c_{t,j} + a_{t,j+1} = x_{t,j} + tr_{t,j} + (1 + r_{t,j}) a_{t,j}, \tag{4}$$

in which

$$x_{t,j} = \begin{cases} w_{t,j} z_j l_{t,j} & \text{for } j \in (j_t^0, j_R - 1) \\ p_{t,j} & \text{for } j \in (j_R, J) \end{cases}, \tag{5}$$

$$l_{t,j} = 0 \text{ for } j \in (j_R, J), \tag{6}$$

$$a_{t,0} = a_{t,J+1} = 0. \tag{7}$$

Further, $q_{t,j} = 1 + \tau_{t,j}^c$, $w_{t,j} = \left(1 - \tau_{t,j}^l\right)\widehat{w}_{t+j}$ and $r_{t,j} = \left(1 - \tau_{t,j}^k\right)\widehat{r}_{t+j}$ are the after-tax prices of consumption, income from labor and income from capital, respectively; $\tau_{t,j}^c$, $\tau_{t,j}^l$ and $\tau_{t,j}^k$ are the corresponding age-dependent tax rates; $\widehat{w}_{t+j}$ and $\widehat{r}_{t+j}$ denote the pre-tax prices of labor and capital; $tr_{t,j}$ are age-related transfers; $p_{t,j}$ is the pension received by retired individuals.

For an individual born in t of age j, the solution to the lifetime maximization problem is the sequence of allocations $\left(c_{t,j}, l_{t,j}, a_{t,j+1}\right)_{j=j_t^0}^{J}$ that for any $t \geq -J$ satisfies the necessary and sufficient conditions:

$$u_{c_{t,j}} = q_{t,j} \lambda_{t,j}, \text{ for } j \in (j_t^0, J), \tag{8}$$

$$u_{1-l_{t,j}} = \lambda_{t,j} z_j w_{t,j}, \text{ for } j \in (j_t^0, j_R - 1), \tag{9}$$

$$\lambda_{t,j} = \beta \lambda_{t,j+1} (1 + r_{t,j+1}), \text{ for } j \in (j_t^0, J - 1), \tag{10}$$

and the constraints in (4)–(7), where $\lambda_{t,j}$ is the Lagrange multiplier associated with an individual's budget constraint.

For the numerical analysis, we make five modifications to household budget constraint in Eq. (4). First, labor productivity is assumed to be also time dependent. Thus we set $z_{t,j} = A_t y_j$, where A_t is the time-varying component of labor productivity growing at the constant rate $g_A \geq 0$ and y_j, with $j \in (j_t^0, j_R - 1)$, is the age-dependent component of labor productivity. Under this specification, pre-tax labor income is given by $\widehat{w}_t A_t y_j l_{t,j}$. Second, taxes are age-independent. Third, households contribute to the pension system through a proportional social security tax levied on wage income at the rate τ_t^p. Thus the after-tax labor income is $\left(1 - \tau_t^p - \tau_t^w\right) A_t y_j \widehat{w}_t l_{t,j}$, with $\tau_t^l = \tau_t^p + \tau_t^w$, for $t \geq 0$. Fourth, transfers are age-independent, $tr_{t,j} = tr_t$ for any $t \geq 0$ and $j \in (j_t^0, J)$. Fifth, pension payments are also age-independent, being set as a constant proportion (replacement ratio) θ of the average labor income in the economy $\widehat{w}_t l_t / \mu^W$, thus $p_t = \theta\left(\widehat{w}_t l_t / \mu^W\right)$ for any $t \geq 0$.

In equilibrium the household is indifferent between holding assets in the form of physical capital or government debt, since both yield the same (certain) after-tax return. With a single household living for two periods the proportion of asset holdings would be the same at the household and the aggregate level, but with many periods, the portfolio allocation is indeterminate. Consequently, without loss of generality, we assume that each household holds the two assets in the same fixed proportions.

Firms. In each period $t \geq 0$ there is a single produced good that can be used as private consumption, public consumption or capital. Goods are produced by a neoclassical production function with constant returns to scale, $y_t = f(k_t, l_t) - \delta k_t$, where y_t and k_t denote per-capita net output and capital, respectively; δ is the rate of physical depreciation and f is monotonically increasing, strictly concave and satisfies the Inada conditions. Factors of production are paid their marginal products. The before-tax prices of capital and labor are:

$$\widehat{r}_t = f_{k_t} - \delta, \tag{11}$$

$$\widehat{w}_t = f_{l_t}, \tag{12}$$

respectively. Production is described by a Cobb-Douglas function with labor-augmenting technological progress, $f(k_t, l_t) = k_t^\alpha (A_t l_t)^{1-\alpha}$. Under this specification, the balanced-growth rate of the economy is equal to the growth rate of labor productivity, $g_A \geq 0$.

Government. The government finances an exogenous sequence of public consumption, transfers and pension payments, $(g_t, tr_t, p_t)_{t=0}^\infty$, through revenue from taxation, $(tax_t)_{t=0}^\infty$, and by issuing public debt, $(b_t)_{t=1}^\infty$ (all variables are in per-capita terms). The sequence of government budget constraints for $t \geq 0$ is given by:

$$g_t + tr_t + p_t + (1 + \widehat{r}_t) b_t = tax_t + (1 + n) b_{t+1}, \tag{13}$$

where tax revenue in any $t \geq 0$ is given by:

$$tax_t = \sum_{j=0}^{J} \left(q_{t-j,j} - 1 \right) \mu_j c_{t-j,j} + \sum_{j=0}^{j_R-1} \left(\widehat{w}_t - w_{t-j,j} \right) \mu_j z_j l_{t-j,j} \tag{14}$$

$$+ \sum_{j=0}^{J} \left(\widehat{r}_t - r_{t-j,j} \right) \mu_j a_{t-j,j}.$$

Fiscal policy is subject to the solvency condition:

$$\lim_{T \to \infty} \frac{b_T}{\prod_{s=0}^{t} \frac{(1+\widehat{r}_s)}{(1+n)}} = 0. \tag{15}$$

There is a separate balanced-budget for pensions, so that aggregate expenditure on pensions is equal to the aggregate revenue raised through the social security tax:

$$\tau^p \widehat{w}_t l_t = p_t \mu^R. \tag{16}$$

For the numerical analysis, government expenditures (consumption and transfers) grow at the exogenous balanced-growth rate. Government revenue is augmented to include all accidental bequests from households that do not survive. This is equivalent to assume that the government collects all accidental bequests and redistributes them as lump-sums to households as, for example, in Krueger and Ludwig (2007), Braun and Joines (2015) and Holter et al. (2018).

Note how the dependency ratio is implicitly accounted for in the constraints faced by fiscal policy through Eqs. (13), () and (16), since these depend on the same J, $(\mu_j)_{j=0}^J$, n and j_R that determine d in Eq. (1). This observation motivates the derivation of the threshold dependency ratio in the next section.

Market-clearing and Feasibility. The equilibrium conditions for per-capita labor, asset holdings and consumption are:

$$l_t = \sum_{j=0}^{j_R-1} \mu_j z_j l_{t-j,j}, \tag{17}$$

$$a_t = \sum_{j=0}^{J} \mu_j a_{t-j,j} = k_t + b_t, \tag{18}$$

$$c_t = \sum_{j=0}^{J} \mu_j c_{t-j,j}, \tag{19}$$

respectively. The per-capita resource constraint requires

$$y_t + (1 - \delta) k_t = c_t + g_t + (1 + n) k_{t+1}. \tag{20}$$

Transfers and pension payments per-capita are $tr_t = \sum_{j=0}^{J} \mu_j tr_{t-j,j}$ and $p_t = \sum_{j=j_R}^{J} \mu_j p_{t-j,j}$, respectively. All variables, other than labor, are made stationary by expressing them as a proportion of technological progress. The stationary equilibrium is described in more detail in Online Appendix A.1.

2.3. Competitive equilibrium and threshold dependency ratio

First we define the set of competitive equilibria. We then show that the dependency ratio can be calculated as the unique number supporting a specific competitive equilibrium. The threshold dependency ratio is then simply a special case.

Definition 1. *Competitive equilibrium.* Given an initial aggregate endowment of assets $a_0 = k_0 + b_0$, a government spending policy $(g_t, tr_t = \sum_{j=0}^{J} \mu_j tr_{t-j,j}, \ p_t = \sum_{j=j_R}^{J} \mu_j p_{t-j,j})_{t=0}^\infty$, a tax policy $((q_{t,j}, w_{t,j}, r_{t,j})_{j=j_t^0}^J)_{t=-J}^\infty$, a borrowing policy $(b_{t+J+1})_{t=-J}^\infty$ and a dependency ratio $d = d((\mu_j)_{j=0}^J, n, j_R)$, a competitve equilibrium is a sequence of relative prices $(\widehat{r}_t, \widehat{w}_t)_{t=0}^\infty$ and individual allocations $((c_{t,j}, l_{t,j}, a_{t,j+1})_{j=j_t^0}^J)_{t=-J}^\infty$ such that:

a) The sequence of individual allocations satisfies ()–(10), for $t \geq -J$;
b) The sequence of relative prices satisfies (11) and (12), for $t \geq 0$;
c) The dependency ratio and the sequence of government spending, tax and borrowing policies satisfy (13),(14), and (16) for $t \geq 0$, and ();
d) All markets clear, i.e. (17)–(19) hold, for $t \geq 0$;
e) Feasibility (20) holds, for $t \geq 0$.

A competitive equilibrium is computed in two stages. The first consists in determining the sequence of individual allocations (*a*) and relative prices (*b*) that describe the private sector's optimal choices. The sequence of individual allocations in (*a*) is determined taking as given government policy and two of the variables contributing to the determination of the dependency ratio in (1), namely, the maximum life duration J and the age of retirement j_R.[8] In the second stage, government policy (spending, tax and borrowing) in (*c*) and aggregate variables in (*d* and *e*) are determined subject to the constraints set by the private sector choices, the dependency ratio and the government budget constraint.[9] It is at this second stage that all parameters of the dependency ratio enter the computation of the competitve equilibrium through the government budget constraint in (13) and (14) and the market clearing conditions. Crucially, one degree of freedom is missing at this stage, as the dependency ratio and the government policy need to satisfy the sequence of government budget constraints in (13) and (14). As a result, there are many competitive equilibria, each indexed with a different dependency ratio and government policy. This multiplicity implies that for any given fiscal policy, the dependency ratio can be derived as a residual from the solution of the government budget constraint. This, however, would not uniquely identify *d*, which is a highly nonlinear combination of the demographic parameters J, $(\mu_j)_{j=0}^{J}$, n and j_R. The residual solution of the dependency ratio from the government budget constraint relies on the fact that the government can always choose at least one of the variables in Eq. (1). As discussed in Section 2.1, governments in advanced economies typically set the minimum age of retirement.

To highlight the relation between changes in tax revenue and the dependency ratio, consider a government implementing a new tax policy that delivers a higher level of tax revenue. For this new policy to be supported as a competitive equilibrium the government budget constraint has to be satisfied. To this end, the additional tax revenue could be used to pay for a higher level of transfers to the existing cohort of retirees. It could also be used to maintain the current level of pensions per-capita while sustaining a higher number of beneficiaries of the pension system. In this second case, increases in tax revenue can be associated with higher dependency ratios, while still be compatible with a competitive equilibrium. The threshold dependency ratio is a special case, being the dependency ratio $\bar{d}$ obtained when tax policy is set to maximize tax revenue given government spending and borrowing policy. In other words, it measures the maximum number of retirees per worker that the government could sustain through tax policy alone.

A maximum dependency ratio sustainable through changes in tax policy emerges naturally in a life-cycle model as long as there is an upper bound on tax revenue. This is provided by the DLE. The upper bound can be exploited in conjunction with the government budget constraint to give the threshold dependency ratio, $\bar{d}$.

Definition 1 implies that there is a competitive equilibrium where $d = \bar{d}$. Still $\bar{d}$ is not uniquely determined being a nonlinear combination of the four parameters in Eq. (1). The computation of the threshold dependency ratio used in this paper takes J, $(\mu_j)_{j=0}^{J}$ and j_R as given in the second stage of the competitive equilibrium calculation, while determining Table 1 genously the growth rate of the population n. The threshold dependency ratio is therefore defined as follows.

Definition 2. Threshold Dependency Ratio. Given an initial aggregate endowment of assets $a_0 = k_0 + b_0$, a government spending $(g_t, tr_t = \sum_{j=0}^{J} \mu_j tr_{t-j,j}, \ p_t = \sum_{j=j_R}^{J} \mu_j p_{t-j,j})_{t=0}^{\infty}$, consumption tax $((q_{t,j})_{j=j_t^0}^{J})_{t=-J}^{\infty}$ and borrowing $(b_{t+J+1})_{t=-J}^{\infty}$ policy and a set of J, $(\mu_j)_{j=0}^{J}$ and j_R, a threshold dependency ratio is a dependency ratio calculated from (1) for a competitve equilibrium such that:

a) The sequence of individual allocations satisfies ()–(10), for $t \geq -J$;
b) The sequence of relative prices satisfies (11) and (12), for $t \geq 0$;
c) The social security budget (16) clear for $t \geq 0$;
d) The sequence of labor and capital tax policy $((w_{t,j}, r_{t,j})_{j=j_t^0}^{J})_{t=-J}^{\infty}$ maximizes (14);
e) The growth rate of the population satisfy (13), for $t \geq 0$, and (15);
f) All markets clear, i.e. (17)–(19) hold, for $t \geq 0$;
g) Feasibility (20) holds, for $t \geq 0$.

As noted above the threshold dependency ratio can be calculated as a residual from the government budget constraint. This requires measuring one of the parameters in (1) from the government budget constraint, while fixing all others. For this purpose we choose *n*, as this is numerically simpler to compute. In Online Appendix B we employ a restricted version of the model to derive a closed-form solution for the threshold and examine its main determinants more closely.

We are interested in two statistics about the threshold. The first is the distance between any forecast of the dependency ratio at some point in the future, $E_t d_{t+h}$, and the threshold dependency ratio at that point of time. The second is the

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[8] Both J and j_R could either be taken as given or included among the set of choice variables.
[9] The social security budget is already satisfied given j_R.

Table 1
Benchmark calibration.

	n	d	β	κ	α	δ	g/y	tr/y	b/y	θ	τ^l	τ^k	τ^c	τ^p
AUS	0.3	36.6	0.984	3.99	39	7.1	20	10.2	65	78.1	50	24	20	29
BEL	0.5	33.8	0.970	5.14	39	8.4	24	14.9	107	46.6	49	42	17	16
DNK	0.4	33.4	0.970	3.27	40	9.2	28	15.3	50	67.8	47	51	35	23
FIN	0.4	35.7	0.966	3.96	34	7.0	24	16.4	46	55.8	49	31	27	20
FRA	0.5	36.0	0.976	5.18	41	6.9	27	6.6	60	55.4	46	35	18	20
GER	−0.1	41.4	0.960	5.18	37	6.7	21	9.7	62	37.5	41	23	15	16
GRE	0.1	40.6	0.984	3.36	40	6.1	20	1.7	100	66.7	41	16	15	27
IRL	1.3	25.4	0.951	5.66	36	8.6	19	11.4	43	34.7	27	21	26	9
ITA	0.2	40.0	0.981	5.03	39	7.0	21	7.1	110	69.5	47	34	15	28
NET	0.5	35.1	0.992	5.8	38	7.7	27	−3.6	58	90.5	44	29	19	32
PRT	0.1	38.1	0.974	3.4	39	9.8	23	−6.0	57	73.8	31	23	21	28
ESP	0.7	27.7	0.982	5.17	42	8.5	21	−1.2	54	82.1	36	30	14	27
SWE	0.5	35.9	0.975	2.99	36	4.8	30	14.4	58	56.0	56	41	21	20
GBR	0.6	33.2	0.942	4.36	36	6.4	21	13.6	44	21.6	28	46	16	7
USA	0.95	30.6	0.958	3.62	35	8.3	18	5.2	63	35.2	28	36	5	11

Notes: Parameter values equal for all countries: $\hat{r} = 4\%$, $\eta = 2$, $\varphi = 1$ and $g_A = 2\%$. All numbers are in percentage, other than β and κ. The survival probabilities are averages over 1990–2010. Age-productivity z_j equal to 1 in all other countries other than in the USA. Data source is described in the main text.

probability of reaching the threshold, or equivalently exhausting the distance, at some point in the future. Online Appendix C shows that the probability that the h-period ahead dependency ratio exceeds the threshold dependency ratio $\overline{d}$, $\Pr\left(\overline{d}, t+h\right)$, can be written as

$$\Pr\left(\overline{d}, t+h\right) = \Pr\left[\left(\overline{d} - d_{t+h}\right) \leq 0\right]$$
$$= \Pr\left[\frac{\overline{d} - E_t d_{t+h}}{\sigma} \leq u_{t+h}\right], \tag{21}$$

with u_{t+h} measuring innovations of the underlying process for d_{t+h} and σ being a measure of the uncertainty surrounding the forecasts of d_{t+h}. The probability $\Pr\left(\overline{d}, t+h\right)$ can be computed for any stochastic distribution of the expected dependency ratio. We define the *distance from the threshold, $D(\overline{d}, t+h)$* as the number of standard deviations that the h-period ahead dependency ratio is from the dependency ratio threshold $\overline{d}$.[10] This is given by:

$$D\left(\overline{d}, t+h\right) = \frac{\overline{d} - E_t d_{t+h}}{\sigma_{\eta, t+h}}, \tag{22}$$

with $\sigma_{\eta, t+h}$ denoting the standard deviation of the h-period ahead innovation to d_{t+h}. The probability of exceeding the threshold dependency ratio $\Pr\left(\overline{d}, t+h\right)$ is therefore a function of the distance from the threshold $D(\overline{d}, t+h)$. It decreases as the gap between the threshold and the forecasted dependency ratios widens, and the uncertainty surrounding the dependency ratio forecast decreases. This probability changes over time due to changes in the base year and to new information which affect the forecast of the dependency ratio, its uncertainty and the threshold.

To compute the distance and probability statistics we use two measures of the dependency ratio endorsed by the Nations (2015). The first is the Old-Age Dependency Ratio 2 (OADR2), which measures the number of people in the population aged 65 and above as a percentage of those aged between 20 and 64. The second is Old-Age Dependency Ratio 3 (OADR3) which measures the number of people aged 70 and above as a percentage of those aged between 20 and 70. Online Appendix D describes these data in detail.

3. Calibration

3.1. Parameters

As in Holter et al. (2018), the parametrization of the model starts with a benchmark calibration that follows as closely as possible Trabandt and Uhlig (2011). In this way, we can better appraise how DLEs in infinitely-lived-agent models change due to the life-cycle structure of the economy and aging.[11]

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[10] This is the analog of the distance-to-default in corporate finance, which is defined as the number of standard deviations that a firm is away from default.

[11] Although this calibration is based on assumptions that are fairly common in quantitative macro models on the effects of taxation, we note that other calibrations have been used in applied work on DLEs. For example, D'Erasmo et al. (2016) use a two-country model with a limited depreciation tax allowance and endogenous capacity utilization in order to better match empirical estimates of the capital income tax base short-run elasticity

The benchmark calibration of the country-specific parameters is reported in Table 1. The second column reports the annual growth rates of the population rate in each country, using the estimates of the Nations (2015). To be consistent with Trabandt and Uhlig (2011), who use 1995–2007 as the calibration period, we use the average population growth rate prevailing during the years 1990–2010. The 5-year survival probabilities for the 15 different age groups are taken from the Nations (2015). These data show that survival probabilities have increased over time, and display larger rates of growth for the older age groups. For our benchmark simulation, we use the average survival probabilities during the period 1990–2010.[12] The third column reports the equilibrium dependency ratios implied as residuals by these demographic variables in each country. The fourth column reports our estimate of the discount factor in each country. Trabandt and Uhlig (2011) calibrate discount factors within the model so that the real interest rate equals 4 percent on an annual basis. As we are using an OLG model, we recalibrate β in each country to match a rate of interest of 21.9% over a period of 5 years, equivalent to 4% on an annual basis. The two preference parameters, η and φ, are taken from Trabandt and Uhlig (2011). Thus, the intertemporal elasticity of substitution η^{-1} and the Frisch elasticity of labor supply φ are set to 1/2 and 1, respectively, for all countries. Consequently, the values of the parameter κ in the fifth column are calculated within the model to match the equilibrium average working hours, equal to 0.3 across countries. The production parameters in columns 6 and 7 are not affected by the OLG structure of the household sector, and are as in Trabandt and Uhlig (2011). As in most of the OLG literature cited in the Introduction, the annual economic growth rate is kept constant over the simulation, set to 2% in all countries.[13] The remaining columns report country-specific fiscal variables and aggregates. Government consumption-to-GDP and debt-to-GDP (columns 8 and 10 respectively) and the tax rates on labor, capital and consumption (columns 12–14) are taken directly from Trabandt and Uhlig (2011).[14] Thus government transfers-to-GDP ratios in column 9 are determined within the model to satisfy the government budget constraint in each country. Pension replacement rates in column 11 are computed using data on the gross replacement ratios for pensions from the OECD (2015).[15] The social security tax rates τ^p in the last column are determined within the model to close the social security budget constraint in each country. Trabandt and Uhlig (2011) employ effective tax rates on labor income which already include social security contributions. We therefore restrict the tax rate on labor income in column 12 to have the same value of the labor tax rate τ^l used by Trabandt and Uhlig (2011), thus $\tau^l = \tau^w + \tau^p$. We use the hump-shaped age-productivity profiles estimated by Hansen (1993) to measure the labor productivity z_j for the United States. Labor productivity is set equal to 1 for all $j \in (0, J)$ in all other countries.[16]

We employ the benchmark calibration to quantify DLEs and threshold dependency ratios in each country for 2010. The dynamic analysis of the evolution of threshold dependency ratio over the period 2010–2100 is carried out by retaining the benchmark calibration for preferences, production and government spending (consumption, transfers and the cost of debt servicing remain constant at their 2010 steady-state in proportion to GDP), while updating the demographic variables over time. In particular, for the projection of survival probabilities that serve as input into the calculation of the threshold over the period 2010–2100 we continue to use moving averages of 4 periods. For example, the threshold for 2015 is based on average survival probabilities during 1995–2015, the threshold for 2020 is based on average survival probabilities during 2000–2020, and so on. In each period over the projection horizon, tax rates on labor and capital income are set at their Laffer peaks, while the growth rate of the population is computed as the implied residual from the government budget constraint. The resulting equilibrium dependency ratio is thus the threshold, as in Definition 2. Over the transition period 2010–2100 the social security contribution rate τ^p is adjusted to balance the social security budget.

3.2. Computation and extensions

The main focus of the analysis is to employ the threshold dependency ratio as a tool to evaluate the sustainability of public pension systems in aging economies. Quantitative studies employing large-scale OLG models are often concerned with the distributional effects of various forms of macroeconomic policy interventions. These studies therefore account for different forms of heterogeneity among agents (for example, with regard to income shocks, financial wealth distribution, education attainments, health, disability status, sex, marital status and household composition), other than age and productivity.

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[12] For reasons of space we do not report the survival probabilities. These are available upon request from the authors.

[13] In contrast, Heer and Irmen (2014) endogenize growth. In their model, firms have a higher incentive to invest in labor-saving technological progress if labor becomes scarcer (relative to capital). Again, the quantitative effect of aging on the growth rate is sensitive to the particular pension reform considered. They find that the average annual growth rate, which amounted to 1.74% during the period 1990–2000, increases to 2.41% in 2100 when the replacement rate of pensions is held constant and financed by additional contributions. The effect could be larger if the contribution rate is frozen at its 2000 level.

[14] Fève et al. (2013) find that the shape of the Laffer curve depends on the sign of the outstanding debt-to-GDP ratio, being "S-shaped" when this is negative. We do not have this issue in our analysis since debt-to-GDP ratios are all positive the calibration reported in column 10.

[15] These are based on the percentage of pre-retirement income for men.

[16] No data is readily available on age-productivity profiles for the majority of European countries. Where there is no information, an alternative would be to use the age-productivity profile estimated from another European country. This would, however, be equally aribtrary. We re-calibrated the labor productivity age-profile in each European country using the labor-productivity estimates calculated for Germany by Heer and Maussner (2009). The impact on the peaks of the Laffer curves and the thresholds is negligible because the revenue loss from those with productivity below 1, namely those aged between 20 and 39 (first four cohorts of workers) is in part offset by the revenue gain from those with productivity above 1, individuals aged between 40 and 64 (last four cohorts of workers). These results for the European countries are not included for reason of space, but are illustrated as an example for the United States in footnote 19.

Table 2
Tax rates on income from labor, income from capital and Fiscal Space (FS) at
the peak of the Laffer hill, USA, 2010.

	τ^l	τ^k	τ^p	FS
Benchmark				
	28	36	11	–
Laffer				
Constant θ	63.6	47.3	11	46.9
Constant p	60.0	34.5	15.7	31.8

Notes: All numbers are in percentage.

In our judgment, we can, without any loss of generality, dispense with most of these features when defining the threshold dependency ratio in Section 2.3. In principle, all these forms of heterogeneity could be included in our quantitative analysis, depending on the availability and comparability of these data for each country. Doing so would, however, add significantly to the computation time required in a multi-country analysis. We estimate that with existing computer technologies and a version of the model including all the forms of heterogeneity described above, the solution of a single equilibrium takes not less than one hour, and iteration of the algorithm over the tax rates grid would take about four weeks. Added to this, the time required to iterate over the 2010–2100 period and for each country. Under our specification, the solution of a single equilibrium takes about one second and the search over the three-dimensional grid for τ^l, τ^k and d takes about two hours for each country, depending on the grid size and the number of years considered. Our specification is an attempt to balance the accuracy of the results with the feasibility of their computation.[17]

Nevertheless, in Section 6, we consider the implications of four main extensions to the benchmark analysis: income heterogeneity among individuals, non-linear labor income taxation, labor choices at the extensive margin in retirement, variation in college attainment.

For the purpose of the model solution demographic parameters evolve deterministically and policy action is known in advance with certainty. These are a common assumptions of most computational OLG models used to study the effects of demographic transitions cited in the Introduction, see, e.g., Auerbach et al. (1983), Auerbach and Kotlikoff (1987), De Nardi et al. (1999), Kotlikoff et al. (2007), Imrohoroğlu et al. (2016) and Ma and Tran (2016). However, uncertainty about future demographic evolution, as well as the timing and type of policy intervention are also likely to be empirically significant. In Section 7 we discuss these possibilities in more detail as further avenues for extensions of the present analysis.

4. United States

4.1. Fiscal space

We begin by quantifying the size of the fiscal space in the United States. Table 2 reports the tax rates on income from labor and capital at the peak of the Laffer hill in 2010, and the implied fiscal space (*FS*), being measured as the percentage increase of tax revenue when tax rates are at the peak of the Laffer hill relative to the benchmark. We consider two cases: a constant replacement ratio (θ) and a constant pension level (p). For convenience we also report the tax rates under the benchmark calibration.[18] The main result is that DLEs in a life-cycle model depend on how tax revenue is shared among retirees. This follows from the peaks of the Laffer curves. They are higher and the size of the fiscal space is larger for the case of constant replacement ratio because pensions fall in absolute value due to the higher tax rates and the government's commitment to maintain a constant replacement ratio. According to life-cycle theory, this will cause workers to increase savings in order to smooth their consumption when retired. This leads to additional capital accumulation that partly offsets the negative effect of a higher tax burden. In contrast, when the pension level is fixed workers no longer need to increase saving. There is then no additional capital accumulation to partly offset the negative effect from a higher tax burden. Additional capital accumulation is a feature specific to a life-cycle model; it is absent in an infinitely-lived agent model, where agents can change their labor supply in every period of their life.

It is useful to relate the results in Table 2 to other studies. First, the tax rate on income from labor could be increased by as much as 115–130%. This increase is larger than the increases in the labor tax rate measured by De Nardi et al. (1999) or by Kotlikoff et al. (2007). This is not surprising as these studies consider the additional tax burden required to sustain a given demographic structure, whereas the peak of the Laffer hill corresponds to the maximum tax burden sustainable for that demographic structure. Second, the rates of the labor income tax at the peak of the Laffer hill reach values similar (60%) to those calculated by Trabandt and Uhlig (2011) using a neoclassical growth model with infinitely-lived agents and by Holter et al. (2018) using an multi-period overlapping generations model. Third, when the government maintains a constant

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[17] Online Appendix A describes the algorithm used for the numerical solution of the model. The GAUSS code implementing the algorithm is available upon request from the authors.

[18] The results in Table 2 are based on the hump-shaped age-productivity profile estimated by Hansen (1993). The age-productivity profile, however, has little impact on the size of the fiscal space. For the case of constant θ, when assuming equal productivity all agents the revenue-maximizing tax rates become 63.6% and 46.0% for τ^l and τ^k, respectively; τ^p is still equal to 10.8%; additional tax revenues are equal to 46.4%.

Table 3
Tax rates on income from labor, income from capital and Fiscal Space (FS) at the peak of the Laffer hill, USA, 2050.

d_{2050}	τ^l	τ^k	τ^p	FS
		Constant θ		
+2 s.e.	66.1	50.3	16.1	54.8
Mean	64.8	49.7	14.4	60.3
−2 s.e.	64.2	48.5	12.6	66.0
		Constant p		
+2 s.e.	61.2	33.9	21.1	37.6
Mean	60.6	33.9	19.1	43.6
−2 s.e.	60.6	33.3	17.2	49.5

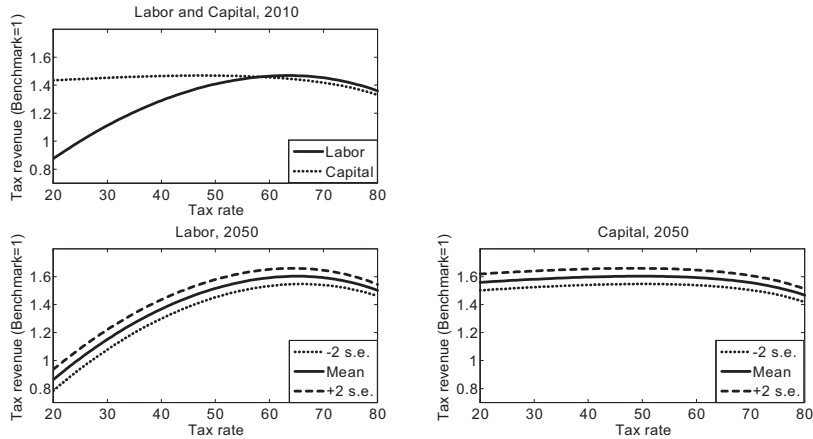Notes: All numbers in percentage.

Fig. 1. Laffer curves on income from labor and capital, USA, 2010 and 2050.

pension level the capital income tax rate is on the "slippery" side of the Laffer hill beyond the peak, as also found by Trabandt and Uhlig (2011). Unless it is explicitly stated, all subsequent results are based on the assumption of a constant replacement ratio.

Table 3 shows how population aging impacts on DLEs. It reports the tax rates on labor and capital income and the fiscal space (*FS*) at the peak of the Laffer hill in 2050, using the mean forecast, the upper and lower two-standard-error bands for the 2050 dependency ratio.[19] For reference we also include the pension contribution rate that, due to population aging, has to increase to balance the social security budget. We highlight two main results. First, the size of the fiscal space declines the more pessimistic is the demographic projection. Second, for any given demographic forecast, the size of the fiscal space increases over time, from about 47% in 2010 (see Table 2) to 55–66% in 2050. This is because due to population aging tax revenue under the benchmark calibration declines more rapidly than the maximum tax revenue. Our analysis of the threshold dependency ratio in the next section illustrates to what extent a larger fiscal space is likely to sustain the increasing cost of the pension system in the United States.

Fig. 1 displays the Laffer curves for labor and capital income taxes in 2010 (top panel) and 2050 (bottom panels).[20] Laffer curves for the capital income tax are flatter than those for the labor income tax, as also found by Trabandt and Uhlig (2011) using a neoclassical growth model with infinitely-lived agents. This suggests that the slope of the Laffer curve is not qualitatively altered when accounting for the life-cycle structure of individuals in the economy. Comparison of the three panels shows that demographic uncertainty affects the position of the Laffer curve but does not significantly alter their shapes.

In summary, these results highlight three dimensions of DLEs in the an OLG life-cycle model: (i) revenue-maximizing tax rates are higher when the government keeps the replacement ratio constant rather than the level of pensions, as this induces further private sector saving and capital accumulation which partly compensates for the negative effects of an increase in

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[19] The equilibrium growth rate of the population is calculated as an implied residual from either the mean forecast, or the upper and lower two-standard-error bands of the 2050 dependency ratio.

[20] The lines for the labor (capital) income tax are obtained by varying the labor (capital) income tax rate while keeping τ^k (τ^l) constant at the value that maximizes total tax revenue.
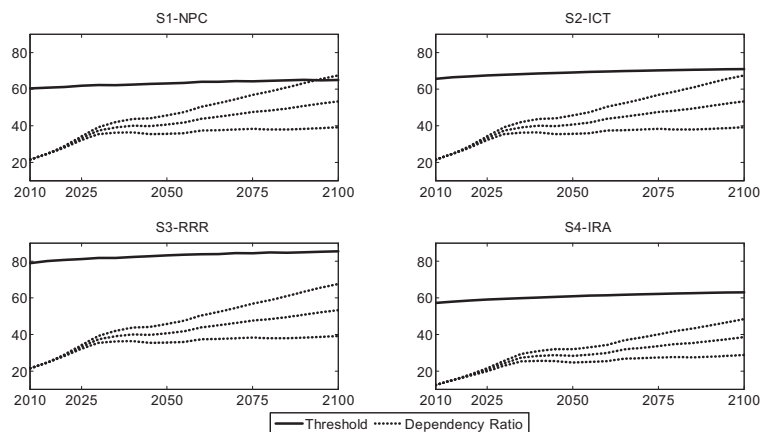
Fig. 2. Threshold dependency ratio, USA, 2010–2100.

taxation; (ii) population aging affects more the position of the Laffer curves than their shape; and (iii) uncertainty about demographic projections has a significant impact on the predicted position of Laffer curves.[21]

4.2. Threshold dependency ratio

We compute a time series of the threshold dependency ratio that is directly comparable with the OADR projections of the Nations (2015) over the period 2010–2100 described in Online Appendix D. Due to population aging, the cost of current pension systems is expected to rise over time. We therefore consider the consequences of four different policy scenarios that are commonly suggested to make pension systems fiscally sustainable, for example De Nardi et al. (1999), Kotlikoff et al. (2007), Nishiyama and Smetters (2007), Council (2012) and Conesa et al. (2016). Under the first scenario the government maintains the replacement ratio of pension as in 2010 and finances increases in the cost of pensions over time by raising the social security tax rate τ^p. This quantifies the threshold dependency threshold ratio under a scenario of no policy change, and is referred to as S1-NPC. The second scenario, S2-ICT, considers the effect of increasing the consumption tax rate by 5 percentage points, from 5 to 10% in the case of the United States. The third scenario, S3-RRR, considers the effect of reducing the replacement ratio of pensions by 10 percentage points, from 35.2 to 25.2% in the case of the United States. The fourth scenario, S4-IRA, considers the effect of increasing the retirement age to 70.

Fig. 2 shows the evolution of the threshold dependency ratio under the four policy scenarios over the period 2010–2100. The dotted lines in the panels denote the dependency ratio forecasts (OADR2 for S1-NPC, S2-ICT and S3-RRR; OADR3 for S4-IRA). In 2010, the threshold dependency ratio under S1-NPC is about three times the actual dependency ratio. Doubling the consumption tax rate (S2-ICT) would change this only marginally. The threshold would be about 4 times higher than the actual dependency ratio following either a reduction in the replacement ratio by 10 percentage points (S3-RRR) or an increase in the retirement age to 70 (S4-IRA).

Over the period 2010–2100, threshold dependency ratios increase, but very little. In contrast, the OADR2 and the OADR3 are forecasted to increase very rapidly over the same period of time. Consequently, the gap between the forecasted dependency ratio and the threshold narrows under each policy scenario. The upper two-standard-error band of the forecast of the dependency ratio rises above the threshold under S1-NPC from 2085, but does not reach the threshold under the other three policy scenarios.

Table 4 reports the threshold dependency ratios and the OARD2 and OADR3 forecasts for 2010, 2015, 2050 and 2100. On average, across the four policy scenarios, thresholds increase by about one percentage point between 2010 and 2015, by about three further percentage points until 2050, and by about two more percentage points by 2100. These increases are consistent with the increase in the size of the fiscal space over this period (compare Tables 2 and 3). Under S4-IRA both the threshold dependency ratio and the projected OADR3 are much lower. Therefore, the effects of the four policy scenarios are not comparable when considering the level of the thresholds in isolations, which motivates the use of our measures of distance and probability.

Fig. 3 shows the evolution of the distance from the threshold and the probability of reaching the threshold under the four policy scenarios between 2050–2100. The distance declines under each policy scenarios due to the dependency ratios increasing more rapidly than the thresholds over the period 2050–2100, thereby reducing the numerator in Eq. (22). The standard deviation of the forecast of the dependency ratio also increases, thereby increasing the denominator. The distance

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[21] Online Appendix E provides a detailed description of the distributional impact of increasing the size of the fiscal space, by analyzing how DLEs impact on the distribution of the tax burden between workers and retirees, across different taxes and age cohorts. We find that as an economy moves towards the peak of the Laffer hill the tax burden shifts further towards workers, with the largest increase in tax revenue generated through the labor income tax.

Table 4
Actual and threshold dependency ratios, USA, selected dates over 2010–2100.

	Actual/Forecasted Dependency Ratio (d)		Threshold Dependency Ratio ($\bar{d}$)			
				Policy Scenarios:		
	OADR2	OADR3	S1-NPC	S2-ICT	S3-RRR	S4-IRA
2010	21.6	14.1	60.4	65.7	79.0	57.3
2015	24.7	15.1	60.9	66.6	80.2	58.0
2050	40.7	28.3	63.3	69.2	83.3	60.9
2100	53.4	38.6	65.0	71.0	85.5	63.0

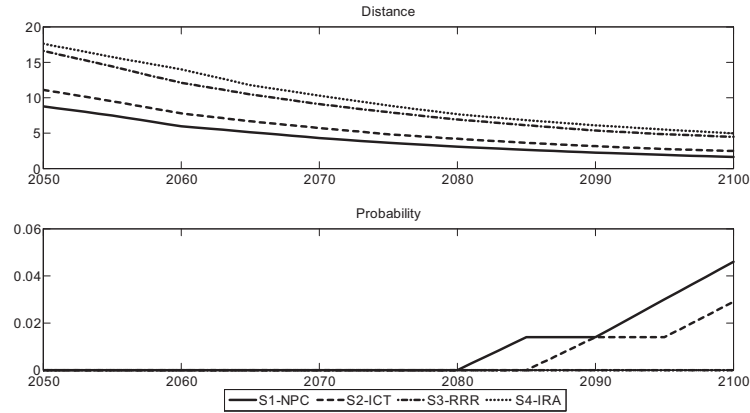Notes: All numbers are in percentage.

Fig. 3. Distance from the threshold dependency ratio and probability of reaching the threshold, USA, 2050–2100.

increases when moving from S1-NPC to S2-ICT to S3-RRR. This is consistent with the increases in the threshold levels under the three policies reported in Table 4. The distance increases even more under S4-IRA. This implies that increasing the retirement age in the United States to 70 would improve the sustainability of the pension system more than would doubling the taxation of consumption or reducing the replacement ratio by 10 percentage points.

Under S1-NPC, the probability of reaching the threshold is strictly positive from 2085 onwards and reaches about 4.5% by 2100. This suggests that without any change in policy, there is a probability of about 5% that the cost of the pension system will become unsustainable in the United States over the medium and long run. Under S2-ICT, the probability of reaching the threshold is positive from 2090 onward, and reaches around 2.5%by 2100. For the other two policy scenarios, the threshold is reached with probability zero.

4.3. Welfare analysis

The choice of policy to increase the distance of the economy from the threshold could have significant welfare implications. We therefore compare the implications of changing in 2010 the consumption tax rate, the replacement ratio of pension and the retirement age in order to deliver the same distance from the threshold dependency ratio by 2050. As shown in Fig. 3, if the retirement age were to increase to 70 the distance from the threshold dependency ratio would be of about 17.66 standard deviations by 2050. The same distance would also result from an increase in the consumption tax rate to 25.4% or a reduction in the replacement ratio to 24.1% We refer to these two adjusted policy scenarios as S2A-ICT and S3A-RRR, respectively.

We begin by calculating the average lifetime utility of the newborn in 2050 under S2A-ICT, S3A-RRR and S4-IRA, using the model equilibrium solution for 2050. In each policy scenario, the labor income tax rate and the pension contribution rate are adjusted to balance the general and the social security government budgets, respectively. We then compute the life-cycle profile of consumption and leisure over the 15 and 9 (10 under S4-IRA) lifetime periods. Instantaneous utility in each lifetime period and lifetime utility are then calculated using Eqs. (3) and (2), respectively.

Table 5 presents the values of aggregate consumption, aggregate labor, the tax rate on income from labor and welfare (U) under the three policies for the 2050 newborn generation. The last row reports the percentage consumption equivalent change (Δ) required for welfare under S4-IRA to be the same as under S2A-ICT and S3A-RRR. The results show that the policy reform based on increasing the consumption tax gives higher welfare than the policy based on reducing the replacement ratio or increasing the retirement age. This policy ranking arises from the tax rate on labor, which is lower under

Table 5
Welfare of generations alive by 2050 under S2A-ICT, S3A-RRR and S4-IRA, USA.

	S2A-ICT	S3A-RRR	S4-IRA
c	0.095	0.096	0.097
l	0.230	0.233	0.238
τ^l	0.168	0.287	0.298
U	−102.78	−103.55	−107.07
Δ	+4.17	+3.40	

Notes: U is lifetime utility. Δ is percentage consumption equivalent change w.r.t. S4-IRA.

Table 6
Tax rates on income from labor, income from capital and Fiscal Space (FS) in percentage at the peak of the Laffer hill, EU14 countries, 2010.

Country	Benchmark			Laffer							
				Constant θ			Constant p				
	τ^l	τ^k	τ^p	τ^l	τ^k	FS	τ^l	τ^k	τ^p	FS	
AUS	50	24	29	62	52	12.4	52	29	30	0.2	
BEL	49	42	16	59	47	3.1	53	**34**	16	0.3	
DNK	47	51	23	57	**46**	1.5	49	**27**	20	2.3	
FIN	49	31	20	60	45	4.6	53	**26**	21	0.3	
FRA	46	35	20	59	52	8.4	50	36	21	0.5	
GER	41	23	16	60	49	16.6	54	35	19	6.1	
GRE	41	16	27	62	52	37.3	52	30	34	7.8	
IRL	27	21	9	56	40	23.8	53	29	12	16.1	
ITA	47	34	28	64	51	14.3	53	**30**	36	0.9	
NET	44	29	32	69	53	31.5	59	**26**	38	6.0	
PRT	31	23	28	67	50	68.9	61	25	40	28.9	
ESP	36	30	27	64	53	44.0	55	**29**	35	1.0	
SWE	56	41	20	60	50	1.0	**50**	**37**	18	1.2	
GBR	28	46	7	57	**43**	15.7	54	**36**	9	11.2	
EU14	42	32	22	61	49	20	53	30	25	6.0	
USA	28	36	11	64	47	47	60	35	16	52	

Notes: All numbers are in percentage. EU14 is the arithmetic average.

S2A-ICT compared to S3A-RRR and S4-IRA. This finding is similar to those of De Nardi et al. (1999), Kotlikof et al. (2007) and Conesa et al. (2016). Thus the policy that achieves the lowest distortion on the production factors brings also higher welfare gains.

The above welfare analysis neglects the (potentially important) welfare effects while the economy is transiting between equilibria from 2010 to 2050. We address this issue in Online Appendix F, where we report the results from the simulation of the evolution of the United States economy as it transits between the 2010 and 2050 equilibria. This shows that reforms designed to improve the sustainability of the pension system in the United States bring welfare gains for all age groups in the long run, but only the young benefit in the short run. The welfare benefits increase over time. This reflects the advantages of having full information about policy changes and the ability to respond to them. The policy change that gives the greatest welfare gains (for all cohorts in the long run and for the young cohorts in the short run) is an increase in consumption taxes.

5. EU14 countries

In this section we extend our quantitative analysis of the threshold dependency ratio to the EU14 countries. First, we provide a cross-country comparison of the size of their fiscal space and highlight how this is related to differences in the labor tax rate, the replacement ratio, the dependency ratio and aging. Second, we present our measurement of the threshold (level, distance and probability) in each country under the same four policy scenarios considered for the United States. Third, we present the results from the welfare analysis. Given the large amount of data involved, we do not report the results for the transition experiments carried out on each individual country, but focus on the 2050 newborn generation. These allow a clear comparison of the effects of policy reforms across countries in the long run, while it is difficult to make any judgment looking at the transition profiles. We conclude our assessment considering the projected impact of pension policy reforms carried out by the EU14 countries on their thresholds using data from the 2018 Ageing Report of the European Commission.
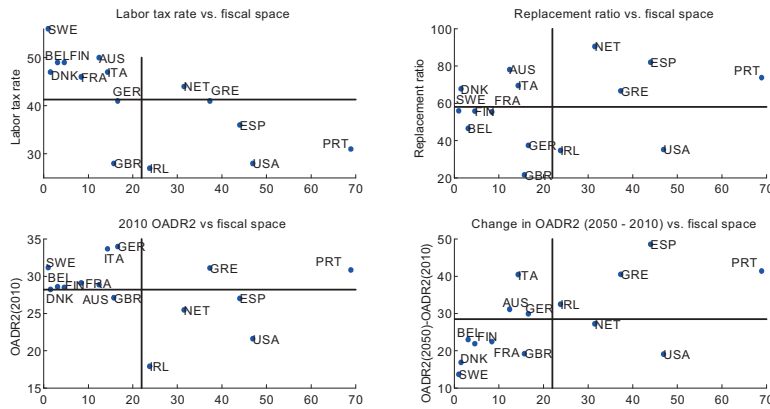
Fig. 4. Fiscal space in 2010 (constant replacement ratio) relative to (i) labor income tax rate, (ii) pension replacement ratio, (iii) 2010 OADR2 and (iv) change in the OADR2 over 2010–2050, EU14 countries.

5.1. Fiscal space

Table 6 reports the tax rates on labor income and capital at the peak of the Laffer hill and the size of the fiscal space (*FS*) for each EU14 country assuming either a constant replacement ratio (θ) or a constant pension level (*p*) in 2010.[22] For reference we also report tax rates under the benchmark calibration. With a constant pension level, the payroll tax rate τ^p adjusts to balance the social security budget. Numbers in bold indicate revenue-maximizing rates that are lower than the corresponding rates under the benchmark calibration. These highlight instances where current tax rates are higher than those at the peak of the Laffer hill, i.e. on the slippery side of the hill.

The main result emerging from the table is that the revenue-maximizing tax rates and the size of the fiscal space are significantly higher in the case of constant replacement ratio. As for the United States, this is because pensions fall in absolute value as tax rates increase and the government maintains a constant replacement ratio. This induces workers to increase savings in order to smooth their consumption when retired, thereby leading to additional capital accumulation that is absent when the government maintains instead a fixed pension level.

The results with a constant replacement ratio are generally in line with those of Trabandt and Uhlig (2011). The labor income tax rates for all countries are lower than at the peak of the Laffer hill. For the capital income tax rate we find that only Denmark and Great Britain are on the slippery side of the Laffer hill. The average size of the fiscal space for the EU14 countries in 2010 is 22%, less than half of that of the United States. There are, however, significant cross-country differences. Portugal has the largest fiscal space of about 70%; Belgium and the three Scandinavian countries have the lowest.

The results for a constant pension level show that for the labor income tax, all countries but Sweden are below the peak and hence on the "right" side of the Laffer hill. For the capital income tax rate, we find that 8 of the 14 countries have a tax rate higher than at the peak of the Laffer hill. The average size of the fiscal space with constant pensions is about a quarter of that with constant replacement ratio. Although there are still cross-country differences in the size of the fiscal space, these are less pronounced.

Fig. 4 relates the fiscal space of the EU14 countries for a constant replacement ratio to the tax rate on labor income, the replacement ratio of pensions, the dependency ratio (OADR2 in 2010) and aging (change in the OADR2 between 2010 and 2050). The United States is included for comparison. In each panel, the vertical line indicates the average fiscal space and the horizontal line is the average value of the variable on the vertical axis.

The negative relation between the fiscal space and the labor tax rate in the top-left panel helps to explain the cross-country differences in the sizes of the fiscal spaces reported in Table 6.[23] The top-right panel shows that the four EU14 countries with the largest fiscal spaces (Portugal, Spain, Greece and the Netherlands) also have among the highest replacement ratios. Six countries (Austria, Denmark, Italy, Sweden, Finland and France) have high replacement ratios but relatively small fiscal spaces. The bottom-left panel shows a negative relation between the fiscal space and the age structure of the population in 2010. Eight countries are concentrated in the top-left corner of the panel. They have relatively high dependency ratios and narrow fiscal spaces. The four countries with the largest fiscal space also have relatively high dependency ratios. The bottom-right panel shows a positive relation between the size of the fiscal space and the increase forecasted in the OADR2 between 2010 and 2050. The four countries with the largest fiscal space are located towards the top-right corner. This suggests that countries which in 2010 have a relatively large fiscal space are likely to exhaust it relatively quickly. From

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[22] As for the United States, we report our calculations for 2010 so that the results can be compared with those based on a neoclassical model with infinitely-lived agents reported by Trabandt and Uhlig (2011).

[23] Differences in the size of the fiscal space across the EU14 countries is also related to the gap between the tax rate on income from capital under the benchmark calibration and at the peak of the Laffer hill. These are not reported for reason of space.
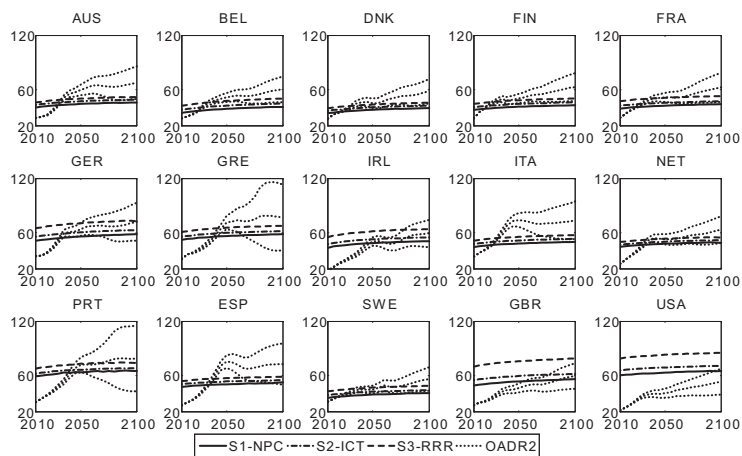
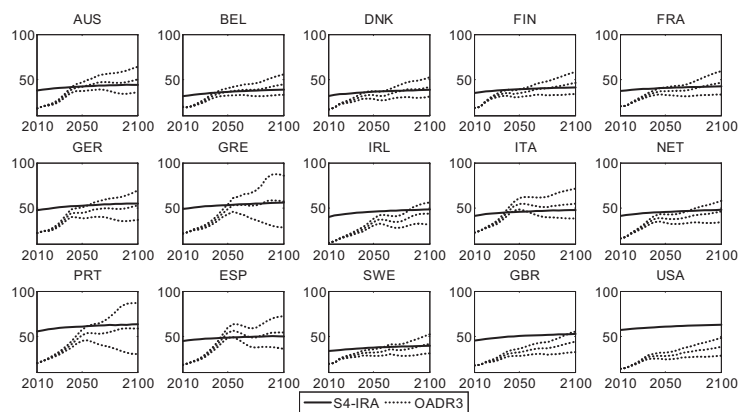**Fig. 5.** OADR2 and thresholds under S1-NPC, S2-ICT and S3-RRR, EU14 countries, 2010–2100.

Fig. 6. OADR3 and thresholds under S4-IRA, EU14 countries, 2010–2100.

this perspective, the fiscal outlooks for Italy, Austria and Germany are the most precarious as they have a relatively narrow fiscal space and are projected to age very rapidly over the period 2010–2050.

In summary, when compared to the United States, the EU14 countries have, on average, narrower fiscal spaces, higher replacement ratios, higher dependency ratios and are expected to age much faster.

5.2. Threshold dependency ratio

Fig. 5 shows the evolution of the threshold dependency ratio in the EU14 countries over 2010–2100. In each panel, the dotted lines denote the OADR2 forecasts (mean and two-standard-deviation bands); the solid line denotes the threshold dependency ratio under the no-policy-change scenario (S1-NPC); the dashed-dotted and dashed lines denote the threshold obtained when the consumption tax rate is increased by 5 percentage points (S2-ICT) and the replacement ratio is reduced by 10 percentage points (S3-RRR), respectively. Fig. 6 shows the evolution of the threshold dependency ratio when the age of retirement is increased from 65 to 70 (S4-IRA) in each EU14 country together with the OADR3 forecasts over the period 2010–2100.

We find that under S1-NPC the majority of the EU14 countries have threshold dependency ratios below the mean forecast of the OARD2 for the largest part of the 2010–2100 period. Under S2-ICT and S3-RRR there is a very modest increase in the threshold in all EU14 countries, other than Great Britain. The outlook appears to improve under S4, though in no country is the threshold above the higher error band of the OADR3.

Table 7 reports our estimate of the year when the OADR2 and OADR3 mean forecast are expected to be higher than the threshold dependency ratio for each EU14 country under the four policy scenarios. Under S1-NPC, the OADR2 is expected to overtake the threshold for all EU14 countries before 2100. Under S2-ICT, S3-RRR and S4-IRA the number of countries overtaking the threshold before 2100 reduces to thirteen, eleven and nine, respectively. The outlook is therefore particularly concerning for this last group of nine, that comprises Austria, Belgium, France, Italy, Greece, Spain and the three Scandina-

Table 7
Years when forecasted dependency ratios are estimated to overtake the thresholds under S1-NPC, S2-ICT, S3-RRR and S4-IRA, EU14 countries.

	S1-NPC	S2-ICT	S3-RRR	S4-IRA
AUS	2030	2035	2035	2055
BEL	2025	2030	2040	2055
DNK	2025	2030	2035	2080
FIN	2025	2025	2030	2080
FRA	2025	2030	2040	2085
GER	2035	2035	–	–
GRE	2040	2045	2050	2085
IRL	2075	2080	–	–
ITA	2030	2030	2035	2040
NET	2035	2035	2065	–
PRT	2045	2045	2070	–
ESP	2035	2040	2040	2050
SWE	2025	2030	2075	2095
GBR	2090	–	–	–
USA	–	–	–	–

Notes: '-' indicates threshold not reached before 2100.

Table 8
Distance from the threshold and probability of reaching the threshold, EU14 countries, 2050.

	S1-NPC		S2-ICT		S3-RRR		S4-IRA	
	D	Pr	D	Pr	D	Pr	D	Pr
AUS	−5.22	100	−4.22	100	−3.29	99.9	0.13	1.7
BEL	−5.11	100	−3.68	100	−1.58	95.2	0.01	47.5
DNK	−2.76	99.7	−1.69	94.8	−1.68	70.6	1.56	5.4
FIN	−3.61	100	−2.39	99	−0.81	79.3	2.08	2.3
FRA	−3.68	100	−2.39	99.5	−0.22	56.7	1.72	5.5
GER	−2.81	98.9	−0.96	84.2	1.79	4.9	2.66	0.9
GRE	−4.07	100	−3.09	99.9	−1.57	95.4	1.76	4.8
IRL	−0.85	80.5	1.02	15.2	4.88	0	6.17	0
ITA	−6.79	100	−5.9	100	−4.85	100	−2.86	99.9
NET	−1.71	95.9	−0.96	84.2	0.23	41.3	3.1	0
PRT	−2.06	99.3	−1.4	92.8	0.27	37.9	3.03	0
ESP	−7.07	100	−6.12	100	−5.13	100	−1.99	98.1
SWE	−2.65	99.7	−1.13	87.6	0.98	15.7	3.02	0.2
GBR	2.95	0	5.4	0	12.17	0	8.82	0
EU14	−3.25	91.0	−1.97	82.7	0.09	56.9	2.09	19.0
USA	8.78	0	11.12	0	16.61	0	17.7	0

Notes: All numbers for Pr are in percentage.

vian countries. On average, under S1-NPC these nine countries are expected to overtake the threshold by 2030. This date is postponed by 5, 15 and 40 years under S2-ICT, S3-RRR and S4-IRA, respectively.

Table 8 reports the distance from the threshold and probability of reaching the threshold in each EU14 country in 2050 under the four policy scenarios. The last two rows report the EU14 average and the United States for reference. Under S1-NPC, for all countries other than Great Britain the distance from the threshold in 2050 is negative implying the dependency ratio exceeds the threshold. The distance is still negative under S2-ICT, though on average smaller than S1-NPC. It becomes positive on average under S3-RRR and S4-IRA. The probability of reaching the threshold by 2050 declines on average from about 90 to about 20% when moving from S1-NPC to S4-IRA. In contrast, there is a zero probability of the United States reaching the threshold by 2050 even under the scenario of no change in policy. These results highlight how pressing reforms of the pension system are in the EU14 countries. They also highlight that the extent of these reforms in EU14 countries should be more radical than for the United States.

5.3. Welfare analysis

Table 9 reports our results on the welfare effects of alternative policy changes that are designed to satisfy the threshold dependency ratio for the EU14 countries based on the lifetime utility of the 2050 newborn generation. The first two columns report for each country the targeted distance from the threshold in 2050 and the corresponding probability (these are the same as in the last two columns of Table 8). The next two columns report the tax rate on consumption and the pension replacement ratio required to achieve the targeted distance in each country under S2A-ICT and S3A-RRR, respectively. The last two columns report the percentage consumption change (Δ) required for welfare under S4-IRA to be equal to welfare

Table 9

Consumption compensations across cohorts required for S2A-ICT and S3A-RRR to yield same equilibrium lifetime utility as under S4-IRA, EU14 countries, 2050.

	S4-IRA		S2A-ICT	S3A-RRR	Δ_1	Δ_2
	D	Pr	τ^c	θ		
AUS	0.13	1.7	46	56	1.49	1.78
BEL	0.01	47.5	34	33	–0.23	2.42
DNK	1.56	5.4	55	51	–8.79	–4.15
FIN	2.08	2.3	50	39	–1.36	2.66
FRA	1.72	5.5	39	41	–0.36	–1.18
GER	2.66	0.9	36	25	3.27	4.49
GRE	1.76	4.8	46	47	13.43	7.97
IRL	6.17	0	45	23	1.23	1.20
ITA	–2.86	99.9	41	49		
NET	3.10	0	48	71	–4.45	–8.01
PRT	3.03	0	55	57	1.52	–3.36
ESP	–1.99	98.1	44	60		
SWE	3.02	0.2	40	42	24.2	–1.45
GBR	8.82	0	28	15	0.40	1.77
EU14	2.09	19.0	43	43		
USA	17.7	0	25	25	4.17	3.40

under S2A-ICT and S3A-RRR, respectively. The results for the United States are also reported in the last row for comparison. Welfare calculations are not provided for Italy and Spain as the distance remains negative for these two countries. We highlight two main results.

First, the increases in the consumption tax rate and/or reduction in the replacement ratio required to achieve the targeted distances are significantly higher than those calculated for the United States. The average consumption tax rate under the benchmark calibration of the twelve countries that have a positive distance is about 21%. This needs to increase on average across these countries to 43%. Similarly the replacement ratio needs to be reduced from 57 to 43%, on average.

Second, the welfare ranking for the three policy reform scenarios is very different across countries. In particular, partial financing using the consumption tax, which is the preferred policy change for the United States, does not yield the higher long-run welfare gains for the majority of EU14 countries. The policy reform S3A-RRR yields higher welfare gains in the long run for Austria, Belgium, Finland, Germany and Great Britain, while S4-IRA yields higher welfare gains for Denmark, France and the Netherlands.

To shed light on some of the factors determining these cross-country differences in the welfare ranking, we calculate how the measured consumption compensations correlate with the deep parameters that determine the distance from the threshold in each country.[24]

Several patterns emerge. Policy reforms based on increasing the taxation of consumption tend to yield higher welfare gains in countries with relatively high debt-to-GDP ratios. Under this policy the government can reduce the level of public spending since the equilibrium stock of capital is high, thus leading to a reduction in the cost of servicing public debt. Policy reforms based on a reduction in pension contributions tend to be preferred to those based on higher taxation of consumption in countries where the taxation of income from labor is relatively high, since the consumption tax rate has the effect of further increasing the tax wedge. For the same reason, in countries with relatively high taxation of income from labor, policy reforms based on the reduction of pension contributions tend to be preferred also to those based on increasing the retirement age.

Cross-country differences are also affected by differences in discount factors and the taxation of saving. Under policy S4-IRA there is a reduction in public transfers at age 65–70 (the pension for the cohort that is required to work under S4-IRA). Under policy S3-RRR, there is on average a reduction in transfers at a later point in life for those age 65–95. Different discount factors across countries weight these two policies differently. For any given policy, individuals in countries with high capital tax rates have lower consumption than those living in countries with low capital tax rates in order to build up their savings. Accordingly, for a lower discount rate and capital income tax rate, policy S3-RRR is preferred to policy S4-IRA.

5.4. EU14 Pension policy reforms

European countries have carried out substantial reforms to enhance the long-term sustainability of their pension systems over the last twenty years. Most of these reforms have modified pension system rules and parameters to reduce the future generosity of pension benefits. Carone et al. (2016) explain how one of the common elements of pension reforms across the European countries has been the introduction of mechanisms that automatically index key pension parameters, like

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[24] We considered tax rates (τ^c, τ^k, τ^l, τ^p), preferences and production parameters (β, κ, α, δ), fiscal expenditure variables and parameters (g/y, tr/y, b/y, θ), the size of the 65-70-year cohort, the OADR2 and OADR3 in 2010, and the change in both of these between 2010 and 2050.
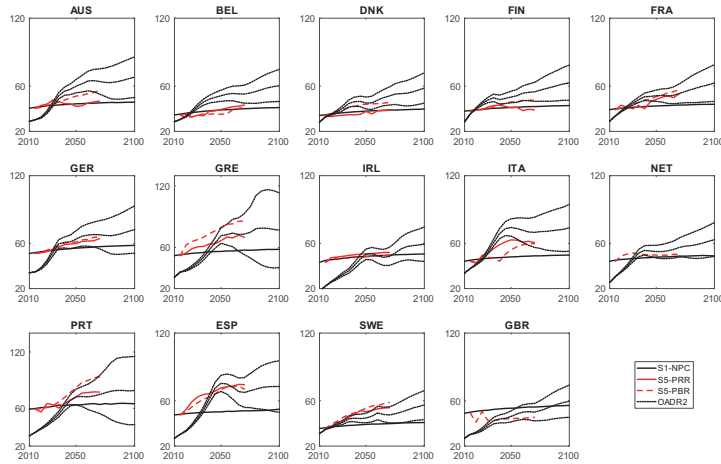
Fig. 7. OADR2 and thresholds under S1-NPC, EU14 countries, 2010–2100. Thresholds under S5-PBR and S5-PRR, EU14 countries, 2015–2070.

retirement age or benefit payments, to demographic changes. According to the 2018 Ageing Report,[25] these reforms are expected to determine sizable decreases in the projected pension generosity of many EU14 countries, thereby stabilizing their pension spending as a share of GDP over the long term. The impact of these reforms is captured, among the others, by (the projected change of) two indicators: the benefit ratio of public pensions and the replacement rate at retirement. The benefit ratio is a measure of the average pension in relation to the average wage. For this reason it captures the overall impact of current and future reforms of the pension system, including indexation, on pension payments. The replacement rate is an additional indicator that focuses on the average first pension as a share of the economy-wide average wage at retirement.[26]

According to the 2018 Ageing Report, benefit ratios and replacement rates are due to decline on average across EU14 countries by 11.3 and 8.9%, respectively, by 2070. There are however widespread country differences. The largest reductions, in percentage points between 2015 and 2070, are projected for Portugal, Spain, Greece, Italy and France. For the other countries, reductions are generally more modest and in some countries these indicators are projected to increase in the short term, before eventually declining by 2070.

To incorporate the effects of EU14 pension policy reforms in the measurement of the threshold dependency ratio we proceeded as follows. First we used the data in the 2018 Ageing report to calculate the rates of change of the projected benefit ratio and the replacement rate over 2015–2070. Second, we applied these rates of change to our estimate of the replacement ratio θ in Table 1. This gives two time series mapping the evolution of θ implied by the projected pension policy reforms described in the 2018 Ageing Report.[27] We then recomputed the threshold dependency ratio in each EU14 country using these time-varying estimates of θ between 2015 and 2070.

The results from this simulation are reported in Fig. 7. The threshold dependency ratio under the scenario that the replacement ratio θ evolves in each country according to the projected changes of the benefit ratio is denoted as S5-PBR (red-dashed lines). The scenario in which the replacement ratio θ evolves in each country according to the projected changes of the replacement rate is denoted as S5-PRR (red-solid line). For each country the corresponding OADR2 (black-dotted line) and the threshold S1-NPC (solid-black line) are also included for reference.[28]

Four main patterns can be observed from Fig. 7. First, in all EU14 countries, other than Great Britain, the threshold dependency ratio increases relative to S1-NPC under both S5 scenarios, at least in the long run (2070). This reflects the fact that pension reforms are projected to achieve reduction of the cost of pension in most countries. It is noticeable however how the increase of the threshold under both S5 scenarios is generally modest across countries, being just sufficient to reach the median projected OADR2. Second, in most countries (other than Belgium, the Netherlands and Great Briain) the thresholds dependency ratio under either S5-PRR or S5-PBR, if not both, appear to follow an increasing trend in line with that of the OADR2. This reflects the impact of the indexation machanisms introduced by european countries on the projected cost of pensions, which is captured by the change in time-varying series for θ over 2015–2070. Third, the thresholds under S5-PBR tend to overlap with those under S5-PRR for most countries over the 2015–2070 period. The main exceptions are Greece and

Table 10
Threshold dependency ratio (level, distance and probability) under benchmark (upper part) and income heterogeneity (income heterogeneity), USA, GER, ITA and ESP, 2050.

	USA	GER	ITA	ESP
Threshold		Benchmark		
$\bar{d}$	63.3	55.4	47.8	50.7
D	8.78	−2.30	−6.84	−7.07
Pr	0	99.1	100	100
Threshold		Heterogeneous income		
$\bar{d}$	58.2	56.2	48.8	42.3
D	6.90	−2.06	−6.59	−9.31
Pr	0	98.4	100	100

Notes: All numbers for $\bar{d}$ and Pr are in percentage.

Portugal, for which the threshold under S5-PBR is well above the threshold under S5-PRR and close to the upper confidence bands of the OADR2 for most of the 2015–2070 period. The opposite can be observed for Italy. The difference between the two S5 thresholds reflects the fact that decrease in the pension cost achieved through reduction in the replacement rates at retirement could be either compounded or partially offset by changes in other rules of the pension systems.

6. Extensions

We study the implications of four possible extensions of the benchmark model. These extensions reflect different features of other studies that might have the greatest impact on our results. The four extensions are (i) income heterogeneity among individuals, (ii) non-linear labor income taxation such as progressive taxation, (iii) labor supply choice at the extensive margin associated with the age of retirement and the costs of early retirement, and (iv) increase in college attainments.

Fehr et al. (2013) and Holter et al. (2018) are examples of life-cycle models that account, among others, for the first three extensions simultaneously. Guner et al. (2016) develop a life-cycle model with heterogeneity and non-linear taxes, but without labor supply decisions at the extensive margin. Cooley and Henriksen (2018) use a life-cycle model including heterogeneous individuals, labor supply decisions at the extensive margin, but without a government sector. Kitao (2014) uses a life-cycle model with heterogeneous individuals, choice at the extensive margin and proportional taxation. Conesa et al. (2019) show that increase in college education would be beneficial for the financing of the pension system.

Our analysis differs from these studies as we cover a wider range of countries, whose economies are simulated under four different policy scenarios based on demographic shifts over the whole 2010–2100 period. For reasons of computational tractability and space, it is not feasible to carry out a quantitative analysis of all of these extensions simultaneously for each of the 15 countries in our study. We therefore carry out a more limited analysis designed to indicate the likely effects of each of the extensions on our measure of the threshold (level, distance and probability). Additional details on the specification, calibration, solution and computation of the model under each of these extensions are provided in Online Appendix G.

6.1. Income heterogeneity

Income heterogeneity among individuals is introduced following Holter et al. (2018) and Cooley and Henriksen (2018). Wages are assumed to depend on an aggregate factor, the aggregate wage per efficiency unit of labor $\widehat{w}$, and an individual-specific productivity factor ω. The latter varies over the life cycle and across individuals due to changes in an individual-age z_j, differences in efficiency ϵ and, over time, to idiosyncratic shocks to individual's productivity u_t. As is usual in the OLG literature, we specify $\omega(j, \epsilon, u_t) = \epsilon z_j e^{u_t}$. The stochastic component u_t follows an AR(1) process $u_{t+1} = \rho u_t + \varepsilon_{t+1}$, where ε_t is identically and independently normally distributed with zero mean and constant variance, i.e. $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Accordingly, the pre-tax wage income of an individual with characteristics (j, ϵ, u_t) is given by $\omega(j, \epsilon, u_t)\widehat{w}_t l_{t,j}$. The permanent efficiency-type, given by $\epsilon \in \{\epsilon_1, \epsilon_2\}$, captures differences in education and ability among individuals. The remaining specification of the model is unchanged. We re-calibrate this new version of the model with heterogeneous individuals for four countries, the United States, Germany, Italy and Spain, making sure that the model still matches the same aggregate moments as under the benchmark calibration for 2010. We then update this to the demographic shift projected for 2050. Our results are presented in Table 10. For convenience, the upper part of the table reports for each country the thresholds level $\bar{d}$, the distance D as in (22), and the probability Pr as in (21) obtained under the benchmark analysis. The bottom part shows the corresponding results when accounting for income heterogeneity.

We make two main observations. First, once accounting for income heterogeneity, the threshold levels change across countries without any regular pattern in terms of signs and magnitudes. Second, due to the uncertainty about demographic forecasts, the calculated changes in the threshold levels cause only small variations in the distance and have negligible effects on the probability of reaching the threshold by 2050. For the United States and Spain, the threshold dependency ratios fall from 63.3 to 58.2% and 50.7 to 42.3%, respectively. The associated distance from the mean-projected dependency

Table 11

Tax rates on income from labor, tax revenue as a proportion to GDP and social security tax revenue as a proportion to GDP in the benchmark model (upper part) and in the model with heterogeneous income, USA, GER, ITA and ESP, 2050.

	USA	GER	ITA	ESP
	Benchmark			
τ^p	22.3	21.0	33.3	36.9
τ^w	36.3	34.6	20.5	16.7
$\widetilde{tax}_t/GDP$	40.6	34.4	36.7	33.4
$\widehat{w}_t l_t \tau^p/GDP$	14.5	13.2	20.3	21.4
	Heterogeneous income			
τ^p	16.8	17.5	35.2	25.4
τ^w	30.6	39.1	20.9	23.0
$\widetilde{tax}_t/GDP$	33.0	36.3	34.4	35.8
$\widehat{w}_t l_t \tau^p/GDP$	10.7	11.0	21.5	14.7

Notes: All numbers are in percentage.

ratio is reduced in both countries by about 1.5 standard deviations, while the probability of reaching the threshold remains unchanged. In the cases of Germany and Italy, the thresholds increase, though by a small amount, from 55.4 to 56.2% and 47.8 to 48.8%, respectively. These results reflect two opposing effects of income uncertainty on the aggregate supply of labor, which in turn alter the size of the fiscal space and the threshold. First, due to income uncertainty, households increase their supply of labor in order to build up precautionary saving. This increases the size of the fiscal space and the level of the threshold. But, second, the higher supply of labor generates a higher fiscal revenue, which brings about higher lump-sum transfers to the households and higher levels of government consumption (which we calibrated to match the empirical ratio with output). Governments therefore need to further increase distortionary taxation to finance higher levels of transfers and public consumption. This crowds-out the effect of the higher supply of labor, in turn reducing the fiscal space and the threshold. Consequently, when comparing the resulting threshold dependency ratios with those from the benchmark model, we are comparing the effects of aging in two models that have different values of G and tr.

Table 11 reports the tax parameters at the threshold levels in 2050 under both the benchmark calibration (upper part) and with income heterogeneity (lower part). These help explaining the key drivers of the results reported in Table 10. In particular, the sum of τ^p and τ^w is the tax rate on labor income at the peak of the Laffer curve, while the sum of $\widetilde{tax}_t/GDP$ and $\widehat{w}_t l_t \tau^p/GDP$ is the total tax revenue as a proportion to GDP generated by the government at the threshold, with $\widetilde{tax}_t/GDP$ denoting total tax revenue in the government budget as a proportion to GDP and $\widehat{w}_t l_t \tau^p/GDP$ denoting social security tax revenue as a proportion to GDP. In the benchmark model, which has homogenous cohorts without income uncertainty, tax rates on labor income at the peak of the Laffer curve vary across the four countries between about 54 and 59%. The total tax revenue as a proportion to GDP raised at the threshold ranges between about 48 and 57% across the four countries. After accounting for income heterogeneity, the peak of the Laffer curve falls for the United States and Spain by about 11 and 5 percentage points, respectively, while it marginally increases for Germany and Italy by about 2 percentage points. Consequently total tax revenue as a proportion to GDP falls significantly in the United States and Spain, while it is almost unchanged for the other two countries. This is why the thresholds of the United States and Spain fall significantly relatively to the benchmark, while are almost unchanged for Germany and Italy. In Spain, the reduction of total tax revenue as a proportion to GDP is characterized by a different composition compared to that of the United States. This is because the fall in the labor income tax rate at the peak of the Laffer curve in Spain is driven by the large fall in the social security contribution tax rate (from 36.9 to 25.4%). This explains why reduction of the social security revenue in Spain (from 21.4 to 14.7%) is larger than that for the United States (from 14.5 to 10.7%).

To explain the opposing effect of heterogeneity on the tax share ($\widetilde{tax}_t/GDP$) in the United States versus Spain, a drop from 40.6 to 33% in the United States versus a rise from 33 to 36% in Spain (even though the threshold dependency ratio shrinks in both countries), it is necessary to take into account the different effects that the pension system has in these two countries. In both countries, aggregate savings go up in the case of heterogeneity because income-rich households save a higher proportion of their income than income-low households.[29] Therefore, the capital-output ratio increases in both countries. The rise of this ratio, however, is much stronger in the United States than in Spain. This is because public saving in the United States is lower than in Spain, given that the replacement ratio is 35.2% in the United States and 82.1% in Spain. As capital deprecation is tax deductible, a smaller portion of GDP, $(Y - \delta K)/Y$, is therefore subject to income taxation in the United States following the increase in K/Y. Therefore, the tax revenue share also falls in the United States while this increases in Spain (where, in addition, the wage tax rate τ^w also increases).

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[29] Huggett and Ventura (2000) show that this result holds both empirically and in the standard OLG model with heterogeneous households.

Table 12
Threshold dependency ratio and associated tax values in the model with heterogeneous income and progressive income taxation under different values of the Frisch elasticity of labor and tax progressivity, USA, 2050.

Frisch elasticity (φ)	1	1	0.6
Tax Progressivity (θ_1)	0.2036	0.1106	0.1106
		Threshold	
$\bar{d}$	43.7	48.9	55.9
D	1.17	3.22	5.99
Pr	15.2	0	0
		Tax statistics	
τ^p	13.6	14.7	16.5
τ^w	21.09	20.08	20.9
$\widetilde{tax}_t/GDP$	25.7	28.8	29.0
$\widehat{w}_t l_t \tau^p/GDP$	8.9	9.7	11.8

Notes: All numbers are in percentage, other than those for D.

6.2. Non-linear taxation of labor income

The second extension considers the effects of non-linear taxation, as in Benabou (2002), Heathcote et al. (2017), Heer and Scharrer (2018) and Holter et al. (2018). In particular, we adopt the approach of the latter and maintain the assumption that the capital income tax, the social security contribution and the consumption tax are levied at flat rates, while labor income is taxed progressively.

It is important to highlight that the household sector in the benchmark model is specified in terms of single individuals and that pension replacement ratios are calibrated on data for single men. As a result, the benchmark model abstracts from two characteristics of the population that are important when fully quantifying the effects of nonlinear taxation, namely, the distinction between men and women (which implies different labor market participation rates and Frisch elasticities of labor supply), and the marital status of individuals.[30] While not accounting for these features explicitly in our model, we try to elucidate their likely effects on the threshold through a sensitivity analysis.

To model the non-linear taxation of labor income, we denote by $\widehat{y} = A\omega(j, \epsilon, u_t)\widehat{w}l$ the pre-tax labor income of an individual with age j, with permanent productivity ϵ, idiosyncratic labor productivity u and labor supply l. After-tax labor income y is formulated as $y = \theta_0 \left(\frac{\widehat{y}}{\bar{y}}\right)^{1-\theta_1} \bar{y}$, where $\bar{y}$ is average income among workers and the parameters θ_0 and θ_1 denote tax-level and tax-progressivity, respectively. This form of the tax function has the advantage that the tax level can be varied by changing θ_0 without affecting the extent of tax progressivity θ_1. The tax burden on an individual labor income is given by $T(\widehat{y}) = \widehat{y} - y$ and this replaces τ^w in the individual and government budget constraints. The remaining specification of the model is unchanged. The model is calibrated for the United States. The tax function is calibrated using the estimates of θ_1 from Holter et al. (2018) and θ_0 is set to match the average labor income tax rate τ^w, as in our benchmark model. We retain the specification of income heterogeneity described in Section 6.1.

Table 12 presents the results obtained from three different simulations involving progressive taxation. In the first, we consider the extent of tax progressivity for the case of a married couple with two children, $\theta_1 = 0.2036$, while retaining the same Frisch elasticity of labor supply used by Trabandt and Uhlig (2011), $\varphi = 1$. In the second simulation, we reduce tax progressivity while leaving the elasticity of labor supply unchanged. Accordingly, the value of the tax progressivity parameter for single men is that used in Holter et al (2018), $\theta_1 = 0.1106$, which we regard as being more consistent with the specification and calibration of the benchmark model. In the third simulation, we consider the effect of reducing the Frisch elasticity of labor supply while leaving progressivity unchanged. To this end we re-calibrated the model with a value of the Frisch elasticity of 0.6, which is in between that for men (0.4) and women (0.8), that was used by Holter et al. (2018). We retained $\theta_1 = 0.1106$.

The results show that, in general, tax progressivity reduces the threshold level. This is consistent with the finding of Holter et al. (2018) that the more progressive the tax system, the smaller is the fiscal space (compare columns 2 and 3). Further, Holter et al. (2018) find that the effect of tax progressivity on the fiscal space depends on the sensitivity of labor supply to changes in tax rates. This result is also reflected in our final simulation (compare columns 3 and 4) as the threshold increases once the Frisch elasticity is aligned to the average between men and women, and reaches levels in line with those reported in Table 10. Under this final simulation, the distance reduces by about 0.9 standard deviations relative to the case of income heterogeneity and proportional taxation, while the probability is still unchanged.

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[30] Holter et al. (2018) quantify the significance of these features for the analysis of progressive tax systems and DLEs.

Table 13
Effects of early retirement on the threshold, USA, 2050.

Financial penalty	0	13.33	25
People retiring			
between age 60–64	74.9	31.77	0.40
		Threshold	
$\bar{d}$	39.0	46.8	57.7
D	−0.68	2.40	6.71
Pr	76.9	0	0

Notes: All numbers are in percentage other than D.

6.3. Extensive margin

The benchmark specification of the model allows for labor decisions at the intensive margin in each period up to the age of retirement of $65 - 69$, after which individuals no longer work. This follows a protocol common in the computational OLG literature. Recent examples include Conesa and Garriga (2016) and Guner et al. (2016). Before discussing the effects for our analysis of allowing labor supply choices at the extensive margin, three preliminary observations are necessary.

First, we note that the age of retirement $65 - 69$ is a close approximation to the average legal (and effective) age of retirement observed in the 15 countries covered in our quantitative analysis. According to OECD data, the legal age of retirement is 65 in the United States and on average 63.4 across the EU14 countries while, the average effective age of retirement is 67 in the United States and 63.9 in the EU14 countries. The gap between the effective and legal retirement age is typically narrow across countries. Notable exceptions are Portugal (6.6) and France (-4.5).[31] These discrepancies between countries are due to pension provisions that allow individuals to take both early and late retirement. The fact that the effective retirement age is higher than the legal age reflects in part the financial cost of early retirement.[32]

Second, the quantitative significance of labor market choices at the extensive margin is unclear. Fehr et al. (2013) and Kitao (2014) specify life-cycle models where individuals also make labor choices at the extensive margin. These works find that labor market choices at the extensive margin can have significant aggregate effects, in particular, once heterogeneity in health status is accounting for. However, Holter et al. (2018) find that labor decisions at the extensive margin have a less significant impact on macroeconomic aggregates, once differences in labor market participation (and Frisch elasticities) between men and women and family composition (single, married couple and married couple with children) are accounting for.

Our third observation is that the results from the benchmark analysis can already be used to infer the likely effects on the threshold of the observed discrepancies between the legal and the effective age of retirement. Thus it is sufficient to compare the distances and probabilities under S1-NPC and S4-IRA for 2050 in Table 8. The threshold distance/probability under S4-IRA can be interpreted as the consequence of everybody in the economy retiring at age 70. The corresponding results under S1-NPC refer to when everybody retires instead five years earlier, at age 65. In general, this shows that early (late) retirement would increase (reduce) the pension cost, as individuals spend longer (shorter) time in retirement, and also negatively (positively) affect the financing of the pension system, as people spend a shorter (longer) life-time working. We note that these results are based on the assumption that people can retire earlier without incurring any financial penalty, which is not the case under the pension provisions of most OECD countries. We would expect that the cost of retiring earlier would be a discouragement from doing so, particularly for those with high earnings.

To evaluate the significance of having both the intensive and extensive margin, we extended the benchmark specification, by modelling retirement as endogenous as in Fehr et al. (2013). In particular, we re-computed the solution for 2050 assuming that individuals can choose whether to retire at the age of 65 (corresponding to $j_R = 10$) or at the early retirement age of 60 (corresponding to $j_{ER} = 9$). If an individual retires early, he/she can spend more time on leisure, but his/her (remaining lifetime) income declines. The resulting optimization problem is a binary choice between the values associated with early and regular retirement. The individual chooses to retire if

$$V^{ER}(a, 9, \epsilon, u) > V^R(a, 9, \epsilon, u),$$

where V^{ER} (V^R) denotes the value function of the household at age $j = 9$ with assets a, permanent productivity ϵ, and stochastic productivity u in the case of early (regular) retirement. We further assume that individuals decide upon early retirement at the beginning of the period at age $j = 9$ after observing the stochastic productivity shock u. The model is calibrated on data for the United States for the demographic structure in 2050 accounting for heterogeneous income and proportional labor income taxation, as in Section 6.1. The cohorts of the 60-64-year old workers constitute 7.09 and 6.84% of the stationary populations in 2010 and 2050, respectively.

Table 13 reports our results for the threshold of allowing for early retirement. We consider three cases. First we assume no penalty for early retirement. Under this specification, about 75% of the workers retire during age 60–64, and the remain-

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ing 25% retires from the age of 65. We then re-calculate the solution accounting for the financial cost of early retirement, using the information provided by the OECD *Pension at a Glance 2017: Country Profiles-United States.*[33] According to this information, individuals are allowed to retire from the age of 62, but their pension is reduced by about 6.7% annually for the first three years. After three years the reduction falls to 5%. This applies to individuals with a statutory retirement age of 65. Therefore, as a second case, we re-computed the model solution under the assumption of an average penalty of 13.33% for those retiring two years earlier (two years penalty at 6.7%). In the final case, we re-computed the model solution under the assumption of a maximum penalty of 25% for those retiring four years earlier (three years penalty at 6.7% plus one year penalty at 5%), as this is the specification most consistent with the model protocol (early retirement at age $j = 9$).

Two main observations may be made about the results reported the Table below regarding the implications of labor choice at the extensive margin for our analysis and pension systems legislations. First, early retirement may increase the pension cost and therefore reduce the threshold. The quantitative effect depends crucially on the financial cost associated with this choice, in particular the legislated reduction in pension payments during the years of early retirement. Second, the effects of labor choices at the extensive margin are likely to vary across countries, possibly resulting in an increase in the threshold in countries where individuals retire late and a reduction in countries where they retire before the legal age. An accurate estimate of these effects is beyond the scope of the present paper. Further exploration of these issues within an OLG model might include having a more detailed demographic specification that accounts for different incentives on labor market choices at the extensive margin other than those provided by the pension system, such as health, a distinction between men and women, and family background.

6.4. Increasing college attainment[34]

Conesa et al. (2019) show that the labor income tax rate required to finance additional pension payments due to the demographic transition in the United States is greatly reduced if college education continues to rise. They find that, if the college attainment rate increases from 22.8% during 1980–2005 to a projected rate of 67.4% in 2100, the required increase in labor income taxes is reduced by 10.1 percentage points.

We follow the analysis of Conesa et al. (2019), considering a simplified version of their model.[35] In particular, we distinguish two types of households, low-skilled and high-skilled, that supply labor $l_{t,j}^1$ and $l_{t,j}^2$ with individual productivities $\epsilon_1 z_j$ and $\epsilon_2 z_j$ at age j, respectively. The skill productivities ϵ_i, $i = 1, 2$, are chosen to reflect permanent income differences between the low- and high-skilled workers, as described in Online Appendix G.1. Accordingly, total labor income, $w_t \epsilon_i z_j A_t l_{t,j}^i$, is the product of the wage rate per efficiency unit, w_t the permanent efficiency ϵ_i of type $i = 1, 2$, the age-efficiency factor z_j, the aggregate productivity level A_t and working hours $l_{t,j}^i$ at age j in period t. The shares of the two types of skills in each cohort j in period t are denoted by μ_t^1 and μ_t^2 (with $\mu_t^1 + \mu_t^2 = 1$) and are assumed to follow a linear increase over the period 2010–2100 as in Concesa et al. (2019). Other model assumptions are identical to those described in Online Appendix G.1.

The new parameter that we need to calibrate in this sensitivity analysis is the share of the college graduates μ_t^2 in population. For the United States, we follow Conesa et al. (2019) and assume that μ_t^2 rises (linearly) from 22.8% during 1980–2005 to a projected rate of 67.4% in 2100. We also extend the analysis to Germany, Italy, and Spain, using data from Roser and Ortiz-Ospina (2019) which are taken from the World Bank EdStats and Unesco Institute of Statistics. In particular, we use linear transition paths between the 2010 levels and the projected levels in 2050. These are, in percentage, 20.3 and 32.0 for Germany, 10.2 and 19.9 for Italy and 9.8 and 19.2 for Spain. Beyond 2050, we extrapolate values linearly.

Fig. 8 shows the estimated effect on the threshold dependency ratio of rising college attainment. The threshold in the case of two skills and rising college attainment (red line) increases relative the S1-NPC scenario (black line) in all four countries. The quantitative effect is most marked in the United States where the threshold rises well above the upper bound of the OADR2 by 2100. The increase is relatively modest for Italy and Spain, since the new threshold lies just above that under the S1-NPC scenario. The threshold for Germany is however higher than that under S1-NPC, being in line with the median OADR2. It is worth observing that these cross-country differences in the shift of the threshold once accounting for rising college attainments are consistent with the differences in the ranking of the projected shares of educated workers by 2100 across the four countries (0.64 for the United States, 0.45 for Germany, 0.31 for Italy and 0.30 for Spain). We conclude commenting on the effects of rising college attainments on the probability of reaching the threshold by 2050 in the four countries. The results under S1-NPC were reported in the upper part of Table 10. The probability is unchanged for the United States, because it is zero under S1-NPC and the threshold further increases once accounting for the higher college attainment. The probability remains virtually unchanged for Italy and Spain, because their threshold are very close to those under S1-NPC. For Germany, given the larger increase in the threshold compared to Italy and Spain, the probability of reaching the threshold by 2050 declines from 99 to 45% in the case of rising college attainment.

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[33] See http://www.oecd.org/social/oecd-pensions-at-a-glance-19991363.htm.

[34] We would like to thank an anonymous referee who suggested this analysis to us.

[35] In addition to our model, these authors also consider skill-specific fertility rates and survival probabilities. Furthermore, health insurance in the form of Medicaid and Medicare as well as private health insurance is also modeled.
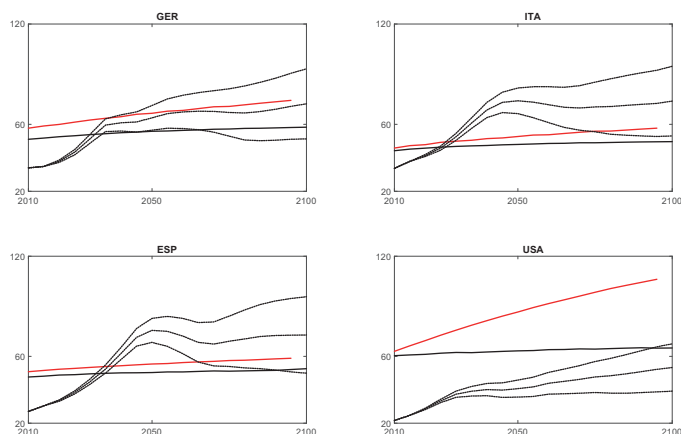
Fig. 8. OADR2, thresholds under S1-NPC, and thresholds under increasing college attainment for Germany, Italy, Spain and the United States, 2010–2100.

7. Conclusion

The paper develops an OLG life-cycle model that explicitly accounts for limits to the ability of governments to increase tax revenues through the distortionary taxation of income from capital and labor. This fiscal limit imposes a constraint on pension provision. As a result, under their current pension arrangements governments may find that there is an upper bound to the size of the dependency ratio that they can sustain solely from the revenues from the taxation of income. This threshold to the dependency ratio is obtained as a competitive equilibrium solution to the model. We measure the distance that an economy is from this threshold and the probability of reaching the threshold at some point in the future. Once this distance falls to zero, reform of the current pension arrangements becomes essential and can no longer be postponed. We consider three possible reforms to pension arrangements: partial financing using consumption taxes, a reduction in pension contributions and an increase in the retirement age.

Many factors affect the ability of a country to sustain its pension system. These include the levels of government spending on consumption, transfers and pensions; the distance of the economy from the peak of the Laffer hill; the demographic structure of the population and its projected change. Countries differ significantly across all of these dimensions. The proposed threshold dependency ratio is a summary measure that takes account of all of these factors simultaneously in a relatively simple and informative way.

In all of the countries we study we find that the threshold is increasing over time but not as rapidly as demographic forecasts of the dependency ratio. As a result, the distance from the threshold is found to decline very quickly and the probability of reaching the threshold is increasing.

There are significant differences in thresholds levels, distances and probabilities among these countries. The outlook for most of the European countries is of particular concern. Compared to the United States, all have, on average, smaller fiscal spaces, more generous pension systems, are older and are projected to age much faster. The European countries are therefore found to be much closer to the threshold than the United States in 2010 and are predicted to reach the threshold well before 2050. In contrast, the United States is found to maintain a positive distance until 2100.

The probability of the United States reaching the threshold can be reduced to be close to zero either by increasing consumption taxation by 5 percentage points, or by reducing the replacement ratio of the pension by 10 percentage points, or by increasing the retirement age to 70. In contrast, such policy changes bring only marginal improvements to the pension outlooks for the EU14 countries and only serve to highlight how pressing an issue pension reform is for these EU14 countries. For the European countries we further consider the impact of planned changes in their pension systems. These reduce the cost of public pensions in most countries, resulting in thresholds being closer to the median values of the projected OADRs rather than their lower bounds.

A further difference is that whereas for the Unites States there is a clear welfare advantage to employing higher consumption taxes to achieve a given distance from the threshold than having a lower replacement ratio or a higher retirement age, there is no such preference ordering for the European countries. Their welfare ranking differ depending on country-specific characteristics, such as the design of the tax system, the current level of public spending, private sector preferences and productivity.

We have also examined to what extent these conclusions might be affected by different modelling asumptions. We considered four extensions: income heterogeneity among individuals; non-linear labor income taxation; labor supply choice at the extensive margin for retirement; rising college attainments. We find that income heterogeneity affects countries differently, that progressive taxation reduces the threshold, that this reduction is greater the small is the elasticity of labor supply, that early retirement is also likely to reduce the threshold but the choice to retire early is affected by the cost involved. In contrast, increasing college attainments have the effect of increasing the threshold.

Although the quantitative results presented in this paper are based on a very stylized model of the economy, we believe that they have identified fundamental problems for the provision of public pensions in advanced economies that require urgent attention. A more complete analysis, beyond the scope of this paper, might also take account of several further implications that population aging may have for the macro-economy and for public finances. For example: (i) the political feasibility of extracting the maximum revenue from the taxation of income; (ii) the cost of non-pension-related components of public spending; and (iii) long-term rates of economic growth. We do not, however, anticipate that taking account of these considerations would alter our main finding, namely, that a large number of European countries are likely to reach their threshold dependency ratio within the next 20–30 years.

In our model demographic parameters evolve deterministically and the policy action is known in advance with certainty. Both assumptions are taken to simplify the analysis. They are also necessary due to the restrictions on the computational time and in light of the high number of equilibria under consideration. However, both kinds of uncertainties are likely to prevail empirically. Uncertainty with respect to the demographics has been studied by J-V (2001) who considers stochastic fertility rates in an OLG model of the United States. He finds that aggregate equilibrium effects of the aging baby boomers depend critically on whether fertility rates are mean-reverting or remain at their current low levels. Uncertainty with respect to the timing of policy has been analysed for the first time by Kitao (2018). He considers the effect of a pension reduction in Japan which takes place with a probability of one third in either 2020, 2030 or 2040. Kitao finds that a delay of policy implementation exacerbates the fiscal burden and welfare costs, in particular among the older individuals. It would be interesting to elaborate on the effects of these two kinds of uncertainty in our present analysis. It would also be interesting to consider the effects of uncertainty about the particular pension policy reform that is chosen by the goverment in the future.[36] Often it is unknown whether the government may implement either a pension reduction or an increase in the retirement rate or a mixture of the two. The solution methods developed by Ríos-Rull (2001) and, in particular, Kitao (2018) provide the necessary tools for such an analysis.

Acknowledgments

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jedc.2020.103913

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