Distributed event-triggered control of transport-reaction systems

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Abstract: We show that decentralized event-trigger can significantly reduce amount of transmitted measurements in network-based control of parabolic systems governed by a semilinear n-dimensional 1D diffusion PDEs. All measurements are sampled in time and space, quantized by a logarithmic quantizer, and are subject to time-varying network-induced delays.

Keywords: partial differential equations; event-triggered control; networked control systems.

1. INTRODUCTION

Networked control systems is a very hot topic due to great advantages they bring, such as low cost, reduced weight, simple installation/maintenance, long distance control, etc. In this paper we study the stability of semilinear parabolic systems under network-based control, which are potentially of a great interest in a long distance estimation/control of chemical reactors (see Smagina and Sheintuch (2006)) or air polluted areas (see, e.g., Koda and Seinfeld (1978); Court et al. (2012)). In such systems it is natural to assume that only point measurements are available, i.e. several sensors measure the output in certain space points. In Fridman and Blighovsky (2012) an estimate of the time-sampling that preserves the stability of a diffusion equation under point measurements has been obtained. However, these conditions for some systems may lead to a small sampling interval resulting in a high workload of the communication network.

In this paper we show that the workload can be significantly reduced by means of decentralized event-trigger. The key idea is to transmit only the measurement whose deviation from the previously sent one is greater than a weighted norm of the current measurement. This approach proved its efficiency in finite-dimensional networked control systems Tabuada (2007); Wang and Lemmon (2011); Mazo and Tabuada (2011); Yue et al. (2013); Peng and Yang (2013). In Hu and Yue (2012); Garcia and Antsaklis (2013) event-triggered control of finite-dimensional systems with quantized measurements has been studied. In Garcia and Antsaklis (2013) a model-based approach has been applied to network-based control. In Hu and Yue (2012) the system similar to the one we consider has been studied but with finite-dimensional plant.

1.1 Notations

The fact that $P \in \mathbb{R}^{n\times n}$ is symmetric positive-definite is denoted by $P > 0$, symbol $*$ stands for the symmetric terms of a matrix. The symbol $\mathbb{N}_0$ stands for a set of nonnegative integers, $\mathcal{C}^1$ denotes a set of smooth functions, $\mathcal{H}^1(0,l)$ is Sobolev space of absolutely continuous functions $z\colon [0,l] \to \mathbb{R}^n$ with the square integrable z_x. Other notations are standard.

1.2 Useful inequalities

Lemma 1. (Halanay's inequality, Halanay (1966)). If $0 < \delta_1 < \delta$ and $V\colon [t_0-\tau_M,\infty) \to [0,\infty)$ is absolutely continuous function such that

$$\dot V(t) \le -\delta V(t) + \delta_1 \sup_{-\tau_M\le\theta\le 0} V(t+\theta), \quad t\ge t_0,$$

then

$$V(t) \le e^{-\alpha(t-t_0)} \sup_{-\tau_M\le\theta\le 0} V(t_0+\theta), \quad t\ge t_0,$$

where $\alpha > 0$ is a unique positive solution of

$$\alpha = \delta - \delta_1 e^{\alpha\tau_M}. \tag{1}$$

Lemma 2. (Wirtinger's inequality, Hardy et al. (1952)). If $f \in \mathcal{H}^1(0,l)$ is a scalar function such that $f(0) = 0$ or $f(l) = 0$ then

$$\int_0^l f^2(\xi)\,d\xi \le \frac{4l^2}{\pi^2}\int_0^l \left[\frac{df}{d\xi}\right]^2 d\xi.$$

2. PROBLEM STATEMENT

We consider the system schematically presented in Fig. 1. Below we describe each block.

2.1 Plant: diffusion PDE

We consider diffusion equation

$$\begin{aligned} z_t(x,t) &= \Delta_D z(x,t) - \beta z_x(x,t) + Az(x,t) \\ &\quad + \phi(z(x,t),x,t) + Bu(x,t), \quad x\in[0,l], \quad t\ge 0 \end{aligned} \tag{2}$$

with the state $z(x,t) = [z^1(x,t),\dots,z^n(x,t)]^T \in \mathbb{R}^n$, control input $u \in \mathbb{R}^r$, constant matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, and a matrix of convection coefficients $\beta = \operatorname{diag}\{\beta_1,\dots,\beta_n\} \in \mathbb{R}^{n\times n}$. The diffusion term is given by

$$\Delta_D z(x,t) = \left[\frac{\partial}{\partial x}(d_1(x)z_x^1(x,t)),\dots,\frac{\partial}{\partial x}(d_n(x)z_x^n(x,t))\right]^T$$

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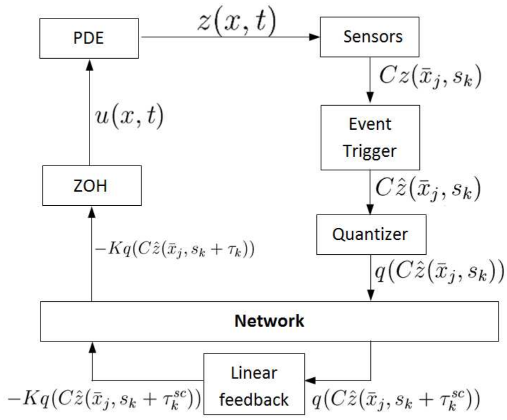

Fig. 1. System representation

with $d_i(x) \in \mathcal{C}^1$ such that $0 < d_i^0 \leq d_i(x)$ for $x \in [0, l]$, $i = 1, \ldots, n$. Following Bar Am and Fridman (2014) we assume that the function $\phi \in \mathcal{C}^1$ satisfies

$$\phi^T(z(x,t),x,t)\phi(z(x,t),x,t) \leq z^T(x,t)Qz(x,t) \qquad (3)$$

with a positive definite $Q \in \mathbb{R}^{n \times n}$.

We consider (2) under the Dirichlet

$$z(0,t) = z(l,t) = 0, \qquad (4)$$

Neumann

$$z_x(0,t) = z_x(l,t) = 0, \qquad (5)$$

or mixed boundary conditions

$$z_x(0,t) = \Gamma z(0,t), \quad z(l,t) = 0 \qquad (6)$$

with $\Gamma = \operatorname{diag}\{\gamma_1, \ldots, \gamma_n\} \geq 0$.

The open-loop system (2) under the above boundary conditions may become unstable for large enough ϕ (see, e.g., Curtain and Zwart (1995)).

2.2 Sampled measurements with event-trigger

Divide the spatial domain into N subdomains

$$0 = x_0 < x_1 < \ldots < x_N = l, \quad x_j - x_{j-1} = \Delta_j \leq \Delta.$$

We assume that there are N sensors with time samplings

$$0 = s_0 < s_1 < \ldots, \quad s_{k+1} - s_k \leq h,$$
$$\lim_{k\to\infty} s_k = \infty, \quad k \in \mathbb{N}_0.$$

The j-th sensor measures the value of $Cz(x,t)$ in the middle of the sampling interval $\bar{x}_j = \frac{x_{j-1}+x_j}{2}$ at time instants s_k, that is, the measurements are

$$y_{j,k} = Cz(\bar{x}_j, s_k), \qquad (7)$$

where $C \in \mathbb{R}^{m \times n}$.

To reduce the workload of the communication network each sensor uses a triggering rule to decide whether to send the newly sampled measurement or not. Denote by $\hat{y}_{j,k}$ the last sent measurement by the sensor j at time instant s_k. Similar to Tabuada (2007); Yue et al. (2013) the newly sampled measurement $y_{j,k}$ is not transmitted if the following relation holds

$$(\hat{y}_{j,k-1} - y_{j,k})^T \Omega (\hat{y}_{j,k-1} - y_{j,k}) \leq \varepsilon\, y_{j,k}^T \Omega y_{j,k}, \qquad (8)$$

where $\varepsilon > 0$, $\Omega \in \mathbb{R}^{m \times m}$ is a positive definite matrix. Therefore,

$$\hat{y}_{j,k} = \begin{cases} \hat{y}_{j,k-1}, & \text{if (8) is valid,} \\ y_{j,k}, & \text{if (8) is not valid,} \end{cases} \qquad (9)$$

where $j = 1, \ldots, N$, $k \in \mathbb{N}_0$, $\hat{y}_{j,-1} = 0$.

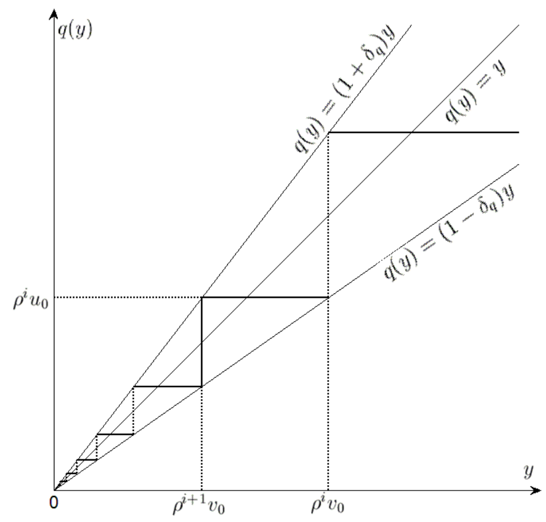

Fig. 2. Logarithmic quantizer

2.3 Logarithmic quantizer

Let us choose some $\rho \in (0,1)$, $u_0 > 0$ and define

$$v_0 = \frac{1+\rho}{2\rho} u_0, \quad \delta_q = \frac{1-\rho}{1+\rho}.$$

Following Elia and Mitter (2001) we introduce a logarithmic quantizer with a density ρ as a mapping $q\colon \mathbb{R} \to \mathcal{U} = \{\pm\rho^i u_0 \mid i \in \mathbb{Z}\} \cup \{0\}$

$$q(y) = \begin{cases} \rho^i u_0, & \rho^{i+1} v_0 < y \leq \rho^i v_0, \\ 0, & y = 0, \\ -q(-y), & y < 0. \end{cases}$$

For a vector $y = (y^1, \ldots, y^m)^T \in \mathbb{R}^m$ we define $q(y) = (q_1(y^1), \ldots, q_m(y^m))^T$, where q_i are scalar logarithmic quantizers with densities ρ_i.

Logarithmic quantizer implements a simple idea: to stabilize the system one should reduce quantization error near the origin by increasing the density of quantization levels, while far from the origin quantization levels can be sparse (see Fig. 2).

2.4 Networked controller

We assume that the measurements from all sensors are sent synchronously at time instants s_k. The quantized measurements from the j-th sampling interval are transmitted to the controller through a communication network and, therefore, are subject to time-varying bounded delays τ_k^{sc}. Linear feedback signals are transmitted through a communication network to a ZOH with time delays τ_k^{ca}. Thus, the updating time of the ZOH is $t_k = s_k + \tau_k$, where the overall network induced delay $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ is assumed to be such that $t_k \leq t_{k+1}$. We set

$$u(x,t) \equiv 0, \quad x \in [x_{j-1}, x_j), \quad t < t_0.$$

Then the overall dynamics of the closed-loop system for $x \in [x_{j-1}, x_j)$ can be presented in the form

$$\begin{aligned} z_t(x,t) &= \Delta_D z(x,t) - \beta z_x(x,t) + Az(x,t) \\ &\quad + \phi(z(x,t),x,t), \quad t \in [0, t_0), \\ z_t(x,t) &= \Delta_D z(x,t) - \beta z_x(x,t) + Az(x,t) \\ &\quad + \phi(z(x,t),x,t) - BKq(\hat{y}_{j,k}), \quad t \in [t_k, t_{k+1}), \end{aligned} \qquad (10)$$

where $K \in \mathbb{R}^{r\times m}$, $k \in \mathbb{N}_0$.

The existence of a continuable for $t \geq 0$ strong solution to the system (10) under the boundary conditions (4), (5), or (6) can be proved by arguments of Fridman and Bar Am (2013) for any $z(\cdot, 0) \in \mathcal{H}^1(0, l)$ satisfying the corresponding boundary conditions.

3. EVENT-TRIGGERED STABILIZATION UNDER POINT MEASUREMENTS

Denote by $\hat{z}(\bar{x}_j, s_k)$ the state that corresponds to $\hat{y}_{j,k}$, that is, $\hat{y}_{j,k} = C\hat{z}(\bar{x}_j, s_k)$. Define the following quantities

$$\begin{aligned} &v_{j,k} = q(\hat{y}_{j,k}) - C\hat{z}(\bar{x}_j, s_k),\\ &e_{j,k} = \hat{z}(\bar{x}_j, s_k) - z(\bar{x}_j, s_k),\\ &\sigma_k(x) = z(\bar{x}_j, s_k) - z(x, s_k), \quad x \in [x_{j-1}, x_j). \end{aligned} \tag{11}$$

Here $j = 1, \dots, N$, $k \in \mathbb{N}_0$. Note that the quantity $e_{j,k}$ is defined following Liu et al. (2012). These quantities can be interpreted as errors due to quantization, triggering, and space sampling, respectively. Denote the overall measurements delay by

$$\tau(t) = t - s_k, \quad t \in [t_k, t_{k+1}).$$

Then

$$\sup_{t\geq t_0} \tau(t) \leq h + \sup_k \tau_k \triangleq \tau_M.$$

Using these notations we rewrite the quantized measurements as

$$q(\hat{y}_{j,k}) = v_{j,k} + Ce_{j,k} + C\sigma_k(x) + Cz(x, t - \tau(t)). \tag{12}$$

Then the closed-loop system (10) can be rewritten in the following form

$$\begin{aligned} z_t(x, t) =&\Delta_D z(x, t) - \beta z_x(x, t) + Az(x, t)\\ &+ \phi(z(x, t), x, t) - BKCz(x, t - \tau(t))\\ &- BK\left[v_{j,k} + Ce_{j,k} + C\sigma_k(x)\right],\\ &x \in [x_{j-1}, x_j), \quad t \in [t_k, t_{k+1}),\\ &j = 1, \dots, N, \quad k \in \mathbb{N}_0. \end{aligned} \tag{13}$$

To study the stability of (13) in the absence of event-trigger, measurements sampling, and quantization ($v_{j,k} = 0$, $e_{j,k} = \sigma_k(x) \equiv 0$) one can use an n-dimensional extension of Lyapunov-Krasovskii functional from Fridman and Orlov (2009):

$$V(t) = V_1(t) + V_2(t) + V_S(t) + V_R(t) + V_B(t), \tag{14}$$

where

$$\begin{aligned} &V_1(t) = \int_0^l z^T(x, t)P_1 z(x, t)\, dx,\\ &V_2(t) = \sum_{i=1}^n \int_0^l p_3^i d_i(x)(z_x^i(x, t))^2\, dx,\\ &V_S(t) = \int_0^l \int_{t-\tau_M}^t e^{\delta(s-t)} z^T(x, s)Sz(x, s)\, ds\, dx,\\ &V_R(t) =\\ &\quad \tau_M \int_0^l \int_{-\tau_M}^0 \int_{t+\theta}^t e^{\delta(s-t)} z_s^T(x, s)Rz_s(x, s)\, ds\, d\theta\, dx,\\ &V_B(t) = b\sum_{i=1}^n p_3^i d_i(0)\gamma_i(z^i(0, t))^2 \end{aligned}$$

with $P_1 > 0$, $p_3^i > 0$, $S > 0$, $R > 0$, $b = 0$ for (4), (5) and $b = 1$ for (6). In Fridman and Blighovsky (2012) spatially sampled measurements have been considered ($v_{j,k} = 0$, $e_{j,k} = 0$, $\sigma_k(x) \not\equiv 0$), where Halanay's inequality (Lemma 1) has been applied to "compensate" the term $\sigma_k(x)$ in the derivative of Lyapunov-Krasovskii functional (14). In this paper to compensate the cross terms with $v_{j,k}$ and $e_{j,k}$ we apply S-procedure (see, e.g., Yakubovic (1977)) to appropriate quadratic forms. Namely, we note that each component of $v_{j,k} = (v_{j,k}^1, \dots, v_{j,k}^m)^T$ satisfies the sector inequality (see Fig. 2 and, e.g., Fu and Xie (2005); Zhou et al. (2010))

$$0 \leq \lambda_q^i \left(\delta_q^i \hat{y}_{j,k}^i - v_{j,k}^i\right)\left(v_{j,k}^i + \delta_q^i \hat{y}_{j,k}^i\right), \tag{15}$$

with $\lambda_q^i \geq 0$, $\delta_q^i = (1-\rho_i)/(1+\rho_i)$. Furthermore, triggering condition (8), (9) is equivalent to

$$\begin{aligned} 0 \leq \varepsilon&[z(x, t - \tau(t)) + \sigma_k(x)]^T C^T\Omega C\times\\ &[z(x, t - \tau(t)) + \sigma_k(x)] - e_{j,k}^T(t)C^T\Omega Ce_{j,k}(t). \end{aligned} \tag{16}$$

Nonnegative quadratic forms (15) and (16) contain the terms $-\lambda_q^i(v_{j,k}^i)^2 \leq 0$ and $-e_{j,k}^T C^T\Omega Ce_{j,k} \leq 0$ that will compensate the cross terms with $v_{j,k}$ and $e_{j,k}$.

Now we are in position to formulate the stability conditions.

Theorem 1. (i) Let there exist positive definite $n \times n$ matrices P_1, $P_3 = \mathrm{diag}\left\{p_3^1, \dots, p_3^n\right\}$, R, S, $m \times m$ nonnegative matrices Ω, $\Lambda_q = \mathrm{diag}\left\{\lambda_q^1, \dots, \lambda_q^m\right\}$, $n \times n$ matrices $P_2 = \mathrm{diag}\left\{p_2^1, \dots, p_2^n\right\}$, G, and scalars $\lambda_\phi \geq 0$, $0 < \delta_1 < \delta$ such that

$$\Xi \leq 0, \quad \begin{bmatrix} R & G\\ G^T & R\end{bmatrix} \geq 0, \tag{17}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & P_2 & \Xi_{17} & \Xi_{18} & \Xi_{19}\\ * & \Xi_{22} & -P_3\beta & 0 & \Xi_{25} & P_3 & \Xi_{27} & \Xi_{28} & \Xi_{29}\\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & 0 & 0 & 0\\ * & * & * & * & \Xi_{55} & 0 & \Xi_{57} & 0 & \Xi_{59}\\ * & * & * & * & * & -\lambda_\phi I_n & 0 & 0 & 0\\ * & * & * & * & * & * & \Xi_{77} & 0 & \Xi_{79}\\ * & * & * & * & * & * & * & -\Lambda_q & 0\\ * & * & * & * & * & * & * & * & \Xi_{99} \end{bmatrix}$$

$$\begin{aligned} &\Xi_{11} = S - e^{-\delta\tau_M}R + P_2A + A^TP_2 + \lambda_\phi Q + \delta P_1,\\ &\Xi_{12} = P_1 - P_2 + A^TP_3, \quad \Xi_{13} = 0, \quad \Xi_{14} = e^{-\delta\tau_M}G^T,\\ &\Xi_{15} = e^{-\delta\tau_M}(R - G^T) - P_2BKC,\\ &\Xi_{17} = \Xi_{19} = -P_2BKC, \quad \Xi_{18} = -P_2BK,\\ &\Xi_{22} = \tau_M^2 R - 2P_3, \quad \Xi_{25} = \Xi_{27} = \Xi_{29} = -P_3BKC,\\ &\Xi_{28} = -P_3BK, \quad \Xi_{33} = D_0(\delta P_3 - 2P_2),\\ &\Xi_{44} = -e^{-\delta\tau_M}(S + R), \quad \Xi_{45} = e^{-\delta\tau_M}(R - G),\\ &\Xi_{55} = -2e^{-\delta\tau_M}R + e^{-\delta\tau_M}[G + G^T] + C^T\Lambda_q\Delta_q^2C\\ &\qquad + \varepsilon C^T\Omega C - \delta_1 P_1,\\ &\Xi_{57} = C^T\Lambda_q\Delta_q^2C + \varepsilon C^T\Omega C,\\ &\Xi_{59} = \Xi_{79} = C^T\Lambda_q\Delta_q^2C, \quad \Xi_{99} = C^T\Lambda_q\Delta_q^2C - C^T\Omega C,\\ &\Xi_{77} = C^T\Lambda_q\Delta_q^2C + \varepsilon C^T\Omega C - \delta_1 P_3 D_0\frac{\pi^2}{\Delta^2}. \end{aligned}$$

$D_0 = \mathrm{diag}\left\{d_1^0, \dots, d_n^0\right\}$, $\Delta_q = \mathrm{diag}\left\{\delta_q^1, \dots, \delta_q^m\right\}$, $\delta_q^i = (1-\rho_i)/(1+\rho_i)$. Then a unique strong solution to the Dirichlet boundary value problem (4), (7), (8), (9), (10) initialized with

$$z(\cdot, 0) \in \mathcal{H}^1(0, l)\colon\ z(0, 0) = z(l, 0) = 0$$

for $t \geq t_0$ satisfies the inequality

$$\int_0^l z^T(x,t)P_1 z(x,t)\,dx + \sum_{i=1}^n \int_0^l p_3^i d_i(x)(z_x^i(x,t))^2\,dx$$
$$\leq e^{-\alpha(t-t_0)}\Bigg[\int_0^l z^T(x,t)P_1 z(x,t)\,dx$$
$$+\sum_{i=1}^n \int_0^l p_3^i d_i(x)(z_x^i(x,t))^2\,dx + b\sum_{i=1}^n p_3^i d_i(0)\gamma_i(z^i(0,t))^2\Bigg] \tag{18}$$

with $b = 0$, where α is a unique positive solution of (1).

(ii) If conditions of (i) are satisfied with $\Xi_{13} = -P_2\beta$ then a unique strong solution to the Neumann boundary value problem (5), (7), (8), (9), (10) initialized with

$$z(\cdot,0) \in \mathcal{H}^1(0,l):\; z_x(0,0) = z_x(l,0) = 0$$

for $t \geq t_0$ satisfies (18) with $b = 0$, where α is a unique positive solution of (1).

(iii) If in addition to the conditions of (i)

$$(\delta p_3^i - 2p_2^i)d_i(0)\gamma_i + p_2^i\beta_i \leq 0, \quad i = 1,\ldots,n,$$

then a unique strong solution to the mixed boundary value problem (6), (7), (8), (9), (10) initialized with

$$z(\cdot,0) \in \mathcal{H}^1(0,l):\; z_x(0,0) = \gamma z(0,0),\; z(l,0) = 0$$

for $t \geq t_0$ satisfies (18) with $b = 1$, where α is a unique positive solution of (1).

Proof is an extension of the proof from Fridman and Blighovsky (2012) that uses Wirtinger's and Halanay's inequalities (Lemmas 1 and 2).

Remark 1. Here we assume that all sensors are synchronized. If this is not the case then one should define different measurement delays $\tau_j(t)$ for each space interval $[x_{j-1}, x_j)$. Then to use Halanay's inequality one could consider

$$-N\delta_1 \sup_{\theta\in[-\tau_M,0]} V(t+\theta) \leq -\delta_1 \sum_{j=1}^N V(t-\tau_j(t))$$
$$\leq -\delta_1 \sum_{j=1}^N \int_{x_{j-1}}^{x_j} z^T(x,t-\tau_j(t))P_1 z(x,t-\tau_j(t))\,dx$$
$$-\,\delta_1 \sum_{j=1}^N \int_{x_{j-1}}^{x_j} \sum_{i=1}^n p_3^i d_i^0 [z_x^i(x,t-\tau_j(t))]^2\,dx.$$

But this approach seems to be quite restrictive since the terms

$$-\int_{x_{j-1}}^{x_j} z^T(x,t-\tau_k(t))P_1 z(x,t-\tau_k(t))$$
$$-\int_{x_{j-1}}^{x_j} \sum_{i=1}^n p_3^i d_i^0 [z_x^i(x,t-\tau_k(t))]^2\,dx \leq 0$$

with $j \neq k$ are ignored.

Remark 2. Instead of the decentralized triggering rule (8) one can think of a centralized event-trigger of the form

$$\sum_{j=1}^N (\hat{y}_{j,k-1} - y_{j,k})^T\,\Omega\,(\hat{y}_{j,k-1} - y_{j,k}) \leq \varepsilon \sum_{j=1}^N y_{j,k}^T \Omega y_{j,k}, \tag{19}$$

where all the measurements $y_{j,k}$ are transmitted to the event-trigger and if (19) is violated all the measurements are quantized and transmitted to the controllers. In the case of uniform space samplings $\Delta_j = \Delta$ the results of Theorem 1 hold for (19). However, as the example demonstrates, decentralized event-trigger mechanism (8) (that is more realistic if the sensors are not close to each other) is more effective.

T (sec.)	1	2	3	4	5	6
No event-trigger	112	223	334	445	556	667
Event-trigger (19)	70	139	209	278	348	417
Event-trigger (8)	70	137	202	264	330	408

Table 1. Average amount of sent measurements

4. EXAMPLE: CHEMICAL REACTOR

Consider the chemical reactor model from Smagina and Sheintuch (2006) governed by (2) under the mixed boundary conditions (6) with $n = 2$, $r = m = 1$, $l = 10$, $D_0 = \operatorname{diag}\{0.01, 0.005\}$, $\beta = \operatorname{diag}\{0.011, 1.1\}$, $K = 1$, $\Gamma = \operatorname{diag}\{6, 111\}$, $Q = \operatorname{diag}\{10^{-4}, 0\}$, $u_0 = 1$, $\rho_i = \rho = 0.9$,

$$A = \begin{bmatrix} 0 & 0.01 \\ -0.45 & -0.2 \end{bmatrix},\; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\; C = [1\;\,0],\; \phi = \begin{bmatrix} \phi_1(z^1) \\ 0 \end{bmatrix}.$$

This model accounts for an activator temperature z^1 that undergoes reaction, advection, and diffusion, and for a fast inhibitor concentration z^2, which may be advected by the flow. The elements of β are convective velocities.

We divide $[0, l]$ into $N = 20$ intervals, set $\delta = 2$, $\delta_1 = 0.9\,\delta$ and consider uniform time samplings $s_k = kh$, $k \in \mathbb{N}_0$.

For $\varepsilon = 0$ conditions of Theorem 1 (iii) are satisfied with $\tau_M = \tau_M^0 = 0.009$ ($\alpha \approx 0.1968$). If $\tau_k^{sc} = \tau_k^{ca} = 0$ this implies that each sensor transmits $\lceil T/\tau_M \rceil + 1$ measurements on the time interval $[0, T]$. For $\varepsilon = 10^{-4}$ we find $\tau_M = \tau_M^\varepsilon = 0.0072$ ($\alpha \approx 0.1974$). In Table 1 one can see the average amount of the sent measurements by one sensor in case of the system without event-trigger ($\varepsilon = 0$), with event-trigger (19), and with decentralized event-trigger (8). Though $\tau_M^\varepsilon < \tau_M^0$, the amount of sent measurements is reduced by approximately 40%. Note that the decentralized event-trigger (8) has a slight advantage over (19). Moreover, according to (1) the decay rate α gets larger with decrease of τ_M. That is, *the event-trigger allows to reduce significantly the workload of a networked control system while the decay rate is preserved.*

5. CONCLUSIONS

We derived stability conditions for the networked diffusion control by point measurements with decentralized event-trigger and quantization. By an example we demonstrated that event-trigger mechanism can significantly decrease the network workload preserving decay rate of convergence.

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