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Kosny, M, Silber, J and Yalonetzky, G orcid.org/0000-0003-2438-0223 (2020) On the Measurement of Multi-Period Income Mobility. Inequality, Redistribution and Mobility (Research on Economic Inequality, 28. pp. 97-122. ISSN 1049-2585

https://doi.org/10.1108/S1049-258520200000028004

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On the Measurement of Multi-Period Income Mobility

by

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March 2020

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Abstract

We propose a framework for the measurement of income mobility over several time periods, based on the notion that multi-period mobility amounts to measuring the degree of association between the individuals and the time periods. More precisely we compare the actual income share of individuals at a given time in the total income of all individuals over the whole period analysed, with their "expected" share, assumed to be equal to the hypothetical income share in the total income of society over the whole accounting period that an individual would have had at a given time, had there been complete independence between the individuals and the time periods.

We then show that an appropriate way of consistently measuring multi-period mobility should focus on the absolute rather than the traditional (relative) Lorenz curve and that the relevant variable to be accumulated should be the difference between the "a priori" and "a posteriori" shares previously defined. Moving from an ordinal to a cardinal approach to measuring multi-period mobility, we then propose classes of mobility indices based on absolute inequality indices.

We illustrate our approach with an empirical application using the EU-SILC rotating panel dataset. Our empirical analysis seems to vindicate our approach because it clearly shows that income mobility was higher in the new EU countries (those that joined the EU in 2004 and later). We also observe that income mobility after 2008 was higher in three countries that were particularly affected by the financial crisis: Greece, Portugal and Spain.

J.E.L. Classification: D31 – D63

Key Words: Absolute Lorenz curve; EU-SILC; Multi-period mobility

Acknowledgements

The authors would like to thank an anonymous referee, Francesco Andreoli, John Bishop, Alessio Fusco, Thesia Garner, and the participants in the 83rd Southern Economic Association Conference in Tampa for their comments and suggestions on previous versions of this paper. Marek Kośny would like to acknowledge the financial support of the Polish National Science Centre (Project No. 3804/B/H03/2011/40). Marek Kośny and Gaston Yalonetzky gratefully acknowledge the financial support of InGRID and the European Commission as well as the hospitality of LISER, Luxembourg during a two-week visit. The research leading to these results has received support under the European Commission's 7th Framework Programme (FP7/2013-2017) under grant agreement no 312691, InGRID – Inclusive Growth Research Infrastructure Diffusion.

1. Introduction

In a survey of income mobility Jäntti and Jenkins (2015) make a basic distinction between intergenerational mobility that refers to income change between generations of parents and children, and intra-generational mobility which deals with the change of income of individuals between one year and another during their lifetime. In the latter case a distributional change can have up to three components: (average) income growth, structural mobility (change in relative inequality) and exchange mobility (re-ranking of individuals[1]). Ruiz-Castillo (2000) and Silber and Weber (2005) attempted indeed to isolate these three components.

The focus of many of the measures of intra-generational income mobility that have appeared in the literature has in fact been on mobility between two periods, with notable exceptions including the works of Shorrocks (1978), Maasoumi and Zandvakili (1986), and Tsui (2009). Tsui (2009) offers a coherent framework to analyse multi-period income mobility. His approach is closely related to his previous work on multi-dimensional income inequality (Tsui, 1995, 1999). Tsui (2009) chose a relatively "weak" definition of complete immobility since he assumed that there is immobility if there is no exchange mobility. A stronger definition of complete immobility is that adopted, for example, by Shorrocks (1978) and Maasoumi and Zandvakili (1986), because they took as immobility benchmark a situation where "the relative income of each individual does not change over time" (Shorrocks, 1978, p. 381). In other words, complete immobility requires the absence of structural as well as of exchange mobility.[2]

Building on this notion of immobility, we propose a new framework for the measurement of income mobility over several time periods, whereby multi-period

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[1] It is interesting to note that in the framework of a two periods analysis, Ruiz-Castillo (2000; 2004) makes a distinction between two types of re-rankings: rank reversals between the first- and second-period income distributions, which he calls 1/2-rerankings, and rank reversals between the first-period and the aggregate income distributions, which he calls 1/1+2-rerankings. In a certain way our approach to multi-period mobility adopts the second approach of Ruiz-Castillo since de facto we compare the income distribution of each period with the aggregate income distribution (distribution of the total income received by the individuals, all periods included).

[2] In this case, structural mobility is related to changes in relative inequality. Note that an even stronger definition of complete immobility would require that at all periods, all the individuals have the same income, so that there would be neither income growth, nor structural and exchange mobility.

mobility can be interpreted as the degree of association between the individuals and the time periods.

To operationalize this idea, let y_{it} be the income of individual i at time t. Our chosen benchmark amounts in fact to the existence of independence between the rows and the columns of the matrix of the incomes y_{it}, that is, between the individuals and the time periods. In other words there will be complete income immobility if for all individuals i and time periods t the observed share $s_{it} = \left(\frac{y_{it}}{\sum_i \sum_t y_{it}}\right)$ is identical to the expected share $w_{it} = \left(\frac{\sum_i y_{it}}{\sum_i \sum_t y_{it}}\right)\left(\frac{\sum_t y_{it}}{\sum_i \sum_t y_{it}}\right)$ that would be observed under independence between the rows and the columns of the matrix of incomes y_{it}. As will be shown in Section 2, our definition of complete immobility turns out to be identical to that adopted by Shorrocks (1978) and Maasoumi and Zandvakili (1986), but the way we formulate this definition of immobility allows us to derive new measures of multi-period mobility.

At this stage, we should note that the gaps between the observed shares s_{it} and the expected shares w_{it} may be positive or negative. We will assess mobility as inequality across these gaps and will adopt an absolute inequality measurement framework and use absolute Lorenz curves (Moyes, 1987) for partial orders. The analytical framework we adopt reminds us somehow of the proposals made by Joe (1985) and Greselin and Zenga (2004) in the context of contingency tables, but neither of them is intended or suited for measuring mobility. We should also stress that, unlike Tsui (2009), we do not need to provide an axiomatic characterization of the mobility indices we introduce, since these are adaptations of previously characterized absolute inequality indices.

To test the usefulness of the new multi-period income mobility indices we look at income mobility in Europe, with the rotating panels of the EU-SILC dataset, covering the period 2005-2012, i.e. years around the financial crisis of 2008. Besides computing mobility indices for the countries involved, we are interested in two specific issues. First, we analysed whether “old” EU members exhibit more or less mobility than “new” EU members (those who joined in 2004 or afterwards). The results indicate higher mobility among “new” EU members; though differences between “new and “old” states diminished over time. Second, we studied the impact of the financial crisis of 2008 on income mobility in EU countries. We found that changes in income mobility were closely related to the extent to which the financial crisis affected a given country. While in most countries a decline in income mobility was observed, there was an increase in mobility in Southern Europe between 2008 and 2011.

The paper is organized as follows. Section 2 presents the basic setting where multi-period income mobility is related to the notion of independence between the rows and the columns of the matrix of individual incomes. This section then proceeds to lay out the desirable properties that mobility indices ought to satisfy in our framework. Section 3 introduces the multi-period mobility partial order based on the concept of absolute Lorenz curves, which emerges naturally from the mobility concepts discussed in the previous section. Section 4 compares our approach with those of previous studies that proposed indices of multi-period mobility. Section 5 provides an empirical illustration based on EU-SILC data. Some concluding comments are given in Section 6.

2. A new approach to measuring multi-period mobility

2.1. The basic setting

As previously mentioned, let $y_{it} \in \mathbb{R}_+$ represent the income received by individual i at time t. Let $Y \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it}$ where N is the total number of individuals in the panel and T the total number of periods (both positive natural numbers).

Define also a $N \times T$ matrix $\mathcal{S}$ of the shares s_{it} defined previously as $s_{it} \equiv (y_{it}/Y)$. The margins of this matrix $\mathcal{S}$ are then $s_{i.} \equiv (\sum_{t=1}^{T} y_{it}/Y) > 0$ and $s_{.t} \equiv (\sum_{i=1}^{N} y_{it}/Y) > 0$. Let us also define a $N \times T$ matrix $\mathcal{W}$ whose typical element w_{it} was also defined previously and may be expressed as $w_{it} \equiv (s_{i.} \times s_{.t})$. Matrices $\mathcal{S}$ and $\mathcal{W}$ are elements of set $\mathcal{X}_{NT}$, which contains all possible matrices of shares with dimensions $N \times T$, excluding any matrix with any zero margins. $\mathcal{X}$ is the union-set containing all subsets $\mathcal{X}_{NT}$ for different values of N and T.

We start with some key definitions that will be useful in what follows:

Complete Immobility: As indicated before, we will assume that complete immobility takes place if and only if all incomes are ranked the same way across all time periods (absence of exchange mobility) and all time periods feature the same level of Lorenz-consistent inequality (absence of structural mobility): $y_{it} = k_t y_i \ \forall i = 1,2, \dots N; , t = 1,2, \dots, T$, where $k_t \in \mathbb{R}_{++}$ and $y_i \in \mathbb{R}_{++}$.

Since the elements of matrix $\mathcal{S}$ fulfil the standard definition of relative frequencies (they take values between 0 and 1 and add up to 1 together), some elementary rules analogous to probability rules can be applied. For instance, if the income trajectories are independent of time periods then $\forall i = 1,2, \dots, N;\ t = 1,2, \dots T$: $s_{it} = s_{i.} s_{.t} \equiv w_{it}$.

More precisely, we can derive the following key proposition establishing the correspondence between complete immobility and contingency table independence.

Proposition 1: $s_{it} = w_{it} \ \forall i, t$ if and only if $y_{it} = k_t y_i \ \forall i = 1,2, \dots, N; t = 1,2, \dots T$, where $k_t \in \mathbb{R}_{++}$ and $y_i \in \mathbb{R}_{++}$.

Proof: See Appendix A.

According to Proposition 1, there is complete independence between people and time if and only if the income distribution in a given period can be expressed as a positive multiple of the income distribution in any other period. Alternatively, independence is achieved if and only if, in the absence of any re-rankings, all distributions preserve the same level of relative inequality (as measured by any Lorenz-consistent, scale-invariant inequality measure) across time.

Complete independence coincides therefore with a lack of structural and exchange mobility, save for proportional transformations of the distributions. Hence the degree of association or dependence between the rows and the columns, i.e. between the individuals and time, can serve as a metric for multi-period mobility in the population.

2.2. Desirable properties of a multi-period mobility index

Let us define a NxT matrix $\mathcal{V} \equiv NT(\mathcal{S} - \mathcal{W})$ whose typical element v_{ij} is equal to the gap between observed shares and expected shares under independence, defined by $v_{it} \equiv NT(s_{it} - w_{it})$. The reason why we multiply by NT will become apparent below.

In what follows we rely on these gaps, v_{it}, since we know that $v_{it} = 0 \ \forall i = 1,2, \dots N, t = 1,2, \dots, T$ if and only if there is table independence, i.e. complete immobility. Otherwise, some gaps will be positive while others will be negative. In this framework, we will assess mobility as inequality across the gaps, since the gaps are only equal among each other (and equal to 0) whenever there is table independence, which in turn characterises complete immobility.

Note that the mean value of the gaps is zero, $\bar{v} \equiv \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} v_{it} = 0$. Therefore, if we want to measure mobility as inequality across these gaps (since these can only be equal under complete immobility), we cannot rely on a relative approach. We must adopt an absolute inequality measurement framework, which implies the use of absolute inequality indices and absolute Lorenz curves (Moyes, 1987) for partial orders.[3]

Let $I: \mathcal{X} \rightarrow \mathbb{R}_+$ be a mobility index mapping from share matrices of any size onto the non-negative real line. This index, I, will still retain the property of scale invariance (i.e. it will not be affected by scalar multiplications of incomes, e.g. by changing its measurement currency) because it maps from income shares as opposed to the incomes themselves.

Note also that, if we represent our data table via gaps v_{it} as opposed to the original shares s_{it}, we are gaining comparability in the sense that we can compare tables with different margins. However, we must take into account the fact that larger tables are bound to have smaller absolute gaps of the form $(s_{it} - w_{it})$, since the expected value $\left(\frac{1}{NT}\right)$ of both s_{it} and w_{it} becomes smaller when the table gets larger.[4]

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[3] Although we use a relative income inequality approach when selecting the benchmark of complete immobility, we use an absolute approach to the measurement of inequality among the gaps v_{it} between the observed and expected shares under complete immobility. Combining these two approaches to inequality measurement is however not a problem, because in one case we look at the distribution of incomes and in the other at that of the gaps in income shares. In principle, we could also use an intermediate approach to the measurement of inequality among the gaps. However only an absolute approach guarantees consistency (Lambert and Zheng, 2011), a necessary property in our measurement framework as explained in section 2.4.

[4] When we have unadjusted shares like s or (s-w) in a table, two things happen when we replicate the shares: (1) we have more shares; (2) each share is smaller in size. If we use indices that satisfy the replication invariance property, then only the change in the number of shares will be corrected for (point 1), but not the size of each share (point 2). Therefore, we will get smaller index values for replications of shares. Hence if we want to impose full replication invariance, we need to change the variable from (s-w) to v.

When defining an index $I(\mathcal{V})$ measuring the inequality of these gaps, we thus need the "population principle" for tables:

Table population principle (TPP): If table $\mathcal{V}_2$ is obtained from table $\mathcal{V}_1$ by replicating its NT shares so that individuals are replicated $\lambda_N \in \mathbb{N}_+$ times and periods are replicated $\lambda_T \in \mathbb{N}_+$ then: $I(\mathcal{V}_1) = I(\mathcal{V}_2)$.

An interesting consequence of the dilution of gaps when tables grow in size, is that, in order to render gaps from tables with different sizes comparable, we need to 'inflate' all gaps by the table size, i.e. NT. Hence, we need to measure mobility via the variables $v_{it} \equiv NT(s_{it} - w_{it})$. This is a necessary but insufficient requirement for making sure that the mobility index satisfies the TPP property.

We also want the mobility indices to satisfy the following two properties:

Permutation of individuals (PI): If table $\mathcal{V}_B$ is obtained from table $\mathcal{V}_A$ by permutations of individuals (i.e. rows), then: $I(\mathcal{V}_B) = I(\mathcal{V}_A)$.

Time Symmetry (TS): If table $\mathcal{V}_B$ is obtained from table $\mathcal{V}_A$ by permutations of time periods (i.e. columns), then: $I(\mathcal{V}_B) = I(\mathcal{V}_A)$.

While the assumption PI is a generally accepted property in the analysis of income mobility because individual features other than income are irrelevant, time symmetry is clearly a stronger property. But such an assumption is also made by the very popular income mobility index suggested by Shorrocks (1978) since the index he proposed compares the inequality in total income (defined by the sum of the incomes received in each period), against the weighted sum of the inequalities recorded for the different period-specific incomes. However, we could also decide not to impose time symmetry, by allowing, for example, for time-specific income discounting coefficients. The implications of such procedure for our measurement framework are beyond the scope of this paper and left for future research.

Another desirable property of a multi-period mobility index is that it should react to progressive transfers among gaps. This is a sensible way to operationalise an immobility-inducing transformation, as progressive transfers among gaps would render gaps' values closer to each other and to the complete immobility benchmark characterised by equality across all gaps. We suggest the following version of a progressive transfers property:

Sensitivity to progressive transfers among gaps (PR): $I(\mathcal{V}_2) < I(\mathcal{V}_1)$ if $\mathcal{V}_2$ is obtained, ceteris paribus, from $\mathcal{V}_1$ through a rank-preserving progressive transfer of $\delta \in \mathbb{R}_{++}$ involving gaps v_{it} and $v_{j\tau}$, with $v_{it} < v_{j\tau}$, so that $v_{it} + \delta, \leq v_{j\tau} - \delta$.

In section 2.4 we show how some income transfers relate to progressive transfers among gaps, and therefore to the PR property.

2.3. Multi-period mobility partial orderings with an absolute Lorenz curve

Let A and B be two populations. Following Moyes (1987) we define an absolute Lorenz curve (ALC), $L: [0,1] \rightarrow (-1,0]$, which maps from population percentiles, p, of v_{it} in ascending order, where the ordered v_{it} are represented by v_j^*, to the actual cumulative values of $\frac{1}{NT} v_j^*(p)$ up to percentile p. Hence the ALC is:

$$L^{\mathcal{V}^*}(p) \equiv \frac{1}{NT}\sum_{j=1}^{pNT}[v_j^*(p) - \bar{v}] = \frac{1}{NT}\sum_{j=1}^{pNT} v_j^*(p) \quad (1)$$

Note that in (1) the mean $\bar{v} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} v_{it}$ is absent from the formula because $\bar{v} = 0$. We can now state an absolute Lorenz-consistency condition akin to those used in the inequality literature (e.g. see Chakravarty, 2009):

Theorem 1: The following statements are equivalent:

(i) For any elements A and B of $\mathcal{X}$, table A exhibits more mobility than table B according to all mobility indices satisfying, PI, TS, TPP and PR, i.e. $I[A] > I[B]$.

(ii) For any elements A and B of $\mathcal{X}$, table B can be obtained from table A by a finite sequence of operations including permutations of individuals (rows), permutations of time periods (columns), replications of shares, and progressive transfers among gaps.

(iii) For any elements A and B of $\mathcal{X}$, $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p \in [0,1] |\ L^A(p) < L^B(p)$.

Proof: See Appendix A. ■

Illustration

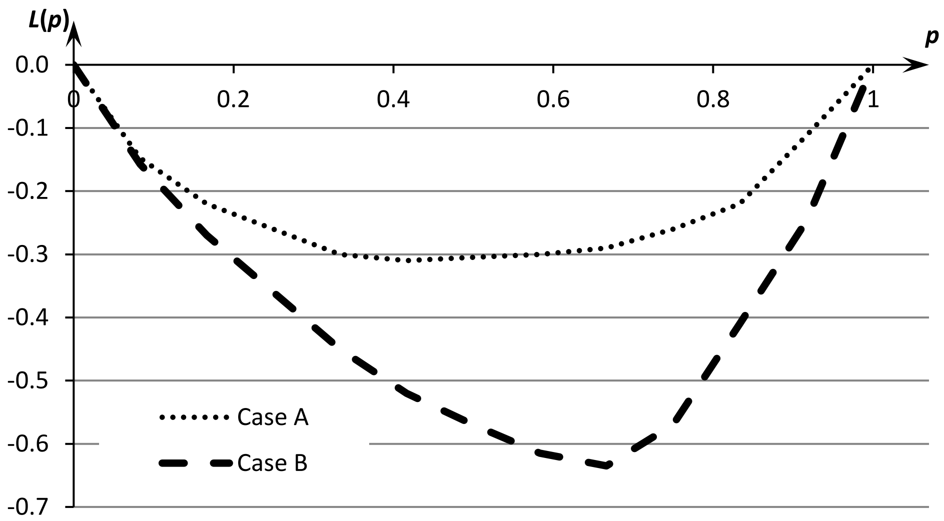
Consider in Table 1 the following two mobility tables that refer to two cases, A and B, and have identical margins.

Table 1. A simple illustration

	Period 1A	**Period 2A**	**Period 3A**	**Period 1B**	**Period 2B**	**Period 3B**
Person 1	0.01	0.04	0.2	0	0	0.25
Person 2	0.01	0.05	0.04	0	0	0.1
Person 3	0.05	0.05	0.1	0.2	0	0
Person 4	0.13	0.31	0.01	0	0.45	0

Their absolute Lorenz curves are drawn in Figure 1.

Figure 1. Two Absolute Lorenz curves



Hence any mobility index satisfying the properties stipulated in Theorem 1 should rank B as more mobile than A. This partial ordering should allow us to compare not only tables with different sizes, but also tables with different margins, since we are mapping from absolute gaps. In fact, all gap tables of the form $\mathcal{V}$ have every margin equal to 0.

In a sense, our definition of mobility is related to deviations from situations in which a table of gaps is full of zeroes.[5]

In Appendix B we provide simple illustrations of exchange mobility, structural mobility and pure growth.

2.4. Satisfaction of other desirable properties

Consistency

We defined the gaps as $v_{ij} \equiv NT(s_{ij} - w_{ij})$. However, this is arbitrary in the sense that we could equally measure mobility as inequality among the elements $-v_{ij} \equiv NT(w_{ij} - s_{ij})$. Therefore, a minimum requirement for a mobility index evaluated at these gaps is that its rankings of share matrices should be independent from the way we defined the gaps.

Consistency (C): Let I be a consistent mobility index. Then $I[\mathcal{V}_A] > I[\mathcal{V}_B]$ if and only if $I[-\mathcal{V}_A] > I[-\mathcal{V}_B]$.

This consistency property is always satisfied when we use absolute inequality indices and share gaps in order to measure mobility.

Proposition 2: Any inequality index I consistent with the absolute Lorenz curve in equation 1 is a consistent mobility index.

Proof: Despite the resemblance in measurement frameworks, we cannot apply the consistency results of Lambert and Zheng (2011) directly, because v_{ij} and $-v_{ij}$ do not add up to an upper bound of either variable. See Appendix C for the proof. ■

Normalization

Since we have a clear benchmark of complete immobility, it is also sensible to make the mobility index satisfy the following normalisation property:

[5] The literature on discrete multivariate analysis has long recognized that two equally sized contingency tables may differ either in their margins or in the degree of independence between rows and columns (or both). Since the two aspects are interrelated, statistical techniques have been devised to isolate them from each other. See, for instance, Silber and Spadaro (2011) who use iterative proportional fitting to isolate the dependence component in contingency tables representing social mobility patterns. Such procedures are not necessary in our setting since our tables of gaps always have margins equal to 0 by construction.

Normalization (NN): for every $\mathcal{V} \neq \mathcal{V}_0$, $I(\mathcal{V}) > I(\mathcal{V}_0) = 0$, where each and every element of $\mathcal{V}_0$ is characterised by $v_{it} = 0\ \forall i = 1,2, \ldots N; t = 1,2, \ldots T$.

As Proposition 3 states, we can normalise any mobility index satisfying the properties mentioned in Theorem 1:

Proposition 3: Any inequality index I consistent with the absolute Lorenz curve in equation 1 can be standardised in order to fulfil the normalisation property (NN).

Proof: Clearly $L^{\mathcal{V}^*}(p) = \frac{1}{NT}\sum_{j=1}^{pNT} v_j^*(p) = 0\ \forall p \in [0,1] \leftrightarrow v_j^*(p) = 0\ \forall p \in [0,1]$ because $v_1^*(p) \leq v_2^*(p) \leq \cdots \leq v_{NT}^*(p)$ and $\frac{1}{NT}\sum_{j=1}^{NT} v_j^*(p) = 0$. Therefore, for any share matrix $\mathcal{V} \neq \mathcal{V}_0$ (where $\mathcal{V}_0$ is defined in the statement of property NN above) it must be the case that $I(\mathcal{V}) > I(\mathcal{V}_0)$ due to the lack of crossing in their respective absolute Lorenz curves. Then, if $I(\mathcal{V}_0) = a > 0$, we could always redefine $I^*(\mathcal{V}) = I(\mathcal{V}) - a$, in order to get $I^*(\mathcal{V}_0) = 0$.

Sensitivity to loans

The property PR is useful to formally represent departures from complete immobility, but it is not directly relatable to specific forms of income transfers. Here we briefly explore the potential effects of a particular type of margin-preserving income transfer, which we call loans, on mobility as measured in our framework:

Loans: A loan between individuals i and j is a pair of transfers between two time periods t and τ, whereby at time $t < \tau$: $\widetilde{s_{it}} = s_{it} + \Delta$ and $\widetilde{s_{jt}} = s_{jt} - \Delta$, and then at time τ: $\widetilde{s_{i\tau}} = s_{i\tau} - \Delta$ and $\widetilde{s_{j\tau}} = s_{j\tau} + \Delta$ (where the tilded symbols refer to post-transfer variables).[6]

In other words, when $\Delta > 0$, a loan involves a transfer from j to i in period t followed by an equivalent reimbursement in period τ.[7] Since loans do not affect the margins, i.e.

[6] Note that Δ does not refer to a transfer in dollar terms but in percentage terms. In other words, Δ is the ratio of the dollar terms transfer over Y, the sum of incomes across individuals and across time.

[7] This is both conceptually and mathematically equivalent to two matching inter-temporal transfers involving i bringing Δ from the future to the present and j performing the opposite operation.

individuals' total incomes or total period incomes, the only gaps affected by the transfer will be the four stemming from intersecting individuals i and j with periods t and τ.

A loan could have different effects on mobility depending on initial conditions and the size of the loan. For example, with $\Delta > 0$, $\left(\widetilde{v_{jt}} - \widetilde{v_{it}}\right) = \left(v_{jt} - v_{it}\right) - 2NT\,\Delta$ and $\left(\widetilde{v_{i\tau}} - \widetilde{v_{j\tau}}\right) = \left(v_{i\tau} - v_{j\tau}\right) - 2NT\,\Delta$. Therefore, if $\left(\widetilde{v_{jt}} - \widetilde{v_{it}}\right) > 0$ and $\left(\widetilde{v_{i\tau}} - \widetilde{v_{j\tau}}\right) > 0$ then mobility will decrease, as the differences between the gaps of i and j have narrowed in both time periods. Intuitively, this example could reflect, inter alia, a situation in which the two period-specific inequality levels move closer (hence less structural mobility). But other situations could produce similar mobility reduction scenarios.

Alternatively, loans could also increase mobility, e.g. if $\left(v_{jt} - v_{it}\right) < 0$ and $\left(v_{i\tau} - v_{j\tau}\right) < 0$ with $\Delta > 0$. The a priori ambiguity of the net effect of loans on mobility makes sense because, depending on initial conditions and loan size, loans could, for instance, produce re-rankings among income shares in the same period that reduce the degree of association of ranks across time periods (i.e. increase exchange mobility) or the other way around (i.e. income shares becoming more similarly arranged in the terminology of Boland and Proschan, 1988). Likewise, loans could alter the degree of relative inequality of the two period distributions involved in ways that could reduce structural mobility (if the levels of inequality get closer to each other and to the other time periods') or the other way around.

2.5. Examples of multi-period mobility indices based on absolute inequality indices

As it turns out, any consistent absolute inequality indices including those classes identified by Lambert and Zheng (2011), as well as Chakravarty et al. (2013), may be used to measure multi-period mobility, as they fulfil the properties previously discussed. These indices include both rank-independent and rank-dependent families. Note that in defining these indices we omit the mean $\bar{v}$ because $\bar{v} = 0$. Here are some of the suitable indices:

Rank-dependent:

A class of generalised absolute Ginis:

$$R(\mathcal{V}^*) = -\frac{2}{(NT)^2}\sum_{j=1}^{NT}\alpha_j v_j^* \qquad (2),$$

where $\alpha_1 > \alpha_2 > \cdots > \cdots \alpha_{NT} > 0$. When $\alpha_j = (NT - j + 1)$ we get

$R(\mathcal{V}^*) = -\sum_{p=0}^{1} L^{\mathcal{V}^*}(p)$, i.e. the absolute Gini coefficient, which coincides with the area between absolute Lorenz curve and the horizontal axis. We can also consider the class of Chakravarty et al. (2013):

$$C_\theta(\mathcal{V}^*) = \frac{1}{\theta} ln[\left(\frac{1}{NT}\right)^2 \sum_{i=1}^{NT}\sum_{j=1}^{NT} exp\left(\theta|v_i^* - v_j^*|\right)] \qquad (3)$$

Rank-independent:

A family of generalized means:

$$M_\rho(\mathcal{V}) = \left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}|v_{it}|^\rho\right]^{\frac{1}{\rho}} \; \forall\rho > 1 \qquad (4)$$

An example of which is the variance:

$$\sigma^2(\mathcal{V}) = (M_2)^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(v_{it})^2 \qquad (5)$$

2.6. Connection to previous measurement proposals in the literature

2.6.1. The Shorrocks multi-period mobility indices

Shorrocks (1978) defined a mobility index M based on a Lorenz-consistent inequality index I:

$$M_{SHORROCKS} = 1 - \frac{I(Y_1,\ldots,Y_i,\ldots Y_N)}{\sum_{t=1}^{T} s_{.t} I\left(y_{1t},\ldots,y_{it},\ldots,y_{Nt}\right)} \qquad (6)$$

where $Y_i \equiv \sum_{t=1}^{T} y_{it}$. If we restrict the class I to that of scale-invariant indices then we can write (6) as:

$$M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t} I\left(s_{1t},\ldots,s_{it},\ldots,s_{Nt}\right) - I(s_{1.},s_{2.},\ldots s_{N.})}{\sum_{t=1}^{T} s_{.t} I\left(s_{1t},\ldots,s_{it},\ldots,s_{Nt}\right)}$$

$$M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t}\left[I\left(s_{1t,}\dots,s_{it},\dots,s_{Nt}\right) - I\left(s_{1.},s_{2.},\dots s_{N.}\right)\right]}{\sum_{t=1}^{T} s_{.t} I\left(s_{1t,}\dots,s_{it},\dots,s_{Nt}\right)} \quad (7)$$

Invoking the scale invariance property again we can further write:

$$M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t}\left[I\left(\frac{s_{1t,}}{s_{.t}}\dots,\frac{s_{it,}}{s_{.t}},\dots,\frac{s_{Nt,}}{s_{.t}}\right) - I\left(s_{1.},s_{2.},\dots s_{N.}\right)\right]}{\sum_{t=1}^{T} s_{.t} I\left(s_{1t,}\dots,s_{it},\dots,s_{Nt}\right)} \quad (8)$$

Finally we conclude that $M_{SHORROCKS} = 0$ if and only if there is table independence. That is, $M_{SHORROCKS}$ also considers table independence as the benchmark of complete immobility.

Regarding the differences between the approach proposed by Shorrocks (1978) and ours, we highlight that $M_{SHORROCKS}$ does not distinguish between tables characterized by a uniform distribution of lifetime shares, i.e. $s_{1.} = s_{2.} = \cdots = s_{N.}$. This is sensible in Shorrocks' framework given its interest in measuring mobility as equalization of lifetime incomes. Yet we can easily produce examples of pairs of tables sharing the same uniform column margin (lifetime shares) but differing in the level of inequality within their respective distributions of absolute gaps. Therefore, our approach will distinguish within the set of matrices characterized by equalized lifetime shares those whose gaps indicate further departure from table independence. As an example, Table 2 provides two sets of distributions, A and B, both characterized by $s_{i.} = 0.25\ \forall i = 1,\dots,4$. Clearly $M_{SHORROCKS}(A) = M_{SHORROCKS}(B) = 1$.

By contrast, not surprisingly, we find, using our mobility index R, that matrix A corresponds to a higher mobility than matrix B since $R(A) = 0.7083 > R(B) = 0.3458$.

Table 2. Two mobility tables

	Period 1A	**Period 2A**	**Period 3A**	**Period 1B**	**Period 2B**	**Period 3B**
Person 1	0.25	0.0	0.0	0.1	0.1	0.05
Person 2	0.0	0.25	0.0	0.15	0.0	0.1
Person 3	0.0	0.0	0.25	0.05	0.15	0.05
Person 4	0.25	0.0	0.0	0.2	0.0	0.05

2.6.2. The Maasoumi and Zandvakili mobility indices

These indices start from Shorrocks' idea of comparing inequality of lifetime incomes against a weighted sum of snapshot income inequality across several periods, but they differ in: (1) Explicitly using the generalized entropy family of inequality indices for I; (2) Using a generalized mean as a measure of lifetime income, i.e. $Z_i = \left[\sum_{t=1}^{T} a_t {y_{it}}^{\gamma}\right]^{\frac{1}{\gamma}}$, with $\sum_{t=1}^{T} a_t = 1$. With our notation and a few rearrangements, we can express the indices as:

$$M_{MZ} = 1 - \frac{\sum_{i=1}^{N}\left[\left(\frac{Z_i}{\bar{Z}}\right)^{\delta} - 1\right]}{\sum_{t=1}^{T} s_{.t} \sum_{i=1}^{N}\left[\left(\frac{Ns_{it}}{s_{.t}}\right)^{\delta} - 1\right]}, \delta \neq 0{,}1 \quad (9)$$

where $\bar{Z} = \frac{1}{N}\sum_{i=1}^{N} Z_i$. Thanks to scale invariance we can actually use

$Z_i = \left[\sum_{t=1}^{T} a_t {s_{it}}^{\gamma}\right]^{\frac{1}{\gamma}}$. It is then easy to show that $\frac{Z_i}{\bar{Z}} = Ns_{i.}$ if and only if there is table independence. Since $\frac{Ns_{it}}{s_{.t}} = Ns_{i.}$ under those same circumstances, then it follows naturally that $M_{MZ} = 0$ if and only if there is table independence. Hence M_{MZ} is also measuring mobility with complete immobility as the same benchmark. We can establish similar results and conclusions for the two Theil versions of M_{MZ}.

However, again, we can find pairs of distributions for which M_{MZ} would yield the same value, whereas our approach clearly ranks one distribution as featuring more mobility (as departure from table independence) than the other distribution. For example, consider the choice $a_1 = a_2 = ... = a_T$. Now consider distributions A and C in Table 3. In distribution C every row-individual has different positive entries, but every row is a time-column permutation of any other row-individual. Meanwhile in distribution A every individual enjoys positive income in only one period. Moreover, all individuals enjoy that same income (albeit in different periods in order to render all time margins positive). Then, clearly $Z_1 = Z_2 = \cdots = Z_N$ and $\sum_{t=1}^{T} s_{.t} \sum_{i=1}^{N}\left[\left(\frac{Ns_{it}}{s_{.t}}\right)^{\delta} - 1\right] > 0$ in both A and C. Therefore: $M_{MZ}(A) = M_{MZ}(C) = 1$. By contrast, our mobility indices will indicate that the degree of mobility in matrix A is higher than that corresponding to matrix C; indeed, for instance: $R(A) = 0.7083 > R(C) = 0.3133$.

Table 3. Two other mobility tables

	Period 1A	Period 2A	Period 3A	Period 1C	Period 2C	Period 3C
Person 1	0.25	0.0	0.0	0.15	0.08	0.02
Person 2	0.0	0.25	0.0	0.15	0.02	0.08
Person 3	0.0	0.0	0.25	0.08	0.15	0.02
Person 4	0.25	0.0	0.0	0.02	0.15	0.08

2.6.3. The mobility indices of Tsui (2009)

Tsui (2009) derived a multi-period income mobility index which, in our notation is expressed as:

$$M_{TSUI} = \frac{\rho}{N}\sum_{i=1}^{N}\left[\prod_{t=1}^{T}\left(\frac{Ns_{it}}{s_{.t}}\right)^{c_t} - 1\right] \quad (10)$$

where ρ and c_t are parameters.[8]

We note that only under independence: $\frac{s_{it}}{s_{.t}} = s_{.i}\ \forall(i,j,t)$. Then, clearly, for $c_t \neq 0$, $M_{TSUI} = 0$ if and only if $s_{it} = \frac{1}{NT}\ \forall i,t$, i.e. if all the shares are equal to each other. While this situation would certainly qualify as one of table independence, it is not the only one that would. Therefore, other situations of table independence, e.g. any in which $s_{it} = s_{i.}s_{.t}$, will not minimize the value of M_{TSUI}. Hence this index does not set table independence generally as its benchmark of complete immobility. Implicitly, M_{TSUI} considers any common growth factor between two periods as a source of mobility. By contrast, in our proposed framework, if the only difference between all snapshot income distributions is a common growth factor, i.e. a multiplication in period 2 of each period 1 income by the same positive scalar, then we are in a situation of complete immobility and table independence.

[8] See Tsui (2009) for more details on the choice of these parameters.

Moreover, again, we can find pairs of distributions for which M_{TSUI} would yield the same value, whereas our approach clearly ranks one distribution as featuring more mobility as departure from table independence than the other distribution. For example, note that M_{TSUI} yields the same value for all tables characterized by rows in which every individual has at least one null income, i.e. $\forall N : \exists t | s_{it} = 0$.

Now consider distributions A and D in Table 4.

In distribution D every individual has zero income in one period. Meanwhile in distribution A every individual enjoys positive income in only one period. Therefore: $M_{TSUI}(A) = M_{TSUI}(D) = -3$. By contrast our mobility indices will indicate that the degree of mobility in matrix A is higher than that corresponding to matrix D; indeed, for instance: $R(A) = 0.70833 > R(D) = 0.3970$.

Table 4. A third illustration of mobility tables

	Period 1A	**Period 2A**	**Period 3A**	**Period 1D**	**Period 2D**	**Period 3D**
Person 1	0.25	0.0	0.0	0.0	0.1	0.1
Person 2	0.0	0.25	0.0	0.15	0.0	0.1
Person 3	0.0	0.0	0.25	0.1	0.15	0.0
Person 4	0.25	0.0	0.0	0.2	0.0	0.1

In summary, even though some of the proposals from the literature agree with ours on certain key axioms (mainly NN, i.e. hitting a value of zero if and only if there is complete immobility, which is satisfied by the proposals of both Shorrocks, and Maasoumi-Zandvakili), the three proposals are inconsistent with our measurement framework. This should not come as a major surprise, or be deemed an indictment on the previous literature, since none of the reviewed contributions had as its stated purpose the measurement of mobility as departure from table independence.

3. An empirical application: multi-period mobility in European countries

3.1.Data description

We use income data from the EU-SILC study, which was launched in 2003. In the first year, however, it covered only 6 countries. In subsequent years, the number of countries underwent a gradual increase. Thus, currently, it is carried out in all member states of the European Union and several European countries outside the EU, including Switzerland, Norway and Turkey. However, our analysis will concentrate only on European Union countries.[9]

In most countries, households participating in the EU-SILC are surveyed on the basis of four-year rotational panels. This means that each year about one-fourth of the whole sample is replaced by a new group of households. As a consequence of such sampling design, panel data are only available for periods no longer than 4 years.

Due to the low number of countries participating in the EU-SILC survey at the beginning, income mobility analysis was performed for selected countries of the European Union for the period 2005-2012. This period includes two non-overlapping 4-year sub-periods: 2005-2008 and 2009-2012. However, since we have a rotating panel, it is possible to carry out the analysis for 4-year periods which partially overlap. This allows for a more detailed assessment of the impact of the data coming from consecutive rounds of EU-SILC study.

We used data on nominal income of individuals in households (variable PY010G – gross employee cash or near cash income) available in the longitudinal personal data file – full samples, representative for populations of analysed countries. This income was recorded for all current household members aged 16 and above (for details see Description of target variables, 2008; and more recent documents for consecutive years). Income values are expressed in Euros, which means that for countries outside the euro zone income levels have been converted at current exchange rates.

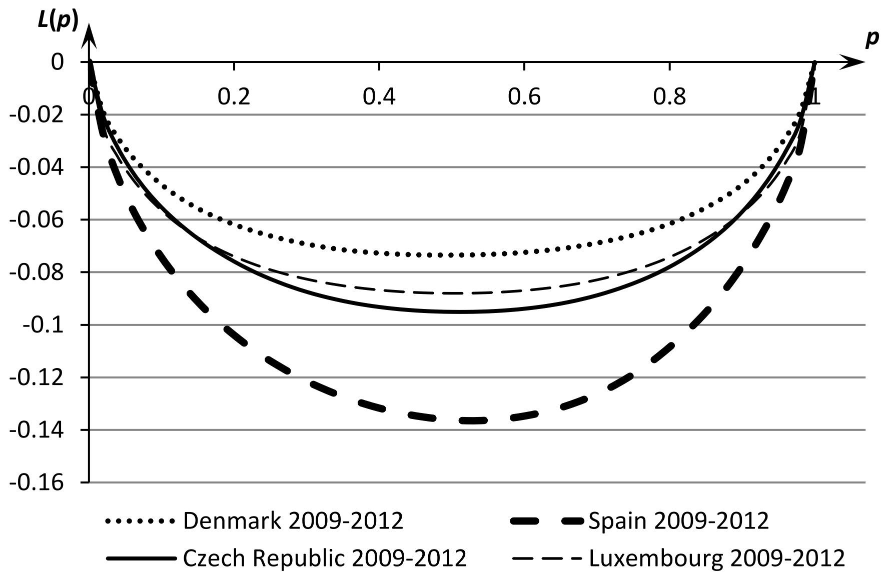
[9] Data for some countries (including Germany) was not available in the dataset shared by LISER (during the INGRiD research stay) which served as a basis for the empirical analysis.

3.2. Results

According to Theorem 1 we can rank countries in terms of income mobility on the basis of the absolute-Lorenz-consistency condition. If $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p |\ L^A(p) < L^B(p)$ for countries A and B, respectively, then all proposed indices will judge income in country A to be relatively more mobile than income in country B.

To illustrate this relationship, absolute Lorenz curves for selected countries are presented in Figure 2.

Figure 2. Absolute Lorenz Curves for Denmark, Czech Republic, Luxembourg and Spain in 2009-2012



The curves in Figure 2 indicate that Denmark is robustly the least mobile country since its curve "dominates" those of the other three countries (Luxembourg, the Czech Republic and Spain). On the other hand, Spain is the most mobile country and its curve is "dominated" by those of the Czech Republic, Luxembourg and Denmark. The curves of the Czech Republic and Luxembourg cross so that the comparative assessment of their income mobility depends on the choice of the mobility index.

In what follows we use the absolute Gini coefficient for the assessment of income mobility. This index can then be interpreted as the expected value of the absolute differences between all the elements of the matrix $\mathcal{V}$.

The detailed results of the assessment of income mobility levels are shown in Table 5. It appears that on average the level of income mobility is higher among the new EU member states (states which joined the European Union after January 1, 2004). The average level of income mobility among the old and new EU members is illustrated in Figure 3.

Table 5: Income mobility in selected European Union countries

Country	Income mobility in the following periods 2005-2008	2006-2009	2007-2010	2008-2011	2009-2012	Country characteristics
Austria	0.196	0.222	0.228	0.208	0.171	Euro zone*
	(0.006)	(0.008)	(0.014)	(0.007)	(0.006)	
Belgium	0.139	0.187	0.183	0.182	0.151	Euro zone *
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	
Bulgaria		0.289	0.249	0.233	0.205	New EU country**
		(0.009)	(0.008)	(0.006)	(0.006)	
Cyprus	0.124	0.119	0.123	0.127	0.116	New EU country**
	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	
Czech Republic	0.170	0.171	0.166	0.181	0.149	New EU country**
	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	
Denmark	0.103	0.123	0.114	0.115	0.118	
	(0.004)	(0.005)	(0.004)	(0.004)	(0.005)	
Estonia	0.210	0.243	0.238	0.216	0.206	New EU country**
	(0.011)	(0.007)	(0.007)	(0.006)	(0.006)	
France	0.139	0.184	0.180	0.169	0.135	Euro zone *
	(0.003)	(0.003)	(0.005)	(0.004)	(0.002)	
Greece		0.139	0.162	0.181	0.208	Euro zone *
		(0.008)	(0.006)	(0.007)	(0.008)	
Hungary	0.226	0.248	0.219	0.230	0.191	New EU country**
	(0.007)	(0.007)	(0.006)	(0.006)	(0.005)	
Italy		0.171	0.180	0.199	0.172	Euro zone *
		(0.004)	(0.004)	(0.004)	(0.019)	
Latvia		0.212	0.228	0.240	0.221	New EU country**
		(0.008)	(0.007)	(0.007)	(0.007)	

Lithuania	0.177	0.182	0.223	0.219	0.207	New EU country**
	(0.007)	(0.006)	(0.007)	(0.008)	(0.006)	
Luxembourg	0.141	0.169	0.173	0.160	0.143	Euro zone *
	(0.004)	(0.011)	(0.010)	(0.004)	(0.004)	
Malta		0.187	0.160	0.164	0.129	New EU country**
		(0.008)	(0.007)	(0.007)	(0.006)	
Poland	0.207	0.226	0.219	0.219	0.191	New EU country**
	(0.004)	(0.004)	(0.006)	(0.005)	(0.004)	
Portugal		0.198	0.198	0.210	0.163	Euro zone *
		(0.009)	(0.008)	(0.009)	(0.006)	
Romania			0.161	0.132	0.131	New EU country**
			(0.005)	(0.004)	(0.005)	
Slovakia	0.204	0.194	0.186	0.181	0.173	New EU country**
	(0.007)	(0.005)	(0.005)	(0.006)	(0.006)	
Slovenia	0.148	0.163	0.152	0.156	0.136	New EU country**
	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	
Spain	0.188	0.225	0.212	0.212	0.190	Euro zone *
	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	
Sweden	0.137	0.140	0.139	0.145		
	(0.004)	(0.005)	(0.004)	(0.005)		
United Kingdom	0.186	0.210	0.211	0.234	0.187	
	(0.011)	(0.006)	(0.007)	(0.010)	(0.008)	

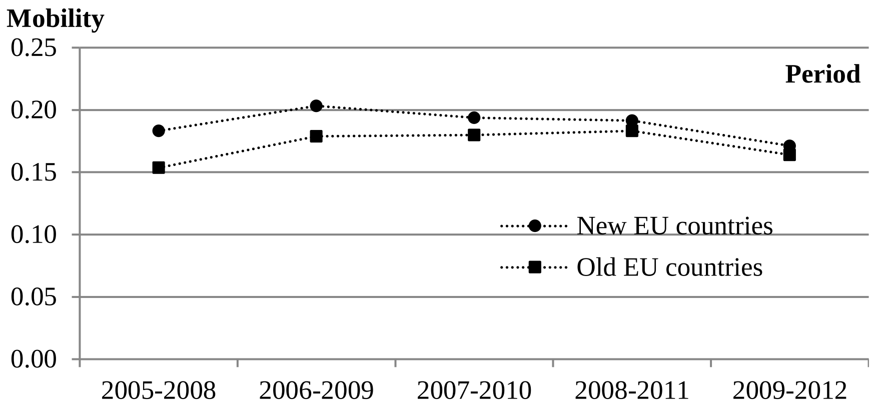
Estimated standard errors in parentheses (based on 1,000 bootstrap samples)

* Countries belonging to the euro zone before January 1, 2005.

** Countries that joined the European Union after January 1, 2004.

Source: own calculations

Figure 3: A*verage income mobility in "Old" and "New" European Union* countries



Although the differences between the old and the new EU gradually decreased over time, income mobility is systematically higher in the new EU countries. One may think of various reasons for such a higher level of mobility. Firstly, these new EU countries are characterized by a lower average level of income and, at the same time, a generally higher rate of economic growth. In conjunction with the continued process of economic transformation, such a combination may lead to major changes in relative incomes and a lower stability. Another factor which could play an important role in income mobility assessment is a floating exchange rate of national currencies. During the financial crisis, currencies of the new EU countries were significantly devaluated, and this affected the relative incomes (in Euros). Poland is a good illustration. Despite a positive rate of GDP growth and increasing average wages (as expressed in national currency), the average income in Euro terms declined between 2009 and 2010. Detailed information on average income levels in the analysed countries is presented in Table D-1 in Appendix D.

Data on changes in average income levels will also help in discussing the second noteworthy issue which concerns the impact of the financial crisis (which began in 2008) on the level of income mobility in the various countries. Among countries particularly affected by the crisis we can mention Greece, Spain and Portugal.

We do not add to this group other countries, especially Latvia and Lithuania, even though the impact of the crisis on income levels was also very serious in their case. In these countries, however, an additional factor influencing the change in average income was the exchange rate. To neutralize the role played by this factor, the analysis concentrates on countries which belonged to the euro zone at the beginning of the period (January 1, 2005). The results are presented in Figure 4.

Figure 4: Average income mobility in Greece, Portugal and Spain and other euro zone countries

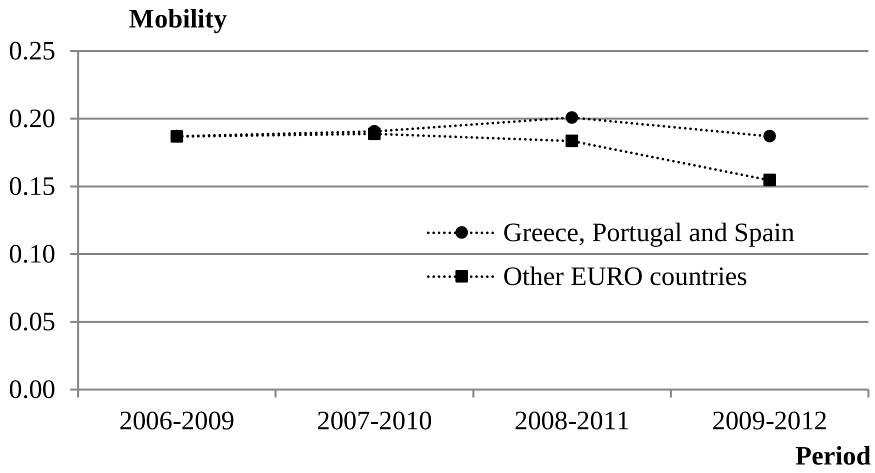


Figure 4 shows that while initially Greece, Portugal and Spain had levels of income mobility similar to those of other Euro countries, in subsequent years the trends were different. The relatively high average level of income mobility gradually decreased in the group of other countries, while remained high in Greece, Portugal and Spain. In these latter countries, the crisis resulting from the significant level of public debt led to budgetary adjustments. The consequences of these adjustments were observed in the following years, in terms of both income levels and mobility. The higher levels of income mobility observed in Greece, Portugal and Spain suggest a lack of stability and income insecurity (like a higher risk of losing a job or bankruptcy).

4. Concluding comments

Although some suggestions have been made in the past to measure multi-period income mobility, most studies of income mobility, in particular those with an empirical analysis, considered only two periods. Initially, the basic idea of the approach proposed in the present paper was to compare "expected" with "actual" income shares. A typical "actual" share would refer to the income share of some individual at a given time in the total income of all individuals over the whole period analysed. The corresponding "expected" share would be the hypothetical income share in the total income of society over the whole accounting period that an individual would have had at a given time, had there been complete independence between the individuals and the time periods.

Previous proposals of multi-period mobility in the literature also identified the benchmark of complete immobility with independence between individuals and time periods, often implicitly. However, as we showed in the paper, these approaches, unlike our proposal, measure, explicitly or implicitly, alternative notions of mobility, different from our concept of mobility as departure from independence between the individuals and the time periods.

A thorough examination of such an approach based on shares' comparisons showed, however, that one should be more careful, and that a more appropriate way of consistently measuring multi-period mobility should focus on the absolute rather than the traditional (relative) Lorenz curve and that the relevant variable to be accumulated should be the difference between the "a priori" and "a posteriori" shares previously defined. Moving from an ordinal to a cardinal approach to measuring multi-period mobility, we then proposed classes of mobility indices based on absolute inequality indices. For the sake of simplicity, we only used one index in the empirical illustration of our paper, the one which is directly related to the absolute Gini index.

The empirical analysis seems to have vindicated our approach because it clearly showed that income mobility was higher in the new EU countries (those that joined the EU in 2004 and later). We also observed that income mobility after 2008 was higher in three countries that were particularly affected by the financial crisis: Greece, Portugal and Spain.

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Appendix A: Proof of Proposition 1 and of Theorem 1.

Proof of Proposition 1:

Sufficiency: if $y_{it} = k_t y_i$ then: $s_{it} = \frac{k_t y_i}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]}$, $s_{i.} = \frac{y_i \sum_{t=1}^{T} k_t}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]} = \frac{y_i}{\sum_{i=1}^{N} y_i}$ and $s_{.t} = \frac{k_t \sum_{i=1}^{N} y_i}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]} = \frac{k_t}{\sum_{t=1}^{T} k_t}$. Then clearly: $s_{it} = s_{i.} s_{.t}$.

Necessity: if $s_{it} = s_{i.} s_{.t}$, then: $\frac{y_{it}}{Y} = \frac{\sum_{t=1}^{T} y_{it}}{Y} \frac{\sum_{i=1}^{N} y_{it}}{Y}$, which leads to: $y_{it} = \frac{\sum_{t=1}^{T} y_{it} \sum_{i=1}^{N} y_{it}}{Y}$. Setting $y_i = \sum_{t=1}^{T} y_{it}$ and $k_t = \frac{\sum_{i=1}^{N} y_{it}}{Y}$, it is clear to see that independence requires y_{it} to be of the form $k_t y_i$.

Proof of Theorem 1:

(ii)→(i): Since I satisfies PI and TS, if A' is obtained from A through a series of permutations of rows and columns then we get $I[A] = I[A']$. Since I satisfies TPP, then if A'' is obtained from A' through a replication of its elements we get $I[A'] = I[A'']$. Since I satisfies PR, then if B is obtained from A'' through a series of progressive transfers among gaps we get $I[A''] > I[B]$. Therefore $I[A] = I[A'] = I[A''] > I[B]$.

(iii)→(ii): Let $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p \in [0,1] |\ L^A(p) < L^B(p)$, with A having N_A individuals and T_A time periods, and B having N_B individuals and T_B time periods. Let B' be obtained by replicating the rows of B N_A times and the columns of B T_A times. Since $v_{it}^{B'} = N_A N_B T_A T_B \left(\frac{s_{it}^B - w_{it}^B}{N_A T_A}\right) = N_B T_B (s_{it}^B - w_{it}^B) = v_{it}^B$, then we get $L^{B'}(p) = L^B(p)$ for all p. Therefore $L^A(p) \leq L^{B'}(p)\ \forall p \in [0,1]$ and $\exists p \in [0,1] |\ L^A(p) < L^{B'}(p)$. Hence, we need to show now that we can obtain B' from A through permutations, replications and progressive transfers among gaps. Permutations do not matter much because the absolute Lorenz curves that we use order all gaps ascendingly. If we replicate the rows of A, N_B times and the columns of A T_B times then we get A', which has the same number of gap elements as B'. Plus, we know that

$L^{A'}(p) = L^{A}(p)$ for all p, and $L^{A'}(1) = L^{B'}(1)$. Let $\delta = \frac{1}{N_A N_B T_A T_B} v_1^*(A') \leq 0$, i.e. $v_1^*(A')$ is the lowest, most negative gap in matrix A'. Then it must be true that $L^{A'}(p) \leq L^{B'}(p)\ \forall p \in [0,1]$ and $\exists p \in [0,1] |\ L^{A'}(p) < L^{B'}(p)$ if and only if $0 \leq L^{A'}(p) - (pNT)\delta \leq L^{B'}(p) - (pNT)\delta\ \forall p \in [0,1]$ and $\exists p \in [0,1] |\ 0 \leq L^{A'}(p) - (pNT)\delta < L^{B'}(p) - (pNT)\delta$, where $N = N_A N_B$ and $T = T_A T_B$. Therefore, since $L^{A'}(1) - NT\delta = L^{B'}(1) - NT\delta > 0$, we can apply Muirhead's theorem (as rendered by Marshall et al., 2011, pp. 7-8) to conclude that B' can be obtained from A' through a finite sequence of progressive transfers among gaps.[10]

(i)→(iii): We prove by contradiction. We know that the first statement implies the third if and only if the negation of the third statement implies the negation of the first statement. Therefore, we start by assuming that $\exists p' \in [0,1] |\ L^{A}(p') > L^{B}(p')$, i.e. the contradiction of statement (iii). Then we must (and it suffices to) find at least one mobility index I satisfying the properties listed in statement (i) such that $I[A] < I[B]$; namely, the contradiction of statement (i). Perhaps the easiest choice is the absolute Gini coefficient $R(\mathcal{V}^*) = -\sum_{p=0}^{1} L^{\mathcal{V}^*}(p)$, i.e. the area between absolute Lorenz curve and the horizontal axis. Let $\Delta R \equiv R(A) - R(B) = \sum_{p=0}^{1}[L^{B}(p) - L^{A}(p)]$. Then, clearly, if $\exists p' \in [0,1] |\ L^{A}(p') > L^{B}(p')$ then there is no guarantee that $\Delta R > 0$ for any possible pair of distributions, as $L^{B}(p') - L^{A}(p') < 0$ could be too large (to the point of being greater in absolute value than the sum of all the other Lorenz-curve gaps even if $L^{A}(p) \leq L^{B}(p)\ \forall p \neq p'$). That is, $\Delta R > 0$ for any possible pair of distributions if and only if statement (iii) holds. Therefore if statement (i) holds, then statement (iii) must hold as well.

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[10] Let $A = (a_1, a_2, \dots, a_n)$ such that $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n$. Same for B with $0 \leq b_1 \leq b_2 \leq \cdots \leq b_n$. Now we say that A majorizes B if and only if $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} b_i$ for all $k = 1,2, \dots n-1$ and $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$. Then Marshall et al. (2011, pp. 7-8) show that A majorizes B if and only if B can be obtained from A through a sequence of rank-preserving progressive transfers (i.e. Pigou-Dalton transfers). (In turn these conditions are equivalent to $B = AD$ where D is a doubly stochastic matrix; i.e. a square matrix characterized by non-negative entries and all rows and columns adding up to one.) So, in the proof we have subtracted δ from every gap element added in $L(p)$ for every p. That way $L(p) - pNT\delta$ is akin to the sum $0 \leq \sum_{i=1}^{k} a_i$. Likewise $L^{A'}(1) - NT\delta = L^{B'}(1) - NT\delta > 0$ resembles $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$. Hence A' majorizes B', which implies that B' can be obtained from A' through a sequence of Pigou-Dalton transfers (bearing in mind that both have an equal number of elements $NT = N_A N_B T_A T_B$).

Appendix B: Simple illustrations[11]

Consider the following simple income matrices:

$$y_A = \begin{pmatrix} 100 & 50 & 25 \\ 50 & 25 & 100 \\ 25 & 100 & 50 \end{pmatrix}$$

$$y_B = \begin{pmatrix} 100 & 200 & 400 \\ 50 & 50 & 50 \\ 25 & 25 & 25 \end{pmatrix}$$

$$y_C = \begin{pmatrix} 100 & 200 & 400 \\ 50 & 100 & 200 \\ 25 & 50 & 100 \end{pmatrix}$$

Each matrix represents a different component of distributional change:

− y_A represents exchange mobility (a re-ranking of individuals over time),

− y_B represents structural mobility (an increase in relative inequality over time due to an income growth for the richest person), and

− y_C represents income growth at equal rates for all individuals over time.

The proposed approach to multi-period mobility involves constructing a NxT matrix with typical element, $v_{ij} \equiv NT(s_{it} - w_{it})$, where $s_{it} = y_{it}/\Sigma_i\Sigma_t\, y_{it}$ is the (observed) income of individual i in period t as a share of the total income of all individuals across all time periods and w_{it} = $[\Sigma_i y_{it}/\Sigma_i\Sigma_t\ y_{it}][\Sigma_t y_{it}/\Sigma_i\Sigma_t\ y_{it}]$ is the expected income share of individual i in period t (the total income of all individuals in period t as a share of the total income of all the individuals across all periods times the total income of individual i in all periods as a share of the total income of all individuals across all periods).

- For y_A, $v_{it} \neq 0$, $\forall i$ = 1,2,…, N and $\forall t$ = 1,2,…, T. If we order these values from smallest to largest and use them to construct an absolute Lorenz curve, it will

[11] We are very thankful to an anonymous referee who provided us with these three illustrations. This Appendix is an almost exact transcription of his/her comment.

lie below the horizontal axis (the smallest values of vit are negative), because there is exchange mobility.

- For y_B, $v_{it} \neq 0$, $\forall i = 1,2,\ldots, N$ and $\forall t = 1,2,\ldots, T$. If we order these values from smallest to largest and use them to construct an absolute Lorenz curve, it will lie below the horizontal axis, because there is structural mobility.
- For y_C, $v_{it} = 0$, $\forall i = 1,2,\ldots, N$ and $\forall t = 1,2,\ldots, T$. If we order these values from smallest to largest and use them to construct an absolute Lorenz curve, it will coincide with the horizontal axis. Here, we would conclude that there is no mobility. Note that this case is covered by Proposition 1, as $k_t = 2^t$ in $\mathcal{Y}c$.

The case represented by y_C will never arise in practice, because the incomes of all individuals never grow at the same rate. Any deviation from a uniform growth rate will result in mobility – of the structural variety for sure, and possibly the exchange variety. For example, if we change y_C by letting $y_{23} = 100$, it will no longer be the case that $v_{it} = 0$, $\forall i = 1,2,\ldots, N$ and $\forall t = 1,2,\ldots, T$, because this change introduces structural mobility.

Appendix C: Proof of Proposition 2

Let the gaps of table $-A$ be obtained from A by multiplying each of its elements by -1. Same for tables B and $-B$. We need to prove that $L^{A}(p) \leq L^{B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^{A}(p) < L^{B}(p)$ if and only if $L^{-A}(p) \leq L^{-B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^{-A}(p) < L^{-B}(p)$. Then, if inequality index I ranks consistently with the absolute Lorenz curve, it should be the case that $I[A] > I[B]$ if and only if $I[-A] > I[-B]$.

Note that both $L^{A}(p)$ and $L^{-A}(p)$ rely on the same gaps. The difference being that the ordered sequence of gaps in A is the exact opposite of the ordered sequence of gaps in $-A$. Hence $L^{A}(p) = L^{-A}(1-p)$ and $L^{B}(p) = L^{-B}(1-p)$. Then it clearly follows that: $L^{A}(p) \leq L^{B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^{A}(p) < L^{B}(p)$ if and only if $L^{-A}(1-p) \leq L^{-B}(1-p)\ \forall p \in [0,1]$ and $\exists p|\ L^{-A}(1-p) < L^{-B}(1-p)$, which is tantamount to $L^{-A}(p) \leq L^{-B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^{-A}(p) < L^{-B}(p)$.

Appendix D: Table D-1: Average nominal personal income

Country	Average nominal personal gross income in consecutive years [EUR]							
	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Austria	21033	19761	20234	21228	21549	22709	23031	25593
Belgium	24201	23870	23653	24017	25809	26626	27565	28822
Bulgaria		1183	1353	2251	2819	2682	2861	2898
Cyprus	13547	12063	12683	13289	14621	16130	18069	20443
Czech Republic	4714	5146	5810	6561	7961	7479	7980	8710
Denmark	28971	27432	29091	30617	32459	33087	33859	35140
Estonia	4402	4292	4533	5935	6887	6211	6664	7338
France		17983	18193	19326	19840	20170	21058	21944
Greece			13470	14525	15435	14906	13930	12373
Hungary	3832	4166	4342	4811	5107	4658	4953	5105
Italy			16822	17036	17484	16984	17556	17551
Latvia			3538	5346	6522	5136	4995	5279
Lithuania	3017	3540	4507	5328	6009	4786	4295	5327
Luxembourg	35797	36130	35923	36543	29833	33199	37391	43045
Malta		7223	8603	9267	12121	12622	14112	14565
Poland	3485	4299	5014	6001	7216	6068	6791	6970
Portugal			10059	10757	11179	11380	11350	11059
Romania			2672	3399	3689	3302	3345	3471
Slovakia	3031	3341	3913	4775	5838	5974	6320	6630
Slovenia	10032	9671	10043	10868	12267	12698	13406	13829
Spain	14609	14460	14452	15432	15534	14799	14703	14358
Sweden	21509	21537	22392	23387	22904	20859	24213	
United Kingdom	27966	27859	29546	26332	22893	24260	23997	25107

Source: own calculations