White Rose Research Online

eprints@whiterose.ac.uk
https://eprints.whiterose.ac.uk
Universities of Leeds, Sheffield and York

Deposited via The University of York.

White Rose Research Online URL for this paper:
https://eprints.whiterose.ac.uk/id/eprint/158217/

Version: Published Version

Article:
Hou, Aijun, Wang, Weining, Chen, Cathy et al. (2020) Pricing cryptocurrency options. Journal of Financial Econometrics. pp. 250-279. ISSN: 1479-8409

https://doi.org/10.1093/jjfinec/nbaa006

Reuse
This article is distributed under the terms of the Creative Commons Attribution (CC BY) licence. This licence allows you to distribute, remix, tweak, and build upon the work, even commercially, as long as you credit the authors for the original work. More information and the full terms of the licence here: https://creativecommons.org/licenses/

Takedown
If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.

UNIVERSITY OF LEEDS
University of Sheffield
UNIVERSITY of York

Journal of Financial Econometrics, 2020, Vol. 18, No. 2, 250–279
doi: 10.1093/jjfinec/nbaa006
Article

OXFORD

Pricing Cryptocurrency Options*

Ai Jun Hou[1], Weining Wang[2,4], Cathy Y.H. Chen[3,4] and Wolfgang Karl Härdle[4,5,6]

[1]Stockholm University, [2]University of York, [3]University of Glasgow, [4]Humboldt-Universität zu Berlin, [5]Singapore Management University and [6]Charles University in Prague

Address correspondence to Ai Jun Hou, Stockholm Business School, Stockholm University, Kräftriket 5, 106 91 Stockholm, Sweden, or email: aijun.hou@sbs.su.se

Received January 14, 2018; revised March 9, 2020; editorial decision March 26, 2020; accepted April 1, 2020

Abstract

Cryptocurrencies (CCs), especially bitcoin (BTC), which comprises a new digital asset class, have drawn extraordinary worldwide attention. The characteristics of the CC/BTC include a high level of speculation, extreme volatility and price discontinuity. We propose a pricing mechanism based on a stochastic volatility with a correlated jump (SVCJ) model and compare it to a flexible cojump model by Bandi and Renò (2016). The estimation results of both models confirm the impact of jumps and cojumps on options obtained via simulation and an analysis of the implied volatility curve. We show that a sizeable proportion of price jumps is significantly and contemporaneously anticorrelated with jumps in volatility. Our study comprises pioneering research on pricing BTC options. We show how the proposed pricing mechanism underlines the importance of jumps in CC markets.

Key words: CRIX, bitcoin, cryptocurrency, SVCJ, option pricing, jumps

JEL classification: C32, C58, C52

Bitcoin (BTC), the network-based decentralized digital currency and payment system, has garnered worldwide attention and interest since it was first introduced in 2009. The rapidly growing research related to BTC shows a prominent role in this new digital asset class in

* We thank Bandi and Renò (2016) for providing us with the codes used to implement the BR cojumps model in this study. We acknowledge the helpful comments from the editor and two anonymous referees. We thank Xiaohao Ji for help with data processing. This research is supported by the Deutsche Forschungsgemeinschaft through the International Research Training Group 1792 "High Dimensional Nonstationary Time Series" (http://irtg1792.hu-berlin.de). In addition, it has been funded by the Natural Science Foundation of China (fund number 71528008). A.J.H. acknowledges the financial support from the Jan Wallender and Tom Hedelius Foundation of Handelsbanken (P2019-0264). W.K.H. acknowledges the financial support from the Czech Science Foundation.

© The Author(s) 2020. Published by Oxford University Press.
This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted reuse, distribution, and reproduction in any medium, provided the original work is properly cited.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

contemporary financial markets.[1] Several studies have suggested econometric methods to model the dynamics of BTC prices, including cross-sectional regression models involving the major traded cryptocurrencies (CCs) and also multivariate time series models.[2] Scaillet, Treccani, and Trevisan (2020) show that jumps are much more frequent in the BTC market than, for example, in the U.S. equity market (see, e.g., Eraker, 2004; Bajgrowicz, Scaillet, and Treccani, 2015; Bandi and Renò, 2016 among others). These earlier studies suggest that jumps should be considered when modeling BTC prices.

However, research on the BTC derivative markets is still limited despite the rapidly growing availability of BTC futures and options traded on an unregulated exchange platform (i.e., Deribit). Especially, the Chicago Mercantile Exchange (CME) Group, the world's leading derivatives marketplace, launched BTC futures based on the CME CF BTC Reference Rate (BRR) on December 18, 2017. The limited research on pricing and hedging BTC derivatives is partly attributed to the fact that pricing BTC derivatives (e.g., options) encounters econometric challenges from the extraordinary occurrence of jumps as this market is unregulated, lacks of central settlement and is highly speculation driven. This calls for a more flexible model to capture the sudden jumps appearing in both the returns and variance processes.

In this article, we contribute to the existing literature by exploring the stochastic and econometric properties of BTC dynamics and then pricing the BTC options based on these properties. The investigation is carried out by using the most advanced stochastic volatility (SV) models, that is, the stochastic volatility with a correlated jump (SVCJ) model of Duffie, Pan, and Singleton (2000) and the SV with the possible nonlinearity structure of Bandi and Renò (2016)(BR hereafter). The employed SVCJ model incorporates jumps in both returns and the SV process, while the BR model captures the possible nonlinearity of return and variance processes and characterizes a nonaffine structure. We aim to provide a theoretical foundation for the future development of derivative markets on CCs.

Numerous empirical studies have applied the SVCJ model in different markets. For example, Eraker, Johannes, and Polson (2003) and Eraker (2004) use the SVCJ model to describe equity market returns and estimate equity option pricing. They find strong evidence of jumps in returns and volatility in the U.S. equity market. We further compare the SVCJ estimates to the simplified versions such as Bates (2000; SVJ hereafter) and the SV model.

For the purpose of robustness check, we compare our results with those from the BR model. Bandi and Renò (2016) propose a price and variance cojump model that generalizes the SVCJ model to capture the possible nonlinearity in the parameters of the returns and variance processes. The BR model characterizes independent and correlated jumps and allows for a nonparametric parameter structure, and estimates the parameters by using

1 See, for example, Becker et al. (2013), Segendorf (2014), Dwyer (2015), also studies on economics (Kroll, Davey, and Felten, 2013), alternative monetary systems (Rogojanu and Badea, 2014; Weber, 2016), and financial stability (Ali, 2014; Badev and Chen, 2014; ECB, 2015). An analysis of the legal issues involved in using BTC can be found in Elwell et al. (2013).

2 For example, Hayes (2017) performs a regression using a cross-section dataset consisting of sixty-six traded digital currencies to understand the price driver of CCs. Kristoufek (2013) proposes a bivariate vector autoregression model for the weekly log returns of BTC prices. Bouoiyour, Selmi and Tiwari (2015) investigates the long- and short-run relationships between BTC prices and other related variables using an autoregressive distributed lag model.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

high-frequency data. We also apply this model to the dynamics of BTC. We base our option pricing on an experimental simulation where the parameters used to execute a simulation are from the SVCJ and BR model, respectively.

We summarize our main empirical findings as follows. *First*, as in the existing literature, the results from the SVCJ and BR models indicate that jumps are present in the returns and variance processes and adding jumps to the returns and volatility improves the goodness of fit. *Second*, in contrast to existing studies that commonly report a negative leverage effect, we find that the correlation between the return and volatility is significantly positive in the SVCJ model. However, we cannot find significant negative relations between risk and return in the BR model. This implies that a rise of price is not associated with a decrease in volatility, which is consistent with the "inverse leverage effect" found in the commodity markets (Schwartz and Trolled, 2009).

Third, we find that the jump size in the return and variance of BTC is anticorrelated. The parameter estimates of the jump size (ρ_j) from both the SVCJ and BR models are negative (though the SVCJ estimate is insignificant). It is worth noting that the correlation between the price jump size and the volatility jump size turns out to be significant with a negative coefficient with high-frequency data, while tending to be insignificant for the SVCJ fitting using daily prices. This finding is in line with existing studies of the stock market from Eraker (2004), Duffie, Pan, and Singleton (2000), and Bandi and Renò (2016), among others. For example, Bandi and Renò (2016) report an anticorrelation with the nonaffine structure. Eraker (2004) finds a negative correlation between jump size only when augmenting return data with options data, and the negative correlation between cojump size being identified in the implied volatility (IV) smirk. Using high-frequency data, Jocod and Todorov (2009) and Todorov and Tauchen (2010) also report that the large jump size of prices and volatility are strongly anticorrelated.

Finally, we observe that the option price level is prominently dominated by the level of volatility and therefore overwhelmingly affected by jumps in the volatility processes. The results from the plots of IV indicate that adding jumps in the return increases the slope of the IV curves. The greater steepness of the IV curve can be strengthened by the presence of jumps in volatility. The presence of cojumps enlarges the IV smile further. As evidenced from the IVs curve, options with a short time to maturity are more sensitive to jumps and cojumps. To fulfill a hedge or speculation need from institutional investors, we replicate the entire analysis for the CRyptocurrency IndeX (CRIX), a market portfolio comprising leading CCs (see more detail in www.thecrix.de). A recent volatility index, VCRIX, created by Kim, Trimborn, and Härdle (2019) also shows the evidence of jumps in CRIX.

To summarize our contributions, this study is the first paper to extensively investigate the stochastic and econometric properties of BTC and incorporate these properties in the BTC options pricing. Our results have practical relevance in terms of model selection for characterizing the BTC dynamics. We document the necessity of incorporating jumps in the returns and volatility processes of BTC, and we find that jumps play a critical role in the option prices. Our approach is readily applicable to pricing BTC options in reality. Our results are also important for policymakers to design appropriate regulations for trading BTC derivatives and for institutional investors to launch effective risk management and efficient portfolio strategies.

The article is organized as follows. Section 1 briefly introduces the BTC market. Section 2 studies the BTC return and variance dynamics with the SV, SVJ, and SVCJ models. Fitting of the BR model is investigated in Section 3. Section 4 implements the option-pricing exercises. Section 5 documents an examination of the CRIX, while Section 6

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

concludes the study. A few preliminary econometrics analysis and estimation results for the CRIX are in Appendix. The codes for this research can be found in www.quantlet.de.

1 The BTC Dynamic

We start by briefly introducing BTC. BTC was the first open-source distributed CC released in 2009, after it was introduced in a paper "Bitcoin: A Peer-to-Peer Electronic Cash System" by a developer under the pseudonym Satoshi Nakamoto. It is a digital, decentralized, partially anonymous currency, not backed by any government or other legal entity. The system has a preprogrammed money supply that grows at a decreasing rate until reaching a fixed limit. Since all is based on open source, the design and control are open for all. Traditional currencies are managed by a central bank, while BTCs are not regulated by any authority; instead, they are maintained by a decentralized community. The transactions of BTCs are recorded in the ledgers (known as the blockchain), which is maintained by a network of computers (called "miners"). Since BTC is not a country-specific currency, international payments can be carried out more economically and efficiently.

Our empirical analyses are carried out based on both daily closing (SVCJ model) prices and five minutes intradaily (BR model) prices. The data cover the period from August 1, 2014 to September 29, 2017 and are collected from Bloomberg. The dynamics of BTC daily prices (left panel) and BTC returns (right panel) are depicted in Figure 1. It shows that the BTC return is clearly more volatile than the stock return, along with more frequent jumps or the scattered volatility spikes. BTC's price spent most of the year 2015 relatively stable. The BTC price in the first four months of 2016 was in the range of 400–460 USD. It moved upward dramatically after 2016 and increased to almost 5000 USD by the end of our sample period in 2017. At the time of the writing of this article, the BTC market capitalization is more than USD 7 billion (*source:* Coinmarketcap 2017).

Both the BTC prices and returns react to big events in the BTC market. A dramatic surge observed after March 2017 was due to the widespread interest in CCs. The subsequent drop in June 2017 was caused by a sequence of political interventions. Several governmental announcements of bans on initial coin offerings (ICOs) have spurred intensive movements on CC markets. For example, the Chinese Securities and Exchange Commission denied permission for a BTC exchange-traded fund (ETF) on March 10, 2017; and BTC crashed down after China banned ICOs on September 4, 2017. The large upward movements in BTC prices caused the returns of BTC displaying extremely high volatility and with scattered spikes/jumps. Several large jumps triggered by a series of big events in the BTC market can be detected from the returns series, see also Kim, Trimborn, and Härdle (2019). We have implemented a number of time series models to the BTC returns and the results are shown in Appendix A.1 and Appendix A.2. We find that the standard set of stationary models, such as autoregressive integrated moving average(ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH), cannot fit the BTC returns well due to the presence of jumps.

2 SVCJ: Affine Specification

In this section, we estimate the SVCJ model using BTC prices. We begin with a simple SVCJ jump specification, and switch to the BR model in Section 3. We focus the analysis on BTC and then introduce CRIX in Section 5.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

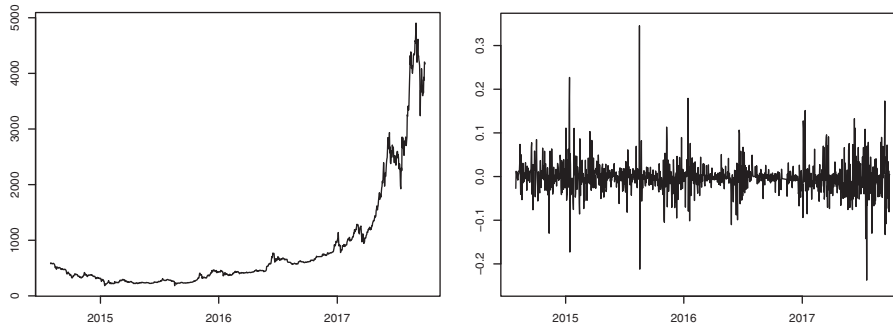

Figure 1 BTC prices and returns.

Notes: This figure graphs the BTC daily price (left panel) from August 1, 2014 to September 29, 2017 and BTC returns (right panel). The returns (R_t) are calculated as $R_t = \log(P_t) - \log(P_{t-1})$, where P_t is the BTC price at time t.

2.1 Models

In order to estimate the BTC dynamics with the SV and SVCJ models regarding returns and volatility, we employ the continuous-time model of Duffie, Pan, and Singleton (2000) that encompasses the standard jump diffusion and the SV with jumps in returns only (SVJ) model of Bates (1996). More precisely, let $\{S_t\}$ be the price process, $\{d\log S_t\}$ the log returns, and $\{V_t\}$ be the volatility process. The SVCJ dynamics are as follows:

$$d\log S_t = \mu dt + \sqrt{V_t} dW_t^{(S)} + Z_t^y dN_t \tag{1}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^{(V)} + Z_t^v dN_t \tag{2}$$

$$Cov\left(dW_t^{(S)}, dW_t^{(V)}\right) = \rho dt \tag{3}$$

$$\mathrm{P}(dN_t = 1) = \lambda dt. \tag{4}$$

Like in the Cox–Ingersoll–Ross model, κ and θ are the mean reversion rate and mean reversion level, respectively. $W^{(S)}$ and $W^{(V)}$ are two correlated standard Brownian motions with correlation denoted as ρ. N_t is a pure jump process with a constant mean jump-arrival rate λ. The random jump sizes are Z_t^y and Z_t^v. Since the jump-driving Poisson process is the same in both Equation (1) and (2), the jump sizes can be correlated. The random jump size Z_t^y conditional on Z_t^v is assumed to have a Gaussian distribution with a mean of $\mu_y + \rho_j Z_t^v$ and standard deviation set to σ_y. The jump in volatility Z_t^v is assumed to follow an exponential distribution with mean μ_v:

$$Z_t^y | Z_t^v \sim \mathrm{N}\left(\mu_y + \rho_j Z_t^v, \sigma_y^2\right); \;\; Z_t^v \sim \exp(\mu_v). \tag{5}$$

The correlation ρ between the diffusion terms is introduced to capture the possible leverage effects between returns and volatility. The jumps may be correlated as well. The correlation term ρ_j takes care of that. The SV process $\sqrt{V_t}$ is modeled as a square root process. With no jumps in the volatility, the parameter θ is the long-run mean of V_t, and the process reverts to this level at a speed governed by the parameter κ. The parameter σ_V is referred to

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

as the volatility of volatility, and it measures the variance responsiveness to diffusive volatility shocks. In the absence of jumps, the parameter μ measures the expected log-return.

SVCJ is a rich model since it encompasses the SV and SVJ approaches. If we set $Z_t^v = 0$ in Equation (5), then jumps are only present in prices, we obtain the SVJ model of Bates (1996). Taking $\lambda = 0$ such that jumps are not present, the model reduces to the pure SV model originally proposed by Heston (1993). If we set $\kappa = \theta = \sigma_V = 0$ and define $Z_t^v = 0$, the model reduces to the pure jump diffusion introduced in Merton (1976).

2.2 Estimation: Markov Chain Monte Carlo

There are plenty of different methods to estimate the diffusion process to real data. The generality of simulation-based methods offers obvious advantages over the method of simulated moments of Duffie and Singleton (1993), the indirect inference methods of Gourieroux, Monfort, and Renault (1993), and the efficient method of moment method of Gallant and Tauchen (1996). For example, Jacquier, Polson, and Rossi (1994) show that Markov Chain Monte Carlo (MCMC) is particularly well suited to deal with SV models. Eraker, Johannes, and Polson (2003) and Eraker (2004) identify several advantages of using the MCMC approach over other estimation models because MCMC methods are computationally efficient and the estimating is more flexible when using simulations. The MCMC method also provides more accurate estimates of latent volatility, jump sizes, jump times, etc. A general discussion and review of the MCMC estimation of continuous-time models can be found in Johannes and Polson (2009).

For the reasons discussed above, we estimate the SVCJ model using the MCMC method. Doing this allows for a wide class of numerical fitting procedures that can be steered by a variation of the priors. Given that there are no BTC options yet, the MCMC method is more flexible in estimating the stochastic variance jumps and thus able to reflect the market price of risk (Franke, Härdle, and Hafner, 2019). The estimation is based on the following Euler discretization:

$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t \tag{6}$$

$$V_t = \alpha + \beta V_{t-1} + \sigma_V \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t, \tag{7}$$

where $Y_{t+1} = \log(S_{t+1}/S_t)$ is the log return, $\alpha = \kappa\theta$, $\beta = 1 - \kappa$ and ε_t^y, ε_t^v are the $\mathrm{N}(0,1)$ variables with correlation ρ. J_t is a Bernoulli random variable with $\mathrm{p}(J_t = 1) = \lambda$ and the jump sizes Z_t^y and Z_t^v are distributed as specified in Equation (5). The daily data sample from August 1, 2014 to September 29, 2017 is used to estimate the model. All returns are in decimal form.

Let us present a brief description on how to estimate the SVCJ model with MCMC (see also Tsay, 2005; Asgharian and Bengtsson, 2006; Johannes and Polson, 2009 for more details). Define the parameter vector as $\Theta = \{\mu, \mu_y, \sigma_y, \lambda, \alpha, \beta, \sigma_v, \rho, \rho_j, \mu_v\}$ and $X_t = \{V_t, Z_t^y, Z_t^v, J_t\}$ as the latent variance, jump sizes, and jump. Recall that Y_t is the log returns.

The MCMC method treats all components of Θ and $X \stackrel{\text{def}}{=} \{X_t\}_{t=1,\ldots,T}$ as random variables. The fundamental quantity is the joint pdf $p(\Theta, X|Y)$ of parameters and latent variables conditioned on data using the Bayes formula:

$$\mathrm{p}(\Theta, X|Y) = \mathrm{p}(Y|\Theta, X)\mathrm{p}(X|\Theta)\mathrm{p}(\Theta). \tag{8}$$

The Bayes formula can be decomposed into three factors: $\mathrm{p}(Y|\Theta, X)$, the likelihood of the data, $\mathrm{p}(X|\Theta)$ the prior of the latent variables conditioned on the parameters and $\mathrm{p}(\Theta)$ the

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

prior of the parameters. The prior distribution $p(\Theta)$ has to be specified beforehand and is part of the model specification. In comfortable settings, the posterior variation of the parameters, given the data, is robust with respect to the prior.

The posterior is typically not available in the closed form, and therefore simulation is used to obtain random draws from it. This is done by generating a sequence of draws, $\{\Theta^{(i)}, X_t^{(i)}\}_{i=1}^{N}$ which form a Markov chain whose equilibrium distribution equals the posterior distribution. The point estimates of parameters and latent variables are then taken from their sample means.

We use the same priors specified in Asgharian and Nossman (2011), who estimate a large group of international equity market returns with jump-diffusion models using the MCMC method, that is, $\mu \sim N(0, 25), (\alpha, \beta) \sim N(0_{2\times1}, I_{2\times2}), \sigma_2^V \sim IG(2.5, 0.1), \mu_y \sim N(0, 100)$, $\sigma_2^y \sim IG(10, 40), \rho \sim U(-1, 1), \rho_j \sim N(0, 0.5), \mu_V \sim IG(10, 20)$ (Inverse Gaussian) and $\lambda \sim Be(2, 40)$ (Beta Distribution). The full posterior distributions of the parameters and the latent-state variables can be found in Asgharian and Nossman (2011) and Asgharian and Bengtsson (2006). We have varied the variance of the priors and found stable outcomes, that is, the reported mean of the posterior that is taken as an estimate of Θ is quite robust relative to changes in variance of the prior distributions. The posterior for all parameters except σ_V and ρ are all conjugate (meaning that the posterior distribution is of the same type of distribution as the prior but with different parameters). The posterior for J_t is a Bernoulli distribution. The jump sizes Z_t^y and Z_t^v follow a posterior normal distribution and a truncated normal distribution, respectively. Hence, it is straightforward to obtain draws for the joint distribution of J_t, Z_t^y and Z_t^v. However, the posteriors for ρ, σ_2^V, and V_t are nonstandard distributions and must be sampled using the Metropolis–Hastings algorithm. We use the random-walk method for ρ and V_t, and independence sampling for σ_V^2. For the estimation of posterior moments, we perform 5000 iterations, and in order to reduce the impact of the starting values, we allow for a burn-in for the first 1000 simulations.

The SVCJ model is known for being able to disentangle returns related to sudden unexpected jumps from large diffusive returns caused by periods of high volatility. For the BTC situation that we consider here, we are particularly interested in linking the latent historical jump times to news and known interventions. The estimates $\hat{J}_t =^{\text{def}} (1/N) \sum_{i=1}^{N} J_t^i$ (where N is the total number of iterations and i refers to each draw) indicate the posterior probability that there is a jump at time t. Unlike the "true" vector of jump times, it will not be a vector of ones and zero. Following Johannes, Rohit, and Polson (1999), we assert that a jump has occurred on a specific date t if the estimated jump probability is sufficiently large, that is, greater than an appropriately chosen threshold value:

$$\tilde{J}_t = 1\{\hat{J}_t > \zeta\}, \quad t = 1, 2, \ldots, T \tag{9}$$

In our empirical study, we choose ζ so that the number of inferred jump times divided by the number of observations is approximately equal to the estimate of λ.

We first estimate the BTC returns by taking the log first differences of prices, then use returns to estimate the SVCJ model. The parameter estimates (mean and variance of the posterior) of the SVCJ, SVJ, and SV models for BTC are presented in Table 1. The estimate of μ is positive. The correlation between returns and volatility ρ is significant and positive. This is remarkable and worth noting since it is different from a negative leverage effect observed over a sequence of studies in stock markets (see, e.g., French and Stambaugh,

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Table 1 BTC parameters for SVCJ, SVJ, and SV models

	SVCJ	SVJ	SV
μ	0.041	0.029	0.030
	(0.022, 0.060)	(0.011, 0.046)	(0.014, 0.046)
μ_y	−0.084	−0.562	–
	(−0.837, 0.670)	(−1.280, 0.155)	–
σ_y	2.155	2.685	–
	(1.142, 3.168)	(1.519,3.850)	–
λ	0.041	0.029	–
	(0.025, 0.056)	(0.019, 0.047)	–
α	0.010	0.010	0.009
	(0.008, 0.012)	(0.006, 0.015)	(0.006, 0.012)
β	−0.132	−0.116	−0.033
	(−0.151, −0.114)	(−0.137, −0.094)	(−0.052, −0.013)
ρ	0.407	0.321	0.169
	(0.232, 0.583)	(0.225, 0.417)	(0.066, 0.271)
σ_v	0.008	0.011	0.017
	(0.007, 0.010)	(0.007, 0.014)	(0.014, 0.021)
ρ_j	−0.573	–	–
	(−1.832, 0.685)	–	–
μ_v	0.620	–	–
	(0.426, 0.813)	–	–
MSE	0.735	0.757	0.763

Notes: This table reports posterior means and 95% finite sample credibility intervals (in parentheses) for parameters of the SVCJ, SVJ, and SV models. All parameters are estimated using BTC daily returns calculated as the log-first difference based on the prices from August 01, 2014 to September 29, 2017.

1987; Schwert, 1989). The effect is named the "inverse leverage effect" and has been discovered in commodity markets (see Schwartz and Trolled, 2009). In other words, the "inverse leverage effect" (associated with a positive ρ) implies that increasing prices are associated with increasing volatility. The reason for this positive relationship between risk and returns might be due to BTC prices being different from conventional stock prices. The digital currency price may be dominated by the "noise trader" behavior described by Kyle (1985) and DeLong et al. (1990). Such investors, with no access to inside information, irrationally act on noise as if it were information that would give them an edge. This positive leverage effect has been also reported by such as Hou (2013) on other highly speculative markets, for example, the Chinese stock markets.

Moreover, the estimates for the SVCJ model are much less extreme than for the SVJ and SVCJ models. More precisely, the volatility of variance σ_v is substantially reduced from 0.017 (SV) to 0.011 (SVJ) and 0.008 (SVCJ). The mean of the jump size of the volatility μ_v is significant and positive. The jump intensity λ is also significant. The jump correlation ρ_j is negative but insignificant, which parallels the results of Eraker, Johannes, and Polson (2003) and Chernov et al. (2003) for stock price dynamics. This effect might be due to the fact that even with a long data history, jumps are rare events. (The evidence is stronger for the BR specifications considered in Section 3.) In summary, the SVCJ model fits the data well by an MSE that is smaller than those of the SVJ and SV models.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Figure 2 shows the estimated jumps in returns (first row) and the estimated jumps in volatility (middle row) together with the estimated volatility (last row). One sees that estimated jumps occur frequently for those of the returns and volatility. The estimated jumps size in returns and variance are different. Figure 3 presents the in-sample fitted volatility processes for the SVCJ and SVJ models, respectively. It is not hard to see that both models lead to a similar overall pattern for the volatility process, though the SVCJ model produces sharper peaks for BTC.

A useful model diagnosis is to examine the standardized residuals obtained from the discrete model, which estimates,

$$\varepsilon_t^y = \frac{Y_t - \mu - Z_t^y J_t}{\sqrt{V_{t-1}}} \tag{10}$$

The normality would be violated if the jumps are not perfectly estimated. However, several previous researches such as Larsson and Nossman (2011), Asgharian and Bengtsson (2006), and Asgharian and Nossman (2011) have estimated the SVCJ model with the MCMC in the equity market and use the normal plot as a diagnostic tool to visualize the model performance. We follow this literature calculating these standardized residuals based on the estimated parameters, then show the QQ plots of the standardized residuals from the fitting of different models in Figure 4. From these diagnostics, it is evident that the GARCH and even the SV models are mis-specified. For the SVJ and SVCJ models, the QQ plot diagnostics are substantially improved. However, it is apparent that the SVCJ model is the preferred choice which is consistent with the MSE reported in Table 1.

3 SV Model with Jumps: High-Frequency Data

3.1 BR Model in Return-Volatility Cojumps

Imposing a specific structure in the stochastic process as documented in Section 2 may produce a specification error. Defining S_t and $\sigma_t = \sqrt{V_t}$ as the price and volatility process, respectively, following the notation of BR, we therefore consider the BR affine jump-diffusion model:

$$\begin{aligned} d\log(S_t) &= \mu_r dt + \sigma_t\left\{\rho_t dW_t^1 + \sqrt{1-\rho_t^2}dW_t^2\right\} \\ &\quad + c_{r,t}^J dJ_{r,t} + c_{r,t}^{JJ} dJ_{r,\sigma,t}, \\ d\xi(\sigma_t^2) &= \left\{m_0 + m_1\log(\sigma_t^2)\right\}dt + \Lambda dW_t^1 + c_{\sigma,t}^J dJ_{\sigma,t} + c_{\sigma,t}^{JJ} dJ_{r,\sigma,t}, \\ \rho_t &= \max\left\{\min(\rho_0 + \rho_1\sigma_t, 1), -1\right\}, \end{aligned} \tag{11}$$

where $\xi(\cdot)$ is an increasingly monotonic function (we will choose it as $\log(\cdot)$ in the following discussions), $W = \left\{W^1, W^2\right\}$ is a bivariate standard Brownian motion vector and $J = \{J_{r,t}, J_{\sigma,t}, J_{r,\sigma,t}\}$ is a vector of mutually independent Poisson processes with constant intensities, which are denoted as λ_r, λ_σ, and $\lambda_{r,\sigma}$, respectively. Thus we allow for common and independent jumps in the system. The Poisson processes are also assumed to be independent from the Brownian motion.

The BR model is estimated through a GMM-like procedure based on infinitesimal cross-moments dubbed by the authors Nonparametric Infinitesimal Method of Moments

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

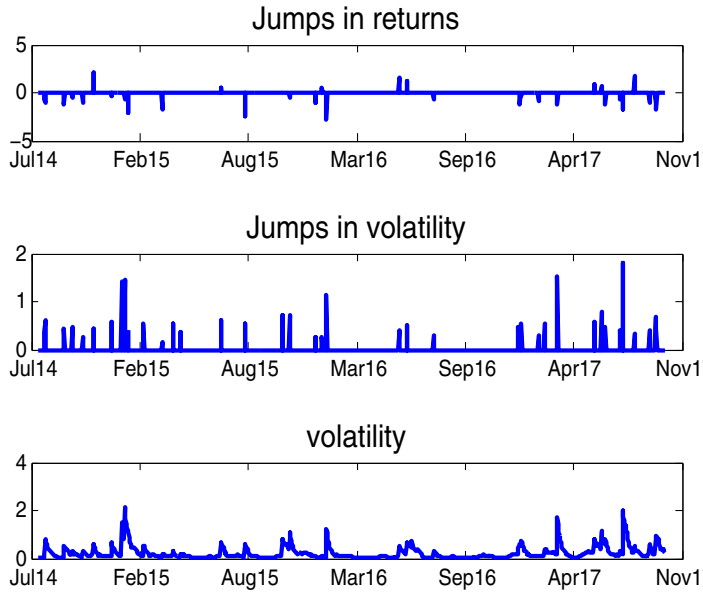

Figure 2 Jumps estimated in returns and volatility from the SVCJ model.

Notes: This figure graphs the estimated jumps in returns and volatility from the SVCJ model. The model is estimated using BTC daily returns calculated as the log-first difference based on the prices from August 1, 2014 to September 29, 2017. The first-, second-, and third-subfigures plot jumps in returns, jumps in volatility and the estimated volatility, respectively.

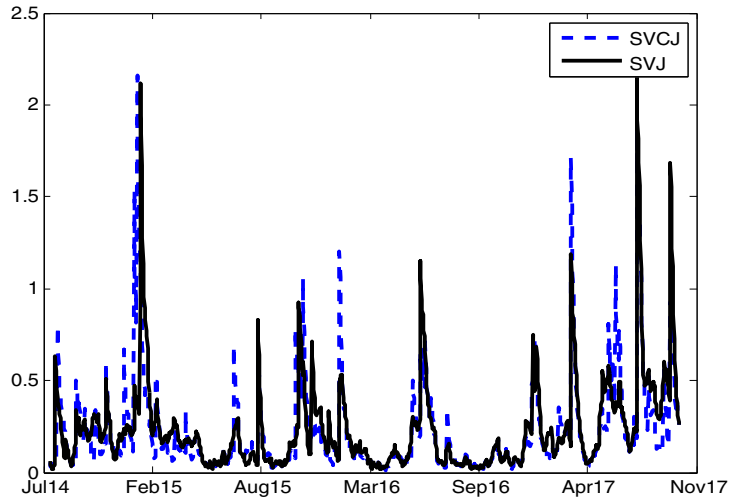

Figure 3 Estimated volatility from the SVCJ and SVJ models.

Notes: This figure plots the estimated volatility from the SVCJ (dotted blue) and SVJ (solid black) models. All models are estimated using BTC daily returns calculated as the log-first difference based on the prices from August 1, 2014 to September 29, 2017.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

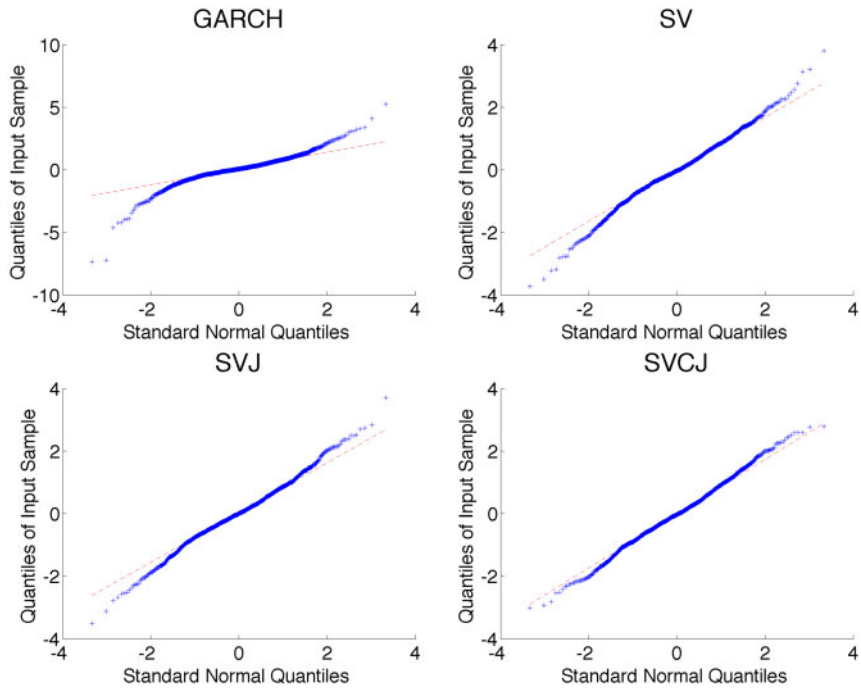

Figure 4 QQ plots for the SVCJ, SVJ, and SV models.

Notes: This figure graphs the QQ plots versus standard normal for fitted standardized residuals from the SVCJ, SVJ, and SV models using BTC daily returns calculated as the log-first difference based on the prices from August 1, 2014 to September 29, 2017. We also include the QQ plot for the GARCH model using the same sample period.

(NIMM). We assume the distribution of the jumps to be normal, that is, $(c^J_{r,t}, c^J_{\sigma,t}) \sim N(\mu^J, \Sigma^J)$ and $(c^{JJ}_{r,t}, c^{JJ}_{\sigma,t}) \sim N(\mu^{JJ}, \Sigma^{JJ})$, with

$$\mu^J = \begin{bmatrix} \mu_{J,r} \\ \mu_{J,\sigma} \end{bmatrix}, \quad \mu^{JJ} = \begin{bmatrix} \mu_{JJ,r,0} + \mu_{JJ,r,1}\sigma_t \\ \mu_{JJ,\sigma} \end{bmatrix},$$
$$\Sigma^J = \begin{bmatrix} \sigma^2_{J,r} & 0 \\ 0 & \sigma^2_{J,\sigma} \end{bmatrix}, \quad \Sigma^{JJ} = \begin{bmatrix} \left(\sigma_{JJ,r,0} + \sigma_{JJ,r,1}\sigma_t^{\sigma_{JJ,r,2}}\right)^2 & \rho_J\left(\sigma_{JJ,r,0} + \sigma_{JJ,r,1}\sigma_t^{\sigma_{JJ,r,2}}\right)\sigma_{JJ,\sigma} \\ \rho_J\left(\sigma_{JJ,r,0} + \sigma_{JJ,r,1}\sigma_t^{\sigma_{JJ,r,2}}\right)\sigma_{JJ,\sigma} & \sigma^2_{JJ,\sigma} \end{bmatrix}. \tag{12}$$

For any $p_1 \geq p_2 \geq 0$, the generic infinitesimal cross-moment of order p_1 and p_2 is defined as:

$$\theta_{p_1,p_2}(\sigma) = \lim_{\Delta \to 0} \frac{1}{\Delta} E\Big\{[\log(S_{t+\Delta}) - \log(S_t)]^{p_1}[\log(\sigma^2_{t+\Delta}) - \log(\sigma^2_t)]^{p_2} | \sigma_t = \sigma\Big\}. \tag{13}$$

In particular, $\theta_{p_1,0}$ helps to identify features of the price process, and θ_{0,p_2} helps to identify those of the variance process, while the genuine cross-moments with $p_1 \geq p_2 \geq 1$ are required to identify the common parameter shared by the two processes $\rho_0, \rho_1, \lambda_{r,\sigma}$, and ρ_J.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

To conduct the NIMM estimation in BR, we first need to estimate the cross-moments that are in theory functions of parameter of interest. The cross-moments are estimated via a nonparametric kernel method. In particular, denote the day index as $t = 1, \ldots, T$ and the equispaced time index as $i = 1, \ldots, N$ within each day. Denote $r_{t,i,k}$ as the high-frequency log returns for day t, knot i, and minute k. We define the closing logarithmic prices as $\log(p_{t,i})$ and logarithmic spot variance estimates as

$$\hat{\sigma}^2_{t,i} = \frac{T}{T-1-n_j}\zeta_1^{-2}\sum_{k=2}^{T}|r_{t,i,k}||r_{t,i,k-1}|1_{\{|r_{t,i,k}|\le\theta_{t,i,k}\}}1_{\{|r_{t,i,k-1}|\le\theta_{t,i,k-1}\}}, \tag{14}$$

where $\zeta_1 \approx 0.7979$, $\theta_{t,i,k}$ is a suitable threshold, and n_j is the number of returns whose absolute value is greater than $\theta_{t,i,k}$. Then the generic cross-moment estimator $\hat{\theta}_{p_1,p_2}(\sigma)$ is defined as

$$\hat{\theta}_{p_1,p_2}(\sigma) = \frac{\sum_{t=1}^{T-1}\sum_{i=1}^{N}K\left(\frac{\hat{\sigma}_{t,i}-\sigma}{h}\right)\{\log(S_{t+1,i})-\log(S_{t,i})\}^{p_1}\left\{\log\left(\hat{\sigma}^2_{t+1,i}\right)-\log\left(\hat{\sigma}^2_{t,i}\right)\right\}^{p_2}}{\Delta\sum_{t=1}^{T}\sum_{i=1}^{N}K\left(\frac{\hat{\sigma}_{t,i}-\sigma}{h}\right)} \tag{15}$$

where $K(\cdot)$ is a kernel function and h is the bandwidth. Finally, with the estimated cross-moments, one can estimate the parameters of interest via the NIMM method, see the details as in Bandi and Renò (2016) for the parametric estimation.

3.2 Correspondence between SVCJ and BR Model

In this section, we fit the BR model using high-frequency data and discuss the comparison with the estimation of the SVCJ model. We collect high-frequency BTC prices from Bloomberg. The price data range is from July 31, 2014 to July 29, 2017, and we collect raw data at a frequency of 60 seconds 24 hours a day. Following Section 3.1, we aggregate the logarithm returns of BTC over a 60-minute time range, namely $r_{t,i,k} = \log S_{t,i,k} - \log S_{t,i,k-1}$, with $k = 1, \ldots, 60$. In addition, we also obtain the spot variance estimates for each day t and each knot i by applying the jump robust threshold bipower variation estimator as in Equation (14).

To compare the data of the high-frequency aggregated volatility and the daily BTC volatility, we plot the average daily spot volatility from the high-frequency data and the daily spot volatility estimates from the SVCJ model together as in Figure 5. We observe that the two sequences sometimes peak at different time points despite that the general pattern agrees.

In Table 2, we show the full model estimation results. The drift parameter μ_r is estimated to be small and insignificant. The linear mean reversions, which can be seen as m_0 and m_1, are both negative. However, they are both insignificant. The volatility of volatility Λ is estimated to be very significant with a value of 0.6766. The average number of independent jumps in volatility is estimated at an annual rate of 0.0519×252, which is around 13. The estimated number of cojumps is around $0.0584 \times 252 \approx 17$. The mean of the independent variance jumps is significant at a level of –0.2783. $\mu_{JJ,r,0}$ is small (−0.0187) and negative, and $\mu_{JJ,r,1}$ is 0.1265. Both parameters are insignificant at the 95% level of confidence. We do not see an obvious tendency for the jumps to be downward, as observed in Bandi and Renò (2016).

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

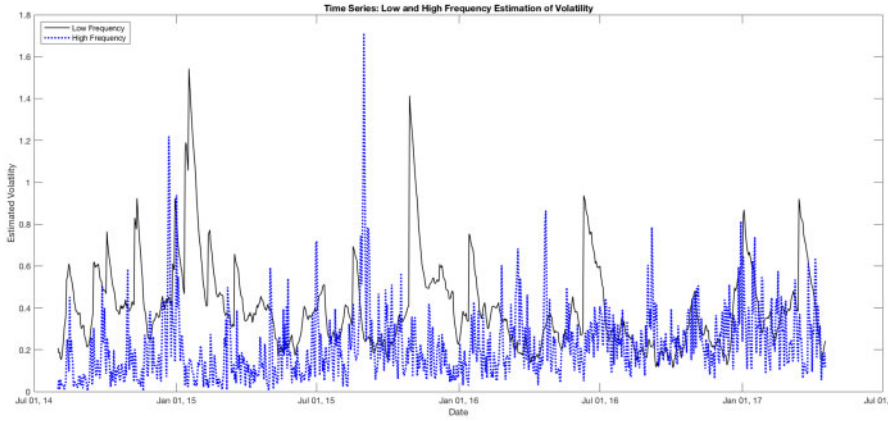

Figure 5 The average daily spot volatility and the daily spot volatility estimates.

Notes: This figure plots the average daily spot volatility from the high-frequency data (dotted line in blue) and the daily spot volatility estimates (solid line in black)

We find that the leverage ρ_0 is estimated to be negative, that is, –0.1485, though insignificant. The leverage would increase with an increasing volatility level as ρ_1 is estimated to be significant and with a value of 0.9292. The standard deviation of the jumps in return $\sigma_{J,r}$ is estimated to be significant with a value of 0.6890. When fitting a nonlinear structure to the standard deviation of the common price jumps, the parameters $\sigma_{JJ,r,1}$ and $\sigma_{JJ,r,2}$ are both significant. The standard deviation of jumps in volatility $\sigma_{J,\sigma}$ is estimated to be 0.8619 with significance. The standard deviation of the common volatility jump $\sigma_{JJ,\sigma}$ is estimated to be insignificant. Notably, the correlation of jumps ρ_J is estimated to be negative and significant with a value of –0.5257, which is in line with BR. This negative and significant cojump size correlation is discovered by Duffie, Pan, and Singleton (2000), who conclude that the price and the volatility jump sizes are "nearly perfectly anti-correlated." Eraker (2004) finds a statistically significant correlation between the jump sizes only when employing option data in addition to stock returns data. Bandi and Renò (2016) also report a "nearly perfect anti-correlation" of -1.

4 Option Pricing

In the previous sections, we have shown that the SVCJ and the BR models can well describe the log-returns dynamics of BTC. In this section, we discuss option pricing for BTC based on the SVCJ and BR models, respectively.

4.1 BTC Options

After fitting the SVCJ and the BR model, we advance with a numerical technique called Crude Monte Carlo (CMC) to approximate the BTC option prices. Derivative securities such as futures and options are priced under a probability measure Q commonly referred to as the "risk neutral" or martingale measure. Since our purpose is to explore the impact of model choice on option prices, we follow Eraker, Johannes, and Polson (2003) and set the

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Table 2 BR parametric estimates and their 95% confidence intervals. The parametric model is specified as in Equations (11) and (12). The first column specifies that $J_{r,\sigma} = 0$ (no cojumps) and the second column specifies that $J_r = J_\sigma = 0$ (no independent jumps)

	No cojumps	No ind. jumps	Full model
μ_r	0.0021	0.0027	0.0082
	(−0.1939, 0.1981)	(−0.1933, 0.1987)	(−0.0444, 0.0608)
ρ_0	0.0044	−0.0148	−0.1485
	(−0.1150, 0.1237)	(−0.1401, 0.1105)	(−0.4851, 0.1882)
ρ_1	−0.3744	−0.2237	0.9292
	(−0.8513, 0.1025)	(−0.7088, 0.2614)	(0.5884, 1.2699)
m_0	−0.0500	−0.0500	−0.0495
	(0.1275, 0.0275)	(−0.1275, 0.0275)	(−0.1475, 0.0485)
m_1	−0.0168	−0.0125	−0.0600
	(−0.2128, 0.1792)	(−0.2085, 0.1835)	(−0.2560, 0.1360)
Λ	0.7634	0.7853	0.6766
	(0.5674, 0.9594)	(0.5893, 0.9813)	(0.6570, 0.6963)
$\mu_{J,r}$	0.1577	0	2.5486
	(0.0372, 0.2782)	–	(2.3526, 2.7446)
$\mu_{JJ,r,0}$	0	−0.0804	−0.0187
	–	(−0.5383, 0.3774)	(−0.1085, 0.0711)
$\mu_{JJ,r,0}$	0	0.0192	0.1265
	–	(−0.6850, 0.7234)	(−0.4183, 0.6713)
$\sigma_{J,r}$	0.6801	0	0.6890
	(0.5453, 0.8148)	–	(0.4930, 0.8850)
$\sigma_{JJ,r,0}$	0	0.0864	0.0043
	–	(−0.3242, 0.4971)	(−0.5459, 0.5544)
$\sigma_{JJ,r,1}$	0	1.8713	1.2159
	–	(1.8436, 1.8991)	(1.0199, 1.4119)
$\sigma_{JJ,r,2}$	0	2.6521	3.9590
	–	(2.5377, 2.7664)	(3.7630, 4.1550)
$\mu_{J,\sigma}$	−0.5000	0	−0.2783
	(−0.5364, −0.4636)	–	(−0.4992, −0.0574)
$\mu_{JJ,\sigma}$	0	−1.9181	−0.4927
	–	(−2.0805, −1.7557)	(−0.6429, −0.3425)
$\sigma_{J,\sigma}$	0.7945	0	0.8619
	(0.7379, 0.8511)	–	(0.7237, 1.0001)
$\sigma_{JJ,\sigma}$	0	1.0705	0.0717
	–	(0.8716, 1.2693)	(−0.0767, 0.2202)
ρ_J	0	−1.0000	−0.5257
	–	(−1.4648, −0.5351)	(−0.7217, −0.3297)
λ_r	0.0002	0	0.0000
	(−0.1958, 0.1962)	–	(−0.1960, 0.1960)
λ_σ	0.0700	0	0.0519
	(0.0504, 0.0896)	–	(0.0323, 0.0715)
$\lambda_{r,\sigma}$	0	0.0060	0.0584
	–	(−0.0136, 0.0256)	(0.0564, 0.0603)

Notes: This table reports the parameter estimates of the model specified in Equations (11) and (12) using the intradaily BTC returns. For each parameter, we report the estimate and the corresponding 95% finite sample credibility intervals in parentheses. The full model is shown in the fourth column, and the second and third columns report the same model with the restriction of no cojumps and no independent jumps, respectively.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

risk premia to zero. This choice can be disputed, but for the lack of existence of the officially traded options a justifiable path to pricing BTC contingent claims. Suppose we have an option with a payoff at time of maturity T as $C(T)$, and typically for call option $C(T) = (S_T - K)^+$. The price of this option at time t is denoted as:

$$E\,Q\big[\exp\{-r(T-t)\}C(T)|\mathcal{F}_t\big], \tag{16}$$

where $\mathcal{F}_t$ is a set that represents information up to time t. We approximate the European option prices of BTC using the CMC technique. The CMC simulation is done for 20,000 iterations to approximate the option price using the parameters reported in Table 1 for the SVCJ, SVJ, and SV models and in Table 2 for the BR (assuming a daily interval) model. Since no BTC option market exists yet, we do not have real market option prices for comparison. Thus, we chose July 2017 randomly as the experimental month in our option-pricing simulation analysis. Throughout our entire analysis of option pricing, the moneyness for strike K and S at t is defined to be K/S_t. The pricing formula is a function of moneyness and time to maturity $\tau = (T - t)$ where T is the maturity day.

In Figure 6, we plot the simulated volatility of various models based on the parameters reported in Table 1 (for the SVCJ, SVJ, and SV models) and in Table 2 (for the BR model) for the month of July 2017. It can be seen from this figure that the approximated volatility on July 15, 2017 had a large jump (there was a large increase observed on July 15, 2017 in the BTC historical prices). The sudden jump is perfectly captured by the BR, SVCJ, and SVJ models, while the SV model cannot characterize the volatility as well as the other three models. The BR model estimates the jump more than the SVCJ and the SV models, this could be attributed to the uncorrelated jumps, which is not considered by the SVCJ and the SVJ models. Assuming a BTC spot price $S_t = 2250$, the estimated BTC call option prices across moneyness and time to maturity on July 17, 2017, obtained using the SVCJ model[3] are presented in Table 3. We see that, for example, a call option on BTC with the strike $K = 1250$ and time to maturity of 90 days would be traded at 1157.95 on July 17, 2017.

To further understand how the option price changes with respect to changes in time to maturity and moneyness for different models, we show in Figure 7 the one-dimensional contour plot of the option prices surface across time to maturity and moneyness estimated from the SVCJ, SVJ, SV, and BR models for the month of July 2017. When examining moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is fixed at at-the-money (ATM). We can see from the contour plot that the relationship between the option price and the time to maturity or moneyness varies over time for all four models. The BR model and the SVCJ models have more volatile patterns than those of the SCJ and SV models. This figure conveys a homogeneous message as we can see from Figure 6 in the volatility plots. For example, for the BTC price, we see a drastic change in the contour structure on, for example, July 15, 2017 as the price suddenly drops from 2232.65 USD on July 15, 2017 to 1993.26 USD. The sudden drop in price should be attributed to the big jump in volatility shown in Figure 6, and we can also observe this jump on July 15 in Figure 7.

Figure 8 displays the estimated BTC call option price differences between the SVCJ and SVJ models with respect to changes in moneyness and across time to maturity for July

3 We have also calculated option prices for the SVJ and SV models. These results are available upon requests. The codes for this research can be found in www.quantlet.de.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

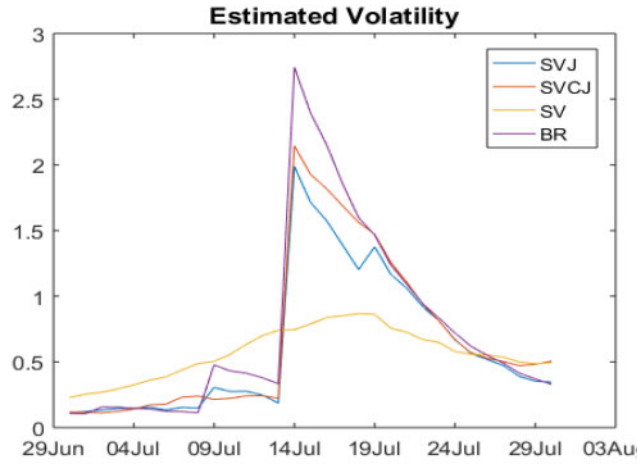

Figure 6 Estimated volatility of BTC for July 2017: BTC.

Notes: This figure plots the estimated volatility of the SVCJ, SVJ, SV, and the BR models. The volatility is approximated based on the parameters reported in Tables 1 and 2 for the month of July 2017. The *x*-axis notes the dates in July 2017. The blue/red/orange/purple line plots the volatility from the SVJ/SVCJ/SV/BR models.

Table 3 Call option price of BTC on July 17, 2017 from the SVCJ model

K τ	1	7	30	60	90	180	360	720
1250.00	1069.18	1017.81	1099.87	1125.90	1157.95	1248.98	1361.04	1365.96
1350.00	959.02	959.02	1006.02	1066.67	1094.08	1224.48	1302.60	1316.03
1450.00	885.20	860.15	929.32	995.45	1046.89	1099.35	1258.83	1438.90
1550.00	802.38	791.34	901.27	950.34	1015.76	1114.94	1192.24	1332.08
1650.00	707.97	739.10	825.07	882.17	902.32	1062.17	1175.59	1282.36
1750.00	625.86	678.22	786.88	856.72	896.56	962.79	1192.61	1338.49
1850.00	552.26	618.94	697.11	785.62	862.83	897.74	1110.36	1289.51
1950.00	502.28	545.58	663.47	740.32	819.72	903.60	1052.09	1229.45
2050.00	425.46	511.28	629.14	741.65	772.51	905.30	1027.76	1193.43
2150.00	358.30	460.57	597.44	683.55	740.64	870.66	1036.76	1164.23
2250.00	302.88	408.62	543.02	633.31	720.57	872.42	938.68	1051.71
2350.00	265.91	378.10	492.86	594.01	651.03	783.37	887.62	1064.33
2450.00	211.26	347.79	470.85	580.30	657.43	761.39	940.90	1085.75
2550.00	193.69	304.13	437.06	547.15	608.36	766.19	914.62	1101.72
2650.00	156.38	266.64	421.86	518.27	571.42	719.92	827.17	992.20
2750.00	136.24	247.38	397.92	484.70	556.31	651.86	863.10	1066.75
2850.00	135.28	228.47	345.42	465.75	541.61	672.76	788.25	955.97
2950.00	100.02	202.57	341.11	413.75	488.15	627.52	780.53	917.27
3050.00	103.45	179.93	313.83	424.23	496.05	619.88	758.99	911.33
3150.00	82.59	162.72	290.90	371.20	450.85	593.10	752.88	888.89
3250.00	72.93	140.40	273.97	358.26	442.91	571.96	726.49	933.57

Notes: This table reports the approximated call option prices at different time to maturity τ and strike prices K the SVCJ model on July 17, 2017 based on the parameters reported in Table 1. The numbers in the first row are the time to maturity. The numbers in the first column are the strike prices. The spot BTC price is assumed to be 2250.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

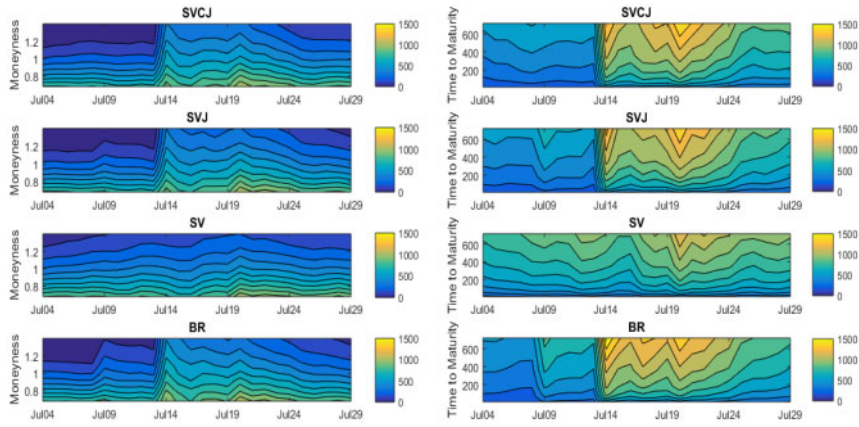

Figure 7 Call option prices across moneyness and time to maturity: BTC.

Notes: This figure graphs the call option prices surface counter plot across different moneyness and different times to maturities for the month of July 2017, as shown in the right-hand side labels. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The color in the graph represents the price level; the brighter the color, the higher the price.

2017. It is not hard to see that the pattern is similar to the fitted volatility shown in Figure 6. The difference between the SVCJ and SVJ models is similar besides on July 15 when there is a large spike in the estimated volatility. Therefore, the price differences between the SVCJ and SVJ models are mainly caused by the jumps in the volatility process and the volatility level, which reflects the necessity of adopting the SVCJ model in practice.

4.2 BTC Implied Volatility Smiles

It is well known that SV determines excess kurtosis in the conditional distribution of returns. The excess kurtosis causes symmetrically higher implied Black–Scholes volatility when strikes are away from the current prices, for example, the level of moneyness is away from the ATM level. This phenomenon is called the "volatility smile". It is well documented in the existing literature that the effect is stronger for short and medium maturity options than for long maturity options for which the conditional returns are closer to normal (Das and Sundaram, 1999). The presence of cojumps and the negative correlation between the presence of cojumps sizes yield additional sources of skewness in the conditional distribution of stock returns (Bandi and Renò, 2016).

To further examine the option-pricing property of BTC, we approximate the implied Black–Scholes volatility from various models for different degrees of moneyness (strike/spot) and different times to maturity. First, the European call option prices are simulated using the model parameters reported in Table 1 for the SVCJ, SVJ, and SV models and Table 2 for the BR model. Then the volatility from various models is implied from the Black–Scholes model based on the options approximated from different models. We consider four times to maturity: one week, one month, three months, and one year. We report

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

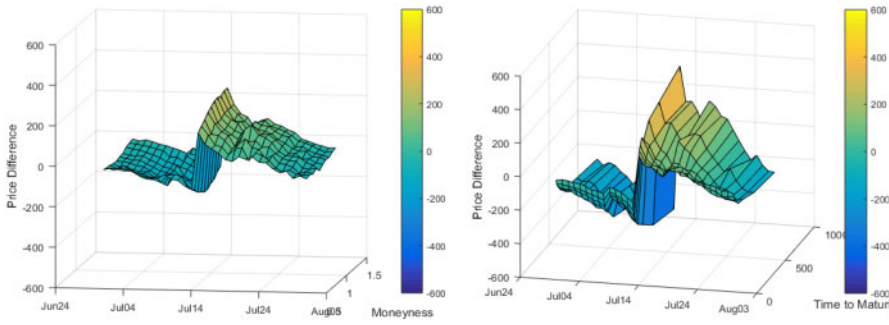

Figure 8 Call option price differences between the SVCJ and SVJ models: BTC.

Notes: This figure plots the option price differences between the SVCJ and SVJ models for July 2017. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The color in the graph represents the price difference level; the brighter the color, the larger the difference between the price from the SVCJ and SVJ models.

the IV surface as a function of moneyness and time to maturity. The results indicate that jumps in returns and volatility include important differences in the shape of the IV curves, especially for the short maturities options.

Figure 9 shows the IV curves for the SVCJ, SVJ, and SV models for four different maturities and across moneyness. It can be seen from Figure 9 that adding jumps in returns steepens the slope of the IV curves. Jumps in volatility further steepen the IV curves. For short maturity options, the difference between the SVCJ, SVJ, and SV models for far ITM options is quite large, with the SVCJ model giving the sharpest skewness among the three models. The difference between the SVCJ and SV volatility is approximately 2–3% for up to one month. All three models have one-side volatility skewness. This could be due to the skewness in the conditional distribution of BTC returns (Das and Sundaram, 1999) and/or that the negative cojump size yields an additional source of skewness (Bandi and Renò, 2016). As time to maturity increases, the volatility curve flattens for all models. According to Das and Sundaram (1999), jumps in returns result in a discrete mixture of normal distributions for returns, which easily generates unconditional and conditional non-normalities over short frequencies such as daily or weekly. Over longer intervals, for example, more than a month, a central-limit effect results in decreases in the amount of excess and kurtosis. Indeed, diffusive SV models may generate very flat curves, such as a flat BTC IV for the three-month and the one-year time to maturity.

However, for the SVCJ model, the curve flattens at a slightly higher level. The IV of the SVJ model is closer to the SVCJ model than the SV model. The difference between the SVCJ, SVJ, and SV models becomes larger with short time to maturity options, that is, the one-week and one-month times to maturity. Similar results have been documented in other studies in which these models have been applied to equity index data. Eraker, Johannes, and Polson (2003), Eraker (2004), and Duffie, Pan, and Singleton (2000) find that jumps in returns and variance are important in capturing systematic variations in Black–Scholes volatility. In general, although the BTC market has the unique feature of having more

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

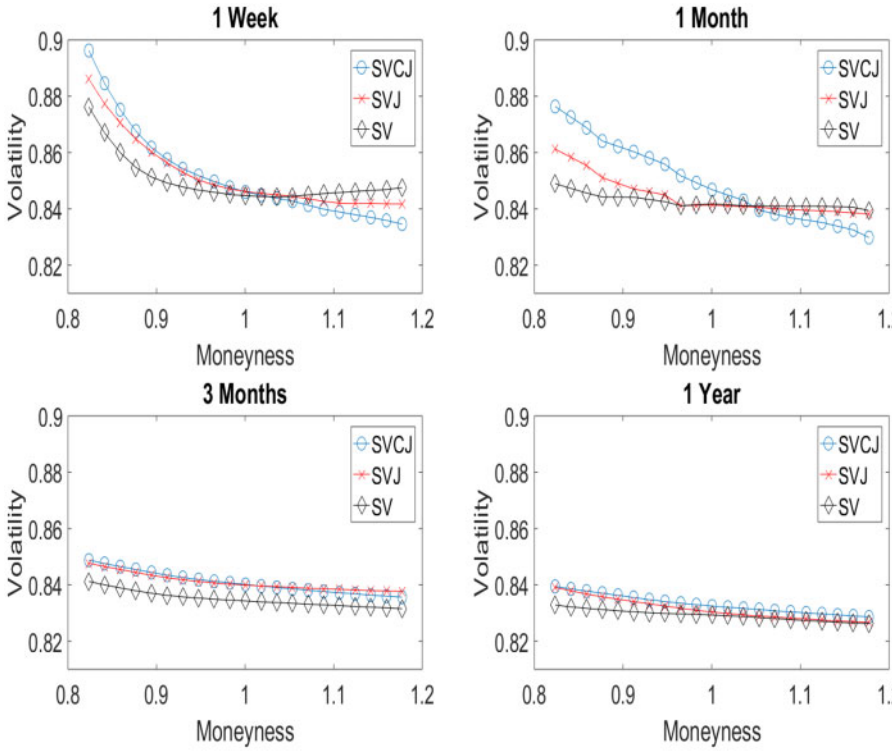

Figure 9 The IV for the BTC market: the SVCJ, SVJ, and SV models.

Notes: This figure plots the Black–Scholes IV for the BTC market based on the SVCJ, SVJ, and SV models. The *x*-axis shows moneyness and the *y*-axis shows the IV. Four times to maturity have been considered: one week, one month, three months, and one year. The lines with $\circ$, *, and $\diamond$ plot the IVs of the SVCJ, SVJ, and SV models, respectively.

jumps, which makes it different from other mature markets (e.g., equity), the option prices and the IV from the affine models generally follow the conventional characteristics reported from other option markets.

We have also estimated the BR IVs with the same time to maturity and moneyness used for the SVCJ IVs. We simulate the option prices using the model parameters reported in column 4 of Table 2. We distinguish the case of ρ_J, which is set to be a model-fitted parameter from the SVCJ fit or to be zero, that is, the IV surface corresponds to a case with a correlation between jump sizes equaling -0.5257 or a correlation between jump sizes equaling to zero. The IVs as a function of moneyness from the BR model are plotted in Figure 10. We can see that the IVs of the BR model agree with the SVCJ model. We see a one-side volatility skewness, that is, the ITM call option prices are higher than the OTM call options. However, due to the significantly negative jump-size correlation ρ_J, the slope of the IVs from the BR full model is steeper than the BR model with a case of uncorrelated jump sizes. The impact of the negative jump size correlation is stronger for short time to maturity

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

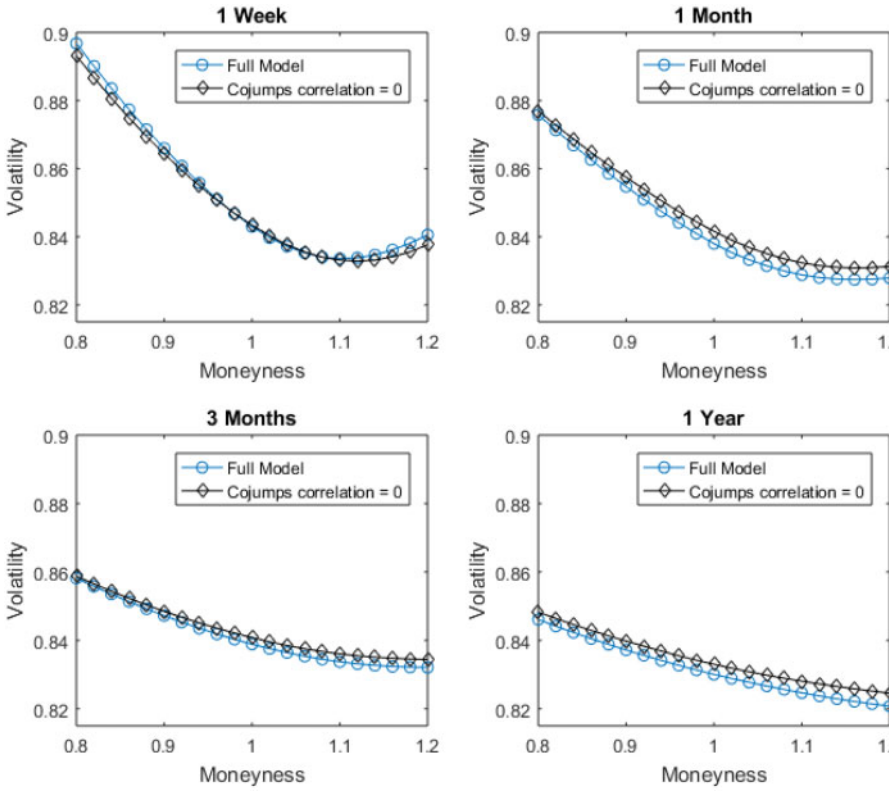

Figure 10 The IV for the BTC market: BR model.

Notes: This figure plots the implied Black–Scholes volatility for the BTC option prices based on the BR model. The *x*-axis shows moneyness, and the *y*-axis shows the IV. Four times to maturity have been considered: one week, one month, three months, and one year. The IVs are based on the simulated option prices using the model parameters reported in Table 2. The full model uses parameters from column 4 of Table 2. A cojumps correlation of 0 means that ρ_J is set to 0 while the other parameters remain the same as in the full model.

options, that is, the one-week and one-month times to maturity. This is mentioned in the results of Duffie, Pan, and Singleton (2000) as well, who find a superior fit of the IV smirk when calibrating a more negative correlation between jump sizes. Similarly, Eraker (2004) finds a statistically significant correlation between jump sizes only when employing option data in addition to returns data. Bandi and Renò (2016) also show that anticorrelated jump sizes are a fundamental property of prices and volatility. However, the use of high-frequency data is sufficient to reveal this property with no further need for option data.

5 The CRIX

The CRIX, a value-weighted CC market index with an endogenously determined number of constituents using some statistical criteria, is described in Härdle and Trimborn (2015)

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

and further sharpened in Trimborn and Härdle (2018). It is constructed to track the entire CC market performance as closely as possible. The representativity and the tracking performance can be assured as CRIX considers a frequently changing market structure. The reallocation of the CRIX happens on a monthly and quarterly basis (see Trimborn and Härdle, 2018 and thecrix.de for details). CRIX has been widely investigated in the pioneering research on CCs, including by Chen et al. (2017), Hafner (2020), Chen and Hafner (2019), and da Gama Silva et al. (2019).

There are two advantages of holding a portfolio comprising a wide variety of CCs like CRIX. The first advantage is the diversification benefit. The evidence from Härdle, Harvey, and Reule (2020) shows that the correlations among the most leading coins are around 0.5, indicating a promising potential of diversification. The correlations among coins vary over time, as shown in Härdle, Harvey, and Reule (2020). It shows that the diversification effect through forming a portfolio is beneficial, although this effect may vary over time.

The second advantage underscores that the efficient portfolio, like CRIX, entails a higher Sharpe ratio than that of BTC. From the view of institutional investors, a smart strategy is to hold a market portfolio comprising of the coins with sufficient liquidity and market capitalization to leverage between profitability and risk-sharing. A simple calculation of the annual Sharpe ratio for both BTC- and CRIX-based portfolios sheds some light. The Sharpe ratios of CRIX in 2016 and 2017 are, respectively, 0.094 and 0.194, however, the ratios of BTC are relatively lower (0.085 in 2016 and 0.149 in 2017). It suggests that investors should rather look at all possible portfolios in an investment opportunity set that potentially optimize their mean–variance preference.

Given the merits of portfolio deployment over a single altcoin investment rule, institutional investors may demand the corresponding derivatives for hedging position risk. The options with a CRIX as underlying may fulfill such needs in practice. Apart from hedging purposes, and for speculators without any position, such index options are quite precious and enable them to bet on future movement.

Therefore, we perform an analysis for CRIX. All econometric models have been estimated with the CRIX data. We summarize our major findings here and place the supplementary parts in Appendix. In brief, all the model parameters estimated with CRIX convey a similar configuration as estimated with BTC, for example, the mean jump size of the CRIX volatility process reported in Table 4 is 0.709, which is 0.620 for BTC shown in Table 1. The estimated volatility from the SVCJ and SVJ models (see Figure 11) shows that the jumps are better captured by the SVCJ than the SVJ model. In addition, Figure 12 displays the call option prices surface contour plot from the SVJ, SV, and SVCJ models with respect to changes in moneyness and time to maturity. It shows that the SVCJ model has more volatile patterns than those of the SVJ and SV models with the BTC options. In general, we confirm the consistency between BTC and the CRIX.

6 Conclusion

"The Internet is among the few things that humans have built that they do not truly understand" according to Schmidt and Cohen (2017). CC, a kind of innovative Internet-based

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Table 4 Parameters for the SVCJ, SVJ, and SV models: CRIX

	SVCJ	SVJ	SV
μ	0.042	0.0437	0.017
	(0.030, 0.054)	(0.027, 0.061)	(0.000, 0.034)
μ_y	−0.0492	−0.515	–
	(−0.777, 0.678)	(−1.110, 0.079)	–
σ_y	2.061	2.851	–
	(1.214, 2.907)	(1.349, 4.354)	–
λ	0.0515	0.035	–
	(0.038, 0.065)	(0.017, 0.052)	–
α	0.0102	0.026	0.010
	(0.009, 0.012)	(−0.012, 0.063)	(0.007, 0.012)
β	−0.188	−0.240	−0.038
	(−0.205, −0.170)	(−0.383, −0.096)	(−0.056, −0.020)
ρ	0.275	0.214	0.003
	(0.140, 0.409)	(0.014, 0.415)	(−0.130, 0.136)
σ_v	0.007	0.016	0.018
	(0.005, 0.009)	(−0.001, 0.033)	(0.014, 0.022)
ρ_j	−0.210	–	–
	(−0.924, 0.503)	–	–
μ_v	0.709	–	–
	(0.535, 0.883)	–	–
MSE	0.673	0.707	0.736

Notes: The table reports posterior means and 95% credibility intervals (in parentheses) for the parameters of the SVCJ, SVJ, and SV models. All parameters are estimated using CRIX daily returns calculated as the log difference based on the prices from August 01, 2014 to September 29, 2017.

asset, brings not only new challenges but also new ways of thinking for economists, cliometricians, and financial specialists. Unlike classic financial markets, the BTC market has a unique market microstructure created by a set of opaque, unregulated, decentralized, and highly speculation-driven markets.

This study provides a way of pricing CC derivatives using advanced option-pricing models such as the SVCJ and BR models. We find that in general, the SVCJ model performs as well as the nonaffine BR model. We especially find that the correlation between the jump sizes in returns and the volatility process is anticorrelated. The jump-size correlation is statistically (marginally) negative in the BR (SVCJ) model. Deviating from the equity market, we cannot obtain a significant negative "leverage effect" parameter ρ, which implies a nonnegative relation between returns and volatility. The reason for this relationship might be that BTC is different from the conventional stock market, not only because the BTC market is highly unregulated but also due to the fact that the BTC price is not informative (as there are no fundamentals allowing the BTC market to set a "fair" price) and is driven by emotion and sentiment. This speculative behavior can be explained by the "noise trader" theory from Kyle (1985). The positive relation might result from the fact that BTC investors irrationally act on noise as if it were information that would give them an edge.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

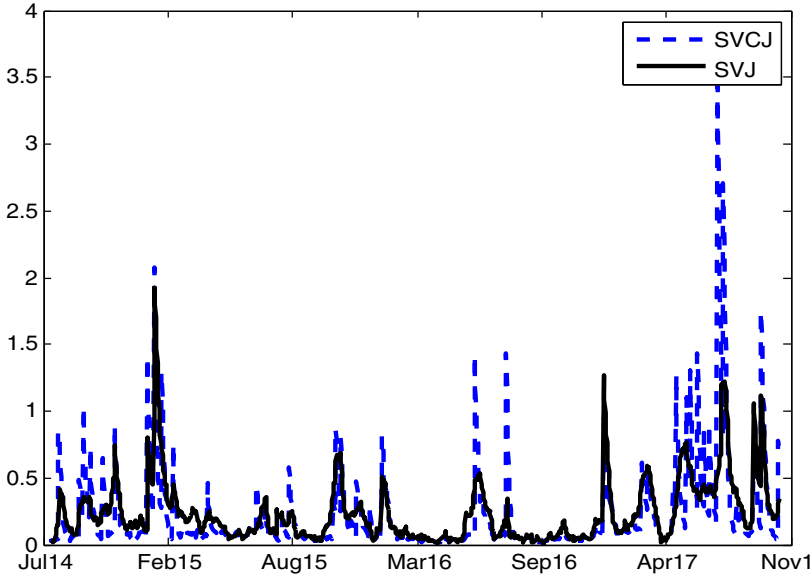

Figure 11 Estimated volatility from the SVCJ and SVJ models: CRIX.

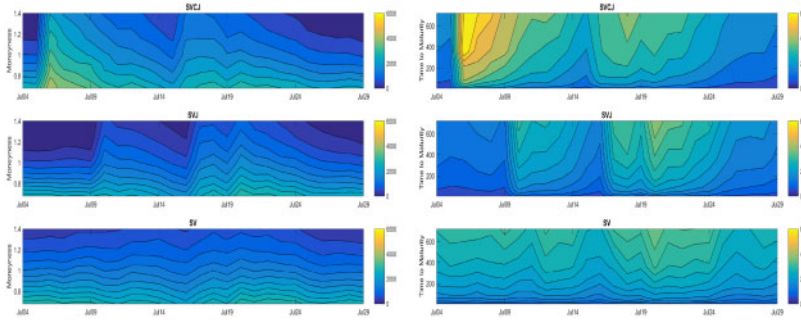

Figure 12 Call option prices across moneyness and time to maturity: CRIX.

Notes: This figure graphs the call option prices surface counterplot across moneyness and time to maturities for the month of July 2017 for CRIX. When looking at moneyness, the time to maturity is fixed at 30 days, and when looking at the time to maturity, moneyness is ATM. The color in the graph represents the price level; the brighter the color, the higher the price.

We find that option prices are very much driven by jumps in the returns and volatility processes and cojumps between the returns and volatility. This can be seen from the shape of the IV curves. This study provides a grounding base, or an anchor, for future studies that aim to price CC derivatives. This study provides useful information for establishing an options market for BTC in the near future.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Appendix

We provide preliminary fit results of econometric models on the BTC time series. We also collect results on analysis of the CRIX.

A.1 ARIMA

We first fit an ARIMA model. After an inspection through the ACF and PACF plot in Figure A.1, we start with an ARIMA(p, d, q) model,

$$a(L)\Delta y_t = b_L \varepsilon_t \tag{17}$$

where y_t is the variable of interest, $\Delta y_t = y_t - y_{t-1}$, L is the lag operator and ε_t a stationary error term. Model selection criteria such as AIC or BIC indicates that the ARIMA(2,0,2) is the model of choice. The parameters estimated from the ARIMA(2,0,2) are reported in Table A.1. The significant negative signs in a_1 and a_2 indicate an overreaction, that is, a promising positive return today leads to a return reversal in the following two days or vice versa. Hence, the CC markets tend to overreact to good or bad news, and this overreaction can be corrected in the following two days. An ARIMA model for the CC assets, therefore, suggests predictability due to an "overreaction." The Ljung–Box test confirms that there is no serial dependence in the residuals based on the ARIMA(2,0,2) specification. Note that the squared residuals carry incremental information that is addressed in the following GARCH analysis.

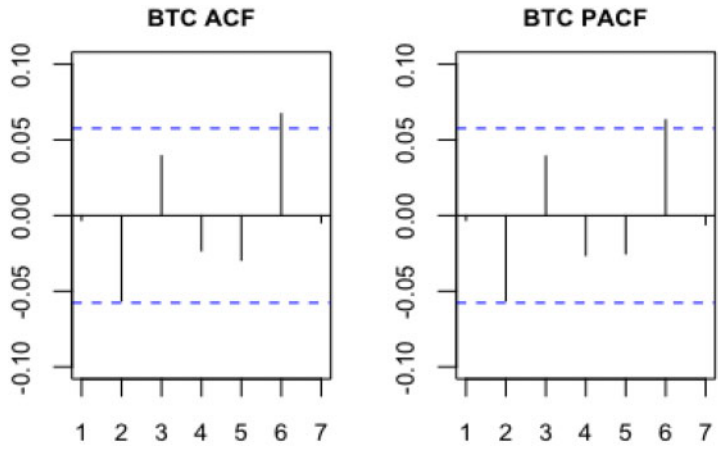

Figure A.1 ACF and PACF of BTC.

Notes: This figure plots the ACF and PACF for BTC returns. The returns are the log-first difference calculated based on the price from August 1, 2014 to September 29, 2017. The *x*-axis plots the lags, and the *y*-axis plots the ACF and PACF values.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Table A.1 Estimation result of ARIMA(2,0,2)

	BTC	
Coefficients	Estimate	Standard error (robust)
Intercept c	0.002	0.001
a_1	−0.867	0.304
a_2	−0.596	0.177
b_1	0.868	0.321
b_2	0.539	0.190

Notes: This table reports the parameter estimated from ARIMA (2,0,2) with BTC daily returns. The residual distributions are assumed to be Gaussian. The maximized likelihood value is 2231.7. The AIC and BIC are −4451.4 and −4415.74, respectively.

A.2 GARCH Model

The GARCH model, introduced first by Bollerslev (1986), reflects the changes in the conditional volatility of the underlying asset in a parsimonious way. The volatility properties of digital currency assets have been studied in a vast amount of literature that applies GARCH-type methods (Chan et al., 2017; Chu et al., 2017; Conrad, Custovic, and Ghysels, 2018; Hotz-Behofsits, Huber, and Zörner, 2018).

Let us start with a GARCH-type model for characterizing the conditional variance process of BTC. The ARIMA-t-GARCH model with t-distributed innovations used to capture fat tails is as follows:

$$a(L)\Delta y_t = b_L \varepsilon_t \qquad (18)$$
$$\varepsilon_t = Z_t \sigma_t, \quad Z_t \sim t(\nu)$$

$$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \qquad (19)$$

where σ_t^2 represents the conditional variance of the process at time t and $t(\nu)$ refers to the zero-mean t distribution with ν degrees of freedom. The choice of the t-distribution rather than the Gaussian distribution is supported by Hotz-Behofsits, Huber, and Zörner (2018) and Chan et al. (2017).

The covariance stationarity constraint $\alpha_1 + \beta_1 < 1$ is imposed. As shown in Table A.2, the β_1 estimate from BTC indicates a persistence in the variance process, but its value is relatively smaller than those estimated from the stock index returns (see Franke, Härdle, and Hafner, 2019). Typically, the persistence-of-volatility estimates are very near to one, showing that conditional models for stock index returns are very close to being integrated. By comparison, BTC places a relatively higher weight on the α_1 coefficient and relatively lower weight on the β_1 to imply a less-smooth volatility process and striking disturbances from the innovation term. This may further imply that the innovation is not pure white noise and can occasionally be contaminated by the presence of jumps.

In addition to the property of leptokurtosis, the leverage effect is commonly observed in practice. According to a large body of literature, starting with Engle and Ng (1993), the leverage effect refers to an asymmetric volatility response given a negative or positive shock. The leverage effect is captured by the exponential GARCH (EGARCH) model by Nelson (1991),

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Table A.2 Estimated coefficients of *t*-GARCH(1,1)

Coefficients	Estimates	Robust std	*t*-value
BTC			
ω	$3.92e-05$	$1.49e-05$	2.63
α_1	$2.28e-01$	$4.46e-02$	5.12
β_1	$7.70e-01$	$5.13e-02$	14.98
ν	$3.64e+00$	$4.08e-01$	8.91

Notes: This table reports the estimated parameters from the *t*-GARCH(1,1) model. The robust version of standard errors (robust std) is based on the method of White (1982).

Table A.3 Estimated coefficients of *t*-EGARCH(1,1) model

Coefficients	Estimates	Robust std	*t*-value
BTC			
ω	$3.84e-05$	$1.47e-05$	2.61
α_1	$1.05e-03$	$5.10e-02$	0.98
β_1	$9.52e-01$	$1.54e-02$	61.73
ϕ_1	$4.16e-01$	$6.64e-02$	6.25
ν	$3.26e+00$	$4.16e-01$	7.82

Notes: This table reports the estimated parameters from the *t*-EGARCH(1,1) model. The robust version of standard errors (robust std) is based on the method of White (1982).

$$\begin{aligned} \varepsilon_t &= Z_t\sigma_t \\ Z_t &\sim t(\nu) \\ \log\left(\sigma_t^2\right) &= \omega + \sum_{i=1}^{p}\beta_i \log\left(\sigma_{t-i}^2\right) + \sum_{j=1}^{q} g_j\left(Z_{t-j}\right) \end{aligned} \tag{20}$$

where $g_j(Z_t) = \alpha_j Z_t + \phi_j\left(|Z_{t-j}| - E|Z_{t-j}|\right)$ with $j = 1, 2, \ldots, q$. When $\phi_j = 0$, we have the logarithmic GARCH (LGARCH) model from Geweke (1986) and Pantula (1986). To accommodate the asymmetric relation between stock returns and volatility changes, the value of $g_j(Z_t)$ must be a function of the magnitude and the sign of Z_t. Over the range of $0 < Z_t < \infty$, $g_j(Z_t)$ is linear in Z_t with slope $\alpha_j + \phi_j$, and over the range $-\infty < Z_t \leq 0$, $g_j(Z_t)$ is linear in Z_t with slope $\alpha_j - \phi_j$.

The estimation results based on the ARIMA(2,0,2)-*t*-EGARCH(1,1) model are reported in Table A.3. The estimated α_1 is no longer significant, showing a vanished sign effect. However, a significant positive value of ϕ_1 indicates that the magnitude effect represented by $\phi_1(|Z_{t-1}| - E|Z_{t-1}|)$ plays a bigger role in the innovation in $\log(\sigma_t^2)$.

We compare the model performances between two types of GARCH models through information criteria, and a *t*-EGARCH(1,1) model is suggested. Note that, as shown in Figure A.2, the QQ plots demonstrate a deviation from the student *t*. In Chen et al. (2017), GARCH and variants such as *t*-GARCH, EGARCH have been reported, and, while they

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

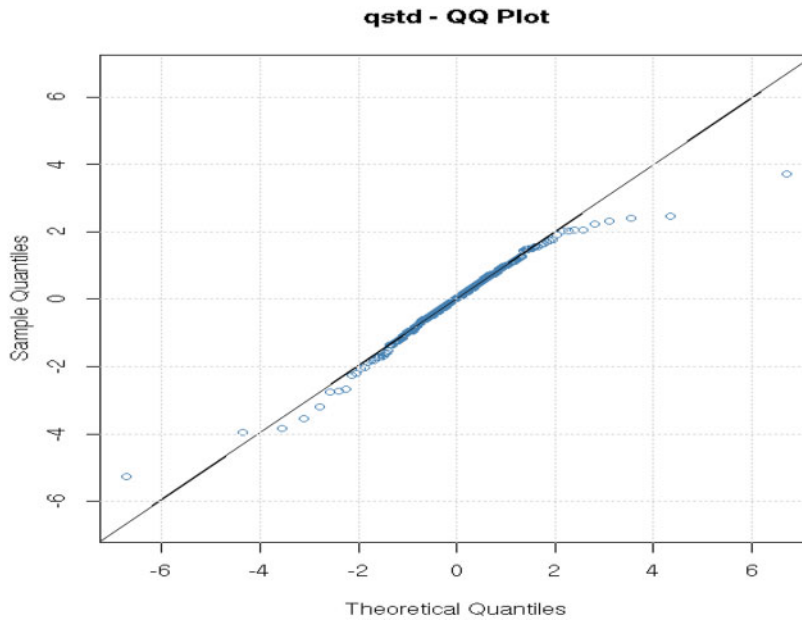

Figure A.2 The QQ plot for BTC based on the residuals of *t*-GARCH(1,1) model.

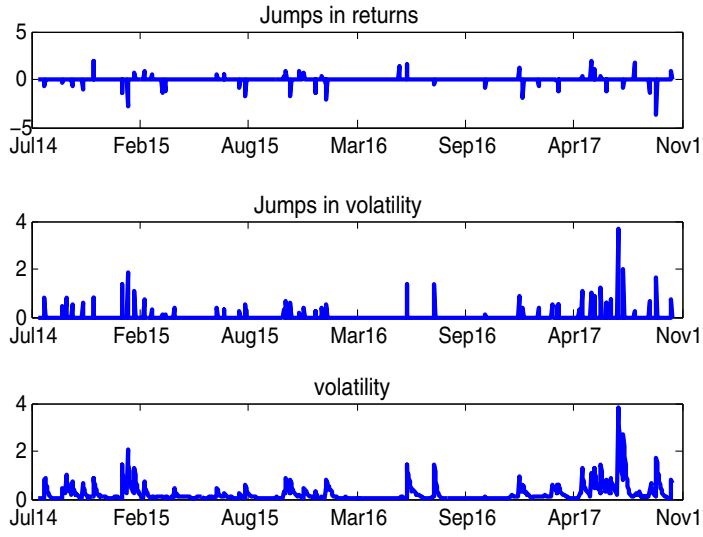

Figure A.3 Jumps estimated in returns and volatility from the SVCJ model: CRIX.

are seen to fit the dynamics of BTC nicely, they still could not handle the extreme tails in the residual distribution. Equipped with these findings and taking into account the occasional interventions, we opt for the models with jumps for better characterization of CC dynamics. The presence of jumps is indeed more likely in this decentralized, unregulated, and

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

illiquid market. Numerous political interventions also suggest the introduction of the jump component into a pricing model.

A.3 CRIX

Appendix presents the empirical results of CRIX covering (1) jumps in returns and volatility from the SVCJ model shown in Figure A.3 and (2) the estimated volatility from the SVCJ and SVJ models shown in Figure 11. (3) The estimated call options across moneyness and time to maturity in Figure 12. In general, a general consistency can be found between CRIX and BTC. Other results are available upon request.

References

Ali, R. B. J. C. R. S. J. 2014. The Economics of Digital Currencies. *Bank of England Quarterly Bulletin* Q3: 276–287.

Asgharian, H., and C. Bengtsson. 2006. Jump Spillover in International Equity Markets. *Journal of Financial Econometrics* 4: 167–203.

Asgharian, H., and M. Nossman. 2011. Risk Contagion among International Stock Markets. *Journal of International Money and Finance* 30: 22–38.

Badev, A., and Chan, M. 2014. "Bitcoin: Technical Background and Data Analysis." *Finance and Economics Discussion Series* n. 2014-104, Divisions of Research and Statistics and Monetary Affairs, Federal Reserve Board, Washington, DC.

Bajgrowicz, P., O. Scaillet, and A. Treccani. 2015. Jumps in High-Frequency Data: Spurious Detections, Dynamics, and News. *Management Science* 62: 2198–2217.

Bandi, F. M., and R. Renò. 2016. Price and Volatility Co-Jumps. *Journal of Financial Economics* 119: 107–146.

Bates, D. S. 1996. Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies* 9: 69–107.

Bates, D. S. 2000. Post-'87 Crash Fears in the S & P 500 Futures Option Market. *Journal of Econometrics* 94: 181–238.

Becker, J., D. Breuker, T. Heide, J. Holler, H. P. Rauer, and R. Böhme. 2013. "Can We Afford Integrity by Proof-of-Work? Scenarios Inspired by the Bitcoin Currency." In *The Economics of Information Security and Privacy*. Springer Berlin Heidelberg, 135–156.

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31: 307–327.

Bouoiyour, J., Selmi R., and Tiwari A.K. 2015. Is Bitcoin Business Income or Speculative Foolery? New Ideas through an Improved Frequency Domain Analysis. *Annals of Financial Economics* 10: 1550002.

Chan, S., J. Chu, S. Nadarajah, and J. Osterrieder. 2017. A Statistical Analysis of Cryptocurrencies. *Journal of Risk and Financial Management* 10: 12.

Chen, C. Y.-H., and C. M. Hafner. 2019. Sentiment-Induced Bubbles in the Cryptocurrency Market. *Journal of Risk and Financial Management* 12: 53.

Chen, S., C. Y.-H. Chen, W. K. Härdle, T. Lee, and B. Ong. 2017. "*A First Econometric Analysis of the Crix Family*." In D.L.K. Chuen and R. Deng (eds.), Handbook of Digital Finance and Financial Inclusion: Cryptocurrency, FinTech, InsurTech, and Regulation. Vol. 1, Academic Press, Elsevier.

Chernov, M., A. R. Gallant, E. Ghysels, and G. Tauchen. 2003. Alternative Models for Stock Price Dynamics. *Journal of Econometrics* 116: 225–257.

Chu, J., S. Chan, S. Nadarajah, and J. Osterrieder. 2017. GARCH Modelling of Cryptocurrencies. *Journal of Risk and Financial Management* 10: 17.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Conrad, C., A. Custovic, and E. Ghysels. 2018. Long-and Short-Term Cryptocurrency Volatility Components: A GARCH-MIDAS Analysis. *Journal of Risk and Financial Management* 11: 23.

da Gama Silva, P. V. J., M. C. Klotzle, A. C. F. Pinto, and L. L. Gomes. 2019. Herding Behavior and Contagion in the Cryptocurrency Market. *Journal of Behavioral and Experimental Finance* 22: 41–50.

Das, S. R., and R. K., Sundaram 1999. Of Smiles and Smirks: A Term Structure Perspective. *The Journal of Financial and Quantitative Analysis* 34: 211–239.

DeLong, J., A. Shleifer, L. Summers, and R. Waldmann. 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy* 98: 703–738.

Duffie, D., J. Pan, and K. Singleton. 2000. Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica* 68: 1343–1376.

Duffie, D., and K. Singleton. 1993. Simulated Moments Estimation of Markov Models of Asset Prices. *Econometrica* 61: 929–952.

Dwyer, G. P. 2015. The Economics of Bitcoin and Similar Private Digital Currencies. *Journal of Financial Stability* 17: 81–91.

ECB 2015. "Virtual Currency Schemes-A Further Analysis." ECB report, February.

Elwell, C. K., M. M. Murphy, M. V. Seitzinger, and E. V. Murphy. 2013. Bitcoin: Questions, Answers, and Analysis of Legal Issues. Congressional Research Service 7-5700. R43339.

Engle, R. F., and V. K. Ng. 1993. Measuring and Testing the Impact of News on Volatility. *The Journal of Finance* 48: 1749–1778.

Eraker, B. 2004. Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. *The Journal of Finance* 59: 1367–1403.

Eraker, B., M. Johannes, and N. Polson. 2003. The Impact of Jumps in Volatility and Returns. *The Journal of Finance* 58: 1269–1300.

Franke, J., W. K. Härdle, and C. M. Hafner. 2019. *Statistics of Financial Markets: An Introduction. 5th edn*. Heidelberg: Springer.

French, K. S. W., and R. Stambaugh. 1987. Expected Stock Returns and Volatility. *Journal of Financial Economics* 19: 3–29.

Gallant, A., and G. Tauchen. 1996. Which Moments to Match?. *Econometric Theory* 12: 657–681.

Geweke, J. 1986. Modelling the Persistence of Conditional Variances: A Comment. *Econometric Reviews* 5: 57–61.

Gourieroux, C., A. Monfort, and E. Renault. 1993. Indirect Inference. *Journal of Applied Econometrics* 8: S85–S118.

Hafner, M. C. 2020. Testing for Bubbles in Cryptocurrencies with Time-Varying Volatility. *Journal of Financial Econometrics* 18: 233–249.

Härdle, W. K., C. R. Harvey, and R. C. Reule. 2020. Understanding Cryptocurrencies. *Journal of Financial Econometrics* 18: 181–208.

Härdle, W. K., and S. Trimborn. 2015. "Crix or Evaluating Blockchain Based Currencies." Technical report, Oberwolfach report 42/2015, The Mathematics and Statistics of Quantitative Risk. DOI: 10.4171/OWR/2015/42.

Hayes, A. S. 2017. Cryptocurrency Value Formation: An Empirical Study Leading to a Cost of Production Model for Valuing Bitcoin. *Telematics and Informatics* 34: 1308–1321.

Heston, S. L. 1993. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies* 6: 327–343.

Hotz-Behofsits, C., F. Huber, and T. O. Zörner. 2018. Predicting Crypto-Currencies Using Sparse Non-Gaussian State Space Models. *Journal of Forecasting* 37: 627–640.

Hou, A. J. 2013. Asymmetry Effects of Shocks in Chinese Stock Markets Volatility: A Generalized Additive Nonparametric Approach. *Journal of International Financial Markets, Institutions & Money* 23: 12–32.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020

Jacquier, E., N. Polson, and P. Rossi. 1994. Baysian Analysis of Stochastic Volatility Models. *Journal of Business & Economic Statistics* 12: 371–389.

Jocod, J., and V. Todorov. 2009. Testing for Common Arrivals of Jumps for Discretely Observed Multidimensional Processes. *The Annals of Statistics* 37: 1792–1838.

Johannes, M., and N. Polson. 2009. "MCMC Methods for Financial Econometrics". In Y. Ait-L.P., Sahalia and Hansen (eds.), *Handbook of Financial Econometrics. Amsterdam: North Holland, pp. 1–72.*

Johannes, M., K. Rohit, and N. Polson. 1999. "State Dependent Jump Models: How Do U.S. Equity Markets Jump?" Working paper, University of Chicago.

Kim, A., S. Trimborn, and W. K. Härdle. 2019. Vcrix-A Volatility Index for Crypto-Currencies. Available at SSRN 3480348. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3480348 (accessed March 8, 2020).

Kristoufek, L. 2013. Bitcoin Meets Google Trends and Wikipedia: Quantifying the Relationship between Phenomena of the Internet Era. *Scientific Reports* 3. Article number: 3415.

Kroll, J. A., I. C. Davey, and E. W. Felten. 2013. The Economics of Bitcoin Mining, or Bitcoin in the Presence of Adversaries. *Proceedings of WEIS* 2013: 11.

Kyle, A. 1985. Continuous Auctions and Insider Trading. *Econometrica* 53: 1315–1336.

Larsson, K., and M. Nossman. 2011. Jumps and Stochastic Volatility in Oil Prices: Time Series Evidence. *Energy Economics* 33: 504–514.

Merton, R. C. 1976. Option Pricing When Underlying Stock Returns Are Discontinuous. *Journal of Financial Economics* 3: 125–144.

Nelson, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica* 347–370.

Pantula, S. G. 1986. Modelling the Persistence of Conditional Variances – Comment. *Econometric Reviews* 5: 71–74.

Rogojanu, A., and Badea, L. 2014. The Issue of Competing Currencies. Case Study-Bitcoin. *Theoretical and Applied Economics* 21: 103–114.

Scaillet, O., A. Treccani, and C. Trevisan. 2020. High-Frequency Jump Analysis of the Bitcoin Market. *Journal of Financial Econometrics* 18: 209–232.

Schmidt, E., and J. Cohen. 2017. *The New Digital Age Reshaping the Future of People, Nations and Business*. Hodder & Stoughton Limited.

Schwartz, E., and A. Trolled. 2009. Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives. *Review of Financial Studies* 22: 4423–4461.

Schwert, W. 1989. Why Does Stock Market Volatility Change over Time. *The Journal of Finance 44:* 1115–1153.

Segendorf, B. 2014. What is Bitcoin?. *Sveriges Riksbank Economic Review* 2: 71–87.

Todorov, V., and G. Tauchen. 2010. Activity Signature Functions for High-Frequency Data Analysis. *Journal of Econometrics* 54: 125–138.

Trimborn, S., and W. K. Härdle. 2018. Crix an Index for Cryptocurrencies. *Journal of Empirical Finance* 49: 107–122.

Tsay, R. S. 2005. *Analysis of Financial Time Series, 3rd edn*. Hoboken, NJ: John Wiley & Sons, Inc.

Weber, B. 2016. Bitcoin and the Legitimacy Crisis of Money. *Cambridge Journal of Economics* 40: 17–41.

White, H. 1982. Maximum Likelihood Estimation of Misspecified Models. *Econometrica* 50: 1–25.

Downloaded from https://academic.oup.com/jfec/article-abstract/18/2/250/5837734 by University of York user on 20 May 2020