Article:

Broekaert, JB orcid.org/0000-0002-4039-8514, Busemeyer, JR and Pothos, EM (2020) The Disjunction Effect in two-stage simulated gambles. An experimental study and comparison of a heuristic logistic, Markov and quantum-like model. Cognitive Psychology, 117. 101262. ISSN 0010-0285

https://doi.org/10.1016/j.cogpsych.2019.101262

The Disjunction Effect in two-stage simulated gambles. An experimental study and comparison of a heuristic logistic, Markov and quantum-like model. REVISED

J.B. Broekaert, J.R. Busemeyer

Indiana University, Department of Psychological and Brain Sciences, Bloomington, IN 47405-7007, USA.

E.M. Pothos

City, University of London,Department of Psychology, London, EC1V 0HB, UK

Abstract

Savage's rational axiom of decision making under uncertainty, called the 'Sure Thing' principle, was purportedly falsified in a two-stage gamble paradigm by Tversky and Shafir (1992). This work revealed that participants would take a second-stage gamble for both possible outcomes of the initial-stage gamble, but would significantly depress this choice when *no* information was available on the outcome of the initial-stage gamble. Subsequent research has reported difficulty to replicate this Disjunction Effect in the two-stage gamble paradigm. We repeated this simulated two-stage gamble paradigm in an online study (N=1119) but adapted the range of payoff amounts, and controlled the order of the blocks of two-stage gambles with, respectively without, information on the outcome of the first-stage gamble. The main empirical contributions of this study are that more risk averse participants produced i) a reliable order effect in relation to the Disjunction Effect and the violation of the Law of Total Probability, and ii) a novel inflation effect on gambling in the Unknown outcome condition analogous but opposite to the Disjunction Effect when Unknown outcome conditioned two-stage gambles precede the Known outcome conditioned ones. By contrast, we found that less risk averse participants produced neither of these effects. We discuss the underlying choice processes and compare the effectiveness of a logistic model, a Markov model and a quantum-like model. Our main theoretical findings are i) a standard utility model and a Markov model using heuristic linear utility, contextual influence and carry-over effect cannot accommodate the present empirical results, and ii) a model based on quantum dynamics, matched in form to the Markov model, can successfully describe all major aspects of our data.

Keywords: Two-stage Gambles, Disjunction Effect, Order Effect, carry-over Effect, Markov Process, Quantum Probability, Interference Effect

1. Introduction

At the core of the question of people's rationality when choosing between risky alternatives is Savage's truism: should you choose to do something in a given circumstance and also choose to do so when that circumstance does not occur, then you should definitely take that action even when no information about that circumstance is available (Savage, 1954). This seemingly straightforward normative rule is known as the Sure Thing Principle, and unexpectedly not withstanding its name nor mundanity, there have been numerous reports on the violation of this principle in the literature (Tversky and Shafir, 1992; Shafir and Tversky, 1992; Croson, 1999; Busemeyer et al., 2006a; Lambdin and Burdsal, 2007; Pothos and Busemeyer, 2009; Pothos et al., 2011; Busemeyer and Bruza, 2012). Our focus will be on a choice between risky prospects in a controlled environment, namely the two-staged gamble paradigm of the seminal paper of Tversky and Shafir (1992). In that paradigm each participant is exposed to three conditions of a choice between risky prospects. In each of the three scenarios the gamble was the same: even odds to win \$200 or lose \$100. Tversky and Shafir offered the participant to take the gamble or stop it, just after having taken that identical gamble. More specifically, they proposed three scenarios for that repeated gamble: one with a Win outcome of the initial gamble, one with a Lose outcome of the initial gamble, and a third scenario in which the outcome of the initial gamble was not disclosed. In this third scenario the initial stage gamble outcome was therefore Unknown to the participant. Their study fascinatingly showed that the probability of taking the second gamble was high at .69 and .59 when the first-stage gamble had been Won, respectively Lost, while a significant depression to .36 was observed for the probability when the outcome of the first-stage gamble was Unknown (Table 1). Tversky and Shafir argued their participants were inclined to take the second-stage gamble under Win condition, and also under Lose condition, but would not take the second-stage gamble under Unknown previous outcome condition. They reasoned this was a violation of the Sure Thing Principle, and named the occurrence of this depressed gamble probability in the Unknown previous outcome condition a Disjunction Effect. Since their original accomplishment, the corroboration of the Sure Thing

	Within-subject (N = 98)	Between-subject (N = 3*71)
Win	.69	.69
Lose	.59	.57
Unknown	.36	.38

Table 1: Tversky and Shafir (1992) observed average two-stage gamble probabilities for Known outcome and Unknown outcome conditions. Within-participants version: Win condition, followed by the Lose condition and finally the Unknown condition with respectively 7 and 10 days in between. The between-participants version shows closely matching results.

Principle in the two-stage gamble paradigm has been rather uncertain. Kühberger et al. (2001) replicated the two-stage gamble experiment but found no significant indication of the Disjunction Effect, neither when using the original payoff scheme (the equivalent in Austrian Shilling), nor when using lower payoffs (both executed in between-participants design), and nor for lowered payoff in a within-participants design, nor when real monetary payoff was used. Later Lambdin and Burdsal (2007) reported that the outcome results on the Disjunction Effect by Kühberger et al. (2001) may still be significant, by arguing that only the within-participants design should be considered relevant and that only those participants that play on Win and play on Lose should be retained to test the Sure Thing Principle. We are critical of the latter claim since it erroneously identifies such outcome patterns as deterministic cognitive responses to the proposed gamble instead of stochastically realised outcome responses (Subsection 1.1). To clarify our discussion of the relation between the Sure Thing Principle (STP), the Disjunction Effect (DE) and the Law of Total Probability (LTP) we expound the definition for each explicitly.

First, the Sure Thing Principle (STP) is the rational rule that a decision that is made under the assumption of either of two mutually exclusive and exhaustive events should then also be made when no information is available on these complementary situations. In particular, winning or losing the gamble are the disjoint and exhaustive outcomes that make up the full event space of the first-stage gamble.

Second, the Law of Total Probability (LTP) expresses the principle that the marginal probability of an event is the weighted average of the conditional probabilities of the event on disjoint conditions. Again, since winning or losing the initial-stage gamble make up all the disjoint conditions, the marginal gamble probability satisfies

$$p(gamble|W\,or L) \quad = \quad p(W)\,p(gamble|W) + p(L)\,p(gamble|L), \tag{1}$$

where the weights for both disjoint conditions satisfy $p(W) + p(L) = 1$. One needs to heed that the cognitive representation of the Unknown condition may not fully conform to the *formal* disjunction $W\,or L$, of the cognitive Win representation and the Lose representation. The semantics of the chosen formulation of the Unknown condition could cause the observed conditional probability relations to be at variance with the Law of Total Probability, eq. (1). Due to this issue, in our study we have meticulously formulated the description of the Unknown condition in the two-stage gamble, Section 2, to express it exactly as the *disjunction*, 'or', of the Win and Lose condition. In this article every mention of the 'violation of the LTP' should more carefully be understood as the 'violation of a prediction derived from the LTP'.

Third, the Disjunction Effect (DE) is a *statistical* pattern in the empirical conditional gamble probabilities. The Disjunction Effect can be expressed as

$$p(gamble|W\,or L) < p(gamble|W, X) \quad \text{and} \quad p(gamble|W\,or L) < p(gamble|L). \tag{2}$$

The Disjunction effect is essentially the finding that the marginal probability $p(gamble)$, which is formally equal to $p(gamble|W\,or L)$, is not bounded between the conditional probabilities, $p(gamble|W)$ and $p(gamble|L)$. Therefore the Disjunction Effect is a violation of a prediction derived from the Law of Total Probability.

Notice that Tversky and Shafir considered the Disjunction Effect to be the statistical pattern in which a majority of participants take the decision to gamble in the Known cases, but most participants reject the gamble in the Unknown case; $\{p(gamble|W), p(gamble|L)\} > .5$ and $p(gamble|W\,or L) < .5$ (Tversky and Shafir, 1992, p.306). We have defined the Disjunction Effect, eq. (2), more generally as any deflative violation of the LTP. Our definition of the DE encompasses the specification by Tversky and Shafir, while allowing us to use the designation for the exact same statistical anomaly in empirical gamble probabilities occurring for any particular payoff size – as we will show to be the case in our observations.

It is clear that the Sure Thing Principle and the Disjunction Effect are phenomena that are not necessarily related, as was pointed out by Lambdin and Burdsal (2007) and which we will further discuss in Subsection (1.1).

One key objective of the present work is to provide a thorough examination of the Disjunction Effect that resolves the above ambiguities, partly by extending the original paradigm by incorporating factors which might offer insight regarding the inconsistencies in related empirical work. Notably, we aim to examine whether the Disjunction Effect might be dependent, i) on the size of the gamble payoff, ii) on the order of presentation of the second-stage gambles with Win, Lose and Unknown previous outcome conditions, iii) on the context brought about by how second-stage gambles are conditioned in a particular block and, iv) on the risk attitude of the participant. In order to realize these additional manipulations, as well as ensure variability regarding the pertinent individual characteristics of participants, we decided for a large sample, online implementation of our main experiment (which follows a smaller exploratory one). This type of approach –using 'Mturk' (Amazon Mechanical Turk) and a short time constraint on the experiment– required us to use a design with a small number of choices provided by each participant, so that an emphasis on analysis by grouped data was built-in into our method.[1] However a partitioning of participants by risk attitude (and by set of played gamble patterns, SM 4) will still allow a more granular analysis of individual differences among participants.

In brief, our study did find a Disjunction Effect which depends on the risk attitude of the participant and on the order of presentation of the two-stage gambles with Known or Unknown previous outcome conditions, Section (3). Remarkably, for more risk averse participants we even found that the *direction* in which the prediction derived from the law of total probability is violated is directly related to the order of Known outcome and Unknown outcome second-stage gambles. No evidence however was found for the violation of the Sure Thing Principle itself.

From a theoretical perspective we found that the model of Tversky and Shafir (1992) based on Prospect Theory – the original explanation they offered for the Disjunciton Effect– cannot properly explain the observed data from our present study, Section (4) (and Appendix B). Previously empirical observations with 'non-classical' probability structure –as in the present two-stage gamble experiment– have been modeled using quantum probability theory (Pothos and Busemeyer, 2009; Accardi et al., 2009). But neither of these models for the two-stage gamble paradigm covered the order effects which we observed at present.

We address this theoretical challenge by developing two stochastic models, premised on an assumption that any choices are probabilistic functions of latent, dynamically evolving belief-action states informed by outcome conditions and payoff values. Both these models are decision process models, one based on the classical framework of Markov dynamics and the other on quantum dynamics.

The Markov dynamical process model we employ is based on the Kolmogorov differential equation to describe the dynamics and adheres to classical probability theory (Sonnenberg and Beck, 1993; Busemeyer et al., 2009). This is formally related to a diffusion model (for example, Ratcliff's model, Smith and Ratcliff (2015)). A diffusion model is also a Markov process which uses the Kolmogorov differential equation. The difference between our Markov model and a diffusion model is that we are using a discrete state Markov model and the diffusion model is a continuous state model. Both models finally produce a predicted probability of making a response. We use that

[1] A Bayesian cognitive model based on the idea that each participant has a prior probability of gambling (which can change from experienced wins or losses) is complicated by the fact that a simple updating scheme cannot cover the observed inflation and deflation of probabilities in the Unknown condition nor the order effect. Should an updating scheme provide a change of a prior probability to gamble depending on the first-stage outcome condition - *up* on W, *down* on L and *neutral* on U - then a net approximately zero effect would result over the block of Known-outcome gambles. No deflation nor inflation of the Unknown-outcome gamble probability would therefore result from this updating scheme. Moreover, a basic Bayesian cognitive model does not a priori offer an implementation for order effects, and requires supplementary events describing the order of choices (Trueblood and Busemeyer, 2011).

predicted probability to calculate the likelihood of the observed response (subsection 4.2). Note, empirical results – superficially at least – at odds with the prediction derived from the Law of Total Probability may undermine our expectations of the suitability of a classical model. Nevertheless it is possible that violations of the predictions derived from the Law of Total Probability could be classically accounted for in a Markov model – for example through the inclusion of noise in the mapping between belief-action states and decisions.

The quantum model was designed in an analogous way to the Markov model. It is also a dynamic process model. Its dynamics is based on the Schrödinger differential equation. A quantum-like model is motivated by the observation of classically 'irrational' choice behaviour. A quantum-like model is based on logical rules alternative to those from Boolean logic, allowing for events to be order dependent (noncommutative) and context dependent (non-distributive). In full deployment, quantum-like models are both probabilistic and dynamic, and as such can reflect the stochastic process of decision making, subsection (4.3).

For both the Markov and the quantum model, dynamical processes were driven by mathematical objects embodying parameters relevant to the psychology of the problem (e.g., heuristic utility, contextuality, carry-over and belief mixing), which produce a state of probabilities for different choices. Crucially, the quantum model represents an alternative philosophy in how the dynamics of a decision evolve: with a Markov model, at any given point in time there is a definite value to the internal state that will ultimately drive the response. With the quantum model it is impossible to assign a specific value to the internal state, prior to a response. That is, the response 'collapses' the inherent uncertainty in the quantum state; in quantum theory, a decision/response/measurement brings potentiality into being. This special kind of quantum uncertainty is the key characteristic of the theory which leads to the emergence of interference effects (see also, Busemeyer et al. (2006b)).

For both the Markov and the quantum-like models, we solve the dynamical differential equations up to a certain time point to compute the predicted probability of a response at that time point, which can be used to calculate the likelihood of the observed responses. Note, we did not collect response times. In past work on the disjunction effect, no one has looked at response times. Modeling data on response times is something to look at in future research. As a final note, regarding how a response is generated from predicted probabilities, we assume that if the model produces a state which indicates e.g. $p(gamble) = 0.7$, a response is sampled by a Bernoulli process producing either of the responses 'gamble' or 'stop' with probabilities 0.7 and 0.3 respectively. Therefore, no additional mechanism is required to go from probabilities to responses and model probabilities translate directly to predictions about choice proportions. Note, diffusion models are always applied to choice proportions in exactly the same way.

Finally, a third model is included as a comparison baseline to the dynamical process models, Section (4.4). The baseline model is essentially a Logistic regression which, as in latent trait modeling, will relate heuristic utility to theoretical gamble probabilities and which is further parametrized to mimic contextual and carry-over features shared by the two dynamical decisions models, Section (4.4).

1.1. The Sure Thing principle, the Disjunction Effect and the Law of Total Probability.

The Disjunction Effect and the violation of the Sure Thing Principle are not necessarily related. In principle the aggregation of individual gamble strategies could mask the violation of the Sure Thing principle, and vice versa, a dominant strategy of violation of the Sure Thing principle should not necessarily lead to a Disjunction Effect in the aggregate data. Tversky and Shafir asserted a strong relation between the occurrence of the violation of the Sure Thing principle and the Disjunction effect. In support of this they observed that when the Disjunction Effect occurred the dominant gamble pattern followed by their participants was 'gamble - gamble - stop', where the notation order of the decisions always follows the fixed order 'Win - Lose - Unknown' of the conditions. We will use a letter notation to denote these choices as $(g|W, g|L, s|U)$, where 'g' stands for 'do the gamble' while 's' stands for 'stop the gamble'. In this notation adhering to the STP by replying 'gamble - gamble - gamble' corresponds to the pattern $(g|W, g|L, g|U)$. In their study precisely 26 out of 98 participants (Table 2, col. TS92) violated the STP by performing pattern $(g|W, g|L, s|U)$, while by the same token 14 out of 98 abided to the STP by performing $(g|W, g|L, g|U)$. Assessing the

W	L	U	TS92 N = 98	KKP-Exp3 N = 35	LB-Coin N = 55	BBP-Exp1 N_{att} = 94	BBP-Exp2$_{KtoU}$ N_{att} = 407	BBP-Exp2$_{UtoK}$ N_{att} = 415
g	g	g	.14	.29	.22	.23	0.371*	0.323*
s	g	g	.06	.0	.04	.24*	0.194	0.195
g	s	g	.11	.11	.11	.07	0.076	0.113
s	s	g	.04	.03	.02	.06	0.032	0.070
g	g	s	.27*	.09	.16	.05	0.081	0.065
s	g	s	.12	.0	.05	.14	0.086	0.068
g	s	s	.17	.31*	.15	.10	0.076	0.077
s	s	s	.08	.17	.25*	.11	0.084	0.089

Table 2: Observed gamble patterns with notation order Win-Lose-Unknown in two-stage gamble experiments. TS92 is the original Tversky and Shafir study with consecutive delayed presentation of conditions(Tversky and Shafir, 1992). KKP-Exp3 is the within-participants ('third') experiment of Kühberger, Komunska, and Perner with random order juxtaposed conditions (Kühberger et al., 2001). LB-Coin is the coin experiment of Lambdin and Burdsal with random order juxtaposed conditions (Lambdin and Burdsal, 2007). BBP-Exp1 is extracted from our Experiment 1 study for payoff parameter $X = 2$, with randomly ordered conditions. BBP-Exp2$_{KtoU}$ are the extracted $X = 2$ results from our Experiment 2 study with blocked Win and Lose outcome conditioned gambles preceding the block of Unknown outcome gambles. BBP-Exp2$_{UtoK}$ has the blocked conditions in reverse order. Dominant patterns are tagged with an asterisk.

validity of the STP by a frequency comparison treats each outcome gamble pattern as composed of three logically correlated –consequentially evaluated– rational decisions. We assert that each gamble decision, hence each gamble triplet, should be considered a stochastic realisation of a latent trait.

What choice patterns then lie at the origin of the Disjunction Effect? In terms of gamble patterns, how does a violation of the Law of Total Probability come about? Strictly the Law of Total probability can be violated in two ways, the gamble probability under Unknown outcome condition either undershoots or overshoots the span of the probabilities for gambling under Known outcome conditions.

The occurrence of these violations is related to the frequency of specific gamble patterns.[2] With three conditions in the two-stage gamble paradigm there are eight possible gamble patterns in

[2]A single binary response pattern to gambles neither indicates a violation of nor consistency with the LTP. The intrinsic probability –the individual's probability to gamble, given some conditions (e.g., payoffs)– generates binary random variable responses. That is, a priori any *individual* pattern is not a reliable indicator with respect to putative violations of the LTP nor the STP. The intrinsic probabilities produce binary choices that lead to choice proportions when averaged across participants and a law of large numbers assures the convergence of observed choice proportions to the intrinsic probabilities.

total, Figure 1. Each conditional gamble probability can be expressed as the marginal of the joint probabilities identified by their corresponding gamble patterns

$$p(g|W) = p_{ggg} + p_{gsg} + p_{ggs} + p_{gss}, \tag{3}$$
$$p(g|L) = p_{ggg} + p_{sgg} + p_{ggs} + p_{sgs}, \tag{4}$$
$$p(g|U) = p_{ggg} + p_{gsg} + p_{sgg} + p_{ssg}. \tag{5}$$

From which it is clear that e.g. an increased presence of the $(g|W, g|L, s|U)$ pattern could push the span of $p(g|W)$ and $p(g|L)$ above $p(g|U)$ to create a DE. But it is also clear that a few other patterns could cause this to happen. A Disjunction Effect expressed as a deflative violation of the

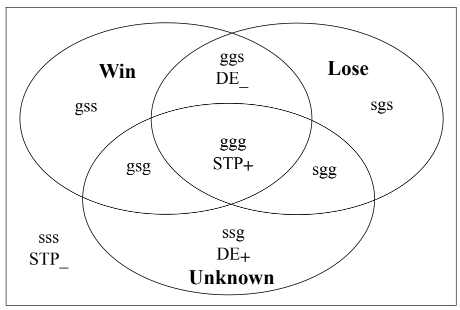

Figure 1: Gamble patterns in WLU notation order. The Sure Thing Principle pattern $(g|W, g|L, g|U)$ is violated by the Disjunction Effect pattern $(g|W, g|L, s|U)$, denoted respectively as STP_+ and DE_-. The index $+$ indicates that the gamble is accepted under U, while the $-$ index indicates it is not accepted. The decision-mirrored Sure Thing Principle pattern $(s|W, s|L, s|U)$ is violated by the mirror Disjunction Effect pattern $(s|W, s|L, g|U)$.

LTP, eqs. (2), can be expressed as

$$p_{ggs} - p_{ssg} \quad > \quad p_{sgg} - p_{gss}, \tag{6}$$
$$p_{ggs} - p_{ssg} \quad > \quad p_{gsg} - p_{sgs}, \tag{7}$$

where we substituted the marginal expressions using eqs. (3, 4, 5). These inequalities show that the Disjunction Effect has no tie to the frequency of the Sure Thing Principle pattern, $(g|W, g|L, g|U)$, itself. It is contrary to the original analysis based on the frequencies of patterns $(g|W, g|L, g|U)$ and $(g|W, g|L, s|U)$ by Tversky and Shafir (1992) and some of the corresponding subsequent work (Lambdin and Burdsal, 2007). Instead we remark that all of the gamble patterns $(g|W, g|L, s|U)$, $(s|W, g|L, s|U)$ and $(g|W, s|L, s|U)$ will contribute to deflation, while all of $(s|W, s|L, g|U)$, $(s|W, g|L, g|U)$ and $(g|W, s|L, g|U)$ contribute to inflation of the gamble probability under Unknown outcome condition. That is, except for the neutral patterns $(g|W, g|L, g|U)$ and $(s|W, s|L, s|U)$, each gamble pattern contributes to the average gamble probability under Unknown outcome towards either inflative ('upward') or deflative ('downward') effect on $p(g|U)$ with respect to $p(g|W)$ or $p(g|L)$, and potentially average to a violation of the Law of Total Probability in either sense (see SM 3). In a more detailed elaboration of participant behaviour we will use the inflative/deflative potential of each gamble pattern to characterise participants (SM 8).

2. Material and Methods.

The application of an experimental gamble paradigm to evaluate Expected Utility Theory goes back to Edwards (1954) and Samuelson (1963). The specific method to probe the Disjunction Effect in our study stays close to the Tversky and Shafir (1992) two-stage gamble paradigm. The major difference consists of examining whether the Disjunction Effect might be dependent on the size of payoff of the proposed gamble, on the order in which the Win, Lose and Unknown outcome conditioned two-stage gambles are presented, on the context of other two-stage gambles in which a gamble is taken and on whether participants are more or less risk averse. Also the fact that the participants were crowdsourced with Mturk and the short time frame in which all the gambles were performed will be considered in the interpretation of the results.

In order to understand the potential factors that lead up to the Disjunction Effect we also control for between-participants and within-participants variation of the study design. From the between-participants design for conditions –{W,L,U}– of the first-stage outcome we could infer whether the Disjunction Effect would emerge even if each participant is exposed to only one specific level of outcome condition. We have detailed in Section 1.1 however that an analysis of the Sure Thing Principle itself is not possible in a between-participants design since it needs to be interpreted as a consequential and rational decision at the level of the individual participant and so requires exposure to the three outcome conditions. In a within-participants design a participant will be exposed to all three conditions, which would in principle allow us to analyse the Sure Thing Principle over and above the Disjunction Effect. But at the same time a within-participants design may suppress the Disjunction Effect – and also the STP– or, on the other hand this design may altogether induce these effects by cross-correlating the participant's decisions over the different conditions. To assess these issues we have explored both within- and between-participants designs in our study, as well as considered other key factors which may inform our understanding of the DE, across Experiments 1 and 2.

Experiment 1 essentially tested the gamble paradigm i) in a within-participants design for all outcome conditions randomly mixed and, ii) in a between-participants design for Win and Lose first-stage gamble outcome conditions in comparison with Unknown first-stage gamble outcome conditions. Contrary to Experiment 1, in Experiment 2 we explored the impact of ordering of first-stage gamble outcome conditions. This was done using a design in which either the first-stage Win and Lose outcome gambles preceded the first-stage Unknown outcome gambles, or the other way round with the first-stage Unknown outcome gamble block preceding the first-stage Win and Lose outcome gamble block.

For both experiments the script of the task was developed in Qualtrics and transferred to MTurk for online data gathering. The participants taking the survey were MTurk Workers located in the US and received \$.90 for their participation. Participants needed to be at least 18 years of age and have a good command of the English language. Precautions against bot responses included an upfront Captcha test –a 'Completely Automated Public Turing test to tell Computers and Humans Apart', and post hoc checking of known GPS- location anomalies (multiple location repetitions or locations documented for bot fraud). Participant engagement was monitored by the inclusion of 'hidden' attention tests. These tests were presented as a normal second-stage gamble but had one sentence inserted that indicated that the present gamble was in fact an attention test and that the participant needed to respond in a specified manner.

All participants were informed that all amounts won or lost in each gamble needed to be imagined, there would be no monetary implication in reality. There were four types of gamble; the second-stage gambles conditioned on Win, Lose, Unknown and, the single-stage (unconditional) gamble.

Participants saw the following text for the various kinds of gambles: [3]

Second-stage gamble, Win [Lose] condition, with payoff parameter X in $\{.5, 1, 2, 3, 4\}$

You just played a new game that gave you a chance
to win $\$100X$ *on heads and to lose* $\$100X/2$ *on tails.*
You tossed the coin and **won** $\$100X$ [**lost** $\$100X/2$].
You are now offered an identical gamble:
– On heads, you win $\$100X$.
– On tails, you lose $\$100X/2$.
Will you toss the coin or not?

toss the coin ○ stop playing ○ (8)

Second-stage gamble, Unknown condition, with payoff parameter X in $\{.5, 1, 2, 3, 4\}$

You just played a new game that gave you a chance
to win $\$100X$ *on heads and to lose* $\$100X/2$ *on tails.*
You tossed the coin but you will **not know** *whether*
you have **won** $\$100X$ *or* **lost** $\$100X/2$ *until you make your next decision.*
You are now offered an identical gamble:
– On heads, you win $\$100X$.
– On tails, you lose $\$100X/2$.
Will you toss the coin or not?

toss the coin ○ stop playing ○ (9)

The single-stage gamble consisted of a gamble without any information that would result from a previous gamble. In practice it was presented as the last four lines of the Known-outcome second-stage gamble but with the word 'identical' replaced by 'new'. Notice there are five levels of payoff, the gamble with lowest value of the payoff parameter $X = .5$ corresponds to Win \$50 or Lose \$25, while at its highest value $X = 4$ the gamble corresponds to Win \$400 or Lose \$200.

First **Experiment 1** was carried out to compare gamble decisions in a between and within-participants design of the first-stage gamble outcome conditions, as closely replicating Tversky and Shafir (1992) as possible, but with the addition of multiple, variable payoff amounts. Experiment 1 had three participant groups assigned to three different tasks. One group was assigned to the Win and Lose conditions for all values of the payoff parameter X and also took the single-stage gambles, all in random order of outcome conditions and payoff amounts (N=118). This group received 10 second-stage gambles, of which 5 were Win-conditioned and 5 were Lose-conditioned. They also received 5 single-stage gambles. Out of 168 participants 118 passed the attention test. For the entire

[3] We note that the wording "stop playing", to mean "do not gamble on this particular gamble" would not appear ambiguous to participants. Participants were first exposed to three explained gambles and practiced three gambles. All of these used the same wording and clearly showed that "stop playing" did not exit them from the survey, and did not apply to some set of gambles, but instead completed the consideration of the current gamble and allowed participants to proceed to the next gamble. Additionally, each new second-stage gamble would always begin with the sentence "You just played a new game that [. . .]", which also emphasizes that "stop playing" only applies to the present gamble.

task, participants required a median time of $461s$. The mean age of the participants was $35.2y$ while the random assignment of participants produced a gender skewed participant cohort, $m_{gender} = 0.60$ (male = 1, female = 0). A second group was assigned to the Unknown outcome condition for all values of the payoff parameter X and also took the single-stage gambles, all in random order of outcome conditions and payoff amounts. This group of participants also received 5 single-stage gambles, while, regarding second-stage gambles, participants received only the 5 Unknown gambles, Out of 134 participants 114 passed the attention test. For the entire task, participants required a median time of $460s$. The mean age of the participants was $36.5y$, again the cohort was slightly gender skewed, $m_{gender} = 0.44$.

The third group took the two-stage task in all three outcome conditions and for all values of the payoff parameter X, all in random order. This group of participants received 5 Win, 5 Lose and 5 Unknown conditioned second-stage gambles, but did not take any single-stage gambles. Out of 126 participants 94 passed the attention test. For the entire task, participants required a median time of $512s$ to finish the task. This group had a mean age of $35.1y$, and a mean gender of 0.44.

Experiment 1 had a single Attention check and those participants who failed it were eliminated from the analysis. This check appeared as a normal second-stage gamble but had a supplementary sentence towards the end of the text on the screen that informed that this particular game (with high payoff) was an attention test and that the participant had to respond mandatorily by clicking the gamble button.

Note, surveying the results of Experiment 1, an indication for a violation of the LTP by a probability discrepancy about the size 0.1 was estimated from the data (see below and Figure S5 in Supplementary Materials, SM 9). For Experiment 2 the number of required participants was estimated accordingly. With a hypothetical average gamble probability of $p \approx .5$ and targeted standard error of $SE \approx 0.025$ the size of the sample, using $N = p \cdot (1-p)/SE^2$, was $N \approx 400$. With about 1 in 3 participants missing the attention test, for each between-participants condition –corresponding to the ordering of the Known outcome gambles relative to the Unknown outcome ones– we would require about 600 participants, and for the full experiment twice as many participants would be needed.

In **Experiment 2** the task progression was structured by grouping trials in blocks of specifically conditioned gambles. One block contained randomly ordered Win and Lose conditioned second-stage gambles, and also single-stage gambles. The other block contained randomly ordered Unknown outcome conditioned second-stage gambles and again single-stage gambles (see diagram Figure 2). All these gambles appeared in five variations, based on the five values of the payoff parameter X. The participants had to reply by clicking the radio button under "toss the coin" or "stop playing", after which a new screen appeared with a new gamble.

In Experiment 2, the examination of the DE is *within*-participants for the $\{W, L, U\}$ conditions, but it is *between*-participants for the key manipulation of order, that is, whether participants first saw the second-stage gambles with a known outcome for the first-stage gamble or they saw first the second-stage gambles with an unknown outcome for the first-stage gamble. All participants received 15 second-stage gambles, composed of 5 Win, 5 Lose and 5 Unknown gambles. They further received 10 single-stage gambles; 5 of these single-stage gambles were presented together with Known second-stage ones and 5 of these with Unknown ones. In total the block with Known outcome conditions - Win or Lose - thus had fifteen gambles, while the block with Unknown outcome conditions had ten gambles.

Each participant was randomly assigned to an ordering of first-stage outcome conditions: either first the Known block and then the Unknown block, or first the Unknown block and then the Known block. These two ordering conditions will be referred to as the 'K-to-U' flow and 'U-to-K' flow respectively. Notice that each flow order stands as a within-participants design evaluation of the Disjunction Effect. A comparison of 'K-to-U' and 'U-to-K' order effects occurs between participants.

Out of N=1230 participants having taken the survey for Experiment 2, the response data of 65 participants needed to be eliminated because the task ordering information was not recorded and data from 46 more participants were deleted because the response records were not complete in the Win, Lose or Unknown fields. This left N=1119 complete participant records divided over both gamble order conditions, of which N=822 passed the attention tests. Experiment 2 included two attention checks, both presented in the format of second-stage gambles (one task was in the Known context and the other in the Unknown one). The specific format of the attention checks was as above. Participants that failed to correctly respond to these attention tasks were eliminated from the analysis. The choice behaviour of participants that failed the attention test (N=297) is discussed in more detail in the Supplementary Materials section, (SM 11). The gamble probabilities of these participants show indifference to payoff amounts, even for the 'More risk averse' group where a strong dependence on payoff is expected, see SM 4. This payoff indifference shows the implemented attention test correctly identifies participants that engage only superficially with the gamble descriptions and warrants their removal from the main analysis.

Finally we remark that, separated by order condition, the demographics were sufficiently similar over both order conditions. The 'K-to-U' flow received 407 participants who took a median time of 562s to finish the task. Their median age was 34y and mean gender .543. The 'U-to-K' flow received 415 participants who took a median time of 539s to finish. Here the median age was 35y and the mean gender was .489. A non-significant gender bias was present in the random assignment of participants in the U-to-K vs K-to-U flows.[4]

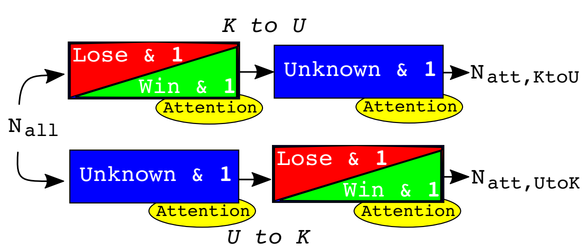

Figure 2: Blocked conditions and flow order of the two-stage gamble Experiment 2: participants were randomly assigned to the 'U-to-K' or 'K-to-U' flow. In the U-to-K flow participants first took all five second-stage gambles with Unknown outcome information, and then proceeded to take all ten second-stage gambles with Known outcome information. In the K-to-U order, the order of the two blocks was reversed. In each block each participant also took five single stage gambles and was exposed to an attention test. Each flow order leads to a within-participants design for the evaluation of the Disjunction Effect. The evaluation of the order effect occurs in a between-participants design.

We point out that in our experiment no actual rewards were being given nor were true monetary risks being taken by the participants. Only a flat fee for participation to the task was given

[4]Based on contingency counts a Fisher test for non-random association between flow order and gender showed $p = .13$ (two-tailed), with odds ratio 0.81 and confidence interval CI = [0.61, 1.06] for $\alpha = .05$.

to each participant. There is no evidence that true monetary rewards would produce less random performance in this paradigm. The pioneering study of Tversky and Shafir (1992), which showed a strong DE, did not involve real monetary risks, and the same applies to most subsequent replications. An exception is the work of Kühberger et al. (2001) who used real monetary risks in one of their conditions. Specifically, their procedure involved tricking students into risking their proper money first and then surprise remitting their debts at the end. [5] At face value, such a procedure should encourage more attentive behavior. However, Kühberger et al. (2001) did not find any differences in behavior between the condition with real monetary risks and a condition with gambles on hypothetical money. Such a powerful null result casts doubt on the effectiveness of (small) monetary risks and rewards in affecting participants' behavior in the two-stage gamble paradigm. This conclusion is consistent with the general impression that small monetary incentivization is not effective in biasing behavior. For example, Camerer and Hogarth (1999) concluded that "there is no replicated study in which a theory of rational choice was rejected at low stakes in favor of a well-specified behavioral alternative and accepted at high stakes" (p.23)

3. Experimental results.

3.1. Experiment 1

The within-participants segment of **Experiment 1** showed no indication for a Disjunction Effect at any of the payoff values of X (see Figure S5 in SM 9). We recall that contrary to the approach of Tversky and Shafir (1992) we distinguish between the violation of the LTP and the violation of the STP, Section 1.1. The aggregate probability to take the second-stage gamble in the Unknown previous outcome condition consistently satisfied the LTP over the payoff range. Also the gamble pattern distributions showed no indication for STP violation, which should have been apparent through the statistical dominance of gamble pattern $(g|W, g|L, s|U)$ (the distribution data corresponding to $X = 2$ are included in Table 2).

In the between-participants segment of Experiment 1 we found a marginally significant inflative violation of LTP for lower values of the payoff variable X, most clearly at $X = .5$ with $p(g|W) = 0.68$, $p(g|L) = 0.71$ and $p(g|U) = 0.87$. Based on contingency counts a left-tailed Fisher test for increased association of Gamble/Stop by Win/Unknown at pay-off at $X = .5$ showed a significance of $p = .0176$, with odds ratio 0.45 and CI = [0.22, 0.91] for $\alpha = .05$. Applying the Holm-Bonferroni correction for the five measurements over the payoff range renders the inflative effect of $p(g|U)$ at X=.5 non-significant (uncorrected p-values over the X-range: $\{.02, .22, .85, .96, .92\}$).

Finally we remark that in both the within-participants and the between-participants design the participants clearly distinguished the Lose outcome and Win outcome condition such that a larger preference was given to taking the gamble under Lose outcome condition than under Win outcome –an observation which runs counter to reported data on the two-stage gamble experiment (Tversky and Shafir, 1992; Kühberger et al., 2001; Lambdin and Burdsal, 2007; Surov et al., 2019).

3.2. Experiment 2

The full participant group was subdivided by condition flow order in **Experiment 2**. This allows an observation of a major effect on choice probability due to the block ordering, Figure

[5] Note, such a procedure may introduce a sampling bias, since the participants who decide to take part in such a study would be expected to be more risk seeking.

3. In the flow order when the Unknown conditioned block precedes the Known conditioned block the LTP is violated over the whole X-range in an inflative manner $p = 6.25e - 06$, (N=415), by Wilcoxon signed rank test. The Wilcoxon signed-rank test was used to assess the paired differences –Lose conditioned outcome response versus Unknown conditioned outcome response– from repeated measurements on a single sample. The test allows to compare the effect of two conditions on paired outcomes –here in particular to test whether the participants gamble more on Unknown vs Known outcome conditions. It is a non-parametric test which does not assume a normally distributed population (the data range from 0 to 1 in fractions 1/5), nor does it require equal variance, and independence of the errors. For each participant the X-averaged score under Lose and Unknown outcome conditions was compared and tested for H_0 hypothesis that $\langle p(g|U,X)\rangle_X < \langle p(g|L,X)\rangle_X$. On the contrary, in the flow order where the Known block precedes the Unknown block the LTP is satisfied over the whole X-range (N=407), Figure 3. The decreasing tendency to gamble under increasing payoff and the differing reaction of participants to second-stage gambles conditioned on W or L are observed in both gamble order conditions.

The design and sample size of Experiment 2 allowed us to analyse the gamble probabilities for different categories of participants. In the first instance we looked at the observed gamble probabilities for the full group by flow order, in the next sections we partition those two flow ordered groups by risk attitude –more vs less risk averse. [6]

3.2.1. More versus less risk averse participants

Considering that it is behavior relative to gambles that is at stake, it seems a shortcoming in both the original Tversky and Shafir (1992) work and later extensions that the risk aversion of participants has not been taken into account. In order to operationally characterise the risk aversion of participants their choices in the *single-stage* gambles were used. We recall the single-stage gambles are the same as the condition-free *first* stage of the two-stage gamble. By experimental design in Experiment 2 each participant is twice presented with all the single-stage gambles, once in the Known block and once in the Unknown block. We use the sum total of the instances a participant accepts the initial gamble as the operational measure of risk attitude. In our design this 'single-gamble score' can vary from 0 to 10. A high single-gamble score indicates a participant who frequently chooses to gamble, despite the potential loss, hence expressing low risk aversion. A low single-gamble score indicates a participant with higher risk aversion since these participants do refrain more often from a risky choice with potential loss.

From Figure S3, we observe that a substantial fraction of the participants obtained a single-gamble score of 10. Participants that *always take the initial gamble*, regardless the payoff, can be considered more risk-seeking than participants that will not always take it. This criterion warrants a partitioning of the sample by either obtaining a single-gamble score less than 10 or the maximum of 10, resulting in a 'More risk averse' group (N=429) and a 'Less risk averse' group (N=393) which are approximately of the same size.[7] A first overall observation of the second-stage gamble probabilities separated along our criterion for risk attitude, Figure 4 confirms some basic

[6] A further partitioning by gamble pattern range, based on ID-score, eq. (S3), is provided in the Supplementary Materials, SM 4.

[7] The demographics for the 'Less risk averse' group with respectively $N_{KU} = 200$ and $N_{UK} = 193$ participants revealed respective gender means $m_{gender} = 0.61$ and $m_{gender} = 0.52$, while the 'More risk averse' group had $N_{KU} = 207$ and $N_{UK} = 222$ participants, with respective gender means $m_{gender} = 0.48$ and $m_{gender} = 0.46$. Therefore a small gender bias was present due to our risk-aversion partitioning, ($p = .006$, two-tailed) where the

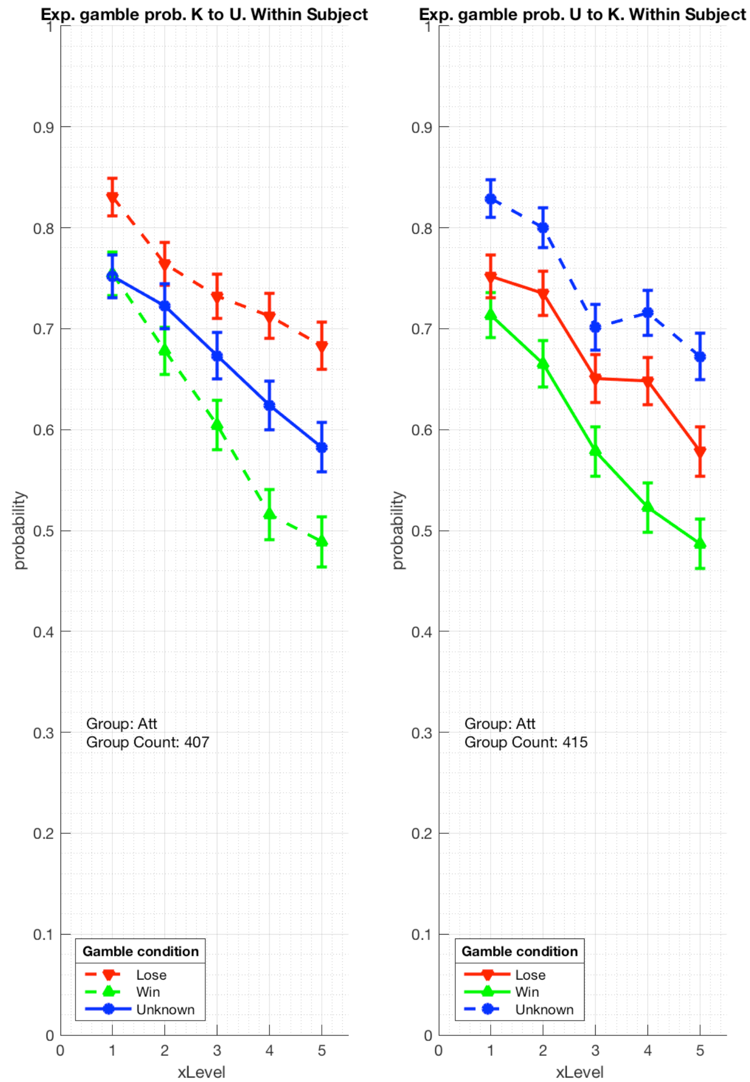

Figure 3: Experimental gamble probabilities, on the left for participants in K-to-U order, on the right for U-to-K order. In the U-to-K order an inflative Disjunction Effect occurs. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

expectations. The defined 'More risk averse' group is indeed more risk averse than the defined 'Less risk averse' group, since all gamble probabilities for all payoff values X and for all outcome conditions are lower in the 'More risk averse' group in comparison to the same gambles taken by the 'Less risk averse' group. Moreover, the 'More risk averse' participants have a faster diminishing motivation to take the second-stage gamble for increasing payoff X when the first-stage gamble outcome was Lose or Unknown. However, when the first-stage gamble outcome was Win, this diminishing motivation to gamble over X remains the same for both groups. The 'Less risk averse' participants also show a significant distinction between gamble choices under W and L condition, while the 'More risk averse' participants hardly discriminate between these two conditions.[8]

A major and rather surprising observation for the 'More risk averse' group is the strong flow-order with outcome-condition cross-over interaction (Figure 4, right panel). Remarkably the 'More risk averse' participants show a tendency for a Disjunction Effect in K-to-U order and a significant inflative violation of the LTP in U-to-K order. Notice that this is a between-participants effect of flow order. A mixed ANOVA with unbalanced design (N=207/N=222) and with a dependent variable gamble probabilities (averaged across all payoffs X) and independent variables first-stage gamble outcome condition {W, L, U} and order {'K-to-U', 'U-to-K'} revealed a significant interaction, $F(2, 1281) = 12.78$, $p = 3.20\text{e-}06$.

By contrast, there were no main effects for either first-gamble outcome condition or order in the 'More risk averse' group. That is, surprisingly, there appears to be no effect on choice behavior from whether the first-stage gamble was indicated as Won or Lost.

The 'Less risk averse' participants do not show any tendency for a Disjunction Effect in the order K-to-U, while in the U-to-K order a non-significant tendency for an inflative violation of the LTP occurs. A mixed ANOVA with unbalanced design (N=200/N=193), testing for factors of condition {W, L, U} and order {'K-to-U', 'U-to-K'}, revealed a significant main effect of condition {W, L, U}, $F(2, 1173) = 43.88$, $p = 4.2\text{e-}19$. Therefore, in this group a substantial difference in gambling probability is observed, depending on whether the first-stage gamble was Won or Lost.

In the U-to-K order the 'More risk averse' participants are mostly indifferent to choice under the Win or Lose first-stage gamble outcome condition, therefore the inflative violation of the LTP has to be tested with respect to both choices of the two Known outcome conditions. To test the statistical significance of the violation of the LTP the Wilcoxon test for repeated measurements on a single sample was applied. The test was used to assess the paired difference from measurements on Known and Unknown conditions for each participant. The Wilcoxon test shows a significant violation of the LTP, with p=1.7e-06 (N=222) for H_0 that $\langle p(g|U, X)\rangle_X < \langle p(g|L, X)\rangle_X$ and $p = .0002$ (N=222), for H_0 that $\langle p(g|U, X)\rangle_X < \langle p(g|W, X)\rangle_X$.

In the K-to-U order the 'More risk averse' participants show a small but consistent diminished choice probability under Win in comparison to the Lose first-stage gamble outcome condition. In this case therefore the Disjunction Effect is tested between the choices in the Unknown and Win outcome conditions only. The Wilcoxon test shows a significant Disjunction Effect, with $p = .045$

odds ratio is 0.68 and the confidence interval CI = [0.52, 0.89] for $\alpha = .05$.

[8]In SM 10, we analyse the effect of informing the participant about the Unknown outcome of the first-stage in a two-stage gamble by comparing the probability of taking the *single*-stage gamble $p(g)$ (thus without any condition set by an earlier stage gamble) and the second-stage gamble $p(g|U)$ (in which the participant is informed that the outcome is Unknown). This analysis is done for 'more risk averse' participants, since for 'less risk averse' participants $p(g) = 1$ for all X, hence further analysis is not pertinent for the latter participant group.

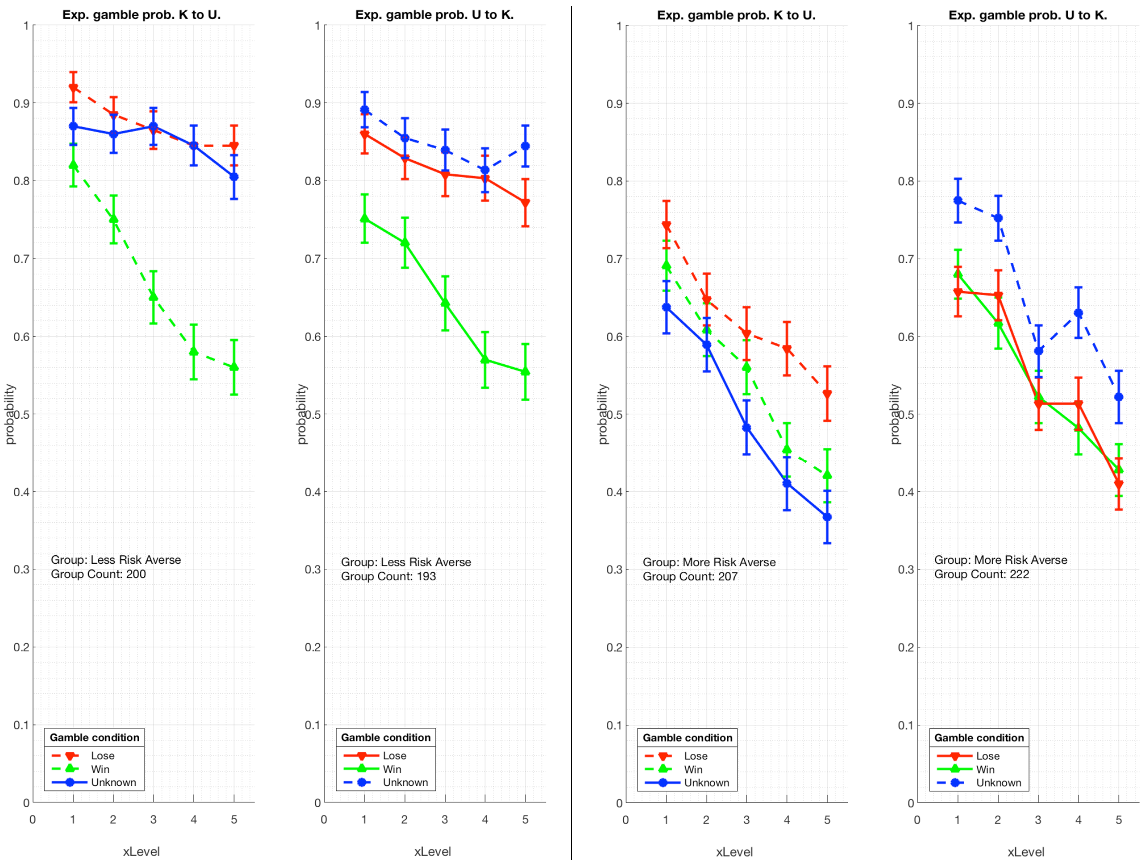

Figure 4: Observed gamble probabilities for sample partitions into 'Less risk averse' (left panel), and 'More risk averse' (right panel). Within each panel, on the left are the observations for the K-to-U order, on the right the U-to-K order. The 'More risk averse' participants show a significant inflative violation of the Law of Total Probability in U-to-K order, and a marginally significant Disjunction Effect, or deflative violation of the Law of Total Probability, in K-to-U order. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

(N=207), for H_0 that $\langle p(g|U,X)\rangle_X > \langle p(g|W,X)\rangle_X$. Since this result seems marginally significant, we also applied the Pratt correction to the Wilcoxon test (by modification of Matlab code in Cardillo (2006)). The Pratt correction is required for samples with frequent ties, which typically occur in discrete distributions like in our present data set where the compared X-averaged gamble response values are fractions ranging from 0/5 till 5/5. While the Wilcoxon test eliminates all zero differences of measurement outcomes, the Pratt correction keeps the zero differences in the ranking procedure of the statistical test (Pratt, 1959). Using the Pratt correction, the Disjunction Effect for 'More risk averse' participants in the K-to-U order is marginally not statistically significant anymore at $p = .062$ (N=207).

The size of the sample allows insight in decision patterns besides aggregate gamble probability. In particular we can consider the prevalence of particular gamble strategies expressed as WLU gamble patterns, Figure 5. Three gamble patterns have a deflative effect on the average gamble probability under Unknown condition –$(g|W,s|L,s|U)$, $(s|W,g|L,s|U)$ and $(g|W,g|L,s|U)$– and three have an inflative effect – $(g|W,s|L,g|U)$, $(s|W,g|L,g|U)$ and $(s|W,s|L,g|U)$, Table (S1). The probability distribution over the patterns causes the occurrence of either a Disjunction Effect or an inflative violation of the Law of Total Probability. It is therefore important to analyse the distribution of the gamble patterns over the spectrum of payoff parameter X, Figure 5.

A remarkable difference between the pattern distribution of the 'Less risk averse' and 'More risk averse' participants occurs over the range of increasing payoff X. In 'Less risk averse' participants the modal strategy remains 'always play', $(g|W,g|L,g|U)$, throughout the X range. In the 'More risk averse' participants the modal pattern changes from 'always play' at the lowest payoff to 'never play', or $(s|W,s|L,s|U)$, at the highest payoff.

The pattern $(s|W,g|L,g|U)$ ('only stop on Win' strategy) is the second most common pattern over the X range for 'Less risk averse' participants. In the 'More risk averse' participants, the patterns with 'stop on U' strategy become more frequent only for higher values of X (reflected in the decreasing of $p(g|U,X)$ with increasing X). In general we observe that 'More risk averse' participants resort to a larger variety of gamble patterns when X increases. In the 'Less risk averse' group the near inflative $p(g|U,X)$ originates mainly from the probability mass in the pattern $(s|W,g|L,g|U)$, in both flow orders.

In sum, in the 'More risk averse' group the marginally significant DE in the K-to-U order emerges due to the empirical preponderance of *all* patterns with deflative effect over patterns with inflative effect, while in the U-to-K order the inflative violation of the LTP emerges through the preponderance of *all* patterns with inflative effect over the deflative patterns. The Disjunction Effect and the inflative violation of the Law of Total Probability are therefore not caused by their purported association to $(g|W,g|L,s|U)$ or $(s|W,s|L,g|U)$ patterns. This still leaves open the question of whether some individual participants might adhere to specific deflative or inflative strategies over the X-range of payoffs, and whether these tendencies are masked by the aggregation of data (Estes, 1956). This issues is addressed in Supplementary Materials section SM 3.

To end this section we discuss the concern that these participants whom we labeled 'less risk averse' would simply 'click through' the experiment rather than informedly choose to always play a single-stage gamble. To avoid the possibility that this type of 'lazy responding' effect could take place, in our survey code in Qualtrics we implemented a random Display Order of the gamble button and the stop button. The gamble button could appear on either of two locations, on the left or the right of the screen. With each new gamble, it was randomly determined whether the gamble button

Figure 5: Experimental gamble pattern probabilities in the WLU order arranged from $(s|W, s|L, s|U)$ to $(g|W, g|L, g|U)$; in the left panel we show 'Less risk averse' participants, in the right panel 'More risk averse' participants. The patterns have been ordered in WLU order with 1 for 'gamble' and 0 for 'stop'. The patterns for payoff parameter $X = .5$ is shown at the top and increasing to $X = 4$ at the bottom. The four 'gamble-on-U' patterns (XYg) are grouped to the right on the X-axis, the four 'stop-on-U' patterns (XYs) are grouped to the left. The probabilities for the K-to-U order appear on the left (pink color) and for U-to-K on the right (teal color). For comparison the yellow markers in the $X = 2$ panel indicate the pattern probabilities of Tversky and Shafir (1992). In the right panel for 'More risk averse' participants one observes the probability mass shifting from XYg to XYs patterns for increasing payoff, corresponding to the decreasing gamble probability $p(g|U, X)$ with increasing X. Error bars represent the standard error of the mean.

was on the right or on the left. A lazy participant would be expected to click through at the same location, which would lead to equivalent proportions of gamble and stop decisions. By contrast, the 'less risk averse' participants would need to hunt the gamble button, at the different screen locations where it would appear, in order to adhere to an 'always gamble' strategy. This commitment is not in line with laziness for it requires attention and takes more time to perform than robotically clicking the same button appearing randomly below their cursor. In fact the average task duration for 'Less risk averse' participants is indeed somewhat longer than for 'More risk averse' participants, $M_{duration,Lra} = 652s$ while $M_{duration,Lra} = 621s$.[9]

Additionally, from the perspective of Expected Value the choices made by the 'less risk averse' participants make sense. This is because all these gambles have an Expected Value which exceeds not playing the gambles by an amount of $25X$ (see SM 12). Therefore it makes sense to always play the single-stage and even to always play the second-stage gambles as well.[10]

Another relevant point is that participants defined as 'less risk averse' do not always play all gambles and show a decreasing tendency to take the second-stage gambles for higher pay-offs (see Figure 4). They meaningfully (i.e., on the basis of a non-random pattern) change their choices depending on outcome condition and payoff size (less so on order).

Finally, it is worth bearing in mind that participants that always play single-stage and second-stage gambles have no influence in creating inflation nor deflation of the probability $P(g|U)$. The $(g|W, g|L, g|U)$ pattern contributes equally to each gamble probability. So, the small percentage of true always takers have a perfectly neutral effect on the ordering of the probabilities (their elimination would scale up the small inflation effect in the U-to-K flow for the less risk averse participants).

In sum, these arguments show the defined 'less risk averse' participants properly engage with their task and show rational behaviour in their decisions. Their behaviour does not warrant elimination from the participant pool.

4. Theoretical analysis

Prospect Theory, for risky decision making under uncertainty, was set forth by Kahneman and Tversky (1979), (Tversky and Shafir, 1992). Essentially their theoretical approach provides Expected Utility theory of losses and gains with shifting reference values. We first shortly expose how Prospect Theory provides a theoretical interpretation for the Disjunction Effect, after which we will show why this model is problematic with respect to our observations.

In order not to commit our discussion to a specific formal expression of utility we will denote the expected utility of the second-stage gamble respectively as $EU(X|W)$, $EU(X|L)$ and $EU(X|U)$ for the three first-stage gamble outcome conditions Win, Lose and Unknown, and payoff value X.

---

[9] A two-sample t-test for the null hypothesis that the task duration in 'less risk averse' and 'more risk averse' players have equal means and equal but unknown variances accepts the null hypothesis; p = .4, ci =[-45.9, 107.0], t= 0.79, df = 820, sd = 558.

[10] A quote of the voluntary feedback of one of the 'true' always takers reveals this participant's motivation: *"I'm not usually a gambler but unless I misunderstood the directions, it was always monetarily wise to flip the coin again. If you win, you have a chance to win again and if you lose you only lose half the amount and have a better chance of winning on the next."* Similarly 'never takers' can act by consistent strategy as well, even if it is against the expected value of each gamble. It is of interest to quote the voluntary feedback of two of those 'never players': *"I think gambling is foolish. I would never put money at risk"*, and *"My parents are addicted to gambling so I really don't like to gamble myself."*

The expected utility is weighted on the utility of wealth, $U_w(X)$, given a won monetary amount parametrized by X, and on the utility of debt, $U_l(X)$, given a lost monetary amount parametrized by X. The choice to take the second-stage gamble is motivated by comparing the expected utility of accepting the second-stage gamble with the utility of stopping the gamble and settling with the outcome of the first-stage gamble. In both Known outcome cases of the first-stage gamble, the expected utility compounds the possible future outcomes with the imparted Win X of the first-stage according to the gamble payoffs, texts (8, 9)

$$EU(X|W) \;=\; .5\,U_w(2X) + .5\,U_w(X/2), \qquad \text{`Win case}' \tag{10}$$

or the incurred Loss $X/2$ of the first-stage

$$EU(X|L) \;=\; .5\,U_w(X/2) + .5\,U_l(X). \qquad \text{`Loss case}' \tag{11}$$

In the Unknown outcome case of the first-stage gamble however the uncertain payoff is shifted to zero in the evaluation of the utility of the second-stage gamble

$$EU(X|U) \;=\; .5\,U_w(X) + .5\,U_l(X/2). \qquad \text{`Unknown case}' \tag{12}$$

The argument goes that the loss of acuity due to conflicting rationales in the Unknown first-stage outcome condition impedes the evaluation of the present bankroll. Therefore in evaluating the utility difference between taking the second-stage gamble and stopping, the uncertain payoff of the first-stage Unknown outcome gamble has its utility shifted to zero

$$EU(X|U) - 0 \;=\; .5\,U_w(X) + .5\,U_l(X/2) - 0. \qquad \text{`Unknown case}' \tag{13}$$

In the Known outcome cases of the first-stage gamble the utility difference that motivates taking the second-stage gamble for an incurred Win X of the first-stage gamble is given by

$$EU(X|W) - U_w(X) \;=\; .5\,U_w(2X) + .5\,U_w(X/2) - U_w(X), \qquad \text{`Win case}' \tag{14}$$

and for an imparted Loss $X/2$ of the first-stage gamble by

$$EU(X|L) - U_l(X/2) \;=\; .5\,U_w(X/2) + .5\,U_l(X) - U_l(X/2). \qquad \text{`Lose case}' \tag{15}$$

Applying a power law for utility and the principle that ‘losses loom larger than gains’, Tversky and Shafir (1992) elegantly showed that Prospect Theory predicts the utility to take the second-stage gamble in both Known outcome cases to be larger than in the Unknown outcome case and hence the choices made by participants would violate the STP. Prospect Theory therefore provided a theoretical framework and principle to understand the Disjunction Effect.

Our present observations of the choice probabilities in the two-stage gamble experiment evidence violations of a prediction derived from the Law of Total Probability both in deflative and inflative sense. Clearly the framework of Prospect Theory cannot be maintained to cover our present study. Moreover we show that the approach of Tversky and Shafir (1992) predicts an increasing probability to take the second-stage gamble when the payoff X increases, (Appendix B). In fact, empirically we observe the opposite, Figure 4. In the present context it will therefore not be correct to express utility of money amount x by its commonly used power law form x^a, with $a < 1$. In Appendix B, we show that neither logarithmic utility nor exponential utility can remedy this payoff dependence of the gamble probability. Lacking an effective alternative form of utility we did not pursue any further the Prospect Theoretic approach of our study.

The inadequacy of prospect theory in covering our empirical results (and as we shall later see likewise for a static logistic regression model) in part motivates the adoption of more sophisticated modeling frameworks, which derive from assumptions about the dynamics of the decision process. As noted earlier, we developed two such models. In one approach we will apply elements from quantum probability theory, since such an approach has proven effective in covering non-classical probability results in experimental paradigms involving uncertainty, ordering and contextuality (Busemeyer et al., 2011; Busemeyer and Bruza, 2012; Pothos and Busemeyer, 2013; Wang et al., 2013, 2014; Broekaert et al., 2006; Aerts and Aerts, 1995; Khrennikov, 2010; Atmanspacher and Filk, 2013; Fuss and Navarro, 2013; Asano et al., 2015; Kvam et al., 2015; Martínez-Martínez and Sánchez-Burillo, 2016; Broekaert et al., 2016) In the second dynamic model we base the stochastic process on continuous-time Markov chain theory. Both the quantum and Markov dynamic models can describe the change of a participant's belief-action state over time as they process the different stages of the gamble. Recall that, in order to assess the effectiveness of the process dynamical approaches a third model is included which will serve as a baseline comparison. This baseline model will mimic the context effects and carry-over features of the two dynamic models, but will produce the gamble probabilities through a logistic function of a heuristic utility function.

4.1. Shared features of the theoretical models

The cognitive process for decision making involves the perception of cues, judgement based on this information, correlation with prior beliefs and rumination about consistency or change and its implications, to finally lead up to a decision. Making a decision is therefore a dynamic process that involves the participant's belief-action state. Both the Markov model and the quantum-like model focus on the description of the evolving belief-action state. These models implement a principle of stochasticity in the cognitive process of decision making, implying that responses of the participants are considered as probabilistic outcomes of the process. The belief-action states will convey the support for specific choices in probabilistic terms. In both models the information embedded in the cue –the description and previous outcome of the gamble– will inform the specific composition of the operator that drives the change of the participant's belief-action state. In Markov theory this core operator is the transition rate matrix or 'intensity' matrix and, in quantum theory this operator is the Hamiltonian. In both modeling approaches these 'generators of change' implement the high-level cognitive process of the decision making. As we have mentioned in Section (1) the Markov and quantum-like approach also differ fundamentally, notwithstanding close formal resemblance (see Appendix A for a concise explanation). The essential difference between the two approaches in the present application is the respective probability theory to which the models abide, namely classical and quantum probability theory (Busemeyer et al., 2009).

The comparative baseline model is a Logistic model which, as in latent trait modeling, relates the observed gamble probabilities to heuristic utility, eqs. (16), and is parametrized to mimic the contextual features we implemented in the two dynamical decisions models, Section (4.4).
The core features that are shared by the dynamical models and are mimicked by the baseline model are directly related to the experimental two-stage gamble paradigm;
— The decision to take the second-stage gamble is driven by its utility which depends on the W, L and U first-stage outcome condition and on the payoff X of the gamble, according to the heuristic linear expression

$$u_W(X) = \delta_{0W} + \delta_{1W} \cdot X \quad , \qquad u_L(X) = \delta_{0L} + \delta_{1L} \cdot X \tag{16}$$

where $X \in [.5, 1...4]$.
— The decision to take the second-stage gamble is influenced by the *context* of the outcome condition block.
— Within a gamble block the participant regains an initial 'averaged' belief-action state before each newly presented second-stage gamble.
— The flow order of condition blocks will lead to a carry-over effect on the belief-action state (Hogarth and Einhorn, 1992).

We will first develop the Markov model as it is a more conventional approach to dynamical modeling in decision making (Sonnenberg and Beck, 1993; Busemeyer et al., 2009). Its formal structure closely resembles the formalism of quantum mechanics and as such it will be easier to understand particularities of the quantum-like model subsequently (see also Appendix A)

4.2. The Markov model

In a Markov model the probabilities for specific beliefs are represented in a vector which encompasses the appropriate features of the paradigm at hand. In our paradigm the minimal representation requires crossing the Win or Lose condition of the first-stage gamble with the Gamble or Stop decision of the second-stage gamble. The belief evaluations Win and Lose crossed with the action potential for the decisions Gamble and Stop make up the full event space. The expression p_{WG} is defined as the probability that the participant's belief is Win first stage and take gamble on second stage. Similarly defined expressions p_{WS}, p_{LG} and p_{LS} and interpretations apply. The probabilities of the full event space add up to unity:

$$p_{WG} + p_{WS} + p_{LG} + p_{LS} \quad = \quad 1 \tag{17}$$

These four joint probabilities are the components of the belief-action state Π in the Markov model,

$$\Pi \quad = \quad (p_{WG}, p_{WS}, p_{LG}, p_{LS})^{\tau} \tag{18}$$

where for simplicity of notation, we write the column vector as a row vector with the transpose operation τ. The probability for the participant to take the second-stage gamble is obtained by adding the two components 'Gamble in the second-stage and Won-first-stage belief' and 'Gamble in the second-stage and Lost-first-stage belief'

$$p(g) \quad = \quad p_{WG} + p_{LG}. \tag{19}$$

The belief-action state changes by a process based on the available information and is formally controlled by the transition rate matrix K. The specific composition of this matrix causes the transfer of probability between the different belief state components. A main source of transfer in the belief state is the gamble outcome information. Given the belief for 'Win' a re-distribution of 'Gamble' or 'Stop' components will result, and an analogous redistribution will occur given a belief for 'Lose'. Within the subspace of Win this re-distribution requires a transition rate sub matrix K_W, and within the subspace for Lose a transition rate sub matrix K_L

$$K_W = \begin{pmatrix} -1 & \delta_W \\ 1 & -\delta_W \end{pmatrix} \quad , \qquad K_L = \begin{pmatrix} -1 & \delta_L \\ 1 & -\delta_L \end{pmatrix} \tag{20}$$

$\delta \geq 0$, and where in each column all rows need to add up to zero for the conservation of probability. Depending on the magnitude of δ, these transition rate matrices will either increase action potential for Gamble ($\delta > 1$) or increase action potential for Stop ($\delta < 1$).[11] The encompassing transition matrix for Win and Lose is given by

$$K_{W\&L} = \begin{pmatrix} -1 & \delta_W & 0 & 0 \\ 1 & -\delta_W & 0 & 0 \\ 0 & 0 & -1 & \delta_L \\ 0 & 0 & 1 & -\delta_L \end{pmatrix} \tag{22}$$

which acts separately on each of the subspaces since the upper left matrix quadrant only engages the two first W probability components of a vector to produce the first two components of the output vector and similarly, the lower right matrix quadrant only engages the last two L probability components of a vector to produce the two last components of the output. Under such a transition matrix Win-related and Lose-related belief changes are fully independent.

The rate of transfer, parameter δ, between the vector components is made to depend on the utility, eqs. (16) of the gamble by a logistic function

$$\delta_W = s \cdot \left(1 + e^{-u_W(X)}\right)^{-1} \quad , \qquad \delta_L = s \cdot \left(1 + e^{-u_L(X)}\right)^{-1} \tag{23}$$

where $X \in [.5, 1...4]$ is the parameter for the size of the payoff. The parameter s controls the sensitivity to the linear utility expression, $s \geq 0$.

The assumed cognitive process mixes the W and L beliefs under all first-stage gamble outcome conditions, but most extensively so under Unknown first-stage outcome condition. In the latter case the uncertainty about being inflicted a loss or endowed a win engenders an uncertainty about Gambling or Stopping, but also in the Known first-stage outcome gambles mixing of Win and Lose beliefs will occur as a contextual effect. Therefore a mixing operator was implemented to cause an attention switching between Win and Lose and concurrently switching the decision between Gambling and Stopping. In practice, this operator thus transfers action-potential from 'Gamble on Win' (p_{GW}) to 'Stop on Lose' (p_{SL}) and, from 'Stop on Win' (p_{SW}) to 'Gamble on Lose' (p_{GL}). These two re-distributions are respectively implemented by the two matrices

$$K_{Mix} \quad = \quad \gamma \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{24}$$

in which the first matrix causes a transfer between p_{WG} and p_{LS} and the second matrix a transfer between p_{WS} and p_{LG}. The magnitude of the mixing process is monitored by the parameter γ.

The full transition rate matrix K, which implements the cognitive process for Win, Lose and

[11]For a two-dimensional Markov model with transition matrix, eq. (20), the propagator $T(t)$, eq. (26), can be easily calculated analytically

$$T(t) \quad = \quad \mathbf{1} + \frac{K}{1+\delta}(1 - e^{-(1+\delta)t}). \tag{21}$$

One can verify that independent of the initial belief-action state the time-asymptotic state is $(\delta/(1+\delta), 1/(1+\delta))^\tau$. Hence for $\delta > 1$ the first component (Gamble) dominates the second (Stop), and vice versa for $\delta < 1$.

Unknown condition is then composed of all four matrices together

$$K = \begin{pmatrix} -1-\gamma & \delta_W & 0 & \gamma \\ 1 & -\delta_W - \gamma & \gamma & 0 \\ 0 & \gamma & -1-\gamma & \delta_L \\ \gamma & 0 & 1 & -\gamma - \delta_L \end{pmatrix} \tag{25}$$

The time evolution driving matrix for the belief-action state under the transition rate K is the transition matrix T, which is a solution of the Kolmogorov Forward equation (Busemeyer and Bruza, 2012):

$$T(t) = e^{Kt} \tag{26}$$

The belief-action state at time t and under condition C of the initial stage gamble outcome is then given by

$$\Pi_C(t) = T(t)\Pi(0, C). \tag{27}$$

From the belief-action state at the moment of decision the probability for taking the second-stage gamble is then obtained by selecting and adding the gamble components, eq. (19). This can be operationally expressed using the selection matrix M_{Gamble}

$$M_{Gamble} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

to produce the gamble probability

$$p(gamble|X, Cond) = |M_{gamble}T(\pi/2)\Pi(0, C)|_1, \tag{29}$$

where $|\cdot|_1$ is shorthand notation for summing of the (absolute) values of the vector components. We have fixed the time of measurement to the conventional choice $t = \pi/2$, which is a standard procedure that is also applied in the quantum-like model as well (Busemeyer and Bruza, 2012). This procedure –of setting a conventional time of measurement– is typically applied in a Markov dynamical approach in order to avoid independence of the final belief-action state on the initial belief-action state. (One can easily check this independence from initial conditions at larger time scales, eq. (21).)

For each of the two periods for decision making in each flow order (K-to-U and U-to-K in Figure 2) a separate evolved belief-action state will be obtained. Each of these evolved states will differ due to their respective initial belief-action states. Therefore, even if the transition rate matrix K is the same in both flow orders and for all outcome conditions, the theoretical gamble probabilities will be different.

In the **first** period, the initial belief-action state on a Win and Lose condition of the first-stage outcome are formally given by the vectors;

$$\Pi_{0,W} = \left(\frac{\nu}{2}, \frac{\nu}{2}, \frac{(1-\nu)}{2}, \frac{(1-\nu)}{2}\right)^{\tau} \quad , \qquad \Pi_{0,L} = \left(\frac{(1-\nu)}{2}, \frac{(1-\nu)}{2}, \frac{\nu}{2}, \frac{\nu}{2}\right)^{\tau} \tag{30}$$

where ν is a weight parameter, $0 \le \nu \le 1$. Should $\nu = 1$ then the states $\Pi_{0,W}$ and $\Pi_{0,L}$ are precisely allocated to their Win and Lose components respectively, while at the same time for both states a uniform probability to Gamble or Stop is assumed. Due to the context effect from the other gambles in the block, regulated by ν, these belief-action states respectively express that Win or Lose information only partially determines the belief state in the block where the outcome is Known. This ambiguous belief-action state reflects incompletely registered information notwithstanding unambiguous Win, or Lose, information in the gamble description. This belief-action state occurs because of its embedding in the mixed context of the Win-outcome and Lose-outcome block. It implements an effect of contextual anchoring which leads to compounding information of the present gamble outcome condition and outcome conditions of previously taken gambles within the same block.

The initial belief-action state –in first period– on Unknown outcome of the first-stage gamble is

$$\Pi_{0,U} \quad = \quad \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)^{\tau} \tag{31}$$

which expresses the belief-action state with uniformly weighted Win and Lose outcomes and is similarly indifferent to either Gamble and Stop decisions due to lack of previously experienced gambles. The state is caused by the uncertainty due to missing information on the first-stage outcome in the Unknown-outcome condition in first period.

In the **second** period similar effects of context are at play, but now due to the carry-over effect the initial state in the second period will depend also on the participant's history of gambling in the first period. The initial belief-action states for Win and Lose first-stage outcome conditions will also contain residual belief support for the opposite condition. The magnitude of the context effect will be changed by the carry-over effect

$$\Pi_{00,W} = \left(\frac{\mu}{2}, \frac{\mu}{2}, \frac{(1-\mu)}{2}, \frac{(1-\mu)}{2}\right)^{\tau} \quad , \qquad \Pi_{00,L} = \left(\frac{(1-\mu)}{2}, \frac{(1-\mu)}{2}, \frac{\mu}{2}, \frac{\mu}{2}\right)^{\tau} \tag{32}$$

where μ is a weight parameter, $0 \le \mu \le 1$. The weighting parameter μ in the second period differs from ν in first period.

Because of previous exposure to Known outcome conditioned first-stage gambles, the initial belief-action state on Unknown conditioned first-stage gambles is not uniform anymore

$$\Pi_{00,U} \quad = \quad \kappa \Pi_{0,W}(\nu) + (1-\kappa)\Pi_{0,L}(\nu). \tag{33}$$

This is a belief-action state weighted by κ, $0 \le \kappa \le 1$, on Win and Lose states of the first period, which is caused by a carry-over effect of the belief tendencies about the two possible outcomes of Win and Lose in first period.

The Markov model processes the belief-action state for each outcome condition, payoff and period by evolving from the appropriate initial state. Each time a new second-stage gamble is proposed the participant will thus first regain a dedicated initial belief-action state. In our experimental paradigm, the participant is assumed to do so, eqs. (32, 33), for each of the five payoff values and for each of the three types of first-stage outcome condition $\{W, L, U\}$. During the experiment each participant thus produces fifteen final belief-action states which lead up to the appropriate gamble decisions according to the first-stage gamble outcome condition, payoff size and flow order.

Parametrization. The Markov model requires four dynamical parameters for the utility expression of the second-stage gambles. Two parameters - intercept and slope - for each condition of Win and Lose express the different motivational utility of the two conditions, denoted by $\{\delta_{0W}, \delta_{1W}\}$ and $\{\delta_{0L}, \delta_{1L}\}$. The effect of this utility difference on the decision is controlled by the sensitivity parameter $\{s\}$ in the logistic form, eqs. (23). The 'coupled-switching' dynamics that implements the attention switching from Win to Lose and its reversal of related Gamble or Stop decision is controlled by the strength of the mixing parameter $\{\gamma\}$. The context effect on the belief-action state is implemented by the weight parameters $\{\nu, \mu\}$ on the Win and Lose states, for first and second period respectively.

Finally the carry-over effect from first to second period on the U condition belief-action state is implemented by the weight parameter $\{\kappa\}$.

The Markov model therefore relies on 9 parameters to cover the process dynamics and the initial beliefs in both flow orders, both periods and all payoffs, amounting to providing theoretical values to 30 data points. In the Supplementary Materials section (SM 1) the full temporal evolution description of the belief-action state is provided for the full sample of participants who passed the attention test.

4.3. The quantum-like model

The quantum-like model applies a state vector to represent the belief-action state of the participant but instead of having probability components like in the Markov approach, it has probability *amplitude* components. These components can be complex valued and only lead to probabilities after taking the squared norm. In Appendix A, an elementary introduction to the application of the quantum formalism in cognition is given, which also provides an exposition of its close resemblance to the Markov formalism.

The similarity with the Markov model allows a fairly straightforward formulation of the quantum-like model that runs parallel to the previous section on the Markov model and only requires some clarification for a few distinct features.

The minimal representation of the gamble paradigm crosses the conditions for Win or Lose and the decision to Gamble or Stop. The associated belief-action state will be denoted as

$$\Psi = (\psi_{WG}, \psi_{WS}, \psi_{LG}, \psi_{LS})^{\tau}, \tag{34}$$

where the amplitude components represent the respective belief support for first-stage gamble outcome condition combined with action-potential for different gamble decisions in the second-stage gamble. In the quantum-like model the probability for the participant to take the second-stage gamble is obtained by adding the modulus squared of the components for 'Gamble in the second-stage and Won-first-stage belief' and 'Gamble in the second-stage and Lost-first-stage belief'

$$p(g) = |\psi_{WG}|^2 + |\psi_{LG}|^2, \tag{35}$$

In general, since the belief-action states covers the full event space for the decisions Gamble and Stop and categories Win and Lose, the corresponding probabilities add up to unity:

$$1 = |\psi_{WG}|^2 + |\psi_{LG}|^2 + |\psi_{WS}|^2 + |\psi_{LS}|^2. \tag{36}$$

which is the normalization of the belief-action state vector. In the quantum-like model the belief-action state at the moment of decision is realized through a measurement operation. In particular the outcome state for the decision to gamble is obtained through the corresponding projector M_{Gamble} for the question 'Take the second-stage gamble?' and the projector M_{Stop} for 'Stop the second-stage gamble?'.[12]

$$M_{Gamble} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; M_{Stop} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{37}$$

Notice that formally this projection matrix is identical to the selection matrix in the Markov model, eq. (28). The modulus square of the projected vector for the measurement 'Take the second-stage gamble?' then gives the gamble probability, eq. (35).

In the quantum-like model the process of change of the belief-action state occurs at the level of the probability amplitudes. The transforming effect of incoming information is controlled by the Hamiltonian operator H. Now the specific parameter positions in this matrix will cause the transfer of probability amplitude between the different vector components of the belief-action state. In the quantum-like case - due to the original relation of the Hamiltonian operator to the real-valued 'energy' of a system - the operator for change has to be Hermitian. This means the component H_{ij} for transferring probability amplitude from vector component with index j to i has to be complex conjugated with respect to the component H_{ji}, which transfers probability amplitude from component i to j. The Hermiticity requirement is expressed as $H = H^\dagger$. In the two stage gamble paradigm the main factor of transfer in the belief-action state depends on the condition of the outcome of the initial gamble. This information will re-distribute the Gamble or Stop components in the Win subspace and also the Gamble or Stop component in the Lose subspace. The transformation within the subspace of Win and subspace of Lose requires the two respective Hamiltonian sub matrices –satisfying the Hermitian condition;[13]

$$H_W = \begin{pmatrix} 1 & \delta_W \\ \delta_W & -1 \end{pmatrix} \quad , \quad H_L = \begin{pmatrix} 1 & \delta_L \\ \delta_L & -1 \end{pmatrix} \tag{39}$$

where $\delta \in \mathbb{R}$. The encompassing Hamiltonian, with H_W in the upper left matrix quadrant and with H_L in the lower right quadrant, acts separately on the subspaces for Win and Lose

$$H_{W\&L} = \begin{pmatrix} 1 & \delta_W & 0 & 0 \\ \delta_W & -1 & 0 & 0 \\ 0 & 0 & 1 & \delta_L \\ 0 & 0 & \delta_L & -1 \end{pmatrix} \tag{40}$$

This type of Hamiltonian would keep Win-related and Lose-related belief amplitudes fully independent.

[12] Note that a projector is any matrix M which is idempotent, $M^2 = M$. The projection occurs on the span of its eigenvectors. See also (Appendix A) for an elementary introduction to quantum modeling.

[13] A two-dimensional quantum-like model with Hamiltonian matrix, eq. (39), allows to analytically calculate the unitary propagator $U(t)$, eq. (44), (Broekaert et al., 2016),

$$U(t) \quad = \quad \cos(\sqrt{1+\delta^2}\, t) - i \frac{\sin(\sqrt{1+\delta^2}\, t)}{\sqrt{1+\delta^2}} H \tag{38}$$

In contrast to the Markov propagator, the oscillatory evolution of the quantum-like propagator requires a choice of measurement time that remains within the system's period.

Similarly as in the Markov model the magnitude of the transfer process will depend on the utility of the second-stage gamble. In contrast to the driving parameters in the transition rate matrix of the Markov process, eq. (23), in the Hamiltonian the driving parameters can be positive or negative valued. More generally we could also parametrise the Hamiltonian with complex valued parameters while assuring Hermiticity. To accommodate both signs, the parameters are modeled by a hyperbolic tangent (version of the logistic) function of the linear utility expression:

$$\delta_W = s \cdot \left(2(1 + e^{-u_W(X)})^{-1} - 1\right) \quad , \quad \delta_L = s \cdot \left(2(1 + e^{-u_L(X)})^{-1} - 1\right) \tag{41}$$

with $X \in [.5, 1...4]$ and with scaling parameter s.

The assumed cognitive process –similar as to the Markov model– will mix the W and L beliefs under all first-stage gamble outcome conditions. In the Unknown first-stage outcome condition the uncertainty about loss or win engenders an uncertainty about Gambling or Stopping, but also in the Known first-stage outcome gambles mixing of Win and Lose beliefs will occur due to a contextual effect in the block. The mixing operator will cause attention switching between Win and Lose beliefs to happen concurrently with switching decisions for Gambling or Stopping. In practice the mixing Hamiltonian thus transfers action-potential from 'Gamble on Win' (ψ_{GW}) to 'Stop on Lose' (ψ_{SL}) and, from 'Stop on Win' (ψ_{SW}) to 'Gamble on Lose' (ψ_{GL}). The mixing dynamics corresponds to an explorative attention switching between potential outcomes of the gamble in which a switch between Win and Lose belief always correlates with a switch in the decision between to Gamble and to Stop in the second-stage gamble. These two correlated attention switching processes are implemented by

$$H_{Mix} \quad = \quad \gamma \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{42}$$

in which the first matrix controls the transfer between ψ_{WG} and ψ_{LS} and the second matrix controls transfer between ψ_{WS} and ψ_{LG}, and where γ monitors the magnitude of the mixing process.

The full Hamiltonian matrix H, which implements the cognitive process for Win, Lose and Unknown condition is then composed of all four matrices together

$$H \quad = \quad \begin{pmatrix} 1 & \delta_W & 0 & \gamma \\ \delta_W & -1 & \gamma & 0 \\ 0 & \gamma & 1 & \delta_L \\ \gamma & 0 & \delta_L & -1 \end{pmatrix}. \tag{43}$$

The temporal change of the belief-action state is produced by the unitary evolution operator U, and is itself driven by Hamiltonian operator H. The unitary operator satisfies the Schrödinger equation (Busemeyer and Bruza, 2012), in accordance with the dynamics of quantum theory

$$U(t) \quad = \quad e^{-iHt}. \tag{44}$$

In the quantum-like model, the belief-action state at time t and under condition C of the initial stage gamble outcome is given by

$$\Psi_C(t) \quad = \quad U(t)\Psi(0, C). \tag{45}$$

The probability for taking the second-stage gamble under condition C is then obtained from the evolved belief-action state at the time of measurement, by projecting with M_{gamble} for 'taking the second-stage gamble' and taking the modulus square of that outcome

$$p(gamble|X, Cond) \quad = \quad ||M_{gamble}U(\pi/2)\Psi_{0,C}||^2. \tag{46}$$

The time of measurement is fixed to the conventional choice $t = \pi/2$, corresponding to the choice of measurement time in the Markov model, eq. (29). Since the time-scale of the evolution, eq. (45), for the cognitive realm is undefined and since no response time observations are involved, a designated time of measurement can be fixed by convention. Notice too that both in the Markov model and in the quantum-like model the optimized parameter fitting will be adapted to this conventional time choice.

For each of the two periods for decision making in each flow order (K-to-U and U-to-K) a separate final belief-action state will be obtained. These final states will differ due to their respective initial belief-action states. Since the quantum-like model uses vectors of probability amplitudes - that require modulus squaring for probabilities - it is more transparent to write the 4-dimensional vectors as a tensor product of two 2-dimensional vectors, the first one for category Win/Lose and the second one for decision Gamble/Stop (see Supplementary Materials, eq. (A2) for details).

In the **first** period, the initial belief-action states on Win, respectively Lose, outcome condition of the first-stage gamble are formally given by the vectors;[14]

$$\Psi_{0,W} = \begin{pmatrix} \nu \\ \sqrt{1-\nu^2} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad , \qquad \Psi_{0,L} = \begin{pmatrix} \sqrt{1-\nu^2} \\ \nu \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix},$$

where ν is a weight parameter, $0 \leq \nu \leq 1$. Should $\nu = 1$ then these respective states are precisely allocated to the Win and Lose components, while the probability to Gamble or Stop for each of them is uniformly .5, as can easily be verified by squaring the entries in the Gamble/Stop vector. In the block with Known outcome conditions the participant is exposed to both Win and Lose outcome gambles. These two conditions create a mutual context for each gamble. The context effect will be present when $\nu < 1$ and expresses the idea that the information on the condition of the first-stage gamble is only partially integrated into the belief-action state.

The initial belief-action state –in first period– in the Unknown outcome case of the first-stage gamble is expressed as

$$\Psi_{0,U} \quad = \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \tag{47}$$

which reveals that the belief support for Win or Lose is uniform and also the action-potential for the Gamble or Stop decision is indifferent due to lack of any prior experience with gambles. The

[14]The tensor product notation is used here to distinguish more easily the effect of the parameter on the belief support in the Win/Lose evaluation. The two dimensional vector for Win/Lose appears as the left factor of the tensor product, the right factor is the two dimensional vector for Gamble/Stop. Both subspace vectors can be blended into the four dimensional vector according to the usual rule

$$\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} \quad = \quad \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}.$$

state is caused by the uncertainty due to missing information on the first-stage outcome in the Unknown-outcome condition.

In the **second** period the context effect is modified by the carry-over effect, hence the initial state now depends on the block's gamble condition as well as on the participant's history of the first period. The initial belief-action states for Win and Lose conditions will reflect residual belief support for the opposite condition modified by the carry-over from the previous period

$$\Psi_{00,W} = \begin{pmatrix} \mu \\ \sqrt{1-\mu^2} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad , \qquad \Psi_{00,L} = \begin{pmatrix} \sqrt{1-\mu^2} \\ \mu \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{48}$$

Since the initial belief-action state in the second period is influenced by the first block's condition, in the Unknown condition the belief-action state will be a superposition of the two states for the possible outcome conditions W and L of the Known outcome conditioned gambles block. In particular, the quantum-formalism allows to weight both conditions equally but also to include a relative complex phase $\kappa\pi$ between the two states for Win and Lose. The sign and amplitude of this phase allows for constructive or destructive interference between the two states and thus brings about a subjective tendency towards either of the known outcome beliefs

$$\Psi_{00,U} \quad = \quad \left(\Psi_{0,W}(\nu) + e^{i\kappa\pi}\Psi_{0,L}(\nu)\right)/\mathcal{N}, \tag{49}$$

where the normalization of the initial state requires $\mathcal{N} = \sqrt{2 + 4\nu\sqrt{1-\nu^2}\cos\kappa\pi}$. From the quantum-like perspective, the potential for interference of belief-action states indicates a susceptibility for an amplifying or reducing relation between beliefs. In the event of 'decoherence' between these belief-action states –by their reduction to separate contexts– interference between them will be diminished or impeded. In the second period the probability for taking the second-stage gamble under either of conditions $\{W, L, U\}$ is again obtained according to eq. (46), by projecting the evolved belief-action state using M_{gamble}, the projector for 'taking the second-stage gamble', and taking the modulus squared.

In the Supplementary Materials, (SM 2), a graph of the time development of the probability of the decision process for the second-stage gamble shows the build up of the gamble probabilities emerging from each of the initial belief-action states.

Parametrization. The quantum-like model requires a parametrization that closely resembles the parametrization of the Markov model. It requires the four dynamical parameters for the driving utility difference of the second-stage gamble for the two conditions of Win and Lose, namely $\{\delta_{0W}, \delta_{1W}\}$ and $\{\delta_{0L}, \delta_{1L}\}$. Also the effect of the utility difference on the decision is controlled by a sensitivity parameter $\{s\}$, however this parameter is now monitoring a hyperbolic tangent version of the logistic function, eqs. (41). A similar 'coupled-switching' dynamics, controlled by a mixing parameter γ, is implemented for the attention switching from Win to Lose correlated to switching between Gamble or Stop decision. The mixing Hamiltonian, eq. (42), and the Markov intensity rate mixing, eq. (24), are structured differently due to Hermeticity instead of probability conservation requirements. In the gamble block with Win and Lose outcome conditions the context effect is implemented by the weight parameters $\{\nu, \mu\}$ for first period and second period respectively.

In the quantum-like model the carry-over effect from first to second period is implemented differently in the belief-action state for the Unknown condition; instead of weighting two components the parameter $\{\kappa\}$ now causes an interference between the two components by implementing a relative complex phase.

The quantum model, just like the Markov model, relies on 9 parameters to cover the process dynamics and the initial belief-action states in both flow orders, both periods and all payoffs amounting to theoretical values for 30 data points. The Supplementary Materials section, (SM 2) provides more details on the temporal evolution in this model.

4.4. Logistic model

In order to compare the performance of the Markov and quantum-like process models, we devised a third model which aims to heuristically reproduce the observed gamble probabilities. Similarly to the Markov and quantum-like models, this baseline model is also made context and order sensitive but does not comprise a dynamic process for the belief-action state. Instead the baseline model implements for each gamble condition ad hoc weightings of the two utilities for Win outcome and Lose outcome, eqs. (16). The gamble probability is then simply obtained from a logistic function of the heuristic utility

$$p(gamble|X, Cond) \quad = \quad \frac{1}{1+e^{-s\cdot U(X,Cond)}}, \tag{50}$$

where the parameter s functions as a sensitivity parameter. In the **first** period, the utility of taking the second-stage gamble in the Win and in the Lose condition will be set according to

$$U_{KU}(W) = \omega_{KU_K} u_W + (1-\omega_{KU_K})u_L \quad , \qquad U_{KU}(L) = (1-\omega_{KU_K})u_W + \omega_{KU_K} u_L, \tag{51}$$

where ω_{KU_K} is a weight parameter, $0 \leq \omega_{KU_K} \leq 1$, expressing the participant's inclination towards Win and Lose beliefs. Hence also in the Logistic model we allow for partial adherence to the information in the outcome condition, but now this occurs on the level of utility instead of belief probability amplitude. It can be argued that each presented gamble is embedded between Win-outcome and Lose-outcome games, and this engenders residual utility-based support.

In the **first** period the utility of the Unknown outcome of a first-stage gamble is

$$U_{UK}(U) \quad = \quad \omega_{UK_U} U_{UK}(W) + (1-\omega_{UK_U})U_{UK}(L) \tag{52}$$

which expresses the resulting utility is a weighting of Win and Lose conditioned utility assessments, by the parameter ω_{UK_U}.

In the **second** period the context effect is now modified by the carry-over effect, which changes the weighting in the utility of the Win and Lose outcome gambles

$$U_{UK}(W) = \omega_{UK_K} u_W + (1-\omega_{UK_K})u_L \quad , \qquad U_{UK}(L) = (1-\omega_{UK_K})u_W + \omega_{UK_K} u_L \tag{53}$$

in which the weight parameter now is ω_{UK_K}.

In the Unknown first-stage outcome gamble the utility weighting, ω_{KU_U}, is now changed because of the carry-over effect

$$U_{KU}(U) = \omega_{KU_U} U_{KU}(W) + (1-\omega_{KU_U})U_{KU}(L) \tag{54}$$

with $0 \leq \omega_{KU_U} \leq 1$.

Parametrization. In contrast to the Markov and quantum-like models, the Logistic model does not rely on belief-action states but rather on heuristically adapted utility functions. For each condition of Known or Unknown outcome, gamble payoff and period a dedicated utility weighting drives a logistic function to render the probabilities for the second-stage gamble. The logistic model requires the same four dynamical parameters for the driving utility difference as in the two dynamical models, namely $\{\delta_{0W}, \delta_{1W}\}$ and $\{\delta_{0L}, \delta_{1L}\}$. The effect of the utility difference on the decision is controlled by a sensitivity parameter $\{s\}$ on a logistic function, eq. (50). In the logistic model the carry-over effect and the context effect are covered by four ad hoc weighting parameters, $\{\omega_{KU_K}, \omega_{KU_U}, \omega_{UK_K}, \omega_{UK_U}\}$.

Like the Markov and the quantum-like model, the logistic model uses 9 parameters to produce the gamble probability for both flow orders, both periods and all payoffs amounting to theoretical values for 30 data points.

5. Theoretical model performance

The three models have been parametrized for maximum likelihood statistical estimation on the data set. With three initial gamble outcome conditions and the five variable payoff amounts the survey produces fifteen observed proportions for each block ordering. For each model the objective function G for the parameter optimalisation is

$$G = 2N_{KU} \sum_{i=1}^{15} \left(o_i \ln\left(\frac{o_i}{e_i}\right) + (1-o_i)\ln\left(\frac{1-o_i}{1-e_i}\right)\right) + 2N_{UK} \sum_{i=1}^{15} (\text{idem}) \qquad (55)$$

where N_{KU} and N_{UK} are the number of participants for each flow order, the o_i and e_i are the observed and expected probabilities, and the sums cover the order condition 'K-to-U' and 'U-to-K' respectively. The G statistic expresses the lack of fit of the model predictions with the observed values. The numerical optimization was executed in Matlab using a 3^9 grid for the initial parameter vectors (Supplementary materials, SM 8).

The model performance is compared using the Bayesian Information Criterion. The BIC penalizes a model for complexity through the number of free parameters. With $BIC = G + p \ln N$, and the three models sharing the same number of parameters and data points the BIC comparison reduces to a G value comparison.

5.1. Model comparison for risk attitude partitioning.

In the sample partitioning approach by risk attitude the observed gamble proportions are separated along 'Less risk averse' and 'More risk averse' attitude. The BIC comparison favours the quantum-like model as the best fit for the risk partitioned observations. For the Logistic model G=123.15, for the Markov model G=123.73 and for the quantum-like model G=42.19 (Table 3). In this partitioning of participants both the Markov model and the Logistic model perform equally unsatisfactorily (Figures 6, 7).[15] This is in the line with expectation since both the Logistic and

[15] The more fine-grained partitioning using the played gamble patterns of each participant, Supplementary Materials section SM 5, reveals the Markov model performs better than the Logistic model only in four out of eight subgroups, namely for 'More risk averse' attitude in the ID-ranges [-2,2], [0,0] and [-1,0] and 'Less risk averse' attitude in the ID-range [-2,2] (the ID-ranges are defined in Table S2). This analysis also shows that the two subgroups with pronounced contributions to the violation of the Law of Total Probability are the ones best modeled by the quantum model (namely for 'More risk averse' attitude in the ID-ranges]-2,2] for UK and [-2,2[for KU).

the Markov models are abiding by classical logical constraints on the probabilities which prevent these models from providing inflative or deflative Disjunction Effects.

	'Less risk averse'			'More risk averse'		
group	Markov	quantum-like	Logistic	Markov	quantum-like	Logistic
all	70.96	18.96	62.15	52.77	23.23	61.00

Table 3: G-statistic for lack of fit test for the Markov, quantum-like and Logistic model fitted to the partitioning of less and more risk averse participants

.

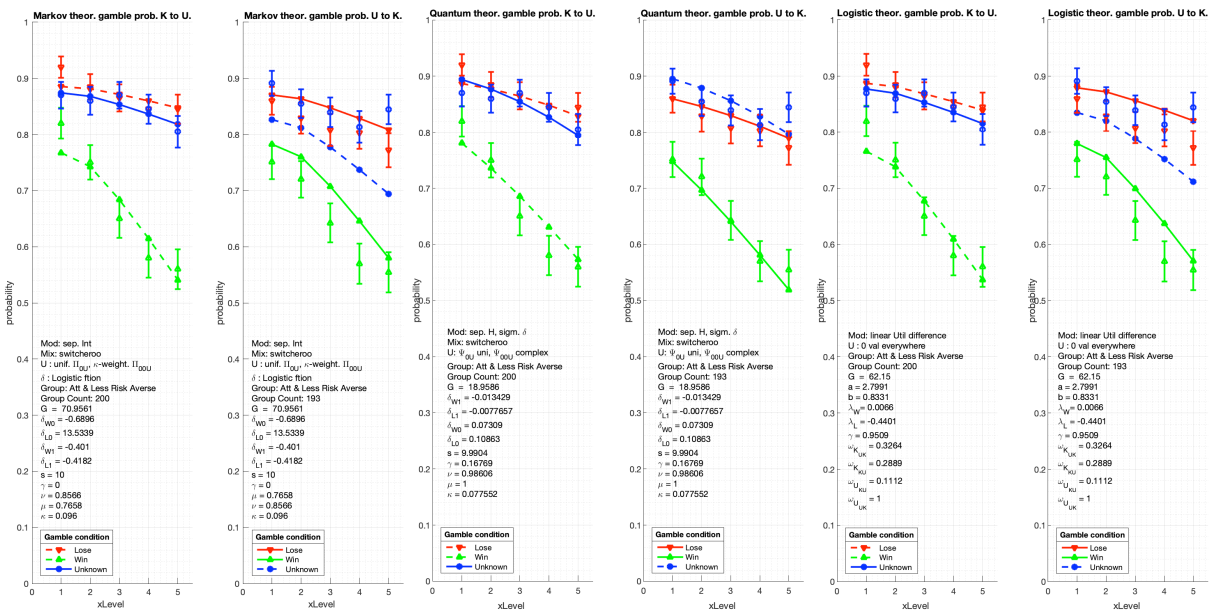

Figure 6: Observed and theoretical gamble probabilities for Less risk averse participants, for each of the three models Markov, Quantum and Logistic for K-to-U (left) and U-to-K (right) flow orders. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

6. Discussion

We defined the Disjunction Effect as a deflative violation of the Law of Total Probability, an effect which appears at a level of aggregate probabilities. We pointed out that the Sure Thing Principle and its violation can contribute to the Disjunction Effect but it does not significantly do so in the data of our present study on the two-stage gamble.

Our study revealed some new empirical findings, set forward some theoretical issues and provided model solutions in the two-stage gamble paradigm. The two main empirical contributions concern 'more risk averse' participants. For these participants, we found a reliable order effect in relation

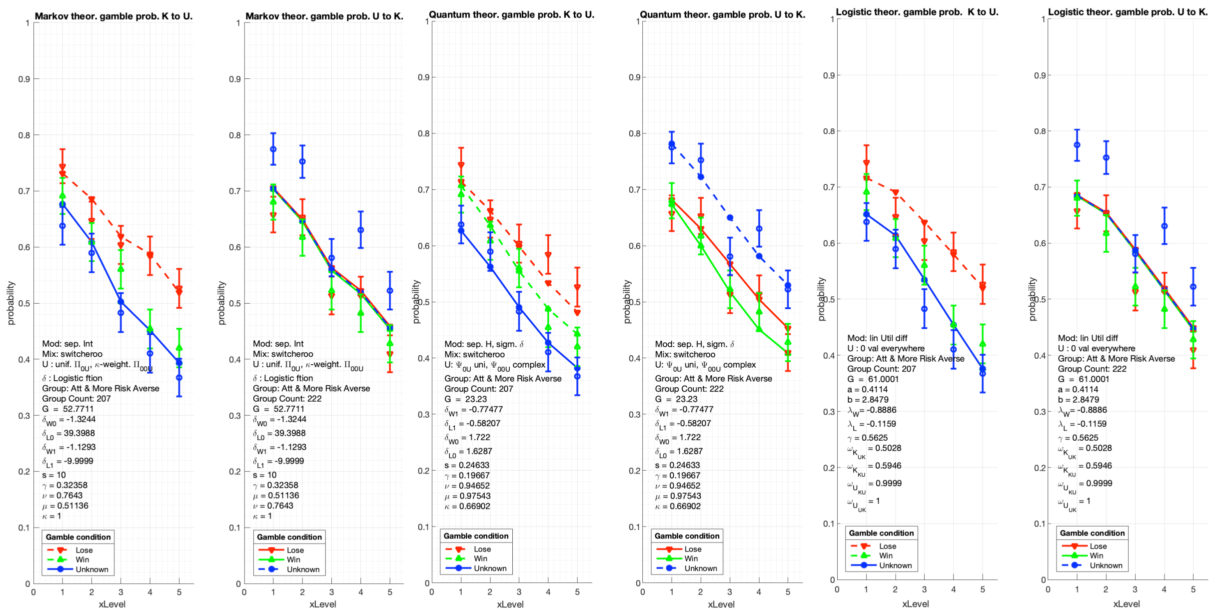

Figure 7: Observed and theoretical gamble probabilities for More risk averse participants, for each of the three models Markov, Quantum and Logistic for K-to-U (left) and U-to-K (right) flow orders. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

to the Disjunction Effect and the violation of the Law of Total Probability. Also a novel inflation effect on gambling in the Unknown outcome condition was observed, analogous but opposite to the Disjunction Effect when Unknown conditioned two-stage gambles precede the Known outcome conditioned ones. We found that 'less risk averse' participants did not produce either of these effects.

Specifically our replication of the two-stage gamble experiment, with variation of the payoff amount and the blocking and ordering of outcome conditions, showed a significant inflative violation of a prediction based on the Law of Total Probability for 'More risk averse' participants when the block of gambles with Unknown outcome conditions preceded the block of gambles with Known outcome conditions. The classical deflative Disjunction Effect was observed with only marginal statistical significance for this same group of 'More risk averse' participants when the grouped gambles with Known outcome conditions preceded the grouped gambles with Unknown outcome conditions. The factors of gamble outcome condition and condition-block ordering showed a strong cross-over interaction in the gamble probability for this 'More risk averse' group.

The group of 'Less risk averse' participants did not show any indication for the deflative Disjunction Effect in Known to Unknown outcome order, and only a very weak indication for an inflative Disjunction Effect in Unknown to Known order.

We mention that in the conclusion of their paper Tversky and Shafir (1992) comment on an additional test (N=87) in which the Unknown outcome gamble case appears on the same page just after Win outcome and Lose outcome. This showed an 'inflative' violation of the Law of Total Probability (see also Kühberger et al. (2001), p. 256). Tversky and Shafir (1992) argued the concurrent presentation allows participants to realize that they accept the repeated gamble in both Known outcome cases compelling them to accept the repeated gamble in the Unknown outcome case. Their observation is at odds with our present results of a similar inflative effect occuring when participants have not previously made decisions on Known previous outcome gambles.

In sum, the factors of previous gamble outcome condition and condition-block ordering showed a strong cross-over interaction in the gamble probability of the ‘More risk averse’ group. In contrast to the Tversky and Shafir (1992) result our observation shows the Disjunction Effect –and its inflative variant– is fully dependent on the order in which the outcome conditions are cued to the participant and moreover only appears in ‘More risk averse’ participants. In none of the cases was Tversky and Shafir’s signature gamble pattern for the Disjunction Effect –to play the gamble on Win outcome and on Lose outcome but to stop the gamble on Unknown outcome– a significant contribution to the effect. Therefore the violation of the Sure Thing Principle –through the named signature gamble pattern– did not contribute to the Disjunction Effect in our observations.

Tversky and Shafir argued their participants were inclined to take the second-stage gamble under Win condition for one reason, e.g. ‘house money’, (Thaler and Johnson, 1992), and also under Lose condition for another reason, e.g. ‘make up for a loss ’, but would not take the second-stage gamble under Unknown previous outcome condition because they lost their acuity to process the differing reasons. By contrast, in our study we clearly found that individual participant risk attitude and first-stage gamble outcome order condition play a crucial role in the occurrence of the deflative and inflative violation of the Law of Total Probability. This shows prior experience of Winning and Losing gambles can carry over into a Disjunction Effect, but lacking such prior experience on the contrary can still lead to a violation of the Law of Total Probability.

In relation to the general sample characteristics it may be important to note that our participants were recruited among MTurk workers, taking on this task in a short duration of time and for a small monetary compensation. The lowered probability to gamble on Win and the relatively increased probability to gamble on Lose indicates these participants are conservative with respect to risk; keeping for sure what was gained and risking to regain what was lost. The gamble strategies may thus well be influenced by socio-cultural traits besides general risk attitude (Surov et al., 2019).

In Experiment 2 we observed the intriguing pattern of results that ’more risk averse’ participants are less likely to accept the second-stage gambles under Unknown conditions in the K to U order, than ’less risk averse’ participants. A plausible (but currently speculative) explanation is that in the U to K order, decisions about second-stage gambles are informed vaguely from first-stage gambles, since there was no experience of gain or loss; participants were just told that a gamble was played, but the outcome of the gamble was Unknown. By contrast, in the K to U order, participants would be offered a more direct experience of loss and gain. ’More risk averse’ participants are likely to be disproportionately influenced by the experience of loss than the experience of gain, in the K to U order, compared to ’less risk averse’ participants, depressing the relative tendency for accepting a second-stage gamble under Unknown conditions in the K to U order for such participants.

From a theoretical point of view we presented two dynamical models for the decision process and provided one static logistic utility model that mimicked relevant decision features in an ad hoc manner. More specifically, the underlying principles of the decision process are i) a utility drive which is condition dependent both on a gamble outcome and payoff size, ii) a context influence of the conditioned gamble block causing partial acceptance of the Win or Lose information, iii) an assumption to regain an effective initial belief-action state prior to each second-stage gamble, and iv) a carry-over influence on the belief-action state due to the ordered flow of condition blocks.
All these –cognitively very plausible– features can be incorporated in formally very similar ways in

both the Markov and the quantum-like model, but the latter was shown clearly superior in most participant groups. The Markov and the quantum-like models were formally matched carefully, so that their difference essentially concerned the use of classical probability principles in the former and quantum ones –like superposition of states belief-action states– in the latter. Our results clearly show that, at least in some cases, the alternative probabilistic principles of quantum theory are required for a satisfactory explanation of decision behavior. The carry-over effect from first to second period in the Unknown condition was implemented by using quantum superposition. This principle causes a belief and action potential interference between the two Known outcome components, and reflects the ambiguous belief in the Unknown condition. It is this aspect of the quantum-like approach that makes it an appropriate and efficient formalism to capture aspects of 'irrationality' in human decision making.

Authors contributions

J.B.B. and E.M.P conceived the initial phase of the study and designed Experiment 1, J.R.B advised order conditioning in Experiment 2 and supervised the completion of the study. The construction of theoretical models and data analysis were done by J.B.B. and discussed with J.R.B and E.M.P.. J.B.B. wrote the paper with both J.R.B. and E.M.P. providing feedback.

Ethical clearance

The survey received ethical clearance in the framework of 'Risky Decision Making', PSYETH (S/L) 16/17 51, City, University of London.

Funding

This research was partly funded by AFOSR FA 9550-12-1-0397 to J.R.B and J.B.B., Leverhulme Trust grant RPG-2015-311 to E.M.P. and J.B.B. and ONRG grant N62909-19-1-2000 to E.M.P.. The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the funding agencies.

Supplementary Material

Supplementary data associated with this article are presented here as appendices. These will appear with the online version at URL: $\cdots$

Acknowledgements

The authors thank the anonymous referees for discussing the possibility of a 'lazy responding' effect in less risk averse participants, and the observed differences between the unconditional first-stage gamble probability and the second-stage Unknown-conditioned gamble, as well as many other comments.

References

Accardi, L., Khrennikov, A., Masanori, O., 2009. Quantum markov model for data from shafir-tversky experiments in cognitive psychology. Open Systems & Information Dynamics 16, 371–385. doi:10.1142/S123016120900027X.

Aerts, D., 2009. Quantum structure in cognition. Journal of Mathematical Psychology 53, 314–348. doi:10.1016/j.jmp.2009.04.005.

Aerts, D., Aerts, S., 1995. Applications of quantum statistics in psychological studies of decision processes. Foundations of Science 1, 85–97. doi:10.1007/978-94-015-8816-4_11.

Asano, M., Khrennikov, A., Ohya, M., Tanaka, Y., Yamato, I., 2015. Quantum Adaptivity in Biology: From Genetics to Cognition. Springer Netherlands.

Asano, M., Ohya, M., Tanaka, Y., Khrennikov, A., Basieva, I., 2011. On application of gorini kossakowski sudarshan lindblad equation in cognitive psychology. Open Systems and Information Dynamics 18, 55–69. doi:10.1142/S1230161211000042.

Atmanspacher, H., Filk, T., 2013. The necker-zeno model for bistable perception. Topics in Cognitive Science 5, 800–817.

Broekaert, J., Aerts, D., D'Hooghe, B., 2006. The generalised liar paradox: a quantum model and interpretation. Foundations of Science 11, 399–418. doi:10.1007/s10699-004-6248-8.

Broekaert, J., Basieva, I., Blasiak, P., Pothos, E., 2016. Quantum-like dynamics applied to cognition: a consideration of available options. Philosophical Transactions of the Royal Society A 20160387. doi:10.1098/rsta.2016.0387.

Busemeyer, J., Bruza, P., 2012. Quantum models of cognition and decision. Cambridge, UK: Cambridge University Press.

Busemeyer, J., Matthew, M., Wang, Z., 2006a. Quantum game theory explanation of disjunction effects. In R. Sun & N. Miyake (Eds.), Proceedings of the 28th annual conference of the cognitive science society, Mahwah, NJ: Erlbaum. , 131–135doi:10.3389/10.1037/a0022542.

Busemeyer, J., Pothos, E., Franco, R., Trueblood, J., 2011. A quantum theoretical explanation for probability judgment errors. Psychological Review 118, 193–218. doi:10.3389/10.1037/a0022542.

Busemeyer, J., Wang, Z., Lambert-Mogiliansky, A., 2009. Empirical comparison of markov and quantum models of decision-making. Journal of Mathematical Psychology 53, 423–433. doi:10.1016/j.jmp.2009.03.002.

Busemeyer, J., Wang, Z., Townsend, J., 2006b. Quantum dynamics of human decision-making. Journal of Mathematical Psychology 50, 220–241. doi:10.1016/j.jmp.2006.01.003.

Camerer, C., Hogarth, R., 1999. The effects of financial incentives in experiments: A review and capital-labor-production framework. Journal of Risk and Uncertainty 19, 7–42. doi:10.1023/A:1007850605129.

Cardillo, G., 2006. Wilcoxon test: non parametric wilcoxon test for paired samples. Mathworks URL: http://www.mathworks.com/matlabcentral/fileexchange/12702.

Croson, R., 1999. The disjunction effect and reason-based choice in games. Organizational Behavior and Human Decision Processes 80, 118–133. doi:10.1006/obhd.1999.2846.

Edwards, W., 1954. The theory of decision making. Psychological bulletin 51, 380–417. doi:10.1037/h0053870.

Estes, W., 1956. Inference from curves based on group data. Psychological Bulletin 53, 134–140.

Fuss, I., Navarro, D., 2013. Open parallel cooperative and competitive decision processes: A potential provenance for quantum probability decision models. Topics in Cognitive Science 5, 818–843. doi:10.1111/tops.12045.

Hogarth, R., Einhorn, H., 1992. Order effects in belief updating: the belief-adjustment model. Cognitive Psychology 24, 1–55.

Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of choice under risk. Econometrica 47, 263–291.

Khrennikov, A., 2010. Ubiquitous Quantum Structure: From Psychology to Finances. Berlin: Springer.

Khrennikova, P., Haven, E., Khrennikov, A., 2014. An application of the theory of open quantum systems to model the dynamics of party governance in the us political system. International Journal of Theoretical Physics 53, 1346–1360.

Kühberger, A., Komunska, D., Perner, J., 2001. The disjunction effect: Does it exist for two-step gambles? Organizational Behavior and Human Decision Processes 85, 250–264. doi:10.1006/obhd.2000.2942.

Kvam, P., Pleskac, T., Yu, S., Busemeyer, J., 2015. Interference effects of choice on confidence: Quantum characteristics of evidence accumulation. Proceedings of the National Academy of Sciences 112, 10645–10650. doi:10.1073/pnas.1500688112.

Lambdin, C., Burdsal, C., 2007. The disjunction effect reexamined: relevant methodological issues and the fallacy of unspecified percentage comparisons. Organizational Behavior and Human Decision Processes 103, 268–276. doi:10.1016/j.obhdp.2006.04.001.

Martínez-Martínez, I., 2014. A connection between quantum decision theory and quantum games: The hamiltonian of strategic interaction. Journal of Mathematical Psychology 58, 33–44. doi:10.1016/j.jmp.2013.12.004.

Martínez-Martínez, I., Sánchez-Burillo, E., 2016. Quantum stochastic walks on networks for decision-making. Scientific Reports 6, 23812. doi:10.1038/srep23812.

Pothos, E., Busemeyer, J., 2009. A quantum probability explanation for violations of 'rational' decision theory. Proceedings of the Royal Society B 276, 2171–2178. doi:10.1098/rspb.2009.0121.

Pothos, E., Busemeyer, J., 2013. Can quantum probability provide a new direction for cognitive modeling? Behavioral and Brain Sciences 36, 255–274. doi:10.1017/S0140525X12001525.

Pothos, E., Perry, G., Corr, P., Matthew, M., Busemeyer, J., 2011. Understanding cooperation in the prisoner's dilemma game. Personality and Individual Differences 51, 210–215. doi:10.1016/j.paid.2010.05.002.

Pratt, J., 1959. Remarks on zeros and ties in the wilcoxon signed rank procedures. Journal of the American Statistical Association 54, 655–667.

Reyna, V., Brainerd, C., 1991. Fuzzy-trace theory and framing effects in choice: Gist extraction, truncation, and conversion. Journal of Behavioral Decision Making 4, 249–262.

Samuelson, P., 1963. Risk and uncertainty: A fallacy of large numbers. Scientia 98, 108–113.

Savage, L., 1954. The Foundations of Statistics. John Wiley & Sons Inc., New York.

Shafir, E., Tversky, A., 1992. Thinking through uncertainty: nonconsequential reasoning and choice. Cogn. Psychol. 24, 449–474. doi:10.1016/0010-0285(92)90015-T.

Smith, P., Ratcliff, R., 2015. An introduction to the diffusion model of decision making. In B. U. Forstmann & E.-J. Wagenmakers (Eds.), An introduction to model-based cognitive neuroscience, New York, NY, US: Springer Science + Business Media. , 49–70doi:10.1007/978-1-4939-2236-9_3.

Sonnenberg, F., Beck, J., 1993. Markov models in medical decision making: A practical guide. Medical Decision Making 13, 322–338. doi:10.1177/0272989X9301300409.

Surov, I., Pilkevich, S., Alodjants, A., Khmelevsky, S., 2019. Quantum phase stability in human cognition. Frontiers in Psychology 10. doi:10.3389/fpsyg.2019.00929.

Thaler, R., Johnson, E., 1992. Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice. Management Science 36, 643–766. doi:10.1287/mnsc.36.6.643.

Trueblood, J., Busemeyer, J., 2011. A quantum probability account of order effects in inference. Cognitive Science 35, 1518–1552. doi:10.1111/j.1551-6709.2011.01197.x.

Tversky, A., Shafir, E., 1992. The disjunction effect in choice under uncertainty. Psychological Science 3, 305–310. doi:10.1111/j.1467-9280.1992.tb00678.x.

Wang, Z., Busemeyer, J., Atmanspacher, H., Pothos, E., 2013. The potential of using quantum theory to build models of cognition. Topics in Cognitive Science 5, 672–688. doi:10.1111/tops.12043.

Wang, Z., Solloway, T., Shiffrin, R.M., Busemeyer, J., 2014. Context effects produced by question orders reveal quantum nature of human judgments. Proceedings of the National Academy of Sciences 111, 9431–9436. doi:10.1073/pnas.1407756111.

Yearsley, J., 2017. Advanced tools and concepts for quantum cognition: A tutorial. Journal of Mathematical Psychology 78, 24–39. doi:10.1016/j.jmp.2016.07.005.

Yearsley, J., Busemeyer, J., 2016. Quantum cognition and decision theories: A tutorial. Journal of Mathematical Psychology 74, 99–116. doi:10.1016/j.jmp.2015.11.005.

Appendix A: Introduction to quantum modeling.

To implement a quantum-like process in cognitive modeling an adaptation and simplification of the quantum mechanical formalism is required (Busemeyer and Bruza, 2012; Khrennikov, 2010; Yearsley, 2017; Yearsley and Busemeyer, 2016). In practice the dimensions and features of the model are set by the measured properties, the operational order of the experimental paradigm and theoretical presuppositions. A participant's belief-action state is assumed to be probabilistic over its potential realisations - quite similar to a Markov approach. In the quantum-like approach a participant's response is a realisation of one of its potentials, in a Markov approach there is no difference between the realised and inherent state. In a quantum-like model the belief-action state is represented by a vector in a Hilbert space, which is a regular vector space - of any dimension - with an inner product and a completeness property that assures limits will exist.

In most models this space will just be the finite dimensional complex valued space $\mathbb{C}^n$, and its points $\Psi = (\psi_1 \dots \psi_j \dots \psi_n)$ represent a belief-action state for n properties. Typically this state will be written as a column vector $|\psi\rangle$, or complex-transposed as a row vector $\langle\psi|$.
When different properties are combined in the belief-action state, it becomes more practical to describe each property by its own vector and then combine these vectors using the tensor product. This will result in an encompassing vector in which each entry is the joint probability amplitude of outcome values for the different features. In our model we have used this tensor decomposition to emphasize the dynamics that deploys in each feature, separately from the 'mixing' dynamics which engages between different features. For instance we take the action-potential for Gambling and Stopping as a vector for the decision, and the belief supporting a Win or Lose state as a vector for the category.

$$\Psi_{G,S} = \begin{pmatrix} \psi_G \\ \psi_S \end{pmatrix} \quad , \quad \Psi_{W,L} = \begin{pmatrix} \psi_W \\ \psi_L \end{pmatrix} \tag{A1}$$

The overall vector for this model becomes

$$\Psi \quad = \quad \Psi_{W,L} \otimes \Psi_{G,S} = \begin{pmatrix} \psi_W \psi_G \\ \psi_W \psi_S \\ \psi_L \psi_G \\ \psi_L \psi_S \end{pmatrix} = \begin{pmatrix} \psi_{WG} \\ \psi_{WS} \\ \psi_{LG} \\ \psi_{LS} \end{pmatrix} \tag{A2}$$

Notice that the tensor product is not commutative, and that the choice of order must be maintained for the propagators of this encompassing belief-action state.

The response of a participant is the realisation of a binary outcome –'gamble' or 'stop'– for a given property, which occurs by a measurement enacted by a measurement operator, say M. The outcome state of the measurement by M –e.g. 'Do you take the gamble?'– is one of the eigenvectors of the operator M. The probability p_j of a given outcome value λ_j is obtained by projecting the belief-action state $|\psi\rangle$ on the corresponding eigenvector of M and taking the norm squared. With M_j being this projector on the eigenstate, the probability for outcome value λ_j is thus given by:

$$p_j \quad = \quad ||M_j|\psi\rangle||^2 = \langle\psi|M_j|\psi\rangle \tag{A3}$$

This is the conventional Born's rule for outcome probabilities in quantum mechanics.
Most importantly the belief-action state $|\psi\rangle$ of a participant changes over time by cognitive input

through the cues in the experimental paradigm. In our present work we use the Schrödinger equation to capture the temporal evolution of the dynamic process of cognition. This approach needs temporal constraining since by its nature a finite dimensional and energetically closed system will be periodical. Alternatively open system dynamics with Lindblad evolution have been used (Asano et al., 2011; Khrennikova et al., 2014; Martínez-Martínez, 2014; Martínez-Martínez and Sánchez-Burillo, 2016) but require additional parametrization to fit the experimental paradigm.

In other 'time-less' quantum-like models that emphasize complementarity of features this evolution is abstracted and ad hoc unitary matrices are used to propagate the belief-action state Aerts (2009). These models would render a change of belief-action state by changing the base of the Hilbert space. These changes of basis correspond to changes of 'cognitive perspective' on the decision. For example, to change the perspective from Win to Lose, a Win base $\{e_{W_j}\}$ and Lose base $\{e_{L_k}\}$ would be related according to:

$$|e_{Wj}\rangle \quad = \quad \left(\sum_k |e_{Lk}\rangle\langle e_{Lk}| \right) |e_{Wj}\rangle = \sum_k \langle e_{Lk}|e_{Wj}\rangle \, |e_{Lk}\rangle \tag{A4}$$

Then

$$U_{jk} \quad = \quad \langle e_{Lk}|e_{Wj}\rangle \ , \quad \text{with} \quad U^\dagger U = \mathbb{I} \tag{A5}$$

In the Schrödinger approach a temporal parameter, 'time', orders the subsequent belief-action states, and evolves them according to the Hamiltonian operator:

$$-i\frac{d}{dt}\Psi(t) = H\Psi(t) \tag{A6}$$

where we have replaced the partial differential towards time with its the total derivative. Also Planck's constant, which is the unit of action, was set equal to 1. This renders both the 'energy' and 'time' into dimensionless variables. This simplified Schrödinger equation is solved easily

$$\psi(t) \quad = \quad U_t\psi(0) \ , \quad \text{with} \quad U_t = e^{-iHt}.$$

Central to developing a quantum-like model is the construction of the Hamiltonian operator. Only the Hamiltonian components that express the transport of probability amplitude in line with cognitive theoretical principles need to be implemented. Formally the Hamiltonian needs to fulfill the Hermitian property $H^\dagger = H$, a property related to its original function as the energy operator, with real-valued energy eigenvalues.

	Markov	Schrödinger
State vector	$\Pi = \begin{pmatrix} p_1 \\ \vdots \end{pmatrix}$	$\Psi = \begin{pmatrix} \psi_1 \\ \vdots \end{pmatrix}$
Vector components	probabilities, $\in \mathbb{R}$	probability amplitudes, $\in \mathbb{C}$
Entity's state	$(0 \cdots 1_j \cdots 0)$	$(r_1 e^{i\theta_1} \cdots r_j e^{i\theta_j} \cdots r_N e^{i\theta_N})$
	"always at some single j"	"at j only after pos. meas. for j"
Normalization	$\sum_j p_j = 1$	$\sum_j \lvert\psi_j\rvert^2 = 1$
Propagator	$T(t) = e^{Kt}$	$U(t) = e^{-iHt}$
	$\sum_i T_{ij} = 1$	$U^\dagger U = I$
Change operator	Transition rate matrix K	Hamiltonian H
	$\sum_i K_{ij} = 0$, $K_{ij} \geq 0$, $i \neq j$	$H^\dagger = H$
Dynamics	$\Pi(t) = T(t)\Pi(0)$	$\Psi(t) = U(t)\Psi(0)$
Measurement for j	component selector M_j for j	subspace projector M_j for j
Probability for j	$\lvert M_j \Pi(t)\rvert_1$	$\lvert\lvert M_j \Psi\rvert\rvert^2$

Table A1: Comparison of main model features in Markov and Quantum-like approach

Appendix B: Prospect Theory and formal Utility functions.

The motivation for taking the second-stage gamble is quantified by the utility difference between taking the second-stage gamble and stopping the gamble after the first-stage gamble, eqs. (14,15,13). In the Prospect Theory approach by Tversky and Shafir (1992) the utility function is given the formal expression of a power law. The utility expression x^a, with $a < 1$, expresses the diminished utility of monetary value due to risk aversion with respect to gains. Similarly for the utility of losses $-|x|^b$, with $b < 1$, expressing risk seeking, and with $b < a$, implementing the principle that losses loom larger than gains. Using this power law utility expression, and anchoring the outcome of the first-stage gamble in Unknown condition to zero, Tversky and Shaffir provided a theoretical argument for how a participant's choices would produce a violation of the STP.

Besides the observation of both an inflative and deflative violation of the LTP, our study also shows systematic decreasing tendencies to take the second-stage gamble when the payoff increases. In this section we show that this observed decreasing tendency is not only at odds with the power law form of the utility function (Tversky and Shafir, 1992), but also with the logarithmic utility form and the exponential utility form.

Power law Utility.

A gain x is evaluated with utility x^a and a loss $-x$ at $-x^b$ ($x > 0$, $0 \leq b \leq a \leq 1$). The motivation to take the second-stage gamble is expressed by the difference of Expected Utility and utility of stopping the gamble, eqs. (14,15,13):

$$\begin{aligned} EU(X|W) - U_w(X) &= .5\,(2x)^a + .5\,(x/2)^a - x^a, && \text{(B1)} \\ EU(X|L) - U_l(X) &= .5\,(x/2)^a - .5\,x^b + (x/2)^b, && \text{(B2)} \\ EU(X|U) - 0 &= .5\,x^a - .5\,(x/2)^b - 0. && \text{(B3)} \end{aligned}$$

One can easily show that this approach predicts increasing probability to take the second-stage gamble when the payoff x increases (loss $-x/2$ decreases). Moreover this is the case for both Known previous outcome conditions W and L. The derivatives of the utility differences in the W and L condition are

$$\begin{aligned} (EU(X|W) - U_w(X))' &= a\frac{x^{a-1}}{2^{a+1}}(2^a - 1)^2, && \text{(B4)} \\ (EU(X|L) - U_l(X))' &= a\frac{x^{a-1}}{2^{a+1}} + bx^{b-1}\left(\frac{1}{2^b} - \frac{1}{2}\right). && \text{(B5)} \end{aligned}$$

Both expressions are non negative, which indicates increasing motivation to take the second-stage gamble with increasing payoff X. According to the Prospect Theoretic approach (Tversky and Shafir, 1992) the probability to take the second-stage gamble should thus increase with payoff, for the W and L condition.

For the U condition the derivative of the utility motivation is

$$(EU(X|U) - 0)' = .5\,ax^{a-1}\left(1 - \frac{b}{2^b a}x^{b-a}\right) \qquad \text{(B6)}$$

in this case we find the probability to take the gamble will decrease for payoffs x larger than the constant $\sqrt[b-a]{\frac{2^b a}{b}}$, which thus allows in principle for a decreasing gamble probability.

Our observations of the gamble probabilities, Figure 4 and Figure 3, show a persistent trend of diminished playing for higher payoffs.[16]

Logarithmic utility.
A gain x is evaluated at logarithmic utility $U(X) = \log(x + b)$, with $b > 0$. The motivation to take the second-stage gamble in the Win condition is evaluated by the difference of Expected Utility and utility of stopping the gamble, eq. (14),

$$EU(X|W) - U_w(X) \quad = \quad .5\,\log(2x + b) + .5\,\log(x/2 + b) - \log(x + b). \tag{B8}$$

The first derivative towards payoff X is

$$(EU(X|W) - U_w(X))' \quad = \quad \frac{b}{4(2x + b)(x/2 + b)(x + b)}(-x + b), \tag{B9}$$

which can produce in principle decreasing probabilities with increasing payoff for larger X.

For a loss $x < 0$ the logarithmic utility is evaluated at $U(x) = -\log(|x| + c)$, with $c > 0$. For continuity of the utility function over the domains of loss and gain we can set $b = c$. The motivation to take the second-stage gamble in the Lose condition is evaluated by the difference of Expected Utility and utility of stopping the gamble, eq. (15)

$$\begin{aligned}(EU(X|L) - U_l(X)) \quad &= \quad .5\,(\log(|x|/2 + b)) + .5\,(-\log(|x| + b)) - (-\log(|x|/2 + b)),\\ &= \quad 1.5\log(|x|/2 + b) - .5\log(|x| + b).\end{aligned} \tag{B10}$$

The first derivative towards payoff X is

$$(EU(X|L) - U_l(X))' \quad = \quad -\frac{1.5}{|x| + 2b} + \frac{.5}{|x| + b}, \tag{B11}$$

which is negative and thus shows the utility increases with $|x|$, and larger losses should lead to a higher probability to gamble. Thus the logarithmic utility expression contradicts the observed gamble probabilities as well, Figure 4.

Exponential utility.
A gain x is evaluated at exponential utility $U(X) = (1 - e^{-ax})/a$, with $a > 0$. The motivation to take the second-stage gamble in the Win condition is evaluated by the difference of Expected Utility and utility of stopping the gamble, eq. (14,

$$EU(X|W) - U_w(X) \quad = \quad \frac{-.5\,e^{-a2x} - .5\,e^{-ax/2} + e^{-ax}}{a}. \tag{B12}$$

[16] Our observations show that the probability to take the second-stage gamble on Lose is larger than on Win, Figure 4. This feature is correctly predicted by Prospect Theory of Kahneman and Tversky (1979), and by Reyna and Brainerd (1991) in their fuzzy-trace theory. One can easily see that by Prospect Theory, eqs. (B1,B2), for a given payoff x the gamble under L has more utility than under W:

$$(EU(X|W) - U_w(X)) - (EU(X|L) - U_l(X)) \quad = \quad x^a(2^{a-1} - 1) + x^b(.5 - (.5)^b) \; < \; 0, \tag{B7}$$

since $0 \leq b \leq a \leq 1$ both summands are negative. Notice that the gamble probabilities in the original experiment of Tversky and Shafir (1992) do not adhere to this ordering, Table (1).

The first derivative towards payoff X is

$$(EU(X|W) - U_w(X))' \quad = \quad e^{-a2x} + .25\, e^{-ax/2} - e^{-ax} = e^{-ax}(e^{-ax} + .25\, e^{ax/2} - 1). \quad \text{(B13)}$$

The latter expression is a third-degree polynomial in $e^{ax/2}$. It has three real-valued roots, one negative and two positive. The derivative has therefore two zero-points. One can easily see from the sign of the bracketed expression, that the first derivative will be positive for payoffs $x > 2/a$.

x	0	$1/a$	$2/a$
$(EU(X\|W) - U_w(X))'$	+	-	+

Again we conclude larger payoffs will lead to a higher probability to gamble. The exponential utility expression thus also contradicts the observed gamble probabilities, Figure 4.

Supplementary Materials.

SM 1. The Markov model time evolution

The temporal evolution of the belief-action state in the Markov model is driven by the transition rate matrix K. The transition matrix at each instance of time t is given by e^{Kt} and evolves the initial belief-action state into the present state. The gamble probability is obtained by applying the selector matrix M_{Gamble} to the evolved belief-action state and taking the sum of the vector components

$$p(gamble, t|X, Cond) \quad = \quad |\,(M_{Gamble}T(t)\Pi_{0,C})\,|_1. \tag{S1}$$

The dynamic build-up of the gamble probability can thus be monitored quantitatively till the time of decision $t = \pi/2$. This probability evolution of the belief-action state is implemented separately for the two temporal periods. It is essential to this model that the parameters have been optimized for measurement at the conventional value $\pi/2$, therefore any measurement at intermediate times would change the protocol of the formalism and the model parameter values.

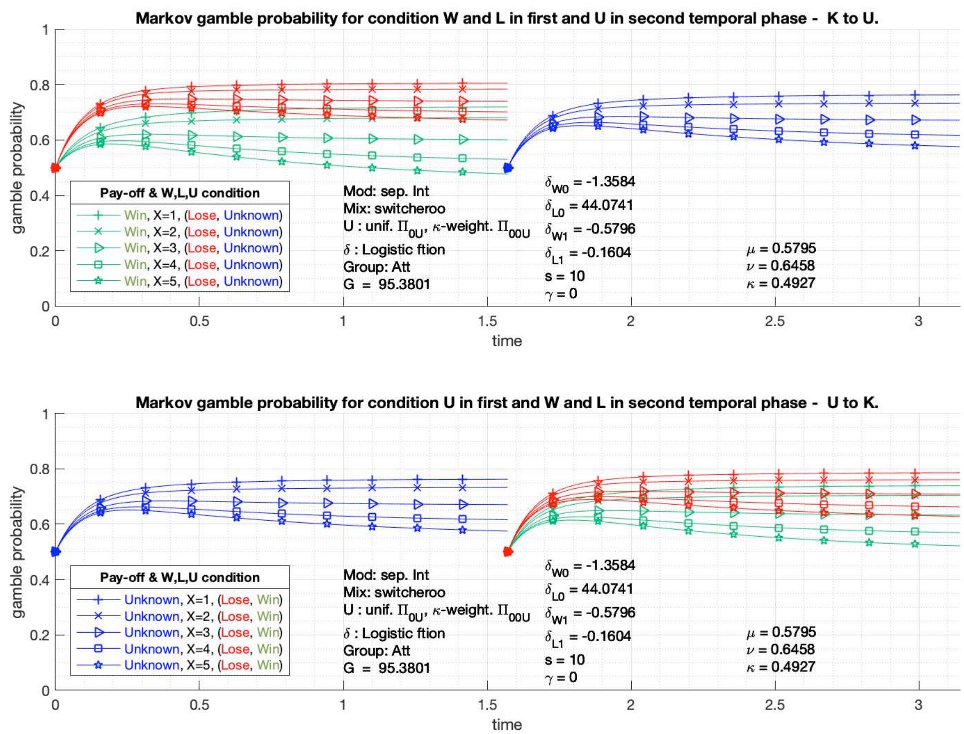

Figure S1: Markov temporal evolution towards the second-stage gamble probability. The group of attentive participants in K-to-U flow order ($N_{KU} = 407$) in the top panel and, U-to-K order in the lower panel ($N_{UK} = 415$). Participants cycle through the first period $[0,\pi/2]$ until all payoffs and conditions are taken and move on to cycle through the second period $[\pi/2,\pi]$ until all payoffs and conditions are taken.

SM 2. The quantum-like model time evolution

The temporal evolution of the belief-action state in the quantum model is driven by the Hamiltonian matrix H. The unitary evolution operator at each instance of time t is given by e^{-iHt} and evolves the initial belief-action state into the present belief-action state at time t. The gamble probability is obtained by applying the measurement operator M_{Gamble} to the evolved belief-action state and taking the sum of squared moduli of the vector components

$$p(gamble, t|X, Cond) \quad = \quad ||M_{gamble} U(t) \Psi_{0,C}||^2 \tag{S2}$$

Like in the Markov model, in the quantum model the dynamic build-up of the gamble probability can be monitored quantitatively till the time of decision $t = \pi/2$. This probability evolution of the belief-action state is implemented separately for the two temporal periods. The model parameters have been optimized for measurement at the conventional time value $\pi/2$, therefore any measurement at intermediate times would change the protocol of the formalism and the model parameter values. Moreover in the quantum model each intermediate measurement would reduce the belief-action state to a specific outcome and require Lüder updating of the wave function for further evolution.

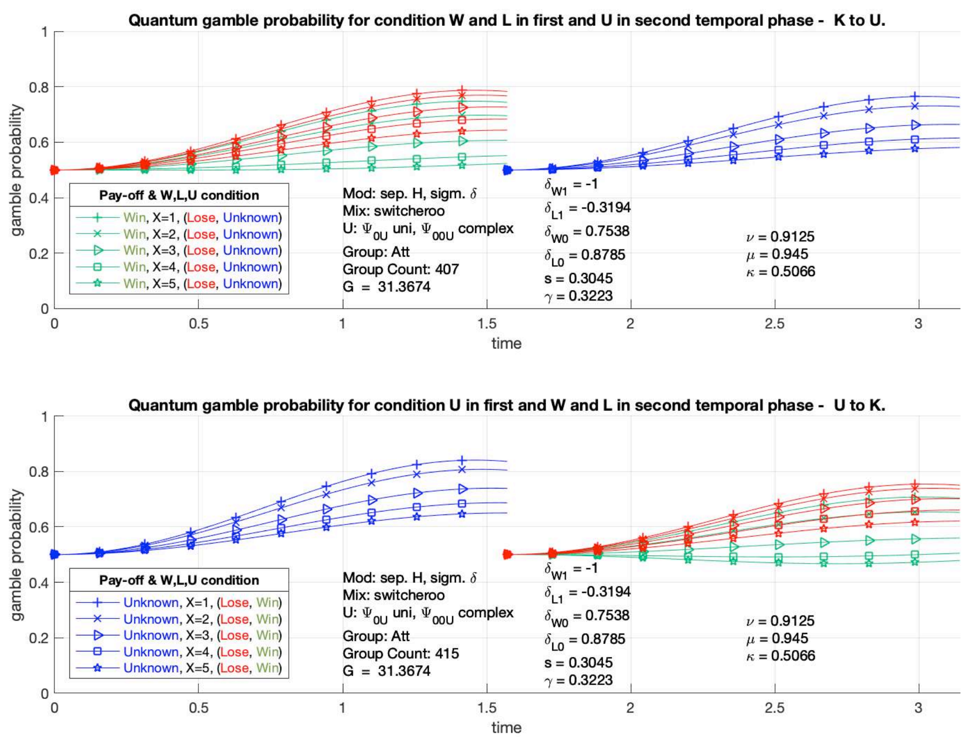

Figure S2: Quantum-like temporal evolution towards the second-stage gamble probability. The participants in K-to-U flow order ($N_{KU} = 407$) in the top panel and U-to-K order in the lower panel ($N_{UK} = 415$). Participants cycle through the first period $[0,\pi/2]$ until all payoffs and conditions are taken and move on to cycle through the second period $[\pi/2,\pi]$ until all payoffs and conditions are taken.

SM 3. Gamble patterns and Inflation-Deflation score.

We define an 'Inflation-Deflation' score, or ID-score, for each gamble pattern which determines their contribution in inflative ('upward') or deflative ('downward') sense to the violation of the Law of Total Probability by the aggregate probability $p(g|U)$. The ID-score is computed from individual gamble patterns, by assigning a +1 point for each choice conditioned on W or L and inconsistent with 'gamble' on U condition; and a -1 point for each choice conditioned on W or L and inconsistent with 'stop' on U condition (Table 3). The ID-score of a pattern is thus equivalent to the linear expression $2R_U - R_W - R_L$ of the responses ('R') under W, L and U condition. For example the pattern $(g|W, g|L, s|U)$ has an ID-score of -2 because both the choice '1' on Win and '1' on Lose are in excess of the choice '0' on Unknown. A participant's average ID-score therefore indicates the participant's *tendency* to deviate from the Law of Total Probability. More precisely by averaging a participants' ID-scores –per WLU triplet– over the set of the participant's played triplets, one obtains

W	L	U	ID$_{score}$
s	s	g	2
s	g	g	1
g	s	g	1
g	g	g	0
s	s	s	0
s	g	s	-1
g	s	s	-1
g	g	s	-2

Table S1: The eight gamble patterns in WLU notation order and their corresponding Inflation-Deflation or ID-score evaluates the tendency for inflation and deflation of the choice under Unknown condition. It attributes +1 point for each of W and L inconsistent with 'g' under Unknown, and -1 point for each of W and L inconsistent with '0' under Unknown.

$$\langle ID \rangle = 2 \left(\langle p(g|U) \rangle - \frac{\langle p(g|W) \rangle + \langle p(g|L) \rangle}{2} \right) \tag{S3}$$

where $\langle ID \rangle = \sum_i ID_i/n$ is the average ID-score over n WLU-patterns, and where $\langle p(g|W) \rangle$, $\langle p(g|L) \rangle$ and $\langle p(g|U) \rangle$ are the participants average gamble probabilities on W, L and U conditions respectively. One can easily check –both at the individual level or the group level– that the precise deflative violation condition of $\langle p(g|U) \rangle$ with respect to $\langle p(g|W) \rangle$ and $\langle p(g|L) \rangle$, eq. (2), can be expressed as

$$\langle ID \rangle < \pm(\langle p(g|L) \rangle - \langle p(g|W) \rangle) \tag{S4}$$

The reverse inequality holds for the inflative violation condition of $\langle p(g|U) \rangle$. The ID-score will be used to characterise each participant's contribution to the Disjunction Effect and the violation of the prediction based on the Law of Total Probability (Section SM 4).

SM 4. Partioning by ID-range and risk-attitude

Having pursued the characterisation of gamble strategies that caused violations of the Law of Total Probability we now examine the response patterns of each participant in more detail. We will consider how particular response patterns contribute to deflative ('DE') or inflative violations of the Law of Total Probability. In particular we now also check each participant's pattern variability in terms of the resulting ID-scores, since this allows us to identify whether this participant would contribute to inflation or deflation of the gamble probability under the Unknown condition.

We recall, that a single binary response pattern does not expose a violation of –or abidance by– the LTP or the STP. The participants' intrinsic probability to gamble (given some conditions, like payoffs) leads to the stochastic actualisation of binary responses; each single pattern does not reliably reflect possible violations of the LTP nor the STP. The intrinsic probabilities produce binary choices that lead to choice proportions. Only when averaged across participants and by the law of large numbers do we obtain the convergence of observed choice proportions to the intrinsic probabilities.

The ID-score assesses a participant's *tendency* to deviate from classical probabilistic constraints in the repeated gamble, eq. (S4), Subsection 1.1. The total ID-score for each individual ranges from -10 to 10, since each participant generates five gamble patterns across all payoff values (see Table S1). Given scores for risk attitude and for inflation-deflation behaviour we created a heat map which allows a perception of the correlation between the two characteristics, Figure S3. The

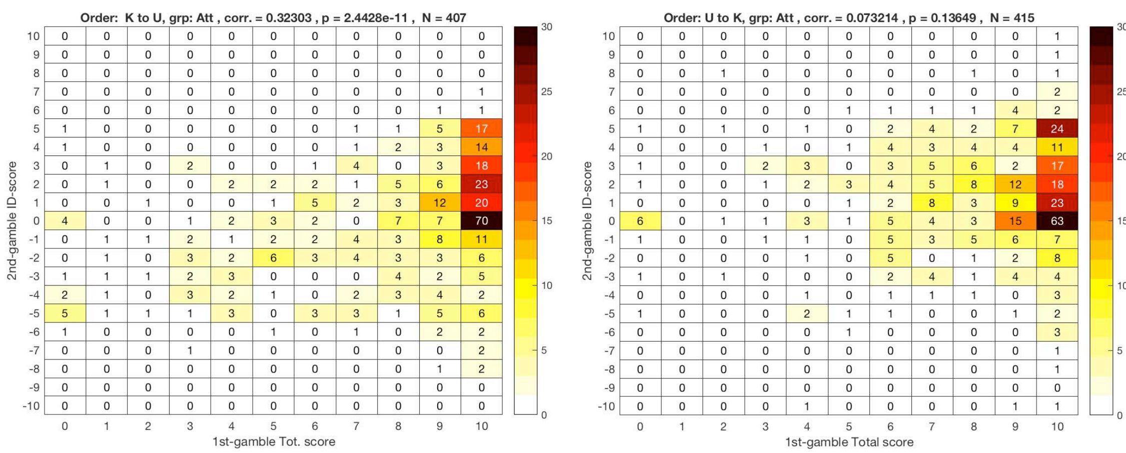

Figure S3: Total single-stage gamble score versus total ID-score for attentive participants in K-to-U order (left panel) and U-to-K order (right panel). The key observations are a concentration of participants with total single-stage score 10 in both flow orders, small tendency of negative ID-score for lower total single-stage score in K-to-U, and small tendency of positive ID-score for lower total single-stage score in U-to-K.

heat maps of the total ID-score against the total single-stage gamble score shows that there is a majority of participants always taking the initial gamble, i.e. for all X and on both occasions the gamble is presented, resulting in a total single-stage gamble score of 10. A small correlation between total ID-score and total single-stage gamble score (r=.32 , p=2.44e-11) is present for the K-to-U group. The heatmap, Figure S3, reveals that 'Less risk averse' participants show a small average tendency of negative ID-scores -0.86 (SD=2.95) in the K-to-U order, while the average tendency of the ID-score is positive 1.05 (SD=2.92) in the U-to-K order. These observed ID-scores reflect –by definition– the violation of the LTP by the aggregate choices, but also show that participants rarely make consistent highly inflative or deflative gamble choices (the individual ID-scores seldom reach extreme values near ± 10).

To obtain a more granular insight in each participant's choices we traced their generated ID-scores at each value of the payoff parameter X. Each participant generated five gamble patterns that lead to a particular way the ID-score depends on payoff parameter X, and which we will refer to as trajectories. First of all, an observation of the generated individual trajectories, Figure S4, supports the presumption of larger stochasticity of choice in the 'More risk averse' participants than in the 'Less risk averse' participants. Our aim is now to classify this large variety of trajectories so that we can assess their contribution to the inflative or deflative violation of the LTP in a well-arranged manner.[17] Hereto we devised an ID-score based classification method of the trajectories. We defined limits in the ID-ranges of the trajectories such that either symmetrical ID-ranges or positive biased or negative biased ID-ranges can be identified. This partitioning into negative or positive biased, or symmetric ID-ranges aims to easily identify respectively the participants that are mainly causing the Disjunction Effect, the inflative violation of the LTP, or abide by the LTP, Table (S2). This reduced number of participant classes, further subdivided by risk attitude and flow order, is the final level of our granular analysis. We now review the observed prevalence for

	ID-range	Included patterns	Excluded patterns	colour code
	[-2,2]	ggs, sgg	-	Yellow
symmetric	[-1,1]	sgs or gss, sgg or gsg	ggs, ssg	Green
	[0,0]	ggg or sss	sgs, gss, sgg, gsg, ggs, ssg	Black
pos. bias	]-2,2]	ssg	ggs	Red
	[0,1]	ggg or sss, sgg or gsg	ggs, sgs, gss, ssg	Magenta
neg. bias	[-2,2[	ggs	ssg	Blue
	[-1,0]	sgs or gss, ggg or sss	ggs, sgg, gsg, ssg	Cyan

Table S2: The ID-score partitioning of participant trajectories. Each participant generates five gamble patterns, with respect to the W, L or U condition of the first-stage gamble, over the gamble payoff values. Patterns are denoted in WLU order with g for 'gamble' and s for 'stop'. The Inflation-Deflation range of these five patterns have been captured in mutually exclusive symmetric, positive and negative biased ID-score intervals. Included patterns are necessary and excluded patterns must be absent for the given ID-range.

each of the ID-ranges and their contribution to the Disjunction Effect or the inflative violation of the LTP.

For participants in the symmetric ID-range we distinguish the closed intervals [-2,2], [1,1] and [0,0], by which we mean that these participants explicitly play both extrema of the ID-interval at least once. For instance, the largest possible symmetric ID-range [-2,2] is composed of participants for whom the trajectory includes both $(g|W, g|L, s|U)$ and $(s|W, s|L, g|U)$ at least once. These participants therefore showed superficially contradictory choice behaviour with respect to uncertainty at least once. The participants in ID-range [0,0] by definition do not contribute to any inflation or deflation of the gamble probability under Unknown outcome condition. As a whole the symmetric

[17] For each of the five payoff values a participant could generate five possible ID-scores, therefore in total 5^5 distinct trajectories are possible.

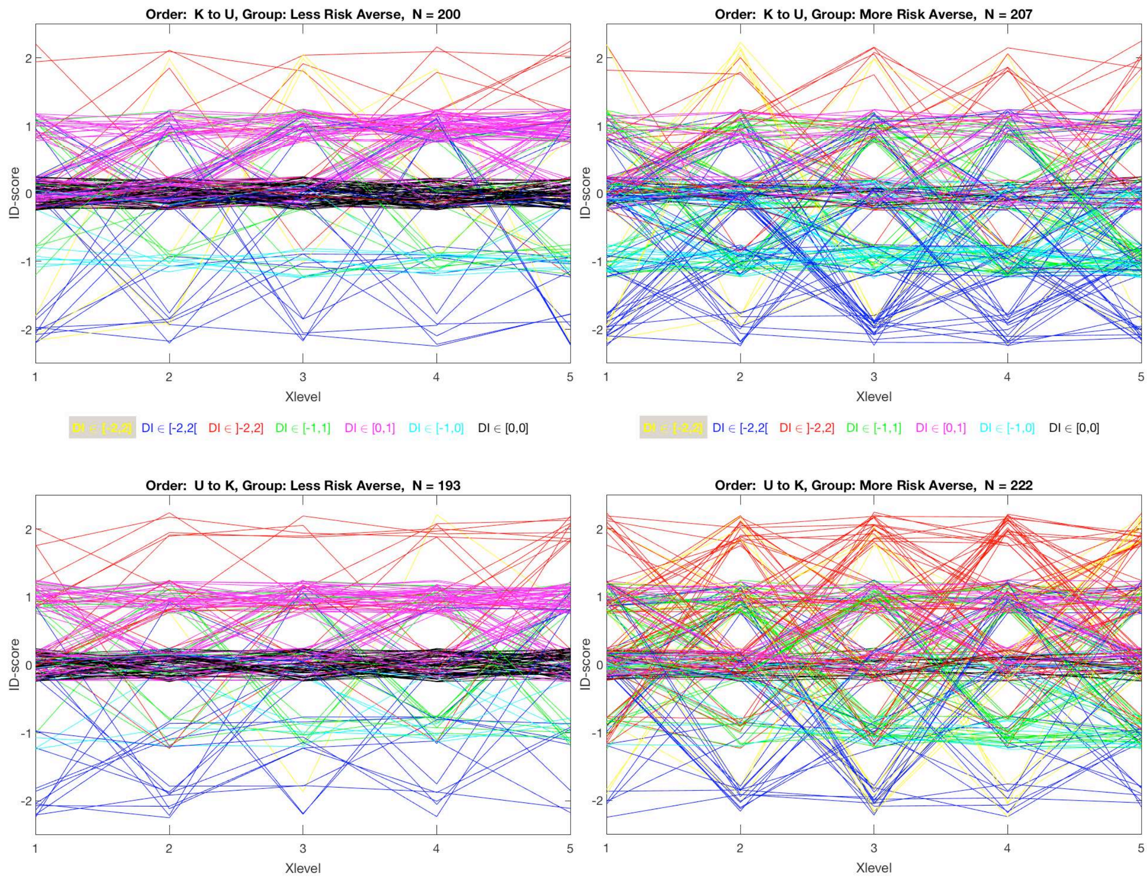

Figure S4: Participant ID-score trajectories over X payoff range, by partition 'Less risk averse' (left panels) and 'More risk averse' (right panels) and by flow order K-to-U (top panels) an U-to-K (bottom panels). A small amount of random jitter was added to the ID-scores in order to visualise the density of the trajectories. 'Less risk averse' participants show more constancy, in terms of ID-score, over the payoff range. 'More risk averse' participants on the other hand show high variability of ID-score over the payoff range. Moreover one can observe that Blue (N=62) and Cyan (N=35) trajectories build the main drive for the marginal DE in the 'More risk averse' partition (K-to-U), while Red (N=60) and Magenta (N=35) build up the drive for the inflative violation of the LTP in the 'More risk averse' partition (U-to-K). Similarly one can observe that the Magenta (N=71 and N=70) participants drive the inflative tendency of the 'Less risk averse' partition in both flow orders.

ID-range participants make only a small contribution to the inflation of $p(g|U,X)$. The largest contribution to the total ID-score by these participants amounts to merely 0.017, i.e. in ‘U to K’ order, Table (S3). [18]

In the positive biased ID-ranges, the dominant inflative component results from the ID-range [0,1] in the ‘Less risk averse’ participants and contributes the second largest inflative component in the ‘More risk averse’ participants, in both order conditions, Table (S3).

In the negative biased ID-ranges, the dominant deflative component results from the ID-range [-2,2[in the ‘More risk averse’ participants in the K-to-U order at -0.195. The [-1,0] ID-range gives the second largest deflative component in the K-to-U order of the ‘More risk averse’ participants. This analysis shows that the ‘More risk averse’ participants with ID-ranges [-2,2[(N=62) and [-1,0] (N=35) provide the main drive for the marginal Disjunction effect in the K-to-U flow, while the participants with ID-ranges]-2,2] (N=60) and [0,1] (N=35) provide the drive for the inflative violation of the LTP in the U-to-K order. Interestingly the participants that generate the signature patterns $(g|W, g|L, s|U)$ and $(s|W, s|L, g|U)$ constitute the largest contribution respectively to the Disjunction Effect and the inflative violation of the LTP in the ‘More risk averse’ participants, which lends some support to the original assumptions from Tversky and Shafir (1992). By contrast, the participants composing the [0,1] ID-range constitute the major contribution for the inflative violation of the LTP by the ‘Less risk averse’ participants in the U-to-K flow.

In sum, we observe that ID-range characterisation of participants provides insight in the stochasticity of choice behaviour and contributions to the Disjunction Effect and the violation of the Law of Total Probability. It is at this level of granular partitioning by risk attitude, order condition and ID-range that we further examine the performance of the theoretical models in SM 5. (In the main text we look at model performance by risk attitude and order condition, subsec. 3.2.1.)

[18] We note that merely based on the symmetry or bias of the played ID-range it need not to be the case that the resulting ID-score should be small or biased according to the ID-range bias.

participant ID-range	'Less risk averse' KU %(N)	ID_{comp}	UK %(N)	ID_{comp}	'More risk averse' KU %(N)	ID_{comp}	UK %(N)	ID_{comp}
[-2,2] - Y	2.5 (5)	-0.001	0.5 (1)	0.003	4.8 (10)	-0.007	05.9 (13)	0.017
[-1,1] - G	8.0 (16)	0.004	9.8 (19)	0.017	19.3 (40)	-0.006	22.1 (49)	0.017
[0,0] - K	32.5 (65)	0.000	29.5 (57)	0.000	4.8 (10)	0.000	6.3 (14)	0.000
]-2,2] - R	5.0 (10)	0.044	8.8 (17)	0.089	9.2 (19)	0.056	27.0 (60)	0.198
[0,1] - M	35.5 (71)	0.210	36.3 (70)	0.225	15.0 (31)	0.077	15.8 (35)	0.086
[-2,2[- B	12.5 (25)	-0.066	10.9 (21)	-0.078	30.0 (62)	-0.195	16.7 (37)	-0.078
[-1,0] - C	4.0 (8)	-0.035	4.2 (8)	-0.021	16.9 (35)	-0.099	6.3 (14)	-0.031
Tot	100 (200)	0.156	100 (193)	0.235	100 (207)	-0.173	100 (222)	0.209
$\langle p_L \rangle - \langle p_W \rangle$		0.200		0.167		0.121		0.101

Table S3: Distribution of participants over all defined ID-ranges and their X-averaged contribution to the full ID-score. Participants are first partitioned by their risk attitude and the order condition and then subdivided by the range of gamble patterns they generated over the payoff values X. The trajectory through the five generated gamble patterns gives rise to a minimal and maximal ID-score per participant, and hence ID-range, Table (S2). The colour code corresponds to the ID-X trajectories in Figure S4.

SM 5. Model performance comparison by ID-range and risk attitude.

Using the risk attitude and ID-score categories from (SM 4), we classified participants with particular gamble tendencies, Table (S3). Given the strongly different characteristic ID-score tendencies of these partitions we tested the performance of the Markov, quantum-like and Logistic models on each of them separately. Using the maximum likelihood statistic, the better performing model for each partition was identified, Table (S4). Across all ID-ranges and risk attitudes the performance of the quantum-like model comes out best with G=555.34, the second better performance comes from the Logistic model, with G=767.12, and the least performing is the Markov model G=862.93, Table (S4). The ID-range partitioning of the participants nuances the relative performance of the three models. In the participant subgroup partitions that cause the main contributions to the violation of the Law of Total Probability, Table (S4), the quantum-like model comes out with smaller G values as the better model namely for the 'More risk averse' attitude in ID-range]-2,2] and [-2,2[. Only in the three subgroups –with ID-ranges [-2,2], [0,0] and [-1,0], all 'More risk averse'– does the Markov model perform better than the Logistic model. By contrast, when partitioning merely by risk attitude, Section 5.1, the Logistic model performs on par with the Markov model.

		G					
		'Less risk averse'			'More risk averse'		
ID_{var}	(N_{LessRA}, N_{MoreRA})	Markov	quantum-like	Logistic	Markov	quantum-like	Logistic
[-2,2] - Y	(6, 23)	36.78	30.52	40.30	32.79	31.56	33.43
[-1,1] - G	(35, 89)	19.89	16.52	18.80	35.91	33.97	25.24
[0,0] - K	(122, 24)	7.40	57.60	4.93	4.76	2.59	5.22
]-2,2] - R	(27, 79)	97.60	35.45	78.40	161.64	58.56	139.48
[0,1] - M	(141, 66)	266.42	190.10	123.23	91.68	40.36	36.63
[-2,2[- B	(46, 97)	78.10	41.03	56.98	120.66	44.99	110.20
[-1,0] - C	(16, 49)	30.30	38.20	14.63	79.00	33.90	79.64

Table S4: Model fit comparison by G-statistic, or log-likelihood ratio test, for partitioned participants by single-stage gamble risk behavior 'Less risk averse' versus 'More risk averse' participants and according to ID-score range. Symmetric ID-range Yellow (Y), Green (G) and Black (K). Positive ID-range Red (R) and Magenta (M). Negative ID-range Blue (B) and Cyan (C).

In the two cases with pronounced contributions to the violation of the Law of Total Probability the quantum-like model is most efficient in producing these effects. This is in the line with the expectation since both the Logistic and the Markov model are abiding to classical logical constraints on the probabilities which prevents these models from providing for inflative or deflative Disjunction Effects.

SM 6. Distribution of gamble pattern probabilities, Less vs More risk averse.

payoff	order	Less risk averse							
		sss	*gss*	*sgs*	*ggs*	*ssg*	*gsg*	*sgg*	*ggg*
.5	KU	0.020	0.010	0.040	0.060	0.010	0.040	0.110	0.710
.5	UK	0.026	0.016	0.021	0.047	0.016	0.083	0.187	0.606
1	KU	0.025	0.010	0.060	0.045	0.020	0.060	0.145	0.635
1	UK	0.026	0.036	0.036	0.047	0.026	0.083	0.192	0.554
2	KU	0.020	0.020	0.060	0.030	0.030	0.065	0.240	0.535
2	UK	0.047	0.031	0.057	0.026	0.026	0.088	0.228	0.497
3	KU	0.020	0.030	0.075	0.030	0.020	0.085	0.305	0.435
3	UK	0.042	0.031	0.083	0.031	0.042	0.083	0.264	0.425
4	KU	0.030	0.030	0.095	0.040	0.025	0.070	0.290	0.420
4	UK	0.047	0.036	0.057	0.016	0.062	0.083	0.280	0.420

Table S5: Second-stage gamble pattern probabilities for participant group 'Less risk averse', by payoff and order condition, shown in Figure 5.

payoff	order	More risk averse							
		sss	*gss*	*sgs*	*ggs*	*ssg*	*gsg*	*sgg*	*ggg*
.5	KU	0.087	0.092	0.082	0.101	0.015	0.063	0.126	0.435
.5	UK	0.081	0.068	0.045	0.032	0.050	0.144	0.144	0.437
1	KU	0.101	0.135	0.106	0.068	0.039	0.077	0.145	0.329
1	UK	0.081	0.063	0.032	0.072	0.090	0.113	0.180	0.369
2	KU	0.145	0.130	0.111	0.130	0.034	0.087	0.150	0.213
2	UK	0.126	0.117	0.077	0.099	0.108	0.135	0.167	0.171
3	KU	0.174	0.150	0.174	0.092	0.034	0.058	0.164	0.155
3	UK	0.135	0.108	0.068	0.059	0.104	0.140	0.212	0.176
4	KU	0.213	0.159	0.174	0.087	0.034	0.068	0.159	0.106
4	UK	0.216	0.149	0.077	0.036	0.117	0.108	0.162	0.135

Table S6: Second-stage gamble pattern probabilities for participant group 'More risk averse', by payoff and order condition, shown in Figure 5.

SM 7. Experimental and Theoretical gamble probabilities

		all											
		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
	.5	0.820	0.920	0.870	0.751	0.860	0.891	0.691	0.744	0.638	0.680	0.658	0.775
	1	0.750	0.885	0.860	0.720	0.829	0.855	0.609	0.647	0.589	0.617	0.653	0.752
Obs.	2	0.650	0.865	0.870	0.643	0.808	0.839	0.560	0.604	0.483	0.523	0.514	0.581
	3	0.580	0.845	0.845	0.570	0.803	0.814	0.454	0.585	0.411	0.482	0.514	0.631
	4	0.560	0.845	0.805	0.554	0.772	0.845	0.420	0.527	0.367	0.428	0.410	0.523
	.5	0.800	0.891	0.882	0.811	0.879	0.845	0.677	0.731	0.677	0.703	0.705	0.704
	1	0.781	0.888	0.877	0.794	0.874	0.834	0.609	0.686	0.609	0.646	0.649	0.647
Th.M.	2	0.735	0.880	0.866	0.753	0.861	0.807	0.503	0.619	0.503	0.558	0.563	0.561
	3	0.678	0.870	0.852	0.703	0.846	0.774	0.452	0.587	0.452	0.517	0.522	0.520
	4	0.613	0.859	0.836	0.644	0.828	0.736	0.394	0.519	0.394	0.454	0.459	0.456
	.5	0.787	0.882	0.900	0.753	0.853	0.902	0.700	0.708	0.617	0.666	0.674	0.778
	1	0.739	0.880	0.882	0.700	0.849	0.885	0.648	0.673	0.572	0.612	0.641	0.735
Th.Q.	2	0.666	0.868	0.845	0.624	0.836	0.848	0.548	0.596	0.487	0.511	0.567	0.644
	3	0.625	0.850	0.815	0.583	0.820	0.818	0.482	0.529	0.426	0.448	0.502	0.573
	4	0.604	0.829	0.791	0.563	0.800	0.794	0.449	0.482	0.391	0.417	0.456	0.529
	.5	0.766	0.887	0.877	0.780	0.879	0.780	0.651	0.716	0.651	0.684	0.686	0.684
	1	0.738	0.881	0.870	0.755	0.872	0.755	0.614	0.691	0.614	0.652	0.655	0.652
Th.L.	2	0.677	0.869	0.853	0.699	0.857	0.699	0.535	0.637	0.535	0.586	0.589	0.586
	3	0.609	0.855	0.835	0.637	0.840	0.637	0.454	0.580	0.454	0.515	0.519	0.515
	4	0.537	0.840	0.816	0.570	0.821	0.570	0.376	0.520	0.376	0.445	0.449	0.445

Table S7: Second-stage gamble probabilities for participant group 'all' (passed the attention test), by risk attitude (N_{Lra} = 429, N_{Mra} = 393), order condition (Lra: N_{KU} = 200, N_{UK} = 193; Mra: N_{KU} = 207, N_{UK} = 222) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

		$[-2, 2]$ - Yellow											
		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
Obs.	.5	1.000	1.000	0.400	0	1.000	1.000	0.500	0.700	0.300	0.615	0.615	0.692
	1	0.400	0.800	0.400	1.000	0	1.000	0.500	0.500	0.700	0.539	0.615	0.769
	2	0.200	0.200	0.800	1.000	1.000	0	0.400	0.700	0.400	0.539	0.692	0.539
	3	0.400	0.800	0.800	0	0	1.000	0.500	0.600	0.500	0.692	0.692	0.539
	4	0.600	0.800	0.600	0	1.000	1.000	0.700	0.600	0.600	0.154	0.154	0.846
Th.M.	.5	0.843	0.708	0.708	0.775	0.775	0.775	0.502	0.685	0.502	0.564	0.623	0.594
	1	0.463	0.607	0.607	0.535	0.535	0.535	0.507	0.685	0.507	0.567	0.625	0.596
	2	0.325	0.628	0.628	0.476	0.477	0.477	0.515	0.685	0.515	0.573	0.627	0.600
	3	0.344	0.672	0.672	0.508	0.509	0.508	0.524	0.671	0.524	0.574	0.621	0.597
	4	0.351	0.690	0.690	0.520	0.521	0.521	0.532	0.426	0.532	0.496	0.462	0.479
Th.Q.	.5	0.884	0.885	0.375	0.565	0.590	0.884	0.544	0.659	0.534	0.507	0.626	0.708
	1	0.732	0.733	0.581	0.634	0.648	0.733	0.541	0.646	0.526	0.504	0.613	0.700
	2	0.504	0.503	0.839	0.733	0.679	0.503	0.534	0.614	0.506	0.498	0.580	0.682
	3	0.618	0.617	0.761	0.718	0.690	0.618	0.528	0.572	0.482	0.492	0.537	0.658
	4	0.772	0.773	0.617	0.663	0.694	0.773	0.522	0.525	0.455	0.486	0.490	0.632
Th.L.	.5	0.625	0.627	0.627	0.625	0.628	0.628	0.503	0.700	0.503	0.542	0.665	0.665
	1	0.590	0.637	0.637	0.570	0.656	0.656	0.503	0.679	0.503	0.538	0.648	0.648
	2	0.516	0.657	0.657	0.454	0.710	0.710	0.502	0.637	0.502	0.528	0.612	0.612
	3	0.441	0.677	0.677	0.344	0.759	0.759	0.501	0.593	0.501	0.519	0.576	0.576
	4	0.369	0.696	0.696	0.249	0.801	0.801	0.500	0.547	0.500	0.509	0.538	0.538

Table S8: Second-stage gamble probabilities for participant group 'Yellow', by risk attitude ($N_{Lra} = 6$, $N_{Mra} = 23$), order condition (Lra: $N_{KU} = 5$, $N_{UK} = 1$; Mra: $N_{KU} = 10$, $N_{UK} = 13$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

		[−1, 1] - Green											
		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
Obs.	.5	0.688	0.750	0.750	0.526	0.842	0.947	0.575	0.750	0.700	0.796	0.633	0.837
	1	0.688	0.750	0.750	0.526	0.684	0.737	0.575	0.700	0.675	0.714	0.612	0.796
	2	0.688	0.625	0.875	0.368	0.790	0.632	0.625	0.600	0.675	0.551	0.449	0.531
	3	0.500	0.688	0.563	0.316	0.737	0.474	0.375	0.700	0.475	0.612	0.408	0.531
	4	0.375	0.750	0.438	0.368	0.737	0.579	0.400	0.600	0.350	0.510	0.367	0.327
Th.M.	.5	0.699	0.783	0.764	0.646	0.836	0.741	0.684	0.760	0.717	0.722	0.722	0.722
	1	0.645	0.751	0.727	0.579	0.817	0.698	0.621	0.728	0.668	0.675	0.675	0.675
	2	0.547	0.692	0.660	0.455	0.784	0.619	0.492	0.664	0.567	0.578	0.578	0.578
	3	0.476	0.649	0.611	0.367	0.759	0.563	0.409	0.622	0.501	0.515	0.515	0.515
	4	0.432	0.621	0.579	0.313	0.740	0.527	0.330	0.522	0.414	0.426	0.426	0.426
Th.Q.	.5	0.728	0.816	0.789	0.582	0.784	0.789	0.602	0.725	0.697	0.744	0.744	0.744
	1	0.675	0.790	0.749	0.514	0.779	0.749	0.620	0.708	0.674	0.688	0.688	0.688
	2	0.574	0.729	0.666	0.394	0.750	0.666	0.546	0.636	0.595	0.600	0.600	0.600
	3	0.502	0.676	0.602	0.321	0.718	0.602	0.406	0.576	0.495	0.502	0.502	0.502
	4	0.468	0.639	0.565	0.300	0.691	0.565	0.314	0.528	0.426	0.432	0.432	0.432
Th.L.	.5	0.677	0.777	0.760	0.584	0.839	0.758	0.638	0.776	0.708	0.770	0.647	0.732
	1	0.642	0.754	0.735	0.541	0.824	0.733	0.595	0.744	0.670	0.737	0.604	0.696
	2	0.568	0.703	0.680	0.453	0.791	0.677	0.505	0.672	0.587	0.663	0.515	0.616
	3	0.491	0.647	0.619	0.368	0.753	0.616	0.415	0.590	0.498	0.581	0.425	0.529
	4	0.414	0.586	0.554	0.290	0.710	0.551	0.330	0.503	0.410	0.493	0.339	0.440

Table S9: Second-stage gamble probabilities for participant group 'Green', by risk attitude ($N_{Lra} = 35$, $N_{Mra} = 89$), order condition (Lra: $N_{KU} = 16$, $N_{UK} = 19$; Mra: $N_{KU} = 40$, $N_{UK} = 49$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

[0, 0] - Black

	X	'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
		W	L	U	W	L	U	W	L	U	W	L	U
Obs.	.5	0.969	0.969	0.969	0.947	0.947	0.947	0.600	0.600	0.600	0.500	0.500	0.500
	1	0.969	0.969	0.969	0.947	0.947	0.947	0.500	0.500	0.500	0.500	0.500	0.500
	2	0.969	0.969	0.969	0.947	0.947	0.947	0.300	0.300	0.300	0.500	0.500	0.500
	3	0.969	0.969	0.969	0.947	0.947	0.947	0.400	0.400	0.400	0.429	0.429	0.429
	4	0.969	0.969	0.969	0.947	0.947	0.947	0.300	0.300	0.300	0.429	0.429	0.429
Th.M.	.5	0.952	0.952	0.952	0.952	0.952	0.952	0.547	0.545	0.543	0.544	0.544	0.544
	1	0.952	0.952	0.952	0.952	0.952	0.952	0.495	0.507	0.495	0.504	0.501	0.501
	2	0.952	0.952	0.952	0.952	0.952	0.952	0.398	0.440	0.398	0.419	0.419	0.419
	3	0.952	0.952	0.952	0.952	0.952	0.952	0.394	0.437	0.394	0.415	0.415	0.415
	4	0.952	0.952	0.952	0.952	0.952	0.952	0.362	0.394	0.362	0.378	0.378	0.378
Th.Q.	.5	0.917	0.917	0.917	0.917	0.917	0.917	0.506	0.569	0.547	0.517	0.517	0.517
	1	0.917	0.917	0.917	0.917	0.917	0.917	0.517	0.528	0.525	0.519	0.519	0.519
	2	0.917	0.917	0.917	0.917	0.917	0.917	0.372	0.324	0.288	0.486	0.486	0.486
	3	0.917	0.917	0.917	0.917	0.917	0.917	0.370	0.394	0.374	0.403	0.403	0.403
	4	0.917	0.917	0.917	0.917	0.917	0.917	0.398	0.416	0.401	0.422	0.423	0.423
Th.L.	.5	0.956	0.962	0.962	0.955	0.963	0.955	0.517	0.529	0.529	0.527	0.519	0.519
	1	0.956	0.962	0.962	0.955	0.963	0.955	0.499	0.501	0.501	0.500	0.499	0.499
	2	0.956	0.962	0.962	0.955	0.963	0.955	0.463	0.444	0.444	0.447	0.461	0.461
	3	0.956	0.962	0.962	0.955	0.963	0.955	0.428	0.389	0.389	0.395	0.422	0.422
	4	0.956	0.962	0.962	0.955	0.963	0.955	0.393	0.337	0.337	0.345	0.385	0.385

Table S10: Second-stage gamble probabilities for participant group 'Black', by risk attitude ($N_{Lra} = 122$, $N_{Mra} = 24$), order condition (Lra: $N_{KU} = 65$, $N_{UK} = 57$; Mra: $N_{KU} = 10$, $N_{UK} = 14$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

$]-2,2]$ - Red

		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
	.5	0.700	0.700	1.000	0.706	0.529	0.941	0.737	0.842	1.000	0.583	0.533	0.767
	1	0.400	0.600	0.900	0.412	0.471	0.882	0.421	0.684	0.842	0.417	0.417	0.767
Obs.	2	0.300	0.600	0.800	0.471	0.294	0.882	0.263	0.526	0.790	0.333	0.283	0.717
	3	0.200	0.300	0.800	0.177	0.412	0.824	0.211	0.211	0.526	0.267	0.333	0.783
	4	0.100	0.300	0.800	0.118	0.177	0.882	0.105	0.211	0.474	0.250	0.283	0.650
	.5	0.637	0.842	0.842	0.738	0.741	0.740	0.572	0.773	0.773	0.672	0.672	0.672
	1	0.416	0.768	0.768	0.589	0.595	0.592	0.387	0.700	0.700	0.5430	0.544	0.543
Th.M.	2	0.288	0.701	0.701	0.491	0.498	0.494	0.291	0.634	0.634	0.462	0.463	0.463
	3	0.261	0.636	0.636	0.446	0.451	0.448	0.263	0.565	0.565	0.414	0.414	0.414
	4	0.229	0.529	0.529	0.377	0.381	0.379	0.226	0.460	0.460	0.343	0.343	0.343
	.5	0.594	0.558	0.910	0.594	0.558	0.910	0.553	0.560	0.848	0.531	0.538	0.848
	1	0.539	0.542	0.892	0.540	0.542	0.892	0.490	0.520	0.799	0.468	0.498	0.799
Th.Q.	2	0.418	0.508	0.815	0.418	0.508	0.815	0.386	0.440	0.683	0.366	0.420	0.683
	3	0.342	0.466	0.718	0.342	0.466	0.718	0.333	0.378	0.590	0.315	0.361	0.590
	4	0.315	0.420	0.642	0.315	0.420	0.642	0.312	0.340	0.537	0.296	0.325	0.537
	.5	0.611	0.757	0.757	0.624	0.746	0.746	0.510	0.7029	0.703	0.534	0.682	0.682
	1	0.516	0.732	0.732	0.536	0.716	0.716	0.452	0.6706	0.671	0.478	0.647	0.647
Th.L.	2	0.331	0.677	0.677	0.359	0.649	0.649	0.341	0.6012	0.601	0.370	0.571	0.571
	3	0.186	0.618	0.617	0.213	0.577	0.577	0.245	0.5274	0.527	0.273	0.491	0.491
	4	0.096	0.554	0.554	0.116	0.501	0.501	0.169	0.4524	0.452	0.194	0.412	0.412

Table S11: Second-stage gamble probabilities for participant group 'Red', by risk attitude ($N_{Lra} = 27$, $N_{Mra} = 79$), order condition (Lra: $N_{KU} = 10$, $N_{UK} = 17$; Mra: $N_{KU} = 19$, $N_{UK} = 60$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

$[0, 1]$ - Magenta

		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
Obs.	.5	0.761	0.930	0.986	0.643	0.871	1.000	0.645	0.936	1.000	0.686	0.857	1.000
	1	0.676	0.887	0.986	0.629	0.871	1.000	0.548	0.839	0.968	0.686	0.943	1.000
	2	0.437	0.901	0.986	0.514	0.843	0.986	0.516	0.742	0.871	0.429	0.800	0.943
	3	0.338	0.831	1.000	0.400	0.843	1.000	0.355	0.774	0.839	0.400	0.714	0.886
	4	0.310	0.859	0.986	0.443	0.814	1.000	0.323	0.710	0.807	0.314	0.771	0.829
Th.M.	.5	0.722	0.946	0.946	0.754	0.914	0.834	0.734	0.915	0.915	0.742	0.907	0.824
	1	0.676	0.944	0.944	0.715	0.906	0.810	0.681	0.904	0.904	0.690	0.895	0.793
	2	0.573	0.941	0.941	0.626	0.889	0.757	0.561	0.876	0.876	0.574	0.863	0.719
	3	0.465	0.938	0.938	0.533	0.870	0.702	0.443	0.837	0.837	0.459	0.821	0.640
	4	0.368	0.935	0.935	0.449	0.853	0.651	0.345	0.782	0.782	0.363	0.764	0.564
Th.Q.	.5	0.674	0.805	0.903	0.669	0.801	0.903	0.674	0.806	0.901	0.689	0.820	0.901
	1	0.643	0.810	0.900	0.638	0.806	0.900	0.624	0.799	0.897	0.642	0.815	0.897
	2	0.576	0.820	0.888	0.571	0.816	0.888	0.518	0.781	0.874	0.539	0.801	0.874
	3	0.506	0.829	0.868	0.500	0.825	0.868	0.408	0.759	0.833	0.431	0.781	0.833
	4	0.437	0.837	0.843	0.430	0.833	0.843	0.308	0.731	0.7772	0.332	0.754	0.777
Th.L.	.5	0.666	0.945	0.945	0.693	0.938	0.938	0.664	0.933	0.933	0.652	0.936	0.936
	1	0.615	0.942	0.942	0.647	0.934	0.934	0.614	0.917	0.917	0.601	0.922	0.922
	2	0.507	0.935	0.935	0.546	0.925	0.925	0.508	0.877	0.877	0.494	0.883	0.883
	3	0.398	0.928	0.928	0.441	0.915	0.915	0.401	0.820	0.820	0.387	0.829	0.829
	4	0.299	0.920	0.920	0.341	0.904	0.904	0.302	0.746	0.746	0.291	0.756	0.756

Table S12: Second-stage gamble probabilities for participant group 'Magenta', by risk attitude ($N_{Lra} = 141$, $N_{Mra} = 66$), order condition (Lra: $N_{KU} = 71$, $N_{UK} = 70$; Mra: $N_{KU} = 31$, $N_{UK} = 35$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

		[−2, 2[- Blue											
		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
Obs.	.5	0.920	0.960	0.640	0.952	0.857	0.429	0.855	0.726	0.516	0.730	0.730	0.649
	1	0.800	0.880	0.600	0.857	0.857	0.333	0.823	0.661	0.484	0.757	0.838	0.568
	2	0.760	0.880	0.560	0.713	0.810	0.476	0.742	0.742	0.339	0.838	0.676	0.270
	3	0.640	0.880	0.560	0.714	0.857	0.381	0.661	0.694	0.307	0.676	0.757	0.568
	4	0.680	0.760	0.400	0.571	0.762	0.571	0.597	0.661	0.307	0.757	0.460	0.378
Th.M.	.5	0.711	0.865	0.711	0.755	0.820	0.788	0.835	0.602	0.602	0.741	0.696	0.718
	1	0.683	0.855	0.683	0.733	0.805	0.769	0.813	0.584	0.584	0.720	0.676	0.698
	2	0.626	0.834	0.626	0.686	0.774	0.730	0.757	0.547	0.547	0.673	0.632	0.652
	3	0.570	0.814	0.570	0.640	0.743	0.692	0.687	0.511	0.511	0.616	0.582	0.599
	4	0.511	0.775	0.511	0.587	0.698	0.643	0.604	0.474	0.474	0.551	0.526	0.539
Th.Q.	.5	0.777	0.797	0.659	0.777	0.797	0.659	0.768	0.709	0.571	0.768	0.709	0.571
	1	0.757	0.795	0.636	0.757	0.795	0.636	0.754	0.696	0.538	0.754	0.696	0.538
	2	0.710	0.790	0.589	0.709	0.790	0.589	0.718	0.668	0.471	0.718	0.668	0.471
	3	0.654	0.786	0.540	0.654	0.786	0.540	0.674	0.637	0.404	0.674	0.637	0.404
	4	0.593	0.781	0.491	0.593	0.781	0.491	0.622	0.606	0.339	0.622	0.606	0.339
Th.L.	.5	0.717	0.898	0.717	0.689	0.910	0.689	0.831	0.618	0.618	0.806	0.658	0.658
	1	0.696	0.885	0.696	0.667	0.898	0.667	0.810	0.596	0.597	0.784	0.636	0.636
	2	0.651	0.854	0.651	0.621	0.869	0.621	0.762	0.553	0.553	0.734	0.589	0.589
	3	0.602	0.817	0.602	0.573	0.834	0.573	0.706	0.508	0.508	0.678	0.541	0.541
	4	0.552	0.773	0.552	0.524	0.792	0.524	0.643	0.464	0.464	0.616	0.492	0.492

Table S13: Second-stage gamble probabilities for participant group 'Blue', by risk attitude ($N_{Lra} = 46$, $N_{Mra} = 99$), order condition (Lra: $N_{KU} = 25$, $N_{UK} = 21$; Mra: $N_{KU} = 62$, $N_{UK} = 37$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

		[−1, 0] - Cyan											
		'Less risk averse'						'More risk averse'					
		'K-to-U'			'U-to-K'			'K-to-U'			'U-to-K'		
	X	W	L	U	W	L	U	W	L	U	W	L	U
	.5	0.125	0.875	0.125	0.500	0.875	0.500	0.629	0.600	0.371	0.786	0.786	0.714
	1	0.250	0.875	0.125	0.625	0.750	0.500	0.486	0.457	0.200	0.786	0.786	0.643
Obs.	2	0.250	0.875	0.125	0.375	0.625	0.250	0.486	0.343	0.086	0.643	0.429	0.214
	3	0.125	0.875	0	0.500	0.375	0.125	0.400	0.343	0.057	0.500	0.429	0.143
	4	0	0.875	0	0.125	0.500	0	0.343	0.257	0	0.571	0.286	0.071
	.5	0.202	0.913	0.202	0.450	0.664	0.557	0.684	0.539	0.539	0.645	0.578	0.612
	1	0.163	0.897	0.163	0.419	0.641	0.530	0.625	0.394	0.394	0.564	0.456	0.510
Th.M.	2	0.124	0.847	0.124	0.377	0.595	0.486	0.526	0.235	0.235	0.448	0.313	0.381
	3	0.111	0.762	0.111	0.338	0.534	0.436	0.443	0.182	0.182	0.373	0.252	0.312
	4	0.106	0.632	0.106	0.290	0.448	0.369	0.367	0.159	0.159	0.312	0.215	0.263
	.5	0.394	0.792	0.341	0.394	0.792	0.341	0.600	0.570	0.439	0.676	0.643	0.439
	1	0.358	0.781	0.295	0.358	0.781	0.295	0.555	0.509	0.347	0.651	0.602	0.347
Th.Q.	2	0.299	0.735	0.216	0.299	0.735	0.216	0.468	0.390	0.193	0.591	0.508	0.193
	3	0.255	0.660	0.160	0.255	0.660	0.160	0.393	0.294	0.104	0.519	0.413	0.104
	4	0.229	0.563	0.132	0.229	0.563	0.132	0.338	0.240	0.093	0.441	0.337	0.093
	.5	0.235	0.925	0.235	0.537	0.765	0.537	0.657	0.506	0.506	0.634	0.530	0.530
	1	0.178	0.915	0.178	0.469	0.725	0.470	0.621	0.435	0.435	0.592	0.464	0.464
Th.L.	2	0.098	0.892	0.098	0.340	0.633	0.340	0.545	0.303	0.303	0.505	0.338	0.338
	3	0.051	0.863	0.051	0.231	0.531	0.231	0.467	0.197	0.197	0.418	0.231	0.231
	4	0.026	0.828	0.026	0.149	0.427	0.149	0.391	0.122	0.122	0.336	0.150	0.150

Table S14: Second-stage gamble probabilities for participant group 'Cyan', by risk attitude ($N_{Lra} = 16$, $N_{Mra} = 49$), order condition (Lra: $N_{KU} = 8$, $N_{UK} = 8$; Mra: $N_{KU} = 35$, $N_{UK} = 14$) and payoff size X. Ordered in row blocks for empirical observations, and theoretical values produced by the Markov, Quantum and Logistic models, further subdivided by payoff parameter X.

SM 8. Maximum likelihood parameter optimization

Quantum

group	G	ν	μ	δ_{0W}	δ_{0L}	γ	δ_{1W}	δ_{1L}	κ	s
Yellow - More RA	31.56	0.99	1.00	0.15	2.00	0.15	-0.08	-0.53	0.65	0.19
Yellow - Less RA	30.52	0.71	1.00	0.00	0.00	0.55	0.40	0.46	0.99	2.82
Green - More RA	33.97	0.99	0.71	2.00	1.99	0.28	-0.98	-0.14	0.38	3.10
Green - Less RA	16.52	0.82	0.95	0.33	1.01	0.57	-0.38	0.45	0.00	1.16
Black - More RA	2.59	1.00	0.71	2.00	1.27	0.66	-1.23	-0.89	0.64	9.51
Black - Less RA	57.60	0.85	0.85	0.79	0.79	0.00	0.00	0.00	0.50	1.78
Red - More RA	58.56	0.99	1.00	0.77	0.68	0.36	-1.00	-0.72	0.00	0.38
Red - Less RA	35.45	1.00	1.00	1.37	0.87	0.43	-1.00	-0.48	0.00	0.44
Magenta - More RA	40.36	1.00	0.99	0.07	0.11	0.30	-0.03	-0.01	0.00	9.91
Magenta - Less RA	190.10	0.99	0.99	0.07	0.12	0.38	-0.03	-0.00	0.00	7.65
Blue - More RA	44.99	1.00	1.00	0.10	0.08	-0.34	-0.02	-0.01	0.00	9.99
Blue - Less RA	41.03	1.00	1.00	0.09	0.09	-0.27	-0.02	-0.00	0.00	9.99
Cyan - More RA	33.90	0.99	1.00	0.10	0.09	-0.46	-0.04	-0.04	0.00	9.99
Cyan - Less RA	38.20	1.00	1.00	0.00	0.12	-0.38	-0.02	-0.03	0.00	9.99

Table S15: Optimized parameters for the Quantum model. Utility difference of gambles parameters $\{\delta_{0W}, \delta_{0W}\}$ and $\{\delta_{1L}, \delta_{1W}\}$. Hyperbolic tangent sensitivity parameter $\{s\}$. Dynamical mixing parameter γ. Context effect parameters $\{\nu, \mu\}$ and carry-over effect complex phase parameter $\{\kappa\}$.

Markov

group	G	ν	κ	μ	γ	δ_{0W}	δ_{0L}	δ_{1W}	δ_{1L}	s
Yellow - More RA	32.79	1.00	1.00	0.66	0.00	-0.19	12.83	0.07	-3.38	2.19
Yellow - Less RA	36.78	0.70	0.00	0.50	0.00	6.38	-2.97	-10	1.02	20.0
Green - More RA	35.91	0.67	0.57	0.50	0.00	-0.28	39.98	-1.06	-9.99	5.08
Green - Less RA	19.89	0.72	0.22	0.99	0.15	-2.19	0.78	-0.74	-0.27	19.99
Black - More RA	4.76	1.00	1.00	0.50	0.83	3.93	39.73	-4.02	-9.96	1.45
Black - Less RA	7.40	0.53	0.94	0.98	0.00	9.96	45.51	9.59	-7.12	20.0
Red - More RA	161.64	0.74	0.00	0.50	0.00	-0.87	1.32	-2.96	-0.78	10.0
Red - Less RA	97.60	0.77	0.00	0.50	0.00	-1.27	0.98	-2.90	-0.81	20.0
Magenta - More RA	91.68	0.88	0.00	0.85	0.00	-1.68	1.76	-0.60	-0.68	20.0
Magenta - Less RA	266.42	0.97	0.00	0.84	0.00	-1.69	7.23	-0.49	-0.83	20.0
Blue - More RA	120.66	1.00	0.00	0.60	0.00	-0.88	-2.42	-0.40	-0.16	20.0
Blue - Less RA	78.10	0.73	1.00	0.60	0.00	-2.23	40.30	-0.34	-9.97	19.99
Cyan - More RA	79.00	0.83	0.00	0.65	0.00	-1.70	-2.14	-0.40	-1.60	20.0
Cyan - Less RA	30.30	1.00	1.00	0.65	0.00	-4.20	0.44	-1.10	-0.70	20.0

Table S16: Optimized parameters for the Markov model. Utility difference of gambles parameters $\{\delta_{0W}, \delta_{0L}\}$ and $\{\delta_{1L}, \delta_{1W}\}$. Logistic sensitivity parameter $\{s\}$. Dynamical mixing parameter γ. Context effect parameters $\{\nu, \mu\}$ and carry-over effect weight parameter $\{\kappa\}$.

Logistic

group	G	ω_{UK_U}	ω_{UK_K}	ω_{KU_K}	ω_{KU_U}	δ_{0W}	δ_{0L}	δ_{1W}	δ_{1L}	s
Yellow - More RA	33.4	0.00	0.57	0.62	0.99	-0.58	0.94	0.11	-0.19	2.61
Yellow - Less RA	40.3	0.00	0.67	0.60	0.00	0.37	0.03	-0.39	0.32	2.85
Green - More RA	25.2	0.67	0.42	0.59	0.53	-0.30	1.21	-0.16	-0.14	2.37
Green - Less RA	18.8	0.39	0.63	0.55	0.19	-0.50	1.37	-0.21	-0.01	2.61
Black - More RA	5.22	0.00	0.46	0.55	0.00	-0.084	0.23	0.07	-0.22	2.59
Black - Less RA	4.93	0.99	0.46	0.46	0.00	1.54	0.68	-0.00	0.00	2.85
Red - More RA	139.5	0.00	0.77	0.86	0.00	0.41	3.82	-1.65	-0.88	0.30
Red - Less RA	78.4	0.00	0.88	0.96	0.00	1.72	2.73	-1.68	-0.50	0.47
Magenta - More RA	36.6	0.00	0.80	0.79	0.00	0.17	3.54	-0.43	-0.44	1.01
Magenta - Less RA	123.2	0.00	0.15	0.01	0.00	2.08	0.44	-0.05	-0.32	1.52
Blue - More RA	110.2	0.00	0.34	0.27	0.00	-0.24	4.37	-0.20	-0.63	0.56
Blue - Less RA	57.0	1.00	0.90	0.83	0.99	1.01	3.82	-0.27	-0.42	0.69
Cyan - More RA	79.6	0.00	0.24	0.13	0.00	0.52	2.03	-1.40	-0.61	0.44
Cyan - Less RA	14.6	1.00	0.43	0.26	0.99	2.15	-1.30	-0.01	-0.44	2.12

Table S17: Optimized parameters for the Logistic model. Utility difference of gambles parameters $\{\delta_{0W}, \delta_{0L}\}$ and $\{\delta_{1W}, \delta_{1L}\}$. Logistic sensitivity parameter $\{s\}$. carry-over effect and the context effect parameters $\{\omega_{KU_K}, \omega_{KU_U}, \omega_{UK_K}, \omega_{UK_U}\}$.

SM 9. Experimental gamble probabilities Experiment 1.

Regarding Experiment 1, we provide some additional detail here. In that experiment the two-stage gamble paradigm was done i) between-participants with each group exposed to one single condition and randomized order for payoff quantity (N=96/79/101), and ii) within-participants with exposure to all three conditions in randomized order and payoff quantity in randomized order (N=81).

The within-participants run did not provide an indication for any Disjunction Effect, in contrast to Tversky and Shafir's (1992) observation of an inflative effect in juxtaposed conditions (see Section 1). The between-participants run provided an indication for an inflative effect when conditions were separated. This effect is confirmed in Experiment 2 with flow order U-to-K.

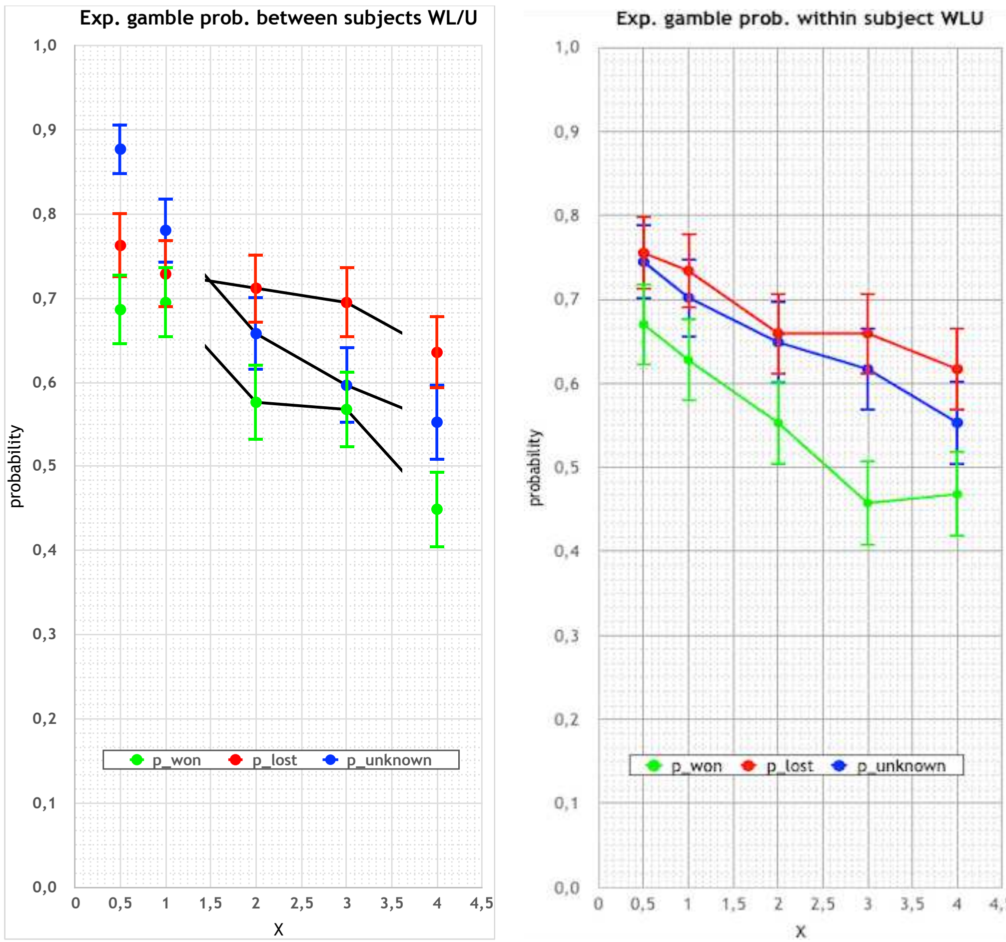

Figure S5: Experimental gamble probabilities. Left panel: between-participants for condition Win-and-Lose (N=118) with respect to Unknown (N=114). Right panel: within-participants with respect to conditions Win, Lose and Unknown (N=94), all gambles cued in random order. An indication for inflative violation of the LTP in the between-participants design at lower values of the payoff parameter X, and a tendency of decreasing gamble probabilities for increasing payoff parameter X. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

SM 10. Experimental gamble probabilities single-stage vs second-stage Unknown in Experiment 2.

In order to analyse the difference between the gamble probability of a second-stage gamble in the Unknown condition, $p(g|U)$, and the gamble probability of the single-stage gamble, $p(g)$ (without condition), we need recall the distinction between Unknown condition and Unknown context. The Unknown condition refers to second-stage gambles for which the outcome of the first-stage gamble was not known. Context refers to the type of conditioned second-stage gambles amongst which the single-stage gamble was taken, e.g., Unknown context means that single-stage gambles were taken together with second-stage gambles conditioned on an unknown outcome from corresponding first-stage gambles. In the ordering manipulation in Experiment 2, participants received in blocked order the second-stage gambles conditionalized on known outcomes of the previous gamble (Known context) and the second-stage gambles conditionalized on unknown outcomes of the previous gamble (Unknown context). We provide a short comparison of $p(g|U))$ and $p(g)$ for the 'more risk averse' participants. In doing this analysis, we observed some interesting results. In order to examine the factors potentially affecting single-stage gamble probabilities we ran a mixed ANOVA with unbalanced design (N=207/N=222), with single-stage gamble probabilities (averaged across all payoffs X) as the dependent variable, and with gamble context {K, U} and order {'K-to-U', 'U-to-K'} as independent variables. It revealed a main effect due to order, $F(1, 854) = 7.52$, $p = .0062$ (in the figure below, variation by line type). There was no significant main effect of context. There was also significant cross-over interaction, $F(1, 854) = 12.12$, $p =$ 5.2e-04. Note, the order effect means that 'more risk averse' participants accepted the first-stage gamble more often in the first block (dashed light-blue line) than in the second block (solid light-blue line).The cross-over effect means that the single-stage gamble probability is larger in context U (vs. K) when it comes first, but it is smaller in context U (vs. K) when it comes second ($p(g)$ in $U_{UK} > p(g)$ in K_{UK} and $p(g)$ in $U_{KU} < p(g)$ in K_{KU}).

In order to examine how conditioning a second-stage gamble on an unknown first-stage gamble (Unknown condition) compares to the unconditioned single-stage gamble probability, we adopted the following approach. We ran a mixed ANOVA with unbalanced design (N=207/N=222), with gamble probabilities as the dependent variable (averaged across all payoffs X), and with independent variables as, first, context or condition {single-stage-gamble-in-K, single-stage-gamble-in-U, second-stage-gamble-in-U } and, second, order {'K-to-U', 'U-to-K'} . Both independent variables revealed significant main effects. For the context/condition variable, we observed $F(2, 1281) = 8.62$, $p =$ 1.9e-4 (in the figure below, variation by line color). The second-stage $p(g|U)$ thus differs from p(g) under the same ordering condition, that is, in the Unknown context and under the same order condition (U to K and, separately, K to U) we observe a higher value of $p(g)$ with respect to $p(g|U)$ (in the graph: light-blue curves are systematically higher than corresponding dark-blue curves). For the order variable main effect, we observed $F(2, 1281) = 27.7$, $p =$ 1.67e-7 (in the figure below, variation between line type, solid vs. perforated). As also described above, the order effect shows a significant larger acceptance of the first-stage and second-stage (U condition) gambles in the first block (dashed lines) than in the second block (solid lines). Finally, we also observed a significant crossover interaction, $F(2, 1281) = 9.71$, $p =$ 6.5e-5, which is interpreted in the same way as for the previous ANOVA.

This series of results further supports the following, remarkable, conclusion, namely that the context within which a gamble is taken can have a potent effect on the gamble itself. Context is shown to influence behavior both in terms of proximal, but unrelated, gambles (the Known vs. Unknown context), but also in terms of previously seen gambles (the order condition, that is, whether the block of Known context gambles was first or second). It is this strong contextuality that essentially provides the motivation for the quantum dynamical model in the present work.

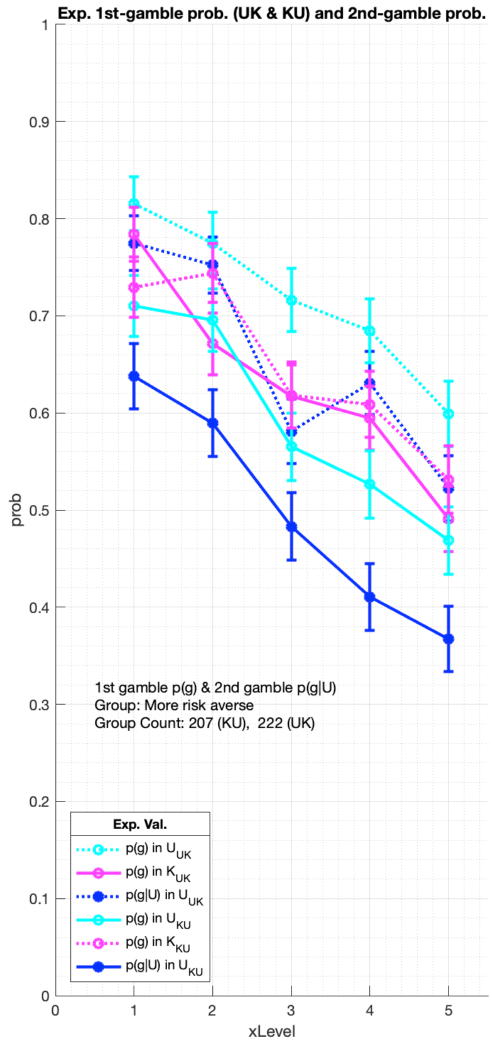

Figure S6: Experimental gamble probabilities for single-stage and second-stage gambles. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

SM 11. Experimental gamble probabilities and Attention test failing in Experiment 2.

Online crowdsourcing of data requires a means to monitor the participant's engagement. One way to verify the participant's attention is to include 'hidden' cues in the task. These cues appeared as a supplementary sentence inserted in the text to describe a second-stage gamble, Section (2). This cue informed the participant that this particular gamble was actually an attention test and that the participant needed to respond in a specified manner. Each participant had to process two of these gambles with a hidden cue. We rejected the data from the participants that failed one or both of the attention tests.

These 'inattentive' participants manifestly showed no significant payoff dependence in their choices (see fig. S7). When this group is partitioned by risk attitude a mixed ANOVA with unbalanced design to test the dependent variable 'X-averaged gamble probability' for factors of condition {W, L, U} and order {'K-to-U', 'U-to-K'}, shows both risk attitude groups make the gamble decision based on first-stage outcome condition {W, L, U}, $F(2, 708) = 6.78$ with $p = .0012$ (More risk averse, N= 121+117) and $F(2, 171) = 21.49$ with $p = 4.74\text{e-}9$ (Less risk averse, N= 31+28). The 'More risk averse' participants still show a significant ($p = .0015$) crossover interaction of outcome condition and flow order.

This pattern of behaviour suggests that the inattention of the participants concerned only 'the numbers' of the payoff size X but not the condition of the gamble. This suggestion is further supported by the fact that the 'attention' sentence was inserted in the text *after* describing the outcome condition of the previous stage, causing the participants to miss that directive.

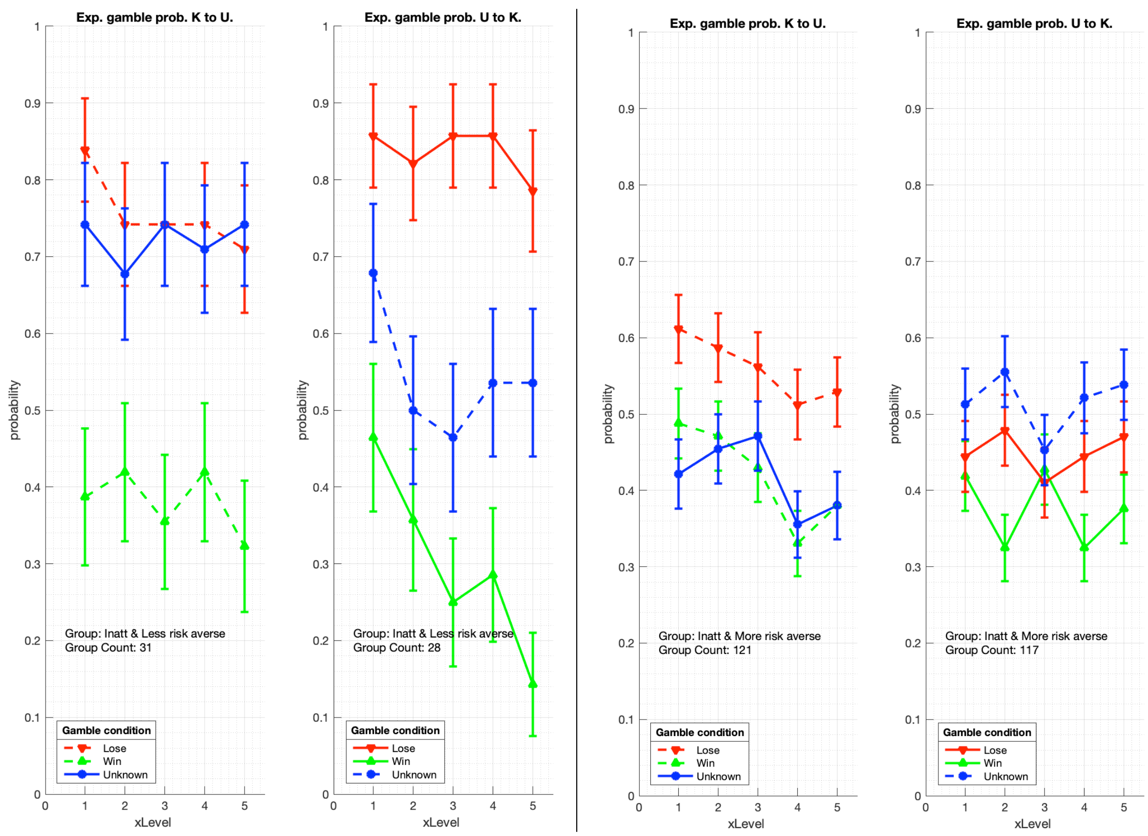

Figure S7: Observed gamble probabilities for participants that failed the attention tests. Participants are partitioned by 'less risk averse' attitude (left pair), and 'more risk averse' attitude (right pair). Within each panel, on the left are the observations for K-to-U order, on the right U-to-K order. The payoff parametrised by $X_{Level} \in [1, 5]$ appears on the x-axis. Error bars represent the standard error of the mean.

SM 12. Expected value of outcome conditioned and single-stage gambles

From the perspective of Expected Value, it makes sense to always play both the single-stage and the second-stage gambles, texts (8, 9). This is because these gambles have an Expected Value which exceeds not playing the gambles by an amount of 25X. The choice response of the small group of 'always takers' is consistent with the modal gamble pattern $(g|W, g|L, g|U)$ of the full participant cohort.

condition	V_1	EV_2	ΔEV
W	$100 \cdot X$	$125 \cdot X$	$25 \cdot X$
L	$-50 \cdot X$	$-25 \cdot X$	$25 \cdot X$
U	$25 \cdot X$ (exp.)	$50 \cdot X$	$25 \cdot X$
single-stage	$25 \cdot X$	•	•

Table S18: Payoffs of the first-stage of the $\{W, L, U\}$ outcome conditioned two-stage gambles, the single-stage gamble and expected value of the second-stage gamble of the corresponding first-stage. Accepting the second-stage gamble leads to the same positive change in expected value, irrespective the outcome condition. Accepting the single-stage gamble offers numerically the same value.