eprints@whiterose.ac.uk
https://eprints.whiterose.ac.uk/

A Robust Coordinated Expansion Planning Model for Wind Farm-Integrated Power Systems with Flexibility Sources Using Affine Policies

Shahab Dehghan, *Senior Member, IEEE*, Nima Amjady, *Senior Member, IEEE*, and Petros Aristidou, *Member, IEEE*

Abstract—This paper presents a two-stage adaptive robust coordinated generation and transmission expansion planning model for a wind farm-integrated power system. Also, dynamic thermal rating (DTR) systems, energy storage (ES) systems, and optimal line switching (OLS) maneuvers are considered as various flexible sources to enhance the flexibility of the power system in response to uncertain variations of net system demand. The proposed approach characterizes the uncertainty of demands, wind power, and DTRs in each representative day by a polyhedral uncertainty set. Additionally, the k-means clustering technique is used to obtain upward/downward variations of correlated uncertain parameters in each representative day and to construct the uncertainty set. The proposed model is inherently intractable as it includes infinite constraints modeling enforced techno-economic limitations for all realizations of uncertain parameters. To resolve this limitation, the proposed intractable model is recast as a tractable mixed-integer linear programming (MILP) problem using affine policies. The proposed approach is implemented on the Garver 6-bus and IEEE 73-bus test systems. Simulation results illustrate its flexibility, practicality, and tractability.

Index Terms—Dynamic Thermal Rating, Energy Storage, Planning, Robust Optimization, Switching, Wind Power.

NOMENCLATURE

A) Indices

b	Index of buses.
d	Index of demands.
l	Index of lines.
$line(r)$	Index of line for installing DTR system r.
o	Index of representative days.
o^s	Index of simulated daily profiles.
r	Index of DTR systems.
$r(l)/s(l)$	Index of receiving/sending bus of line l.
s	Index of ES systems.
t/t'	Index of hours.
u	Index of thermal units.
w	Index of wind farms.

B) Parameters

eoc_o	Expected operation costs of representative day o ($).
$e_s^{\max}/e_s^{\min}$	Maximum/minimum capacity of ES system s (MWh).
e_s^0	Initial stored energy of ES system s (MWh).
ic_l	Annualized installation cost of line l ($).
ic_r	Annualized installation cost of DTR system r ($).
ic_s	Annualized installation cost of ES system s ($).
ic_u	Annualized installation cost of thermal unit u ($).
ic_w	Annualized installation cost of wind farm w ($).
m_l	A sufficiently large positive number.
n_E	Number of constraints in (2b).
n_U	Number of elements in $\boldsymbol{U}$.
oc_u	Operation cost of thermal unit u ($/MWh).
oc_w	Operation cost of wind farm w ($/MWh).
$p_s^{c\max}/p_s^{d\max}$	Maximum charging/discharging rate of ES system s (MW).
$\bar{p}_{dto}$	Forecasted demand d in hour t of representative day o (MW).
$\hat{p}_{dto}^-/\hat{p}_{dto}^+$	Downward/upward variation range of $\tilde{p}_{dto}$ (MW).
$p_l^{\max}$	Capacity (i.e., static rating) of line l (MW).
$\Delta\bar{p}_{lto}$	Forecasted increase in capacity of line l in hour t of representative day o.
$\Delta\hat{p}_{lto}^-/\Delta\hat{p}_{lto}^+$	Downward/upward variation range of $\Delta\tilde{p}_{lto}$ (MW).
$p_u^{\max}$	Capacity of thermal unit u (MW).
$\bar{p}_{wto}$	Forecasted power production of wind farm w in hour t of representative day o (MW).
$\hat{p}_{wto}^-/\hat{p}_{wto}^+$	Downward/upward variation range of $\tilde{p}_{wto}$ (MW).
rd_u/ru_u	Ramp-down/ramp-up limit of thermal unit u (MW/h).
tic	Total investment costs ($).
toc_{o^s}	Total operation costs of simulated daily profile o^s for demands, wind power, and DTRs ($).
vol_b	Value of lost demand at bus b ($/MWh).
y_l	Susceptance of line l (mho).
ρ_o	Weighting factor of representative day o.
$\varsigma_s^c/\varsigma_s^d$	Charging/discharging efficiency of ES system s.
$\theta^{\max}$	Maximum permitted value of θ_{bto}.
Υ	Budget of uncertainty.

C) Sets

Ω^B	Set of buses.
Ω^D	Set of demands.
Ω^{D_b}	Set of demands connected to bus b.
Ω^L	Set of lines.
Ω^O	Set of representative days.
Ω^R	Set of DTR systems.
Ω^S	Set of ES systems.
Ω^{S_b}	Set of ES systems connected to bus b.
Ω^T	Set of hours of each representative day.
Ω^U	Set of thermal units.
Ω^{U_b}	Set of thermal units connected to bus b.
Ω^W	Set of wind farms.
Ω^{W_b}	Set of wind farms connected to bus b.
Ω^Γ	Polyhedral uncertainty set.

S. Dehghan and P. Aristidou are with the School of Electronic and Electrical Engineering, University of Leeds, Leeds, UK.

N. Amjady is with the Department of Electrical and Computer Engineering, Semnan University, Semnan, Iran.

Ω^{Γ_D}	Polyhedral uncertainty set of uncertain demands.
Ω^{Γ_R}	Polyhedral uncertainty set of uncertain wind power.
Ω^{Γ_R}	Polyhedral uncertainty set of uncertain DTRs.

D) Variables

p_{bto}	Loss of demand at bus b in hour t of representative day o (MW).
$\tilde{p}_{dto}$	Uncertain demand d in hour t of representative day o (MW).
p_{lto}	Power flow of line l in hour t of representative day o (MW).
$\Delta\tilde{p}_{lto}$	Uncertain increase in maximum capacity of line l in hour t of representative day o.
p^c_{sto}/p^d_{sto}	Charging/discharging power of ES system s in hour t of representative day o (MW).
p_{uto}	Power production of thermal unit u in hour t of representative day o (MW).
p_{wto}	Power production of wind farm w in hour t of representative day o (MW).
$\tilde{p}_{wto}$	Uncertain power production of wind farm w in hour t of representative day o (MW).
x_{lto}	Switching status of line l in hour t of representative day o (1: closed; 0: otherwise)
x^c_{sto}/x^d_{sto}	Charging/discharging status of ES system s in hour t of representative day o (1: charging/discharging; 0: otherwise).
z_l	Binary variable indicating installation status of line l (1: installed; 0: otherwise).
z_r	Binary variable indicating installation status of DTR System r (1: installed; 0: otherwise).
z_s	Binary variable indicating installation status of ES system s (1: installed; 0: otherwise).
z_u	Binary variable indicating installation status of thermal unit u (1: installed; 0: otherwise).
z_w	Binary variable indicating installation status of wind farm w (1: installed; 0: otherwise).
θ_{bto}	Phase angle of bus b in hour t of representative day o (rad).
θ^{ref}_{to}	Phase angle of reference bus in hour t of representative day o (rad).
σ, Δp_{lto}, p^+_{dto}, p^-_{dto}, p^+_{wto}, p^-_{wto}, $\Delta p^+_{lto}, \Delta p^-_{lto}$	Continuous auxiliary variables.

E) Vectors and Matrices

$\boldsymbol{E}, \boldsymbol{J}, \boldsymbol{Q}$	Vectors of coefficients/requirements.
$\boldsymbol{F}, \boldsymbol{G}, \boldsymbol{H}, \boldsymbol{I}$, $\boldsymbol{q}$	Matrices of coefficients.
$\boldsymbol{U}$	Vector of second-stage continuous independent variables (i.e., $\forall p^c_{sto}$, $\forall p^d_{sto}$, $\forall p_{uto}$, $\forall p_{wto}$).
$\boldsymbol{U}^a$	Matrix of adjustable second-stage independent variables.
$\boldsymbol{U}^n$	Vector of non-adjustable second-stage independent variables.
$\boldsymbol{V}$	Vector of second-stage continuous dependent variables (i.e., $\forall p_{bto}$, $\forall p_{lto}$, $\forall\theta_{bto}$).
$\boldsymbol{Z}$	Vector of first-stage binary variables (i.e., $\forall x_{lto}$,

	$\forall x^c_{sto}$, $\forall x^d_{sto}$, $\forall z_l$, $\forall z_r$, $\forall z_s$, $\forall z_u$, and $\forall z_w$).
γ	Vector of uncertain parameters (i.e, $\forall\tilde{p}_{dto}$, $\forall\tilde{p}_{wto}$, and $\forall\Delta\tilde{p}_{lto}$).

I. INTRODUCTION

A. Motivation and Background

THE need for a more flexible power system has been intensified in recent years due to the increasing integration of intermittent renewable energy sources (RES) (e.g., solar and wind parks) [1]. *"Flexibility"* refers to the capability of optimally utilizing generation and transmission facilities in power system aiming at adequately responding to any uncertain variation in net system demand (i.e., any portion of system demand not supplied by RES) [2]. Specifically, the power system flexibility can be increased by providing higher ramp-rates/ramp-ranges for power generations (e.g., by installing energy storage (ES) systems) and providing higher ratings for power flows (e.g., by installing dynamic thermal rating (DTR) systems) [1], [3], [4]. Also, optimal line switching (OLS) maneuvers may enhance power system flexibility. Hence, it is viable to proficiently expand the existing power system and to appropriately characterize uncertain variations in net system demand aiming at attaining a higher flexibility.

Robust optimization (RO) is a proficient non-deterministic optimization method recently used in the literature to characterize different types of uncertain parameters and to obtain a robust expansion plan for generation system [5], transmission system [6]–[11], and both generation and transmission systems [12], [13]. However, to the best of the authors' knowledge, there is no two-stage adaptive robust coordinated planning model for flexible investment in new lines, DTR systems, ES systems, thermal units, and wind farms considering OLS maneuvers.

B. Contributions

The main contributions of this paper are as follows:

- A coordinated generation and transmission planning (G&TP) model is introduced for flexible investment in new lines, DTR systems, ES systems, thermal units, and wind farms considering OLS maneuvers.
- A two-stage adaptive robust approach is presented for the proposed G&TP model to characterize the uncertainty of demands, wind power, and DTRs within a polyhedral uncertainty set. In addition, the k-means clustering technique is used in this paper to obtain upward/downward variations of uncertain parameters in each representative day based on correlated daily patterns of uncertain parameters.
- The proposed two-stage adaptive robust model is reformulated into a tractable mixed-integer linear programming (MILP) problem by introducing proficient affine polices.
- A structural analysis as well as an out-of-sample analysis are carried out to evaluate the flexibility and the robustness of the proposed G&TP model, respectively.

C. Paper Organization

The rest of this paper is organized as follows. In Section II, the main characteristics of the proposed G&TP model are presented. In Section III, the deterministic G&TP (DG&TP)

model is introduced. In Section IV, the two-stage adaptive robust G&TP (RG&TP) model and its affine policies are presented. In Section V, the DG&TP and RG&TP models are implemented on the Garver 6-bus and the IEEE 73-bus test systems. Finally, the main conclusions of the paper are summarized in Section VI.

II. PROBLEM DESCRIPTION

The main characteristics of the proposed model are:

- The planning horizon is single period to attain a compromise between accuracy and tractability.
- The patterns of hourly demands, wind power, and DTRs are characterized by representative days to more accurately evaluate the impact of ramp-rates/ramp-ranges on the operation of ES systems, thermal units, and wind farms.
- The uncertainties pertaining to the patterns of demands, wind power, and DTRs in each hour of every representative day are characterized by a polyhedral uncertainty set. The lower, nominal, and upper estimates of the uncertain parameters are obtained by the k-means clustering technique [14] to consider the inter-temporal correlations of the uncertain parameters.
- For the sake of brevity in presentation, all existing and candidate equipment are considered as installation candidates where the installation costs and binary status variables for all existing equipment are set to 0 and 1, respectively.

Note that the DG&TP model only incorporates the nominal estimates of the uncertain parameters while the RG&TP model incorporates the bounded intervals of the uncertain parameters.

III. THE PROPOSED DG&TP MODEL

The proposed DG&TP model is formulated as an MILP optimization problem given below:

$$
\begin{aligned}
\min \quad & \sum_{l\in\Omega^L} ic_l \cdot z_l + \sum_{r\in\Omega^R} ic_r \cdot z_r + \sum_{s\in\Omega^S} ic_s \cdot z_s \\
& + \sum_{u\in\Omega^U} ic_u \cdot z_u + \sum_{w\in\Omega^W} ic_w \cdot z_w \\
& + \sum_{o\in\Omega^O}\sum_{t\in\Omega^T}\sum_{u\in\Omega^U} \rho_o \cdot oc_u \cdot p_{uto} \\
& + \sum_{o\in\Omega^O}\sum_{t\in\Omega^T}\sum_{w\in\Omega^W} \rho_o \cdot oc_w \cdot p_{wto} \\
& + \sum_{o\in\Omega^O}\sum_{t\in\Omega^T}\sum_{b\in\Omega^B} \rho_o \cdot vol_b \cdot p_{bto}
\end{aligned}
\tag{1a}
$$

s.t.

$$
\begin{aligned}
& p_{bto} - \sum_{s\in\Omega^{S_b}} p_{sto}^c + \sum_{s\in\Omega^{S_b}} p_{sto}^d - \sum_{l\in\Omega^L|s(l)=b} p_{lto} \\
& + \sum_{l\in\Omega^L|r(l)=b} p_{lto} + \sum_{u\in\Omega^{U_b}} p_{uto} + \sum_{w\in\Omega^{W_b}} p_{wto} \\
& = \sum_{d\in\Omega^{D_b}} \bar{p}_{dto} \quad b\in\Omega^B, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1b}
$$

$$
0 \le p_{uto} \le p_u^{\max}\cdot z_u \quad u\in\Omega^U, t\in\Omega^T, o\in\Omega^O
\tag{1c}
$$

$$
-rd_u \le p_{uto} - p_{u(t-1)o} \le ru_u \quad u\in\Omega^U, t\in\Omega^T, o\in\Omega^O
\tag{1d}
$$

$$
0 \le p_{wto} \le \bar{p}_{wto}\cdot z_w \quad w\in\Omega^W, t\in\Omega^T, o\in\Omega^O
\tag{1e}
$$

$$
0 \le p_{bto} \quad b\in\Omega^B, t\in\Omega^T, o\in\Omega^O
\tag{1f}
$$

$$
\begin{aligned}
e_s^{\min}\cdot z_s \le & \sum_{t'=1}^{t}\left(\varsigma_s^c\cdot p_{st'o}^c - \frac{1}{\varsigma_s^d}\cdot p_{st'o}^d\right) \\
& + e_s^0\cdot z_s \le e_s^{\max}\cdot z_s \quad s\in\Omega^S, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1g}
$$

$$
\sum_{t'=1}^{24}\left(\varsigma_s^c\cdot p_{st'o}^c - \frac{1}{\varsigma_s^d}\cdot p_{st'o}^d\right) = 0 \quad s\in\Omega^S, o\in\Omega^O
\tag{1h}
$$

$$
0 \le p_{sto}^c \le p_s^{c\max}\cdot x_{sto}^c \quad s\in\Omega^S, t\in\Omega^T, o\in\Omega^O
\tag{1i}
$$

$$
0 \le p_{sto}^d \le p_s^{d\max}\cdot x_{sto}^d \quad s\in\Omega^S, t\in\Omega^T, o\in\Omega^O
\tag{1j}
$$

$$
0 \le x_{sto}^c + x_{sto}^d \le z_s \quad s\in\Omega^S, t\in\Omega^T, o\in\Omega^O
\tag{1k}
$$

$$
\begin{aligned}
& \frac{p_{lto}}{y_l} - \left(\theta_{s(l)to} - \theta_{r(l)to}\right) \le m_l\cdot(1-x_{lto}) \\
& \qquad\qquad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1l}
$$

$$
\begin{aligned}
& \frac{p_{lto}}{y_l} - \left(\theta_{s(l)to} - \theta_{r(l)to}\right) \ge -m_l\cdot(1-x_{lto}) \\
& \qquad\qquad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1m}
$$

$$
\begin{aligned}
& -p_l^{\max}\cdot x_{lto} - \Delta p_{lto} \le p_{lto} \le p_l^{\max}\cdot x_{lto} + \Delta p_{lto} \\
& \qquad\qquad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1n}
$$

$$
\begin{aligned}
& 0 \le \Delta p_{lto} \le \Delta\bar{p}_{lto}\cdot z_{r|line(r)=l} \\
& \qquad\qquad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\end{aligned}
\tag{1o}
$$

$$
0 \le \Delta p_{lto} \le \Delta\bar{p}_{lto}\cdot x_{lto} \quad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\tag{1p}
$$

$$
x_{lto} \le z_l \quad l\in\Omega^L, t\in\Omega^T, o\in\Omega^O
\tag{1q}
$$

$$
-\theta^{\max} \le \theta_{bto} \le \theta^{\max} \quad b\in\Omega^B, t\in\Omega^T, o\in\Omega^O
\tag{1r}
$$

$$
\theta_{to}^{ref} = 0 \quad t\in\Omega^T, o\in\Omega^O
\tag{1s}
$$

The objective function (1a) minimizes: 1) the total installation costs of new lines, DTR systems, ES systems, thermal units, and wind farms, 2) the total operation costs of thermal units and wind farms, and 3) the total costs of loss of demands. Constraint (1b) ensures the nodal production-consumption balance during the planning horizon. Constraint (1c) limits the power production of every thermal unit between zero and its maximum capacity. Also, constraint (1d) enforces the ramp-limits of thermal units. Constraint (1e) limits the power production of every wind farm between zero and its available capacity (i.e., $\bar{p}_{wto}$) considering the wind farm spillage. Constraint (1f) ensures the non-negativity of loss of demands. Constraint (1g) limits the stored energy of each ES system in every hour of each representative day within the allowable limits. Also, constraint (1h) ensures that the stored energies of each ES system at the initial and final hours of each representative day are identical. Constraints (1i) and (1j) limit the charging and discharging rates of ES systems, respectively. Constraint (1k) is used to avoid simultaneous charging and discharging of each ES system considering its installation status. Constraints (1l)-(1m) and constraints (1n)-(1p) enforce the power flow and the capacity of each line in every hour of each representative day, respectively, where m_l is a sufficiently large positive number (i.e., $|m_l| \ge 2\cdot\theta^{\max}$ [15]). The dynamic thermal capacity of any line equipped with DTR system is typically higher than its static thermal capacity [16] and thus the auxiliary variable Δp_{lto} and the constraints (1n)-(1p) are used to model this property. If any switchable line is closed (i.e., $x_{lto} = 1$), constraint (1l)-(1m) and constraints (1n)-(1p) lead

to $p_{lto} = y_l \cdot \left(\theta_{s(l)to} - \theta_{r(l)to}\right)$ and $|p_{lto}| \leq p_l^{\max} + \Delta p_{lto}$, respectively; otherwise, constraints (1l)-(1m) are neutralized and constraints (1n)-(1p) lead to $|p_{lto}| = 0$. Also, if any closed switchable line is equipped with DTR system (i.e., $z_{r|line(r)=l} = 1$ and $x_{ltb} = 1$), constraints (1o) and (1p) lead to $0 \leq \Delta p_{lto} \leq \Delta \bar{p}_{lto}$; otherwise, $\Delta p_{lto} = 0$. Constraint (1q) avoids considering non-built lines closed. Constraint (1r) bounds the permitted variation ranges of phase angles [15]. Also, constraint (1s) sets the phase angle of the reference bus to zero. For notational brevity, the proposed DT&GP model in (1a)-(1s) can be compactly rewritten in epigraph form as:

$$\min_{\sigma, \boldsymbol{Z}, \boldsymbol{U}} \quad \sigma \tag{2a}$$

s.t.

$$\boldsymbol{E} \cdot \sigma \geq \boldsymbol{F} \cdot \boldsymbol{Z} + \boldsymbol{G} \cdot \boldsymbol{U} + \boldsymbol{H} \cdot \boldsymbol{V} + \boldsymbol{I} \cdot \boldsymbol{\gamma} + \boldsymbol{J} \tag{2b}$$

where the objective function (1a) and the constraints (1b)-(1s) are represented as constraints (2b). For the DG&TP model, the vector of uncertain parameters $\boldsymbol{\gamma}$ (i.e., $\tilde{p}_{dto}$, $\tilde{p}_{wto}$, $\Delta \tilde{p}_{lto}$) is fixed on the forecasted values of the uncertain demands, wind power, and DTRs (i.e., $\bar{p}_{dto}$, $\bar{p}_{wto}$, $\Delta \bar{p}_{lto}$).

IV. ROBUST PLANNING MODEL

A. Uncertainty Characterization

In practice, the forecasted demands, wind power, and DTRs (e.g., $\bar{p}_{dto}$, $\bar{p}_{wto}$, $\Delta \bar{p}_{lto}$) are subject to uncertainty. Although different types of uncertainty sets (i.e., box, polyhedral, and ellipsoidal) can be considered in the proposed RG&TP model, a polyhedral uncertainty set is used in this paper aiming at [17]–[19]: 1) controlling the conservatism of the optimal solution, 2) retaining the linearity of the robust counterpart, 3) using duality in linear optimization to obtain a tractable robust counterpart by means of affine policies. Accordingly, similar to other research works in the area that use a polyhedral uncertainty set [5]–[14], a polyhedral uncertainty set is presented here to characterize the uncertainties pertaining to the patterns of demands, wind power, and DTRs in each representative operating day by means of the lower, nominal, and upper estimates of each uncertain parameter:

$$\Omega^\Gamma = \left\{ \Omega^{\Gamma D}, \Omega^{\Gamma W}, \Omega^{\Gamma R} \right\} \tag{3}$$

where

$$\Omega^{\Gamma D} = \left\{ \begin{array}{c} \tilde{p}_{dto} = \bar{p}_{dto} + p_{dto}^+ - p_{dto}^- \quad \forall d, t, o \\ 0 \leq p_{dto}^+ \leq \hat{p}_{dto}^+ \\ 0 \leq p_{dto}^- \leq \hat{p}_{dto}^- \end{array} \right\} \tag{4a}$$

$$\Omega^{\Gamma W} = \left\{ \begin{array}{c} \tilde{p}_{wto} = \bar{p}_{wto} + p_{wto}^+ - p_{wto}^- \quad \forall w, t, o \\ 0 \leq p_{wto}^+ \leq \hat{p}_{wto}^+ \\ 0 \leq p_{wto}^- \leq \hat{p}_{wto}^- \end{array} \right\} \tag{4b}$$

$$\Omega^{\Gamma R} = \left\{ \begin{array}{c} \Delta \tilde{p}_{lto} = \Delta \bar{p}_{lto} + \Delta p_{lto}^+ - \Delta p_{lto}^- \quad \forall l, t, o \\ 0 \leq \Delta p_{lto}^+ \leq \Delta \hat{p}_{lto}^+ \\ 0 \leq \Delta p_{lto}^- \leq \Delta \hat{p}_{lto}^- \end{array} \right\} \tag{4c}$$

Also, constraint (5) can be added to the polyhedral uncertainty set Ω^Γ to control the conservatism of the optimal solution:

$$\begin{aligned} &\sum_{d \in \Omega^D} \left(\frac{p_{dto}^+}{\hat{p}_{dto}^+} + \frac{p_{dto}^-}{\hat{p}_{dto}^-} \right) + \sum_{w \in \Omega^W} \left(\frac{p_{wto}^+}{\hat{p}_{wto}^+} + \frac{p_{wto}^-}{\hat{p}_{wto}^-} \right) + \\ &\sum_{l \in \Omega^L} \left(\frac{\Delta p_{lto}^+}{\Delta \hat{p}_{lto}^+} + \frac{\Delta p_{lto}^-}{\Delta \hat{p}_{lto}^-} \right) \leq \Upsilon \quad t \in \Omega^T, o \in \Omega^O \end{aligned} \tag{5}$$

The budget of uncertainty Υ in (5) can vary from zero to the total number of uncertain parameters in each hour of every representative day. By means of the budget of uncertainty Υ, the size of the polyhedral uncertainty set can be controlled. For notational brevity, Ω^Γ can be compactly rewritten as follows:

$$\Omega^\Gamma = \{ \boldsymbol{\gamma} \geq \boldsymbol{0} \mid \boldsymbol{q} \cdot \boldsymbol{\gamma} \leq \boldsymbol{Q} \} \tag{6}$$

B. Robust Model Based on Affine Policies

The proposed RG&TP model can be compactly presented as given below:

$$\min_{\sigma, \boldsymbol{Z}, \boldsymbol{U}} \quad \sigma \tag{7a}$$

s.t.

$$\begin{aligned} \forall \boldsymbol{\gamma} \in \Omega^\Gamma \; : \; \exists \boldsymbol{U} \geq \boldsymbol{0} \; : \; \boldsymbol{E} \cdot \sigma &\geq \boldsymbol{F} \cdot \boldsymbol{Z} + \boldsymbol{G} \cdot \boldsymbol{U} \\ &+ \boldsymbol{H} \cdot \boldsymbol{V} + \boldsymbol{I} \cdot \boldsymbol{\gamma} + \boldsymbol{J} \end{aligned} \tag{7b}$$

Unlike the DG&TP model in (2a)-(2b) which only considers the forecast values of the uncertain parameters $\boldsymbol{\gamma}$ (i.e., $\bar{p}_{dto}$, $\bar{p}_{wto}$, $\Delta \bar{p}_{lto}$), the proposed RG&TP model in (7a)-(7b) is immunized against *any realization* of the uncertain parameters $\boldsymbol{\gamma}$ belonging to Ω^Γ. However, this optimization problem is intractable since it includes infinite constraints because of the universal quantifier in (7b) (i.e., $\forall \boldsymbol{\gamma} \in \Omega^\Gamma$). To come up with a tractable optimization problem, the following three-step procedure is presented:

First Step: It is assumed that the independent operation decisions are affine in $\boldsymbol{\gamma}$ as given below [19]:

$$\boldsymbol{U}(\boldsymbol{\gamma}) = \boldsymbol{U}^n + \boldsymbol{U}^a \cdot \boldsymbol{\gamma} \tag{8}$$

Hence, the proposed RG&TP model in (7a)-(7b) can be rewritten as:

$$\min_{\sigma, \boldsymbol{Z}, \boldsymbol{U}^a, \boldsymbol{U}^n} \quad \sigma \tag{9a}$$

s.t.

$$\begin{aligned} \forall \boldsymbol{\gamma} \in \Omega^\Gamma \; : \; \boldsymbol{E} \cdot \sigma &\geq \boldsymbol{F} \cdot \boldsymbol{Z} + \boldsymbol{G} \cdot (\boldsymbol{U}^n + \boldsymbol{U}^a \cdot \boldsymbol{\gamma}) \\ &+ \boldsymbol{H} \cdot \boldsymbol{V} + \boldsymbol{I} \cdot \boldsymbol{\gamma} + \boldsymbol{J} \; ; \quad \boldsymbol{U}^n + \boldsymbol{U}^a \cdot \boldsymbol{\gamma} \geq \boldsymbol{0} \end{aligned} \tag{9b}$$

Second Step: This optimization problem with affine policies in (9a)-(9b) is still intractable because of including infinite constraints for all realizations of uncertain parameters. Thus, it is reformulated to obtain a tractable optimization problem immunized against the worst-case realization of the uncertain parameters. It is noteworthy that the reformulated optimization problem is feasible for all realizations of the uncertain parameters as it is feasible for the worst-case realization of the uncertain parameters. Hence, each constraint k for $k = 1, ..., n_E + n_U$ in (9b) (i.e., $\boldsymbol{E}_k \cdot \sigma \geq \boldsymbol{F}_k \cdot \boldsymbol{Z} + \boldsymbol{G}_k \cdot (\boldsymbol{U}^n + \boldsymbol{U}^a \cdot \boldsymbol{\gamma}) + \boldsymbol{H}_k \cdot \boldsymbol{V} + \boldsymbol{I}_k \cdot \boldsymbol{\gamma} + \boldsymbol{J}_k$ for $k = 1, ..., n_E$ and $\boldsymbol{U}_k^n + \boldsymbol{U}_k^a \cdot \boldsymbol{\gamma} \geq 0$ for

$k = n_E + 1, ..., n_E + n_U$) can be rewritten to be immunized against the worst-case realization of $\boldsymbol{\gamma}$ as given below:

$$\boldsymbol{E}_k \cdot \sigma - \boldsymbol{F}_k \cdot \boldsymbol{Z} - \boldsymbol{G}_k \cdot \boldsymbol{U}^n - \boldsymbol{H}_k \cdot \boldsymbol{V} - \boldsymbol{J}_k - \overbrace{\max_{\forall \boldsymbol{\gamma} \in \Omega^\Gamma} \left[(\boldsymbol{G}_k \cdot \boldsymbol{U}^a + \boldsymbol{I}_k) \cdot \boldsymbol{\gamma} \right]}^{\Theta_k^E} \geq 0 \quad k = 1, ..., n_E \tag{10a}$$

$$\boldsymbol{U}_k^n - \overbrace{\max_{\forall \boldsymbol{\gamma} \in \Omega^\Gamma} \left[-\boldsymbol{U}_k^a \cdot \boldsymbol{\gamma} \right]}^{\Theta_k^U} \geq 0 \quad k = n_E + 1, ..., n_E + n_U \tag{10b}$$

where the protection function Θ_k^E in constraint (10a) and the protection function Θ_k^U in constraint (10b) find the worst-case realization of $\boldsymbol{\gamma}$ by a maximization problem over $\forall \boldsymbol{\gamma} \in \Omega^\Gamma$. Therefore, this reformulation guarantees feasibility against any realization of $\boldsymbol{\gamma}$ belonging to Ω^Γ.

Third Step: Given a specific $(\boldsymbol{G}_k \cdot \boldsymbol{U}^a + \boldsymbol{I}_k)$ for $k = 1, ..., n_E$, $\Theta_k^E = \max_{\forall \boldsymbol{\gamma} \in \Omega_\Gamma} \left[(\boldsymbol{G}_k \cdot \boldsymbol{U}^a + \boldsymbol{I}_k) \cdot \boldsymbol{\gamma} \right]$ with $\Omega^\Gamma = \left\{ \boldsymbol{\gamma} \geq \boldsymbol{0} \mid \boldsymbol{q} \cdot \boldsymbol{\gamma} \leq \boldsymbol{Q} : \boldsymbol{\Psi}^k \right\}$ can be recast by duality theory [20] as a minimization problem given in (11):

$$\Theta_k^E = \min_{\forall \boldsymbol{\Psi}^k \in \Omega^{\Psi^k}} \boldsymbol{Q}' \cdot \boldsymbol{\Psi}^k \tag{11}$$

with

$$\Omega^{\Psi^k} = \left\{ \boldsymbol{\Psi}^k \geq \boldsymbol{0} \mid \boldsymbol{q}' \cdot \boldsymbol{\Psi}^k \geq (\boldsymbol{G}_k \cdot \boldsymbol{U}^a + \boldsymbol{I}_k)' : \boldsymbol{\gamma} \right\} \tag{12}$$

where $\boldsymbol{\Psi}^k$ and $\boldsymbol{\gamma}$ represent the vector of dual variables corresponding to the constraints of Ω^Γ in (6) and the constraints of Ω^{Ψ^k} in (12) for $k = 1, ..., n_E$, respectively. Also, the symbol $(')$ denotes the transpose of a matrix/vector. Consequently, each constraint k for $k = 1, ..., n_E$ in (10a) can be rewritten as:

$$\boldsymbol{E}_k \cdot \sigma - \boldsymbol{F}_k \cdot \boldsymbol{Z} - \boldsymbol{G}_k \cdot \boldsymbol{U}^n - \boldsymbol{H}_k \cdot \boldsymbol{V} - \boldsymbol{J}_k - \boldsymbol{Q}' \cdot \boldsymbol{\Psi}^k \geq 0 \quad k = 1, ..., n_E \tag{13a}$$

$$\boldsymbol{q}' \cdot \boldsymbol{\Psi}^k \geq (\boldsymbol{G}_k \cdot \boldsymbol{U}^a + \boldsymbol{I}_k)' \quad k = 1, ..., n_E \tag{13b}$$

where superscript/subscript k represents the kth column/row of a matrix. Similarly, given a specific $-\boldsymbol{U}_k^a$ for $k = n_E + 1, ..., n_E + n_U$, $\Theta_k^U = \max_{\forall \boldsymbol{\gamma} \in \Omega^\Gamma} \left[-\boldsymbol{U}_k^a \cdot \boldsymbol{\gamma} \right]$ with $\Omega^\Gamma = \left\{ \boldsymbol{\gamma} \geq \boldsymbol{0} \mid \boldsymbol{q} \cdot \boldsymbol{\gamma} \leq \boldsymbol{Q} : \boldsymbol{\Lambda}^k \right\}$ can be recast by duality theory as a minimization problem given in (14):

$$\Theta_k^U = \min_{\forall \boldsymbol{\Lambda}^k \in \Omega^{\Lambda^k}} \boldsymbol{Q}' \cdot \boldsymbol{\Lambda}^k \tag{14}$$

with

$$\Omega^{\Lambda^k} = \left\{ \boldsymbol{\Lambda}^k \geq \boldsymbol{0} \mid \boldsymbol{q}' \cdot \boldsymbol{\Lambda}^k \geq -\boldsymbol{U}_k^{a'} : \boldsymbol{\gamma} \right\} \tag{15}$$

where $\boldsymbol{\Lambda}^k$ and $\boldsymbol{\gamma}$ represent the vector of dual variables corresponding to the constraints of Ω^Γ in (6) and the constraints of Ω^{Λ^k} in (15) for $k = n_E + 1, ..., n_E + n_U$, respectively. Consequently, each constraint k for $k = n_E + 1, ..., n_E + n_U$ in (10b) can be rewritten as follows:

$$\boldsymbol{U}_k^n - \boldsymbol{Q}' \cdot \boldsymbol{\Lambda}^k \geq 0 \quad k = n_E + 1, ..., n_E + n_U \tag{16a}$$

$$\boldsymbol{q}' \cdot \boldsymbol{\Lambda}^k \geq -\boldsymbol{U}_k^{a'} \quad k = n_E + 1, ..., n_E + n_U \tag{16b}$$

By reformulating every constraint k for $k = 1, ..., n_E + n_U$, the RG&TP problem in (9a)-(9b) can be presented as an MILP

problem given below:

$$\min_{\sigma, \boldsymbol{Z}, \boldsymbol{U}^a, \boldsymbol{U}^n, \boldsymbol{\Psi} \geq \boldsymbol{0}, \boldsymbol{\Lambda} \geq \boldsymbol{0}} \sigma \tag{17a}$$

s.t.

$$\boldsymbol{E} \cdot \sigma - \boldsymbol{F} \cdot \boldsymbol{Z} - \boldsymbol{G} \cdot \boldsymbol{U}^n - \boldsymbol{H} \cdot \boldsymbol{V} - \boldsymbol{J} - \left(\boldsymbol{Q}' \cdot \boldsymbol{\Psi} \right)' \geq \boldsymbol{0} \tag{17b}$$

$$\boldsymbol{q}' \cdot \boldsymbol{\Psi} \geq (\boldsymbol{G} \cdot \boldsymbol{U}^a + \boldsymbol{I})' \tag{17c}$$

$$\boldsymbol{U}^n - \left(\boldsymbol{Q}' \cdot \boldsymbol{\Lambda} \right)' \geq 0 \tag{17d}$$

$$\boldsymbol{q}' \cdot \boldsymbol{\Lambda} \geq -\boldsymbol{U}^{a'} \tag{17e}$$

where $\boldsymbol{\Psi}$ and $\boldsymbol{\Lambda}$ represent the matrices of dual variables. The problem of infinite constraints is resolved by the reformulated MILP form given in (17a)-(17e). Due to space limitations, the extended formulation of the proposed RG&TP model is presented as a separate supplementary document in [21].

V. Case Studies

In this section, the proposed DG&TP and RG&TP models are applied to the Garver 6-bus [22] and the IEEE 73-bus [23] test systems. The patterns of demands and wind power in electric reliability council of Texas (ERCOT) during 2016 [24], [25] are used to derive representative days by means of the k-means clustering technique [14]. Also, the pattern of DTR systems is simulated by means of the weather data of Texas state during 2016 [25] and the IEEE standard 738 [26] and is used to derive appropriate representative days. In all case studies, the values of lost demands are obtained zero in the final solution, where the penalization cost at each bus is equal to 1000 $/MWh (i.e., $vol_b = 1000$ $/MWh). All data sets are available online on [21]. Also, the CPLEX solver in general algebraic modeling system (GAMS) is used to run all case studies on a windows-based server with 120 Intel Xeon processors and 102 GB RAM.

A. Garver 6-Bus Test System

The modified Garver test system [22] includes 8 existing lines, 10 existing thermal units, and 2 existing wind farms connected to buses 1 and 2 as depicted in Fig.1. Moreover, 15 candidate lines, 15 candidate DTR systems, 12 candidate ES systems, 12 candidate thermal units, and 12 candidate wind farms are considered. Note that 2 identical candidate ES systems and 2 identical candidate wind farms are considered at each bus. Also, the uncertainty set includes 5+2+12+15 = 34 uncertain parameters as there are 5 buses with uncertain demands, 2 existing and 12 candidate wind farms with uncertain power productions, and 15 candidate DTR systems with uncertain ratings. Hence, the budget of uncertainty Υ can vary between [0,34]. In addition, the penetration level of wind farms is assumed to be 30% of the installed capacity.

1) Structural Analysis: To highlight the impact of DTR systems, ES systems, and OLS on enhancing the flexibility of the power system and reducing the total expansion and operation costs, a structural analysis is carried out here. In this study, 12 representative days are considered for demands, wind power, and DTRs. Also, identical patterns are considered for representative days of power productions in both 100-MW and 200-MW wind farms as well as representative days of ratings in 100-MW and 200-MW lines. For the sake of simplicity, the

TABLE I
OPTIMAL SOLUTIONS IN THE STRUCTURAL ANALYSIS FOR THE GARVER 6-BUS TEST SYSTEM

Case #	Built Lines (From/To Bus)	Built DTR Systems (From/To Bus)	Built ES Systems (Bus)	Built Thermal Units (Bus|Type)	Built Wind Farms (Bus)	OLS (From/To Bus)	Costs (M$) Expansion	Operation
1	(1-6),(5-6)	-	-	(1|1),(2|1),(3|1), (4|1),(5|1)	2×(3),(4),2×(5)	-	200.16	111.65
							Total Costs: 311.81	
2	(1-6),(3-6),(5-6)	-	(5)	(1|1),(2|1),(4|1), (5|1)	(3),(4),2×(5)	-	172.08	111.33
							Total Costs: 283.41	
3	(3-6),(5-6)	(2-6),(4-6),(5-6)	(5)	(1|1),(2|1),(4|1), (5|1)	2×(1),(2),(5)	-	169.40	112.22
							Total Costs: 281.63	
4	(3-6),(5-6)	(2-6),(4-6),(5-6)	(5)	(1|1),(2|1),(4|1), (5|1)	(1),(4),2×(5)	(1-2),(1-4),(1-5), (2-3)	169.40	111.85
							Total Costs: 281.25	

TABLE II
OPTIMAL SOLUTIONS VS. BUDGET OF UNCERTAINTY FOR THE GARVER 6-BUS TEST SYSTEM

| Υ	Built Lines (From/To Bus)	Built DTR Systems (From/To Bus)	Built ES Systems (Bus)	Built Thermal Units (Bus|Type)	Built Wind Farms (Bus)	OLS (From/To Bus)	Costs (M$) Expansion	Operation
0	(3-6),(5-6)	(2-6),(4-6),(5-6)	(5)	(1|1),(2|1),(4|1), (5|1)	(1),(4),2×(5)	(1-2),(1-4),(1-5), (2-3)	169.40	111.85
							Total Costs: 281.25	
1	(2-3),(3-5),(3-6),(5-6)	(2-6),(3-5),(4-6), (5-6)	(2),(4),(5)	(1|1),(2|1),(4|1), (5|1)	(1),(4),2×(5)	(1-2),(1-4),(1-5)	189.01	122.10
							Total Costs: 311.11	
2	(1-5),(1-6),(5-6)	(2-3),(2-6),(3-5), (4-6),(5-6)	-	(1|1),(2|1),(3|1), (4|1),(5|1)	(1),(2),(4), 2×(5)	(1-4),(1-5)	201.02	136.65
							Total Costs: 337.67	
3	(1-3),(1-6),(5-6)	(2-3),(2-6),(3-5), (4-6),(5-6)	(5)	(1|1),(2|1),(3|1), (4|1),(5|1)	(2),(3),(4), 2×(5)	(1-4)	210.74	152.57
							Total Costs: 363.30	
4	(1-5),(1-6),(2-3), (3-5),(5-6)	(2-3),(2-6),(3-5), (4-6),(5-6)	(2),(4),(5)	(1|1),(2|1),(3|1), (4|1),(5|1)	(2),(3),(4), 2×(5)	-	229.62	169.59
							Total Costs: 399.21	
5	(3-4),(3-6),(5-6)	(1-5),(2-3),(2-6), (3-5),(4-6),(5-6)	-	(1|1,2),(2|1),(3|1), (4|1),(5|1)	2×(1),(2),2×(5)	(1-4)	250.79	179.62
							Total Costs: 430.40	
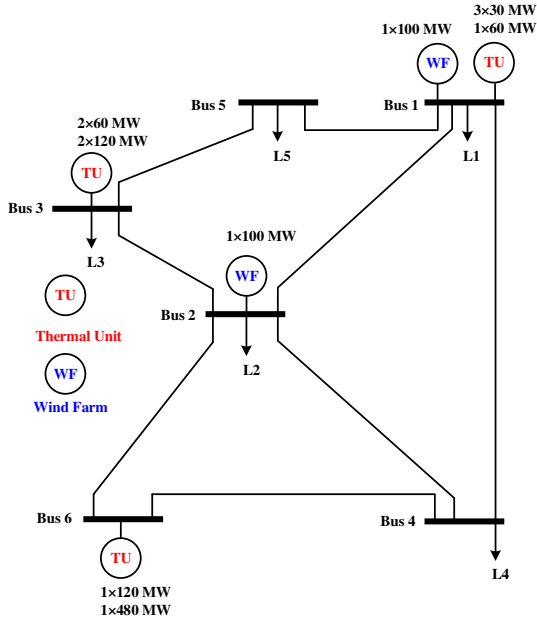
Fig. 1. The modified Garver 6-bus test system.

budget of uncertainty Υ is set to zero. Accordingly, the optimal solutions of the DG&TP and RG&TP models are identical. Four different cases are considered in the structural analysis as given below:

Case 1) Lines, thermal units, and wind farms are considered as expansion candidates.

Case 2) Lines, thermal units, wind farms, and ES systems are considered as expansion candidates.

Case 3) Lines, thermal units, wind farms, ES systems, and DTR systems are considered as expansion candidates.

Case 4) Lines, thermal units, wind farms, ES systems, and DTR systems are considered as expansion candidates with OLS.

The results of the structural analysis are given in Table I. According to Table I, the total costs are decreased from 311.81 M$ in Case 1 to 281.25 M$ in Case 4 (i.e., 30.56 M$ decrease), thanks to increasing flexibility from Case 1 to Case 4 by adding ES systems in Case 2, DTR systems in Case 3, and OLS in Case 4. The optimal expansion plans in both Case 3 and Case 4 are identical except for four 200-MW wind farms installed at buses 1, 2, and 5 in Case 3 and buses 1, 4, and 5 in Case 4. Hence, the total expansion costs in both Case 3 and Case 4 are equal to 169.40 M$ while the total operation costs in Case 4 are reduced by 0.37 M$ in comparison with Case 3 as a result of OLS. In Case 4, OLS opens line (1-2) in representative days 1 and 9, line (1-4) in representative days 6 and 8, line (1-5) in days 2 and 7, and line (2-3) in representative day 4, respectively.

2) Optimal Solution vs. Budget of Uncertainty: The proposed RG&TP model is capable of controlling the robustness of the optimal expansion plan by means of the budget of uncertainty. Therefore, to evaluate the impact of varying the budget of uncertainty on the optimal expansion plan, Υ is increased in this study from 0 to 34 as its minimum and maximum values, respectively. Similar to Case 4 in Section V-A1, lines, DTR systems, ES systems, thermal units, and wind farms are considered as expansion candidates with OLS. Also, 12 representative days are considered for demands, wind power, and DTRs. According to Table II, increasing the value of Υ from 0 to 5 provides higher robustness for the optimal expansion plans where the total expansion and operation costs are increased from 281.25 M$ for $\Upsilon = 0$ to 430.40 M$ for $\Upsilon = 5$. However, increasing the value of Υ from 5 to 34 has no effect on the optimal expansion plan and its total costs as the robustness

level are saturated. Hence, the effective variation range of Υ in this study is from 0 to 5. A similar issue is discussed by other robust planning tools in the literature [5], [11].

To further evaluate and compare the robustness/conservatism level of the optimal expansion plans in Table II, the expected total costs (i.e., ETC) during a one-year period is calculated for every expansion plan by an out-of-sample analysis. The out-of-sample analysis for every expansion plan simulates 365 chronological daily profiles for demands, wind power, and DTRs using proficient autoregressive moving average (ARMA) time series and calculates the total costs for 365 simulated daily profile using a linear programming (LP) problem. In this study, the ARMA models are derived using the profiles of demands, wind power, and DTRs of Texas state in 2016, respectively [24], [25]. Also, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are used to find the optimal degrees of autoregressive (AR) and moving-average (MA) lag parameters for ARMA models pertaining to demands, wind power, and DTRs [27], [28]. Accordingly, ARMA(4,3), ARMA(4,4), and ARMA(3,4) are found by AIC and BIC for demands, wind power, and DTRs, respectively. It is worthwhile to note that both AIC and BIC return identical optimal degrees for AR and MA lag parameters. The out-of-sample analysis is outlined in the Appendix. In Fig. 2, ETC versus Υ is illustrated. Although the total expansion and operation costs in Table II are monotonically increased by increasing Υ from 0 to 5, ETCs are decreased by increasing Υ from 0 to 2 and increased by increasing Υ from 2 to 5. Therefore, the optimum value of Υ, leading to minimum ETC, is equal to 2 as depicted in Fig. 2. In addition, it is seen that the ETC of the RG&TP model with $\Upsilon = 2$ is significantly lower than the ETC of the DG&TP model which is obtained when $\Upsilon = 0$.

3) Optimal Solution vs. Number of Representative Days: To evaluate the impact of varying the number of representative days (i.e., $|\Omega^O|$) on the optimal solution, the number of representative days are increased from 1 to 12 where expansion candidates are similar to Case 4 in Section V-A1 and the budget of uncertainty is set to its maximum effective value (i.e., $\Upsilon = 5$ as indicated in Section V-A2). According to Table III, the total expansion and operation costs are decreased by increasing the number of representative days from $|\Omega^O| = 1$ to $|\Omega^O| = 12$ at the expense of higher computation times. Its reason is that the modeling accuracy of various operation conditions and the characterization accuracy of uncertain parameters are increased by increasing the number of representative days.

B. IEEE 73-Bus Test System

To evaluate the tractability of the proposed planning tool in a larger test system, it is also implemented on the modified IEEE 73-bus test system [23] including 120 existing lines, 96 existing thermal units, and 9 existing wind farms. Moreover, 30 candidate lines, 30 candidate DTR systems, 14 candidate ES systems, 30 candidate thermal units, and 9 candidate wind farms are taken into account for expansion. In addition, 30 different lines are considered as candidates for switching maneuvers. Also, the uncertainty set includes 51+9+9+30 = 99 uncertain parameters as there are 51 buses with uncertain demands, 9 existing and 9 candidate wind farms with uncertain power productions, and
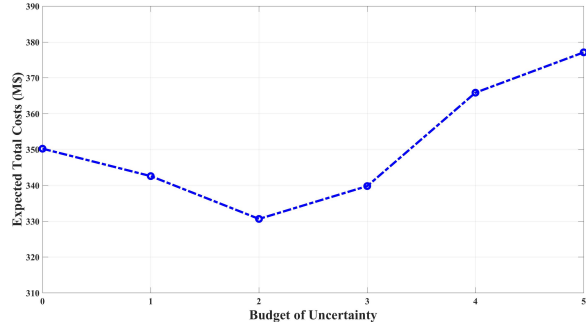

Fig. 2. Expected total costs versus the budget of uncertainty in the Garver 6-bus test system.

30 candidate DTR systems with uncertain ratings. Hence, the budget of uncertainty Υ can vary between [0,99]. In addition, the penetration level of wind farms is assumed to be 10% of the installed capacity. In this study, 4 representative days for demands, wind power, and DTRs are considered. Moreover, the budget of uncertainty Υ is increased from 0 to 99 as its minimum and maximum values, respectively. The optimal solutions for the IEEE 73-bus test system and its corresponding expansion and operation costs versus the budget of uncertainty are illustrated in Table IV. According to Table IV, increasing the value of Υ from 0 to 15 provides higher robustness for the optimal expansion plans where the total expansion and operation costs are increased from 871.49 M\$ for $\Upsilon = 0$ to 1630.05 M\$ for $\Upsilon = 15$. Similar to Table II for the Garver 6-bus test system, increasing the value of the budget of uncertainty after its maximum effective value (i.e., $\Upsilon = 15$) in Table IV has no effect on the optimal solution. Also, the CPU time for this case study is about 1 hour justifying the tractability of the proposed planning model in larger test systems.

VI. CONCLUSION

In this paper, a two-stage adaptive robust planning tool based on affine policies is introduced for optimal installation of new lines, DTR systems, ES systems, thermal units, and wind farms considering OLS maneuvers. The proposed approach characterizes the uncertainty of demands, wind power, and DTRs by means of a polyhedral uncertainty set. Also, a structural analysis as well as an out-of-sample analysis are performed to highlight the flexibility and the robustness of the proposed planning tool. The structural analysis evaluates the impact of utilizing DTR systems, ES systems, and switching maneuvers on increasing the flexibility of the power system and reducing the total cost while the out-of-sample analysis evaluates the impact of varying the budget of uncertainty on the robustness of the power system. Moreover, simulation results justify the tractability of the proposed model on both small and large test systems. In future works, the proposed planning tool can be extended to model network losses and evaluate the impact of DTR systems, ES systems, and OLS on them.

VII. APPENDIX

A. The Clustering Algorithm

Given 365 vectors of historical observations pertaining to 365 daily 24-hour patterns of demands, wind power productions,

TABLE III
OPTIMAL SOLUTIONS VS. NUMBER OF REPRESENTATIVE DAYS FOR THE GARVER 6-BUS TEST SYSTEM

$|\Omega^O|$	Built Lines (From/To Bus)	Built DTR System (From/To Bus)	Built ES System (Bus)	Built Thermal Units (Bus|Type)	Built Wind Farms (Bus)	OLS (From/To Bus)	Costs (M$) Expansion	Operation	CPU Time (s)
1	(1-5),(1-6),(3-4),(3-6),(5-6)	(2-3),(2-6),(3-5)	(1),(2),(4),2×(5)	(1|1),(2|1),(3|1),(4|1),(5|1)	2×(1),(3),(4),(6)	-	232.27	266.51	67
							Total Costs: 498.78		
4	(1-6),(5-6)	(2-3),(2-6),(3-5),(4-6),(5-6)	-	(1|1),(2|1),(3|1),(4|1),(5|1,2)	2×(2),2×(4),(5)	(1-5)	249.22	195.52	183
							Total Costs: 444.74		
8	(3-4),(3-6),(5-6)	(1-5),(2-3),(2-6),(3-5),(4-6),(5-6)	-	(1|1,2),(2|1),(3|1),(4|1),(5|1)	2×(1),(2),2×(5)	(1-2),(1-4)	250.79	180.28	427
							Total Costs: 431.07		
12	(3-4),(3-6),(5-6)	(1-5),(2-3),(2-6),(3-5),(4-6),(5-6)	-	(1|1,2),(2|1),(3|1),(4|1),(5|1)	2×(1),(2),2×(5)	(1-4)	250.79	179.62	871
							Total Costs: 430.40		

TABLE IV
OPTIMAL SOLUTIONS VS. BUDGET OF UNCERTAINTY FOR THE IEEE 73-BUS TEST SYSTEM

Υ	Built Lines (From/To Bus)	Built DTR Systems (From/To Bus)	Built ES Systems (Bus)	Built Thermal Units (Bus|Type)	Built Wind Farms (Bus)	OLS (From/To Bus)	Costs (M$) Expansion	Operation
0	(2-6),(7-8)	(16-17),(36-47),(56-58)	-	3×(15|12),(55|100),(63|12)	(8),(46)	-	51.42	820.07
							Total Costs: 871.49	
3	(2-6),(7-8),(7-27)	(16-17),(35-37),(36-47),(56-58),(60-61)	-	(2|20),(7|100),3×(15|12),(61|197)	(7),(8)	(2-4)	93.52	913.36
							Total Costs: 1006.88	
6	(2-6)	(16-17),(36-47),(56-58)	-	(1|20),(2|20),(13|197),3×(15|12),(55|100),(61|197)	(7),(8),(45)	(2-4)	147.69	1011.03
							Total Costs: 1158.72	
9	(2-6)	(16-17),(26-28)	-	(2|76),2×(13|197),(15|12),(55|100),(61|197)	(7),(8),(45)	-	193.73	1114.70
							Total Costs: 1308.43	
12	(2-6)	(16-17),(26-28),(36-47),(56-58)	-	(2|76),2×(13|197),(15|12),(55|100),2×(61|197),(63|12)	(7),(8),(45),(46)	-	242.55	1223.75
							Total Costs: 1466.30	
15	(1-5),(2-6),(7-8),(7-27),(11-13)	(11-13),(15-16),(16-17),(26-28),(35-37),(36-37),(36-47),(39-48),(40-41),(56-58),(60-61)	(9)	(1|20),(2|20),(2|76),(7|100),2×(13|197),5×(15|12),(55|100),2×(61|197),(63|12)	(7),(8),(45),(46)	-	297.62	1332.43
							Total Costs: 1630.05	

and DTR systems, the application of the k-means clustering algorithm to obtain representative days can be summarized as follows:

Step 1) Define the number of clusters to obtain representative days.

Step 2) Initialize the centroid of all clusters by randomly adding one historical observation to each cluster.

Step 3) Calculate the distance between the centroid of each cluster and all historical observations. In this paper, similar to [29] and [30], a quadratic distance is utilized.

Step 4) Add each historical observation to its nearest cluster using distances calculated in Step 3.

Step 5) Update the centroid of all clusters using historical observations added to each cluster in Step 4.

Step 6) Iterate between Steps 3-5 until all clusters remain unchanged in two successive iterations.

After clustering all historical observations, the centroid of each cluster represents nominal estimates for uncertain parameters in one representative day. Additionally, the σ-quantile and $(1 - \sigma)$-quantile of the empirical cumulative probability distribution of historical observations in each cluster represent the lower and upper estimates of uncertain parameters, respectively. In this paper, it is assumed that $\sigma = 0.05$.

B. The Out-of-Sample Analysis

The out-of-sample analysis, calculating the expected total costs (i.e., ETC), can be summarized as follows:

Step 1) Solve the RG&TP problem for a specific budget of uncertainty value, find its total investment costs (i.e., tic), and simulate 365 chronological daily profiles by means of the ARMA models pertaining to demands, wind power, and DTRs.

Step 2) Set $o = 1$.

Step 3) Set $o^s = 1$, ignore investment costs in (1a)-(1s), and fix its binary variables to their optimal values obtained from

Step 1 for the representative day o. Accordingly, the MILP problem in (1a)-(1s) is recast into an LP problem minimizing the total operation costs for every simulated daily profile of demands, wind power, and DTRs.

Step 4) Solve the LP problem obtained from Step 3 and find the total operation costs for the simulated daily profile o^s (i.e., toc_{o^s}) where $\bar{p}_{dto}$ in (1b), $\bar{p}_{wto}$ in (1e), and $\Delta\bar{p}_{lto}$ in (1o) and (1p) are fixed on the realized values of demands, wind power, and DTRs for the simulated daily profile o^s obtained from Step 1 (out of 365 chronological daily profiles).

Step 5) If $o^s < 365$, set $o^s = o^s + 1$ and go to Step 4. Otherwise, calculate $eoc_o = \sum_{o^s=1}^{365} toc_{o^s}$ and go to Step 6.

Step 6) If $o < |\Omega^O|$, set $o = o + 1$ and go to Step 3. Otherwise, calculate $ETC = tic + \sum_{o=1}^{|\Omega^O|} eoc_o/|\Omega^O|$ and report its value.

REFERENCES

[1] EPRI, "Electric power system flexibility: challenges and opportunities," *Technical Report*, 2016.

[2] E. Lannoye, D. Flynn, and M. O'Malley, "Evaluation of power system flexibility," *IEEE Transactions Power Systems*, vol. 27, no. 2, pp. 922–931, May 2012.

[3] H. Saboori, R. Hemmati, S. M. S. Ghiasi, and S. Dehghan, "Energy storage planning in electric power distribution networks – a state-of-the-art review," *Renewable and Sustainable Energy Reviews*, vol. 79, pp. 1108–1121, Nov. 2017.

[4] M. A. Bucher and G. Andersson, "Robust corrective control measures in power systems with dynamic line rating," *IEEE Transactions Power Systems*, vol. 31, no. 3, pp. 2034–2043, May 2016.

[5] S. Dehghan, N. Amjady, and A. Kazemi, "Two-stage robust generation expansion planning: a mixed integer linear programming model," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 584–597, March 2014.

[6] R. A. Jabr, "Robust transmission network expansion planning with uncertain renewable generation," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4558–4567, Nov. 2013.

[7] B. Chen, J. Wang, L. Wang, Y. He, and Z. Wang, "Robust optimization for transmission expansion planning: minimax cost vs. minimax regret," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 3069–3077, Nov. 2014.

[8] C. Ruiz and A. J. Conejo, "Robust transmission expansion planning," *European Journal of Operational Research*, vol. 242, no. 2, pp. 390–401, April 2015.

[9] R. Mínguez and R. García-Bertrand, "Robust transmission network expansion planning in energy systems: Improving computational performance," *European Journal of Operational Research*, vol. 248, no. 1, pp. 21–32, Jan. 2016.

[10] B. Chen and L. Wang, "Robust transmission planning under uncertain generation investment and retirement," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 5144–5152, Nov. 2016.

[11] S. Dehghan, N. Amjady, and A. J. Conejo, "Adaptive robust transmission expansion planning using linear decision rules," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 4024–4034, Sept. 2017.

[12] C. Roldán, A. A. S. de la Nieta, R. García-Bertrand, and R. Mínguez, "Robust dynamic transmission and renewable generation expansion planning walking towards sustainable systems," *International Journal of Electrical Power and Energy Systems*, vol. 96, pp. 52–63, March 2018.

[13] J. Li, Z. Li, F. Liu, H. Ye, X. Zhang, S. Mei, and N. Chang, "Robust coordinated transmission and generation expansion planning considering ramping requirements and construction periods," *IEEE Transactions on Power Systems*, pp. 268–280, Jan. 2018.

[14] S. Dehghan, N. Amjady, and A. J. Conejo, "Reliability constrained robust power system expansion planning," *IEEE Transactions on Power Systems*, vol. 31, no. 3, pp. 2383–2392, May 2016.

[15] G. Vinasco, M. J. Rider, and R. Romero, "A strategy to solve the multistage transmission expansion planning problem," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2574–2576, Nov. 2011.

[16] M. Jabarnejad and J. Valenzuela, "Optimal investment plan for dynamic thermal rating using benders decomposition," *European Journal of Operational Research*, vol. 248, no. 3, pp. 917–929, Feb. 2016.

[17] D. Bertsimas, D. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Review*, vol. 53, no. 3, pp. 464–501, 2011.

[18] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows," *Mathematical Programming*, vol. 98, no. 1, pp. 49–71, Sep 2003.

[19] D. Bertsimas and F. Ruiter, "Duality in two-stage adaptive linear optimization: faster computation and stronger bounds," *INFORMS Journal on Computing*, vol. 28, no. 3, pp. 500–511, May 2016.

[20] M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali, *Linear Programming and Network Flows*. Wiley, 2010.

[21] S. Dehghan, N. Amjady, and P. Aristidou, "A robust coordinated expansion planning model for wind farm-integrated power systems with flexibility sources using affine policies: extended model and data sets," Sep. 2019. [Online]. Available: https://doi.org/10.5281/zenodo.3459592

[22] L. L. Garver, "Transmission network estimation using linear programming," *IEEE Trans. Power App. Syst.*, vol. PAS-89, no. 7, pp. 1688–1697, Sep. 1970.

[23] The Reliability Test System Task Force, "IEEE reliability test system," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999.

[24] http://www.ercot.com.

[25] Alliance for Sustainable Energy, Department of Energy (DOE), National Renewable Energy Laboratory (NREL), System Advisor Model, Version 2016.

[26] IEEE Standard 738, "IEEE standard for calculation of bare overhead conductor temperature and ampacity under steady-state conditions," *IEEE Standard 738-1986*, pp. 1–19, Dec 1985.

[27] G. E. P. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time Series Analysis: Forecasting and Control*, ser. Wiley Series in Probability and Statistics. Wiley, 2015.

[28] S. Dehghan, N. Amjady, and H. Zareipour, "Optimal integration of multiple wind farms into bulk electric system considering wind speed correlation uncertainties," *International Transactions on Electrical Energy Systems*, vol. 26, no. 5, pp. 1085–1102, May 2016.

[29] L. Baringo and A. Conejo, "Correlated wind-power production and electric load scenarios for investment decisions," *Applied Energy*, vol. 101, pp. 475 – 482, 2013.

[30] L. Baringo and A. J. Conejo, "Risk-constrained multi-stage wind power investment," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 401–411, Feb 2013.