Sparse Planar Antenna Array Design for Directional Modulation

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Abstract. Directional modulation (DM) has been applied to sparse linear antenna arrays to increase security of signal transmission. In this work, we extend the DM design to sparse planar antenna arrays and provide the corresponding design formulations. In previous studies, group sparsity technique was used for sparse antenna array design, but no quantitative analyses were given. In this paper, both designs with and without group sparsity are provided, and the corresponding optimised antenna locations are shown explicitly. Design examples are provided to verify the effectiveness of the proposed design.

Keywords: Directional modulation, group sparsity, planar antenna array

1 Introduction

Directional modulation (DM) as a physical layer security technique was introduced to keep known constellation mappings in a desired direction or directions, while scrambling them for the remaining ones [1]. Both reconfigurable arrays [2] and phased antenna arrays [3–5] were employed in its design. In [6], a phased antenna array was combined with a reflecting surface to achieve positional modulation (PM), where the signal can only be received at desired locations instead of directions. Similarly, multiple phased antenna arrays [7] were also proposed for the PM design. From the algorithm's aspect, dual beam DM [8], bit error rate (BER) DM transmitter synthesis [9], artificial-noise-aided zero-forcing synthesis approach [10], a multi-relay design [11] and a pattern synthesis approach [12, 13] were introduced to the DM design area.

Recently, the so-called artificial noise (AN) was introduced with an aim to create a noise component which does not affect the signal received by desired receivers, but distorts the signal intercepted by eavesdroppers. Two methods were proposed for the AN design, including the orthogonal vector method [14,

15], where the added AN vector is orthogonal to the steering vector of the desired direction, and the AN projection matrix method [16, 17], where by designing an artificial noise projection matrix, the AN vector is projected into the zero space of the derivative of the desired direction.

However, to our best knowledge, almost all of the existing studies are focused on one dimensional DM. In this work, we extend the DM design to a two-dimensional (2-D) planar antenna array, and to further reduce the number of antenna, a sparse planar antenna array design method is proposed with optimised antenna locations. In previous studies, group sparsity was used for sparse antenna array design, but no quantitative analyses were given. In this paper, both designs with and without group sparsity are provided, and the corresponding optimised antenna locations are shown explicitly.

The remaining part of this paper is structured as follows. A review of planar antenna array based beamforming is given in Sec. 2. The proposed sparse planar antenna array design for DM is presented in Sec. 3. In Sec. 4, design examples are provided, with conclusions drawn in Sec. 5

2   Review of Planar Antenna Array Based Beamforming

A narrowband planar antenna array for transmit beamforming is shown in Fig. 1, consisting of N omni-directional antennas with the spacing $d_{x,n}$ along the x-axis, and K omni-directional antennas with the spacing $d_{y,k}$ along the y-axis, where $d_{x,n}$ and $d_{y,k}$ ($n = 0, \ldots, N - 1$ and $k = 0, \ldots, K - 1$) represent the spacing between the first antenna to its subsequent antennas, respectively. The elevation angle $\theta \in [0°, 180°]$, and azimuth angle $\phi \in [0°, 180°] \cup [0°, -180°]$. For the antenna on the n-th position of the x-axis and k-th position of the y-axis, the corresponding weight coefficient is represented by $w_{n,k}$. Here we gather all weight coefficients together to form a vector represented by $\mathbf{w}$,

$$\mathbf{w} = [w_{x_0,y_o}, w_{x_0,y_1}, \ldots, w_{x_0,y_{K-1}}, \ldots, w_{x_{N-1},y_{K-1}}]^T. \tag{1}$$

where $\{\cdot\}^T$ is the transpose operation. Then the corresponding steering vector is given by

$$\begin{aligned}
\mathbf{s}(\omega, \theta, \phi) = [1, & e^{j\omega(d_{x,0}\sin\theta\cos\phi + d_{y,0}\sin\theta\sin\phi)/c}, \ldots, \\
& e^{j\omega(d_{x,0}\sin\theta\cos\phi + d_{y,K-1}\sin\theta\sin\phi)/c}, \ldots, \\
& e^{j\omega(d_{x,N-1}\sin\theta\cos\phi + d_{y,K-1}\sin\theta\sin\phi)/c}]^T,
\end{aligned} \tag{2}$$

where c is the speed of propagation. The beam response of the array can be written as

$$p(\omega, \theta, \phi) = \mathbf{w}^H \mathbf{s}(\omega, \theta, \phi), \tag{3}$$

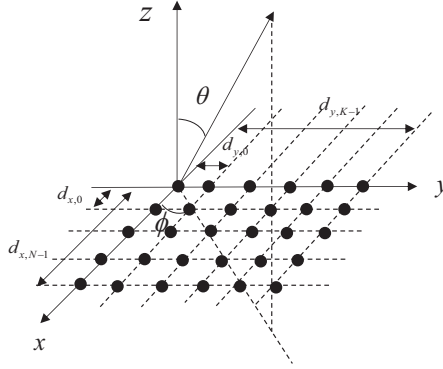where $\{\cdot\}^H$ represents the Hermitian transpose.

Fig. 1. A planar antenna array.

3 Sparse Planar Antenna Array Design for DM

The method to achieve DM is to find the corresponding weight vector of the antenna array for each symbol. Here we assume the weight vector for the m-th symbol is given by

$$\mathbf{w}_m = [w_{m,x_0,y_0}, w_{m,x_0,y_1}, \ldots, w_{m,x_0,y_{K-1}}, \ldots, w_{m,x_{N-1},y_{K-1}}]^T, \qquad (4)$$

$m = 0, \ldots, M - 1$. The desired response $p_m(\omega, \theta, \phi)$ for the m-th symbol can be categorised into two regions: the mainlobe response $\mathbf{p}_{m,ML}$ and the sidelobe response $\mathbf{p}_{m,SL}$. Without loss of generality, we assume R elevation angles are sampled for each azimuth angle ϕ_v $(v = 0, 1, \ldots, V - 1)$, and the desired directions are $\theta_0, \theta_1, \ldots, \theta_{r-1}$ with ϕ_0. Then, the desired beam responses and steering vectors for the mainlobe and sidelobe regions can be written as

$$
\begin{aligned}
\mathbf{p}_{m,ML} &= [p_m(\omega, \theta_0, \phi_0), p_m(\omega, \theta_1, \phi_0), \ldots, p_m(\omega, \theta_{r-1}, \phi_0)], \\
\mathbf{p}_{m,SL} &= [p_m(\omega, \theta_r, \phi_0), p_m(\omega, \theta_{r+1}, \phi_0), \ldots, p_m(\omega, \theta_{R-1,\phi_0}), p_m(\omega, \theta_0, \phi_1), \\
&\qquad \ldots, p_m(\omega, \theta_{R-1}, \phi_1), \ldots, p_m(\omega, \theta_{R-1}, \phi_{V-1})], \\
\mathbf{S}_{ML} &= [\mathbf{s}(\omega, \theta_0, \phi_0), \mathbf{s}(\omega, \theta_1, \phi_0), \ldots, \mathbf{s}(\omega, \theta_{r-1}, \phi_0)], \\
\mathbf{S}_{SL} &= [\mathbf{s}(\omega, \theta_r, \phi_0), \mathbf{s}(\omega, \theta_{r+1}, \phi_0), \ldots, \mathbf{s}(\omega, \theta_{R-1,\phi_0}), \mathbf{s}(\omega, \theta_0, \phi_1), \\
&\qquad \ldots, \mathbf{s}(\omega, \theta_{R-1}, \phi_1), \ldots, \mathbf{s}(\omega, \theta_{R-1}, \phi_{V-1})].
\end{aligned}
\qquad (5)
$$

The CS-based sparse antenna array design is to make the designed response close to the desired one with the minimum number of non-zero valued weight coefficients; those antennas with zero-valued coefficients can then be removed from the array, leading to a sparse solution. To achieve this goal, we employ the reweighted l_1 norm minimisation method, which provides a closer approximation to the required l_0 norm than the original l_1 norm [18–20]. Then, the weight vector

$\mathbf{w}_m$ for the m-th symbol is given by

$$
\begin{aligned}
\min_{\mathbf{w}_m} \quad & \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} \delta_{m,n,k}^u ||w_{m,x_n,y_k}^u||_2 \\
\text{subject to} \quad & ||\mathbf{p}_{m,SL} - (\mathbf{w}_m^u)^H \mathbf{S}_{SL}||_2 \leq \alpha \\
& (\mathbf{w}_m^u)^H \mathbf{S}_{ML} = \mathbf{p}_{m,ML},
\end{aligned}
\tag{6}
$$

where $|| \cdot ||_2$ represents the l_2 norm, the superscript u indicates the u-th iteration, and $\delta_{m,n,k}$ is the reweighting term for the coefficient at the n-th location of the x-axis and the k-th location of the y-axis for the m-th symbol, given by $\delta_{m,n,k}^u = (||w_{m,x_n,y_k}^{u-1}||_2 + \xi)^{-1}$ ($\xi > 0$ provides numerical stability to prevent $\delta_{m,n,k}$ becoming infinity). The inequality constraint is to make the difference between desired and designed responses in sidelobe regions under a given threshold value α, while the equality constraint is to set the same value between designed and desired responses in mainlobe directions.

However, the corresponding optimised locations deduced from the weight vectors $\mathbf{w}_0, \ldots, \mathbf{w}_{M-1}$ in (6) may not be the same, i.e., the optimised antenna locations for one symbol may be the redundant locations for other symbols. Therefore, a common set of active antenna locations for all symbols is needed, and the group sparsity technique can provide the solution [21]. Here, we introduce $\tilde{\mathbf{w}}_{x_n,y_k}$, representing weight coefficients for all M symbols at the n-th location of the x-axis and the k-th location of the y-axis,

$$
\tilde{\mathbf{w}}_{x_n,y_k} = [w_{0,x_n,y_k}, w_{1,x_n,y_k}, \ldots, w_{M-1,x_n,y_k}].
\tag{7}
$$

Then the weight vectors and the corresponding optimised locations for all symbols based on group sparsity can be obtained by solving the following problem

$$
\begin{aligned}
\min_{\mathbf{W}} \quad & \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} \delta_{n,k}^u ||\tilde{\mathbf{w}}_{x_n,y_k}^u||_2 \\
\text{subject to} \quad & ||\mathbf{P}_{SL} - (\mathbf{W}^u)^H \mathbf{S}_{SL}||_2 \leq \alpha \\
& (\mathbf{W}^u)^H \mathbf{S}_{ML} = \mathbf{P}_{ML},
\end{aligned}
\tag{8}
$$

where $\mathbf{W}$, $\mathbf{P}_{SL}$ and $\mathbf{P}_{ML}$ are three matrices for weight coefficients, beam responses in sidelobe regions and beam responses in mainlobe directions,

$$
\mathbf{W} = [\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_{M-1}],
\tag{9}
$$

$$
\mathbf{P}_{SL} = [\mathbf{p}_{0,SL}, \mathbf{p}_{1,SL}, \ldots, \mathbf{p}_{M-1,SL}]^T,
\tag{10}
$$

$$
\mathbf{P}_{ML} = [\mathbf{p}_{0,ML}, \mathbf{p}_{1,ML}, \ldots, \mathbf{p}_{M-1,ML}]^T.
\tag{11}
$$

The above problem can be solved by the CVX toolbox in MATLAB, a package for specifying and solving convex problems [22, 23].
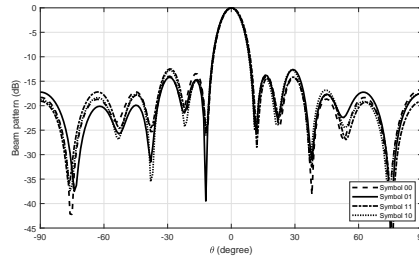
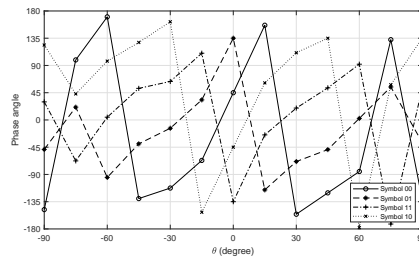**Fig. 2.** Resultant beam pattern based on the sparse planar array design in (8).

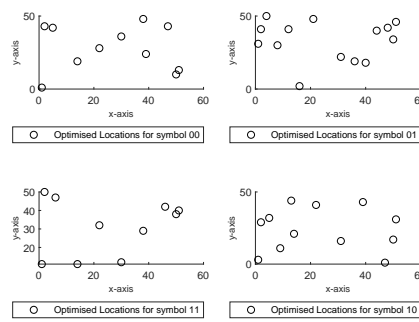**Fig. 3.** Resultant phase pattern based on the sparse planar array design in (8).

Fig. 4. Optimised locations for the planar antenna array without group sparsity in (6).
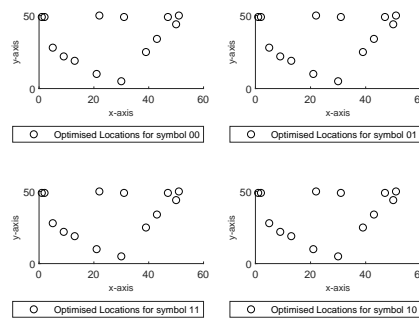

Fig. 5. Optimised locations for the planar antenna array with group sparsity in (8).

4   Design Examples

In this section, we consider a $5\lambda \times 5\lambda$ uniform planar antenna array with a 0.1λ spacing between adjacent antennas. Without loss of generality, the desired direction $\theta_{ML} = 0°$ with $\phi = 90°$. The sidelobe regions $\theta_{SL} \in [5°, 90°]$ for $\phi = \pm 90°$. The desired response in the mainlobe direction is a value of one (magnitude) with $90°$ phase shift (QPSK), i.e.,

$$\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} \qquad (12)$$

for symbols '00', '01', '11', '10', and a value of 0.1 (magnitude) with random phase shifts over the sidelobe regions. The given threshold $\alpha = 2$ for the design without group sparsity (location optimisation for each symbol separately) in (6), while $\alpha = 4$ for the design with group sparsity (a common set of optimised locations for all symbols) in (8). Bit error rate (BER) is also calculated based
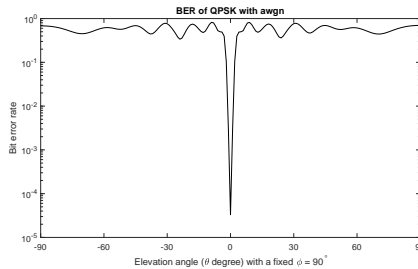

Fig. 6. BER based on the sparse planar array design in (8).

on in which quadrant the received signal lies in the IQ complex plane, and 10^6 randomly generated bits are transmitted, with signal to noise ratio (SNR = 12 dB) in the desired direction.

The resultant beam and phase patterns in (8) for all symbols are shown in Figs. 2 and 3, respectively, where we can see that all main beams are exactly pointed to the desired direction $0°$ with a low sidelobe level, and the phase only in the desired direction follows the required QPSK modulation, with random values in other directions. Fig. 4 shows the optimised locations for the design without group sparsity technique. It can be seen that the set of optimised locations for all symbols are not the same, and not even for a single optimised location, which means we have to keep all these optimised locations (the total number of optimised locations is 47, where for symbol '00' is 11, for symbol '01' is 14, for symbol '11' is 10, and for symbol '10' is 12), while Fig. 5 shows the common set of optimised locations for the design with group sparsity (the number of optimised locations is 14). BER in all transmission angles is shown in Fig. 6, which is down to 10^{-5} in the desired direction, and around 0.5 in other directions.

5   Conclusions

Directional modulation design has been applied to planar antenna arrays with optimised antenna locations for the first time. Satisfactory design results for beam patten, phase pattern and BER were provided, where the main beams for all symbols are pointed to the desired direction, with the given QPSK modulation based phase value, while in other directions power level is low and phase values are random. The BER pattern shows that error bits received in the desired direction is the lowest, while in other directions the BER is about 0.5, indicating that it would be extremely difficult for eavesdroppers located in these regions to crack the information. Moreover, design examples with and without group sparsity were shown in comparison with each other, further demonstrating the effectiveness of the proposed formulations.

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