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Multi-objective Fuzzy Rule-Based Prediction and Uncertainty Quantification of Aircraft Taxi Time

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Abstract—The ever growing air traffic demand and highly connected air transportation networks put considerable pressure for the sector to optimise air traffic management (ATM) related performances and develop robust ATM systems. Recent efforts made in accurate aircraft taxi time prediction have shown significant advancement in generating more efficient taxi routes and schedules, benefiting other key airside operations, such as runway sequencing and gate assignment. However, little study has been devoted to quantification of uncertainty associated with taxiing aircraft. Routes and schedules generated based on deterministic and accurate taxi time prediction for an aircraft may not be resilient under uncertainties due to factors such as varying weather conditions, operational scenarios and pilot behaviours, impairing system-wide performance as taxi delays can propagate throughout the network. Therefore, the primary aim of this paper is to utilise multi-objective fuzzy rule-based systems to better quantify such uncertainties based on historic aircraft taxiing data. Preliminary results reveals that the proposed approach can capture uncertainty in a more informative way, and hence represents a promising tool to further develop robust taxi planning to reduce delays due to uncertain taxi times.

I. Introduction

The continuous growth of air traffic demand on highly connected air transportation networks makes efficient and robust air traffic management (ATM) systems a priority in the air transportation sector. Recently, the research on deterministic aircraft taxi time prediction [1]–[3] has shown promising results which can be utilised for more efficient airport ground movement planning and related operations such as runway sequencing and gate assignment. However, as various factors affect ground movement such as weather conditions, operational scenarios, airport layouts, runway crossing and pilot behaviours, routes and schedules generated based on deterministic and accurate taxi time prediction may not longer be robust. The situation is further compounded in the context of the prevailing hub-and-spoke network structure due to deregulated airline market. In this case, local airport ATM-related inefficiency may have a system-wide impact because taxiing delays may be transmitted throughout the network [4], causing delays and disruptions at both downstream and upstream airports. As a result, the need for quantification of uncertainty associated with taxiing aircraft is apparent.

In our previous works, we have shown that an adaptive Mamdani Fuzzy Rule-based System (FRBS) [1] can be used for accurate aircraft taxi time prediction while preserving certain transparency in the rule base. A detailed comparison [2] between TSK and Mamdani FRBSs with more conventional statistic regression approaches [3], [5]–[9] has shown the superiority of using FRBSs for more accurate predictions. Some of the key aspects of the adaptive Mamdani FRBS are the following: (1) The FRBS is well suited for modelling complex non-linear systems due to several rules with nonlinear membership functions describing the investigated system at the same time. (2) Different regions of the explanatory variable space can be described by different rules. (3) Human expertise can be integrated into the FRBS in a form of rules elicited from airport practitioners. (4) The meaning of rules in the FRBS can be interpreted via linguistic terms involved in rules. (5) The adaptive Mamdani FRBS uses a membership function in the consequent part which expresses certainty of the prediction and thus provides a means for uncertainty quantification. It should be noted that in the previous research [1], [2] the only objective taken into account is to improve prediction accuracy. On the one hand, this may sacrifice the transparency of the rule base, causing problems in convincing airport practitioners to accept the rule base as they will not be able to validate the rule base using their domain knowledge. On the other hand, this may lead to an overall output membership function with a high certainty over a wide support, merely delivering any useful information for uncertainty quantification.

In light of this, in this paper, we endeavour to utilise a multi-objective FRBS based approach to simultaneously improve prediction accuracy and quantify uncertainty. The philosophy behind the approach is that uncertainty can be better captured if certain prediction accuracy is achieved. This philosophy is implemented in the framework of multi-objective optimisation so that the parameters of FRBS are tuned to achieve high prediction accuracy, while at the same time, only moderate fuzziness is allowed for the input space. The implication of the second objective is that there should be a dominant rule with a firing strength close to unity that accounts for the final output, which is achieved in this paper through FRBS structure simplification and a specially devised second objective function. The

aim is to preserve input membership functions from too much overlap and make the overall output membership function more informative, suitable for uncertainty interpretation.

The remainder of the paper discusses the airport ground movement problem and the proposed multi-objective FRBS based approach in Section II. Section III-A describes the utilised datasets and experimental setup; Preliminary results are included in Section III-B and conclusions drawn in Section IV.

II. PROBLEM DESCRIPTION AND METHODOLOGY

A. Problem description

A FRBS for taxi time prediction takes n input variables (explanatory variables) for accurate prediction of taxi times. The FRBS uses 15 explanatory variables, which were identified in [3] and are as follows: airport operating mode (single/two runways in use), type of movement (arrival/departure), total taxi distance and its logarithmic transformation, distance on straight segments, total turning angle along the route and its logarithmic transformation, whether a push-back manoeuvre was performed and the N and Q number of moving aircraft. N represents the number of other aircraft taxiing when the aircraft under consideration starts to taxi. Q represents the number of other aircraft which stop their taxiing during the ground movement of the aircraft under consideration. N and Q are further broken down according to the type of movement (arrival/departure), resulting in 8 variables in total.

B. Adaptive Mamdani FRBS for accurate prediction

For prediction of taxi times, an adaptive Mamdani FRBS is used in this work. The Mamdani-type FRBS for the taxi time prediction is defined as a set of fuzzy if-then rules R_i:

If x_1 is H_i^1 and x_2 is $H_i^2, \ldots,$ and x_j is $H_i^j, \ldots,$
$\ldots$ and x_n is H_i^n Then $y_i = Z_i,$

with values of explanatory variables x_j $(j = 1, 2, \ldots, n)$ as inputs and y_i as the output of the i-th rule. Each input has a linguistic value (fuzzy set) H_i^j associated with it. Z_i is the consequent of the i-th rule, and is defined as the fuzzy set B_i.

Fuzzy set H_i^j is defined in (1) as a Gaussian membership function $\mu_{H_i^j}(x_j)$ for all of the explanatory variables. Fuzzy set B_i is a bell-shaped membership function $\mu_{B_i}(y)$ for the consequents, as defined in (2):

$$\mu_{H_i^j}(x_j) = \exp[-\frac{1}{2} \cdot (\frac{x_j - c_i^j}{\sigma_i^j})^2], \qquad (1)$$

$$\mu_{B_i}(y) = \frac{1}{1 + (\frac{y - c_i^y}{\sigma_i^y})^2}. \qquad (2)$$

where c_i^j and σ_i^j are the centre and the spread of the i-th membership function of the input. Similarly, c_i^y and σ_i^y denote the centre and spread of i-th membership function of the output.

With the link between fuzzy set H_i^j and membership function $\mu_{H_i^j}(x_j)$ and similarly between B_i and $\mu_{B_i}(y)$, each rule can be expressed as linguistic terms, e.g.

If taxi $distance$ is long, and $aircraft$ is departing and
$\ldots$, Then $taxi\ time$ is long.

The defuzzified output y^{crisp} of the Mamdani FRBS with r rules for input X can be calculated as follows:

$$y^{crisp} = \frac{\sum_{i=1}^{r} c_i^y \cdot \mu_i(X) \cdot \int_y \mu_{B_i}(y)\, \mathrm{d}y}{\sum_{i=1}^{r} \mu_i(X) \cdot \int_y \mu_{B_i}(y)\, \mathrm{d}y}, \qquad (3)$$

where $\mu_i(X)$ represents the degree of certainty for a data sample associated with the i-th rule and is defined in (4).

$$\mu_i(X) = \mu_{H_i^1}(x_1) \cdot \mu_{H_i^2}(x_2) \cdot \ldots \cdot \mu_{H_i^j}(x_j) \cdot \ldots \cdot \mu_{H_i^n}(x_n) \quad (4)$$

The overall implied fuzzy set $\hat{B}$ which represents the output membership function is defined in (5):

$$\mu_{\hat{B}}(y) = \mu_{\hat{B}_1}(y) \oplus \mu_{\hat{B}_2}(y) \oplus \ldots \oplus \mu_{\hat{B}_i}(y), \quad i = 1, 2, \ldots, r, \qquad (5)$$

where the probabilistic OR operator is used for $\oplus$ operation. This way, by employing the adaptive Mamdani FRBS, we can obtain not only accurate predictions for the taxi time, but also the membership function which conveys uncertainty information.

The parameter vector $\theta = (c_i^j, \sigma_i^j, c_i^y, \sigma_i^y)$ defines the Mamdani FRBS and determines its prediction capability of the output y^{crisp}. The initial values of θ are derived in this study by applying a clustering algorithm [10]. Furthermore, θ is fine-tuned with a back-error propagation (BEP) algorithm [11] in order to improve the accuracy of prediction. For details of the BEP algorithm, readers are referred to [11].

C. Multi-objective Mamdani FRBS for uncertainty quantification

The fine-tuned initial values of θ can be further tuned with respect to two conflicting objectives, i.e. FRBS prediction accuracy and interpretability. The first objective f_1 focuses on prediction accuracy and is defined in (6) as a root mean square error of the predicted values y_m^{crisp} and real values y_m^{real} for $m = 1, 2, \ldots, M$ data samples. There are different measures that can be used to express interpretability of FRBS. An overview of measures can be found in [12]. In this work, the number of fired rules is used and expressed as the second objective f_2:

$$\min f_1 = \sqrt{\frac{\sum_{m=1}^{M}(y_m^{crisp} - y_m^{real})^2}{M}}, \qquad (6)$$

$$\min f_2 = M - \sum_{m=1}^{M}[max(\mu_i(X_m)) - max_2(\mu_i(X_m))], \quad (7)$$

where X_m is the m-th data sample and $max(\cdot)$, $max_2(\cdot)$ are functions which return the largest value and the second

largest value respectively. If $\mu_i(X_m)$, which is the degree of certainty of the m-th data sample associated with the i-th rule, is high, then the i-th rule is highly relevant. Ideally, $\mu_i(X_m)$ should be high for only one rule, while for other rules it should be low. In such case, data sample X_m is predominantly described by only one rule, and FRBS is interpretable (informative) and may quantify uncertainty more precisely. The term $[max(\mu_i(X_m)) - max_2(\mu_i(X_m))]$ in (7) maximises the difference between the highest $\mu_i(X_m)$ from $i = 1, 2, \ldots, r$ rules and the second highest $\mu_i(X_m)$ and promotes the maximum difference between the degrees of certainty. This should be true for all $m = 1, 2, \ldots, M$ data samples and is calculated as a sum. Finally, the sum is subtracted from M to convert the objective into a minimisation one.

For the search, a Population Adaptive Immune Algorithm (PAIA) [13] and an immune inspired multi-objective fuzzy modelling (IMOFM) framework [11] are employed. IMOFM is a evolutionary search algorithm with specialised operators for FRBS structure simplification to remove the redundancy both in the initial rules and fuzzy sets which are the results of Section II-B. The following simplification steps are performed:

1) Removing Unimportant Rules,
2) Merging Similar Rules,
3) Removing Universal Fuzzy Sets
4) Merging Similar Fuzzy Sets.

For detailed description of the steps, readers are referred to [11].

As rule base parameters θ and its structure are tuned at the same time, a link between the two is needed for effective search. Without this link between the parameter and structure representation, the search process is unaware of vital structural information which is constantly modified during the evolution and leads to inefficient search performance. Therefore, a link consisting of two matrices $FISmap$ and $RULE$ is created. For every parameter in θ, $FISmap$ stores the corresponding identification number of each membership function. $RULE$ indicates which rules are active in the rule base. After simplification steps, $FISmap$ and $RULE$ are updated in order to reflect the current status of the rule base.

The function of $FISmap$ and $RULE$ is illustrated in Fig. 1. As an example, suppose, that fuzzy sets A_1^1 and A_1^3 are similar and are combined into a single set $\breve{A}_1^1$ by the merging similar membership functions step. If there is no link between the FRBS structure and its parameter coding representation, during evolution, $\breve{A}_1^1$ in R_3 may be changed by mutation into different fuzzy set, while $\breve{A}_1^1$ in R_1 stays the same, creating two different rules R_1 and R_3. With the $FISmap$ link, both mutation points are mutated together, preserving the FRBS structure. Similarly, information about active rules in $RULE$, prevents inactive rules from taking part in crossover and mutation.

III. COMPUTATIONAL RESULTS

A. Data instance and experimental setup

The FRBS framework and optimisation algorithm described in Section II were applied on a set of ground movements from
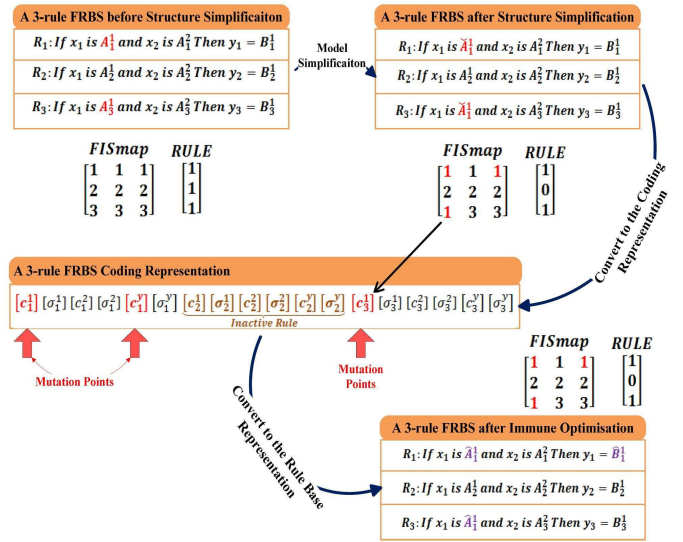

Fig. 1. Illustration of $FISmap$ and $RULE$ functionality during the evolution.

Manchester Airport, UK, which is the third busiest airport in the UK. The taxiway layout is shown in Fig. 2. The airport has two runways and is operated in two operating modes: during busy period, one runway is used for arrivals and the other one for departures, while in less busy period only one runway is used for both arrivals and departures.
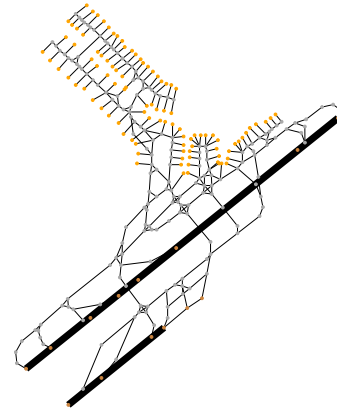

Fig. 2. A graph representation of the airport surface for Manchester Airport.

The ground movement data used for training and optimisation of FRBS was gathered from freely-available data on the website FlightRadar24.com (FR24), with specialised tools described in [14]. The dataset contains 1413 ground movements in total which were recorded during 5–12 November 2013. The data was randomly divided into 2/3 training and 1/3 checking data sets. Only training data was used for obtaining the initial FRBS and the refined FRBS via multi-objective optimisation.

IMOFM was run for 600 generations based on the initial experiments. Other parameters specific to PAIA are the same as in [11].

TABLE I
TRADE-OFF FRBSS.

	Rules	f_1	f_2	3 min	5 min
Training					
FRBS A	10	1.97	574.04	88%	95%
FRBS B	5	3.65	0.17	67%	86%
FRBS C	5	2.31	136.81	86%	96%
Checking					
FRBS A	10	2.63	259.48	82%	95%
FRBS B	5	3.92	0.35	67%	86%
FRBS C	5	2.68	69.98	82%	95%

TABLE II
AVERAGE INDICATOR VALUES FOR 15 RUNS OF ALGORITHMS.

	I_H	I_{GD}	min f_2
IMOFM	1333.5	0.3487	0.2329
IMOFM without $RULE$	**1269.0**	0.3516	**1.7346**
IMOFM without $FISmap$	**1315.9**	0.4017	**1.0437**

B. Results

As described in Section II, firstly, an initial FRBS was obtained using the clustering and BEP algorithms. The predicted taxi times by the initial FRBS was checked with the checking data with $f_1 = 2.63$, $f_2 = 266.26$. For ground movements, another measures of accuracy of taxi times within 3 and 5 minutes are also used [5]. For the initial FRBS, 81% of movements were accurate to within 3 mins and 94% were accurate to within 5 mins.
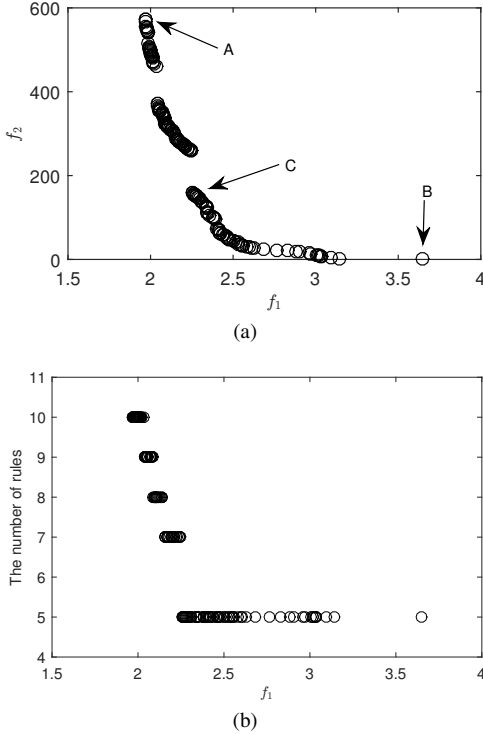

(a)

(b)

Fig. 3. Pareto fronts .

Fig. 3a shows the Pareto front obtained by IMOFM. As can be seen, there is an apparent trade-off between f_1 and f_2, i.e. accuracy and interpretability. This is also supported in Fig. 3b, where a trade-off between accuracy (f_1) and the number of rules (associated with interpretability) can be observed. Table I summarises the objective function values and 3 and 5 min. accuracy for the selected FRBSs (as indicated in Fig. 3) obtained by IMOFM.

For the purpose of comparing Pareto fronts of different algorithms, performance indicators for multi-objective optimisation are used, namely: hypervolume I_H, generational distance I_{GD} and the minimum value of f_2. As the initial solution is generated subject to minimisation of f_1, the real benefit of the proposed IMOFM is in area of objective space with small f_2 (as this is where the initial FRBS structure has been mostly perturbed). Therefore, the minimum value of f_2 can be employed to reflect the performance of IMOFM. For I_H, larger values are preferred, whereas smaller values of I_{GD} and f_2 are better. Table II summarises average indicator values for 15 runs of IMOFM with and without the link between the FRBS parameters and structure, i.e. $FISmap$ and $RULE$. With the link, the algorithm obtained better results for all performance indicators. Statistically significant results (compared to IMOFM) calculated by t-test at the 5% significance level are in boldface.

Fig. 4 shows membership functions $\mu_B(y)$ for the consequent, i.e. taxi time, the output membership function $\mu_{\hat{B}}$ and the defuzzified output y^{crisp} for a single data sample using FRBSs A, B and C. As can be seen, interpretability of the FRBSs has been improved for FRBSs B and C, compared to A. FRBSs B and C have less rules and overlap among membership functions. Furthermore, the output membership function $\mu_{\hat{B}}$ for FRBS A provides little information about uncertainty, as for a large interval of taxi time, the certainty is relatively high. In contrast, for FRBS B and C, a distinct peak in $\mu_{\hat{B}}$ can be seen. This is due to inclusion of f_2 in multi-objective optimisation, which results in only one rule being fired predominantly. Also, it can be noted that FRBS B, the defuzzified output is different from those of FRBSs A and C, as accuracy is low for FRBS B.

IV. CONCLUSION

To the best of our knowledge, this paper represents the first attempt to utilise a multi-objective FRBS based approach for accurate prediction of aircraft taxi times and their associated uncertainty. The results show that by simultaneously simplifying the structure of FRBSs and tuning associated membership function parameters, we kill two birds with one stone. Firstly, prediction accuracy of all simplified FRBSs is maintained. This is to ensure that all FRBSs are credible models. Secondly, as the structure of the FRBS is simplified, gradually, only one predominate rule in the simplified FRBS accounts for one taxiing scenario. The resulting overall output membership function will have high certainty at the defuzzified value (i.e. the predicted taxi time) and a support which gives good quantification of uncertainty.
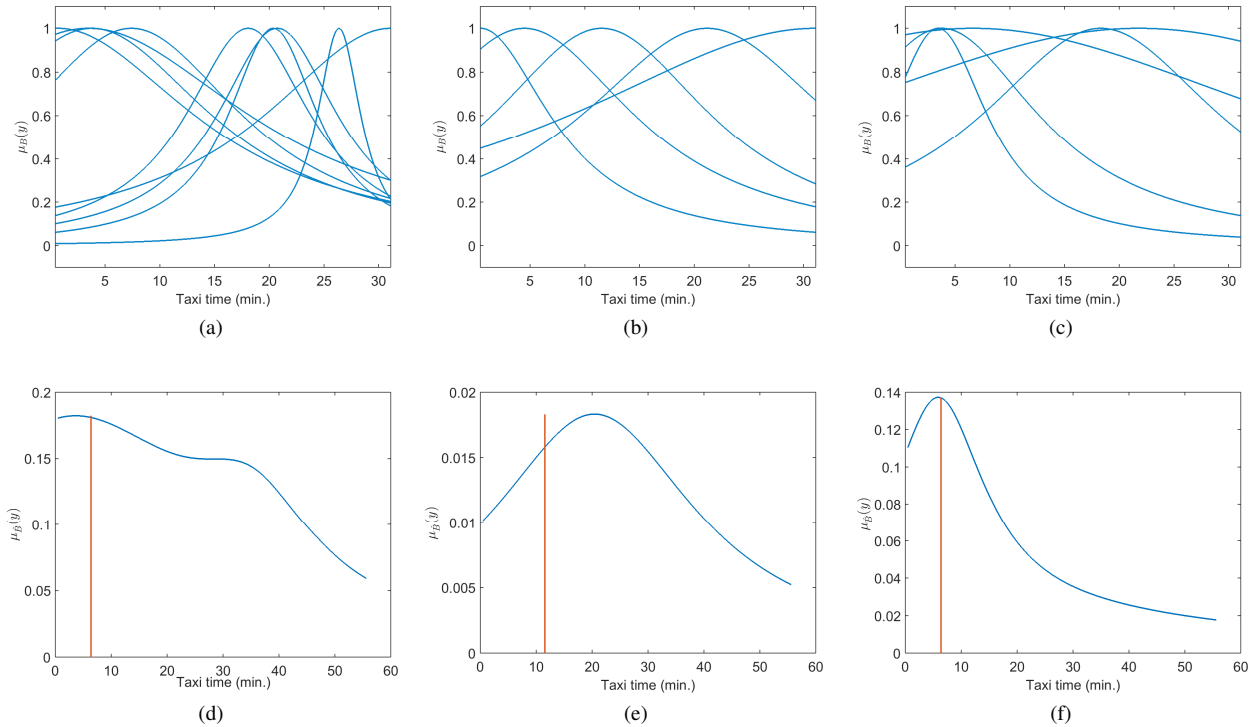
Fig. 4. Membership functions $\mu_B(y)$ for consequent for FRBS A (a), FRBS B (b), FRBS C (c), and the output membership function $\mu_{\hat{B}}$ for a single data sample for FRBS A (d), FRBS B (e), FRBS C (f). The vertical line represents the defuzzified output y^{crisp}.

Building upon this work, we believe, that the proposed approach could be further utilised in developing robust taxi plans by proactively incorporate such uncertainty in the routing and scheduling module. As multiple trade-off FRBSs with varied capability in prediction accuracy and uncertainty interpretation are obtained as the result of IMOFM, further investigation are needed to decide which FRBS should be included in the routing and scheduling module.

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