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NEED ANYTHING FOLLOW FROM A CONTRADICTION?

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ABSTRACT. Classical and intuitionistic logic both validate Ex Contradictione Quodlibet (ECQ), according to which any proposition whatsoever follows from a contradiction. Many philosophers have found ECQ counter-intuitive, but criticisms of the principle have almost universally been directed from a position of support for relevance or other orthodox paraconsistent logics, according to which some, but not necessarily all, propositions follow from a contradiction. This paper draws attention to the historically significant view that nothing whatsoever follows from a contradiction - Ex Contradictione Nihil (ECN). It addresses the key arguments - model-theoretic, proof theoretic, meaning theoretic and holist - deployed by proponents of ECQ against their standard opponents, and shows that each can be defeated from a position of support for ECN. In the course of answering these arguments some understanding is developed of possible motivations for ECN and a solid case is built that ECN deserves further investigation.

Nobody who has taught elementary logic can have escaped student objections when teaching *Ex Contradictione Quodlibet* (ECQ), the metatheorem of classical and intuitionistic logic stating that any proposition whatsoever follows from a contradiction.[1] 'That doesn't make sense', comes the complaint, 'it's absurd'. And however familiar one might be working with ECQ, however adept at responding to students (in one or more of the ways surveyed below), the suspicion remains that there is substance to the protest. Surely, it doesn't follow *as a matter of logic* from an arbitrary contradiction that the moon is made of cheese.[2]

In the recent past ECQ has had to defend itself against objections from proponents of mainstream paraconsistent logics [19][20].[3] These logics allow that, for a given contradiction, *some* propositions may follow logically from that contradiction, but that it needn't be the case that *every* proposition follows logically from it.[4] Typically, for instance, P is a consequence of P together with $\neg P$, whereas an arbitrary Q is not.[5] Historically, by contrast, ECQ developed in opposition to the view that *nothing whatsoever* follows from a contradiction. Arguably present in Aristotle, and certainly supported by Boethius and Abelard, this latter position – which we might call *Ex Contradictione Nihil* (ECN) – is rarely considered in contemporary debates about logical consequence. A small exception is provided by the literature on connexive logics [27].[6] But the exception genuinely is a small one, and someone surveying the state of philosophical logic might be forgiven for thinking that standard

Key words and phrases. contradictions, ex contradictione quodlibet, paraconsistency, logical consequence.

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[1]By using the word 'proposition' here I don't mean to commit myself to any particular view on the nature of the relata of the consequence relation. It should be viewed as a placeholder, designating whatever kind of things stand in that relation.

[2]The instinct that something is 'up' with ECQ persists at the highest levels of logical competence. So, for example, during much of 2015 there was a discussion thread on the *Foundations of Mathematics* e-list entitled 'The Unbearable Ghastliness of EFQ' ('Ex Falso Quodlibet' functioning as a practical synonym for ECQ.).

[3]*Terminology:* A paraconsistent logic is any logic which does not license ECQ. It follows that a logic in which *nothing* follows from a contradiction is paraconsistent. However the mainstream of paraconsistent logic works with systems that have as a metatheorem $\exists\phi\ (\psi \wedge \neg\psi) \vdash \phi$

[4]This needn't be ruled out, however. Any theory of **LP**, for example, has the trivial model, in which every proposition is true.

[5]I will not be fussy here about whether contradictions are understood as a pair of propositions $P, \neg P$ or as single conjuctive propositions $(P \wedge \neg P)$. I'll work with paired-premise contradictions, but of course an application of $\wedge E$ will convert any one of these into a conjoined contradiction.

[6]See [18] for explicit discussion of ECN in the context of connexive logic.

paraconsistency was the only alternative to acceptance of ECQ.

There are good, as well as bad, reasons for this. Standard paraconsistency, typified in the logic **LP**, has clear motivations and relates naturally to a dialethic response to semantic and other paradoxes [1][19]. It is perhaps less immediately apparent what might motivate acceptance of ECN. In what follows I will go some way towards suggesting reasons in support of the position. That said, the main purpose of this paper is to assess arguments in favour of ECQ from the perspective of belief in ECN. It will turn out that the proponent of ECN has available to her counter-arguments to every argument for ECQ we consider. It follows then that there is an under-explored alternative to explosive consequence which ought to be taken more seriously. The take-home message is that ECN, its formal implementation, motivations, and merits deserve further examination.

The paper proceeds by considering one-by-one arguments for ECQ. In each case they are responded to from the perspective of belief in ECN. §1 considers the well-known proof often attributed to C.I. Lewis. §2 looks at a case for ECQ from an understanding of logical consequence in terms of necessary truth-preservation (as well as discussing model-theoretic accounts). §3 asks whether ECQ can be supported on the basis of a requirement of proof-theoretic harmony, whereas §4 looks at theoretical-holist reasons for accepting ECQ. In each instance, the case for ECQ is found wanting given an antecedent commitment to ECN.

It might be helpful to say something about the dialectical structure of what follows. Each argument for ECQ is met on its own terms with a possible reply on behalf of the ECN-ist. This is all that is needed to defeat the strong claim that there is an irresistible case for ECQ. I will suggest some philosophical support for the positions from which ECN is defended, but I am under no illusion that this is watertight as it stands. Nor am I committed towards the positions of the ECN-ists defended in each section being pairwise coherent; certainly there are issues about the relative priority of proof-theoretic and truth-theoretic understandings of consequence which are left unresolved.[7] These matters are the subject of ongoing work. The purpose of the present paper is modest; to show that ECN gets off the starting blocks against ECQ, motivated by the standard arguments, whilst hinting towards some philosophical reasons ECN might be thought attractive. The conclusion is that ECN deserves more attention than it currently receives.

1. The Lewis Proof

Commonly attributed to C.I. Lewis, the following proof appears in *Symbolic Logic*, co-authored with Langford [13, 248-251]. Taking P and $\neg P$ as premises:

(1)	P	Premise
(2)	$\neg P$	Premise
(3)	$(P \vee Q)$	$(\vee\text{I}, 1)$
(4)	Q	$(3, 2, \text{DS})$

Here *DS* is disjunctive syllogism, which is a derivable rule in any formulation of classical or intuitionistic logic. In actual fact the argument dates back to the middle ages, being clearly present in fourteenth century discussion but, on a plausible reading of John of Salisbury's *Metalogicon*, attributable to William of Soissons in the twelfth [14]. This is significant since it situates the first known appearance of the proof amongst the Parisian schools of the twelfth century, and in particular amongst the Parvipontanians, of whom William was a member: his teacher, Adam Parvipontanus, advocated ECQ against the Abelardian acceptance of ECN.[8] The Lewis proof belongs therefore to the early history of ECQ.

[7] For the record, my sympathies here lie firmly in the proof-theoretic camp.

[8] On other advocacy of ECQ (or more generally, ex *impossibili* quodlibet) in the twelfth century, see [10].

The proof is classically valid and suffices to licence any instance of ECQ.[9] Any attempt to resist this conclusion will therefore require rejection of at least one classically acceptable rule, or else the introduction of new constraints (on which more below). Some relevance logicians have seen $\vee$-introduction as the guilty party (after all, this is the point at which apparently irrelevant material is imported into the argument), whereas others have focused on disjunctive syllogism. Rejecting the transitivity of entailment will also suffice to block the proof.

All of these routes to blocking the Lewis proof are individually consistent with contradictions having consequences (and thus with the non-obtaining of ECN). And indeed proponents of relevant, mainstream paraconsistent, and non-transitive logics do allow that contradictions have consequences (typically, for instance, they will want to allow that P follows from P together with $\neg P$). They therefore find themselves in a significantly different dialectical situation with respect to the Lewis proof from the proponent of ECN (who we will call the *ECN-ist*). Since a relevant logician, say, accepts that there are genuine proofs which make use of both constituent propositions of a contradiction, she needs to explain at which point the Lewis proof takes a step which prevents it from counting as a genuine proof. (Inevitably these kind of discussions oscilate between two related senses of the word 'proof': (1) proof in some formal system, (2) proof which is valid *as a matter of logic*).

From the perspective of antecedent acceptance of ECN things look different. Presented with the Lewis argument she can simply reject it on the basis that it has contradictory premises. She does not therefore need to locate some stage at which the argument goes wrong (and thereby some rule which is unacceptable). Something went wrong when we tried to reason from a contradiction.

To make this clearer, imagine that the ECN-ist takes a standard natural deduction system for classical logic, such as that developed in [12], and takes it to be constrained by the following rule:

> **EC-R:** Once both of a contradictory pair of propositions have been written by themselves on lines of a proof,[10] except within the scope of assumptions on lines depending only on undischarged assumptions, nothing more may be written. The proof may not be cited as a proof of any proposition.

This implementation of the ECN-ist position will no doubt need finessing, but will do for present purposes. In particular, EC-R allows us to see why the ECN-ist will be untroubled by the Lewis proof. The moment the second line has been written down, she will invoke EC-R to the effect that nothing may be proved on this basis. And in general, anyone who holds that nothing follows from a contradiction ought to take the deployment of the Lewis proof against them as question-begging, since the Lewis proof operates by presenting us with a supposed case of something following from a contradiction.

Initially, then, the ECN-ist who incorporates EC-R into her logic seems to have no reason to be moved by the Lewis proof, since in her own terms it does not so much as get off the deductive tarmac. We ought to ask, however, whether there could be a credible motivation for accepting EC-R. We ought to ask, moreover, what kind of rule EC-R is supposed to be. It does not look like an operational rule, yet to call in structural might seem dubious since it makes tacit appeal to a particular connective (what it is to be a contradiction is defined in terms of negation). In what remains of this section, I will suggest one way of answering the first question which permits us to view the second

[9] Since the dispute between classical and intuitionist logicians is not our concern, I'll talk from now onwards simply about classical deductive validity, without prejudice to debates with advocates of intuitionistic or intermediate logics.

[10] The 'by themselves' constraint rules out, e.g. the propositions featuring as the consequents of conditionals counting towards invocation of EC-R. More interestingly, it also rules out propositions of the form $(\phi \wedge \neg\phi)$. EC-R will never allow such a proposition to be *introduced*, but it will e.g. allow them to be worked with as premises. Thus the logic resulting from adding EC-R to classical logic proves $(P \wedge \neg P) \vdash P$. This tolerance might be thought an advantage of working with an understanding of contradictions as pairs of propositions, rather than as conjunctions. On the other hand, it might be doubted that it captures perfectly the thought that *nothing* follows from a contradition. I'd want to defend it against this charge: it is only in a pickwickian sense that P follows from the contradiction as such. It follows from P simpliciter, and so in particular follows from the conjunction of P with anything, even though the conjunct does not feature in the subsequent deduction.

in a new light.

Suppose that our ECN-ist adopts what has been called a *rejectivist* account of the relationship between negation and the speech-act of denial [24]. That is, rather than following Frege and Geach and explaining denial in terms of negation (to deny that P is to assert that $\neg P$), she instead turns the explanatory tables, explaining negation in terms of denial [6][7]. On this view of things, an adequate understanding of the contribution negation makes to the meaning of a proposition requires an antecedent understanding of a primitive speech-act of denial. The canonical conditions under which a speaker is entitled to assert that $\neg P$ are just those under which she is entitled to deny that P. Here there is formal agreement between the rejectivist and the Frege-Geach position, but there is a deeper disagreement. The rejectivist thinks that a grasp of P's denial conditions is prior to a grasp of the assertion conditions for $\neg P$. This view is very naturally cashed out in terms of a bilateral semantics for the language (as in [25]), with the gloss that to fully understand P a speaker needs to grasp both its assertion and its denial conditions. I develop this line of thought in N

How does rejectivism help our ECN-ist? Assertion and denial are incompatible:[11] a speaker cannot both assert and deny the same proposition simultaneously. Moreover an inferential position in which she is committed to both assert and deny the same proposition is simply unintelligible.[12] The unfortunate reasoner who finds herself in such a situation can go no further. Her deductive activity has brought her to a point where she can go no further. She has reached, to use an image of Rumfitt's, a 'full stop' in her reasoning [25]. We should take the image with full seriousness. In order to engage in argument, one needs to have intelligible commitments with respect to assertion and denial. Once one has made argumentative moves which give one an explicit commitment to assert and deny the same proposition, one no longer has intelligible commitments and the business of arguing can go no further.[13]

More would be needed to say to adequately motivate and defend from objections this theoretical underpinning of EC-R. But suppose that we accept it. Then we can adopt a new perspective on the question whether EC-R is a structural or an operational rule. In the background is an assumption that every rule of inference is clearly one or the other. This acquires philosophical significance particularly if one holds (as do I) that the statements of rules of inference fix the meaning of logical expressions. Operational rules – let's consider the introduction and elimination rules of a natural deduction system – dictate how we are to reason with particular connectives and, on one familiar account, exhaustively determine the meaning of the associated connectives. Structural rules, meanwhile, are not concerned with particular connectives, but rather state general permissions for transforming one proof into another. So, for example, in a single-conclusion context the standard rules, formulated Gentzen-style, will be:

$$\frac{\Gamma \vdash \phi}{\psi, \Gamma \vdash \phi}\ \text{(Weakening)}$$

$$\frac{\Gamma, \phi, \psi \vdash \chi}{\Gamma, \psi, \phi \vdash \chi}\ \text{Permutation}$$

[11] Note that even dialetheists are likely to concede this [19].

[12] What *commitment* amounts to here requires fleshing out, of course, but my present purpose is simply to sketch one possible motivation for adopting EC-R. My own favoured approach to commitment is to view it in terms of what an agent must accept in a dialogal situation of the sort appealed to in [16].

[13] Isn't this too quick? Mightn't a reasoner merely be supposing the premises, or entertaining them in a third-personal manner about a hypothetical reasoner? Undoubtedly, and suppositional reasoning will be discussed below. In the background of my defence here is the following view: there is a paradigm case of deductive reasoning, which is a human, dialogal [16], practice in which premises are asserted – put forward for consideration – and the practical commitments they bring with them assessed. This practice is the primary subject matter of logic. There might be other nearby subjects of study for which the situation of ECQ is less clear. That is fine: our ECN-ist can allow that ECQ obtains regarding *schmlogic*, given only that she has a decent case that logic doesn't validate the rule! Clearly there is much more to be said here; I refer readers to the final paragraph of the opening section and thank an anonymous referee for discussion.

$$\frac{\Gamma, \phi, \phi \vdash \psi}{\Gamma, \phi \vdash \psi}\ \text{Contraction}$$

We can remove one or more of these to obtain *substructural* logics [23].[14]

EC-R looks neither obviously operational nor obviously structural. It makes irreducible mention of a particular connective, like an operational rule, yet concerns the general shape of arguments, like a structural rule. It seems wrong to say that EC-R has any affect on the *meaning* of negation;[15] the rules governing them are the same as they always were. The temptation to suppose otherwise issues from the thought that some classically available inferential moves are no longer permitted. This is true, but this is not because any particular operational rule has been restricted (e.g. Disjunction introduction blocked, or quantifier rules). In particular, the rules for negation remain unaltered: the elimination rule being classical *reductio* (or ECQ itself in the intuitionistic case). What EC-R does is not to modify the meanings of connectives but to regulate how we may reason, given the antecedently given meanings of those connectives. This regulation proceeds by blocking any chain of reasoning which would involve a speaker in commitments to incompatible speech-acts. It is therefore an instance of a distinctive type of rule, neither structural nor operational, but serving to safeguard the coherence of a speaker's pragmatic commitments.

A sceptic might think this is too good to be true from the ECN-ist's perspective.[16] EC-R mentions negation. Aren't we simply stipulating that it isn't, in the phraseology of the previous paragraph, one of the *rules for negation*? No. To see why not consider the case of Marian the logic prodigy. Taught a (classical) natural deduction system whilst still in the cradle, she works through proofs all day long. She is proficient at applying the rules again and again, correctly, and proving theorems. If anybody knows how to apply the rules, Marian does. Now, one day, Marian stumbles upon a problem. Choosing premises at random, for the purposes of recreational proof, she accidentally writes down 'Trump is a genius' and 'It is not the case that Trump is a genius'. She is taken aback, and not for the obvious reason. For Marian is no mere mechanical theorem prover. The sentences she works with are meaningful parts of a language and encode pragmatic commitments, commitments to assert and deny in an argumentative context. Her proofs are representations of *how arguments could go*, between speakers[17] or (derivitavely) within her internal monologue. But no argument, she reasons, could go any way whatsoever from those premises. Committed simultaneously to assert and deny the same proposition, a speaker (or at least a competent, reflective speaker, which is the sort of speaker we have in mind when philosophising about logic) would be reduced to silence. 'Ok', Marian says to herself, 'we won't allow reasoning from contradictory premises'. Now, prior to making this stipulation, Marian clearly understood negation.[18] It was because she understood negation that she was appraised of the difficulty with her premises. Nor was her understanding partial; she knew prodigiously how to reason with negation in all circumstances in which reasoning could take place at all. Her introduction, after her mistake in writing premises, of EC-R is merely an instance of a meta-rule: *do not reason in ways which issue in incomptaible pragmatic commitments*. Negation is particularly prone to be implicated in such ways of reasoning, because of its internal relation to a

[14]We'll be able to prove cut-elimination and derive every instance of identity for intuitionistic logic with the rules provided. Note that Rumfitt takes the usual structural rules to be definitional of an implication [26, Ch. 1]. Whether the standard structural rules are preserved in a logic containing ECN, and to what extent, is going to be highly formulation-sensitive. So, for example, if contradictions are taken to be of the form $(\phi \wedge \neg\phi)$, such that further reasoning is blocked once a wff of that form has been entered in a proof, then identity, weakening and cut can easily be seen to fail..

[15]Here I'm assuming, in Dummett's terms, a molecular rather than holist theory of meaning [3].

[16]Grateful thanks to Nick Jones for tokening such scepticism.

[17]See [15].

[18]I can imagine an objection from a position influenced by the later Wittgenstein on mathematics – the meaning of negation *changes* as Marian tries to execute her proof and introduces a rule. I find this implausible – the meanings of logical vocabulary items must be minimally stable in order for us to be able to reason coherently. c.f. 'One would like to say: the proof changes the grammar of our language, changes our conceptions, makes new connexions…' [29, 78].

particular speech-act, but this does not make EC-R, as a particular instance of the meta-rule, constitutive of the meaning of negation.

The ECN-ist then *agrees* with the classicist about the meaning of negation. It is just that she thinks that the classicist makes illicit inferences because of a failure to fully grasp that meaning in practice and take on board the commitments which flow from an assertion of a negated proposition. The Lewis proof is one such illicit inference and the ECN-ist is therefore unmoved by it.

2. Necessary Truth Preservation

One popular and historically venerable analysis of logical consequence has it that,

> **NEC-TP:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ it is not possible for every element of Γ to be true and ϕ false.

This analysis provides a basis for motivating ECQ. For it is simply not possible that both of P and $\neg P$ be true, and so it is not possible for P together with $\neg P$ to be true and Q false, for arbitrary Q. ECQ follows immediately by (NEC-TP).

As an argument for ECQ this is only as good as the necessary truth-preservation analysis of consequence. So one option open to the supporter of ECN is to reject that analysis. For all its impressive ancestry, it is far from unproblematic. It is, after all, not possible that an object be red all over and yellow all over at the same time, or that a bachelor be married, yet it seems wrong to say that these propositions follow from any other as a matter of *logic*, as (NEC-TP) suggests.[19] What is missing is a constraint that necessary truth-preservation be in virtue of logical form alone. The notion of logical form is fraught with controversy, intimately related as it is to issues around the nature and identification of logical vocabulary, and here is not the place to resolve those. However it would be unwise for the ECN-ist to rest on her laurels on the assumption that necessary truth-preservation is an inadequate analysis of logical consequence and that it therefore provides no basis for accepting ECQ. The acceptability of this move turns on deep questions in the philosophy of modality, around the answers to which there is no consensus: what is the relationship between logical necessity and metaphysical and analytic necessities? Is there a modality 'wider' than metaphysical modality in terms of which it *is* possible for an object to be both red all over and yellow all over at the same time? If the answer to this latter question were affirmative then the counter-examples to (NEC-TP) under discussion disappear, and moreover we would have a modality which captures extensionally necessary truth in virtue of logical form, thereby answering the worry that (NEC-TP) fails to do justice to the role of form in determining consequence.[20]

The ECN-ist cannot entrust her fortune to the outright rejection of (NEC-TP). Instead, there is scope for a modest modification of the principle which arguably captures better the underlying understanding of logical consequence than does (NEC-TP) itself. Bracketing for our purposes the issue of logical form, we can reformulate (NEC-TP) in terms of the familiar possible worlds heuristic:

> **TP-PW:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ every world at which every element of Γ is true is also a world where ϕ is true.

Phrasing matter this way allows us to see the connection between modal analyses of consequence and standard model-theoretic approaches. For there is an obvious structural similarity between TP-PW and:

> **TP-MOD:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ every model of Γ is a model of ϕ.

[19] At least as significantly, we want to be able to licence significant discussion of counter-possibles. Suppose Goldbach's conjecture is true. Nevertheless fruitful mathematics can be done by exploring the consequences of $\neg GC$, and those consequences are non-trivial.

[20] More cautiously, appeal to a wide logical modality *could* answer the concern that (NEC-TP) doesn't do justice to the role of form. It might be objected that if (NEC-TP) is supposed to be a *conceptual analysis* of logical consequence then the role of form ought to be made explicit in the statement of the analysis, rather than hidden away in a modal operator. I have a good deal of sympathy with this objection.

This is not the place to adjudicate on whether the similarity between (TP-MOD) and (TP-PW) captures something deeper, for example whether the model-theoretic account succeeds in explicating, or at least delineating extensionally, consequence as understood modally. Rather the similarity points to the availability of a similar case for ECQ within the setting of model-theoretic semantics (there is no model in which a contradiction is true, so every model in which a contradiction is true is a model of an arbitrary proposition) as well as a similar response on behalf of the ECN-ist. It is worth keeping this in mind during the following discussion.

(TP-PW) supports ECQ because the universal quantifier does not have existential import.[21] So a formula of the form $\ulcorner \forall x\, \phi(x) \urcorner$ is true in the case when nothing whatsoever satisfies ϕ. Thus the truth of the claim that every Γ-world is also a ϕ-world does not require that there be any Γ-worlds. The recognition that the universal quantifier is not existentially committing was a step forward made by modern logic in comparison to Aristotelian logic, and nothing that follows is intended to call it into question. Instead, I want to suggest that it is open to the ECN-ist to reject (TP-PW) as an adequate analysis of logical consequence precisely because the quantifier after the biconditional is not committing.

Why might the ECN-ist adopt this position? She is liable to turn the question round on us; why is (NEC-TP), and its cashing out as (TP-PW), an attractive account of logical consequence? Why might it be thought to capture something correct and important in our pre-theoretic notion of *following from*? It is because, where $\Vdash$ expresses pre-theoretic consequence, if

$$\Gamma \Vdash \psi$$

we want to say that the world being such as the elements of Γ claim it to be guarantees that ψ be true. Now, if there is simply no way for the world to be such that the $\gamma \in \Gamma$ describe it correctly, it's not clear what content could be had by the claim that *were* the world that way ψ would be true. Whether sense can in fact be made of this claim is a matter for discussions of counterpossibles and needn't detain us.[22] What does matter for present purposes is that we are far from the concerns of someone engaged in the ordinary practice of reasoning. When we ask ordinarily whether ψ follows from Γ we suppose (in some sense – precisely *which* sense will prove important below) that the world is Γ-like and ask then whether ψ holds. We cannot do this if the world couldn't be Γ-like.

The plausibility of this line is going to turn on the interpretation of the modality in statements such as 'the world couldn't be Γ-like. Before we get onto that we should revisit the schemas (NEC-TP), (TP-PW) and (TP-MOD) in the light of the above line of reasoning:

> **NEC-TP*:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ (a) it is possible for every element of Γ to be true, and (b) it is not possible for every element of Γ to be true and ϕ false.
>
> **TP-PW*:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ (a) there is a world at which every element of Γ is true, and (b) every world at which every element of Γ is true is also a world where ϕ is true.
>
> **TP-MOD*:** ϕ is a logical consequence of the elements of Γ iff$_{df}$ (a) Γ has a model, and (b) every model of Γ is a model of ϕ.

[21] What about reformulating TP-PW trading on the classical equivalence between $\forall x\, \phi$ and $\neg\exists x \neg\phi$ so that the RHS reads 'there is no possible world at which Γ is true and ϕ is also true? The right thing to say here is that, since we're rejecting the standard (non-committing) classical universal quantified form of the condition, we ought to reject the negated existential equivalent form (and this latter rejection can be motivated by reasons like those given in the main body of the text). If we want a negated existential condition for logical consequence, that is fine, but we will have to include an existential requirement ('and there is at least one world. . .') just as in the universally quantified case.

[22] Note though that simply providing a formal representation of the world being Γ-like, perhaps using the machinery of impossible worlds, is not enough here. We are concerned with matters of *meaning*, and in particular with how a speaker could be brought to understand what is being claimed when it is said that every element of Γ is true.

It is an easy exercise to verify that each of these yields ECN on the assumption that can be no true contradictions.

The ECN-ist who advocates a modal or model-theoretic analysis of consequence has therefore the means to give a formal account of the consequence relation as she understands it. To this extent her position is secure relative to that of her ECQ-ist opponent. To properly answer the case for ECQ in a necessary truth-preservationist context, however, the ECN-ist needs to be able to respond to a likely line of objection to her motivation for her account. Remember that she holds that, where Γ is contradictory, we cannot suppose that the world is Γ-like, since Γ doesn't detail a way the world could be. The revised schemata, however, do not make direct appeal to contradictoriness, but rather rule out inference from *impossible* premises (or their possible worlds or model-theoretic surrogates). This might seem to exclude too much. It is impossible that a material object be both entirely red and entirely green at the same time, but that impossibility is not logical,[23] nor is it obvious that someone who thinks that nothing follows from contradictions need deny that anything follows from 'the ball is both red and green' (recall the motivation for ECN in terms of the relationship between denial and negation suggested in the previous section). The ECN-ist can reply here by appealing to a narrow, restrictedly logical modality:

> **Logical Possibility**: Some world w is one of the *logically possible* worlds iff there is no P such that both P and $\neg P$ are true at w.

The ECN-ist can insist that (TP-PW*) be interpreted so that the worlds taken into account are all only the logically possible worlds. It is worth observing that, if any case is to be made for (TP-MOD*) capturing a modal notion of consequence, then things are easy for the ECN-ist at this point, since standardly models correspond to exactly these worlds. No stipulation needs to be made about the intepretation of (TP-MOD*).

Perhaps more serious for the ECN-ist is an apparent threat to her entitlement to reasoning under contradictory assumptions. If she cannot reason under such assumptions, to be discharged later in her reasoning, then severe restrictions on her entitlement to *reductio ad absurdum* and $\vee$-elimination will ensue. Similarly, cases of conditional proof will be inadmissible; because of the close connection between the conditional and consequence, we will address conditional proof separately below, focusing initially on *reduction* and $\vee$-E. There are real problems here. Most ECN-ists will certainly want to be able to argue that $\neg((P \wedge \neg Q) \wedge (Q \wedge \neg P))$ by assuming the negated proposition. Yet as simple and intuitive a theorem as this would be barred if reasoning from contradictory suppositions were ruled out.

Why might reasoning under contradictory suppositions be thought unavailable? Remember how we justified the existential clause in (NEC-TP*). Considering a sequent $\Gamma \vDash \psi$, and considering narrowly logical modality, we argued that

> if there is simply no way the world could be such that the $\gamma \in \Gamma$ describe it correctly, it's not clear what content could be had by the claim that *were* the world that way ψ would be true.

But doesn't this consideration infect reasoning under a supposition? Let ϕ be a supposition. If there is no way in which the world could be such that ϕ correctly describes it, how can we so much as reason under the supposition that ϕ? For are we not in so reasoning indulging the pretense that ϕ, but how can we even do that if ϕ is not even logically coherent? One option here would be to insist that we can grasp contradictiory scenarios; Priest's short story *Sylvan's Box* is intended to persuade us of this [17]. It would be unwise, however, to rest the ECN-ist's access to workaday logical resources on so vulnerable a claim.[24] In the remainder of this section, I'll suggest an alternative approach to reasoning under supposition which will shore up the ECN-ist's position without even committing

[23] Or at least not *narrowly* logical. The contrast here is with what Edgington and others term *broad* logical modality [5, 1].
[24] The loss of classical $\neg$-introduction, in particular, would severely weaken the resulting logic.

her to the claim that we can grasp the combined content of contradictory sets of propositions, let alone imaginatively entertain what the world would be like were they to obtain.

In natural deduction formulations of proofs it is usual not to mark a sharp distinction between premises and assumptions. Premises, on this approach, are simply undischarged assumptions. For technical purposes this is perfectly in order. There is, however, a danger of us assuming unreflectively, on the basis of this formal treatment, that when one executes a deductive argument from premises and when one reasons under temporary supposition one is doing the same sort of thing. And this assumption is not compulsory.

It is true, of course, that one does not need to be prepared to assert all (or for that matter, any) of the premises of an argument to be in a position to prove that a certain consequence follows from those premises. For this reason there is a certain affinity between arguing from premises and reasoning under supposition. Imagine two reasoners, Alice and Bryony. Alice and Bryony both agree that a certain proposition P is false. Nevertheless Alice is perfectly able to show Bryony that if P *were* the case then she ought to believe another proposition Q on the basis of supplying an argument for Q from P. Deductive arguments compel acceptance of the conclusion on the basis of acceptance of the premises, since if the latter are true, the former must be true also. Think of a deductive argument, then, as a series of related considerations advanced to convince a reasoner (or reasoners) that were the world thus-and-so it must also be some other way. On this understanding, Alice could not have got off the starting block in convincing Bryony that P entails Q if there were no intelligible situation to which P corresponded. Imagine Alice commencing 'Right, let's assume that P; I'm going to show you that Q'. There are many circumstances in which Bryony might come back, 'hang on, what? Assume that *what*?' In most of these Alice will be able to explain the dialectical situation to her, and proceed with her argument – if P is obviously false, for instance, or if Bryony doesn't understand P. But if P does not so much as pick out a possible way things could be, even in the widest logical sense, Bryony is entitled to persist with her 'what?' Given what deductive arguments are supposed to be, there is no deductive argument to be had here.

Contrast this with another case. In course of executing an argument, Alice says to Bryony 'Just allow for a moment that R, and then allow that $\neg R$. I know that can't be right, and I know we can't even understand what it would be for R and $\neg R$ to both hold, but humour me. In allowing this, we'll treat the sentences mechanically' – Alice has her white-board out by now – 'we'll apply all our usual rules to them, and I'll show you something important'. And indeed Alice goes on to show that $R \rightarrow \neg\neg R$, or some other outcome of working with the assumptions and then discharging them. The suppositions are not asserted here, there is no requirement that the reasoners be able to conceive of their being the case,[25] they are rather put forward as a purely syntactic staging post in an argument, as a way of demonstrating what issues from the rules governing the language, even though they don't succeed in picking out any intelligible situation.

The ECN-ist can avail herself of the distinction between two different types of speech-act: the putting forward of a premise, which is the argumentative entertaining of an intelligible situation in order to deduce consequences, and the procedural temporary admission of a sentence with no requirement of intelligibility. This latter speech-act would appropriately be called something along the lines of *allowing for the sake of argument*, but corresponds to those parts of proofs which are universally called *suppositions* or *assumptions*. In order to make the distinction between premises and suppositions do the work needed by the ECN-ist, we will have to banish from our minds any suggestion that suppositions need be of contents we can in fact suppose to be the case, as will as of the private, mentalistic, colouring of the words 'supposition' and 'assumption'. With this distinction in hand, the ECN-ist can explain why she does not think there are valid deductive arguments with

[25]On the distinction between conceiving and supposition, with both characterised as mental activities, see [11]. I take what I say here to be continuous with this, although note that I am using the word 'supposition' to designate a type of action in public, rule-governed, reasoning, rather than something private and mental.

contradictory premises (since these are not intelligible in the manner that premises must be, given what one is doing when one puts forward a premise for argument) but admits the usual supposition-discharge rules of deductive reasoning. The latter, she can maintain, involve reasoners in a distinct speech-act from that of putting forward premises, and so constraints operative in the case of the latter cannot be assumed to apply.

These considerations suffice to justify classical-style reasoning with negation and disjunction. But what about the conditional? Here the ECN-ist needs to be cautious, lest she admit explosive reasoning by the back door. For an assumption of:

$$(P \wedge \neg P)$$

She can infer using unrestricted conditional proof the theorem,

$$(P \wedge \neg P) \rightarrow Q$$

But then, using only modus ponens and cut, she can prove the conjunction for of *ex contradictione quodlibet*. Something must be done!

An obvious way forward can be seen if we reflect on the fact that the (material[26]) conditional encodes the consequence relation in the object language. Indeed arguably that is the purpose of the conditional. Of course the distinction between object language and metalanguage is of recent pedigree, and the medieval disputes which heralded the decline of ECN in favour of ECQ were conducted in terms of the word *si*[27]. Once the distinction between object language and metalanguage is in place, it is of course right to mark the conceptual distinction between consequence and conditionality. Quine's animadversions not withstanding, however, this is perfectly compatible with what was correct about the medieval discussion, and what explains our valuing deduction and reduction theorems about logics, namely that the conditional represents the logical consequence relation on a language within the language itself. And, given this, it is eminently reasonable to place constraints on admissible suppositions for conditional proof mirroring the constraints on premises which may be used in deductions. There are syntactic niceties here: we have been understanding contradictions as two propositions, separated on the left of the turnstile by a comma. There is no equivalent of the comma in the object language, so we will have to make do with the conjunction of two contradictories and bar the assumption of $\ulcorner \phi \wedge \neg \phi \urcorner$, for any ϕ for the purposes of conditional proof. The justification is the function of the conditional in representing the consequence relation, and this constraint suffices to block the backdoor route to ECQ.

3. Proof-theoretic harmony

The final two cases for ECQ, based as they are on more general considerations rather than analyses of logical consequence, will be passed over more swiftly. The first appeals to proof-theoretic harmony. In order, amongst other reasons, to rule out-of-order deviant connectives such as Prior's *Tonk* it is often maintained that the introduction and elimination rules for connectives ought to be in harmony [21][2][4]. The precise nature of this harmony has been the subject of wide-ranging discussion in the literature. Suffice it to note that the elimination rule standardly taken to be harmonious[28] with *reductio* as the elimination rule is the intuitionistic ¬-E:

$$\frac{P, \neg P}{Q}\ \neg\text{-E}$$

[26]The rescue proposed here will break the equivalence between $P \rightarrow Q$ and $\neg P \vee Q$, so talk of the materiality of this conditional is somewhat deviant. However, we have an extensional conditional proof-theoretically characterised by conditional proof and modus ponens, which is as material as we're going to get in an ECN context!

[27]Latin: *if*.

[28]Reductio in the sense I intend it here allows that, if a contradiction follows from a supposition (possibly with side-premises), then a reasoner may discharge the supposition and introduce the negation of the supposition.

which of course is just ECQ. Doesn't this show that if we want a decent negation operator we have to admit ECQ?

It does not. First note that the ECN-ist as we have described her *accepts* ¬-E. It is just that she thinks that the rule is frequently inadmissible, since any context in which both P and $\neg p$ have been asserted, that is introduced to reasoning outside the scope of a supposition, is one in which reasoning cannot continue. A rule which is rarely invoked is still a rule, and there is consequently a harmonious E-rule available to match the introduction rule for negation, namely ¬-E displayed above. Nor is never the case for the ECN-ist that ¬-E may be invoked in the course of a proof; it will often be admissible within the scope of a supposition, as in the obvious proof that $P \vdash \neg P \rightarrow Q$.

There is a more fundamental reason for thinking that considerations of harmony do not present a problem for the ECN-ist. Tonk presents us first and foremost with a problem about *meaning*: what constraints are there on the rules governing a purported connective such that they succeed in describing a connective with a coherent sense? Philosophers who have adopted the harmony response to Prior's challenge have generally followed Gentzen in taking the introduction rule alone to be sense-constitutive. The requirement of harmony then follows on the basis suggested by Gentzen,

> The introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. This fact may be expressed as follows: In eliminating a symbol, we may use the formula with whose terminal symbol we are dealing only 'in the sense afforded it by the introduction of that symbol'.[8, 184]

If this is right, then there could be no objection in principle to a connective with *no* elimination rule. So even if the ECN-ist had to somehow reject ¬-E – and I have suggested above that she doesn't – this does not tell against her position, just so long as she agrees with Gentzen about the meaning of logical expressions. She can simply accept the usual ¬-I rule and not admit any rule for eliminating negation.

4. Theoretical Holism

Perhaps, however, all these arguments for ECQ are beside the point. One school of thought, of which Quine is the best known representative, holds that theories about logic, much like theories about anything else, ought not to win our assent in a piecemeal fashion on the basis of individual arguments [22]. Rather we ought to weigh up theories according to their costs and benefits, as determined by a number of criteria for theory choice, before adopting the one with the best overall payoff. Depending on the holist philosopher in question the extent of the theories up for grab might be limited to logic (the theory according to which logic is classical, on this view, competes against the theory according to which logic is intuitionistic, and so on) or might be accounts of total science (on this view, a theory of total science will *include* a view about logic). Either way the result is what has been termed *anti-exceptionalism* about logic [9]; the manner in which we enquire about matters of logic is not different in kind, neither distinctively philosophical nor even restrictedly *a priori*, from other scientific investigations. For logic, along with the rest of philosophy, is in some significant sense a *science* [28].

If this is the correct way to think about logic (and, for the record, I think that it is not), the adoption of a classical theory of logic can be motivated by appeal to the combination of simplicity and strength which classical logic undoubtedly possesses. These undoubted benefits will be taken to outweigh any costs that accrue to the adoption of classical logic. Perhaps, the classicist can concede, validating ECQ is a cost. However, it's not one that outweighs all the very obvious benefits of classical logic. But therein lies the rub; for the ECN-ist, along with relevantists and paraconsistentists in general, thinks that classical logic brings with it considerable costs, namely that it licenses inferences which we do not ordinarily consider valid. If there is a logic which is sufficiently strong for the purposes of ordinary life and science which does not license these inferences then it ought to be prefered to classical logic by the ordinary criteria of theory choice. Graham Priest is a prominent example

of a logician who is both a paraconsistentist and an anti-exceptionalist who has argued for a non-classical theory in precisely this fashion. The ECN-ist can do likewise. Moreover she is in a better position than Priest. The light adaptation of classical rules sketched above is going to yield a logic considerably stronger than Priest's **LP**, with a better recapture of classical mathematics.[29] Unless the classical logician can show, in anti-exceptionalist friendly fashion, that there is some unacceptable cost to the ECN-ist's position, she has nothing to fear.

5. Conclusion

I began by recalling the common lecture-hall experience of students objecting to ECQ. Suppose one of these students has come to a course on classical logic already convinced that *nothing* follows from a contradiction. When the lecturer remarks that the logic he is teaching validates ECQ, the student objects – 'that's not right; nothing follows from a contradiction'. The point of this paper has been to argue that the most likely lines of response from the lecturer are not ones the student need accept in her own terms. The supporter of ECQ, then, needs to develop some better arguments. Meanwhile it looks worth developing and investigating a logic in which ECN holds.

[29]Quite *how much* classical recapture is a matter for future research.

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