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# 1 Shrinkage and Flexural Behaviour of Free and Restrained Hybrid Steel Fibre 2 Reinforced Concrete

## 3 [Abstract](#)

4       The effect of restrained shrinkage on the mechanical performance of concrete and [steel](#)  
5 [fibre reinforced concrete \(SFRC\)](#) requires more investigation, especially when using recycled  
6 [tyre steel fibre \(RTSF\)](#). This paper examines the free and restrained shrinkage strains and the  
7 mechanical performance of seven SFRC mixes. Results show that both free and restrained  
8 average shrinkage strains are very similar in all blends of fibres and they exhibited [non-](#)  
9 [uniform shrinkage through the height of the section](#). All examined blends meet strength  
10 requirements by MC-2010 for fibres to replace part of the conventional reinforcement in RC  
11 structures.

## 12 [Highlights](#)

- 13       • Free shrinkage strains are not affected significantly by addition of steel fibres.
- 14       • GGBS reduces shrinkage strains.
- 15       • Recycled tyre steel fibres can replace manufactured steel fibres partially.

## 16 [1. Introduction](#)

17       In water retaining structures or bridge elements, serviceability limit state (SLS) design  
18 aims to control crack widths to achieve a target life span by providing relatively large  
19 amounts of surface steel reinforcement. In such structures, the additional reinforcement is  
20 required to control cracks induced by restrained shrinkage, which creates further  
21 constructability challenges. To reduce the amount of additional surface reinforcement,  
22 shrinkage can be mitigated by reducing paste/aggregate ratio, minimising C<sub>3</sub>S content in  
23 cement, using expansive or shrinkage reducing additives, and internal curing materials [1].

24       Shrinkage cracking can also be controlled by adding randomly distributed steel fibres as  
25 successfully utilised by the construction industry in pavements and tunnels [2, 3, 4]. Steel

26 fibres can enhance the performance of concrete in flexure, shear and punching whilst at the  
27 same time help control shrinkage cracking and reduce spalling [5, 6, 7], depending on the  
28 amount and characteristics of the steel fibres, such as type, shape and aspect ratio [8, 9, 10,  
29 11]. Recycled tyre steel fibres (RTSF) are also available and were found to be good in  
30 controlling micro-cracks [12, 8]. RTSF can improve flexural toughness and post cracking  
31 performance and can successfully substitute manufactured fibres partially or fully in some  
32 applications [9, 13].

33 In most published research on RTSF [4, 14, 15], a single type of fibre is used as  
34 reinforcement. Recently, some studies investigated blends of manufactured and recycled steel  
35 fibres with different shapes and aspect ratios [9, 16], but the recycled fibres used were not  
36 classified raising reliability and repeatability concerns. The cleaning process of RTSF has  
37 been improved significantly recently and improved classified fibres have become available  
38 [17, 18, 19]. Hence, there is a need to investigate the effect of hybrid steel fibres (both  
39 manufactured and classified RTSF) on concrete exposed to free and restrained shrinkage.

40 The impact of steel fibres on free shrinkage of concrete is not clearly understood, with  
41 some researchers reporting an increase due to the increase in air voids, whilst others reporting  
42 either a decrease due to the internal restraint provided by the fibres or insignificant changes  
43 due to the cancelling effect of the two actions [4, 14, 15, 20]. Nonetheless, the effect of steel  
44 fibres on free shrinkage is known to vary depending on water-to-binder ratio, volume and  
45 type of admixtures, method of concrete laying (conventional, self-compacted concrete (SCC)  
46 or roller compacted concrete (RCC)), time of vibration, etc. [21].

47 In concrete structures, shrinkage of concrete is restrained by different actions internally  
48 and externally. External restraint can arise due to friction or reaction against the ground,  
49 concrete supporting elements or adjacent rigid structures, whilst internal restraint is provided

50 by aggregates and reinforcement [22, 23]. It is also known that aggregates tend to settle and  
51 concentrate at the bottom of the mould whilst water and air rise due to vibration and surface  
52 tamping. These phenomena can cause differences in compressive strength and elastic  
53 modulus at the top and bottom of the element [24, 25]. As more paste and water are found  
54 near the top surface, this can cause much higher shrinkage strains in that region. Non-uniform  
55 distribution of aggregates and water can create non-uniform shrinkage through a section and  
56 lead to additional curvature in concrete elements [4]. RILEM TC 107-CSP [26] determines  
57 shrinkage from the change in the distance between the centres of the two ends of a cylinder,  
58 which means that its approach is unable to capture the effect of aggregate sedimentation. To  
59 the knowledge of the authors, none of the design codes or standards deal with curvature due  
60 to the non-uniform shrinkage and this can lead to underestimate of long-term deflections and  
61 crack widths.

62 Free shrinkage tests on small elements are not normally able to develop enough internal  
63 tensile stresses to crack the concrete, hence, restrained shrinkage tests are needed to  
64 understand the cracking behaviour of restrained concrete [12]. Restraint causes tensile  
65 stresses in the concrete, which theoretically could increase with time due to concrete  
66 maturity, but creep is expected to relieve some of these stresses and reduce the probability of  
67 cracking [23, 27, 28]. Normally, it is difficult to quantify the degree of restraint imposed on  
68 an element, as it depends on the type of application, the location of the member in the  
69 structure and environmental conditions [3, 29]. However, there are several tests to assess the  
70 restrained shrinkage of concrete [1], with the most used being the ring test [30, 31]. Though  
71 simple and popular, this test can only be used for comparison purposes, as it only detects the  
72 stress and time of the first crack. Another disadvantage of this approach is that the sectional  
73 size needs to be kept relatively small (to enable cracking at a reasonable time frame) and this

74 enhances boundary effects and makes the concrete section less representative of sections in  
75 practice.

76 Active systems with larger specimens [32, 33, 34] can be used to restrain concrete  
77 shrinkage by fixing one end of a linear element whilst the other end is attached to an actuator  
78 which keeps the total length constant. In active systems, cracks tend to occur when the strain  
79 is being adjusted and this can affect the time at which cracking takes place [35]. Furthermore,  
80 full and active restraint is rarely found in practice, where restraint depends on the relative  
81 stiffness of the restraining structure and is mitigated by creep. For these reasons, and for  
82 simplicity, passive systems [36, 37] can be used by restraining concrete specimens through  
83 fixing bolts onto rigid structural elements. Younis (2014) [4] proposed the use of a passive  
84 restraining frame able to hold three prisms at the same time. The use of linear elements also  
85 enables shrinkage measurements to be taken at different levels through the section and  
86 examine shrinkage curvature.

87 The aim of this work is to examine the effect of restraint on shrinkage and mechanical  
88 performance of hybrid SFRC mixes. The performance of SFRC prisms comprising different  
89 fibre blends and subjected to a combination of restraining, curing and drying conditions are  
90 studied and compared. Ground granulated blast-furnace slag (GGBS) and RTSF are used,  
91 along with manufactured fibres, to control the amount of shrinkage strains and limit the  
92 propagation of concrete cracking under restrained conditions.

93 This paper comprises three main sections along with an introduction and conclusions.  
94 The first section presents the experimental programme including the examined parameters,  
95 the physical and mechanical characteristics of the examined materials and testing  
96 methodology. This is followed by a discussion on the results obtained from free and  
97 restrained shrinkage tests of hybrid SFRC prisms ([blends of manufactured undulated steel](#)

98 fibres (MUSF) and RTSF). The level of restraint imposed by restraining frames is assessed  
 99 through a finite element numerical analysis and used to gain additional insight into the effect  
 100 of restraint level on overall behaviour. Finally, in the third section the paper discusses the  
 101 effect of restrained shrinkage and different drying conditions on the flexural performance of  
 102 the examined concrete mixes.

## 103 2. Experimental Programme

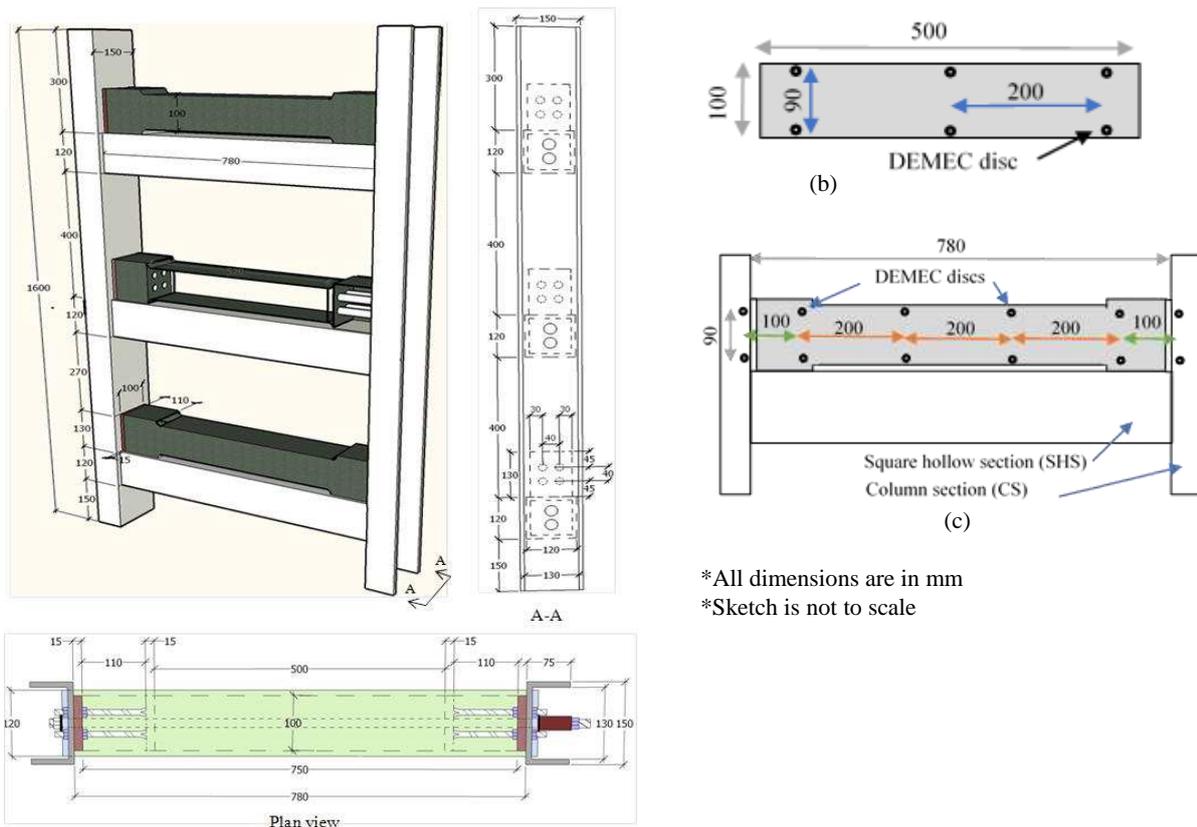
### 104 2.1 Parameters

105 The experimental programme examined seven SFRC mixes in addition to a control mix  
 106 made of plain concrete, as shown in Table 1. Each mix was used to manufacture twelve  
 107 control cubes (100 mm), six prisms (100x100x500mm) for free shrinkage measurement and  
 108 three prisms, which were cast in a restraining steel frame as shown in Figure 1a [4]. Three  
 109 prisms (out of the six) were stored in a mist room (MR) to monitor autogenous shrinkage.  
 110 The other three specimens were stored under controlled environmental (CR) conditions  
 111 (temp: 23±2 °C and RH: 40±5%) to quantify drying shrinkage. The restrained specimens  
 112 (RS) were stored under the same conditions as the CR specimens.

113 *Table 1 Steel fibre types and contents.*

Mix	MUSF L (mm)	MUSF Ø (mm)	MUSF Dose (kg/m <sup>3</sup> )	RTSF Dose (kg/m <sup>3</sup> )	RTPF Dose (kg/m <sup>3</sup> )	Batch number
P	-	-	-	-	-	1, 2, 3
M30	55	0.8	30	-	-	1
M20R10	55	0.8	20	10	-	2
M20R10P1	55	0.8	20	10	1	3
R30	-	-	-	30	-	3
M35	60	1.0	35	-	-	1
M45	60	1.0	45	-	-	1
M35R10	60	1.0	35	10	-	2

114



\*All dimensions are in mm  
 \*Sketch is not to scale

115 Figure 1 Restraining frame used to restrain concrete prisms (a) and layout of shrinkage DEMEC distribution in free (b) and  
 116 restrained prisms (c).

117

118 **2.2 Measurements**

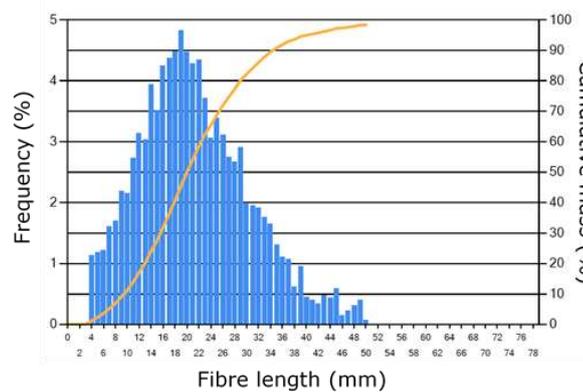
119 Shrinkage measurements were taken using a 200-mm demountable mechanical  
 120 “DEMEC” strain gauge at the top and bottom of both sides of all prisms for 300 days. Figure  
 121 1 (b and c) shows the measurement layout for free and restrained shrinkage, respectively. It  
 122 should be noted that a 100 mm “DEMEC” strain gauge was used to measure the deformation  
 123 at the boundaries between concrete and restraining frame.

124 At the end of the shrinkage measurement period, CR prisms were dried in an oven until  
 125 constant weight was observed. This was always achieved after 3 cycles at 50°C and 3 cycles  
 126 at 100°C, each cycle lasting 24 hours. After that and prior to flexural testing, all prisms were  
 127 notched (on one of the sides as cast) at the centre to 1/6 of the sectional depth. They were  
 128 then tested in three-point flexure by controlling the crack mouth opening displacement

129 (CMOD) [38]. The exact dimensions of the prisms were taken to the nearest 0.5 mm to  
130 account for casting imperfections. Each prism was then split into two pieces and each portion  
131 tested for compressive strength according to BS 1881-119 [39]. Concrete compressive  
132 strength was also obtained from cube test at 7 days, 28 days and 14 months.

### 133 2.3 Materials

134 Two types of steel fibres were used in this programme: manufactured undulated fibres  
135 (MUSF) with a nominal tensile strength of 1450 MPa (two types of undulated  
136 length/diameter (L/Ø) 55/0.8 and 60/1) and recycled tyre steel fibres (RTSF) with a nominal  
137 tensile strength greater than 2000 MPa [40]. The average diameter of RTSF was about 0.2  
138 mm, whilst the average length, determined using a special optical device, at 50% cumulative  
139 mass, was about 20 mm as shown in Figure 2. Both single and blended steel fibres were used  
140 to reinforce the concrete in three amounts of 30 kg/m<sup>3</sup>, 35 kg/m<sup>3</sup> or 45 kg/m<sup>3</sup>. Mix  
141 M20R10P1 also contained 1 kg/m<sup>3</sup> of recycled tyre polymer fibres (RTPF) to examine the  
142 effect of polymer fibres in controlling shrinkage cracking.



143  
144 *Figure 2 Length distribution of classified RTSF.*

145  
146 Three batches of ready mix concrete were used to manufacture the test specimens (see  
147 Table 2). The mix design is based on a design used in Europe for slabs-on-grade. The binder  
148 consisted of 50% CEM 1 and 50% GGBS.

<b>Composition</b>	<b>Quantity (kg/m<sup>3</sup>)</b>
Cement 52.5N CEM1	150
GGBS (BS EN 15167-1:2006)	150
4/20 River aggregates	1097
0/4 River sand	804
Water/binder ratio	0.55
SP (Master Polyheed 410)	1.5 L

150

### 151 3. Results and Discussion on Shrinkage Strains

#### 152 3.1 Free shrinkage strains

##### 153 3.1.1 Drying shrinkage

154 The free shrinkage strains versus time at the top and bottom of the specimen (T for top  
155 and B for bottom) are shown in Figure 3 (a and b), respectively. The small fluctuations in the  
156 curves are a result of small temperature and relative humidity changes in the control room.  
157 Shrinkage strain predictions of Eurocode [41] and fib Model Code [42], shown in dotted  
158 lines, are higher than the experimental strains, possibly due to the high amount of GGBS and  
159 differences in first measurement time. The EC and fib models consider conventional cements  
160 and do not consider other cementitious materials in their predictions. GGBS was found by  
161 some authors to reduce total shrinkage amount (*average of top and bottom measurement*) [35,  
162 43, 44] as the fineness of GGBS can close the concrete pores and prevent water from  
163 escaping the substrate [27]. Codes recommend taking the first shrinkage measurements  
164 within 3 minutes after demoulding, but due to the high amount of DEMEC discs used in this  
165 study, the first shrinkage measurement was taken after 6 hours.

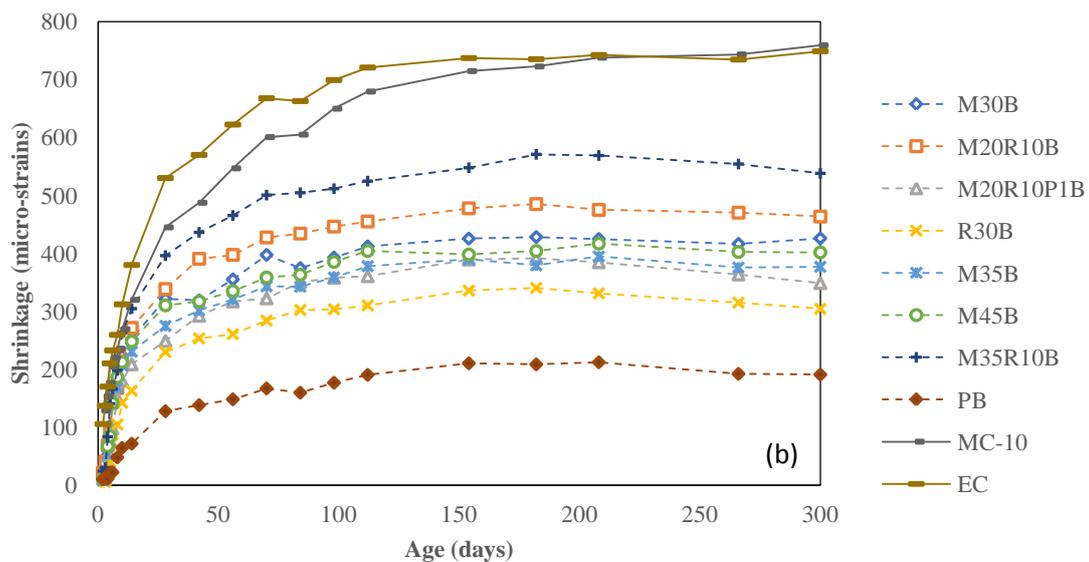
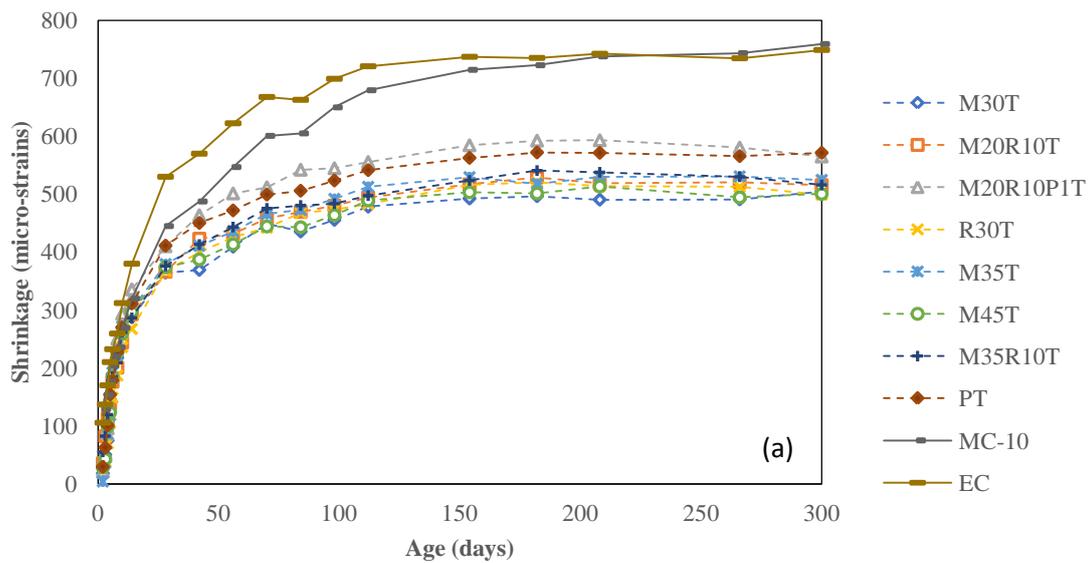


Figure 3 Free shrinkage strains at top (a) and bottom (b).

166

167

168 It is difficult to determine, from the results, the precise effect of steel fibre type/dosage

169 on free shrinkage. However, it is evident that shrinkage strains at the top are overall higher

170 than at the bottom possibly due to non-uniform distribution of concrete constituents. Plain

171 concrete shows higher shrinkage strains at the top than SFRC mixes, whilst showing the

172 lowest strain at the bottom. This may be due to the fact that superplasticiser was added to the

173 plain concrete (to maintain the workability of concrete after the addition of steel fibres) and  
174 this may have led to more bleeding than in the other mixes. Overall, SFRC specimens  
175 experienced higher amounts of average shrinkage strains compared to plain concrete,  
176 possibly due to the air entrainment on the surface of the fibres. Shrinkage strains of SFRC  
177 were between 500 and 600 micro-strains at the top and between 300 and 500 micro-strains at  
178 the bottom. The scatter of the bottom measurements was higher than that of the top  
179 measurements possibly due to the fact that the presence of steel fibres prevented some of the  
180 coarse aggregates from settling to the bottom of the mould [45]. The varying amounts of  
181 coarse aggregates at the bottom of the section resulted in varying degrees of restraint and thus  
182 a higher scatter in shrinkage resistance. Non-uniform shrinkage strains in these rectangular  
183 sections can be the result of non-uniform distribution of the coarse aggregates across the  
184 depth of concrete section, which also creates curvature that will contribute to the global  
185 deformation of the members [25, 45].

186

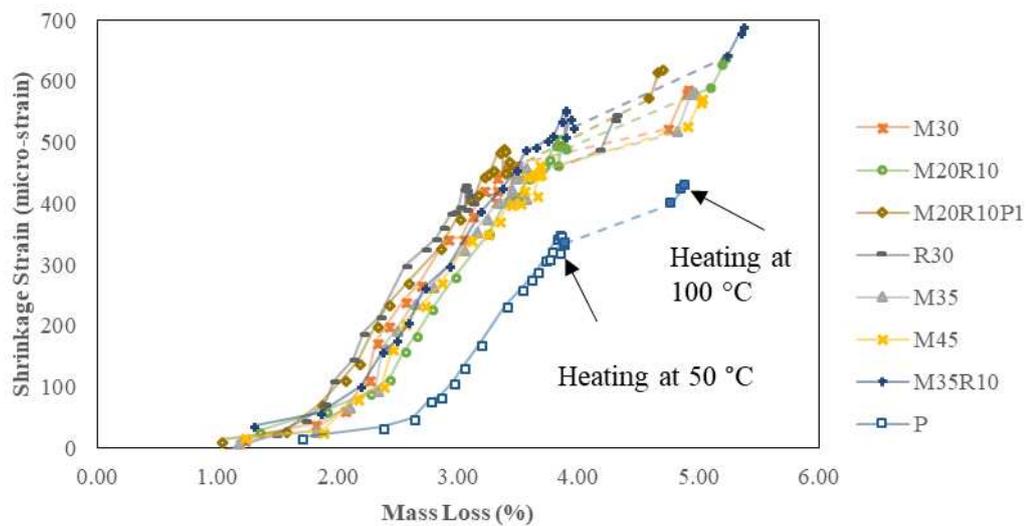
### 187 3.1.2 Drying shrinkage and mass loss relationship

188 The relationship between total free shrinkage and mass loss of the CR specimens is  
189 shown in Figure 4. Though the water content in the original mix was the same for all mixes,  
190 workability decreased after introducing steel fibres as some of the free water was adsorbed in  
191 wetting the surface of the fibres. Hence, it appears that, as a result, the plain concrete mix lost  
192 more free water than the SFRC mixes during the first few days of drying.

193 The behaviour of each mix shows three stages: 1) the first five days of rapid drying, 2)  
194 normal drying and 3) accelerated drying in the oven. The first stage indicates rapid mass loss  
195 possibly due to the evaporation of the free water [46]. The second stage tends to show a linear  
196 trend in mass loss with free shrinkage until mass loss stabilises and the moisture inside the  
197 samples become approximately equal to the relative humidity of the atmosphere [36]. The

203 third stage was created artificially due to accelerated drying of the prisms in the oven initially  
 204 at 50 °C and then at 100 °C. During the first three cycles at 50 °C, there was little change in  
 205 mass loss and shrinkage. However, once the temperature was elevated to 100 °C, there was a  
 206 noticeable increase in mass loss and drying shrinkage.

207



208  
 209 *Figure 4 Free shrinkage and mass loss relationship.*

210

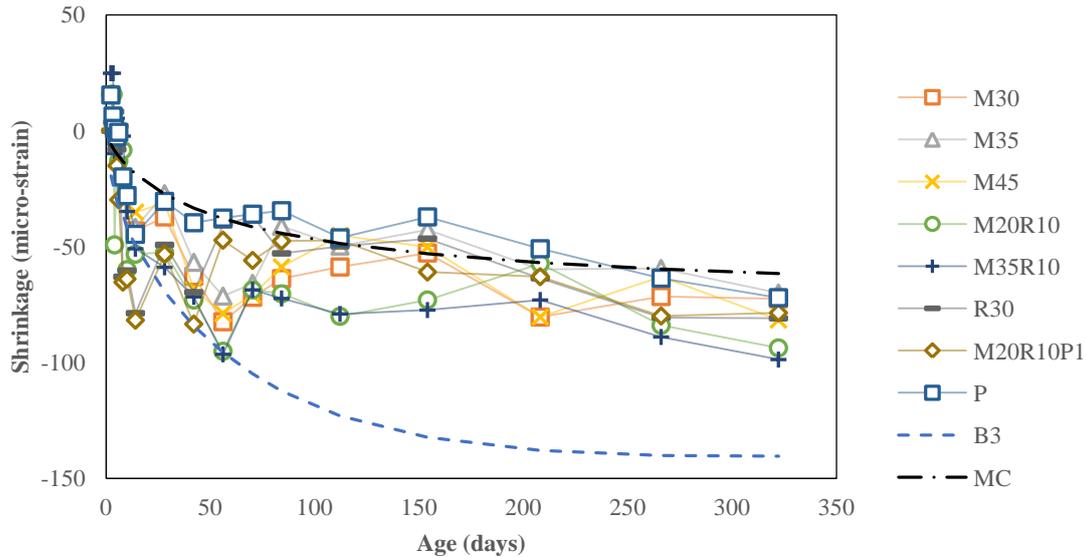
211 The purpose of completely drying the samples was to assess if it is possible to predict the  
 212 ultimate drying shrinkage strains from mass loss by assuming that the relationship between  
 213 shrinkage and mass loss is linear. However, during accelerated drying, there was more mass  
 214 loss (on average 14%) or less shrinkage (on average 19%) than expected under normal drying  
 215 conditions (second stage). This phenomenon may be attributed partly to micro diffusion of  
 water from gel pores to capillary pores, which helps to free larger amounts of water [47], or  
 to the micro-cracking that was caused by differential temperature at the surface of the  
 concrete during cooling. This was evident in the plain concrete that showed the highest  
 number of micro-cracks on the surface. Therefore, heating the samples at 100 °C appears to  
 have altered the mechanism of drying due to micro diffusion of water or micro-cracking,

216 which was not intended by the experiment. However, the ultimate mass loss could be  
217 obtained at lower temperatures, e.g. at about 80 °C without causing damage in the concrete  
218 micro-structure, and could be used to predict the long-term evolution of drying shrinkage  
219 strain and its impact on the health of the structure.

### 220 3.1.3 Humid concrete shrinkage strains

221 Figure 5 shows the evolution of shrinkage strain in specimens conditioned in a mist  
222 room. Negative strain values mean that the samples are swelling. The non-uniformity in the  
223 curves between age of 50 and 70 days was due to unexpected fluctuations in moisture inside  
224 the mist room (due to some mechanical problems). The initial swelling in the samples can be  
225 attributed to swelling in GGBS grains, which can absorb water and lead to disjoining  
226 pressure [48, 49], as they get fully saturated during the hydration process. As a result,  
227 swelling continues until the relative humidity in the matrix becomes less than the relative  
228 humidity in the pores of the grains [50]. However, the plain concrete specimens swelled less  
229 compared to those reinforced with fibres, possibly due to their lower permeability, [which](#)  
230 [prevented the GGBS in the matrix from absorbing any additional water](#) [51].

231 Swelling continued for the entire 11-month period of measurements, which indicates that  
232 swelling due to absorption of moisture is higher than any autogenous shrinkage strains.  
233 Model B3 [47] and fib MC-2010 [42] predict expansion in any concrete stored under relative  
234 humidity of about 100%. Predictions by model B3 and fib MC-2010 are shown in dashed  
235 lines in Figure 5 (indicated as B3 and MC, respectively). B3 is found to be in agreement with  
236 the initial experimental results while MC is close to the plain concrete throughout the  
237 measuring period. It should be noted that this analysis was carried out using CEM I as a  
238 cementitious material in both models as there is no provision for GGBS in the current  
239 formulations. fib MC-2010 was found to predict expansion strains up to two times greater  
240 than those induced by autogenous shrinkage strains.



241  
242

Figure 5 Humid concrete strain results.

### 243 3.2 Restrained shrinkage strains

244 Figure 6 (a and b) shows the restrained shrinkage strains of all tested prism at the top (T)  
245 and bottom (B), respectively. In general, prisms made with different mixes exhibited similar  
246 restrained shrinkage strain development, apart from those made with mixes M35 and R30,  
247 which started deviating from the rest between the age of 14 and 28 days. No significant  
248 development in shrinkage took place in mix M35, possibly due to early age micro-cracking  
249 near the anchors, whilst there was a remarkable increase in mix R30, possibly due to slip at  
250 the interface between concrete and anchors.

251 Shrinkage strains varied between 250 and 300 micro-strains at the top and between 160  
252 and 180 micro-strains at the bottom at the age of 180 days. These strains decreased after 200  
253 days, possibly as a result of creep and the development of micro-cracks inside the concrete.  
254 The similarity in the shrinkage strain levels exhibited by all specimens indicates that the  
255 effect of steel fibre type and dosage is insignificant with respect to restrained shrinkage, as  
256 was also observed in free shrinkage prisms. It should be noted that some of the curvature  
257 induced in the specimens can be attributed to restraint loss at the external boundaries between  
258 the concrete and steel anchors and/or differential aggregate distribution.

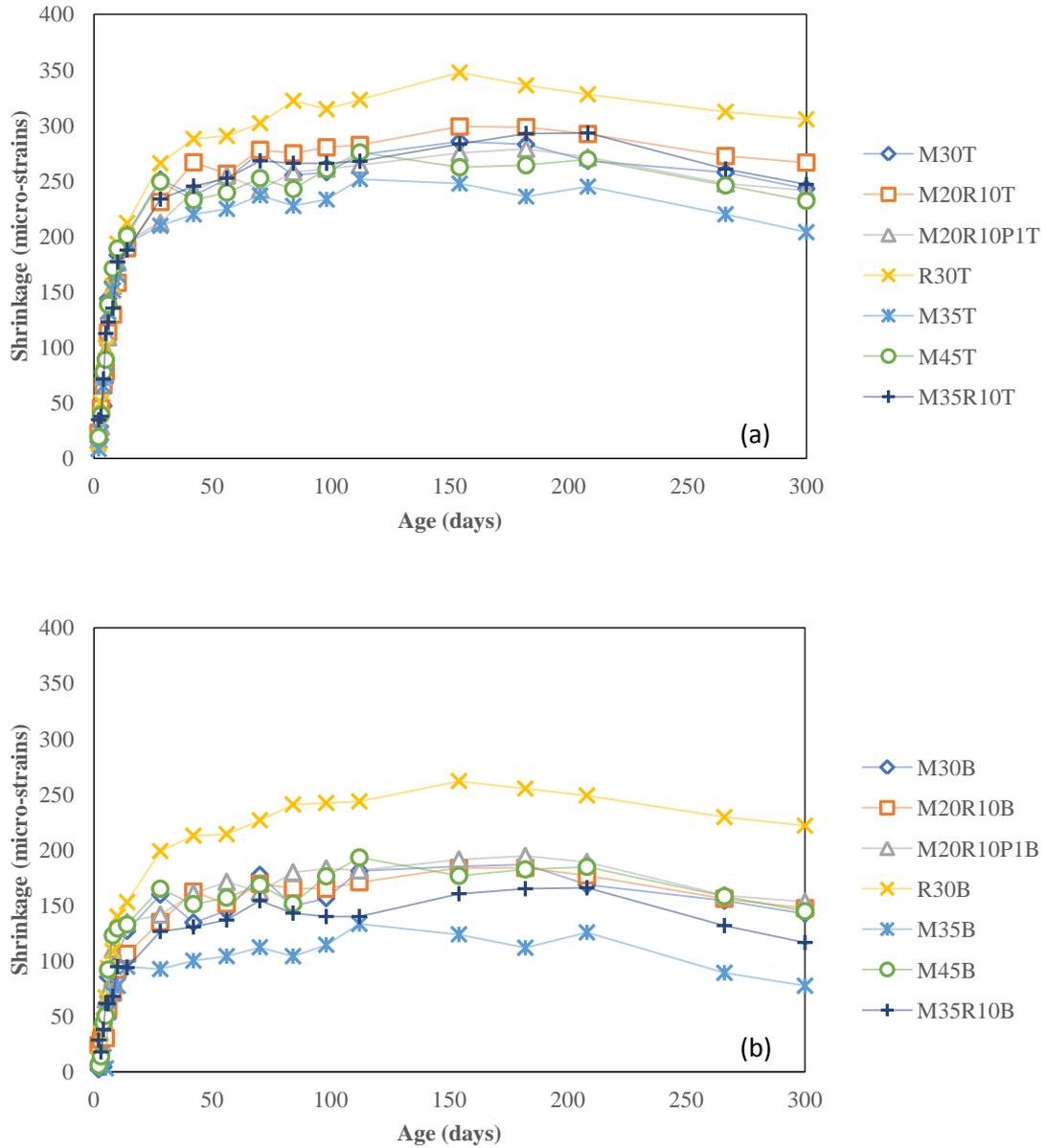


Figure 6 Restrained shrinkage strains at top (a) and bottom (b).

259

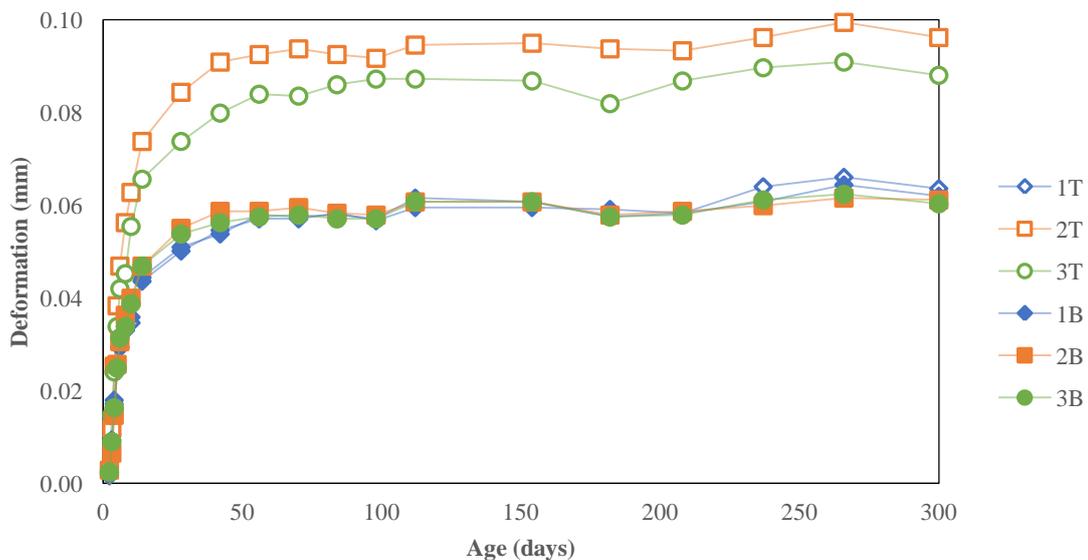
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### 261 3.3 Performance of the restraining frame

262 The degree of restraint (**DOR**) is defined as the difference in strain between the free and  
 263 restrained elements (see Equation (1)). The **DOR** values for the passive restraining frame  
 264 used varied between 0.5 and 0.6. The theoretical values using simple elastic calculations is  
 265 higher at 0.73 [12].

$$DOR = \frac{\varepsilon_{sh,free} - \varepsilon_{sh,restrained}}{\varepsilon_{sh,free}} \quad (1)$$

266 Drying shrinkage induces shortening of the concrete specimens, which are restrained by  
 267 the frame through the anchors. Figure 7 shows that there is some additional deformation at  
 268 the boundaries between concrete and the restraining frame over a gauge length of 100 mm  
 269 (see Figure 1c) for a typical mix (M20R10P1) at the top (T) and bottom (B) of each specimen  
 270 (1 - top, 2 - middle and 3 - bottom prism in the restraining frame). The deformations are  
 271 higher at the top of prisms 2 and 3 whilst they are similar for all prisms at the bottom level.  
 272 Most of the deformation takes place during the first 50 days. This deformation is the result of:  
 273 a) elastic deformation of the concrete, b) anchor elongation, c) slip at the interface between  
 274 concrete and anchors and d) local deformation of the frame. These additional deformations  
 275 b), c) and d) contribute to the differences between the actual (0.57) and theoretical (0.73)  
 276 DOR.



277

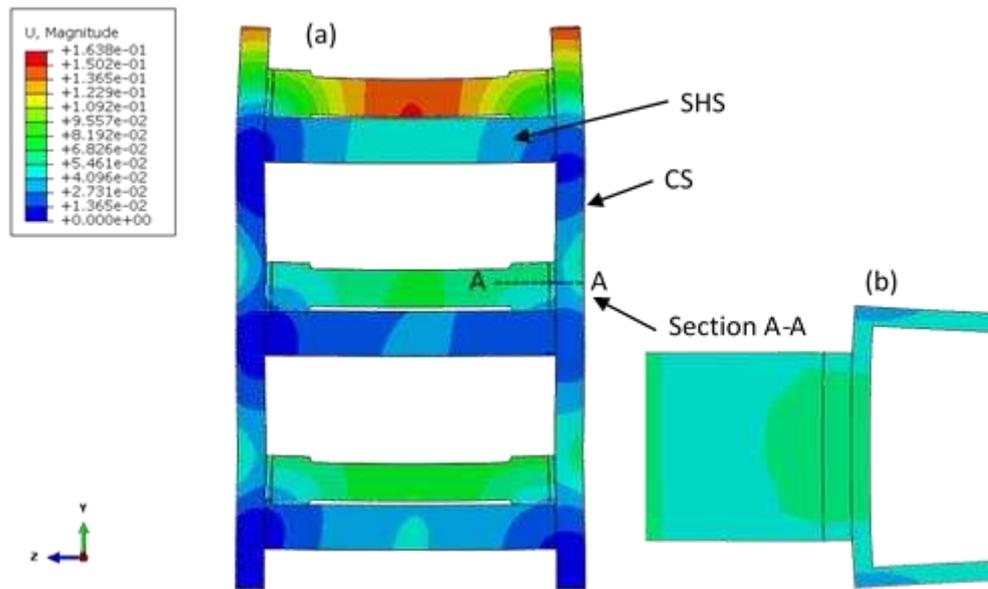
278 *Figure 7 Deformation at the boundary between restraining frame and concrete specimens, mix M20R10P1.*

279

280 A numerical investigation was conducted by Younis (2014) [4] using 3D linear FEA  
 281 models in ABAQUS [52] to estimate the induced deformations by concrete drying shrinkage  
 282 on the restraining frame. Solid (continuum) elements with 8 integration points (CD8R) were  
 283 used. The approximate global mesh size was 20 mm, but a finer mesh was adopted close to

284 the connections. The model was run without the presence of concrete elements and pre-  
285 stressing forces on the rods inside the square hollow sections (SHS). This study adopted this  
286 model, but modified the boundary conditions, modelling of concrete prisms and pre-stressed  
287 forces as follows; the right column (CS) was fixed (welded) to the SHSs and to the base of  
288 the frame whilst the left CS was pinned by pre-stressed forces of 56.25 kN (the result of a  
289 torque of 180 N.m on the bolts) applied to rods inside the SHS. The anticipated induced force  
290 due to drying shrinkage ( $\epsilon_{sh}E_cA_c$ ), at the age of 300 days, was applied uniformly on the  
291 anchors. At this age, the specimens have reached hygral stabilisation and the relative  
292 deformation between concrete and frame can be considered to be approximately stabilised  
293 (see Figure 7). Therefore, the shrinkage induced force can be assumed to be 100% resisted by  
294 the anchors.

295 Figure 8 shows the exaggerated global and local deformations of the restraining frame  
296 obtained by FEA. Concrete shrinkage caused relative translation of the CS and bending in  
297 both CS and SHS. The relative translation between the columns at the level of prism 1, 2 and  
298 3 is 0.127, 0.075 and 0.087 mm, respectively, corresponding to RF of 0.64, 0.79 and 0.75.  
299 The bending deformation of the SHS restraining prism 1 is higher than that at prisms 2 and 3  
300 due to the free end effect. Prism 2 experienced the lowest deformations due to the restraint  
301 contribution of both top and bottom SHSs. Figure 8b shows the local deformation of the CS  
302 at the level of prism 2 and the relative deformation between web and flanges. This highlights  
303 the additional contribution to the boundary zone deformation due to local deformations of the  
304 frame, which can actually account for some of the deformation shown in Figure 7. Much of  
305 this local deformation can be avoided if the CS is locally stiffened to prevent the flange  
306 rotation. The average apparent measured RF at 300 days was 0.57 whilst the theoretical and  
307 numerical DORs are very similar at 0.73.



308

309 *Figure 8 Exaggerated global and local deformation in the restraining frame (a) and supporting column (b).*

310

## 311 4. Results and Discussion on Mechanical Characteristics

### 312 4.1 Compressive strength

313 Table 3 shows the average results of density and compressive strength for the plain  
 314 concrete mixes, for both air and water cured cubes (standard deviations are shown in  
 315 parenthesis). For air cured cubes, there was only a slight increase in the compressive strength  
 316 between 7 and 28 days whereas for water cured specimens, there was a dramatic change in  
 317 compressive strength due to the activation of the GGBS in the presence of water. The GGBS  
 318 is also responsible for the lower in early strength of the water cured samples at 7 days [35,  
 319 53]. At 14 months, the compressive strength for the samples stored in air is similar to that  
 320 measured at 28 days, while there was an increase from 40 MPa to 56 MPa for the water cured  
 321 samples.

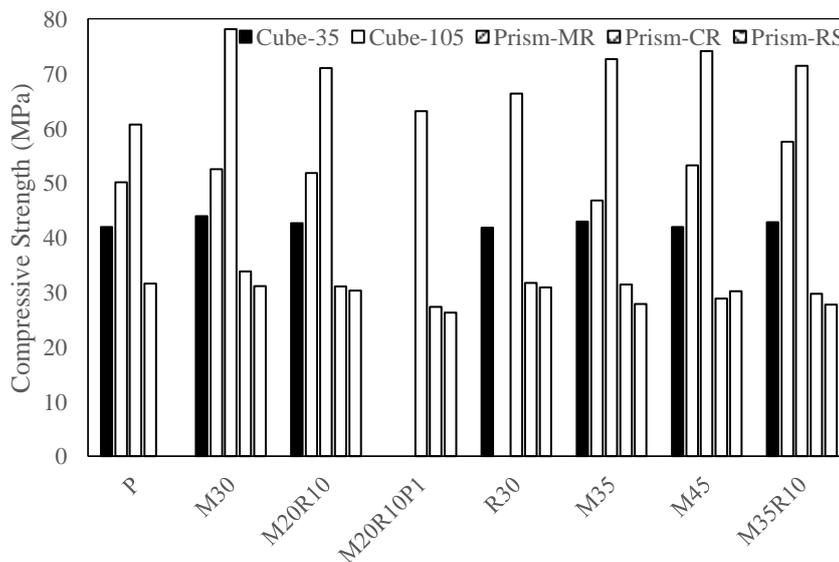
322 *Table 3 Plain concrete density and compressive strength results (standard deviation).*

Curing method	Density (kg/m <sup>3</sup> )			Compressive Strength (MPa)		
	7 days	28 days	14 months	7 days	28 days	14 months
Air cured	2310 (34)	2285 (11)	2284 (42)	20.7 (0.2)	24.5 (1.6)	24.1 (0.3)

323

324        Figure 9 shows the mean compressive strength values, obtained from three cubes (150  
 325 mm) per mix at 35 and 105 days (moisture cured in the laboratory) as well as from six  
 326 samples for each curing condition (MR, CR, RS) obtained from the broken prisms in flexure  
 327 at the age of 14 months. At age of 14 months, SFRC obtained from broken prisms shows  
 328 higher compressive strength compared to plain concrete at the same curing condition.  
 329 However, in all cases, the dose of the steel fibres appears to have no clear effect on  
 330 compressive strength.

331        As expected, prisms stored in the mist room (MR) show much higher compressive  
 332 strength compared to the ones air cured in the control room (CR and RS) *by about 56% on*  
 333 *average*. CR and RS samples resulted in similar compressive strengths despite the fact that  
 334 RS samples were restrained for ten months and experienced drying shrinkage micro-cracks.  
 335 This may be because CR samples were fully dried in an oven (to determine mass loss) which  
 336 may have caused micro-cracks and weakened their structure. *Micro-cracks were observed on*  
 337 *the surface of plain concrete specimens as discussed in subsection 3.1.2.*



338

339 *Figure 9 Compressive strength obtained from cubes at 35 and 105 days and broken prisms in flexure at 14 months.*

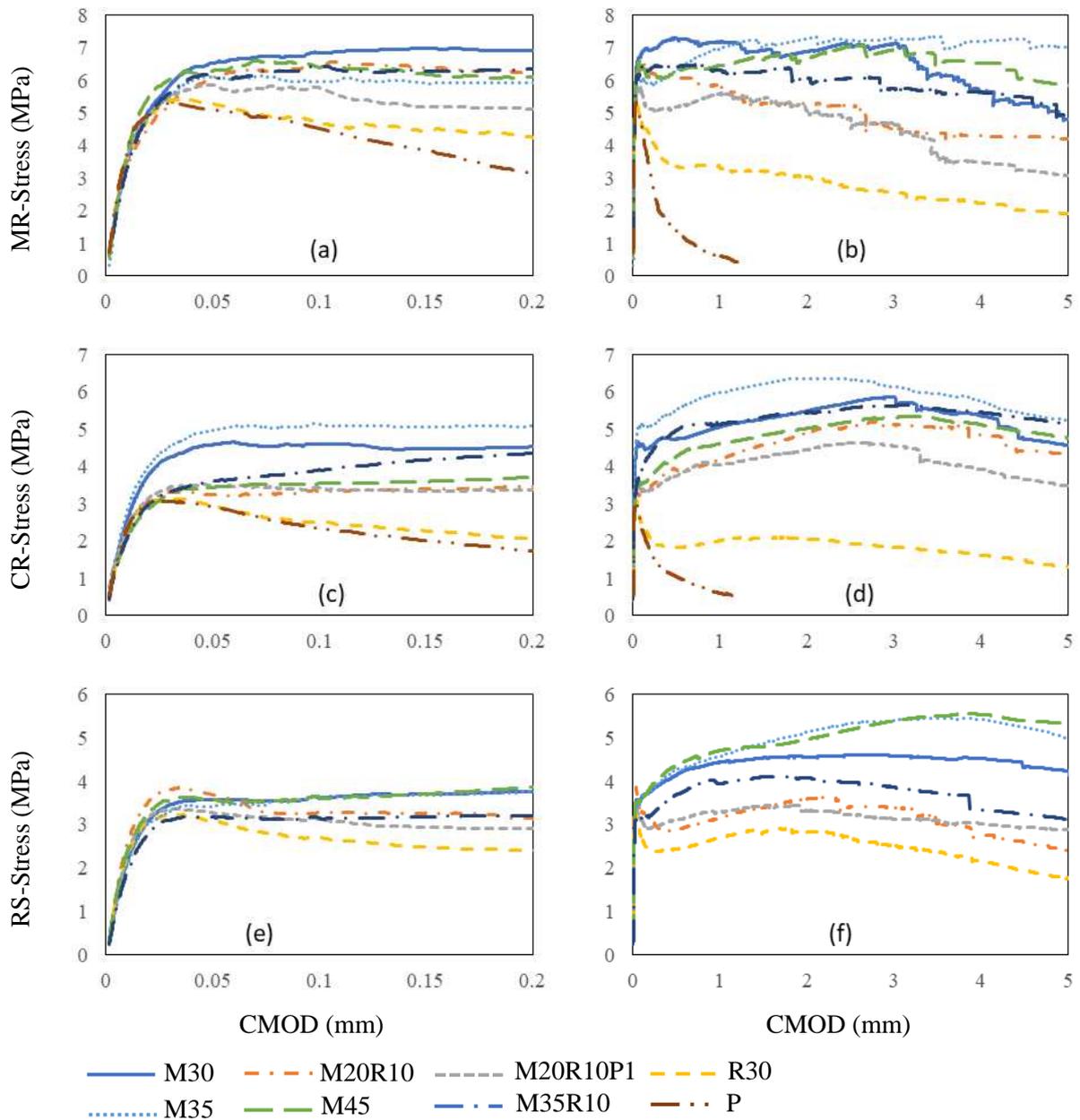
340

## 341 4.2 Flexural Performance

342 Figure 10 (a-f) shows the stress-CMOD results (average of three prisms) for specimens  
343 conditioned in MR, CR and RS environments. The initial elastic behaviour of all specimens,  
344 shown in the graphs up to CMOD of 0.2 mm, is very similar. This confirms that the test  
345 arrangement and measuring method is accurate and reliable and that the fibre content does  
346 not influence much the elastic modulus. The plain concrete mix (P) shows the lowest strength  
347 and least overall toughness. The fibre content seems to have some influences on **residual**  
348 **tensile strength** with some mixes (e.g. CR M35) showing up to 100% increase in strength and  
349 clear strain hardening characteristics. The initiation of cracking in the plain concrete appears  
350 to take place just before the peak load at a CMOD of 0.02 mm. The same applies to all other  
351 specimens and, as expected, fibres get mobilised and control the crack development. In  
352 several cases, there is some initial drop in stress after the opening of the crack at around 0.03  
353 mm until the fibres are mobilised sufficiently and contribute to stiffening the cracked  
354 concrete. Sudden drops in stress are also seen in the post-peak range, due to fibre fracture or  
355 slip.

356 All prisms conditioned in the mist room (MR) show higher strength and toughness than  
357 the CR specimens **by about 40% on average**. This highlights the importance of curing in  
358 strength development as well as the dominance of concrete strength on the flexural strength  
359 of SFRC. Higher concrete strength also results into higher bond strength between the  
360 concrete and fibres, which contributes to higher toughness. However, this also leads to more  
361 fibres fracturing during the post-peak stage than slipping, as indicated by the fracturing  
362 sounds during the test. In the case of the MR conditioned specimens (see Figure 10b), the  
363 higher concrete strength leads to a high flexural strength when the first crack develops, but  
364 due to high bond, more fibres break, leading to mainly flat post-cracking behaviour. On the

365 other hand, CR and RS conditioned specimens, which have a lower concrete strength, show a  
 366 lower flexural strength at first crack, but mobilize more fibres due to slippage, which leads to  
 367 smoother curves with hardening behaviour [54, 55].



368 Figure 10 Stress-CMOD curves a) MR-0.2 mm, b) MR, c) CR-0.2 mm, d) CR e) RS-0.2 mm, f) RS.

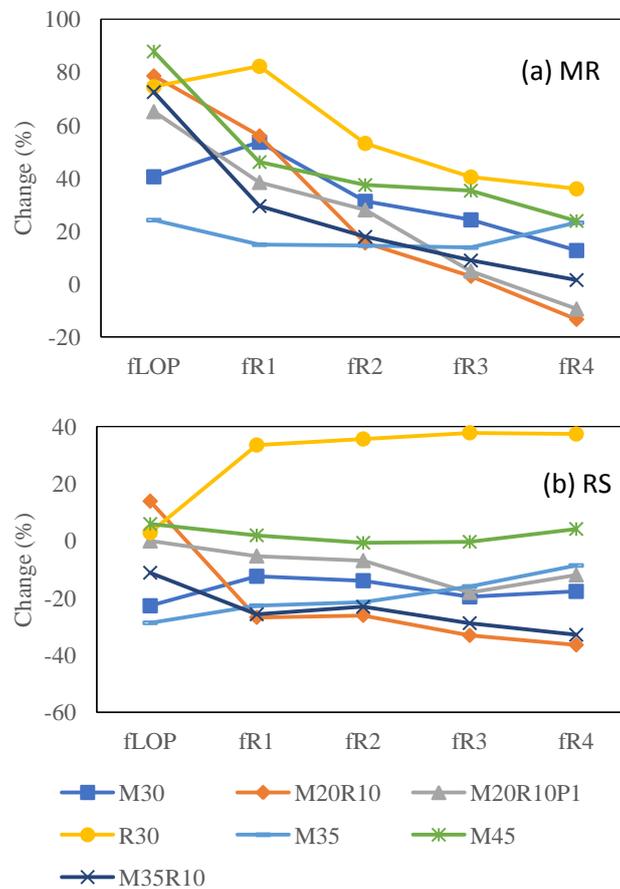
369 **4.3 Residual flexural tensile strength**

370 This sub-section examines the effect of fibre type and dosage, curing and restrain  
 371 condition on the flexural strength at the limit of proportionality ( $f_{LOP}$ ) and residual flexural

372 tensile strength values ( $f_{R1}$ ,  $f_{R2}$ ,  $f_{R3}$ ,  $f_{R4}$ ) at different CMODs (0.5 mm, 1.5 mm, 2.5 mm and  
 373 3.5 mm). In accordance to EN 14651:2005,  $f_{LOP}$  is taken as the maximum stress value up to  
 374 CMOD of 0.05 mm.

375 **4.3.1 Effect of curing**

376 Figure 11 (a and b) shows the change in flexural strength and residual values for the  
 377 specimens subjected to different conditions (MR and RS) relative to the CR condition. Figure  
 378 11a shows that mist curing increases  $f_{LOP}$  by up to 90% (on average 60%), but this increase  
 379 decreases at larger CMODs. This confirms that curing has a significant effect on concrete  
 380 strength development as reflected by the increase in  $f_{LOP}$ . However, as the effect on  $f_R$  values  
 381 reduces with increasing CMOD, curing condition has less impact on the bridging capacity of  
 382 fibres which, at large CMOD, depends more on frictional stresses, geometrical characteristics  
 383 and less on bond strength.



384 *Figure 11 Change in flexural and residual flexural tensile strength relative to CR in a) MR and b) RS.*

385

#### 386 4.3.2 Effect of restraint

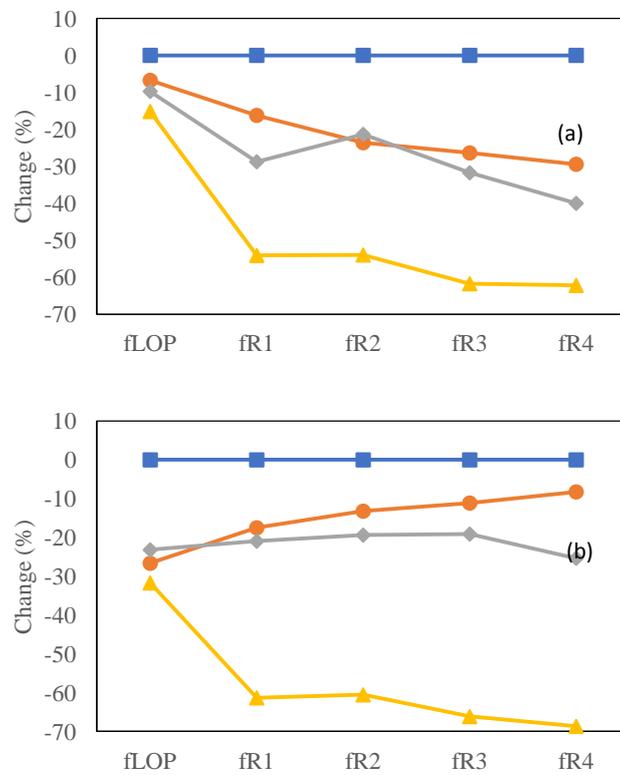
387 Figure 11b shows the relative change in flexural strength due to restraint. The figure  
388 shows an overall loss of  $f_{LOP}$  on average of 10% due to restraint, despite the fact that no  
389 cracks were visible on the RS specimens. However, as tensile strain and stress developed in  
390 the RS specimens, micro-cracking must have taken place and caused some damage to the  
391 concrete. The effect of the damage and micro-cracks appears to overall increase marginally as  
392 the CMOD increases. Specimens manufactured with mix R30 show better performance partly  
393 because they were not well restrained, thus could lead to smaller cracks that self-healed. Self-  
394 healing in restrained concrete was also reported by Younis (2014) [4].

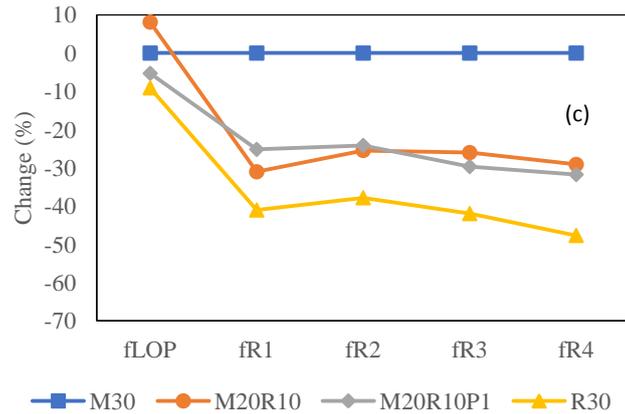
#### 395 4.3.3 Effect of fibres

396 Figure 12 (a-c) shows the change in flexural strength and residual flexural tensile stresses  
397 due to the substitution of MUSF with RTSF for a total fibre content of  $30 \text{ kg/m}^3$  under MR,  
398 CR and RS conditions, respectively. The changes are shown relative to M30 for each  
399 respective condition. In Figure 12a, the effect of substituting MUSF with  $10 \text{ kg/m}^3$  of RTSF  
400 results in about 10% reduction in both  $f_{LOP}$  and residual flexural tensile stresses, as this  
401 substitution did not affect much the concrete tensile strength and concrete-fibre interface  
402 bond strength. In larger specimens (150 mm prisms and slabs), the blends with  $10 \text{ kg/m}^3$   
403 RTSF showed a positive change in  $f_{LOP}$  and  $f_R$  values [40]. This can be attributed to the fact  
404 that fibre alignment is more critical in cast elements with small cross section due to boundary  
405 effect, thus a small reduction in the amount of MUSF (which is longer) can affect  
406 significantly the post cracking behaviour. The highest strength reduction at bigger CMODs  
407 was observed in specimens made with mix R30, partly due to fibre slippage as RTSF have  
408 shorter and thinner geometries compared to MUSF and partly due to their more random  
409 distribution. The reduction in  $f_{LOP}$  for specimens conditioned in CR (Figure 12b) is higher

410 than that found in MR samples by about 20%, but this may be more to do with the high  $f_{LOP}$   
 411 values of M30 than the effect of fibres, as the  $f_R$  values changes are similar to those observed  
 412 for MR samples (Figure 12a).

413 When the concrete is restrained (see Figure 12c), even though there is an overall drop in  
 414  $f_{LOP}$  of 10%, the fibres do not appear to influence  $f_{LOP}$ . However, there is a drop of about 30%  
 415 in the  $f_R$  values of the blended mixes and of about 40% for the R30 mix. This is possibly due  
 416 to the reasons given above to Figure 12a.





417 Figure 12 Change in residual flexural tensile strength due to substitution of MUSF with RTSF conditioned in a) MR, b) CR and  
 418 c) RS.

#### 419 4.4 Characteristic residual flexural tensile strength ratios

420 In order to replace parts of conventional reinforcement with fibres in concrete structures,  
 421 fib MC-2010 imposes that the minimum values of characteristic residual flexural tensile  
 422 strength ratios at serviceability ( $f_{R1k}/f_{Lk}$ ) and ultimate limit state ( $f_{R3k}/f_{R1k}$ ) conditions, be 0.4  
 423 and 0.5, respectively ( $f_{Lk}$ ,  $f_{R1k}$  and  $f_{R3k}$  are the characteristic values at  $f_{LOP}$ ,  $f_{R1}$  and  $f_{R3}$ ,  
 424 respectively). These characteristic values are calculated using RILEM TC 162-TDF (2003)  
 425 [56] and depend on the number of specimens tested per parameter.

426 Figure 13 shows the serviceability characteristic residual flexural tensile strength ratios  
 427 ( $f_{R1k}/f_{Lk}$ ) for all mixes in MR, CR and RS conditions. Mixes with a total of  $30 \text{ kg/m}^3$  of steel  
 428 fibres show ratios less than one, while mixes with a total of  $45 \text{ kg/m}^3$  show ratios mostly  
 429 greater than one. Mixes with  $45 \text{ kg/m}^3$  contain considerably more longer manufactured fibres  
 430 which have a larger diameter and can resist tension more effectively even at larger CMOD.  
 431 CR and RS specimens with a total fibre content more than  $35 \text{ kg/m}^3$  have higher ratios than  
 432 MR specimens, due to their lower  $f_{LOP}$ .

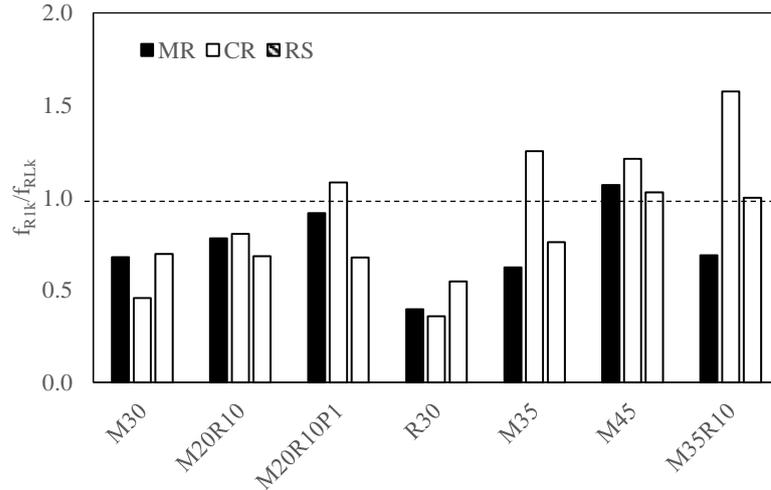


Figure 13 Ratio of characteristic strength,  $f_{R1k}/f_{Lk}$ .

433  
434  
435

436 Figure 14 shows the ultimate characteristic residual flexural tensile strength ratios  
437 ( $f_{R3k}/f_{R1k}$ ) for all specimens subjected to MR, CR and RS conditions. Most of  $f_{R3k}/f_{R1k}$  ratios  
438 for the CR and RS specimens are greater than those for MR samples. This can again be  
439 attributed to the higher  $f_{LOP}$  achieved in the MR samples as a result of better curing. Overall,  
440 all blends satisfied the required ratios of fib MC-2010 for serviceability and ultimate limit  
441 states. Hence, blends of MUSF and RTSF can be used to replace part of conventional  
442 reinforcement in RC structures.

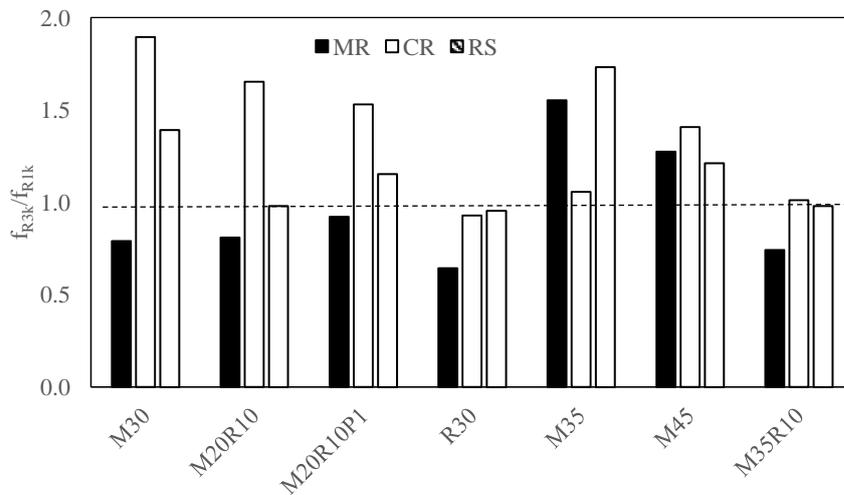


Figure 14 Ratio of residual characteristic strength,  $f_{R3k}/f_{R1k}$ .

443  
444  
445

## 446 5. Conclusions

447 This paper presents the free and restrained shrinkage behaviour of FRC specimens with  
448 different fibre type (MUSF, RTSF and various blends) and their associated mechanical  
449 characteristics. It has been shown that the utilisation of GGBS and RTSF in concrete mixes  
450 contributes to reducing shrinkage strains and controlling cracking. Based on the experimental  
451 results the following conclusions can be drawn:

- 452 • Free shrinkage was much lower than predicted by the design codes by 35% on  
453 average due to the use of GGBS.
- 454 • Non-uniform shrinkage strains through the height of plain and SFRC sections were  
455 observed in free and restrained elements, possibly due to uneven distribution of coarse  
456 aggregates.
- 457 • Average shrinkage strains in SFRC were higher than in plain concrete, possibly due to  
458 an increase in air voids.
- 459 • Drying and end restraint caused the development of micro-cracking in the concrete  
460 which resulted in lower compressive strength (by about 56% on average) and residual  
461 flexural tensile strength (up to 40%).
- 462 • Curing has a significant effect on concrete strength development, but less impact on  
463 the bridging capacity of fibres which depends more on frictional stresses.
- 464 • The high residual flexural tensile strength of SFRC (cured in MR) and the high  
465 frictional stresses between the concrete and the fibres caused the used small dosages  
466 of fibres to break, due to the highly applied tensile stress on fibres.
- 467 • The decay in the stress-CMOD curves show that the hybrid mixes of MUSF and  
468 RTSF satisfy the ratios imposed by the fib MC-2010 and can reduce the required  
469 amount of conventional reinforcement in concrete structures.

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